# Grade 8 Mathematics

Number

# Number and Shape and Space (Measurement)—8.N.1, 8.N.2, 8.SS.1

**Enduring Understandings:**

The square roots of perfect squares are rational numbers.

The square roots of non-perfect squares are irrational numbers.

Many geometric properties and attributes of shapes are related to measurement.

**General Learning Outcomes:**

Develop number sense.

Use direct or indirect measurement to solve problems.

| Specific Learning Outcome(s): | Achievement Indicators: |
| --- | --- |
| **8.N.1** Demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). [C, CN, R,V] | ➛ Represent a perfect square as a square region using materials, such as grid paper or square shapes.<br>➛ Determine the factors of a perfect square, and explain why one of the factors is the square root and the others are not.<br>➛ Determine whether or not a number is a perfect square using materials and strategies such as square shapes, grid paper, or prime factorization, and explain the reasoning.<br>➛ Determine the square root of a perfect square, and record it symbolically.<br>➛ Determine the square of a number. |
| **8.N.2** Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T] | ➛ Estimate the square root of a number that is not a perfect square using the roots of perfect squares as benchmarks.<br>➛ Approximate the square root of a number that is not a perfect square using technology (e.g., calculator, computer). |

*continued*

| Specific Learning Outcome(s): | Achievement Indicators: |
| --- | --- |
| | ➛ Explain why the square root of a number shown on a calculator may be an approximation.<br>➛ Identify a number with a square root that is between two given numbers. |
| **8.SS.1** Develop and apply the Pythagorean theorem to solve problems.<br>[CN, PS, R, T, V] | ➛ Model and explain the Pythagorean theorem concretely, pictorially, or by using technology.<br>➛ Explain, using examples, that the Pythagorean theorem applies only to right triangles.<br>➛ Determine whether or not a triangle is a right triangle by applying the Pythagorean theorem.<br>➛ Solve a problem that involves determining the measure of the third side of a right triangle, given the measures of the other two sides.<br>➛ Solve a problem that involves Pythagorean triples (e.g., 3, 4, 5 or 5, 12, 13). |

## Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of regular and irregular 2-D shapes by
  - recognizing that area is measured in square units
  - selecting and justifying referents for the units $cm^2$ or $m^2$
  - estimating area by using referents for $cm^2$ or $m^2$
  - determining and recording area ($cm^2$ or $m^2$)
  - constructing different rectangles for a given area ($cm^2$ or $m^2$) in order to demonstrate that many different rectangles may have the same area
- Solving problems involving 2-D shapes and 3-D objects
- Designing and constructing different rectangles given either perimeter or area, or both (whole numbers), and drawing conclusions
- Identifying and sorting quadrilaterals, including
  - rectangles

- squares
- trapezoids
- parallelograms
- rhombuses

according to their attributes

- Developing and applying a formula for determining the
  - perimeter of polygons
  - area of rectangles
  - volume of right rectangular prisms
- Constructing and comparing triangles, including
  - scalene
  - isosceles
  - equilateral
  - right
  - obtuse
  - acute

  in different orientations

## Background Information

### Squares and Square Roots

A *square* is a 2-dimensional (2-D) shape with all four sides equal.

The total area the square covers is measured in square units.

To determine the side length of a square when given the area, the square root must be determined.

A *perfect square* can be described as

- a square with whole number sides (e.g., 1 × 1, 2 × 2, 3 × 3)
- a number whose square root is an integer (e.g., $\sqrt{4}$ = 2 or –2)

A *non-perfect square* can be described as

- a square with non-whole number sides (e.g., 1.2 × 1.2)
- a number whose square root is not a whole number (e.g., $\sqrt{2}$)

Rounding is often used to determine the approximate square root of non-perfect squares.

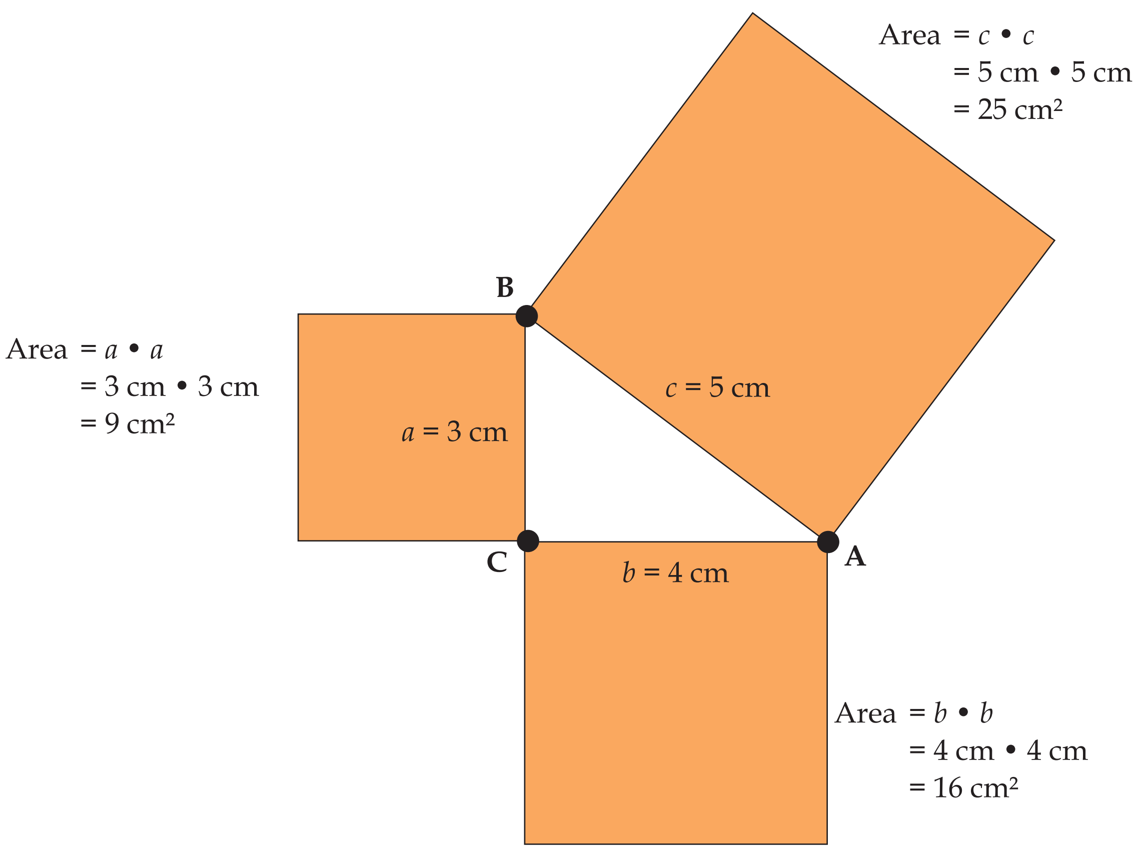
## Pythagorean Theorem

The *Pythagorean theorem* states that, in a right triangle,

- the sum of the squares of the legs is equal to the square of the length of the hypotenuse
- the sum of the area of the squares formed on the legs is equal to the area of the square formed on the hypotenuse
- if the lengths of the legs are $a$ and $b$ and the hypotenuse is $c$, then $a^2 + b^2 = c^2$

*Example:*

We can determine whether triangle ABC, shown below, is a right triangle by checking whether the Pythagorean relationship is present.




