Grade 2 Mathematics
Support Document for Teachers
GRADE 2 MATHEMATICS

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Available in alternate formats upon request.
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INTRODUCTION

Purpose of the Document

*Grade 2 Mathematics: Support Document for Teachers* provides various instructional activities, assessment strategies, and learning resources that promote the meaningful engagement of mathematics learners in Grade 2. The document is intended to be used as an aid to teachers as they work with students in achieving the prescribed outcomes and achievement indicators identified in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)* (Manitoba Education).

Background

*Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes* is based on *The Common Curriculum Framework for K–9 Mathematics*, which resulted from ongoing collaboration with the Western and Northern Canadian Protocol (WNCP). In its work, WNCP emphasizes:

- common educational goals
- the ability to collaborate and achieve common goals
- high standards in education
- planning an array of educational activities
- removing obstacles to accessibility for individual learners
- optimum use of limited educational resources

The growing effects of technology and the need for technology-related skills have become more apparent in the last half century. Mathematics and problem-solving skills are becoming more valued as we move from an industrial to an informational society. As a result of this trend, mathematics literacy has become increasingly important. Making connections between mathematical study and daily life, business, industry, government, and environmental thinking is imperative. The *Kindergarten to Grade 12 Mathematics* curriculum is designed to support and promote the understanding that mathematics is:

- a way of learning about our world
- part of our daily lives
- both quantitative and geometric in nature
Beliefs about Students and Mathematics Learning

The Kindergarten to Grade 8 Mathematics curriculum is designed with the understanding that students have unique interests, abilities, and needs. As a result, it is imperative to make connections to all students’ prior knowledge, experiences, and backgrounds.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with unique knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem-solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

The learning environment should value and respect all students’ experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Conceptual understanding: comprehending mathematical concepts, relations, and operations to build new knowledge. (Kilpatrick, Swafford, and Findell 5)

Procedural thinking: carrying out procedures flexibly, accurately, efficiently, and appropriately.

Problem solving: engaging in understanding and resolving problem situations where a method or solution is not immediately obvious. (OECD 12)
First Nations, Métis, and Inuit Perspectives

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural, and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning, and learn mathematics best when it is contextualized and not taught as discrete content.

Many First Nations, Métis, and Inuit students come from cultural environments where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

Affective Domain

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

Teachers, students, and parents* need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessment of personal goals.

* In this document, the term parents refers to both parents and guardians and is used with the recognition that in some cases only one parent may be involved in a child’s education.
Early Childhood

Young children are naturally curious and develop a variety of mathematical ideas before they enter Kindergarten. Children make sense of their environment through observations and interactions at home, in daycares, preschools, and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home.

Activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities.

Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Mathematics Education Goals for Students

The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between mathematical knowledge and skills, and their application
- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity
CONCEPTUAL FRAMEWORK FOR KINDERGARTEN TO GRADE 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

Mathematical Processes

There are critical components that students must encounter in mathematics in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems
The common curriculum framework incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

- **Communication [C]:** Students communicate daily (orally, through diagrams and pictures, and by writing) about their mathematics learning. They need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. This enables them to reflect, to validate, and to clarify their thinking. Journals and learning logs can be used as a record of student interpretations of mathematical meanings and ideas.

- **Connections [CN]:** Mathematics should be viewed as an integrated whole, rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—concrete, pictorial, and symbolic (the symbolic mode consists of oral and written word symbols as well as mathematical symbols). The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas.

- **Mental Mathematics and Estimation [ME]:** Mental mathematics and estimation are a combination of cognitive strategies that enhance flexible thinking and number sense. Strategies within mental mathematics and estimation enable students to calculate mentally without the use of external aids. In the process, they improve their computational fluency—developing efficiency, accuracy, and flexibility.

- **Problem Solving [PS]:** Students are exposed to a wide variety of problems in all areas of mathematics. They explore a variety of methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problems.

- **Reasoning [R]:** Mathematics reasoning involves informal thinking, conjecturing, and validating—these help children understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

- **Technology [T]:** Technology contributes to the learning of a wide range of mathematical outcomes, and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

- **Visualization [V]:** Mental images help students to develop concepts and to understand procedures. Students clarify their understanding of mathematical ideas through images and explanations.

These processes are outlined in detail in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013).*
Strands

The learning outcomes in the Manitoba curriculum framework are organized into four strands across Kindergarten to Grade 8. Some strands are further subdivided into substrands.

The strands and substrands, including the general learning outcome for each, follow.

**Number**
- Develop number sense.

**Patterns and Relations**
- Patterns
  - Use patterns to describe the world and solve problems.
- Variables and Equations
  - Represent algebraic expressions in multiple ways.

**Shape and Space**
- Measurement
  - Use direct and indirect measure to solve problems.
- 3-D Objects and 2-D Shapes
  - Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
- Transformations
  - Describe and analyze position and motion of objects and shapes.

**Statistics and Probability**
- Data Analysis
  - Collect, display, and analyze data to solve problems.
- Chance and Uncertainty
  - Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.
Outcomes and Achievement Indicators

The Manitoba curriculum framework is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators.

- **General learning outcomes** are overarching statements about what students are expected to learn in each strand/substrand. The general learning outcome for each strand/substrand is the same throughout the grades from Kindergarten to Grade 8.

- **Specific learning outcomes** are statements that identify the specific skills, understanding, and knowledge students are required to attain by the end of a given grade.

- **Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success.

In this document, the word *including* indicates that any ensuing items **must be addressed** to meet the learning outcome fully. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are **not requirements that must be addressed** to meet the learning outcome fully.

Summary

The conceptual framework for Kindergarten to Grade 9 mathematics describes the nature of mathematics, mathematical processes, and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Learning activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes, and lead students to an understanding of the nature of mathematics through specific knowledge, skills, and attitudes among and between strands. The *Grade 2 Mathematics: Support Document for Teachers* is meant to support teachers to create meaningful learning activities that focus on formative assessment and student engagement.
Assessment

Authentic assessment and feedback are a driving force for the suggestions for assessment in this document. The purposes of the suggested assessment activities and strategies are to parallel those found in *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning* (Manitoba Education, Citizenship and Youth). These include the following:

- assessing *for, as, and of* learning
- enhancing student learning
- assessing students effectively, efficiently, and fairly
- providing educators with a starting point for reflection, deliberation, discussion, and learning

Assessment *for learning* is designed to give teachers information to modify and differentiate teaching and learning activities. It acknowledges that individual students learn in idiosyncratic ways, but it also recognizes that there are predictable patterns and pathways that many students follow. It requires careful design on the part of teachers so that they use the resulting information to determine not only what students know, but also to gain insights into how, when, and whether students apply what they know. Teachers can also use this information to streamline and target instruction and resources, and to provide feedback to students to help them advance their learning.

Assessment *as learning* is a process of developing and supporting metacognition for students. Assessment *as learning* focuses on the role of the student as the critical connector between assessment and learning. When students are active, engaged, and critical assessors, they make sense of information, relate it to prior knowledge, and use it for new learning. This is the regulatory process in metacognition. It occurs when students monitor their own learning and use the feedback from this monitoring to make adjustments, adaptations, and even major changes in what they understand. It requires that teachers help students develop, practise, and become comfortable with reflection, and with a critical analysis of their own learning.

Assessment *of learning* is summative in nature and is used to confirm what students know and can do, to demonstrate whether they have achieved the curriculum outcomes, and, occasionally, to show how they are placed in relation to others. Teachers concentrate on ensuring that they have used assessment to provide accurate and sound statements of students’ proficiency, so that the recipients of the information can use the information to make reasonable and defensible decisions.
## Overview of Planning Assessment

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<th>Assessment of Learning</th>
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<tr>
<td><strong>Why Assess?</strong></td>
<td>to enable teachers to determine next steps in advancing student learning</td>
<td>to guide and provide opportunities for each student to monitor and critically reflect on his or her learning and identify next steps</td>
</tr>
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<td><strong>Assess What?</strong></td>
<td>each student's progress and learning needs in relation to the curriculum outcomes</td>
<td>each student's thinking about his or her learning, what strategies he or she uses to support or challenge that learning, and the mechanisms he or she uses to adjust and advance his or her learning</td>
</tr>
<tr>
<td><strong>What Methods?</strong></td>
<td>a range of methods in different modes that make a student's skills and understanding visible</td>
<td>a range of methods in different modes that elicit the student's learning and metacognitive processes</td>
</tr>
<tr>
<td><strong>Ensuring Quality</strong></td>
<td>accuracy and consistency of observations and interpretations of student learning</td>
<td>accuracy and consistency of a student's self-reflection, self-monitoring, and self-adjustment</td>
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<td></td>
<td>clear, detailed learning expectations</td>
<td>engagement of the student in considering and challenging his or her thinking</td>
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<td>accurate, detailed notes for descriptive feedback to each student</td>
<td>the student records his or her own learning</td>
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<td><strong>Using the Information</strong></td>
<td>provide each student with accurate descriptive feedback to further his or her learning</td>
<td>provide each student with accurate, descriptive feedback that will help him or her develop independent learning habits</td>
</tr>
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<td>differentiate instruction by continually checking where each student is in relation to the curriculum outcomes</td>
<td>have each student focus on the task and his or her learning (not on getting the right answer)</td>
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<td>provide parents or guardians with descriptive feedback about student learning and ideas for support</td>
<td>provide each student with ideas for adjusting, rethinking, and articulating his or her learning</td>
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<td>provide the conditions for the teacher and student to discuss alternatives</td>
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<td>the student reports his or her learning</td>
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INSTRUCTIONAL FOCUS

The Manitoba mathematics curriculum framework is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of learning outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Integration of the mathematical processes within each strand is expected.
- Problem solving, conceptual understanding, reasoning, making connections, and procedural thinking are vital to increasing mathematical fluency, and must be integrated throughout the program.
- Concepts should be introduced using manipulatives and gradually developed from the concrete to the pictorial to the symbolic.
- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Use educational resources by adapting to the context, experiences, and interests of students.
- Collaborate with teachers at other grade levels to ensure the continuity of learning of all students.
- Familiarize yourself with exemplary practices supported by pedagogical research in continuous professional learning.
- Provide students with several opportunities to communicate mathematical concepts and to discuss them in their own words.

“Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students” (Ontario 24).
This document consists of the following sections:

- **Introduction**: The Introduction provides information on the purpose and development of this document, discusses characteristics of and goals for Early Years learners, and addresses First Nations, Métis, and Inuit perspectives. It also gives an overview of the following:
  - **Conceptual Framework for Kindergarten to Grade 9 Mathematics**: This framework provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.
  - **Assessment**: This section provides an overview of planning for assessment in mathematics, including assessment *for*, *as*, and *of* learning.
  - **Instructional Focus**: This discussion focuses on the need to integrate mathematics learning outcomes and processes across the four strands to make learning experiences meaningful for students.
  - **Document Organization and Format**: This overview outlines the main sections of the document and explains the various components that comprise the various sections.

- **Number**: This section corresponds to and supports the Number strand for Grade 2 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes* (2013).

- **Patterns and Relations**: This section corresponds to and supports the Patterns and Relations strand for Grade 2 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes* (2013).

- **Shape and Space**: This section corresponds to and supports the Measurement and 3-D Objects and 2-D Shapes substrands of the Shape and Space strand for Grade 2 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes* (2013).

- **Statistics (Data Analysis)**: This section corresponds to and supports the Data Analysis substrand of the Statistics strand for Grade 2 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes* (2013).

- **Blackline Masters (BLMs)**: Blackline Masters are provided to support student learning. They are available in *Microsoft Word* format so that teachers can alter them to meet students’ needs, as well as in *Adobe PDF* format.

- **Bibliography**: The bibliography lists the sources consulted and cited in the development of this document.
Guide to Components and Icons

Each of the sections supporting the strands of the Grade 2 Mathematics curriculum includes the components and icons described below.

**Enduring Understanding(s):**

These summarize the core idea of the particular learning outcome(s). Each statement provides a conceptual foundation for the learning outcome. It can be used as a pivotal starting point in integrating other mathematical learning outcomes or other subject concepts. The integration of concepts, skills, and strands remains of utmost importance.

**Essential Question(s):**

These are used to guide students’ learning experiences and may be useful when planning assessments. Inquiring into essential questions gives teaching and learning purposeful and meaningful focus for achieving the specific learning outcome(s).

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s):</th>
<th>Achievement Indicators:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific learning outcome (SLO) statements define what students are expected to achieve by the end of the grade. A code is used to identify each SLO by grade and strand, as shown in the following example: 2.N.1</td>
<td>Achievement indicators are examples of a representative list of the depth, breadth, and expectations for the learning outcome. The indicators may be used to determine whether students understand the particular learning outcome. These achievement indicators will be addressed through the learning activities that follow.</td>
</tr>
<tr>
<td>The first number refers to the grade (Grade 2). The letter(s) refer to the strand (Number). The last number indicates the SLO number. [C, CN, ME, PS, R, T, V]</td>
<td></td>
</tr>
<tr>
<td>Each SLO is followed by a list indicating the applicable mathematical processes.</td>
<td></td>
</tr>
</tbody>
</table>
PRIOR KNOWLEDGE

Prior knowledge is identified to give teachers a reference to what students may have experienced previously. Teachers should assess students’ prior knowledge before planning instruction.

BACKGROUND INFORMATION

Background information is identified to give teachers knowledge about specific concepts and skills related to the particular learning outcome.

MATHEMATICAL LANGUAGE

Lists of terms students will encounter while achieving particular learning outcomes are provided. These terms can be placed on math word walls or used in a classroom math dictionary. Kindergarten to Grade 8 Mathematics Glossary: Support Document for Teachers (Manitoba Education, Citizenship and Youth) provides teachers with an understanding of key terms found in Kindergarten to Grade 8 mathematics. The glossary is available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/supports.html>.

LEARNING EXPERIENCES

Suggested teaching strategies and assessment ideas for the specific learning outcomes and achievement indicators are provided. In general, learning activities and teaching strategies related to specific learning outcomes are developed individually, except in cases where it seems more logical to develop two or more learning outcomes together. Suggestions for assessment include information that can be used to assess students’ progress in their understanding of a particular learning outcome or learning experience.

Assessing Prior Knowledge:
Observation Checklist:
Assessing Understanding:
Suggestions are provided for assessing prior to and after lessons, and checklists are provided for observing during lessons to direct instruction.
Suggestions for Instruction

- **Achievement indicators appropriate to particular learning experiences are listed.**

The instructional suggestions include the following:

- **Materials/Resources:** Outlines the resources required for a learning activity.
- **Organization:** Suggests groupings (individual, pairs, small group, and/or whole class).
- **Procedure:** Outlines detailed steps for implementing suggestions for instruction.

Some learning activities make use of BLMs, which are found in the Blackline Masters section in *Microsoft Word* and *Adobe PDF* formats.

**Putting the Pieces Together**

Putting the Pieces Together tasks are found at the end of some learning outcomes and consist of a variety of assessment strategies. They may assess one or more learning outcomes across one or more strands and may make cross-curricular connections.
GRADE 2 MATHEMATICS

Number
Grade 2: Number (2.N.1)

**Enduring Understanding:**
Counting is a strategy for finding the answer to how many.

**Essential Question:**
Is there a quicker way to find the answer than counting by ones from one?

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s):</th>
<th>Achievement Indicators:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.N.1 Say the number sequence from 0 to 100 by</td>
<td>➤ Extend a skip-counting sequence by 2s, 5s, or 10s forward and backward.</td>
</tr>
<tr>
<td>➤ 2s, 5s, and 10s, forward and backward, using starting points that are multiples of 2, 5, and 10 respectively</td>
<td>➤ Skip-count by 10s, given any number from 1 to 9 as a starting point.</td>
</tr>
<tr>
<td>➤ 10s using starting points from 1 to 9</td>
<td>➤ Count by 2s starting from 1 or from any odd number.</td>
</tr>
<tr>
<td>➤ 2s starting from 1</td>
<td>➤ Identify and correct errors and omissions in a skip-counting sequence.</td>
</tr>
<tr>
<td>[C, CN, ME, R]</td>
<td>➤ Count a sum of money with pennies, nickels, or dimes (to 100).</td>
</tr>
<tr>
<td></td>
<td>➤ Count quantity using groups of 2s, 5s, or 10s and counting on.</td>
</tr>
</tbody>
</table>

**Prior Knowledge**

Students may have had experience

- counting by
  - 1s forward and backward between any two given numbers (0 to 100)
  - 2s to 30, forward starting at 0
  - 5s and 10s to 100, forward starting at 0

- demonstrating an understanding of counting by
  - using the counting-on strategy
  - using parts or equal groups to count sets
BACKGROUND INFORMATION

Stages of Counting

Rote Counting (Ages 2 to 6): Most preschool children learn some counting words, even though they may not say these words in the correct order.

With experience, they learn the proper sequence (stable order) but may be unable to make one-to-one correspondence between the object being counted and the number names that are applied to them.

Rational Counting (Ages 5 to 7): The students attach the correct number name to each object as it is counted (one-to-one correspondence).

