Grade 11 Pre-Calculus Mathematics (30S)

Grade 11 Pre-Calculus Mathematics (30S) is designed for students who intend to study calculus and related mathematics as part of post-secondary education. It builds on the topics studied in Grade 10 Introduction to Applied and Pre-Calculus Mathematics and provides background knowledge and skills for Grade 12 Pre-Calculus Mathematics.

The course comprises a high-level study of theoretical mathematics with an emphasis on problem solving and mental mathematics. The topics include study of algebra, quadratic functions, reciprocal functions, and trigonometry.

Assessment of Grade 11 Pre-Calculus Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in Grade 11 Pre-Calculus Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The learning outcomes are divided into three topics: Algebra and Number; Trigonometry; and Relations and Functions. For instructional purposes, the learning outcomes could be arranged into units. Learning outcomes from different topics could be taught in the same unit. Some learning outcomes may fit into multiple units, and parts of the learning outcome could be taught in one unit while the remaining parts can be taught later. Two possible sequences of the learning outcomes into units with suggested time allotments follow. The suggested times include time for instruction and assessment. These are not the only possibilities, but they will provide some direction for teachers for their first time through the course.

Regardless of the organization of the learning outcomes into units, students should constantly be looking for and be given opportunities to see connections between the various learning outcomes in Grade 11 Pre-Calculus Mathematics.

Possibility 1			Possibility 2		
Unit	Learning Outcomes	Suggested Hours	Unit	Learning Outcomes	Suggested Hours
Quadratic Equations	R1, R5	12	Algebra	R1, A4, A5	15
Radicals	A2, A3	15	Functions	R3, R4, R11	20
Quadratic Functions	R3, R4	17	Sequences	R9, R10	10
Sequences	R9, R10	10	Absolute Value	A1, R2	10
Rationals	A4, A5, A6, R11	15	Equation Solving	A3, A6, R5, R6	15
Trigonometry	T1, T2, T3	20	Trigonometry	T1, T2, T3	20
Systems	R6	6	Inequalities	R7, R8	10
Inequalities	A1, R2, R7, R8	15	Radicals	A2, A3	10
	Total	**110**		**Total**	**110**

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 11 Pre-Calculus Mathematics

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Algebra and Number		**General Learning Outcome:** Develop algebraic reasoning and number sense.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.A.1.	Demonstrate an understanding of the absolute value of real numbers. [ME, R, V]	■ Determine the distance of two real numbers of the form $\pm a$, $a \in R$, from 0 on a number line, and relate this to the absolute value of a ($\|a\|$). ■ Determine the absolute value of a positive or negative real number. ■ Explain, using examples, how distance between two points on a number line can be expressed in terms of absolute value. ■ Determine the absolute value of a numerical expression. ■ Compare and order the absolute values of real numbers in a set.
11P.A.2.	Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. [CN, ME, PS, R, T]	■ Compare and order a set of radical expressions with numerical radicands. ■ Express an entire radical with a numerical radicand as a mixed radical. ■ Express a mixed radical with a numerical radicand as an entire radical. ■ Perform one or more operations to simplify radical expressions with numerical or variable radicands. ■ Rationalize the denominator of a rational expression with monomial or binomial denominators. ■ Describe the relationship between rationalizing a binomial denominator of a rational expression and the product of the factors of a difference of squares expression. ■ Explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = \|x\|$, *if* $x^2 = a$, then $x = \pm\sqrt{a}$ ■ Identify the values of the variable for which a radical expression is defined. ■ Solve a problem that involves radical expressions.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Algebra and Number *(continued)*		**General Learning Outcome:** Develop algebraic reasoning and number sense.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.A.3.	Solve problems that involve radical equations (limited to square roots). [C, CN, PS, R, T] It is intended that the equations will have no more than two radicals.	■ Determine any restrictions on values for the variable in a radical equation. ■ Determine the roots of a radical equation algebraically, and explain the process used to solve the equation. ■ Verify, by substitution, that the values determined in solving a radical equation algebraically are roots of the equation. ■ Demonstrate that some roots determined in solving a radical equation algebraically are extraneous. ■ Solve a problem by modelling a situation using a radical equation.
11P.A.4.	Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, ME, R]	■ Compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers. ■ Explain why a value is non-permissible for a rational expression. ■ Determine the non-permissible values for a rational expression. ■ Determine a rational expression that is equivalent to a rational expression by multiplying the numerator and denominator by the same factor (limited to a monomial or a binomial), and state the non-permissible values of the equivalent rational expression. ■ Simplify a rational expression. ■ Explain why the non-permissible values of a rational expression and its simplified form are the same. ■ Identify and correct errors in a simplification of a rational expression, and explain the reasoning.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Algebra and Number *(continued)*		**General Learning Outcome:** Develop algebraic reasoning and number sense.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.A.5.	Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, CN, ME, R]	■ Compare the strategies for performing an operation on rational expressions to the strategies for performing the same operation on rational numbers. ■ Determine the non-permissible values when performing operations on rational expressions. ■ Determine, in simplified form, the sum or difference of rational expressions with the same denominator. ■ Determine, in simplified form, the sum or difference of rational expressions in which the denominators are not the same and which may or may not contain common factors. ■ Determine, in simplified form, the product or quotient of rational expressions. ■ Simplify an expression that involves two or more operations on rational expressions.
11P.A.6.	Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, CN, PS, R] It is intended that the rational equations be those that can be simplified to linear and quadratic equations.	■ Determine the non-permissible values for the variable in a rational equation. ■ Determine the solution to a rational equation algebraically, and explain the process. ■ Explain why a value obtained in solving a rational equation may not be a solution of the equation. ■ Solve a problem by modelling a situation using a rational equation.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Trigonometry	**General Learning Outcome:** Develop trigonometric reasoning.