If $a^2 + b^2 = c^2$, then triangle ABC is a right angle triangle:

$$9\text{ cm}^2 + 16\text{ cm}^2 \stackrel{?}{=} 25\text{ cm}^2$$

$$25\text{ cm}^2 = 25\text{ cm}^2 \checkmark$$

Therefore, triangle ABC is a right angle triangle.

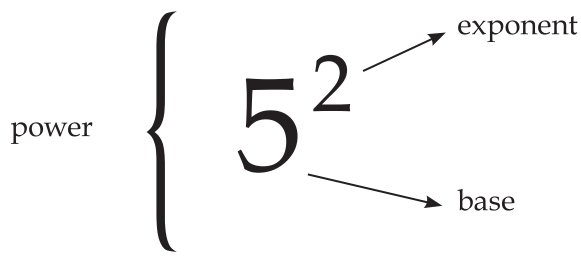
If you are given two of the values ($a$, $b$, or $c$) in a right angle triangle, you can determine the missing value by solving the equation.

### Powers

This is the first formal experience that students will have with powers. A simple introduction will be needed to explain the role of the exponent and the base.

- *Power*: A short-hand, symbolic representation of repeated multiplication (e.g., $5^2 = 5 \bullet 5$).
- *Base*: The factor in a power; what is being repeatedly multiplied (e.g., in $5^2$, 5 is the base).
- *Exponent*: The number in a power that tells how many factors there are; the number of factors in a repeated multiplication (e.g., in $5^2$, 2 is the exponent).

*Example:*




## MATHEMATICAL LANGUAGE

factors

hypotenuse

perfect square

prime factorization

prime numbers

Pythagorean relationship (Pythagorean theorem)

right triangles

square root

**Assessing Prior Knowledge**

**Materials**: Grid paper

**Organization**: Individual

**Procedure**:

1. Tell students that they will be learning about the Pythagorean theorem over the next few lessons; however, you first need to determine what they already know.
2. Provide students with grid paper and have them draw a square with the side length 5 cm.
3. Have students determine the area of the square they just drew. Ask them how they found the area (e.g., Did they count the squares? Did they multiply length times width?).
4. Ask students to determine the side length of a square that has an area of 64 $cm^2$. They may use the grid paper if they need it to determine the length.
5. Have students draw a right triangle with one leg 4 cm and one leg 3 cm.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❐ Determine the area of a square.
- ❐ Find the side length of a perfect square.
- ❐ Understand the property that determines right triangles (90° angle).
- ❐ Draw right triangles.

## Suggestions for Instruction

- **Represent a perfect square as a square region, using materials such as grid paper or square shapes.**
- **Determine the square of a number.**

**Materials:** BLM 5–8.9: Centimetre Grid Paper, straightedge, math journals, BLM 8.N.1.1: Determining Squares, calculators

**Organization:** Individual/whole class

**Procedure:**

1. Tell students that by the end of this lesson they will understand squares and square roots of perfect squares.
2. Each student needs a piece of graph paper (or see BLM 5–8.9: Centimetre Grid Paper).
3. Ask students to explain the difference between a square and a rectangle. (A *square* is a special kind of rectangle in that all four sides are equal.)
4. Ask students to draw a 2 × 2 square and label the length and width of the square 2 × 2. Then have them count the units within the boundary of the drawn square (the area) and write that amount inside the square.
5. Ask students to repeat the above, making a 3 × 3 square, a 4 × 4 square, and a 5 × 5 square.
6. Ask students to explain, in their math journals, any connections they have noticed between the length and width of the square and the area of the square. (Explanations can include words, pictures, diagrams, symbols, and so on.) The hope is that students will notice that the area is determined by multiplying the length by the width and that the two factors are always the same (e.g., 2 × 2, 3 × 3, 4 × 4, 5 × 5).
7. Ask students whether they have ever seen another way to write 2 × 2, 3 × 3, 4 × 4, 5 × 5, and so on. This question will generate discussion on saying 2 squared or $2^2$, and so on.
8. Have a brief class discussion about powers, bases, and exponents to ensure students understand the process of, and the symbolic notation for, squaring a number.
9. Provide students with copies of BLM 8.N.1.1: Determining Squares and ask them to determine the squares of a variety of numbers with or without the use of a calculator.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the square of a number, given a picture.
- ❒ Determine the square of a number symbolically.
- ❒ Use mental mathematics strategies and number sense to determine the square of a number.

## Suggestions for Instruction

- **Determine the square root of a perfect square, and record it symbolically.**

**Materials:** Square tiles, calculators, graph paper, math journals, BLM 8.N.1.2: Determining Square Roots

**Organization:** Pairs/whole class/individual

**Procedure:**

1. Give students square tiles, and allow them a few minutes to explore the manipulatives.
2. Ask students to make squares with varying side lengths. Have them record the side lengths and areas of the squares in a table.
3. Record class data to form a table of side lengths and areas for squares of lengths 1 to 15.
4. Ask students to add to the table a third column titled Area as Power, and have them record the areas of the first 15 squares as powers.

*Example:*

| Side Length (units) | Area (units$^2$) | Area as Power |
|---|---|---|
| 1 | 1 | $1^2$ |
| 2 | 4 | $2^2$ |

5. Ask students whether they have ever heard the term *square root*. Generate discussion, using questions such as the following:
   - What is the relationship between the area of a square and the length of its side?
   - We just learned how to square a number. What do you think *square root* could be?
   - What does the word *root* mean?
6. As a class review, remind students that when they find the square of a number, they are finding the area of a square, which can be written as $2^2$, and so on.
7. Determining the square root of a number is determining what number was multiplied by itself to get the square.

   *Examples:*
   - If the square of a number is 4, what number multiplied by itself is 4?
     Answer: 2
   - If the square of a number is 16, what number multiplied by itself is 16?
     Answer: 4

   Since the square of a number is actually the area of the square, the square root of a number is the same as finding the length and/or width of the square.
8. Ask students to add to the table a fourth column titled Side Length as Square Root, and have them record the side lengths of the first 15 squares as square roots.