The students understand that the final count number indicates the number of objects in a set (cardinality).

Strategic Counting (Ages 5 to 8): Counting on and counting back are two strategies that extend students’ understanding of numbers and provide a basis for later development of addition and subtraction concepts.

In counting on, the students count forwards beginning at any number. Counting back is challenging for many young students, and students need many opportunities to gain skill and confidence in counting backwards from different numbers.

It is important that students realize that skip-counting sequences relate to putting groups of the same number together.

Example of counting by 2s

![Counting by 2s](image)

Therefore the count is 2, 4, 6, 8 …

MATHEMATICAL LANGUAGE

Counting numbers:

one to one hundred  
count back  
count on  
 penny  
skip count  
nickel  
set  
dime  
number  
money  
numeral


Assessing Prior Knowledge: Interview

Rote Counting

1. Ask the student to
   a) count forward by 1s starting at 36 (stop the count at 46)
   b) count backward by 1s starting at 82 (stop at 72)
   c) count by 2s starting at 0 (stop at 30)
   d) count by 5s starting at 0 (stop at 100)
   e) count by 10s starting at 0 (stop at 100)

2. Tell students
   a) “I have a set of 27. Finish the counting so the set will equal 35.”
   b) “Here are 63 pennies. Count in more pennies until there are 80.”

Observation Checklist

Students are able to
- count forward by 1s over the decade
- count backward by 1s over the decade
- count by 2s to 30
- count by 5s to 100
- count by 10s to 100
- count on from a given number in the range 0 to 40
- count on from a given number in the range 60 to 80
Suggestions for Instruction

- Use groups/tallies for everyday counting, for example,
  - by 2s: “If everyone in our class wanted to wear 2 rings, how many rings would be needed?”

  \[
  \begin{array}{ccc}
  \text{OO} & \text{OO} & \text{...} \\
  2 & 4 & \text{...} \\
  \end{array}
  \]

  - by 5s: “How many students are having milk today? How much milk does our class drink at lunch in 1 month?”
  - by 10s: “How many fingers are there at our table/in our group? How many fingers are there in our classroom?”

- Have students create visuals to represent counting by 2s, 5s, and 10s. This could be done with a digital camera, drawings (hand or computer drawn), prints, etc.

- **Skip-Counting Songs:** As a class, make up skip-counting songs by piggybacking the words on to familiar songs.
  Go to websites such as CanTeach at <www.canteach.ca> for examples of songs.
Ten-Strips: Use a set of ten-strips along with strips containing 1 to 9 dots (digit strips) to help students count by 10s off the decade.

- Begin by using only ten-strips. Have students count by 10s, forward and backward, as you add or subtract the strips one at a time.
- Begin with a digit strip and then add ten-strips. Some students will need to count the first couple of strips by 1s at first. Remove strips as they count backward.
- Begin with a ten-strip and then add a digit strip followed by a series of ten-strips. Remove strips as they count backward.

![Diagram of hundred chart with skip-counting patterns shaded]

- **Skip-Counting Patterns on the Hundred Chart**: Have students shade in or place counter on the skip-counting patterns for 2, 5, and 10 on a hundred chart. Ask questions such as the following:
  - What pattern do you see when you count by 2s and begin with 2?
  - What do you notice when you count by 2s and begin with 1? Why?
  - What happens if you skip count by 10s and start with 6? What do you notice? What if you start with 4?

- **Calculator Counting**: Have students use the constant key on the calculator for skip counting. For example,
  - Key in \( \boxed{1} + = \) then keep pressing equal. The calculator continues to count on in steps of 1.
  - Key in \( \boxed{1} + 2 = = \) then keep pressing equal. The calculator counts on in steps of 2, starting at 1.

Students can record the skip-counting patterns on a hundred chart.
Subtraction: Key in the start number then the subtraction sign followed by the number you want to subtract. Continue pressing equal.

Use an overhead calculator or an interactive whiteboard calculator if possible. Use the constant key function to enter the start of a skip-counting pattern. Have students predict the next number.

*Extension:* Have students key in the skip-counting pattern you want to explore, for example, enter

```
3 + 10 =
```

Have them place their finger over the equal symbol without pressing the key and close their eyes. Ask them to press the equal symbol until they think their display will show 83. Open their eyes and check. Repeat with other forward and backward counting sequences.

- **Number Line Jumps:** Use a number line to show the skip-counting patterns, for example, counting by 2s.

- Orally present a skip-counting sequence with an error or an omission (for example, 5, 10, 15, 20, 30, 35, 40). Have students identify the incorrect or missing number.

- Prepare sets of cards for the different skip-counting sequences (2s on and off the multiple, 5s on the multiple, 10s on and off the multiple). Have students
  - work with one set and order them
  - find the error or omission in a pre-arranged sequence
  - work with two sets and sort them according to the sequence (for example, counting by 2s and counting by 5s) (*Note:* Students will recognize that some of the counting sequences share the same numbers. This would be a good opportunity to use a Venn diagram for the sorting.)

- **Problem Solving:** Present the following problem.

  Two friends are counting. One starts at 25 and counts by 5s to 100. The other starts at 20 and counts by 10s to 100. Predict who will say the most numbers. Explain.

  Ask:
  - What numbers do they say?
  - What numbers are the same for both?

  *Extension:*
  - What are the next two numbers they will both say if they count past 100? Explain how you know.
What’s in My Bank? Place a collection of pennies, nickels, or dimes in small containers (representing piggy banks). Have students count the collections and record their results.

Extension: Make collections using two different coins. Have students count from the largest coin and then count on.

It’s a Handful!

Materials: small counters (centimetre cubes, pennies, beans), spinner

Directions: The student takes a handful of counters and then spins the spinner. The counters are grouped by the number shown on the spinner and then counted.

Example:

Assessing Understanding: Interview

Ask the students to

- start at 42 and count by 2s (stop at 60)
- start at 13 and count by 2s (stop at 31)
- start at 78 and count backward by 2s (stop at 64)
- start at 30 and count by 10s (stop at 100)
- start at 7 and count by 10s (stop at 57)
- start at 100 and count backward by 10s (stop at 40)
- start at 15 and count by 5s (stop at 60)
- start at 85 and count backward by 5s (stop at 55)
- count a set of counters by 2s, 5s, or 10s, and count on
Observation Checklist

Students are able to

☐ Count by 2s
  ☐ forward on the multiple
  ☐ forward off the multiple
  ☐ backward on the multiple

☐ Count by 10s
  ☐ forward on the multiple
  ☐ forward off the multiple
  ☐ backward on the multiple

☐ Count by 5s
  ☐ forward on the multiple
  ☐ backward on the multiple

☐ Count a set in groups and count on.
Grade 2: Number (2.N.2, 2.N.3)

Enduring Understandings:
Numbers are used to represent quantities or position.
Numbers can be described in many ways.

Essential Questions:
What makes a quantity odd or even?
How do you know if a number is odd or even?
How are numbers used to describe position?

Specific Learning Outcome(s):

<table>
<thead>
<tr>
<th>2.N.2</th>
<th>Demonstrate if a number (up to 100) is even or odd. [C, CN, PS, R]</th>
<th>➤ Determine if a number is even or odd by using concrete materials or pictorial representations. ➤ Identify even and odd numbers in a sequence, such as in a hundred chart. ➤ Sort a set of numbers into even and odd.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.N.3</td>
<td>Describe order or relative position using ordinal numbers. [C, CN, R]</td>
<td>➤ Indicate the position of an object in a sequence by using ordinal numbers. ➤ Compare the relative position of an object in two different sequences.</td>
</tr>
</tbody>
</table>

Prior Knowledge

Students may have had no formal instruction in this area.
**BACKGROUND INFORMATION**

Even numbers end in 0, 2, 4, 6, or 8. Odd numbers end in 1, 3, 5, 7, or 9. Students should arrive at these generalizations through hands-on experiences.

The terms “odd” and “even” are used in real-life situations as well as in mathematics. Odd means strange, extraordinary, or unusual outside of mathematics. The term “even” can relate to a balance scale. It can also be used in sharing situations; for example, if you are sharing 12 candies with 3 people you could say that they all have an even number. In this case “even” means “the same.” It is important that the vocabulary be explored with students so that they have a clear understanding of the mathematical definitions as they relate to number.

Ordinal numbers should be used in meaningful ways in the classroom (e.g., lining up, giving directions, dates, etc.).

**MATHEMATICAL LANGUAGE**

- odd
- even
- ordinal numbers (to 100)
- position

**LEARNING EXPERIENCES**

**Assessing Prior Knowledge: Informal Classroom Discussion**

Arrange a group of students in a line. Ask students to identify the position of each person. Make note of the language used.

Ask students to tell you what the words odd and even mean. Make note of their interpretation of the terms.
Suggestions for Instruction

- **Determine if a number is even or odd by using concrete materials or pictorial representations.**
- **Identify even and odd numbers in a sequence, such as in a hundred chart.**
- **Sort a set of numbers into even and odd.**

**Read a book such as My Even Day by Doris Fisher and Dani Sneed, or Even Steven and Odd Todd (Hello Math Reader, Level 3) by Kathryn Cristaldi, or The Odds Get Even: The Day the Numbers Went on Strike by Pamela Hall to introduce the terms odd and even as they relate to number.**

- **Partner Up!** Have students line up in pairs. Ask, “Do we have an even number or an odd number of students here today? How do you know?” **Note:** This could be done as a quick attendance routine.

Have the girls line up in partners. Ask, “Do we have an even number or an odd number of girls in the class today? How do you know?” Repeat with the boys.

Use a set of bear counters if available (any type of counter will work as well). In partners, students take turns taking a handful of counters, estimating how many, pairing them up to check, and then recording the results.

**Example:**

<table>
<thead>
<tr>
<th>Turn Number</th>
<th>Estimate</th>
<th>Actual</th>
<th>Odd or Even?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>19</td>
<td>Odd</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>18</td>
<td>Even</td>
</tr>
</tbody>
</table>

Compile the class information, listing the odd numbers in one column and the even numbers in another.

- **Odd or Even?** Use grid pictures and ten frames. Have students identify whether the represented number is odd or even and give reasons for their answer.

**Example:**

---

---
Introduce the poem “Odd and Even” by Marg Wadsworth, which can be found at <www.canteach.ca/elementary/songspoems72.html>.
Discuss the poem with the class.

■ Working through to 50

Materials: centimetre grid paper, chart paper with 50 spaces in rows of 10
Directions: Assign (or have students draw out of a hat) one or two numbers (1 to 50). Using the grid paper, students cut out a two-column grid to represent each number. Glue the grids on to the chart under the appropriate number.
Example:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
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<td>3</td>
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<td>16</td>
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<td>17</td>
<td>18</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>

Ask students to describe the patterns they see on the chart. Possible patterns include
- even, odd, even, odd, . . .
- all the numbers in the 1s column are odd . . .
- all the numbers in the 4s column are even . . .

Assessing Understanding: Paper-and-Pencil/Journal Task
Student Directions:
1. Pick any two numbers between 10 and 100.
2. Tell whether each number is even or odd.
3. Explain how you know.
Indicate the position of an object in a sequence by using ordinal numbers.
Compare the relative position of an object in two different sequences.

Suggestions for Instruction

- Read a book such as Ten Little Rubber Ducks by Eric Carle or The 13 Nights of Halloween by Rebecca Dickinson to introduce or reacquaint students with ordinal numbers.
  - Discuss the use of ordinal numbers in the book and how counting with ordinal numbers is different from “regular” counting. Ask for other examples of when we count using ordinal numbers.
  - Line up 10 students in a row. Have the remainder of the students identify a particular person’s position in the line using ordinal numbers.
  - Have students work in small groups to write their own “ordinal” story.

  **Ordinal Concentration:** Prepare a set of 12 cards—6 cards with matching double (colours, numbers, pictures, etc.) or use the blackline master.
  - Show students the cards and explain that they will be looking for pairs.
  - Have 12 students line up in front of the room. Give each student one of the cards. Have them hold the cards against their chests so that no one can see the front of their card. Go down the row and identify each person’s position (e.g., Paul is first, Mary is second, etc.).
  - Select one student from the “audience” to try to make a match. Have the student identify the position of the students whose cards they want to see (e.g., the second person and the seventh person). Note: Students should not call them by name; they should use only ordinal numbers. If the cards match, the students sit down where they are in line so that they maintain the ordinal positions. If they do not match, they turn the cards around and someone else takes a turn. Continue until all matches have been made.

- Use ordinal numbers in everyday classroom activities—lining up, reading the calendar, giving directions, counting the days in school, et cetera.
Assessing Understanding: Paper-and-Pencil

Provide students with a picture.

Give oral directions such as the following:

- Circle the tenth happy face.
- Draw a line under the third happy face.
- Put a box around the seventh happy face.
- Put a check mark on the twelfth happy face.
- Colour the first happy face.
Grade 2: Number (2.N.4, 2.N.5, 2.N.6, 2.N.7)

**Enduring Understandings:**
Quantities can be represented in a variety of ways with objects, pictures, and numerals.
The position of a digit in a number determines the quantity it represents.

**Essential Questions:**
How can quantities be shown?
How many different ways can you represent a number?

**Specific Learning Outcome(s):**

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s):</th>
<th>Achievement Indicators:</th>
</tr>
</thead>
</table>
| **2.N.4** Represent and describe numbers to 100, concretely, pictorially, and symbolically. [C, CN, V] | ➤ Represent a number using concrete materials, such as ten frames and base-10 materials.  
➤ Represent a number using coins (pennies, nickels, dimes, and quarters).  
➤ Represent a number using tallies.  
➤ Represent a number pictorially.  
➤ Represent a number using expressions (e.g., 24 + 6, 15 + 15, 40 – 10).  
➤ Read a number (0–100) in symbolic or word form.  
➤ Record a number (0–20) in words.  
➤ Determine compatible number pairs for 20 or 50. |
| **2.N.5** Compare and order numbers up to 100. [C, CN, R, V] | ➤ Order a set of numbers in ascending or descending order, and verify the result using a hundred chart, number line, ten frames, or by making reference to place value.  
➤ Identify errors in an ordered sequence.  
➤ Identify missing numbers in a hundred chart.  
➤ Identify errors in a hundred chart. |

(continued)
<table>
<thead>
<tr>
<th>Specific Learning Outcome(s):</th>
<th>Achievement Indicators:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.N.6</strong> Estimate quantities to 100 using referents. [C, ME, PS, R]</td>
<td>➤ Estimate a quantity by comparing it to a referent (known quantity). &lt;br&gt;➤ Estimate the number of groups of 10 in a quantity using 10 as a referent. &lt;br&gt;➤ Select between two possible estimates for a quantity, and explain the choice.</td>
</tr>
<tr>
<td><strong>2.N.7</strong> Illustrate, concretely and pictorially, the meaning of place value for numbers to 100. [C, CN, R, V]</td>
<td>➤ Explain and show with counters the meaning of each digit for a 2-digit numeral with both digits the same (e.g., for the numeral 22, the first digit represents two tens [twenty counters] and the second digit represents two ones [two counters]). &lt;br&gt;➤ Count the number of objects in a set using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings of 10s and 1s. &lt;br&gt;➤ Describe a 2-digit numeral in at least two ways (e.g., 24 as two tens and four ones, twenty and four, two groups of ten and four left over, and twenty-four ones). &lt;br&gt;➤ Illustrate using 10 frames and diagrams that a numeral consists of a certain number of groups of 10 and a certain number of 1s. &lt;br&gt;➤ Illustrate using proportional base-10 materials that a numeral consists of a certain number of tens and a certain number of ones. &lt;br&gt;➤ Explain why the value of a digit depends on its placement within a numeral.</td>
</tr>
</tbody>
</table>
**Prior Knowledge**

Students may have had experience

- representing and describing numbers to 20 concretely, pictorially, and symbolically
- reading number words to 20
- determining compatible number pairs for 5, 10, and 20
- placing numerals on a number line with benchmarks 0, 5, 10, and 20

**Note:** Students may have had no formal instruction in place value.

In Grade 1, students may have had experience “demonstrating, concretely and pictorially, how a number, up to 30, can be represented by a variety of equal groups with and without singles.”

**Background Information**

Part-whole relationships refer to the idea that numbers can be broken down into parts, and that these parts can be compared to the whole. According to John Van de Walle, to conceptualize a number as being made up of two or more parts is the most important understanding that can be developed about number relationships.

A pair of numbers that is easy to work with mentally (also known as friendly or nice numbers) are said to be **compatible**.

\[
3 + 6 + 7 + 4 = \underbrace{10 + 10} = 20
\]

**Note:** When solving this number sentence, it is easier to look for combinations that make 10.

Students are able to represent numbers in different ways before understanding place value. As place value understanding develops, student representations will become more complex.

In order to understand and use place value, students need to be able to “think in groups” or to unitize. They need to see 10 as a unit and not as a collection of 10 individual parts.

