Questions by Strand

Solidifying Prior Learning M–1

Support for Strand:	*Measurement*
Prior Learning for:	*Problems involving measurement using SI and imperial units, estimation strategies, or measurement strategies.*

1. Find several possible values for a and b that make the equation true: $\frac{1}{a} = \frac{2}{b}$. Describe what you notice about a and b.

2. Select 6 different positive integers less than 10. Place each number in one of the boxes. **(TN)**

$$\frac{[\square]}{[\square]} - \frac{[\square]}{[\square]} - \frac{[\square]}{[\square]}$$

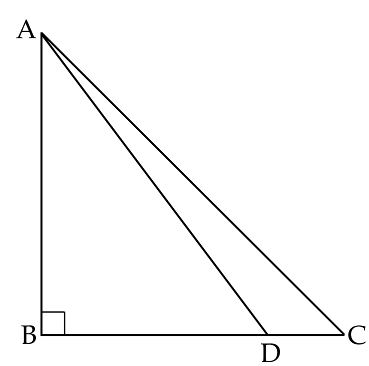
   a) Arrange 6 integers so the difference is a large number. How large of a difference is possible?*

   b) Arrange 6 integers to find a difference that is close to zero. How small of a difference is possible?

3. Ava lives 10 kilometres from Kendall and 20 kilometres from Jordyn.
   How far is it from Kendall's house to Jordyn's house?
   Provide at least two possible answers and diagrams to support the answers.

4. Sarah lives $\frac{3}{4}$ of a mile from Kathryn and twice that distance from Leah.
   How far is it from Kathryn's house to Leah's house?
   Provide at least two possible answers and diagrams to support the answers.

5. ΔABC is a right triangle (as shown, not to scale) with $\angle$ABC = 90°, BD = 6 m, AB = 8 m. The area of ΔABC is 50% greater than the area of ΔABD.
   Determine the perimeter of ΔADC.



* Based on a problem by Robert Kaplinsky and Ellen Metzger. Available at www.openmiddle.com/adding-mixed-numbers-3/.

Solidifying Prior Learning M–2

Support for Strand:	*Measurement*
Prior Learning for:	*Apply proportional reasoning when converting between SI and imperial units of measure.*

1. Find three pairs of positive integers for a and b that make this inequality true: $\left[\frac{2}{a}\right] > \left[\frac{3}{b}\right]$.

   Explain your thinking using an area model or number line.

2. There are 365 days in a year (one more for a leap year), 24 hours in a day, 60 minutes in each hour, and 60 seconds in each minute. **(TN)**
   a) According to the Guinness Book of World Records, the oldest human to ever live was 122 years and 164 days old at the time of her death. Jeanne Louise Calment was born on February 21, 1875, and died at a nursing home in Arles in Southern France on August 4, 1997. How many minutes was this person alive? (What assumptions have you made?)
   b) How old would you be when you celebrate your millionth minute? Billionth minute?

3. Jaswinder says, "If a numerator and a denominator of one fraction are closer together than the numerator and denominator of a second fraction, the first fraction is bigger."
   a) Write a pair of fractions that support his statement.
   b) Write a pair of fractions that show that his statement is not always true.

4. The area of a circle is 20 cm^2. What is the diameter?

5. What is the better deal? One large 18" pizza for $25 or two medium 12" pizzas for $25? **(TN)**

6. Your mother asks you to go to her desk and get your school picture and its enlargement. When you get to her desk, you find five pictures in various sizes:
   i) 9 cm × 10 cm
   ii) 10 cm × 12 cm
   iii) 8 cm × 9.6 cm
   iv) 6 cm × 8 cm
   v) 5 cm × 6.5 cm
   a) Which two could be a picture and an enlargement without distortion?
   b) She wants to print an enlarged picture with a width of 36 cm. Find the length of this enlargement.

Solidifying Prior Learning

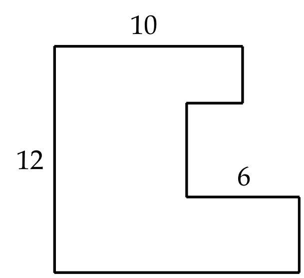
M–3

Support for Strand:	*Measurement*
Prior Learning for:	*Solve problems involving surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres using SI and imperial units.*

1. Gilles has driven $\frac{2}{3}$ of the trip distance in his car. He started with a full tank and his tank is now $\frac{1}{4}$ full. Will he likely run out of gas? How do you know? What assumptions are you making?

2. Six friends shared a bag of cookies. The first person ate $\frac{1}{6}$ of the cookies, the next ate $\frac{1}{5}$ of what remained, and the next ate $\frac{1}{4}$ of what remained. The fourth ate $\frac{1}{3}$ of what remained before the next person ate $\frac{1}{2}$ of what remained, which left three cookies for the sixth friend. How many cookies did the bag contain at the start?

3. Consider the following shape (not drawn to scale): **(TN)**
   a) Calculate a possible answer for the area.
   b) Calculate a possible answer for the perimeter.
   c) Compare your answers to a classmate. Explain how you arrived at your answers.



4. Using one piece of $8\frac{1}{2} \times 11$ paper, construct a cylinder with two ends and a large volume. Assume that no overlap is needed to join the edges.

5. Construct a circle of radius, *r* (any size you want), and then cut out several squares of length *r* (same length as the circle radius). Cut up the squares to fit them inside the circle. How many squares of side length *r* will fit into a circle of radius *r*? **(TN)**

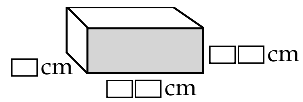
6. Check a variety of different coffee cups. **(TN)**
   a) Which is greater: the height of a coffee cup or the distance around the rim?
   b) How does the distance across the cup compare to the distance around the rim?
   c) What do you notice? What do you wonder?

Solidifying Prior Learning

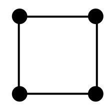
M–3

Support for Strand:	*Measurement*
Prior Learning for:	*Solve problems involving surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres using SI and imperial units. (continued)*

7. Using the numbers from 1 to 9 (at most once each), find the length, width, and height of the rectangular prism (as shown) so that the volume is close to 2500 cm^3.



8. Dylan starts building a 3-D figure as shown:
   a) What figure could Dylan be building?
   b) What figure could Dylan not be building?

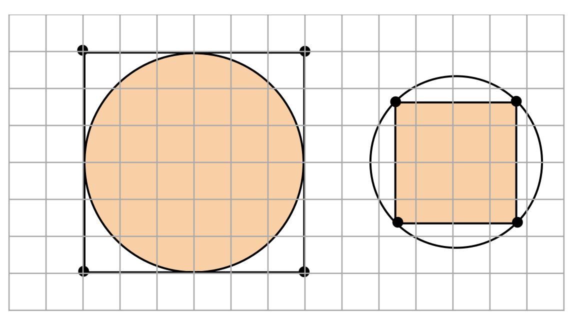


9. A rectangular prism (box) has a length that is twice the width and a height that is twice the length. What algebraic expressions can describe features of this box? **(TN)**

10. A cylinder has an unknown radius and a height of four units.
    What algebraic expressions can describe features of this cylinder?