   *Example:*

| Side Length (units) | Area (units$^2$) | Area as Power | Side Length as Square Root |
|---|---|---|---|
| 1 | 1 | $1^2$ | $\sqrt{1}$ |
| 2 | 4 | $2^2$ | $\sqrt{4}$ |

9. Have students brainstorm, in pairs, what patterns they see in the table. Discuss these patterns as a class.
10. Demonstrate to students the square root button on a calculator so that they can find the square roots of numbers. If they do not have a square root button, brainstorm ideas about how they can use a calculator to determine the square root (guess and check).
11. Have students individually complete BLM 8.N.1.2: Determining Square Roots.
12. Have students insert a piece of graph paper into their math journals, and ask them to represent 81 as a square region on the grid. Check whether they have drawn a 9 × 9 square. Ask them to identify the square and square root of the square region.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the square root of a number, given a model.
- ❒ Determine the square root of a number symbolically.
- ❒ Use mental mathematics strategies and number sense to determine the square root of a number.

## Suggestions for Instruction

- **Determine the factors of a perfect square, and explain why one of the factors is the square root and the others are not.**
- **Determine whether or not a number is a perfect square using materials and strategies such as square shapes, grid paper, or prime factorization, and explain the reasoning.**
- **Determine the square root of a perfect square and record it symbolically.**
- **Estimate the square root of a number that is not a perfect square using the roots of perfect squares as benchmarks.**
- **Approximate the square root of a number that is not a perfect square using technology (e.g., calculator, computer).**

**Materials:** Square paper cut into individual squares (BLM 5–8.6: Blank Hundred Squares or BLM 5–8.9: Centimetre Grid Paper), math journals, calculator or computer

**Organization:** Individual/pairs/whole class

**Procedure:**

*Part A*

1. Give each pair of students a handful of square paper tiles. (Make sure some groups end up with an amount that forms a perfect square and some end up with an amount that does not.)
2. Have students make a square using all the tiles they received. (Students may need to be encouraged to use parts of squares. For example, if a group is given 10 tiles, students would use nine whole tiles and divide the tenth tile to create a square whose sides are slightly larger than three tiles.)
3. Make a class chart stating the area of the square and the length of the side.
4. Have students make observations of what is happening
5. Take pictures of students' squares or have students tape these down for use in Part F.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand that the space covered by a shape is its area.
- ❒ Recognize that a square is a quadrilateral with four equal sides that meet at right angles.
- ❒ Make the connection between having no squares left over and having a perfect square.

*Part B*

1. Tell students that they will learn two more ways, other than using a calculator, to determine the square root of a perfect square.
2. List all the factors of 36 (a perfect square). When students are listing the factors, observe whether they notice a way that will help them to find the square root.

   *Examples:*

   - If students list the factors forming a rectangle, it is easier for them to identify the square root.

     1, 36

     2, 18

     3, 12

     4, 9

     6

     6 is listed only once because it is multiplied by itself to get 36.

     6 is the square root of 36.

     OR

   - If students list the factors in order from lowest to highest, the square root is always the median number.

     1, 2, 3, 4, 6, 9, 12, 18, 36

3. Have students list all the factors of a few more perfect squares to identify the square root of those squares.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the factors of a number.
- ❒ Use factors to determine the square root of a number.
- ❒ Apply mental mathematics and number sense strategies to determine the square root of a number.

*Part C*

1. Ask students to write down the following numbers and categorize them as perfect squares or non-perfect squares: 16, 25, 36, 27, 32, 42.
2. Review with students what a *prime number* is and what a *factor* is. Demonstrate the prime factorization of several numbers.
3. Model, using 16 as an example, how to write numbers as the product of prime factors (2 • 2 • 2 • 2). Have students write the next five numbers as the product of prime factors.
4. After ensuring that all students have the correct response, use a Think-Pair-Share strategy by having students individually examine the prime factorization to determine whether there is any pattern or rule that can be determined, and then having them share it with a partner. Elicit answers from the class to form a general rule. (Perfect squares have an even number of prime factors.)

   *Examples:*

   - Written as a product of prime numbers, 36 is 2 • 2 • 3 • 3. There are two 2s and two 3s, so 36 must be a perfect square.
   - Written as a product of prime numbers, 27 is 3 • 3 • 3. There are three 3s (an uneven number of 3s in the product of prime numbers), so 27 must be a non-perfect square.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the prime factors of a number.
- ❒ Develop an understanding of using prime factors to determine the square root of a number.
- ❒ Apply mental mathematics and number sense strategies to determine the square root of a number.

*Part D*

1. Using the number 36 from the previous learning activity, have students determine the square root of a perfect square. Write 36 as a product of prime factors on the whiteboard.
2. Have students divide the factors 2 • 2 • 3 • 3 into two equal groups (2 • 3 and 2 • 3). Ask them to multiply the two groups and see what number they end up with.
3. Ask students to write 144 as a product of prime factors.
4. Have students separate the prime factors into two equal groups and multiply. Record their results to determine the square root of 144.
5. Ask students whether they can determine the square root of numbers in another way.
6. Discuss students' responses to generate other ways to determine the square root of perfect squares.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the square root of perfect squares using prime factorization.
- ❒ Develop a personal strategy for determining square roots.

*Part E*

1. Have students select one of the following and complete a math journal entry with words, pictures, tables, symbols, and so on.
   - Explain how prime factorization (or factoring) can be used to determine whether a number is a perfect square. Provide examples to help support your response.
   - Select a number that is a perfect square and a number that is not. Select a method for proving that the number is or is not a perfect square. Describe why you chose the method you did.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the square root of perfect squares.
- ❒ Use materials, diagrams, and symbols to determine the square root of a number.
- ❒ Develop a personal strategy for determining square roots.

*Part F*

1. Using the squares created in Part A, have students work in pairs to discuss the difference between squares made from a perfect square number of paper tiles and those made from a number of tiles that is not a perfect square (e.g., 25 squares vs. 30 squares).
2. Facilitate a class discussion about the square roots of numbers that are not perfect squares.
3. Have students work in pairs to estimate the square roots of each group's square and then check responses using technology.
4. Discuss as a class the strategies that students used to determine the square roots of numbers that are not perfect squares.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use number sense to approximate the square root of numbers that are not perfect squares.
- ❒ Reason mathematically.