In 1989, Sharon Ross (cited in Van de Walle and Folk 205) identified five distinct levels of understanding of place value based on responses to the following task:

- Place 36 blocks on the table and have the students count them.
- Have them write the number that tells how many there are.
- Circle the 6. Ask, “Does this part of your 36 have anything to do with how many blocks there are?”
- Circle the 3. Ask, “Does this part of your 36 have anything to do with how many blocks there are?”
- Do not give clues.
Levels of Understanding of Place Value

1. **Single numeral**: The students write 36 but view it as a single numeral. The individual digits 3 and 6 have no meaning by themselves.

2. **Position names**: The students identify correctly the tens and ones positions but still make no connections between individual digits and the blocks.

3. **Face value**: The students match six blocks with the 6 and 3 blocks with the 3.

4. **Transition to place value**: The 6 is matched with the six blocks and the 3 with the remaining 30 blocks but not as three groups of 10.

5. **Full understanding**: The 3 is correlated with three groups of 10 blocks and the 6 with six single blocks.

At the end of Grade 2, level 4 understanding (transition to place value) is expected.

**MATHEMATICAL LANGUAGE**

- ten frame
- represent
- place value
- base-10 blocks
- tally
- coins
- penny
- nickel
- dime
- quarter
- number words to 100
- number sentence
- expression
- order
- ascending
- descending
- greatest
- least

**LEARNING EXPERIENCES**

**Assessing Prior Knowledge: Cooperative Group Activity**

**Materials**: various math materials/manipulatives, chart paper, markers

Put students into small groups (two to four students). Assign each group a number between 15 and 20 (adjust the numbers if needed). Have each group represent their number in as many ways as possible.

Have each group present their representations to the class.
Observation Checklist

Observe students as they work and during their presentation to determine if they are able to represent their number in a variety of ways using:

- materials such as counters, cubes, ten frames, beaded number line, fingers, money, base-10 materials, dominoes, et cetera
- pictures and tallies
- number sentences or expressions

Also determine if students are able to explain their representations.

Record the various representations used on a chart for students to use as a reference. Additional representations can be added over the course of the year.

Suggestions for Instruction

- **Daily Routine—Number of the Day:** Organize students into teams of two. Assign the routine to a different team each day. This can be used as assessment for learning. It also helps to keep the concepts “fresh” in the minds of the students over the course of the year.

  **Note:** The Number of the Day can be done on a laminated chart (although after a while it becomes difficult to erase). Some teachers have put words/phrases from the chart on individual strips of paper with magnetic strip on the back and for use on a whiteboard. This enables the teacher to differentiate for groups of students by adding or deleting representations.
Although there are no specific learning outcomes related to money, students will need to be introduced to the names and values of pennies, nickels, dimes, and quarters in order to be able to use them in their representations.

Provide students with a set of coins. Include dimes, nickels, and pennies initially. Have them do the following:

- Sort the coins according to their value.
- Name the types of coins found in each set. Tell the value of each type of coin.
- Show how many pennies equal one dime and how many equal one nickel.
- Solve a problem such as the following: You want to buy a toy for 104. Use the chart to show three different ways that you can pay for it.

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
<th>Pennies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The Royal Canadian Mint has ceased the distribution of pennies to financial institutions. The penny will retain its value indefinitely. The penny is a representation for one. If you still have real or play pennies, use the penny to represent values and for counting. Pennies are a good support for counting and can be used to help build number sense.
### Assessing Understanding: Paper-and-Pencil Task

Periodically have students fill in a printed version of the Number of Day chart independently.

### Student Self-Assessment

Have students add to the chart several times during the year (perhaps at reporting times).

Sample

<table>
<thead>
<tr>
<th>Name:</th>
</tr>
</thead>
</table>

<p>| Represent and describe numbers to 100, concretely, pictorially, and symbolically. |</p>
<table>
<thead>
<tr>
<th>September</th>
<th>November</th>
<th>March</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>represent</td>
<td>represent</td>
<td>represent</td>
<td>work with</td>
</tr>
<tr>
<td>numbers to 20</td>
<td>numbers to 50</td>
<td>numbers to 100</td>
<td>numbers larger than 100</td>
</tr>
<tr>
<td>I can</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>represent</td>
<td>represent</td>
<td>represent</td>
<td>represent</td>
</tr>
<tr>
<td>numbers using</td>
<td>numbers using</td>
<td>numbers using</td>
<td>numbers using</td>
</tr>
<tr>
<td>pictures, tallies,</td>
<td>money</td>
<td>base-10 blocks</td>
<td>place value</td>
</tr>
<tr>
<td>ten-frames,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cubes, counters,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>and other math</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>materials</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I can</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>write addition</td>
<td>write addition</td>
<td>write addition</td>
<td>write addition</td>
</tr>
<tr>
<td>and subtraction</td>
<td>and subtraction</td>
<td>and subtraction</td>
<td>and subtraction</td>
</tr>
<tr>
<td>number sentences for</td>
<td>number sentences for</td>
<td>number sentences for</td>
<td>number sentences for</td>
</tr>
<tr>
<td>numbers to 20</td>
<td>numbers to 50</td>
<td>numbers to 100</td>
<td>numbers beyond 100</td>
</tr>
</tbody>
</table>
Suggestions for Instruction

- Use word problems to provide meaningful contexts for students to explore compatible number pairs.

Examples:

- Meg needs 20 cards to complete her collection. She already has 12. How many more does she need?
- Tom’s team scored 50 points in the football game. They scored 30 points in the first half. How many points did they score in the second half?

- **Race to Twenty:** Use a double ten-frame mat. Have students roll a dice and place counters on the mat to match the number rolled. As they play ask questions such as:
  - How many counters do you have altogether? How do you know?
  - How many more do you need to make 20? How do you know?

This activity can be extended by changing the ten-frame mats to hold 3, 4, or 5 ten frames. It can also be played in reverse by having students fill the ten frames and then roll to remove counters.

**Assessing Prior Knowledge: Small Group Activity**

Play a game of “I say . . . You say . . .” to review compatible number pairs for 10.

Example:
Target number is 10. The teacher says 6. Students respond by saying 4.

**Observation Checklist**

Students are able to

- confidently give the compatible number for 10 without counting
- count on to determine the answer
- count from 1 to determine the answer
How Many More to Make _____ ? Game

Materials: game grid for each student, game markers, number cards

Directions: Decide on a target number (20, 30, 40, or 50). Have students write numbers between 0 and the target number anywhere on their grid. Numbers can be repeated. The teacher/leader draws a number card and calls it out. Students cover the compatible number on their game board. The first player to get 3 or 4 in a row is the winner.

How Many More?
Target Number 20

<table>
<thead>
<tr>
<th>6</th>
<th>9</th>
<th>10</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>19</td>
<td>18</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>12</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>18</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>20</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: Students do not have to memorize these. Provide supports such as ten frames, counters, and base-10 materials as needed.
Assessing Understanding

Prepare partially filled ten-frame mats. Show them one at a time to students and ask:

- How many counters do you have altogether? How do you know?
- How many more do you need to make 20 (30, 40, 50)? How do you know?

Example:

Teacher: “How many do you have now?”
“How many more do you need to get to 40?”

Student: “I have 27. I need 13 more because one ten frame and 3 more spaces are empty.”

Teacher: “If you rolled 5 on your next turn how many would you have?”

Student: “I would have 32 because 3 would fill the third ten frame and remaining 2 would go on the last ten frame. There would be 8 empty spaces.”

Observation Checklist

Students are able to

- identify the total number without counting by ones
  - to 20
  - to 30
  - to 40
  - to 50
- identify the remaining number without counting by ones
- see the compatible number pairs for 10 and 5 within a ten frame
- make an accurate prediction based on a fictitious dice roll
Order a set of numbers in ascending or descending order, and verify the result using a hundred chart, number line, ten frames, or by making reference to place value.

- Identify errors in an ordered sequence.
- Identify missing numbers in a hundred chart.
- Identify errors in a hundred chart.

Assessing Prior Knowledge: Small Group or Individual Activity

Materials: numeral cards from 10 to 20
- numeral cards with multiples of 10 to 100
- numeral cards with numbers from 32 to 42

Have students order the cards (10 to 20) in order from least to greatest and then read them.
If correct, have them order and read the multiples of 10. Continue with the last group of cards.

Observation Checklist

<table>
<thead>
<tr>
<th>Students</th>
<th>Order and Read Numbers 10 to 20</th>
<th>Order and Read Multiples of 10 to 100</th>
<th>Order and Read Numbers 32 to 42</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Suggestions for Instruction

- **Number Sticks**
  
  **Materials:** 100 tongue depressors numbered from 1 to 100
  
  **Directions:**
  
  Students draw a stick from the container and then order themselves in ascending or descending order.

  **Differentiating instruction:**
  
  - Sticks can be handed out in numerical order or selected randomly.
  
  - The range of numbers can be reduced (e.g., Select sticks from 1 to 30 or from 30 to 60 only).

  **Note:** A small strip magnet on the back of the sticks makes it possible for students to take a handful of the sticks and then order them on a magnetic whiteboard or cookie sheet. This is a good formative assessment task.

- **Guess My Number**
  
  **Materials:** number line from 1 to 100
  
  2 sticks (metre sticks work well) with an arrow on one end

  **Directions:**
  
  The leader selects a number and identifies the range for students (e.g., “I am thinking of a number between 1 and 60.”).

  Students take turns asking questions in order to guess the number.

  The leader can only answer “yes,” “no,” “too small,” or “too large.”

  The metre sticks can be used at either end of the number line and moved as numbers are eliminated. In the example the first metre stick would start the game at the 1 and the second stick would be on the 60. Students can then see that the numbers 61 to 100 are out of play.

  If the first student asks if the number is 20 and the leader’s answers, “too small” the metre stick can be moved from 1 to the 20. The range is then narrowed to between 20 and 60.

  It is important to model “good” questions rather than have students guess random numbers.

  **Example:**
  
  - Is the number greater than ___?
  
  - Is it less than ___?
  
  - Is it an even/odd number?
- **Hundred Chart**: Have students
  - fill in missing numbers
  - identify and correct errors

- Ask questions such as the following:
  - Which is less: 36 or 63? How do you know?
  - A number is between 38 and 42. What could it be?
  - Put a number in the blank so that the three numbers are in order
    
    \[
    25 \quad \_\_\_\_ \quad 31 \quad 75 \quad \_\_\_\_ \quad 68
    \]
  - Which is more: 874 or 794? How do you know?

---

**Assessing Understanding: Small Group or Individual Student Task**

1. Give each student a hundred chart. Ask them to put a marker on
   - a number greater than 36
   - a number less than 84
   - a number between 53 and 60

2. Have each student select 8 number sticks and order them from least to greatest.

3. Ask students, “Which is greater: 76 or 67? Explain/show how you know.”

---

- **Estimate a quantity by comparing it to a referent (known quantity).**
- **Estimate the number of groups of 10 in a quantity using 10 as a referent.**
- **Select between two possible estimates for a quantity, and explain the choice.**

**Suggestions for Instruction**

- **How Many?** Place between 20 and 50 small counters/centicubes on the overhead projector. (Increase the number as students become more confident with their estimating.) Turn the projector on for five seconds and then turn it off. Ask the students to write down their estimate. Turn the projector on and move 10 counters (referent) off to the side (but still on the screen). Tell students, “Here are 10 counters. How many groups of 10 do you think there are altogether? Do you want to change your estimate? If you do, will you change it to be more or less than your first estimate?” Ask students to explain their decision.

  **Note:** In the beginning, give the minimum and maximum number in the set (e.g., between 20 and 50). This will help students make a more reasonable initial estimate.
**Picture Estimation:** Find or create pictures of sets of objects. Use pictures of objects your students show interest in. Show a picture and suggest two possible quantities. Have students make a selection and justify their choice.

- Provide opportunities in the classroom for estimation. These might include:
  - using estimation jars filled with different objects
  - the number of math materials in a container
  - the number of buttons the class has on their clothing
  - the number of pages in a book

**Observation Checklist**

Students are able to
- use a referent to make reasonable estimates in the range
  - 20 to 50
  - 50 to 100
- select an appropriate estimate and justify their choice

**Student Self-Assessment**

**Handfuls:** Provide three different sets of small objects such as centicubes, pennies, or beans. Have students take a handful (or a scoop) of one of the objects, estimate, and then count to check. Record results on the record sheet. Students can then self-assess indicating whether they thought their estimate was too small, just right, or too large. They can then set a goal based on their findings. (Decide as a class what an allowable difference is for an estimate that is just right, for example, ± 4.)

Sample

<table>
<thead>
<tr>
<th>Object</th>
<th>Estimate</th>
<th>Actual</th>
<th>My Estimate Was Too Small, Just Right, Too Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>centicubes</td>
<td>38</td>
<td>45</td>
<td>too small</td>
</tr>
<tr>
<td>pennies</td>
<td>52</td>
<td>41</td>
<td>too large</td>
</tr>
<tr>
<td>beans</td>
<td>68</td>
<td>65</td>
<td>just right</td>
</tr>
</tbody>
</table>

**My goal:** I will try more activities so I know how much my hand (a scoop) can hold.
Suggestions for Instruction

Proportional models for base ten, such as craft (popsicle) sticks, Digi-Blocks, Base-10 Blocks, and bean sticks are proportional in size. The representation for 10 is ten times the size of the representation for 1; 100 is ten times the size of 10, etc. Non-proportional models, such as money, do not have any size relationship.

Counting and Grouping Stations

Station 1: Place different numbers of craft sticks into numbered paper bags. Students count them by grouping them into bundles of 10s with elastic bands. Have them record their answers as a two-digit numeral under the headings of tens and ones.

Station 2: Place a different number of counters, such as bingo chips or beans, into numbered bags. Provide empty ten frame cards/mats. Have students count them using the ten frames to group them. Have them record their answers as a two-digit numeral under the headings of tens and ones.

Station 3: Place a set of two-digit numeral cards and a container of craft sticks at the station. Have students select a numeral card and then represent it using the craft sticks. Record the results pictorially.

Note: Students need opportunities to construct their own understanding of our place value system. In order to do this they should work with materials such as craft sticks that they have to group themselves. Base-10 Blocks and Digi-Blocks are useful materials once their understanding is in place, but they are adult rather than student- or child-developed constructs.

- Explain and show with counters the meaning of each digit for a 2-digit numeral with both digits the same (e.g., for the numeral 22, the first digit represents two tens [twenty counters] and the second digit represents two ones [two counters]).
- Count the number of objects in a set using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings of 10s and 1s.
- Describe a 2-digit numeral in at least two ways (e.g., 24 as two tens and four ones, twenty and four, two groups of ten and four left over, and twenty-four ones).
- Illustrate using 10 frames and diagrams that a numeral consists of a certain number of groups of 10 and a certain number of 1s.
- Illustrate using proportional base-10 materials that a numeral consists of a certain number of tens and a certain number of ones.
- Explain why the value of a digit depends on its placement within a numeral.
Station 4: Place a set of two-digit numeral cards and a collection of empty ten-frame cards/mats at the station. Have students select a numeral card and then represent it using the ten-frame cards/mats. Record the results pictorially.

- Finger representations: Have students show 7, 5, 2, 9, and 6 with their fingers. Ask them to show 12. Observe students to see whether they realize that they will need two people. Have them make 14, 18, and 16.

Extend to represent numbers in the 20s, 30s, and 40s. Have students count to check (e.g., 45 would have 4 groups of ten fingers and 5 additional fingers up). Students count 10, 20, 30, 40, 45.

Change the order of the people by putting the 5 first. Students count 5, 15, 25, 35, 45.

Extend to place value by having the groups of ten stand one behind each other. Use a numeral card to represent the number of tens in the row and the number of ones.

- Present the following problem:

  Mr. Jones asked his students to represent the number 22 with craft sticks. Here are the answers from two students:

  Which student is right?
  Why is the other student wrong? What could you tell them so that they know why they made their mistake?
- **Same Number—Different Representations**: Show students a two-digit numeral such as 37. Have them brainstorm different ways to describe the number. Record their ideas on a chart for future reference.

Examples:
- 37 ones
- 3 tens and 7 ones
- 30 and 7
- 3 groups of 10 and 7 left over
- 2 groups of 10 and 17 ones
- 1 group of 10 and 27 ones

(Grade 3 learning outcome)

Students select a two-digit number and describe it in different ways. Have the class guess their number from the descriptions.

- Ask students to explain/show the difference between the 2 in 24 and the 2 in 32. For example, “The 2 in 24 means 2 tens or 20. The 2 in 32 means 2 ones.”

- Add place value representations to the Number of the Day routine.

- Record the days of school attended on a vertical number line. It serves as a good visual model of the need to shift to the left when an additional digit is needed. It also shows the increasing pattern in our place value system.
Assessing Understanding: Interview
Give the students a collection of 25 counters or pennies. 
Ask them to count the collection and write down the number. 
Point to the 5 and ask them to use the counters to show what the digit 
means/represents. 
Repeat for the 2.