11. Using the model of a cube and starting with a 1 cm × 1 cm × 1 cm cube, what is the ratio of the surface area to the volume (SA : Vol)? Consider growing cubes of various sizes. As the side length of the cube increases, what do you notice about the SA : Vol ratio? Some questions to consider:
    a) Is there a side length where the ratio is exactly 1?
    b) When is the SA : Vol ratio greater than 1? When is it less than 1?
    c) What is ratio of SA : Vol for a cube with a side length 100?
    d) Show algebraically that the ratio of the SA : Vol for a cube with a side length of n is $6 : n$.
    e) What is the SA : Vol ratio for cylinders with height equal to the radius—that is, with radius r and height r?

12. Which is a better fit: a round peg in a square hole or a square peg in a round hole? **(TN)**



Solidifying Prior Learning

M–4

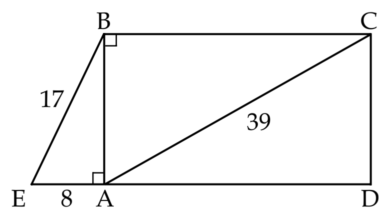
Support for Strand:	*Measurement*
Prior Learning for:	*Develop and apply primary trigonometric ratios to solve right angle triangles.*

1. Using digits from 0 through 9, at most one time each, fill in each of the boxes so that the fraction equals the decimal.*

$$\frac{\square\square}{\square\square} = \square.\square\square$$

2. What is the fewest number of people surveyed if exactly 93.6% of people completed a survey?**

3. Francie owns a garden plot. A diagram of this garden with rectangle ABCD and right triangle ABE is shown. She knows the length of AC is 39 m, the length of side EA is 8 m, and the length of side EB is 17 m. There is a water tap at C. How long must a hose be to ensure Francie can reach any location in the garden? (Calculate to the nearest tenth of a metre.)



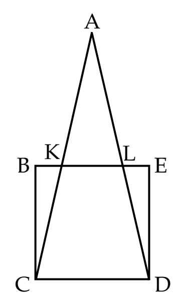
4. Draw a triangle. **(TN)**

   Keeping the shape of the triangle the same, make a much bigger version and a much smaller version of it. Measure the sides and angles of both of your new triangles. What do you notice and wonder as you compare the measures of the angles and the sides?

5. The area of ΔACD is twice the area of the square BCDE. AC and AD intersect BE at K and L, respectively. **(TN)**

   a) The measure of KL is 6 cm. The area of square BCDE is 64 cm^2. Determine the area of the top triangle ΔAKL.

   b) Suppose the measure of KL is unknown. If the area of the square is 144 cm^2, determine the area of trapezoid KCDL.



* Based on a problem by Gisele Garcia. Available at www.openmiddle.com/sum-of-fractions-closest-to-10/.

**Based on a problem by Robert Kaplinsky. Available at www.openmiddle.com/interpretting-percentages/.

Solidifying Prior Learning A–1

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding factors of whole numbers: primes, GCF, LCM, roots*

1. How are these pairs of powers the same? How are they different?

   a) 2^4 4^2 b) 3^2 2^3

2. Greatest Common Factor (GCF):

   a) List the factors of 32. List the factors of 8. What is the GCF of 32 and 8?

   b) List the factors of 24. List the factors of 18. What is the GCF of 24 and 18?

3. Least Common Multiple (LCM):

   a) List some multiples of 32. List some multiples of 8. What is the LCM of 32 and 8?

   b) List some multiples of 12. List some multiples of 8. What is the LCM of 12 and 8?

4. Sara has an unlimited supply of square tiles. Sara has 1 cm × 1cm tiles, 2 cm × 2 cm tiles, 3 cm × 3 cm tiles, and so on, up to 100 cm × 100 cm tiles. (The side length of every tile is a whole number.) A rectangular tabletop with an 84 cm by 112 cm surface is to be completely covered using tiles that are all the same size and no cuts can be made. How many tiles might Sara use to cover the tabletop? Could she use fewer tiles? What is the least number of tiles Sara could use? Provide a diagram. **(TN)**

5. The number 720 is divisible by all the numbers from 1 to 6, since 1 × 2 × 3 × 4 × 5 × 6 = 720.

   a) Find a smaller number that is also divisible by the numbers from 1 to 6. Is it the smallest? Compare your answer with other classmates. Compare your strategies.

   b) Find a number that is divisible by all the numbers from 1 to 20. Can you find one that is smaller? Compare your answer with other classmates. Compare your strategies.

6. A school hallway has 20 lockers assigned to 20 students that are numbered 1, 2, 3 … 19, 20. Their teacher says, “We will play a game.” The first student is to go down the hall and open all the lockers. The second student is to start with the second locker and close every second locker. The third student is to start with the third locker and change every third locker (i.e., open a closed locker or close an open one). The fourth student changes every fourth locker, and so on. The process continues until all 20 students have participated. Which lockers are open at the end of the game? Which lockers were touched the most? **(TN)**

7. Set A is {36, 49, 64, 81, 100, 121}.

   a) What do you notice and wonder about this set of numbers?

   b) Set B is {35, 48, 63, 80, 99, 120}. How do these numbers relate to Set A?

   c) Set C is {20, 33, 48, 65, 84, 105}. What does this set have in common with Sets A and B?

Solidifying Prior Learning

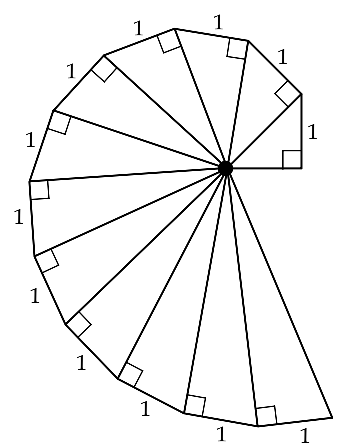
A–2

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding irrational numbers: representing, identifying, simplifying, ordering*

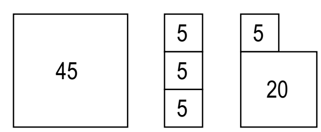
1. Replace x with a rational number to make each of these statements true:
   a) $1 < x < \sqrt{2}$
   b) $\frac{3}{2} < x < \sqrt{3}$
   c) $2 < x < \sqrt{5}$
   d) Do you notice any patterns that help you find more fractions that work?

2. Use all of the digits 0 to 4 (only once each) to create mathematical equations (i.e., $1^2 + 3 = 4 + 0$ will work BUT $(3)(2) + 1 - 4 = 3$ will not work).