## Suggestions for Instruction

- **Estimate the square root of a number that is not a perfect square using the roots of perfect squares as benchmarks.**
- **Approximate the square root of a number that is not a perfect square using technology (e.g., calculator, computer).**
- **Explain why the square root of a number shown on a calculator may be an approximation.**
- **Identify a number with a square root that is between two given numbers.**

**Materials:** Math journals

**Organization:** Small group/whole class

**Procedure:**

1. Tell students that they will be learning how to identify and explain the approximate square root of non-perfect squares.
2. Draw a number line and write the numbers 25 to 36 above it.
3. Underneath the line, have students write the square root of 25 and 36 directly under the numbers. (They are setting the benchmarks for the next step.)
4. Working in small groups, students estimate the square root of 30 and explain their thinking. Have a reporter from each group present to the class the group's decision. It should fall between 5 and 6 (around 5.5) since 30 is approximately midway between 25 and 36.
5. In their groups, students estimate the square root of 54 using benchmarks and justify their responses.
6. As students are working in their groups, listen to the dialogue to observe the level of understanding students have. Record your observations.
7. You may need to repeat this process a few more times with non-perfect squares. Then ask students questions such as the following:
   - Why are the numbers you have been finding the square roots of non-perfect squares?
   - Why do we find the approximate square roots of them?
   - Can you determine an exact value with your calculator? The numbers are non-repeating, non-terminating. Calculators will round the number based on the space available on the screen.
8. Ask students to respond to the following in their math journals:
   - Based on your understanding of perfect and non-perfect squares and their square roots, identify a number that will have a square root between 4 and 5.
   - Use diagrams, tables, materials, symbols, words, and/or numbers to justify your choice.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use number sense to approximate the square root of a non-perfect square.
- ❒ Explain why the square root of a non-perfect square may be an approximation.
- ❒ Reason mathematically.

- **Identify a number with a square root that is between two given numbers.**
- **Determine the square root of a perfect square, and record it symbolically.**
- **Determine the square of a number.**
- **Estimate the square root of a number that is not a perfect square using the roots of perfect squares as benchmarks.**

**Materials:** BLM 8.N.1.3: I Have . . . , Who Has . . . ?

**Organization:** Whole class/small group

**Procedure:**

1. Tell students that they will be playing a square root version of the game, I Have . . . , Who Has . . . ? (see BLM 8.N.1.3). Explain that each student will get one card (some students will need to have more cards if there are fewer than 30 students in the class). One student will start the game by reading his or her card, and the person who has the answer to the question posed by the student reads his or her card. Play continues in this fashion until it gets back to the person who started the game.
2. After students have played the game several times, have them make their own square root game and play it with the other members of the class.

**Variation:** Have students work in groups of two or three, giving them several cards. Play the game as described in the procedure above. This variation gives students the opportunity to engage with more than one card.

**Observation Checklist**

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Determine the square of a number.
- ❒ Determine the square root of a perfect square.
- ❒ Determine the square root of a non-perfect square.

## Suggestions for Instruction

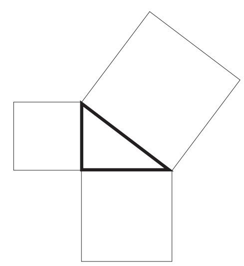
- **Model and explain the Pythagorean theorem concretely, pictorially, or by using technology.**
- **Explain, using examples, that the Pythagorean theorem applies only to right triangles.**
- **Determine whether or not a triangle is a right triangle by applying the Pythagorean theorem.**
- **Solve a problem that involves Pythagorean triples (e.g., 3, 4, 5 or 5, 12, 13).**

**Materials:** Two colours of 1 cm grid paper (BLM 5–8.9: Centimetre Grid Paper), ruler, sharp pencil, scissors

**Organization:** Small group

**Procedure:**

1. Tell students that they are going to determine a unique relationship between the legs of a right triangle and the hypotenuse of the right triangle. They will be using their understanding of squares and square roots.
2. Draw a right triangle and label the triangle so that students understand what the legs of the triangle are and what the hypotenuse is.
3. Have students form small groups, and provide them with the following instructions:



   - Using one colour of grid paper, draw a right triangle with one leg 3 cm and one leg 4 cm.
   - Using the other colour of grid paper, cut out squares large enough to fit along the edge of each of the legs and the hypotenuse.
   - Count the area of each square, combining partial squares to make whole squares.
   - Describe what relationship you see between the areas of the squares.
   - Try the procedure again to see if your theory is correct. This time, draw a right triangle with one leg 6 cm and the other leg 8 cm.
   - Try it one more time, this time with one leg 9 units and the other leg 12 units. (At this point, the small groups should have noticed that the sum of the areas of the squares off the legs is equal to the area of the square off the hypotenuse.)
4. Work with students to help them express their thoughts using mathematical language and symbols. Lead them to the generalization that $a^2 + b^2 = c^2$.
5. Have students draw other non-right angle triangles (e.g., 4 cm, 5 cm, 8 cm or 4 cm, 6 cm, 7 cm). Discuss with students what they notice. Lead them to the generalization that the Pythagorean theorem applies only to right triangles.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Explain the Pythagorean theorem using pictures.
- ❒ Demonstrate that the Pythagorean theorem applies only to right triangles.

## Suggestions for Instruction

- **Solve a problem that involves determining the measure of the third side of a right triangle, given the measures of the other two sides.**

**Materials:** Two colours of 1 cm grid paper (BLM 5–8.9: Centimetre Grid Paper), calculator, ruler, scissors

**Organization:** Pairs/whole class

**Procedure:**

1. Provide students with several triangles from which the length of the hypotenuse is missing.
2. Have students work in pairs to develop a procedure for determining the length of that missing side.
3. Discuss various procedures as a class.
4. If the symbolic procedure does not come up in the discussion, demonstrate this procedure for the class and allow students to practise determining the measure of the hypotenuse.
5. Repeat steps 1 to 4 for triangles from which the length of one of the legs is missing.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply prior knowledge of squares and square roots.
- ❒ Develop a procedure for determining the measure of a missing side in a right angle triangle.
- ❒ Apply a procedure for determining the measure of a missing side in a right angle triangle.

## Suggestions for Instruction

- **Determine whether or not a triangle is a right triangle by applying the Pythagorean theorem.**
- **Solve a problem that involves determining the measure of the third side of a right triangle, given the measures of the other two sides.**

**Materials:** BLM 8.N.1.4: Pythagorean Theorem, calculator

**Organization:** Individual/pairs

**Procedure:**

1. Provide students with a copy of BLM 8.N.1.4: Pythagorean Theorem.
2. Ask students to complete the BLM. Have them check with a learning partner if they experience difficulty.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the measure of a missing side in a right angle triangle.
- ❒ Apply the Pythagorean theorem to determine whether a triangle is a right triangle.

## Putting the Pieces Together

### Pythagorean Theorem in Real Life

**Introduction:**

Students will have the opportunity to apply the Pythagorean theorem to everyday situations by solving everyday problems.

**Purpose:**

Students should have a good understanding of the Pythagorean theorem before being assigned this task. They will need to know how to create equivalent proportions to create a scale model of the scenarios provided below. In addition, they will need to know how to convert metres to centimetres.

**Curricular Links:** Art, English Language Arts (ELA)

**Materials/Resources:** Various art supplies

**Organization:** Individual or small group

If this is an individual learning activity, students can choose one of the following scenarios. Choice allows for adaptations and individual interests.