	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.T.1.	Demonstrate an understanding of angles in standard position [0° to 360°]. [C, R, V]	■ Sketch an angle in standard position, given the measure of the angle. ■ Determine the reference angle for an angle in standard position. ■ Explain, using examples, how to determine the angles from 0° to 360° that have the same reference angle as a given angle. ■ Illustrate, using examples, that any angle from 90° to 360° is the reflection in the x-axis and/or the y-axis of its reference angle. ■ Determine the quadrant in which an angle in standard position terminates. ■ Draw an angle in standard position given any point P (x, y) on the terminal arm of the angle. ■ Illustrate, using examples, that the points P (x, y), P $(-x, y)$, P $(-x, -y)$, and P $(x, -y)$ are points on the terminal sides of angles in standard position that have the same reference angle.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Trigonometry *(continued)*		**General Learning Outcome:** Develop trigonometric reasoning.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.T.2.	Solve problems, using the three primary trigonometric ratios (sine, cosine, and tangent) for angles from 0° to 360° in standard position. [C, ME, PS, R, T, V]	■ Determine, using the Pythagorean theorem or the distance formula, the distance from the origin to a point P (x, y) on the terminal arm of an angle. ■ Determine the value of sin θ, cos θ, or tan θ, given any point P (x, y) on the terminal arm of angle θ. ■ Determine, without the use of technology, the value of sin θ, cos θ, or tan θ, given any point P (x, y) on the terminal arm of angle θ, where $\theta = 0^o, 90^o, 180^o, 270^o$, or 360^o. ■ Determine the sign of a trigonometric ratio for an angle, without the use of technology, and explain. ■ Solve an equation of the form $\sin \theta = a$ or $\cos \theta = a$, where $-1 \leq a \leq 1$, or an equation of the form $\tan \theta = a$, where a is a real number. ■ Determine the exact value of the sine, cosine, or tangent of an angle with a reference angle of $30^o, 45^o$, or 60^o. ■ Describe patterns in and among the values of the sine, cosine, and tangent ratios for angles from 0° to 360°. ■ Sketch a diagram to represent a problem involving trigonometric ratios. ■ Solve a contextual problem, using trigonometric ratios.
11P.T.3.	Solve problems, using the cosine law and sine law, including the ambiguous case. [C, CN, PS, R, T]	■ Sketch a diagram to represent a problem that involves a triangle without a right angle. ■ Solve a non-right triangle using right triangle methods. ■ Explain the steps in a given proof of the sine law or cosine law. ■ Sketch a diagram and solve a contextual problem, using the cosine law. ■ Sketch a diagram and solve a contextual problem, using the sine law. ■ Describe and explain ambiguous case problems that may have no solution, one solution, or two solutions.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Grade 11 Pre-Calculus Mathematics

Strand: Relations and Functions	**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.