3. Use a ruler to draw the first 10 right triangles of the **Spiral of Theodorus** (let 1 unit = 5 cm). The first right triangle has two legs. Each leg measures 1 unit (5 cm). Draw the next right triangle using the hypotenuse of the previous triangle as the first leg with a second leg of 1 unit (5 cm). Repeat the process. Through measurement, determine an approximate value of the lengths. Which hypotenuse lengths are whole units? Predict which triangle will be the next one to have a whole number hypotenuse.



4. The number inside each square represents its area in cm². The area of each square is a natural number. The squares are not drawn to scale. **(TN)**
   a) Should the heights of the stacks of squares be equal if they were drawn to scale? Justify your answer.
   b) Find stacks of squares that are the same height as a square with area 72 cm².



5. Let $x = 0.33333\ldots$, then $10x = 3.33333\ldots$. This is the same as $10x = 3 + 0.33333\ldots$ and the same as $10x = 3 + x$.
   a) Use the equation to write $0.33333\ldots$ as a fraction.
   b) Describe how you can represent the following repeating decimals as fractions (rational numbers).
      i) $0.636363\ldots$
      ii) $5.454545\ldots$

Solidifying Prior Learning

A–3

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding powers with integral and rational exponents*

1. Take any integer greater than 1 and square it. Find two consecutive integers whose sum equals that square number (e.g., 11 squared is 121. Consecutive integers with a sum of 121 are 60 and 61.). For which integers is this possible? For which integers is this not possible? Explain.

2. Replace each box in the radical expressions below with one of the digits from 1 to 9 (used once each) to produce an integer result.*

   $\sqrt{\square}$ $\quad$ $\sqrt{\square\square}$ $\quad$ $\sqrt{\square + \square}$ $\quad$ $\sqrt{\square} + \square$ $\quad$ $\sqrt[\square]{\square}$

3. Without a calculator, determine the value of $\left(5^4\right)\left(20^5\right)$. Rewrite the expression in a different form to help perform the calculation. Why does rewriting help? Write another pair of numbers that will be easier to multiply after rewriting in a similar way.

4. Using digits from 0 to 9 (at most, once each), create an expression by filling in the blanks $(\square)(\square)^{\square}$.

   a) Create two expressions in this form that are equivalent. Are other equivalent expressions possible?

   b) Create two or more expressions with values between 100 and 1000.

5. Find three different positive integers with a sum of 10. Place each number in one of the blanks in the expression $(\square)(\square)^{\square}$ to create a large result. Compare your results with classmates' results.

* Based on a problem by Owen Kaplinsky. Available at www.openmiddle.com/tag/5-nf-1/.

Solidifying Prior Learning A–4

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Multiplication of polynomial expressions concretely, pictorially, symbolically*

1. The product of 11×13 can be represented using an area model as shown on the right. $(11)(13) = (10 + 1)(10 + 3)$

   The product is $100 + 30 + 10 + 3 = 143$.

	10	1
10	100	10
3	30	3

   a) Use the area model to represent the product of 26×14.

   b) What is the product?

   c) Use an area model to illustrate a product of two 2-digit integers that is between 500 and 700.

2. Fill in the missing monomials and then write the product that is represented by this area model:

		7
$2x$	$6x^2$	
		14

3. Fill in the missing monomials and then write the product that is represented by this area model:

	$6x^2y^2$	
3		$15x^2y$
	$30x^6y^3$	

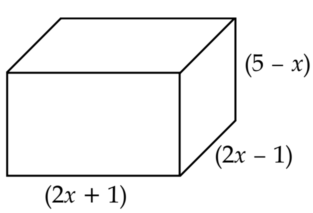
4. Rectangular prisms (boxes):

   a) Determine the surface area and volume of a rectangular prism (box) with a width of 5 metres, a depth of 3 metres, and a height of 3 metres.

   b) Write an expression for the surface area of the rectangular prism shown.

   c) Write an expression for the volume of the rectangular prism shown.

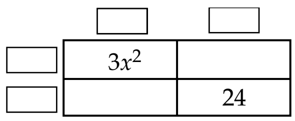
   d) What are the possible whole number values for x?



Solidifying Prior Learning A–4

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Multiplication of polynomial expressions concretely, pictorially, symbolically (continued)*

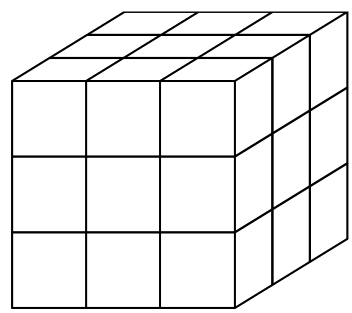
5. Using positive integer coefficients, what might be in the missing parts and the grid?



Write the equation represented:

(______________)(______________) = $3x^2$ + ______________ + ______________ + 24

6. The painted cube:
   a) Construct (or sketch) a $3 \times 3 \times 3$ cube that is made from $1 \times 1 \times 1$ cubes. How many smaller $1 \times 1 \times 1$ cubes are needed?
   b) Imagine the outside of the large $3 \times 3 \times 3$ cube is painted orange. x
      i) How many small cubes have only 1 orange face?
      ii) How many small cubes have 2 faces covered in orange paint?
      iii) How many small cubes have 3 faces covered in orange paint?
      iv) How many small cubes have 4 faces covered in orange paint?
      v) How many cubes have zero faces covered in orange paint?

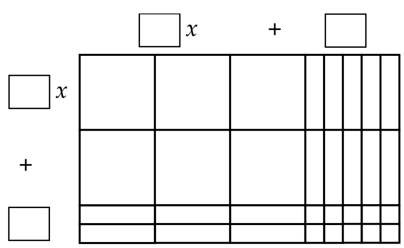


   c) Determine the number of sides painted for the same questions regarding a $4 \times 4 \times 4$ cube, then a $5 \times 5 \times 5$ cube, and, finally, a $6 \times 6 \times 6$ cube. What do you notice or wonder?
   d) How would you answer the questions regarding a $N \times N \times N$ cube? Justify your reasoning.

Solidifying Prior Learning A–5

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding common factors and trinomial factors*

1. Name two integers that
   a) have a product of 24
   b) have a sum of −13
   c) have a product of 24 and a sum of 11
   d) have a product of 15 and a sum of −8
2. The product of two integers is 18; what are all the possible sums of the two integers?
3. The product of two integers is 12; what are all the possible differences of the two integers?
4. The following list is the sums of pairs of integers that share the same product: −17, −7, −3, 3, 7, 17. What is the product? Justify your answer.
5. Write the product of binomials and the equivalent trinomial represented by the area model.