If this is a group learning activity, each member is responsible for one of the scenarios; however, students can receive assistance from their group members.

**Inquiry:**

Students will create a scaled version of one of the following scenarios. They must make the selected scenario appear realistic. They will create the appropriate length of the hypotenuse of the chosen scenario to the nearest tenth. They must complete a one-page explanation of all their mathematical work, including an explanation of how they created their scaled version and how they used the Pythagorean theorem to determine the lengths/distances.

**Scenarios:**

1. Fire fighters are called to an apartment fire. A family is trapped on the second floor. Fire fighters need to rescue the family using their extension ladder. The second-floor apartment is 3 metres from the ground. There are shrubs and a sidewalk jutting out 2 metres from the building. How long must the extension ladder be to reach the second-floor apartment?
2. Joey is trying out as a catcher for the local baseball team. Part of his evaluation involves showing how quickly he can throw the ball to second base. How far does he need to throw the ball if the bases are 90 feet apart?
3. Sam is starting his first day at a new job at a moving company. The truck box he is driving is 2 metres high. The end of the ramp is on the ground 4 metres from the back of the truck. How long is Sam's ramp?

**Assessment:**

The following rubric can be used to assess achievement of the mathematics learning outcomes.

| Pythagorean Theorem in Real Life—Assessment | | | | |
|---|---|---|---|---|
| Criteria | Meeting Expectations | Developing to Meet Expectations | Beginning to Meet Expectations | Incomplete |
| The student | | | | |
| ■ applies the Pythagorean theorem to solve problems | ❒ provides a clear explanation of how the Pythagorean theorem was used to determine the length/distance | ❒ provides a general explanation of how the Pythagorean theorem was used to determine the length/distance | ❒ provides a vague or minimal explanation of how the Pythagorean theorem was used to determine length/distance | ❒ provides no explanation of how the Pythagorean theorem was used to determine length/distance |
| ■ solves problems that involve proportions | ❒ provides a clear explanation of how he or she determined the scale model for the scenario | ❒ provides a general explanation of how he or she determined the scale model for the scenario | ❒ provides a vague or minimal explanation of how he or she determined the scale model for the scenario | ❒ provides no explanation of how he or she determined the scale model for the scenario |

**Extension:**

Students could research other ways in which the Pythagorean theorem is used in everyday life and prepare a presentation on what they have learned. They could present their findings in the form of a demonstration, PowerPoint presentation, video, and so on.

# NOTES

# **Number**—8.N.3

**Enduring Understandings:**

Percents can be thought of as a ratio comparing to 100 or a fraction out of 100.

Percents can range from 0 to higher than 100.

Percents, fractions, decimals, and ratios are different representations of the same quantity.

Percents have the same value as their fraction, decimal, and ratio equivalent, and this can be useful in solving problems with percents.

**General Learning Outcome:**

Develop number sense.

| **Specific Learning Outcome(s):** | **Achievement Indicators:** |
|---|---|
| **8.N.3** Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V] | ➛ Provide a context where a percent may be more than 100% or between 0% and 1%.<br>➛ Represent a fractional percent using grid paper.<br>➛ Represent a percent greater than 100% using grid paper.<br>➛ Determine the percent represented by a shaded region on a grid, and record it in decimal, fractional, or percent form.<br>➛ Express a percent in decimal or fractional form.<br>➛ Express a decimal in percent or fractional form.<br>➛ Express a fraction in decimal or percent form.<br>➛ Solve a problem involving percents.<br>➛ Solve a problem involving combined percents (e.g., addition of percents, such as GST + PST).<br>➛ Solve a problem that involves finding the percent of a percent (e.g., A population increased by 10% one year and then increased by 15% the next year. Explain why there was not a 25% increase in population over the two years.). |

## Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of fractions by using concrete and pictorial representations to
  - create sets of equivalent fractions
  - compare fractions with like and unlike denominators
- Describing and representing decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically
- Relating decimals to fractions (tenths, hundredths, thousandths)
- Comparing and ordering decimals (tenths, hundredths, thousandths) by using
  - benchmarks
  - place value
  - equivalent decimals
- Demonstrating an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically
- Solving problems involving percents from 1% to 100%
- Demonstrating an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions

## Background Information

People regularly encounter practical situations requiring them to understand and solve problems related to percent. These situations include problems related to sports statistics, price discounts, price increases, taxes, polls, social changes and trends, and the likelihood of precipitation. The media provide sources of contextual data for problems involving percent.

Learning outcome 8.N.3 builds on understandings related to fractions, ratios, decimals, percents, and problem solving that students developed in previous grades.

### Fractions and Decimals

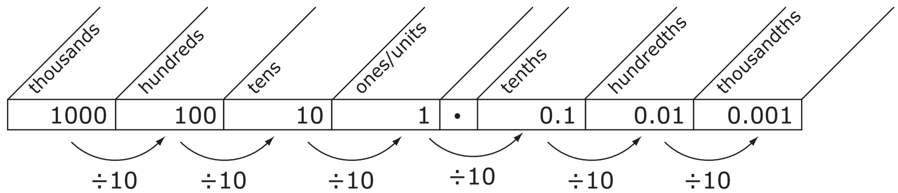
Before students become skilful at solving problems involving percent, they must have a strong conceptual understanding of fractions and decimals.

The term *fraction* has several meanings. An expert blends and separates these meanings for convenience, but this blending can confuse students who lack fluency in applying the different meanings of fraction. *Fraction notation* is used to represent a “cut” or part of a unit, a part of a group or set, a measure or point on a number line, a ratio, and a portion of a turn, and to indicate the division operation.

*Decimals* are a convenient way to represent fractional quantities using a place value system. Fractions may be converted to decimals by dividing the numerator by the denominator, or by finding an equivalent fraction with a denominator of 100.

A *decimal point* separates whole units from parts of units. Each position to the right of the decimal represents a tenth part of one of the previous units. The first position following the decimal represents a tenth part of one whole unit, and the second place represents a tenth part of a tenth or a hundredth part of one unit.

*Example:*




## Problems Involving Percent

When translating standard notation to percent, the decimal point indicates where to read the hundredths in a number. The word *percent* means per hundred and may be substituted for the word *hundredths* when reading a number. Therefore, $\frac{7}{100}$ or 0.07 may be read as 7 hundredths and also as 7 percent.

Percent may also be used to represent fractional quantities that are a little larger than a hundredth. Each successive place value position represents one of the previous units cut into 10. For example, the third position represents a tenth of a hundredth part of a unit, or a thousandth part of one unit.

Our understanding of place value allows us to express any number as a number of selected units. Just as 141 can represent 14 tens and 1 one, 0.141 represents a number that is a little larger than one tenth of one whole. It may be expressed as 1.41 tenths, 14.1 hundredths, or 141 thousandths. When substituting the word *percent* as another word for hundredth, the decimal number 0.141 may be read as 14.1 hundredths, or 14.1 percent (%).