	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.R.1.	Factor polynomial expressions of the form ■ $ax^2 + bx + c, a \neq 0$ ■ $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$ ■ $a(f(x))^2 + b(f(x)) + c, a \neq 0$ ■ $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$ where a, b, and c are rational numbers. [ME, R]	■ Factor a polynomial expression that requires the identification of common factors. ■ Determine whether a binomial is a factor for a polynomial expression, and explain why or why not. ■ Factor a polynomial expression of the form ▪ $ax^2 + bx + c, a \neq 0$ ▪ $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$ ■ Factor a polynomial expression that has a quadratic pattern, including ▪ $a(f(x))^2 + b(f(x)) + c, a \neq 0$ ▪ $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Relations and Functions *(continued)*	**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.

	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.R.2.	Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. [C, PS, R, T, V]	■ Create a table of values for $y = \|f(x)\|$, given a table of values for $y = f(x)$. ■ Generalize a rule for writing absolute value functions in piecewise notation. ■ Sketch the graph of $y = \|f(x)\|$; state the intercepts, domain and range; and explain the strategy used. ■ Solve absolute value equations graphically, with or without technology. ■ Solve, algebraically, equations with a single absolute value, and verify the solution. ■ Explain why the absolute value equation $\|f(x)\| = a$, $a < 0$ has no solution. ■ Determine and correct errors in a solution to an absolute value equation. ■ Solve a problem that involves absolute value functions.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Relations and Functions *(continued)*	**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.

	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.R.3.	Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the ■ vertex ■ domain and range ■ direction of opening ■ axis of symmetry ■ x- and y-intercepts [C, CN, R, T, V]	■ Explain why a function given in the form $y = a(x - p)^2 + q$ is a quadratic function. ■ Compare the graphs of a set of functions of the form $y = ax^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of a. ■ Compare the graphs of a set of functions of the form $y = x^2 + q$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of q. ■ Compare the graphs of a set of functions of the form $y = (x - p)^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of p. ■ Determine the coordinates of the vertex for a quadratic function of the form $y = a(x - p)^2$, and verify with or without technology. ■ Generalize, using inductive reasoning, a rule for determining the coordinates of the vertex for quadratic functions of the form $y = a(x - p)^2 + q$. ■ Sketch the graph of $y = a(x - p)^2 + q$, using transformations, and identify the vertex, domain and range, direction of opening, axis of symmetry, and x- and y-intercepts. ■ Explain, using examples, how the values of a and q may be used to determine whether a quadratic function has zero, one, or two x-intercepts. ■ Write a quadratic function in the form $y = a(x - p)^2 + q$ for a graph or a set of characteristics of a graph.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Relations and Functions *(continued)*		**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.R.4.	Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including ■ vertex ■ domain and range ■ direction of opening ■ axis of symmetry ■ x- and y-intercepts [C, CN, PS, R, T, V]	■ Explain the reasoning for the process of completing the square, as shown in an example. ■ Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y = a(x - p)^2 - q$ by completing the square. ■ Identify, explain, and correct errors in an example of completing the square. ■ Determine the characteristics of a quadratic function given in the form $y = ax^2 + bx + c$, and explain the strategy used. ■ Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$. n Verify, with or without technology, that a quadratic function in the form $y = ax^2 + bx + c$ represents the same function as the quadratic function in the form $y = a(x - p)^2 + q$. ■ Write a quadratic function that models a situation, and explain any assumptions made. ■ Solve a problem, with or without technology, by analyzing a quadratic function.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Relations and Functions *(continued)*		**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11P.R.5.	Solve problems that involve quadratic equations. [C, CN, PS, R, T, V]	■ Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function, and the *x*-intercepts of the graph of the quadratic function.
		■ Derive the quadratic formula, using deductive reasoning.
		■ Solve a quadratic equation of the form $ax^2 + bx + c = 0$ by using strategies such as
		  ■ determining square roots
		  ■ factoring
		  ■ completing the square
		  ■ applying the quadratic formula
		  ■ graphing its corresponding function
		■ Select a method for solving a quadratic equation, justify the choice, and verify the solution.
		■ Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one, or no real roots; and relate the number of zeros to the graph of the corresponding quadratic function.
		■ Identify and correct errors in a solution to a quadratic equation.
		■ Solve a problem by determining or analyzing a quadratic equation.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Relations and Functions *(continued)*	**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.