6. Replace the boxes with numbers to make the equation true:
   a) $4x + 8 = \square(x + 2)$
   b) $\square x^2 + 12x = \square(x + 2)$
   c) $(2x^2 + \square x + 3)(\square x + \square) = 4x^3 + 20x^2 + 30x + 12$
   d) $12x + 9 = \square(\square + \square)$
7. Replace each box with the operations + and −. What are the possible coefficients for the linear (middle) term of the trinomial product? What do you notice or wonder?
   a) $(n \square 2)(n + 6)$
   b) $(x - 3)(x \square 3)$
   c) $(a \square 5)(a \square 4)$
   d) $(2b + 3)(b \square 5)$

Solidifying Prior Learning A–5

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding common factors and trinomial factors (continued)*

8. Find two numbers that
   a) have a product of 36 and a sum of −15
   b) have a product of −18 and a sum of −7

9. Use algebra tiles to represent the product, and then write the trinomial product symbolically:
   a) $(2x+3)(x+2)$
   b) $(2x+1)(x+4)$
   c) $2(x+3)(x+4)$

10. Use algebra tiles to represent the product of two binomials resulting in the trinomial product in as many ways as possible, using different numbers of x-tiles or unit tiles as appropriate. Then write the binomial products symbolically.
    a) $(\_\_\_\_)(\_\_\_\_) = x^2 + \square x + 6$
    b) $(\_\_\_\_)(\_\_\_\_) = x^2 + \square x + 7$
    c) $(\_\_\_\_)(\_\_\_\_) = x^2 + \square x + 9$

11. Use algebra tiles to represent each trinomial and whether it can be written as a product of binomial factors. **(TN)**
    a) $x^2 + 8x + 8$
    b) $x^2 + 7x + 12$
    c) $x^2 + 1$

Solidifying Prior Learning R–1

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Interpret and explain the relationships among data, graphs, and contexts*

1. In the addition shown, P, Q, and R each represent a single digit and the sum is 2009. Find the value of the sum of digits P + Q + R.*

$$\begin{array}{r} PQP \\ +\ RQQQ \\ \hline 2009 \end{array}$$

2. Examine the table of values for white sturgeon fish in Manitoba.

White Sturgeon Fish	
Age	**Weight (pounds)**
26	116
30	148
33	172
154	400

   a) Graph the ordered pairs of data. Why should these points **not** be connected?
   b) Using your graph, predict the weight of a 10-year-old sturgeon.
   c) Using your graph, predict the age of a 200-pound sturgeon.

3. There is a strong relationship between ______________ and ______________.
   a) Fill in the blanks with phrases that make sense (e.g., "the weather" and "number of beachgoers").
   b) Create a graph that could represent the relationship.
   c) Show another version of your graph of these same variables using data values that hide the relationship.

4. Given an algebraic equation, create a story or picture(s). Be sure to define the variables.

   For example: A delivery of building materials will cost $50 plus $0.25 for each kilometre driven from the store.

   $C = 50 + 0.25d$
   C = cost ($)
   d = distance (km)

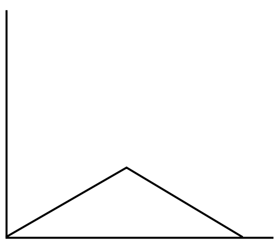
   a) $c = 12 + 2f$
   b) $y = 25x$
   c) $b = a + 12$

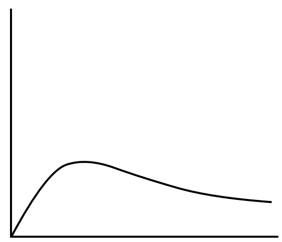
* Based on a problem by CEMC. Available at www.cemc.uwaterloo.ca/contests/past_contests/2009/2009Gauss8Contest.pdf.

Solidifying Prior Learning R–1

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Interpret and explain the relationships among data, graphs, and contexts (continued)*

5. Choose one of the graphs below. Label the axes. Describe the pattern using a story, table, or rule. **(TN)**

   a) 

   b) 

6. David read 40 pages of a novel on day one, 37 pages on day two, and 34 pages on day three. This pattern of daily reading continued until David finished his book.
   a) Predict how long this pattern could continue. Explain your thinking.
   b) What was the total number of pages David read in one week?
   c) David started a new book. It is 150 pages. He maintained the same reading pattern and finished the book on a Tuesday. What day of the week did he start reading his book?

Solidifying Prior Learning

R–2

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Demonstrate an understanding of relations and functions*
Developing Number Sense	

1. Which one doesn't belong? Find a reason and explain why each number does not belong in the set.*

$\frac{1}{20}$	$\frac{20}{25}$
$\frac{2}{3}$	$\frac{5}{4}$

2. Enzo has two piles of buttons. In each pile there are red buttons and blue buttons. In one pile, the ratio of the number of red to the number of blue buttons is 1 : 2. In the second pile the ratio of the number of red to the number of blue buttons is 3 : 5. If Enzo has a total of 20 red buttons, what might the total number of blue buttons be?

3. Grace plants flowers (x) around her tomato plants (•) to protect them from bugs. **(TN)**

N = 1	N = 2	N = 3
x x x *x* • *x* *x x x*	*x x x x* *x* • • *x* *x* • • *x* *x x x x*	*x x x x x* *x* • • • *x* *x* • • • *x* *x* • • • *x* *x x x x x*

   a) Describe how you see the pattern growing. Extend the pattern by drawing the next two shapes.

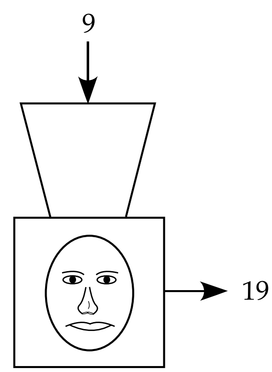
   b) For a large value of N, describe how to find the number of tomato plants (•) and the total number of plants (x + •).

4. Bobo the function robot takes a number as input (x) and gives out another number as output (y). You put in a 9 and you get back 19.

   a) Use words to describe three different possible rules that Bobo could be using.

   b) Represent each rule with an equation.

   c) Create a rule that involves more than one operation.



* Based on a problem by Hélène Matte. Available at www.wodb.ca/numbers.html.

Solidifying Prior Learning

R–3

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Demonstrate an understanding of slope: rise & run, lines, range of change....*

1. Which one doesn't belong? Find a reason and explain why each number does not belong in the set.*

33%	$\frac{1}{3}$
$\frac{5}{3}$	$0.\overline{6}$

2. For each question, use digits from 1 to 9 (without repeats) to make the value of each expression close to 1 (but not ≥ 1).** **(TN)**

   a) $\frac{\square}{\square} + \frac{\square}{\square}$

   b) $\frac{\square}{\square} - \frac{\square}{\square}$

   c) $\frac{\square}{\square} \times \frac{\square}{\square}$

   d) $\frac{\square}{\square} \div \frac{\square}{\square}$

3. Complete the products in the tables below. What do you notice? What do you wonder?

   a)

Expression	Product
(4)(5)	
(3)(5)	
(2)(5)	
(1)(5)	
(0)(5)	
(−1)(5)	
(−2)(5)	

   b)

Expression	Product
(4)(−5)	
(3)(−5)	
(2)(−5)	
(1)(−5)	
(0)(−5)	
(−1)(−5)	
(−2)(−5)	

4. Draw a square with vertices at E(6,1) and G(2,1).

   a) What is the length of each side? What is the perimeter? What is the area?

   b) Sketch another possible square using E and G as vertices. What are the characteristics of the second square?