In Grade 8, students need to work with numbers from 0% to 1%, from 1% to 100%, and percents greater than 100%, including all fractional percents (e.g., $\frac{1}{4}$%, $35\frac{1}{2}$%, and $225\frac{3}{4}$%).

With these various understandings of percent, students have multiple approaches to solving problems that involve percent:

- To find 25% of 80, students may think of the equivalent fraction $\frac{1}{4}$, and then find $\frac{1}{4}$ of 80.

  80 divided by 4 is 20, so 25% of 80 is 20.

- To find 0.1% of 200, students may use their understanding of place value to determine the value.

  100% of 200 is 200.

  10% of 200 is 200 ÷ 10, which is 20.

  1% of 200 is 200 ÷ 100, which is 2.0.

  0.1% of 200 is 200 ÷ 1000, which is 0.2.
- To find 120% of 40, students may use the knowledge that 120% is equivalent to 1.20, and so 120% of 40 is the same as 1.20 • 40, or 48.

Choosing numbers that are easy to work with will enable students to concentrate on the processes involved rather than on the arithmetic.

Where possible, use mental mathematics and the distributive property to find percents:

- Think of 35% as 25% + 10%. In the first problem above, 25% of 80 is 20, 10% of 80 is 8, and 20 + 8 = 28, so 35% of 80 is 28.
- To extend the second problem above, show how the distributive property is used for fractional percents. Think of how to determine 0.2% of 200. Think of 0.2% as 0.1% + 0.1%. Since 0.1% of 200 is 0.2, 0.2% of 200 would be 0.2 + 0.2 = 0.4.
- For the third problem above, think of 120% as 100% + 20%. 100% of 40 is 40 and 20% of 40 is 8 (the double of 10% of 40).

Students need to have a strong conceptual understanding of percent. Always start with hands-on activities to provide opportunities for students to develop that conceptual understanding. When converting fractional percents and percents greater than 100%, start with what students should already know—how to convert whole number percents less than 100% from percent to decimal form. Then, students can apply those same skills to converting fractional percents and percents greater than 100% to decimals and fractions. If students convert from percents to decimals first, they can then write the decimal as a fraction and simplify.

*Example:*

> To change a percent to a decimal, you divide by 100 (since percent means out of 100). This is true of fractional percents. For example, $85\frac{1}{2}$% can be written in decimal form by thinking of it as a decimal percent (85.5%) and then showing this in decimal representation (85.5% ÷ 100% = 0.855).
>
> **Note:** Students often struggle with fractional percents because they see a fraction or decimal (e.g., $15\frac{1}{2}$% or 22.75%) and already think the percent is a fraction or decimal. To clear up misconceptions, ask students whether they see the percent (%) sign. If yes, the number is still in percent form.

When combining percents, use various methods to solve the problems. It is important for students to be aware of the various methods but also know that one way is not better than the other. There may be fewer steps using one method over the other, but, depending on the learning style of the student, a longer method may be necessary. The ultimate goal is efficiency, which means the student is able to get accurate answers consistently and productively, using methods that the student understands.

## Mathematical Language

combined percent
decimal
fraction
fractional percent
percent
GST
PST

## Learning Experiences

**Assessing Prior Knowledge**

**Materials**: BLM 8.N.3.1: Percent Pre-Assessment, BLM 8.N.3.2: Percent Self-Assessment

**Organization**: Individual

**Procedure**:

1. Tell students that you need to find out what they already know about percents and that they are expected to do the best they can. Hand out BLM 8.N.3.1: Percent Pre-Assessment.
2. Have students work individually to complete the pre-assessment.
3. Assess students' work and provide descriptive feedback (rather than a mark) for the work they completed. Return each student's work.
4. Have students complete the Before Instruction column of BLM 8.N.3.2: Percent Self-Assessment prior to any instruction on percents.
5. At the end of the unit, have students complete the After Instruction column of BLM 8.N.3.2.

**Note**:

- This assessment is not to be used for marks but to obtain a benchmark of what students already understand about percents. This will help guide you to make instructional decisions and plan for students' individual needs.
- Have students complete BLM 8.N.3.1: Percent Pre-Assessment throughout the sequence of learning experiences related to percents. It is important for students to be aware of their learning progress.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Represent fractional percents between 1% and 100% on a hundreds grid.
- ❒ Convert fractions to decimals and percents.
- ❒ Convert decimals to fractions and percents.
- ❒ Convert percents to fractions and decimals.
- ❒ Solve problems involving percents.

## Learning Experiences

### Suggestions for Instruction

- **Represent a fractional percent using grid paper.**
- **Represent a percent greater than 100% using grid paper.**
- **Determine the percent represented by a shaded region on a grid, and record it in decimal, fractional, or percent form.**

**Materials:** Graph paper, hundred grids (BLM 5–8.6: Blank Hundred Squares) or BLM 5–8.10: Base-Ten Grid Paper, BLM 8.N.3.3: Percent Grids

**Organization:** Individual

**Procedure:**

1. Tell students that by the end of this lesson they will be able to represent percents on graph paper/hundred grids and record percents from graph paper/hundred grids.
2. Review with students the definition of percent. *Percent* means out of each hundred.
3. Ask students the following questions:
   - How many squares are on the 10 × 10 graph paper or on the hundred grid? (100 squares)
   - How many squares would need to be shaded to represent 100%? (Shade in all squares.)
   - What is the value of one shaded square? (1%)
   - How would you represent 43%? (Shade in 43 squares.)

- How would you represent 74%? (Shade in 74 squares.)
- How would you represent $23\frac{1}{2}$%? (Shade in 23 full squares and $\frac{1}{2}$ of the 24th square.)
- How would you represent 150%? (Shade in one full grid and 50 squares on a second grid.)
- What does 150% mean to you? (Various answers—greater than 1)
- How would you represent $\frac{1}{2}$%? (Shade in half of one square.)
- How would you represent $\frac{3}{4}$%? ($\frac{3}{4}$ is less than one percent, so use only one square of a hundred grid. Divide it into four equal parts. Shade in three squares.)
- How would you represent 0.125%? (0.125% is the same as $\frac{1}{8}$%. $\frac{1}{8}$% is less than 1%, so use only one square of a hundred grid. Divide it into eight equal parts. Shade in one square.)

4. Provide students with graph paper or hundred grids. Ask them to represent the following fractions on their grids: 45%, 230%, $17\frac{2}{3}$%, and 0.2%.
5. Show each of the percent grids from BLM 8.N.3.3: Percent Grids, one at a time. Have students write the percent represented by each grid on an individual whiteboard and ask them to show you their responses.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❐ Represent percents between 0% and 1%.
- ❐ Represent fractional percents less than 100%.
- ❐ Represent percents greater than 100% on graph paper or grids.
- ❐ Determine percents between 0% and 1%.
- ❐ Determine fractional percents less than 100%.
- ❐ Determine percents greater than 100% on graph paper or grids.