	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.R.6.	Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables. [C, CN, PS, R, T, V] It is intended that the quadratic equations be limited to those that correspond to quadratic functions.	■ Model a situation, using a system of linear-quadratic or quadratic-quadratic equations. ■ Relate a system of linear-quadratic or quadratic-quadratic equations to the context of a problem. ■ Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations graphically, with technology. ■ Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations algebraically. ■ Explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations. ■ Explain, using examples, why a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two, or an infinite number of solutions. ■ Solve a problem that involves a system of linear-quadratic or quadratic-quadratic equations, and explain the strategy used.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Relations and Functions *(continued)*	**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.

	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.R.7.	Solve problems that involve linear and quadratic inequalities in two variables. [C, PS, T, V]	■ Explain, using examples, how test points can be used to determine the solution region that satisfies an inequality. ■ Explain, using examples, when a solid or broken line should be used in the solution for an inequality. ■ Sketch, with or without technology, the graph of a linear or quadratic inequality. ■ Solve a problem that involves a linear or quadratic inequality.
11P.R.8.	Solve problems that involve quadratic inequalities in one variable. [CN, PS, V]	■ Determine the solution of a quadratic inequality in one variable, using strategies such as case analysis, graphing the related function, roots and test points, or sign analysis; and explain the strategy used. ■ Represent and solve a problem that involves a quadratic inequality in one variable. ■ Interpret the solution to a problem that involves a quadratic inequality in one variable.
11P.R.9.	Analyze arithmetic sequences and series to solve problems. [C, CN, PS, R, T]	■ Identify the assumption(s) made when defining an arithmetic sequence or series. ■ Provide and justify an example of an arithmetic sequence. ■ Derive a rule for determining the general term of an arithmetic sequence. ■ Describe the relationship between arithmetic sequences and linear functions. ■ Determine the first term, the common difference, the number of terms, or the value of a specific term in a problem involving an arithmetic sequence. ■ Derive a rule for determining the sum of *n* terms of an arithmetic series. ■ Determine the first term, the common difference, the number of terms, or the value of the sum of specific numbers of terms in a problem involving an arithmetic series. ■ Solve a problem that involves an arithmetic sequence or series.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Relations and Functions *(continued)*		**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11P.R.10.	Analyze geometric sequences and series to solve problems. [C, CN, PS, R, T]	■ Identify assumptions made when identifying a geometric sequence or series. ■ Provide and justify an example of a geometric sequence. ■ Derive a rule for determining the general term of a geometric sequence. ■ Determine the first term, the common ratio, the number of terms, or the value of a specific term in a problem involving a geometric sequence. ■ Derive a rule for determining the sum of n terms of a geometric series. ■ Determine the first term, the common ratio, the number of terms, or the value of the sum of a specific number of terms in a problem involving a geometric series. ■ Generalize, using inductive reasoning, a rule for determining the sum of an infinite geometric series. ■ Explain why an infinite geometric series is convergent or divergent. ■ Solve a problem that involves a geometric sequence or series.
11P.R.11.	Graph and analyze reciprocal functions (limited to the reciprocal of linear and quadratic functions). [CN, R, T, V]	■ Compare the graph of $y = \frac{1}{f(x)}$ to the graph of $y = f(x)$. ■ Identify, given a function $f(x)$, values of x for which $y = \frac{1}{f(x)}$ will have vertical asymptotes; and describe their relationship to the non-permissible values of the related rational expression. ■ Graph, with or without technology, $y = \frac{1}{f(x)}$, given $y = f(x)$ as a function or a graph, and explain the strategies used. ■ Graph, with or without technology, $y = f(x)$, given $y = \frac{1}{f(x)}$ as a function or a graph, and explain the strategies used.