Observation Checklist
Observe to determine the level of place value understanding:* 
1. Single numeral: The student writes 25 but views it as a single 
numeral. The individual digits 2 and 5 have no meaning by 
themselves. 
2. Position names: The student identifies correctly the tens and ones 
positions but still makes no connections between individual digits and 
the counters. 
3. Face value: The student matches five counters with the 5 and two 
counters with the 2. 
4. Transition to place value: The 5 is matched with the five counters and 
the 2 with the remaining 20 counters but not as two groups of 10. 
5. Full understanding: The 2 is correlated with two groups of 10 counters 
and the 5 with five single counters.

Journal Entry
In your journal explain/show the difference between 54 and 45.

______________________________
* These levels of place value understanding were determined by Sharon Ross in 1989 (cited in Van de Walle and Folk 205).
“All About Number _____” Flip Book

Have students make a flip book by taking three letter-sized sheets of paper and lining them up one on top of the other leaving about 3 centimetres at the bottom of each one (Figure 1).

![Figure 1](image)

Fold the papers (together) over so that they end up with six flaps.

Put two staples across the top.

With the class determine the different representations to be included in the book (Figure 2).

Assign each student (or pair of students) a number to represent or have them select their own.

Share the completed book with the class.
Enduring Understandings:

Quantities can be taken apart and put together.
Addition and subtraction are inverse operations.

Essential Questions:

How can symbols be used to represent quantities, operations, or relationships?
How can strategies be used to compare and combine numbers?
What questions can be answered using subtraction and/or addition?

Specific Learning Outcome(s):  Achievement Indicators:

2.N.8  Demonstrate and explain the effect of adding zero to or subtracting zero from any number. [C, R]

- Add zero to a number and explain why the sum is the same as the addend.
- Subtract zero from a number and explain why the difference is the same as the number.

2.N.9  Demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by
- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems that involve addition and subtraction
- explaining that the order in which numbers are added does not affect the sum
- explaining that the order in which numbers are subtracted may affect the difference [C, CN, ME, PS, R, V]

- Model addition and subtraction using concrete materials or visual representations, and record the process symbolically.
- Create an addition or a subtraction number sentence and a story problem for a solution.
- Solve a problem involving a missing addend, and describe the strategy used.
- Solve a problem involving a missing minuend or subtrahend, and describe the strategy used.
- Match a number sentence to a missing addend problem.
- Match a number sentence to a missing subtrahend or minuend problem.
- Add a set of numbers in two different ways, and explain that the sum is the same (e.g., \(2 + 5 + 3 + 8 = 2 + 3 + 5 + 8\) or \(5 + 3 + 8 + 2\)).

(continued)
**Specific Learning Outcome(s):**

2.N.10 Apply mental mathematics strategies, including

- using doubles
- making 10
- using one more, one less
- using two more, two less
- building on a known double
- using addition for subtraction to develop recall of basic addition facts to 18 and related subtraction facts.

Achievement Indicators:

- Explain the mental mathematics strategy that could be used to determine an addition or subtraction fact, such as
  - using doubles (e.g., for 4 + 6, think 5 + 5)
  - using doubles plus one (e.g., for 4 + 5, think 4 + 4 + 1)
  - using doubles take away one (e.g., for 4 + 5, think 5 + 5 – 1)
  - using doubles plus two (e.g., for 4 + 6, think 4 + 4 + 2)
  - using doubles take away two (e.g., for 4 + 6, think 6 + 6 – 2)
  - making 10 (e.g., for 7 + 5, think 7 + 3 + 2)
  - building on a known double (e.g., 6 + 6 = 12, so 6 + 7 = 12 + 1 = 13)
  - using addition for subtraction (e.g., for 7 – 3, think 3 + ? = 7)

- Use and describe a personal strategy for determining a sum to 18 and the corresponding subtraction.

Recall of facts to 10, doubles to 9 + 9, and related subtraction facts is expected by the end of Grade 2.

---

**Prior Knowledge**

Students may have had experience

- representing numbers from 2 to 20 in two parts (part-part-whole)
- adding and subtracting numbers up to 20

The following mental math strategies for determining basic addition and subtraction facts to 18 may have been introduced

- counting on or counting back
- using one more or one less
- making 10
- starting from known doubles
- using addition to subtract

To help students become efficient with computational fluency, students need to develop mental math skills and recall math facts automatically. Learning math facts is a developmental process where the focus of instruction is on thinking and building number relationships. Facts become automatic for students through repeated exposure.
and practice. When a student recalls facts, the answer should be produced without resorting to inefficient means, such as counting. When facts are automatic, students are no longer using strategies to retrieve them from memory.

**Background Information**

Zero cannot be represented by concrete items. Create opportunities to discuss adding and subtracting zero using contextual situations. Using zero in story problems is a good way to help children understand zero in addition and subtraction.

There are many different types of addition and subtraction problems. Students should have experience with all types.

<table>
<thead>
<tr>
<th>Result Unknown $(a + b = ?)$</th>
<th>Change Unknown $(a + ? = c)$</th>
<th>Start Unknown $(? + b = c)$</th>
<th>Combine $(a + b = ?)$</th>
<th>Compare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat has 8 marbles. Her brother gives her 4. How many does she have now?</td>
<td>Pat has 8 marbles but she would like to have 12. How many more does she need to get?</td>
<td>Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start with?</td>
<td>Pat has 8 blue marbles and 4 green marbles. How many does she have in all?</td>
<td>Pat has 8 blue marbles and 4 green marbles. How many more blue marbles does she have?</td>
</tr>
<tr>
<td>$(8 + 4 = ?)$</td>
<td>$(8 + ? = 12)$</td>
<td>$(? + 4 = 12)$</td>
<td>$(8 + 4 = ?)$</td>
<td>$(8 - 4 = ?$ or $4 + ? = 8)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Result Unknown $(a - b = ?)$</th>
<th>Change Unknown $(a - ? = c)$</th>
<th>Start Unknown $(? - b = c)$</th>
<th>Combine $(a - b = ?)$</th>
<th>Compare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pat has 12 marbles. She gives her brother 4 of them. How many does she have left?</td>
<td>Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother?</td>
<td>Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start with?</td>
<td>Pat has 8 blue marbles. Eight are blue and the rest are green. How many are green?</td>
<td>Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have?</td>
</tr>
<tr>
<td>$(12 - 4 = ?)$</td>
<td>$(12 - ? = 8)$</td>
<td>$(? - 4 = 8)$</td>
<td>$(12 - 8 = ?)$</td>
<td>$(8 - 4 = ?$ or $4 + ? = 8)$</td>
</tr>
</tbody>
</table>

**Note:** Addition and subtraction should be taught together. This will enable students to see the relationships between the two operations.
**Addend:** One of the numbers in a designated sum of two or more numbers (e.g., $3 + 5 + 1 = 9$; 3, 5, and 1 are addends).

**Minuend:** In a subtraction problem, the number from which another number is to be subtracted (e.g., in $5 - 3 = 2$, the 5 is the minuend).

**Subtrahend:** In subtraction, the number being subtracted from a given number (e.g., in $5 - 2 = 3$, 2 is the subtrahend).

Learning outcome 2.N.9 states that students “Demonstrate an understanding of addition . . . with answers to 100 and the corresponding subtraction by . . . explaining that the order in which numbers are subtracted may affect the difference.” Although negative numbers are not explored in the Early Years mathematics curriculum, students should not hear, for example, that you cannot subtract 11 from 2. Instead, tell students you can do this, but you would end up with a negative number. Students will explore negative numbers in the Middle Years.

### Mathematical Language

<table>
<thead>
<tr>
<th>Operations</th>
<th>Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>addition</td>
<td>counting on</td>
</tr>
<tr>
<td>add</td>
<td>counting back</td>
</tr>
<tr>
<td>sum</td>
<td>one/two more</td>
</tr>
<tr>
<td>total</td>
<td>one/two less</td>
</tr>
<tr>
<td>more</td>
<td>making ten</td>
</tr>
<tr>
<td>subtraction</td>
<td>doubles</td>
</tr>
<tr>
<td>subtract</td>
<td>doubles ±1 or 2</td>
</tr>
<tr>
<td>difference</td>
<td>addition fact</td>
</tr>
<tr>
<td>less</td>
<td>subtraction fact</td>
</tr>
<tr>
<td>story problem</td>
<td></td>
</tr>
<tr>
<td>number sentence</td>
<td></td>
</tr>
<tr>
<td>complementary (compatible)</td>
<td></td>
</tr>
</tbody>
</table>

### Learning Experiences

**Assessing Prior Knowledge: Individual Student or Small-Group Task**

Give students the following questions and have them explain how they solved them.

$$12 + 2 \quad 16 - 1 \quad 9 + 4 \quad 11 - 2 \quad 8 + 9$$
Suggestions for Instruction

- Have students use the pan balance with equal amounts on each side and have the students explore what needs to be added or subtracted to keep the balance.
- Create story problems involving zero to help students understand zero in addition and subtraction.

Observation Checklist

Look for
- counting on
- counting back
- making 10
- using doubles
- one more
- one less
- known fact

Some students will already have committed the fact to memory. This is usually evident when there is no hesitation giving the answer and therefore no opportunity to have used another strategy. If we ask students to “make up” a strategy we are suggesting to students that they should always use a strategy rather than committing facts to memory.

- **Add zero to a number and explain why the sum is the same as the addend.**
- **Subtract zero from a number and explain why the difference is the same as the number.**
Suggestions for Instruction

- Present the following problems.
  - Owen has 53 baseball cards. He got 24 more for his birthday. How many cards does Owen have now?
  - This is the 33rd day of school. How many more days until the 100th day?

Students work with a partner to solve the problems. Provide large sheets of paper for work to be shown. Manipulative materials should be available.

Have students share their solutions and strategies with the class.

**Note:** Students will come up with their own strategies for solving the problems even if they have not experienced two-digit addition and subtraction before. Instruction should build on these beginning strategies.

**Possible Strategies for Addition**

**38 + 26**

**Breaking Up Numbers:** This method requires place value understanding.

\[
\begin{array}{c}
38 \\
30 \quad 8
\end{array}
\begin{array}{c}
+26 \\
20 \quad 6
\end{array}
\]

\[
30 + 14 = 44
\]

**Empty Number Line (Jump Strategy)**

There are many possibilities.

**Note:** In order to be able to use the empty number line students need to be able to count by tens on and off the decade both forward and backward.

Use a beaded hundred string or a unifix number line to help students begin to be able to use an empty number line.
Example: $38 + 26$

Representations of Materials such as Base-10 Blocks or Ten Frames

Compensating (Making “nice” or “friendly” numbers)

\[
38 \quad + \quad 26 \quad \rightarrow \quad (38 + 2) + 24 \quad \rightarrow \quad 64
\]

Note: Students need to use their knowledge of compatible number pairs for 10 to use this strategy.

Strategies for Subtraction

Breaking up Numbers (Split Strategy): Using Place Value

Empty Number Line (Jump Strategy)
Representations of Materials such as Base-10 Blocks or Ten Frames

Compensating (Making “nice” or “friendly” numbers)

\[(38 + 2) - (26 + 2) \rightarrow 40 - 28 = 12\]

or

\[(38 + 4) - (26 + 4) \rightarrow 42 - 30 = 12\]

- Have students use a blank 10 x 10 grid to demonstrate the subtraction process. Students shade in the larger number and cut the smaller number from the shaded part.

Example:

56 – 27

Step 1: Shade in 56.

Step 2: Cut out 27.

Count the remaining shaded squares.

Grids can also be used to demonstrate the addition process.
Suggestions for Instruction

- **Create an addition or a subtraction number sentence and a story problem for a solution.**

**Suggestions for Instruction**

- **“The Answer Is _____” Books:** Make a booklet with six to eight pages. On the front cover write “The answer is 25 (or any other number). What is the question?” Have students create their own addition and subtraction word problems that would result in an answer of 25 and write them on a page in the booklet.

- Build problem writing into class routines by having two students each day responsible for writing an addition and a subtraction problem that would result in an answer equal to the Number of the Day or day of the month.

- **Solve a problem involving a missing addend, minuend, or subtrahend, and describe the strategy used.**

- **Match a number sentence to a missing addend, subtrahend, or minuend problem.**


---

**Suggestions for Instruction**

- Word problems are really like story writing in language arts. They have a beginning, middle, and an end. They are different in that in the problem one of the parts is missing. Using word problems and representing them pictorially will help students with missing addend, minuend, or subtrahend questions.  

For example: Problem strips like these can be used to have students tell the story first.

1.  
   ![Diagram 1](image1.png)

2.  
   ![Diagram 2](image2.png)

3.  
   ![Diagram 3](image3.png)
Example:
Beginning: Four penguins were on the ice floe.
Middle: Two penguins had to leave.
End: Now there are two penguins left on the ice floe.

Make a paper slider that can be moved along the problem strip. The slider should be large enough to cover one of the sections of the strip. Cover the last section of the strip and have the students retell the story leaving out the end. Record the problems.

Beginning: Four penguins were on the ice floe.
Middle: Two penguins had to leave.
End: How many penguins are left on the ice floe?

Repeat leaving out the middle.
Beginning: Four penguins were on the ice floe.
Middle: Some penguins had to leave.
End: How many penguins had to leave the ice floe?

Repeat leaving out the beginning.
Beginning: There were some penguins on the ice floe.
Middle: Two penguins had to leave.
End: Now there are two penguins left on the ice floe.

How many penguins were on the ice floe at the start?

Revisit the problems asking students how they would represent the problem using a number sentence.

End missing: $4 - 2 = \_\_\_\_\_$
Middle missing: $4 - \_\_\_\_ = 2$
Beginning missing: $\_\_\_\_ - 2 = 2$

Discuss student strategies used for solving the missing part. Possible strategies might include counting on, counting back, and thinking addition for subtraction.

- **Screened Problems:** Use an opaque container (e.g., margarine, yogurt). Invert the container and place a number of counters underneath. Have students look away. Remove some of the counters and place them on top of the container. Ask students to identify how many are still under the container.

Example:

![There are six cubes altogether. How many are hidden?](image)
Domino Problems: “If I cover half of a domino, you see only 4 dots. Altogether the domino has 10 dots. Which domino am I thinking about? How do you know?”

Assessing Understanding

A. Match the number sentences and the problems.

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
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</tbody>
</table>

B. In your journal, solve the following number sentences and explain the strategies you used.

12 — _____ = 5

_____ + 6 = 15
Suggestions for Instruction: Mental Math

Note: The development of mental math strategies is greatly enhanced by sharing and discussion. Students should be given the freedom to adapt, combine, and invent their own strategies.

- Explain the mental mathematics strategy that could be used to determine an addition or subtraction fact, such as:
  - using doubles (e.g., for $4 + 6$, think $5 + 5$)
  - using doubles plus one (e.g., for $4 + 5$, think $4 + 4 + 1$)
  - using doubles take away one (e.g., for $4 + 5$, think $5 + 5 – 1$)
  - using doubles plus two (e.g., for $4 + 6$, think $4 + 4 + 2$)
  - using doubles take away two (e.g., for $4 + 6$, think $6 + 6 – 2$)
  - making 10 (e.g., for $7 + 5$, think $7 + 3 + 2$)
  - building on a known double (e.g., $6 + 6 = 12$, so $6 + 7 = 12 + 1 = 13$)
  - using addition for subtraction (e.g., for $7 – 3$, think $3 + ? = 7$)

- Use and describe a personal strategy for determining a sum to 18 and the corresponding subtraction.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Teaching Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using doubles:</td>
<td>■ Use ten frames to help students visualize the strategies (e.g., $4 + 6$).</td>
</tr>
<tr>
<td>for $4 + 6$, think $5 + 5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Students can see that moving the one square (counter) to the other ten frame will make the addition easier by adding $5 + 5$.</td>
</tr>
<tr>
<td>Using doubles plus one or two:</td>
<td>■ Use two-colour counters (beans).</td>
</tr>
<tr>
<td>for $4 + 5$, think $4 + 4 + 1$</td>
<td>Example: $4 + 5$</td>
</tr>
<tr>
<td>for $4 + 6$, think $4 + 4 + 2$</td>
<td></td>
</tr>
</tbody>
</table>
A series of math fact games, activities, and centres can be found in the mathematics group on <www.maple4teachers.ca>. Look under the K–4 Math Resources Wiki.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Teaching Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using doubles</td>
<td>Students can see that they can either add $4 + 4 + 1$ or $5 + 5 - 1$.</td>
</tr>
<tr>
<td>take away one or two:</td>
<td>![Using doubles diagram]</td>
</tr>
<tr>
<td>■ for $4 + 5$, think $5 + 5 - 1$</td>
<td></td>
</tr>
<tr>
<td>■ for $4 + 6$, think $6 + 6 - 2$</td>
<td></td>
</tr>
<tr>
<td>Building on a known double</td>
<td>Use a set of double nine dominoes. Have students sort them into five groups: doubles, doubles $\pm 1$, doubles $\pm 2$, make 10, and other.</td>
</tr>
<tr>
<td>■ $6 + 6 = 12$, so $6 + 7 = 12 + 1$ = 13</td>
<td></td>
</tr>
<tr>
<td>Make 10</td>
<td>Use a double ten frame to help students visualize the strategy.</td>
</tr>
<tr>
<td>■ for $7 + 5$, think $7 + 3 + 2$</td>
<td>Example:</td>
</tr>
<tr>
<td></td>
<td>When adding $9 + 4$, students can see that moving one from the 4 to make 10 makes adding easier. This is a practical application of part-part-whole understanding. Eventually, students will be able to show the steps without the ten frames.</td>
</tr>
<tr>
<td>![Make 10 example]</td>
<td></td>
</tr>
<tr>
<td>Using addition for subtraction</td>
<td>Note: Thinking addition is an efficient strategy for subtraction.</td>
</tr>
<tr>
<td>■ for $7 - 3$, think $3 + ? = 7$</td>
<td>Teaching addition and subtraction at the same time helps students see this relationship between the operations. For example, for $9 - 5$, think “5 and how many more to make 9?” ($5 + ____ = 9$).</td>
</tr>
</tbody>
</table>

- A series of math fact games, activities, and centres can be found in the mathematics group on <www.maple4teachers.ca>. Look under the K–4 Math Resources Wiki.
Assessing Understanding

**Strategy Sort:** Give students a set of addition and subtraction problem cards and strategy cards. Have them sort the problem cards under the strategy headings. Ask students to tell how they would use the strategy to arrive at the answer.