* Based on a problem by Erick Lee. Available at www.wodb.ca/numbers.html.

**Based on a problem by Owen Kaplinsky. Available at www.openmiddle.com/tag/5-nf-1/.

Solidifying Prior Learning

R–3

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Demonstrate an understanding of slope: rise & run, lines, range of change.... (continued)*

5. Draw a rectangle at W(–2,4) and Y(2,1).
   a) Draw a rectangle using these points. What are the coordinates of the other vertices? What is the area?
   b) Sketch another possible rectangle using W and Y as vertices. What are the characteristics of the second rectangle?

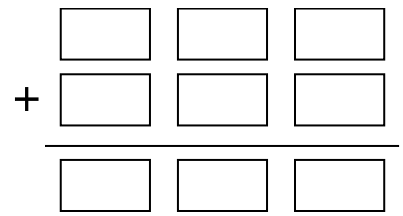
6. Draw a line segment from point A(1, 1) to point B(5, 4). **(TN)**
   a) What are some points that would be on the line extended through AB? Describe the pattern.
   b) Draw another line segment from point X(6, 2) to point Y(14, 6). What are some points that would be on the line extended through XY? Describe the pattern.

Solidifying Prior Learning

R–4

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Describe and represent linear relations, using words, tables, graphs, equations....*

1. Use all of the digits from 1 to 9 once each to complete the three-digit sum. Is there another possibility?*



2. Create a rule for a decreasing pattern over six days.

Day						
Value						

   a) Describe a possible context for the decreasing pattern. What will your numbers represent?
   b) Show the first six days of your pattern in a table of values.
   c) Describe the pattern in words.
   d) Represent the pattern as a graph.

3. The table of values displays a pattern/relationship between x- and y-values.
   a) Find two or more ordered pairs (x, y) that can fill the boxes.
   b) Explain how these points fit the pattern.
   c) If you graphed the points, would they all lie on the same line? Explain.

x	y
4	16
$\boxed{}$	$\boxed{}$
12	44

4. Given the pattern of blocks:
   a) Describe the pattern(s) you see.
   b) How many blocks will be in Figure 4? Figure 5?
   c) How many blocks are required to build Figure 10? Explain your thinking.
   d) Your friend says, "The total number of blocks for each figure does not increase in linear fashion." What do you think your friend means?

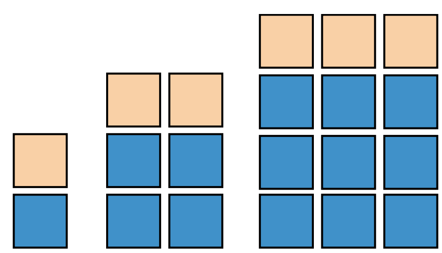


Figure 1 Figure 2 Figure 3

* Based on a problem by Owen Kaplinsky. Available at www.openmiddle.com/tag/5-nf-1/.

Solidifying Prior Learning R–4

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Describe and represent linear relations, using words, tables, graphs, equations.... (continued)*
Developing Number Sense	

5. Given the pattern of blocks:

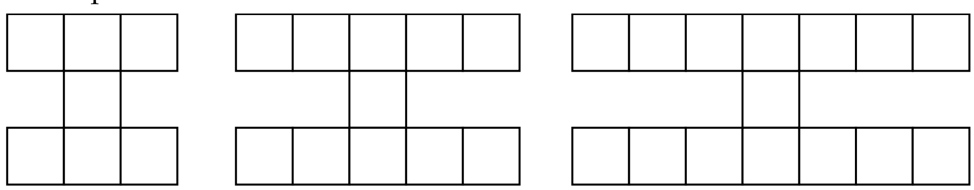


Figure 1 Figure 2 Figure 3

   a) Describe the pattern(s) you see.
   b) How many blocks will be in Figure 4? Figure 5?
   c) How many blocks are required to build Figure 10? Explain your thinking.
   d) Which figure will be closest to having 1000 blocks? Explain.
   e) Your friend says, "The number of blocks for each figure increases in linear fashion." What does your friend mean?

6. At Mathville Middle School, 30 boys and 20 girls entered a math contest. Certificates were awarded to 30% of the boys and 40% of the girls. What percentage of all of the participating students received certificates? Explain the process of your calculation.

7. Without a calculator:
   a) Which expression represents the smallest value when x is a number between 0 and 1? Explain your reasoning.
      i) x
      ii) x^2
      iii) $\frac{1}{x}$
      iv) $2x$
      v) $\sqrt{x}$

* Based on a problem by CEMC. Available at www.cemc.uwaterloo.ca/contests/past_contests/2009/2009Gauss8Contest.pdf.

Solidifying Prior Learning

R–4

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Describe and represent linear relations, using words, tables, graphs, equations.... (continued)*

b) Which expression represents the smallest value when x is a number between 1 and 4? Explain your reasoning.*

i) x

ii) x^2

iii) $\frac{1}{x}$

iv) $2x$

v) $\sqrt{x}$

8. Choose a babysitting fee for each hour of babysitting. As a babysitter, you charge a flat fee of $5 for snacks in addition to the hourly fee.

   a) Graph the relationship between the number of hours babysitting and the total amount earned. Label the axes with what they represent.

   b) Would you describe the relationship as linear? Explain.

   c) Could your graph of this scenario include negative numbers?

   d) The parents just told you they are going to give you a $20 bonus before you start so you can buy dinner. How does this change your graph?

* Based on a problem by CEMC. Available at www.cemc.uwaterloo.ca/contests/past_contests/2009/2009Gauss8Contest.pdf.

Solidifying Prior Learning

R–5

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Characteristics of graphs of linear relations, intercepts, slope, domain, and range*
Developing Number Sense	

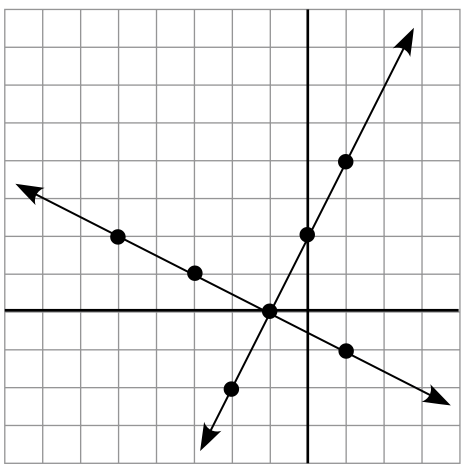
1. Which of the following numbers is closest to 1? Explain.*

   a) $\frac{11}{10}$ b) $\frac{111}{100}$ c) 1.101 d) $\frac{1111}{1000}$ e) 1.011

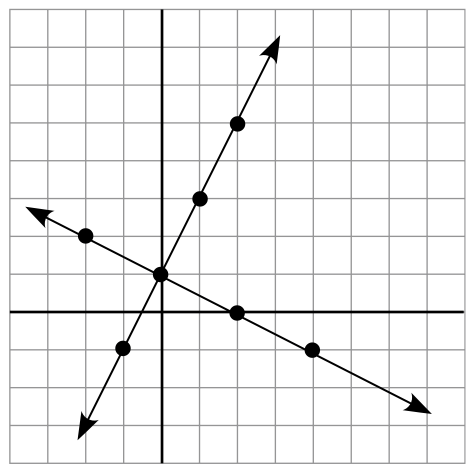
2. One car is travelling at 60 km/h and a motorcycle is travelling at 16 m/s. Which vehicle is travelling faster? How do you know?