- **Provide a context where a percent may be more than 100% or between 0% and 1%.**

**Materials:** BLM 8.N.3.4: Percent Scenarios

**Organization:** Small group/whole class

**Procedure:**

1. Have students form small groups, and provide each group with a copy of BLM 8.N.3.4: Percent Scenarios.
2. Ask students to explain what the scenario statements mean and give reasons for their explanations. One presenter from each group then presents the group's explanation to the class.
3. Record ideas on a whiteboard or an overhead and work toward reaching consensus about the meaning of students' explanations.
4. Ask students to discuss fractional percents and percents greater than 100% with members of their household. Ask them to come to class the next day prepared to share a new real-world example from each category.
5. Have students share their examples, and see whether they are consistent with the meanings that students discussed in the previous class.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a scenario to represent percents between 0% and 1%.
- ❒ Describe a scenario to represent fractional percents greater than 100%.

## Suggestions for Instruction

- **Express a percent in decimal or fractional form.**
- **Express a decimal in percent or fractional form.**
- **Express a fraction in decimal or percent form.**

**Materials**: Small whiteboards, margarine lids (or any other tool that students can use to record their answers and then quickly wipe off their responses in preparation for a new problem)

**Organization**: Whole class

**Procedure:**

1. Tell students that in this lesson they will learn how to represent fractions, decimals, and percents when the percents are fractional percents and when the percents are greater than 100%. They will use what they know from Grade 7 Mathematics (representing fractions, decimals, and percents) and extend it to new percents.
2. Ask students to write 75% on their whiteboard. Then ask them to write 75% as a decimal and a fraction.
3. Ask students what they did to go from the percent to the decimal.
4. Ask students what they did to go from the percent to the fraction.
5. Continue with whole-number percents ranging from 1% to 100% until the class has a solid foundation for how to convert percents to decimals and fractions. The procedures they describe here could be written on the board for reference.
6. Ask students to write 125% on their whiteboard. Then ask them to write 125% as a fraction and a decimal and explain how they did it.
7. Ask students whether they think their answer is reasonable (1.25 and $1\frac{1}{4}$ based on their understanding of percents and decimals).
8. Ask students to write $45\frac{1}{2}$ % on their whiteboard. Then ask them to write $45\frac{1}{2}$ % as a fraction and a decimal and explain how they did it.

   **Note:** For percents that contain fractional parts, students may experience some difficulty, as they may believe that the percent is already a fraction. It may help to get students to express these as a decimal percent (45.5%).
9. Ask students whether they think their answer is reasonable.
10. Ask students to write $\frac{3}{4}$ % on their whiteboard. Then ask them to write $\frac{3}{4}$ % as a fraction and a decimal and explain how they did it.
11. Ask students whether they think their answer is reasonable. (1% would be $\frac{4}{400}$. $\frac{3}{4}$ % is a little smaller than 1%, and $\frac{3}{400}$ is just a little less than $\frac{4}{400}$, so the answer is reasonable.)

12. Continue to provide students with similar questions until they are demonstrating an understanding of the concept.
13. Repeat the above steps
    - with decimals (converting to fractions and percents)
    - with fractions (converting to decimals and percents)

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:
- ❒ Convert fractions to decimals and percents.
- ❒ Convert decimals to fractions and percents.
- ❒ Convert percents to decimals and fractions.
- ❒ Use mathematical reasoning to determine the reasonableness of their answers.

## Suggestions for Instruction

- **Solve a problem involving percents.**

**Materials:** Chart paper, markers, BLM 8.N.3.5: Percent Savings

**Organization:** Small group/whole class

**Procedure:**

1. Tell students that in this lesson they will learn how to solve problems involving percents.
2. Arrange students in groups of three or four, and assign one problem from BLM 8.N.3.5: Percent Savings to each group.
3. Students work together to solve the problem, showing their work on chart paper.
4. Each group presents the solution to their problem to the class. The problems are all similar, so, by the end, the class should have 10 exemplars that can be posted in the classroom for determining the percent saving.
5. Have students make up their own problems involving percents. They may switch questions with other groups and solve them using the solution(s) discussed in class.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply problem-solving skills to determine an appropriate method to solve a problem with percents.
- ❒ Communicate mathematically within their group and with the class.
- ❒ Solve a problem involving percents.

## Suggestions for Instruction

- **Solve a problem involving combined percents (e.g., addition of percent, such as GST + PST).**

**Materials:** Chart paper, markers, BLM 8.N.3.6: Final Cost

**Organization:** Small group/whole class

**Procedure:**

1. Arrange students in groups of three or four again (these groupings may be the same as or different from those of the previous learning experience). Assign each group one of the problems from BLM 8.N.3.6: Final Cost. Students must work together to solve the problem, showing their work on chart paper.
2. Each group presents the solution to their assigned problem to the class. The problems are all similar, so, by the end, the class should have 10 exemplars that can be posted in the classroom for determining how to combine percents.

   **Note:** If all the groups solve the problem the same way, they will have to show other methods of solving problems for combining percents. For example, you can calculate the percents separately, combine the tax percents first (PST + GST = 13%, then add that to the cost of an item), or combine the cost and tax percents (100% represents the cost of an item, 8% for PST and 5% of GST = 100% + 8% + 5% = 113%).

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply problem-solving skills to determine an appropriate method to solve a problem with combined percents.
- ❒ Communicate mathematically within their group and with the class.
- ❒ Solve a problem involving combined percents.

## Suggestions for Instruction

- **Solve a problem that involves finding the percent of a percent (e.g., A population increased by 10% one year and then increased by 15% the next year. Explain why this was not a 25% increase in population over the two years.).**

**Materials:** Chart paper, markers, BLM 8.N.3.7: Percent Increase and Decrease

**Organization:** Small group/whole class

**Procedure:**

1. Tell students that they will determine whether a percent decrease (or increase) one time, and then an additional percent decrease (or increase) a second time, is the same thing as a total percent decrease (or increase).
2. Arrange students in groups of three or four. Give each group one of the two problems presented in BLM 8.N.3.7: Percent Increase and Decrease. Students work together to solve the problem, showing their work on chart paper.
3. Each group presents the solution to their problem to the class. Encourage student discussion and questions during this time. Add other scenarios as necessary.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply problem-solving skills to determine an appropriate method to solve a problem with a percent of a percent.
- ❒ Communicate mathematically within their group and with the class.
- ❒ Solve a problem involving a percent of a percent.

## Percents: My Understanding

**Introduction:**

Students are used to shopping for various items. Provide students with three items with the regular and sale prices included. They will need to determine the percent savings based on the prices, represent those percent savings on graph paper, and determine the total cost when PST and GST are included.