**Observation Checklist**

Use a checklist. Show students a number sentence. Have individual students explain the strategy used to solve the problem. Record the strategy used on the chart.

<table>
<thead>
<tr>
<th>Student</th>
<th>Doubles</th>
<th>Doubles ±1</th>
<th>Doubles ±2</th>
<th>Known Double</th>
<th>Make 10</th>
<th>Addition for Subtraction</th>
<th>Known Fact</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
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</table>
GRADE 2 MATHEMATICS

Patterns and Relations
Grade 2: Patterns and Relations (Patterns) (2.PR.1, 2.PR.2)

Enduring Understandings:
Patterns show order in the world.
Patterns can be found in many different forms.

Essential Questions:
What is the repeating unit (core) in the pattern?
How can we tell if something is a pattern?
How is the pattern increasing/growing?

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s):</th>
<th>Achievement Indicators:</th>
</tr>
</thead>
</table>
| **2.PR.1** Predict an element in a repeating pattern using a variety of strategies. [C, CN, PS, R, V] | ➤ Identify the core of a repeating pattern.  
➤ Describe and extend a pattern with two attributes.  
➤ Explain the rule used to create a repeating non-numerical pattern.  
➤ Predict an element in a repeating pattern using a variety of strategies.  
➤ Predict an element of a repeating pattern, and extend the pattern to verify the prediction. |
| **2.PR.2** Demonstrate an understanding of increasing patterns by  
■ describing  
■ reproducing  
■ extending  
■ creating patterns using manipulatives, diagrams, sounds, and actions (numbers to 100). [C, CN, PS, R, V] | ➤ Identify and describe increasing patterns in a variety of contexts (e.g., hundred chart, number line, addition tables, calendar, a tiling pattern, or drawings).  
➤ Represent an increasing pattern concretely and pictorially.  
➤ Identify errors in an increasing pattern.  
➤ Explain the rule used to create an increasing pattern.  
➤ Create an increasing pattern and explain the pattern rule.  
➤ Represent an increasing pattern using another mode (e.g., colour to shape).  
➤ Solve a problem using increasing patterns.  
➤ Identify and describe increasing patterns in the environment (e.g., house/room numbers, flower petals, book pages, calendar, pine cones, leap years).  
➤ Determine missing elements in a concrete, pictorial, or symbolic increasing pattern, and explain the reasoning. |
**PRIOR KNOWLEDGE**

Students may have had experience
- sorting objects using a single attribute
- copying, extending, describing, and creating a repeating pattern with a core of two to four elements using a variety of materials and modalities
- identifying the pattern core in a repeating pattern
- translating a repeating pattern to another mode
- labelling repeating patterns with letters

**BACKGROUND INFORMATION**

Simple repeating and increasing/growing patterns consist of a series of related elements—each new element related to the previous in some manner. Students must be able to identify the relationship in order to understand the pattern.

Repeating patterns can be extended in both directions. It is difficult to identify a pattern from a small part of the pattern. Therefore, the pattern core should be repeated more than twice. The core is the shortest string of elements that repeats in a repeating pattern.

When presenting a repeating pattern, encourage students to verbalize how the pattern repeats. To encourage students to make number relationships, have them present the pattern with numerical term positions.

Example:

```
  1  2  3  4  5  6  7  8  9 10 11 12
  ○ △ □ ○ △ □ ○ △ □ ○ △ □ ○ △ □
```

Increasing patterns are patterns in which one or more elements of the sequence or arrangement increases. Increasing patterns should be both numerical and non-numerical. Numerical increasing patterns lead to a better sense of number.

When presenting increasing patterns to students, always provide the first three terms. Some students have difficulty identifying an increasing pattern and think of the first terms as being the core of a repeating pattern. Although students may use other language to describe patterns, it is important to model mathematical language and thinking.

```
  1  2  3  4  5  6
  ■ ■ ■ □ ■ ■
```
MATHEMATICAL LANGUAGE

pattern element
repeating pattern extend
increasing pattern reproduce
core rule
predict term

LEARNING EXPERIENCES

Assessing Prior Knowledge
Give students a collection of unifix or interlocking cubes. Have students
- make a pattern
- identify the pattern core
- use the same cubes to make a different pattern
- label the pattern with letters
- translate the pattern into an action or sound pattern

Observation Checklist
Observe students as they work.
- How complex is the pattern? (How many elements in the core?)
- Can students
  - identify the pattern core?
  - make another pattern?
  - label the pattern with letters?
  - translate the pattern?
Suggestions for Instruction

- Show students three patterns in which two have a similar repeating pattern. Ask them to look at the patterns, and to explain which of the patterns are alike and which are different.

  Example:

  ![Pattern Example]

  Then ask students to choose the pattern that is different from the others, and to duplicate it and extend it. Ask students to make a pattern that is different from the one they reproduced and explain how it is different.

- Present a pattern using two attributes (e.g., colour and size).

  ![Pattern Example]

  Have students describe the pattern, identify the core, and then extend the pattern.

- Prepare a “pattern slider.” Use a legal size sheet of paper and fold it to form a flat tube. Tape it together. Cut a V on one side.

  Example:

  ![Pattern Slider]

  Use pattern strips. Slide a strip through the slider. Gradually pull the pattern through until two complete pattern cores can be seen. Have students predict the next element(s) in the pattern.

  Example:
Have students use attribute (logic) blocks to create patterns with two attributes.

**Note:** Attribute blocks are 3-D objects because they have length, width, and thickness. They are, however, described using 2-D vocabulary.

Example:

□ is described as a hexagon and not as a hexagonal prism.

Exchange patterns with a partner. Have the partner identify the pattern rule and extend the pattern.

- **Create a Pattern:** Use the following clues and a set of attribute blocks to create repeating patterns with three repeats of the core.
  
  A. The pattern has a core of four elements.
     - The core has three different elements.
     - The elements that are the same begin and end the core.
     - What might the pattern be?

  B. The pattern core is made up of five elements.
     - The elements differ by shape and size.
     - Three different elements are used.
     - What might the pattern be?

**Extension:** Have students create a pattern and then describe it in riddle form. Exchange riddles and materials used with a partner and have them try to reproduce the pattern.

**Note:** This learning experience can also be used with increasing patterns.
Suggestions for Instruction

- Prepare a large number line with the numbers 1 to 30. Place the number line on the floor or on a table. Use pattern bocks to make a repeating pattern with a two element core. Place each pattern block above a number on the number line.

Example:

```
□□ □□ □□ □□ □□
1 2 3 4 5 6 7 8 9 10 11 12 13 14
```

Ask questions such as the following:

- “Can you describe the pattern?”
- “What part of the pattern repeats? What is the pattern core?”
- “What will the next shape be? How do you know?”
- “If this pattern continues, what shape will be above the number 10? How do you know?”
- “What shape will be above the number 15? How do you know?”
- “If we read all of the numbers that have a hexagon above them, what do we know about these numbers?” (They are even numbers. They are counting by 2s.)
- “If we continue the pattern up to the number 20, how many squares will there be altogether?”

**Extension:** In pairs, have students use a number line and pattern blocks to create a repeating pattern and ask questions to be answered by their partner.

- Students will use different strategies to predict an element in a repeating pattern.

Example:

What will the 20th element be if this pattern continues?

```
△ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △ △
```
Possible strategies include

- using a multiple: “I doubled the pattern and that made 18 elements and then I added two more.”
- using the size of the pattern core and skip counting: “The core has three elements so I counted by 3s until I got to 21, and then I took one element (the small triangle) away.”
- breaking the pattern into “easy to count” segments: “I counted five elements and I know that it takes four fives to get to twenty so the 20th element is a large white triangle.” Note: This strategy works for this pattern but it would not work if they had used four elements and repeated them five times.

Guide students to try out their strategies with different patterns to see if it is a strategy that can be generalized or one that is specific to a particular pattern.

Assessing Understanding: Paper-and-Pencil Task
If the following pattern continues, what will be the 50th bead? Explain your thinking.

Observation Checklist
Look for
- the use of an efficient strategy (using the “fiveness” of the core and skip counting, grouping into tens and counting, etc.)
- the correct use of mathematical language related to patterns
Suggestions for Instruction

- **Introduction:** Group the class into five or six small groups. Begin the activity by having the first group snapping their fingers. The second group takes their turn by snapping their fingers and then adding an action like clapping their hands. The third group repeats the first and second actions and adds a third. This pattern continues until the last group adds their own action. Now have the whole class do the entire sequence together. Ask students to explain what was happening in the activity.
  Model the pattern with materials.

- Read a book (or sing a song) such as *There Was an Old Lady Who Swallowed a Fly* by Michael Twinn. Ask students to describe what is happening in the story/song (the number of animals swallowed increases by one each time). Reread the story. Represent the pattern in the story using unifix cubes or colour tiles.
  Example:

  There was an old lady who swallowed a fly . . .

  There was an old lady who swallowed a spider . . .

  There was an old lady who swallowed a bird . . .

- Have students select a counting book, poem, or song and represent the pattern concretely and pictorially.

- **Patterns on a Number Line:** Increasing patterns can be shown on the number line.
  Example:
  Skip counting by 2s

  ![Number Line Example](image)
Patterns on the Hundred Chart: Possible increasing patterns include
- for each row, left to right, numbers increase by 1
- for each column, top to bottom, numbers increase by 10
- numerous skip-counting patterns
- on the diagonal from left to right numbers increase by 11
- on the diagonal from right to left numbers increase by 9

Patterns in Other Number Charts: Students need opportunities to notice patterns in a variety of number charts, not just one type. Students need to understand that the numbers have a particular order and that there are patterns in the numeration system that enables us to predict a number. After students have explored the patterns in a Hundred Chart, use a 0 to 99 Chart to look for patterns. To explore other patterns, ask students to visualize what the chart would look like if it had five columns, and have them create the chart.
Patterns on the Calendar: Possible increasing patterns include
- in each row numbers increase by 1
- in each column numbers increase by 7
- on the diagonal from left to right numbers increase by 8
- on the diagonal from right to left numbers increase by 6

The increasing patterns on a calendar are naturally occurring because of the size of the grid and the sequence of the numbers. Artificially creating repeating patterns with colours and shapes, etc., is not a meaningful use of the calendar.
Patterns on the Addition Table: Possible increasing patterns include
- in each row and column numbers increase by one
- on the diagonal from left to right numbers increase by two

Addition Table

<table>
<thead>
<tr>
<th>+</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
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</table>
Assessing Understanding: Paper-and-Pencil Task

1. Identify and describe increasing patterns on the following chart.

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<tbody>
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<td>50</td>
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</tbody>
</table>

2. Use the number line to show an increasing pattern. Describe your pattern.

![Number Line]

3. Explain how the pattern shown below grows. If this pattern continues, what would the next term look like?

Term 1   Term 2   Term 3   Term 4
Suggestions for Instruction

- Present students with a variety of increasing patterns. Have them identify the missing elements and explain the rule used to create the pattern.

Examples:

a) ![Pattern Image](image1.png)

   Term 1  Term 2  Term 3  Term 4  Term 5

b) ![Pattern Image](image2.png)

   Term 1  Term 2  Term 3  Term 4  Term 5
c) ![Pattern Image](image3.png)

   Term 1  Term 2  Term 3  Term 4  Term 5
d) 2, 4, 6, 8, ___, 12, 14, ___

As students are working ask questions such as the following:

- How does the pattern grow?
- What changes as the pattern grows?
- What remains the same as the pattern grows?
Suggestions for Instruction

- **Pattern or Not a Pattern:** Prepare a set of increasing pattern strips, some with errors and others without. Students sort the strips under the headings “Pattern” or “Not a Pattern.” Have students justify the placement of each strip.

- Have students work with a partner. Each student creates a pattern that contains an error. Students exchange patterns and identify and correct the errors in each others’ pattern.

- **Create an increasing pattern and explain the pattern rule.**
- **Represent an increasing pattern using another mode (e.g., colour to shape).**

Suggestions for Instruction

- Provide many opportunities for students to create increasing patterns and to explain the pattern rule.

- **Translating Patterns:** Provide students with an increasing pattern.

  Example:

  
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</table>

  Ask the following:

  - How does the pattern grow?
  - What changes as the pattern grows?
  - What remains the same as the pattern grows?
  - What might the pattern look like if we used different shapes?
    Example:
    
    | △ | △ | △ | △ |
    |---|---|---|---|
    | △ | △ | △ | △ |
    | △ | △ | △ | △ |
  
  - numbers?
    Example:
    2, 3, 4, 5
— size?
  Example:
  \[
  \triangle \quad \triangle \quad \triangle \quad \triangle \quad \triangle \quad \triangle \quad \triangle \quad \triangle \quad \triangle \quad \triangle \quad \triangle
  \]

— actions?
  Example:
  snap, clap, snap, clap, snap, clap, snap, clap, snap, clap, snap, clap, snap, clap, clap

— letters?
  Example:
  \[
  \text{AB} \quad \text{ABB} \quad \text{ABBB} \quad \text{ABBBB}
  \]

— people?
  Example:
  boy, girl, boy, girl, girl, boy, girl, girl, girl

**Presto Change-o!** Divide the class or a small group into two teams.

**Materials:**
- a set of cards with possible ways to represent a pattern
  (e.g., letters, sounds, actions, shape, colour, size, numbers, people, attribute blocks, etc.)
- materials such as unifix cubes, pattern blocks, attribute blocks, two-colour counters, et cetera
- a set of pictorial representations of increasing patterns
- game board
- a 1-to-6 dice

**Directions:** Place the increasing pattern in the centre of the game board. Shuffle the cards and place them face down on the board.

Teams take turns drawing a card. The word on the card describes how the team is to change the representation of the pattern. If correct, they roll the dice and move the number of spaces shown. If incorrect they lose a turn. The first team to reach the finish wins.
Suggestions for Instruction

- **People Patterns**: Tell students that they are going to explore increasing patterns using the people in the classroom.

  Ask, “How many eyes are there in our classroom? How can we use increasing patterns to find the answer?” Model the process using pictures and then translating them to numbers.

  Example:

  - Number of Eyes: 2, 4, 6, 8
  - Number of Students: 1, 2, 3, 4

  Ask students to find out how many fingers, how many noses, et cetera, in the classroom. Have them use increasing patterns to explain their findings.

- **Sample problems**:

  1. There are frogs and lily pads in the pond.
     Each lily pad has two frogs sitting on it.
     How many frogs are in the pond if there are six lily pads altogether?
     Use an increasing pattern to explain your answer.
3. There are eight markers in a box.
   We have 40 markers at our table.
   How many boxes do we have?
   Use an increasing pattern to explain your answer.

4. Mr. Jones delivers mail on Centre Street.
   He finds some of the house numbers difficult to see.
   Write in the missing numbers for him.
   Explain your thinking.

   ![House Numbers Pattern]

   *Extension:* Have students find out how the house/apartment numbers increase on their streets/buildings.

5. Look at each increasing pattern.
   Find a similar pattern on the hundred chart.
   a) 😊😊😊😊😊😊😊 (any pattern that increases by 1)
   b) ♡♡♡♡♡♡♡ (skip counting by 2s)
   c) 🌟🌟🌟🌟🌟🌟🌟🌟🌟 (skip counting by 5s)

   **Observation Checklist**
   Observe students as they work on the problems. The student is able to
   - identify the pattern rule and apply it to solving the problem
   - use mathematical language related to increasing patterns
   - use an increasing pattern to solve a problem
   - identify similar patterns in different modes
Suggestions for Instruction

- Have students use a digital camera to take pictures of increasing patterns in the environment (Literacy with ICT connection).

- **Science Observation Centre:** Students bring in objects from nature that have increasing patterns and place them at the science centre. Have students identify and record the increasing patterns observed.