3. How are these graphs similar? How are these graphs different?

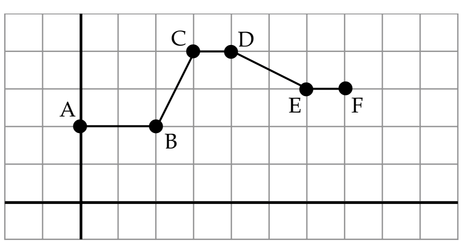
Graph A



Graph B



4. What could this graph represent? Label the axes. Tell the story.



* Based on a problem by CEMC. Available at www.cemc.uwaterloo.ca/contests/past_contests/2011/2011PascalSolution.pdf.

Solidifying Prior Learning R–5

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Characteristics of graphs of linear relations, intercepts, slope, domain, and range (continued)*

5. Your school table tennis team wants to buy team clothing. They have decided to purchase t-shirts costing \$10 and hats costing \$15 (tax is included). The team only has \$300 to spend.
   a) What different combinations of t-shirts and hats can the team buy with the entire \$300?
   b) What is the smallest number of hats that they could buy? What is the largest number of hats that they can buy?
   c) In one sentence, describe a pattern that helps describe all of the possible combinations.

6. Create tables of values for each of the equations. What similarities and differences do you notice? Predict what each of the graphs will look like with a rough sketch. **(TN)**

   Check your predictions by creating a graph (with paper and pencil or technology such as at www.desmos.com).
   a) $y = -3x + 5$
   b) $y = -x + 5$
   c) $y = \frac{1}{3}x + 5$
   d) $y = x + 5$

Solidifying Prior Learning R–6

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Relate linear relations as slope-intercept form, general form, and point-slope form*

1. Write each of the following phrases as algebraic expressions:
   a) Add a number and the square of the number.
   b) Subtract 10 from a number.
   c) Add 1 to a number and then double the result.
   d) Double a number and then add 1.
   e) Subtract a number from 12 and then divide the result by 4.
   f) Add 2 to a number, then triple the result and add one more.

2. Find the value of the missing number in the equation: $-2(1+5)+3\left(6+\square\right)=3$

3. Create two equations with an unknown, with both multiplication and addition (or subtraction), with brackets, and with solutions that are negative numbers. Challenge others to solve them. Create an equation that
   a) uses only integers with an integer solution
   b) includes fractions with an integer solution

4. Given the equation $-3x+4=5x+2$:
   a) Without solving, rewrite an equation with the same solution that has all terms on one side of the equal sign, leaving zero on the other.
   b) Compare your work with others. What is the same? What is different?

5. Create an equation.
   a) The original equation needs to meet all of the following conditions:
      i) contains one variable
      ii) includes at least 4 terms (2 of which are constants)
      iii) uses a different integer for each term's coefficient
   b) Rearrange your equation so that all terms are on one side with zero on the other so that you have a positive coefficient in front of your variable. Check that the solution is the same for the original and the rearranged equations.

Solidifying Prior Learning R–6

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Relate linear relations as slope-intercept form, general form, and point-slope form (continued)*

6. Fill in the blanks with numbers to establish a pattern rule:

   "Start at ____________ and add/subtract ____________ for each new term."

   a) Show your pattern in a table of values, giving the pattern value and the term number.

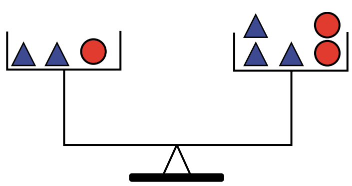
   b) Plot the pattern as a set of points (e.g., term number, pattern value).

   c) Establish a second rule by changing ONLY the start number. Create a new table of values. Plot the new set of points on the same graph with the pattern from the first rule.

   d) Establish a third rule by changing ONLY the number you add/subtract each time. Create a new table of values. Plot the new set of points on the same graph with the patterns from the first and second rule.

   e) What do you notice or wonder about the data in the tables and the graphs associated with the three rules?

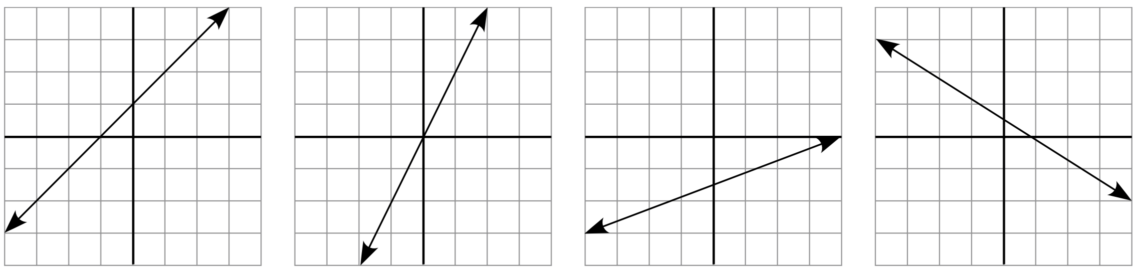
7. Find the relationship between the ▲ and the ● by determining the integer values the ▲ and ● might represent to keep the balance. Find two pairs of integers for the triangle and circle. Describe the general relationship between the value of the triangle and the value of the circle.



Solidifying Prior Learning R–6

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Relate linear relations as slope-intercept form, general form, and point-slope form (continued)*

8. Which one doesn't belong? Find a reason and explain why each graph does not belong in the set.*



9. Joti is saving up to buy tickets to a concert. She has $50 saved already and is earning $15 per week to add to her savings.
   a) Model her savings over time in two different ways (e.g., words, graph, table, equation).
   b) Joti expects the ticket to cost between $120 and $280. How long will it take Joti to save enough money?
   c) Joti has other priorities and she would like to consider an alternative. She will use only $20 of her original savings and $10 of her weekly earnings. How long will it take her to save enough money for the ticket? How do your models change to represent this alternative?

* Based on a problem by Mary Bourassa. Available at https://wodb.ca/graphs.html.

Solidifying Prior Learning

R–7

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Determine the equation of a linear relation from a graph, a point and a slope, two points, a point and the equation of a parallel or perpendicular line, and a scatterplot*

1. For each set of numbers, calculate the mean (average). For each set, represent the values and the mean on a number line. Considering all the cases below, what do you notice (as an observation) or wonder (question)?
   a) 2, 3, 4
   b) −8, −6, −4
   c) −3, −1, 1, 3, 5, 7
   d) −7, 1
   e) −10, 10, 30
2. Calculate the sum:
   a) 9 + 16
   b) 36 + 64
   c) 144 + 25

   How are these expressions alike? Create other expressions that share this property.
3. Using a Cartesian plane:
   a) Draw a polygon with the conditions that it has to have
      i) vertices in three or more quadrants
      ii) at least one pair of perpendicular sides
      iii) at least one pair of parallel sides
   b) Using only verbal communication, instruct a partner to draw an identical polygon. Choose one of the vertices as a starting point. This is the only point you can identify using coordinates. Provide verbal instructions (with no hand waving) to have your partner create a drawing of the polygon.
   c) Evaluate the communication by comparing the drawing to the original polygon.

Solidifying Prior Learning R–7

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Determine the equation of a linear relation from a graph, a point and a slope, two points, a point and the equation of a parallel or perpendicular line, and a scatterplot (continued)*

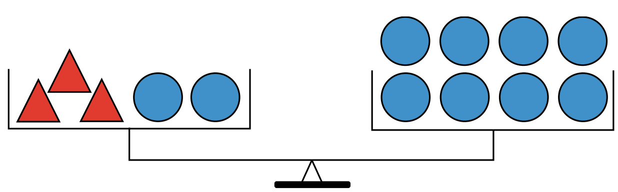
4. The following data shows the number of birds counted in an area on some days over a period of a few weeks in the spring. **(TN)**

Day	3	5	9	11	13	15	21
Bird Count	1	5	6	8	10	14	12

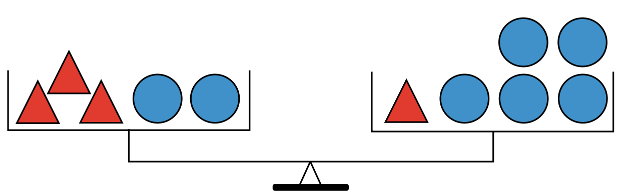
   a) Plot the data points on a graph.
   b) Draw a line that you think best represents the data trend.
   c) Compare your line to a classmate's. How are they the same or different? Justify why you think one line is a better representation of the trend of the data.

5. The weights of the triangles on the equal arm balances are unknown. The weight of each marble (circle) is 7 g. For each scenario, do the following:
   a) Determine the triangle weight without using algebra.
   b) Describe, in words, the steps you took to determine the triangle weight.
   c) Write an equation to represent the scenario and solve the equation algebraically.
   d) How does your description in words connect with your algebraic steps?

i)



ii)



Solidifying Prior Learning

R–7

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Determine the equation of a linear relation from a graph, a point and a slope, two points, a point and the equation of a parallel or perpendicular line, and a scatterplot (continued)*

6. The following steps illustrate the use of the distributive property to mentally calculate 15×13:

   $15(10 + 3)$

   $(15 \times 10) + (15 \times 3)$

   $150 + 45$

   195

   a) What is the distributive property? Describe it in words, algebraically, or with an area model.

   b) Use the distributive property to write equivalent expressions.

      i) 7×14 is the same as (____)(____ + ____).

      ii) $(6)(5x + 2)$ is the same as (____ + ____).

      iii) $120y - 84$ is the same as (____)(____ − ____).

7. On a coordinate plane, draw the graph of a line that goes through a point in the second quadrant and intersects the x-axis at 45°. Draw a second line that is perpendicular and passes through the origin. **(TN)**

Solidifying Prior Learning

R–8

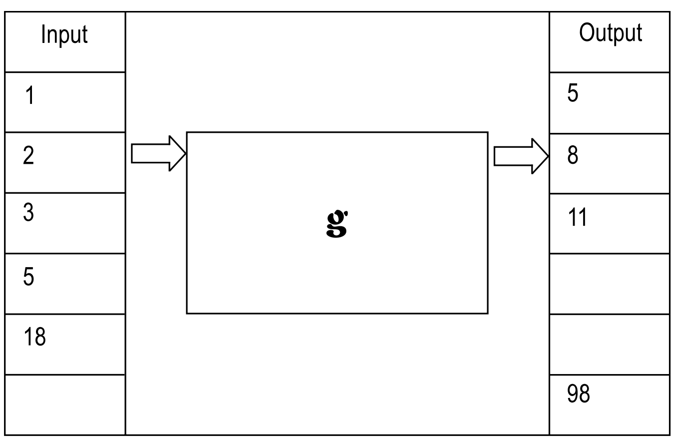
Support for Strand:	*Relations and Functions*
Prior Learning for:	*Represent a linear function using function notation*

1. Use the distributive property to mentally determine the product of each expression below. Explain the thinking you used to arrive at your answer. **(TN)**
   a) 12×16
   b) 26×8
   c) 10.5×12

2. It's Waffle Wednesday and John makes waffles for his friends. The recipe calls for $1\frac{1}{2}$ cups of flour and 3 teaspoons of baking powder. Based on experience, John knows the recipe won't quite be enough to feed everyone and doubling the recipe is too much. He decides to add one-third more to all of the ingredients because he thinks it is easier than adding one-half more. How much flour and how much baking powder should he put in the mixing bowl? Do you agree or disagree with John? Why?

3. Notation is important for communication. Choose one imperial unit and one metric unit, and then write all the notations you know for the related units of length, area, and volume.

4. Use the input and output values of the function machine, g, as shown below, to determine the missing output and input values. Describe what the function machine, g, is doing with the input. **(TN)**



Solidifying Prior Learning R–8

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Represent a linear function using function notation (continued)*

5. Temperature in the USA is reported in °F (Fahrenheit) and temperature in Canada is reported in °C (Celsius). You can input degrees Celsius into a function machine, T, and output degrees Fahrenheit. The operations of the function machine are as follows: divide by 5, multiply by 9, and then add 32.

   a) Write out the operations of T algebraically.

   b) Outside the thermometer reads +40°C. What output value will machine T show?

   c) Outside, the thermometer reads −40°C. What output value will machine T show?

   d) Your oven reads 350°F. What input value in °C gives an output value of 350°F?