**PUTTING THE PIECES TOGETHER**

**Performance Task**

Work with a partner.

Choose two different shapes.

Use the shapes to

**Part A**

- create a repeating pattern with three repeats of the pattern core
- explain your pattern rule
- represent your pattern in another way
- predict the 50th element in your pattern and explain your thinking

**Part B**

- make an increasing/growing pattern with four figures/terms
- explain your pattern rule
- represent your pattern in another way
Grade 2: Patterns and Relations (Variables and Equations) (2.PR.3, 2.PR.4)

**Enduring Understandings:**

“Equals” indicates equivalent sets.

Unknown quantities can be found by using the balance strategy.

**Essential Questions:**

How do you know the sets are equal?

How do you know the sets are not equal?

**Specific Learning Outcome(s):**

<table>
<thead>
<tr>
<th>Achievement Indicators:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.PR.3 Demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100). [C, CN, R, V]</td>
</tr>
<tr>
<td>2.PR.4 Record equalities and inequalities symbolically using the equal symbol or the not-equal symbol. [C, CN, R, V]</td>
</tr>
</tbody>
</table>
**Prior Knowledge**

Students may have had experience
- describing equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).
- recording equalities using the equal symbol (0 to 20).

The not-equal symbol may have not been introduced.

**Background Information**

The equal symbol represents a relation between two equal quantities. In other words, the expression on the left-hand side of the equal symbol represents the same quantity as the expression on the right-hand side of the equal symbol. The equal symbol means “is the same as.” This type of thinking is critical in algebra, and it can enrich the number sense of students by allowing them to be more flexible in applying and developing mental mathematics strategies.

Many students have misconceptions about the equal symbol. Many think that the equal symbol means “give answer.” These students see the equal symbol as an action rather than as a relationship. As a result, they have difficulty.

Examples:
- $4 + \_\_ = 7$
  Students will add across the equal sign and fill the blank with 11.
- $\_\_ = 2 + 5$
  Students will say that the question itself is incorrect because the blank is on the wrong side.
- $3 + 4 = 5 + \_\_\$
  Students will add all the numbers and put 12 in the blank.

Avoid using the equal symbol incorrectly. Present equations in various ways.

Examples:
- $8 = 8$
- $14 = 7 + 7$
- $5 + 1 + 2 = 8$
Exploring the Relationship between Terms

Exploring the relationship between terms on either side of an equal symbol enables students to develop an understanding of the relationship that exists between the terms. It is important that they discover this relationship on their own.

You may wish to provide examples for students to explore the relationship.

Examples:
- $16 + 18 = 18 + 16$
- $13 + 9 = 15 + 7$
- $16 + 26 = 8 + 34$
- $2 + 8 = 1 + 9$

Ask students what relationship they notice between the terms. If students don’t notice any relationship, provide more examples. Observation of the relationship between the numbers has to come from the students. The goal is to engage students in using relational thinking when solving equations. When students are comparing mathematical expressions, encourage them to use relational thinking instead of actually carrying out the calculations. To encourage relational thinking, provide examples that discourage calculations.

**Mathematical Language**

<table>
<thead>
<tr>
<th>same</th>
<th>match</th>
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<tbody>
<tr>
<td>more</td>
<td>equal sign</td>
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<tr>
<td>less</td>
<td>equal symbol</td>
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<tr>
<td>equal</td>
<td>inequality</td>
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<tr>
<td>not equal</td>
<td>equality</td>
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<td>balance</td>
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</table>
Assessing Prior Knowledge

**True or False:** Prepare a classroom set of cards with the word *True* on one side and the word *False* on the other. Present the following equations one at a time. Have students hold up their True/False card to indicate whether the equation is true or false. Ask students to justify their answer using materials, pictures, number lines, numbers, et cetera.

- 6 + 5 = 11
- 8 + 1 = 6 + 4
- 6 = 6
- 9 - 5 = 2 + 2
- 7 + 9 = 10 + 5
- 14 - 5 = 6 + 3

This assessment could be done with a small group or with individual students.

Suggestions for Instruction

- Read the book *Equal Shmequal* by Virginia Kroll.
  
  The story begins with a group of animals attempting to make equal teams for a tug-o-war. The meaning of the word *equal* is discussed. The animals explore using equal numbers for the teams and later use a seesaw to look at equal weight.

  The story provides the basis for a discussion about using objects of the same size and mass when using a balance scale to explore equality and inequality.
- **Equal or Not Equal:** Group the class into small groups. Each group needs a balance scale and a set of number sentences/equations. Have each group use the balance scale and a set of cube or counters to determine whether the number sentences are equal or not equal.
  - $2 + 9 = 11$
  - $10 = 4 + 6$
  - $12 + 4 = 13 + 2$
  - $9 = 9$
  - $7 = 5 + 3$
  - $6 + 8 = 5 + 9$

- Use a double number line to show equality/inequality.
  Example:
  Show that $7 + 4 = 5 + 6$.

- Have students work in pairs. Provide each pair with a balance scale mat. Give each student 10 of one colour of cube and 10 of another colour.
  Student A places any combination of the cubes on their side of the mat (e.g., 4 red cubes and 5 blue cubes, making a total of 9).
  Student B places a different combination of red and blue cubes on their side of the mat so that both buckets have the same total (e.g., 2 red cubes and 7 blue cubes).
  Both students record their findings using drawings, numerals, symbols and/or words.
  Example:
  $4 + 5 = 2 + 7$ or
  $4 + 5$ is equal to $2 + 7$
Extension: Have students use the unequal balance mat.

Student A places any combination of cubes on one side of the mat (e.g., 2 red and 4 blue cubes).

Student B places a combination of cubes on the other side of the mat to match the scale (e.g., 4 red cubes and 4 blue cubes because that side of the scale has to be greater than Student A’s side).

Have students record their work.

Example:

\[ 2 + 4 \neq 4 + 4 \]

Inequality: Place two equal sets of objects on the balance scale. Have students describe what they see. Ask, “What could you do to make the sets not equal?” Responses should highlight either adding objects to or subtracting objects from one side of the scale.

Students work with a partner to make equal sets and then change them to represent an inequality. Have students record their work e.g. \( 6 = 6, 6 + 1 \neq 6 \)

Which Set Does Not Belong? Use a set of dominoes. Select three dominoes—two that are equal and one that is unequal. Have students identify the set that is not equal to the others and explain how they know.

Example:

Classroom Routine—Nifty Number Sentences: Use a laminated chart or white/chalk board. Write a number between 0 and 100 at the top of the chart each day. Students take turns writing a number sentence to equal the number on the chart. Encourage students to try to write a sentence that is different from the ones already on the chart.
Pocket Chart: Copy some of the expressions (the part of the number sentence without the equal symbol e.g., 3 + 8 or 16 − 9) from the Nifty Number Sentences chart each day on to cards. Note: Use the same colour of marker for all of them. Mix the expressions up. Place equal or not equal symbols down the centre of a pocket chart. Have students make true number sentences by placing equivalent expressions on the either side of each symbol.

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<tr>
<th>Make True Number Sentences</th>
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<tbody>
<tr>
<td>23 + 8 ≠ 42 − 1</td>
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<tr>
<td>46 + 29 ≠ 57 + 18</td>
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Assessing Understanding

Have students fill in the Understanding Words charts to demonstrate their understanding of the equality and inequality.

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<thead>
<tr>
<th>What does it mean?</th>
<th>Word equal</th>
<th>Picture</th>
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<tbody>
<tr>
<td>Number examples</td>
<td>Symbol</td>
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<table>
<thead>
<tr>
<th>What does it mean?</th>
<th>Word not equal</th>
<th>Picture</th>
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</thead>
<tbody>
<tr>
<td>Number examples</td>
<td>Symbol</td>
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</tbody>
</table>
True or False Game

Materials:
- game board
- true or false game cards
- game pieces

Directions:
Game cards are placed with number sentences facing up on the table.
Players take turns drawing a card, stating whether the number sentence/equation is true or false. If correct, the player moves their game marker five spaces for a true statement and three spaces for a false statement.

Scenario:
We have been asked to help design a True or False game for Grade 2 students. We already have the game board but we don’t have the game cards. I need each of you to make five cards for the game. Each card needs to have a number sentence. The number sentence can be either true or false. The answer needs to be on the back of the card. Put a T for true or an F for false in the bottom right hand corner.

Sample:

<table>
<thead>
<tr>
<th>Front</th>
<th>Back</th>
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</thead>
<tbody>
<tr>
<td>26 + 32 = 44 + 14</td>
<td>T</td>
</tr>
</tbody>
</table>

To address student needs, do the following:
- Vary the number range assigned to each student.
  - combinations to 20
  - combinations to 50
  - combinations to 100
- Vary the operations used.
  - use only addition
  - use only subtraction
  - use a combination of both operations
- Colour code the cards based on the complexity of the equations.
GRADE 2 MATHEMATICS

Shape and Space
Grade 2: Shape and Space (Measurement) (2.SS.1)

**Enduring Understanding:**
Events are ordered.

**Essential Question:**
How can the order of events be described?

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s):</th>
<th>Achievement Indicators:</th>
</tr>
</thead>
</table>
| 2.SS.1 Relate the number of days to a week and the number of months to a year in a problem-solving context. [C, CN, PS, R] | ➔ Read a date on a calendar.  
➔ Name and order the days of the week.  
➔ Identify the day of the week and the month of the year for an identified calendar date.  
➔ State that there are seven days in a week and twelve months in a year.  
➔ Determine whether a set of days is more or less than a week.  
➔ Identify yesterday’s/tomorrow’s date.  
➔ Identify the month that comes before and the month that comes after a given month.  
➔ Name and order the months of the year.  
➔ Solve a problem involving time that is limited to the number of days in a week and the number of months in a year. |

**Prior Knowledge**

Students may not have had any formal instruction with these concepts.
BACKGROUND INFORMATION

In the real world the calendar is used to plan, keep track of appointments, and measure time. This is how it should be used in the classroom.

Build the calendar at the beginning of the month so that the focus is on the structure of the month and numerical patterns. If possible, have a one page, year-long calendar nearby. Write birthdays and special events on the calendar.

MATHEMATICAL LANGUAGE

<table>
<thead>
<tr>
<th>Monday</th>
<th>January</th>
<th>September</th>
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<tbody>
<tr>
<td>Tuesday</td>
<td>February</td>
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<td>Wednesday</td>
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<td>November</td>
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<tr>
<td>Sunday</td>
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<tr>
<td></td>
<td>August</td>
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LEARNING EXPERIENCES

Assessing Prior Knowledge
Ask students to
- identify today’s date on the calendar
- identify the day of the week
- identify the month
- read the date
- give the month and date of their birthday

Observation Checklist
Use a checklist for recording purposes.
Suggestions for Instruction

- **Through the Year:** Use a year-long calendar as a game board. Begin on January 1st. Student A rolls a dice, moves the number of spaces indicated and reads the date landed on (e.g., Saturday, January 13th). Play continues until a player reaches December 31st.

- **Calendar Routine:** Assign two students each day to the calendar routine.
  
  **Job description:**
  
  1. Write today’s date and be prepared to give the dates for yesterday and tomorrow.
  2. Highlight any special events listed for today.
  3. Locate today’s date on the year-long calendar.
  4. Choose two calendar questions (listed below) and answer them.
  5. Be prepared to share your information with the class.

  **Calendar questions:**
  
  - Is the third Wednesday an even or an odd number?
  - Is the first Thursday an even or an odd number?
  - Write the date of the second Friday.
  - On which day of the week is the first two-digit number?
  - On which day of the week is the last one-digit number?
  - On which day of the week is the 14th?
  - On which day of the week is the ninth?
  - Which day of the week is seven days after the 14th?
  - Which day of the week was five days before the 22nd?
  - How many full weeks are in this month?
  - How many partial weeks are in this month?
  - Find the shortest week this month.
  - What season will it be four months from today?
  - What season will it be nine months from today?
  - Which day of the week is nine days after the sixth?
  - Find the second week. What is Friday’s date?
- How many more months until June?
- How many more months until September?
- How many more months until November?
- How many more months until May?
- How many more months until Christmas?
- If your birthday were in this month, on which day would it fall?
- Find the third week. What is the first date and last date of this week?
- Find the last week. What is the first date and last date of this week?
- On which day of the week does the first two-digit odd number fall?
- On which day of the week is the 25th?
- On which day of the week is the 16th?
- In four months, what month will it be?
- In nine months, what month will it be?
- In 12 months, what month will it be?
- On which day of the week is the first two-digit even number?
- On which day of the week is the 12th?
- How many Saturdays are there in this month?
- Write the date of the second Wednesday.
- Write the date of the last Monday.
- Write the date of the first Thursday.
- In six months, what month will it be?
- If you had homework every Monday this month, how many times would you have had homework?
- What day of the week comes after the 15th?
- What day of the week comes before the 26th?
- What day of the week comes before the 4th?
- What day of the week comes before the 17th?
- What is the last day of the month?
- What month comes after this one?
- What month came before this one?
- What day of the week is four days after the 19th?
- What day of the week is two days before the 20th?
- What day of the week is seven days after the 13th?
- On which day of the week will the first day of the next month fall?
- On what day of the week does the third Saturday fall?
What day of the week is three days before the 11th?
How many Thursdays are in this month?
On what day of the week does the 10th fall?
How many days are in the last week of this month?
How many Fridays are there in this month?
What is the date of the second Monday?
How many days are in this month?
How many school days are in this month?
This month falls in which season?

- **Name and order the days of the week.**
- **Determine whether a set of days is more or less than a week.**

Suggestions for Instruction

- Read a book such as *Today is Monday* by Eric Carle to introduce or reinforce the order of the days of the week.
- Use songs and poems to help students remember the days of the week.
  Example:
  Use the tune to “Clementine.”
  
  There are seven days
  There are seven days
  There are seven days in a week.
  Sunday, Monday,
  Tuesday, Wednesday,
  Thursday, Friday,
  Sa-tur-day.

- **What’s Missing?** Write the days of the week on cards and put them in order in a pocket chart. Read them together. Have students close their eyes. Remove one of the cards. Have students open their eyes and identify the missing day.
  
  *Extension:* Remove two cards each time.
- **Build-a-Week Game**: Students play in groups of two to four to place the days of the week in order to complete a week.

  **Materials:**
  - game board
  - game cards (four of each of the days of the week)

  **Directions:**
  Shuffle the game cards and place them face down on the game board. Student A draws a card and places it in the correct space on their side of the board. If they are unable to use the card they lose a turn and the card is returned to the bottom of the pile. Play continues until one player has completed the week.

  Note: Enlarge the games to fit an 11 x 17 sheet.

**Assessing Understanding: Interview**
Have students order cards with the days of the week written on them.

**Observation Checklist**
Observe students as they do the calendar routine or play the Build-a-Week game.
Suggestions for Instruction

- Use songs and poems to help students remember the months of the year.
  Example:
  Use the tune to “One Little, Two Little ….”
  January (one little)
  February (two little)
  March and (three little)
  April (…)
  May (four little)
  June (five little)
  July and (six little)
  August (…)
  September (seven little)
  October (eight little)
  November (nine little)
  December (ten little)
  12 months in a year (Ten little …)

- What’s the Order? Prepare a set of tent cards with the months of the year written on them. Use a clothesline. Place January, June, and December on the line as referents. Have students take turns drawing a card, placing it on the clothesline and justifying the placement.

- Read a book such as Pepper’s Journal: A Kitten’s First Year by Stuart J. Murphy. In this story the girl keeps a journal describing the changes in a kitten over a year. The book supports an understanding of the relationships among days, weeks, months, and years. It also shows the practical (real world) use of the calendar.

- Provide students with individual calendars and encourage them to focus on the number of months in the year. For example, ask students to write in the dates of special events, such as field trips, classmates’ and family birthdays, and school holidays. If the class has been counting the days at school for Hundred Day, students may want to include this information as well.
Missing Months Game: Use an open-ended game board. Prepare a set of cards. Example:

Players take turns drawing a card and filling in the missing month. If correct, they roll a dice and move the number of spaces shown.

Assessing Understanding: Interview
Have students order cards with the months of the year written on them.

Observation Checklist
Observe students as they do the calendar routine or play the Missing Months game.

State that there are seven days in a week and twelve months in a year.
Solve a problem involving time that is limited to the number of days in a week and the number of months in a year.

Suggestions for Instruction

Connect to Statistics: Have students do a survey to determine the favourite day of the week or month of the year. Make a pictograph of the class birthdays.

Present students with problems such as the following:

1. Bill’s birthday is in September. Maria’s birthday is two months after Bill’s. Paul’s birthday is three months before Bill’s. In what month is Maria’s birthday? Paul’s birthday?
2. Name the seventh month. Who has a birthday in the third month? Which months are in the middle of the year? What month is between October and December?
3. Jason is going on a trip in two weeks. How many more days does he have to wait?
4. Kate’s little sister is two years old. How many months old is she?
5. Anna walks her dog twice each day. How many times does she walk the dog in a week? in two weeks?
Putting the Pieces Together

Keeping Records

Organization:
whole class

Materials:
calendar with spaces large enough to record events or pictures

Context:
Tell students that the class is going to keep a record of the events (birthdays, special lunch days, field trips, visitors, holidays, etc.) that happen over the course of the school year.