Solidifying Prior Learning

R–9

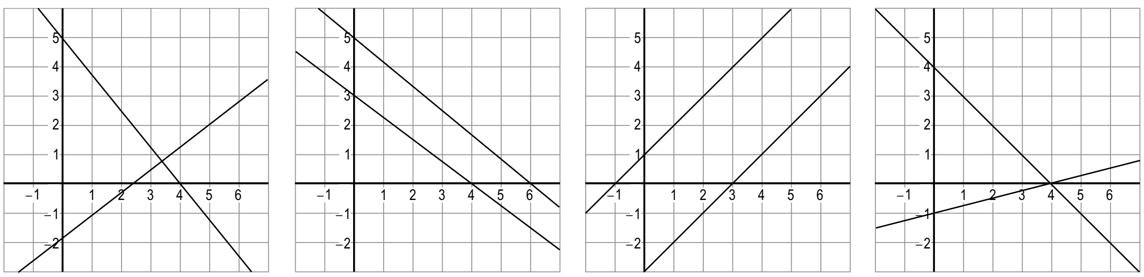
Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving systems of linear equations in two variables, graphically and algebraically*

1. Sansa is 5 years older than Arya. At least one of them is a teenager and the sum of their ages is a prime number. How old might they be?
2. Consider the exchange rate of a USA dollar (USD) and a Canadian dollar (CAD).
   a) When a USA dollar is worth about $1\frac{1}{2}$ times a Canadian dollar, a Canadian dollar is worth $\frac{2}{3}$ of a USA dollar. How are these two fractions related?
   b) If a USA dollar is worth about $1\frac{1}{4}$ times a Canadian dollar, what fraction of a USA dollar would a Canadian dollar be worth?
3. The sum of the ages of Bonnie, her sister, her mother, and her father is currently 89. In relation to Bonnie, her sister is 3 years younger and her mother is 3 times as old. Bonnie's father is 4 years older than her mother. Represent their ages using algebraic expressions on a number line. Is this enough information to determine their ages? Explain.
4. The school track team plans to order hoodies for each team member and needs to decide which company to go with.
   a) Company A charges an initial fee of $350 to print the school logo and $20 per hoodie.
   b) Company B charges an initial fee of $200 to print the school logo and $25 per hoodie.

   Which company would you recommend they choose? Explain your answer.
5. Start with the point (3, 5) on a graph.
   a) Use a ruler to draw two lines that both cross through this point.
   b) Write down several attributes of each of the two lines you just drew. What is the same? What is different?

Solidifying Prior Learning	R–9
Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving systems of linear equations in two variables, graphically and algebraically (continued)*

6. Which one doesn't belong? Find a reason and explain why each one does not belong in the set.*



* Based on a problem by Kyle Ramstad. Available at https://wodb.ca/graphs.html.

Solidifying Prior Learning

R–10

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving the distance between two points and the midpoint of a line segment*

1. Using all of the digits from 1 to 9, complete the following to make a true statement.*

$$\frac{\square}{\square} > \frac{\square\square}{\square} = \frac{\square}{\square} > \frac{\square}{\square}$$

2. If we add two integers, A and B, and get A + B = –1, what do we know about the integers? Demonstrate your answer on a number line.

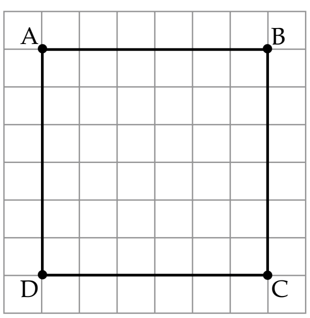
3. My age is 35 years old. How old is someone half my age? How old is someone half your age? What is the age of a person halfway between your age and my age? **(TN)**

4. Mr. Bowe is 42 years old. His age is halfway between your age and Ms. Carlyle. How old is Ms. Carlyle?

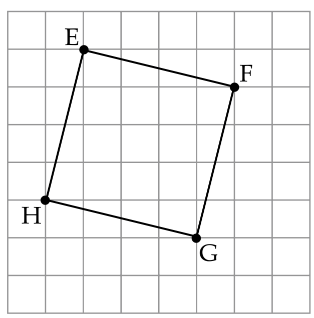
5. Liam is 9 years old. His age is halfway between you and your cousin Sara. How old is Sara? Explain your thinking.

6. Consider the squares in the diagrams below. Describe where lines need to be drawn to divide each square into 4 smaller squares. Is there more than one place to draw the lines?

a)



b)

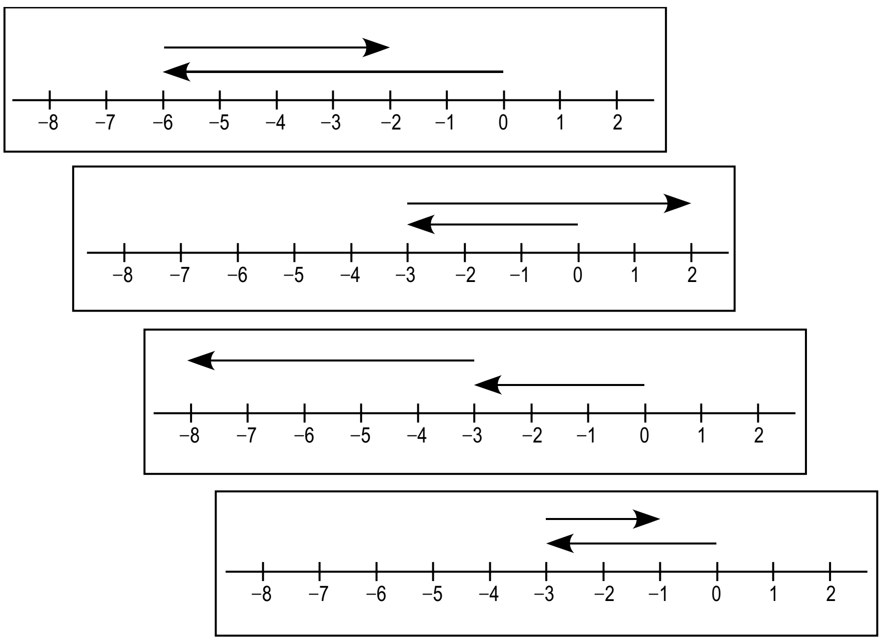


* Based on a problem by Owen Kaplinsky. Available at www.openmiddle.com/tag/5-nf-1/.

Solidifying Prior Learning R–10

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving the distance between two points and the midpoint of a line segment (continued)*

7. Consider the four number lines shown:*
   a) Which one does not belong? Find a reason and explain how each one does not belong.
   b) What expression does each number line represent?



* Based on a problem by Nicole Paris. Available at https://wodb.ca/numbers.html.

Solidifying Prior Learning

R–10

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving the distance between two points and the midpoint of a line segment (continued)*

8. Use the distributive property to mentally determine the quotient of 280 ÷ 8. Explain the thinking you used to arrive at your answer.

9. a) The arithmetic mean of 1, 3, 5, 7, 9 is the same as the median. Explain why.

   b) What two integers can be added to this list that will not change the mean value? How many pairs can you find?

10. The vertices of the base of an isosceles triangle are at A(1, 3) and B(7, 3).

    a) Where might the third vertex, C, be? Where else might C be?

    b) Draw the triangle on a coordinate plane.

    c) Determine the area and perimeter of your triangle.

11. A triangle is formed by joining vertices at A(3, 0), B(10, 0), and C(0, 6). (TN)

    a) Plot ΔABC.

    b) Find the lengths of AB, AC, and BC (exact and to the nearest thousandth).

    c) Plot point D on AC at (1.5, 3). Show that the length of CD equals half of AC.

    d) Draw a line through D parallel to AB and label the line's intersection with BC, E.

    e) What do you notice or wonder?