The calendar can be arranged in a timeline form or made into a class journal. Individual or pairs of students can take turns being responsible for recording and writing about events. Pictures can also be included.
Grade 2: Shape and Space (Measurement) (2.SS.2, 2.SS.3, 2.SS.4, 2.SS.5)

**Enduring Understandings:**
- Objects have distinct attributes that can be measured with appropriate tools.
- Objects can be compared using the same attribute.
- Changing the position of an object does not affect its attributes.

**Essential Questions:**
- Why are units used in measuring?
- How are non-standard units used to measure objects?
- How are measuring units selected?
- How is estimation helpful in measurement?
- How do measurements help compare objects?

**Specific Learning Outcome(s):**

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s):</th>
<th>Achievement Indicators:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.SS.2</strong></td>
<td>Explain why one of two non-standard units may be a better choice for measuring the length of an object. Explain why one of two non-standard units may be a better choice for measuring the mass of an object. Select a non-standard unit for measuring the length or mass of an object, and explain why it was chosen. Estimate the number of non-standard units needed for a measurement task. Explain why the number of units of a measurement will vary depending upon the unit of measure used.</td>
</tr>
<tr>
<td>Relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass (weight). <img src="https://example.com" alt="C, CN, ME, R, V" /></td>
<td><img src="https://example.com" alt="C, CN, ME, R, V" /></td>
</tr>
</tbody>
</table>

| **2.SS.3** | Estimate, measure, and record the length, height, distance around, or mass (weight) of an object using non-standard units. Compare and order the measure of two or more objects in ascending or descending order, and explain the method of ordering. |
| Compare and order objects by length, height, distance around, and mass (weight) using non-standard units, and make statements of comparison. ![C, CN, ME, R, V](https://example.com) | ![C, CN, ME, R, V](https://example.com) |

(continued)
### Specific Learning Outcome(s):

<table>
<thead>
<tr>
<th>2.SS.4</th>
<th>Measure length to the nearest non-standard unit by</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>▪ using multiple copies of a unit</td>
</tr>
<tr>
<td></td>
<td>▪ using a single copy of a unit</td>
</tr>
<tr>
<td></td>
<td>(iteration process)</td>
</tr>
<tr>
<td></td>
<td>[C, ME, R, V]</td>
</tr>
</tbody>
</table>

#### Achievement Indicators:

- Explain why overlapping or leaving gaps does not result in accurate measures.
- Count the number of non-standard units required to measure the length of an object using a single copy or multiple copies of the same unit of measure.
- Estimate and measure an object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results.
- Estimate and measure, using non-standard units, a length that is not a straight line.
- Create different rulers, using non-standard units of measure, and use these rulers to measure length.

<table>
<thead>
<tr>
<th>2.SS.5</th>
<th>Demonstrate that changing the orientation of an object does not alter the measurements of its attributes.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[C, R, V]</td>
</tr>
</tbody>
</table>

#### Achievement Indicators:

- Measure an object, change the orientation, re-measure, and explain the results.

### Prior Knowledge

Students may have had experience looking at measurement as a process of comparing by
- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering, or matching
**Background Information**

**Mass** is the amount of matter in an object. It is measured using a pan balance and standard masses. The mass of an object is measured in grams and kilograms.

**Weight** is a measure comprising a combination of the mass of an object and the pull of gravity on that mass. Weight is measured in newtons. In daily life, the terms mass and weight are virtually interchangeable, but in reality they are not the same. Weight is frequently used when mass is intended.

The use of non-standard units allows students the opportunity to develop an understanding of measurement.

**Iteration** means the act of repeating. In measurement, using a unit smaller than the object being measured and repeating it end-to-end is an example of iteration.

**Referent** is a known quantity used to estimate or compare (e.g., using the width of the baby finger as a referent for a centimetre).

**Mathematical Language**

- length
- mass
- weight
- lighter than
- unit
- measurement
- longer than
- height
- shorter than
- compare
- heavier than
- distance around
- estimate
- non-standard
- ruler
- tall
- short

**Shape and Space (Measurement)**

15
LEARNING EXPERIENCES

Assessing Prior Knowledge
Set up a measurement centre. Provide a collection of objects each with a different length and mass. Have students choose two objects and compare their length and mass. Record their findings using comparative statements.

Observation Checklist
The student is able to

- compare the objects by length
- compare the objects by mass
- write (or state orally if necessary) comparative statements about their findings
- use the language of measurement and comparison

Suggestions for Instruction
Collect a variety of non-standard measurement tools such as craft sticks, paper clips (large and small if possible), bear counters (three sizes), toothpicks, straws, unifix cubes, colour tiles, pennies, blocks, et cetera.

- Explain why one of two non-standard units may be a better choice for measuring the length/mass of an object.
- Select a non-standard unit for measuring the length or mass of an object, and explain why it was chosen.
- Explain why the number of units of a measurement will vary depending upon the unit of measure used.
Comparing Units: Students work in small groups. Provide each group with a different non-standard unit (small paper clips, craft sticks, straws, bear counters [one size], toothpicks). Have students use the unit to measure the length of their table or a desk. Record their results. Debrief the activity with the whole class. Have each group present their findings. Ask questions such as

- Did any group have problems using their measuring unit? (Perhaps a group with a very small unit such as a paper clip might have difficulty handling them or keeping them in a row.)
- Did you find your measuring unit easy to use? Why?
- Each group measured the same object. Why did groups get a different answer?
- Does the size of the measuring unit make a difference?
- Do you think that the size of the measuring unit would make a difference if you were finding the mass of an object?

Repeat the activity for mass to investigate students’ predictions. Use non-standard units (small paper chips, craft sticks, straws, counters, toothpicks, etc.) to measure the mass of small objects in the classroom (blocks, counters, pencils, erasers, etc.).

At this level, students need a lot of practice making and accepting estimates. Estimates are not random guesses, nor are they exact predictions. This learning is ongoing. Model the use of referents at all times (e.g., placing one paper clip beside the pencil or one rod beside the string before estimating).

Select a variety of objects of different lengths. Have students determine the best (most efficient) non-standard measurement unit to use and justify their choice. Repeat the activity for mass.

Assessing Understanding: Journal Entry
Mark and Elly measured the length of the teacher’s desk.

I got an answer of 45 units.

I got an answer of 15 units.

Mark

Elly

Explain how this might have happened.
Suggestions for Instruction

Note: Students will need the measurement process modelled. Use a non-standard unit and demonstrate how to use it to measure an object. Talk about the importance of beginning at the one end of the object and placing the unit end-to-end without leaving spaces or overlapping. Demonstrate the difference in measurement when the objects are lined up end-to-end, when gaps are left, or when units are overlapped.

- Present students with the following problem.

  “I want to measure the length of the table (or any object) with this unit (select something that students have not used; for example, a whiteboard/chalk brush), but I only have one of them. How do you think we can do this?”

  Brainstorm for ideas. Have students try their suggestions.

  If students do not suggest the iteration (repeating) of the unit, model the process.

  Example:

  Place the brush at one end of the table. Make a mark or put a finger at the end of the brush. Move the brush up to the mark/finger. Repeat until the other end is reached. Talk about the need to have some way of keeping track of the number of iterations.

- Have students practise measuring objects with multiple copies of the same unit and then with a single copy of the same unit and compare the results. Ask students to give reasons for any discrepancies.

- Model for the class the language of estimation as well as the act of estimating (e.g., “I think this book is about 15 cubes on this edge.”). Other estimation terms include the following: is almost, is close to, is approximately, is near to.
- Ask questions to demonstrate the reasonableness of estimates, such as “Is this pencil 6 paper clips or 60 paper clips long?” “Is this string 10 rods or 100 rods long?”

- **What Does It Measure?** Have students work with a partner to measure a variety of classroom objects and record their results.

Example:

<table>
<thead>
<tr>
<th>Object</th>
<th>Unit of Measure</th>
<th>Estimate</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>table</td>
<td>craft stick</td>
<td>8 craft sticks</td>
<td>12 craft sticks</td>
</tr>
<tr>
<td>book</td>
<td>unifix cube</td>
<td>10 cubes</td>
<td>9 cubes</td>
</tr>
</tbody>
</table>

- Provide small groups of students with three or four objects, each with a different length and mass. If possible try to have one of the shorter objects weigh more than the others. Have students measure the length of the objects and then order them from the longest to the shortest. Record their results.

Then, have students find the mass of the objects and order them from heaviest to lightest. Record their results.

*Extension:* Have students compare the results. Ask questions such as:

- Is the longest/tallest object the heaviest?
- Is the shortest object the lightest?
- Do you think that you can predict the mass of an object by looking at its height? Why or why not?
- If someone told you the mass of a hidden object do you think that you could predict the height? Why or why not?

**Assessing Understanding**

Kim measured the top of the table using rods.

The table measured 8 rods. Is the answer correct? Explain your thinking.
Suggestions for Instruction

Note: The learning experiences for this achievement indicator are appropriate for measuring distance around (circumference).

- Draw curved or zigzagged lines on large pieces of paper or make them with masking tape on the floor. Ask students how they might measure the length of the lines. Try some of the suggestions. If the use of string or ribbon is not mentioned, introduce the method and demonstrate the procedure.

  Example:

  Lay a piece of string on top of the line. Cut the string when you reach the end. Stretch the string into a straight line and then measure.

  ![Diagram of curved lines](A)

  ![Diagram of zigzag lines](B)

  Have students use the string to measure the distance around (circumference) cylinders, pumpkins, et cetera.
Suggestions for Instruction

- Ask students about any problems they have encountered measuring. If no problems are identified, suggest the following:
  - units being accidentally bumped
  - losing count
  - units are small and hard to handle
  - units are difficult to keep in a straight row
  - hard to measure objects that are vertical (e.g., classroom door)

Ask students for possible solutions.

- Have students make their own rulers. Some possibilities include
  - a string of beads (two colours grouped in fives or tens)
    ![String of beads diagram]
  - a long strip of paper with the actual objects taped to the paper (adding machine tape works very well)
    ![Long strip of paper diagram]

Tape down one of the objects and then mark off iterations.

![Taped object diagram]

Mark only the iterations.

![Marked iterations diagram]
Suggestions for Instruction

- Students work with a partner to measure the length and height of an object such as a box. Have them turn it over, rotate it, or stand it on its end and re-measure. Record their results.

  Have the results shared and discussed with the class.

Assessing Understanding: Measurement Tic-Tac-Toe

Have students select and complete a minimum of one activity from each row.

| Measure the length of three different objects. Order the objects from shortest to longest. Record your results. | Select three different objects. Find their length and their mass. Record your results. | Find the mass of three different objects. Order them from heaviest to lightest. Record your results. |
| Make your own “ruler.” Use it to measure two different objects. Record your results. | Measure the masking tape lines on the floor. Record your results. | Demonstrate how to measure an object using only one unit. |
| Explain how to estimate length. | If you change the position of an object, do its measurements change? Explain your thinking. | Measure an object of your choice. Explain your choice of unit. |
PUTTING THE PIECES TOGETHER

Poster Project

Organization:
partners

Materials:
large sheet of paper
markers, crayons, et cetera

Context:
Tell students that they have been asked to design a measurement poster for a teacher store. If possible, show students a commercially prepared poster. Ask students what they notice about the poster. Possible answers may include
- colourful
- information is easy to read
- not cluttered, neatly done
- divided into sections
- poster topic is prominent

Have students follow the design process (from science) to create their poster (discuss possible solutions, develop criteria, create a plan/diagram, make the prototype).

The poster should
- use/explain measurement vocabulary (unit, length, mass, etc.)
- show/explain how to measure using a set of units or using one unit and repeating it
- include pictures/diagrams
- include an example of ordering

Look for an understanding of
- measurement vocabulary
- length and mass
- ordering
- measurement with a set of non-standard units
- measurement with one unit and repeating it
Grade 2: Shape and Space (3-D Objects and 2-D Shapes) 
(2.SS.6)

**Enduring Understanding:**
Geometric shapes and objects can be classified by attributes.

**Essential Questions:**
What are the attributes of a shape or object?  
What are ways shapes or objects can be sorted?

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s):</th>
<th>Achievement Indicators:</th>
</tr>
</thead>
</table>
| 2.SS.6 Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule. [C, CN, R, V] | ➤ Determine the differences between two pre-sorted sets, and explain the sorting rule.  
➤ Identify and name two common attributes of items within a sorted group.  
➤ Sort a set of 2-D shapes (regular and irregular) according to two attributes, and explain the sorting rule.  
➤ Sort a set of 3-D objects according to two attributes, and explain the sorting rule. |

**Prior Knowledge**
Students may have had experience
- sorting 3-D objects and 2-D shapes using one attribute
- explaining the sorting rule
**BACKGROUND INFORMATION**

Sorting and classifying are basic concepts that help students organize and understand their surroundings. Through sorting and classifying experiences, students come to understand that objects can be grouped in different ways. This supports part-part-whole understanding (e.g., 38 can be grouped as 30 and 8 or 35 and 3).

In order to sort, students need to identify attributes such as colour, shape, or size. This is the basis of patterning.

Regular 2-D shapes are those that are commonly seen. They have equal sides and equal angles.

```
□ △ ○ ◆
```

Irregular shapes are those that are less common. They have sides and angles that are not equal.

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▁▁▁▁
▁▁ 
▁▁▁
```

**MATHEMATICAL LANGUAGE**

colour words
student-chosen vocabulary for shape (specific 2-D and 3-D names, corners, sides, edges, round, etc.)
vocabulary for size (big, small, heavy, light, long, short, etc.)
sort
classify
group
the same as
different
2-D shape
3-D object
set
attribute
Assessing Prior Knowledge

Work with small groups.

Give students a small group of 3-D objects.
1. Have them sort them and then state their sorting rule.
2. Ask them to re-sort the set and then state their new sorting rule.
3. Sort a set of objects into two groups. Have students identify the sorting rule. Hold up another one of the sorted objects and ask them to identify where it should go.

Give students a small group of 2-D shapes.
1. Have them sort them and then state their sorting rule.
2. Ask them to re-sort the set and then state their new sorting rule.
3. Sort a set of objects into two groups. Have students identify the sorting rule. Hold up another one of the sorted objects and ask them to identify where it should go.

Observation Checklist

The students are able to

☑ sort a collection of 3-D objects using self-selected attribute
☑ sort a collection of 2-D shapes using self-selected attribute
☑ state the sorting rule 3-D 2-D
☑ re-sort a set in another way 3-D 2-D
☑ identify the sorting rule of a pre-sorted set 3-D 2-D
☑ identify the placement of an additional object 3-D 2-D
Suggestions for Instruction

- **Game: What’s My Rule?** Sort a small number of students into two groups (e.g., laced shoes, not laced shoes), without telling the students how they are being sorted. One at a time, the rest of the students go to the group to which they think they belong. Tell them whether or not they are in the correct group. When all students are in the groups, ask “What’s my rule?” Repeat using other characteristics of children.

  **Extension:**
  
  Sort students using two attributes.
  
  Play the game using 3-D objects or 2-D shapes.
  
  Let students take turns doing the sorting.

- **Sorting Attribute Blocks:** Show students a set of attribute (logic) blocks. Have students take turns selecting two of the blocks and then stating how they are alike and how they are different.

  Example:
  
  “These shapes are both squares but one is red and the other is blue.”

  Sort the attribute blocks into two groups according to two attributes and have students identify the sorting rule.

  In small groups, a student randomly selects a card and shows it to the group. The group then sorts the attribute blocks according to the attributes on the card.

  **Cards:**

  - colour and size
  - shape and size
  - shape and thickness
  - size and thickness
  - colour and shape
  - colour and thickness

- Determine the differences between two pre-sorted sets, and explain the sorting rule.
- Identify and name two common attributes of items within a sorted group.
- Sort a set of 2-D shapes (regular and irregular) according to two attributes, and explain the sorting rule.
- Sort a set of 3-D objects according to two attributes, and explain the sorting rule.
- **Button Sort:** Sort a set of buttons using two attributes. Have students guess the sorting rule.

Example:

Have small groups of students sort sets of buttons using two (or more) attributes, write the sorting rule on a piece of paper and turn it upside down on the table/desk. Groups then change places and try to guess the sorting rule.

- **Attribute Mats:** Students can use problem-solving attribute mats as they recognize and identify the similarities and differences among attribute (logic) blocks or 3-D objects. Attribute mats consist of circles with lines connecting them. Each line represents a difference between the objects in the two joined circles (e.g., two lines indicate two differences between the connected objects). The teacher can make several attribute mats, each with a different number of circles and numbers of lines connecting the circles. With a given attribute mat, a student selects the attribute blocks (or other objects) to place in the circles.

Sample problem-solving attribute mat:
Use a Venn diagram for sorting.

Example:

This Venn diagram shows a sort by two attributes—four-sided and small. The intersection of the two circles contains figures that share both attributes.

Assessing Understanding

Give students a small group of regular and irregular 3-D objects.
1. Have them sort them according to two attributes and then state their sorting rule.
2. Sort a set of objects into two groups. Have students identify the sorting rule.

Give students a small group of regular and irregular 2-D shapes.
1. Have them sort them according to two attributes and then state their sorting rule.
2. Sort a set of objects into two groups. Have students identify the sorting rule.

Observation Checklist

The students are able to
- sort a collection of 3-D objects according to two attributes 3-D
- sort a collection of 2-D shapes according to two attributes 2-D
- state the sorting rule 3-D 2-D
- identify the sorting rule of a pre-sorted set 3-D 2-D
Shape and Space (3-D Objects and 2-D Shapes) (2.SS.7, 2.SS.8, 2.SS.9)

**Enduring Understandings:**

Geometric shapes can be described and compared using their attributes. A 3-D object can be analyzed in terms of its 2-D parts.

**Essential Questions:**

How can 3-D objects and 2-D shapes be described?
What are the attributes of 3-D objects and 2-D shapes?

### Specific Learning Outcome(s): Achievement Indicators:

| **2.SS.7** Describe, compare, and construct 3-D objects, including: | ➤ Sort a set of 3-D objects, and explain the sorting rule.  
➤ Identify common attributes of cubes, spheres, cones, cylinders, prisms, or pyramids from sets of the same 3-D objects.  
➤ Identify and describe 3-D objects with different dimensions.  
➤ Identify and describe 3-D objects with different orientations.  
➤ Create and describe a representation of a 3-D object using materials such as modelling clay.  
➤ Identify examples of cubes, spheres, cones, cylinders, prisms, or pyramids found in the environment. |
| --- | --- |
|  ➤ cubes  
➤ spheres  
➤ cones  
➤ cylinders  
➤ prisms  
➤ pyramids | [C, CN, R, V] |
| **2.SS.8** Describe, compare, and construct 2-D shapes, including: | ➤ Sort a set of 2-D shapes, and explain the sorting rule.  
➤ Identify common attributes of triangles, squares, rectangles, or circles from sets of the same type of 2-D shapes.  
➤ Identify 2-D shapes with different dimensions.  
➤ Identify 2-D shapes with different orientations.  
➤ Create a model to represent a 2-D shape.  
➤ Create a pictorial representation of a 2-D shape. |
|  ➤ triangles  
➤ squares  
➤ rectangles  
➤ circles | [C, CN, R, V] |
Prior Knowledge

Students may have had experience
- replicating composite 2-D shapes and 3-D objects
- comparing 2-D shapes to parts of 3-D objects in the environment

Background Information

Pierre van Hiele and Dina van Hiele-Geldof, mathematics teachers from the Netherlands in the 1950s, researched the development of geometry thinking (cited in Van de Walle and Folk 427–432). Through their research they identified five sequential levels of geometric thought.

There are four characteristics of these levels of thought:
- The levels of geometric reasoning/understanding are sequential. Students must pass through all prior levels to arrive at any specific level.
- These levels are not age-dependent.
- Geometric instructional experiences have the greatest influence on advancement through the levels.
- Instruction or language at a higher level than the level of the student may inhibit learning.

Level 0 (sometimes labelled as Level 1): Visual

At this level students can name and recognize shapes by their appearance, but cannot specifically identify properties of shapes. Students may think that a rotated square is a “diamond” and not a “square” because it looks different from their visual image of square. Most students in Kindergarten to Grade 3 will be at Level 0 (visualization).

**Specific Learning Outcome(s):**

| 2.SS.9 Identify 2-D shapes as parts of 3-D objects in the environment. [C, CN, R, V] | ➔ Compare and match a 2-D shape, such as a triangle, square, rectangle, or circle, to the faces of 3-D objects in the environment. ➔ Name the 2-D faces of a 3-D object. |

**Achievement Indicators:**
Suggestions for instruction at this level include
- sorting, identifying, and describing shapes
- working with physical models
- seeing different sizes and orientations of the same shape in order to distinguish the characteristics of the shape and to identify features that are not relevant
- building, drawing, making, putting together, and taking apart 2-D shapes and 3-D objects

**Mathematical Language**

<table>
<thead>
<tr>
<th>term</th>
<th>term</th>
</tr>
</thead>
<tbody>
<tr>
<td>cube</td>
<td>square</td>
</tr>
<tr>
<td>cylinder</td>
<td>rectangle</td>
</tr>
<tr>
<td>sphere</td>
<td>circle</td>
</tr>
<tr>
<td>cone</td>
<td>2-D shape</td>
</tr>
<tr>
<td>pyramid</td>
<td>3-D object</td>
</tr>
<tr>
<td>triangle</td>
<td>prism</td>
</tr>
</tbody>
</table>

**Learning Experiences**

**Assessing Prior Knowledge**
1. Give students a composite 2-D shape and have them reproduce it.
2. Give students a composite 3-D object and have them reproduce it.
3. Show students a rectangle and have them give examples of 3-D objects that have parts that are rectangles.

**Observation Checklist**
The students are able to
- reproduce a composite 2-D shape
- reproduce a composite 3-D object
- predict and select the 2-D shapes used to create a 2-D composite shape
- predict and select the 3-D objects used to create a 3-D composite object
- identify 3-D objects in the environment that have rectangular parts
Suggestions for Instruction

- Provide students with sets of cubes, spheres, cones, cylinders, prisms, or pyramids. Each set should contain objects of different sizes. Have students compare the objects and identify the common attributes. Record the attributes on a chart.

Example:

![Attributes of a cube:
- has six square faces (accept informal term “sides”)
- the faces are all the same size
- has eight vertices (accept “corners”)](image)

- **3-D Object Sort:** Provide students with a set of cards with pictures of cubes, cylinders, spheres, cones, prisms, and pyramids in different orientations. Have students sort the pictures under the correct headings.

Example:

![3-D Object Sort:](image)

<table>
<thead>
<tr>
<th>Cylinders</th>
<th>Cones</th>
<th>Pyramids</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Students use play dough, plasticine, or modelling clay to make particular 3-D objects. Have students describe their object orally or in writing.

Scavenger Hunt: Have students go on a scavenger hunt to find examples of the 3-D objects in the environment. A digital camera can be used to record their findings. The pictures could then be made into a class book.

Assessing Understanding
Show two of the following 3-D objects.

![3D Shapes](image)

a) Describe each of them in as many ways as possible.
b) Tell where these objects can be found in the classroom/school.

Observation Checklist
The student is able to

- identify the object
- identify the attributes of the object
- identify examples of the objects in the environment
Suggestions for Instruction

- Provide students with paper or pictorial sets of triangles, squares, rectangles, or circles. Each set should contain shapes of different sizes. Have students compare the shapes and identify the common attributes. Record the attributes on a chart.

  Example:

  Attributes of a triangle
  - has three sides
  - has three corners (accept “points”)
  - is a 2-D shape (is flat)

- **2-D Shape Sort:** Provide students with a set of cards with pictures of triangles, squares, circles and rectangles in different orientations. Have students sort the pictures under the correct headings.

  Example:

<table>
<thead>
<tr>
<th>Triangles</th>
<th>Squares</th>
<th>Rectangles</th>
<th>Circles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
• Have students create paper models of 2-D shapes. Use the paper models to make pictures. Have students describe their creations. Example:

```
  \[ \text{My picture has five rectangles, one circle, and three triangles.} \]
```

• Have students create drawings using particular 2-D shapes. Example:

```
Draw a picture that uses at least three triangles, four rectangles, two squares, and one circle. Label the 2-D shapes in your picture.
```

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**Assessing Understanding: Journal Entry**

a) Explain why these are all triangles.

```
\[ \text{\includegraphics[width=0.5\textwidth]{triangle}} \]
```

b) Which shape does not belong? Explain your thinking.

```
\[ \text{\includegraphics[width=0.5\textwidth]{rectangle}} \]
```
Class Riddle Book

**Organization:**
pairs

**Materials:**
paper
markers, coloured pencils, et cetera
digital camera (optional)

**Context:**
Tell students that they are going to make a class riddle book for 3-D objects and 2-D shapes. Show students an example.

**Example:**
I am a 2-D shape.
The white/chalk board has my shape.
I am found on the covers of most books.
I am a face on a cereal box.
I have four sides, two long and two short.
What am I?

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Observation Checklist

The students are able to

☐ select and correctly name a 3-D object or 2-D shape
☐ identify examples in the environment
☐ identify attributes of the object/shape
☐ put the information together in the form of a riddle
Grade 2 Mathematics

Statistics and Probability
Grade 2: Statistics (Data Analysis) (2.SP.1, 2.SP.2)

Enduring Understandings:
Data can be collected and organized in a variety of ways.
Data can be used to answer questions.

Essential Questions:
Why do we collect data?
How can data be collected and recorded?

Specific Learning Outcome(s):

<table>
<thead>
<tr>
<th>Specific Learning Outcome(s):</th>
<th>Achievement Indicators:</th>
</tr>
</thead>
</table>
| 2.SP.1 Gather and record data about self and others to answer questions. [C, CN, PS, V] | ➤ Formulate a question that can be answered by gathering information about self and others.  
➤ Organize data as it is collected using concrete objects, tallies, checkmarks, charts, or lists.  
➤ Answer questions using collected data. |
| 2.SP.2 Construct and interpret concrete graphs and pictographs to solve problems. [C, CN, PS, R, V] | ➤ Determine the common attributes of concrete graphs by comparing a set of concrete graphs.  
➤ Determine the common attributes of pictographs by comparing a set of pictographs.  
➤ Answer questions pertaining to a concrete graph or pictograph.  
➤ Create a concrete graph to display a set of data and draw conclusions.  
➤ Create a pictograph to represent a set of data using one-to-one correspondence.  
➤ Solve a problem by constructing and interpreting a concrete graph or pictograph. |
Prior Knowledge

Students may have had no formal instruction in statistics.

Background Information

A pictograph uses uniform, representative pictures to depict quantities of objects or people. It is used when the data are discrete (non-continuous). The symbols used must be the same size and shape to avoid misleading the audience.

Example of a pictograph:

<table>
<thead>
<tr>
<th>Types of Shoes</th>
<th>slip-on</th>
<th>laces</th>
<th>velcro</th>
</tr>
</thead>
</table>

= 1 shoe

Pictographs need to have a title, labels, and pictures. Legends/keys are needed when the pictures or symbols are used to represent more than one quantity (many-to-one correspondence).

A concrete graph is made using the actual objects or people on a graphing mat.

Human or Concrete Object Graph

Concrete graphs need to have a title and labels.

A graphing mat is made from thick plastic sheeting (the type that can be bought off the roll at a hardware store). One side has squares large enough for a person to stand on. These are made using masking tape. There is usually room for three columns and 10 rows. The second side has tile-sized squares again made with masking tape. There is usually room for five columns and at least 12 rows.
As children collect objects, they naturally sort, count, and compare. Sorting, counting, and comparing are the basis for understanding statistics. Children also naturally ask questions to gather information. Teachers can use classroom experiences as sources of information to capitalize on children’s interests and to help them see that statistics are a part of everyday life. As well, data collection provides a way to connect mathematics to other subject areas. Good questions are an integral part of data collection. Students need practice formulating questions in more than one way. By examining the possible answers to a set of similar questions, students can determine which one will best provide the desired data.

**Note:** Surveys should be made manageable by obtaining information from a small population (e.g., no larger than a single class), and by limiting the number of categories to two or three.

Teachers need to model and develop the language of statistics in oral and written formats.

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**Mathematical Language**

<table>
<thead>
<tr>
<th>categories</th>
<th>match</th>
<th>least</th>
</tr>
</thead>
<tbody>
<tr>
<td>label</td>
<td>more</td>
<td>concrete graph</td>
</tr>
<tr>
<td>title</td>
<td>less</td>
<td>pictograph</td>
</tr>
<tr>
<td>data</td>
<td>same amount as</td>
<td>compare</td>
</tr>
<tr>
<td>tallies</td>
<td>most</td>
<td>survey</td>
</tr>
</tbody>
</table>

---

**Learning Experiences**

**Assessing Prior Knowledge**

Present a question such as, “Do you have a pet at home?”

Ask students what answers are possible (yes or no). Ask for suggestions as to how you might gather the information. The suggestions they make should reflect their prior experience.

Complete the survey and discuss the results.
Suggestions for Instruction

- Read a book such as *The Best Vacation Ever* by Stuart J. Murphy or *Charlie’s Checklist* by Rory S. Lerman. Both books have characters that formulate questions and gather data. Discuss the questions chosen and the methods used to gather and record the data.

- Model the formulation of questions, such as
  - “I wonder . . .”
  - “How can we find out?”
  - “Whom shall we ask?”

- Use everyday occurrences to formulate questions about the children’s environment.
  - Sample questions:
    - “How do you travel to school?”
    - “Which kind of pizza did you order?”
    - “Which author should we read this week?”
    - “How many times can you hop on one foot?”
    - “What is your favourite animal?”

- Model questions on the same topic in several ways and allow the group to choose the best question for its purpose. This is an important process of data collection that they will need to practise.
  - Examples:
    - “How did you travel to school today?”
    - “Did you walk to school today?”
    - “How many children in our class used the school bus today?”
Select a survey question that can be answered “yes” or “no.” Model ways in which the data can be collected.

Examples:
- two different colours of unifix cubes (one for “yes” and the other for “no”)
- tallies
- a class list and writing “yes” or “no” beside each person’s name
- checkmarks

Have students answer questions about the data.

Examples:
- Which one has the most/least?
- How many more? How many less?
- How many people were surveyed altogether?

Math Routine: Question of the Week

Have pairs of students take turns formulating a survey question, collecting the data, representing it, and then presenting their findings to the class.

This can be used as formative assessment.

Assessing Understanding

Students work in pairs. Have each group formulate a survey question, collect the data, and summarize the results by making statements about the data.
Suggestions for Instruction

- Model the construction of a concrete graph. Have students formulate a question and decide the answer choices. Make a label for each choice. Decide on a title for the graph and write it on a strip of paper. Place the labels at the bottom of each column of the graphing mat and the title at the top or on the side. Have students stand in the appropriate column. Students need to be shown how to line up on the graphing mat—start at the bottom, one person in each square, do not skip squares. Discuss the results. Demonstrate the use of one-to-one matching as a strategy for comparing the data. Have students hold hands with someone from the next column and count students without partners to determine the difference.

**Note:** Data can be transferred to the pictograph using a small grid and student pictures.

Concrete Graph

![Concrete Graph Image]

Pictograph

![Pictograph Image]

**Note:** Although pictographs consist of uniform pictures, use student photographs on a similar background to make a class pictograph.
Use coloured cubes, pasta, cereal, or candies, and a small graphing mat. From the collection of objects have students select one that represents their favourite colour. Make colour labels and a title for the graph. Have students place their object in the correct column on the graph. Change the concrete graph to a pictograph by having students substitute a coloured square or circle for the actual object. Discuss the similarities and differences between the two graph types.

Have students create three different representations of the same set of data.
Example:
Glue coloured pasta on the first grid. On the second grid, draw and color pasta pieces to represent the same information that is on the first one. Lastly, represent the same data with tallies. Discuss how the three grids are the same and how they are different.

Construct two different concrete graphs. Have students compare the graphs and identify the common attributes (title, labels).

Construct two different pictographs. Have students compare the graphs and identify the common attributes (title, labels).

In preparation for the interpretation of data, lead students to ask and answer questions about the information on graphs.
Example:

<table>
<thead>
<tr>
<th>Favourite Apple Colours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Green</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Yellow</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Sample questions:
- What does the pictograph show? How do you know?
- What does this tell about the colours of apples?
- Which do we like most? least?
- How many more are there of our most favourite colour than our least favourite colour?
- Which do we like more—yellow or green? How do you know?
- How many people were surveyed? How do you know?
• Provide meaningful opportunities for students to collect, represent, and interpret data.

Examples:
• vote on a class book to read
• collect data on the number of sunny, cloudy, rainy, snowy days in a particular month
• decide on a game to play for indoor recess

The science, social studies, and health curricula provide meaningful contexts for working with data.

Assessing Understanding

1. Show students a concrete graph or a pictograph. Have students describe, orally or in writing, what the graph is showing (interpret the data).
2. Give students a set of data. Example:

<table>
<thead>
<tr>
<th>Do you have a cat?</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>✦✦✦✦</td>
<td>✦✦✦✦</td>
</tr>
</tbody>
</table>

Have students construct a concrete graph or pictograph using the data.
Planning a Class Celebration

Context
Tell students that they are going to be planning a class celebration/special event. Have them brainstorm for things they would like to have at the celebration (food, games, beverage, music, movie, etc.).

Have students work in partners or small groups. Assign each group (or have groups select) a category from the brainstormed list. Have each group

- formulate a question
- determine the answer choices
- collect the data
- represent the data in graph form
- summarize the data in written form
- present the results to the class
Grade 2 Mathematics

Bibliography