**Purpose:**

Students will represent percents on graph paper and show how they combine percents to find a total cost. This learning task should provide an assessment of many of the achievement indicators for learning outcome 8.N.3.

**Curricular Links:** ELA, Literacy with Information and Communication Technology (LwICT)

**Materials/Resources:** Access to flyers (or use the examples provided), technology (if students are representing their learning using ICT), poster paper (if students are representing their learning on paper)

**Organization:** Individual

**Scenarios:**

The holiday season is right around the corner and you have some presents to purchase. You want to buy your family great gifts, but you also want to get a good price for the items. The following are some gifts you want to buy. The prices are taken from flyers obtained from different stores in your community.

For each of the purchase scenarios below, do the following:

- Determine the sale price of the items (rounded to the nearest cent).
- Represent the percents on a hundreds grid.
- Show the percents as fractions and decimals.
- Determine the total cost of the item once PST and GST are added.

**Purchases:** Round to the nearest tenth of a percent.

> 25% off a GPS system with preloaded North American maps
>
> Regular price: $199.99

45% off a new release of a movie

Regular price: \$23.99

$5\frac{1}{2}$% off a 26″ LCD TV

Regular price: \$419.99

**Assessment:**

The following rubric can be used to assess achievement of the mathematics learning outcomes.

| Criteria | Meeting Expectations | Developing to Meet Expectations | Beginning to Meet Expectations | Incomplete |
|---|---|---|---|---|
| The student | | | | |
| ■ represents percents using grid paper | ❒ accurately represents all three percents using grid paper | ❒ accurately represents one or two percents using grid paper | ❒ represents whole percents on grid paper but has errors with fractional percents | ❒ does not represent percents using grid paper |
| ■ converts between fractions, decimals, and percents | ❒ consistently converts between fractions, decimals, and percents | ❒ converts between fractions, decimals, and percents with few errors | ❒ converts between fractions, decimals, and percents with many errors | ❒ does not convert between fractions, decimals, and percents |
| ■ solves problems that involve percents | ❒ provides a clear explanation, using symbols and words, showing how he or she determined the sale price | ❒ provides a general explanation, using symbols and words, showing how he or she determined the sale price | ❒ provides a vague or minimal explanation showing how he or she determined the sale price | ❒ provides no explanation showing how he or she determined the sale price |

*continued*

| Criteria | Meeting Expectations | Developing to Meet Expectations | Beginning to Meet Expectations | Incomplete |
|---|---|---|---|---|
| The student | | | | |
| ■ solves problems that involve combining percents | ❒ clearly demonstrates how he or she determined the total cost when including PST and GST | ❒ provides some explanation as to how he or she determined the total cost when including PST and GST | ❒ provides minimal explanation as to how he or she determined the total cost when including PST and GST | ❒ provides no explanation as to how he or she determined the total cost when including PST and GST |
| ■ determines the discount | ❒ accurately calculates the discount | ❒ makes rounding errors that affect the final calculation of the discount | ❒ makes calculation and rounding errors that affect the final calculation of the discount | ❒ does not determine the discount |
| ■ determines the sale price | ❒ accurately calculates the sale price | ❒ makes rounding errors that affect the calculation of the sale price | ❒ makes calculation and rounding errors that affect the final calculation of the sale price | ❒ does not determine the sale price |
| ■ determines the new price with PST and GST added | ❒ accurately determines the new price with PST and GST added | ❒ makes rounding errors that affect the calculation of the new price with PST and GST added | ❒ makes calculation and rounding errors that affect the final calculations of the new price with PST and GST added | ❒ does not determine the new price with PST and GST added |

## NOTES

# **Number**—8.N.4, 8.N.5

**Enduring Understandings:**

Ratios and rates are comparisons of two or more quantities.

Ratios can represent part-to-part quantities or part-to-whole quantities.

Ratios and rates can be used to solve proportional reasoning.

Percents, fractions, decimals, and ratios are all different representations of the same quantity.

**General Learning Outcome:**

Develop number sense.

| **Specific Learning Outcome(s):** | **Achievement Indicators:** |
|---|---|
| **8.N.4** Demonstrate an understanding of ratio and rate. [C, CN, V] | ➣ Express a two-term ratio from a context in the forms 3:5 or 3 to 5.<br>➣ Express a three-term ratio from a context in the forms 4:7:3 or 4 to 7 to 3.<br>➣ Express a part-to-part ratio as a part-to-whole ratio (e.g., Given the ratio of frozen juice to water is 1 can to 4 cans, this ratio can be written as $\frac{1}{4}$, or 1:4, or 1 to 4 [part-to-part ratio]. Related part-to-whole ratios are $\frac{1}{5}$, or 1:5, or 1 to 5, which is the ratio of juice to solution, or $\frac{4}{5}$, or 4:5, or 4 to 5, which is the ratio of water to solution.).<br>➣ Identify and describe ratios and rates from real-life examples, and record them symbolically.<br>➣ Express a rate using words or symbols (e.g., 20 L per 100 km or 20 L/100 km).<br>➣ Express a ratio as a percent, and explain why a rate cannot be represented as a percent. |
| **8.N.5** Solve problems that involve rates, ratios, and proportional reasoning. [C, CN, PS, R] | ➣ Explain the meaning of $\frac{a}{b}$ within a context.<br>➣ Provide a context in which $\frac{a}{b}$ represents a<br>▪ fraction<br>▪ rate<br>▪ ratio<br>▪ quotient<br>▪ probability<br>➣ Solve a problem involving rate, ratio, or percent. |

## Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of fractions by using concrete and pictorial representations to
  - create sets of equivalent fractions
  - compare fractions with like and unlike denominators
- Demonstrating an understanding of ratio, concretely, pictorially, and symbolically
- Demonstrating an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically
- Solving problems involving percents from 1% to 100%

## Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of percents greater than or equal to 0%

## Background Information

Rate, ratio, and proportion are three closely linked concepts:

- A *ratio* is a comparison of two or more quantities (e.g., 1 out of 4 people interviewed prefer . . . ).
- A *rate* is a ratio that is often expressed as one quantity per unit of another quantity (e.g., 100 kilometres per hour).
- A *proportion* is a statement of equality between two ratios (e.g., $\frac{2}{3} = \frac{10}{15}$ ).

Each of these concepts is explained in detail below.

### Ratios

A *ratio* is a comparison of two or more quantities.

*Examples:*

- To make this recipe, you need 2 kg of white flour for every 3 kg of whole wheat flour.
- At the airport, there is 1 taxi for every 20 people who arrive on a plane.

Some ratios, such as the ratio in the first example, are comparisons of one part of a whole (the amount of white flour) to another part of a whole (the amount of whole wheat flour). This is sometimes called a *part-to-part ratio*. For example, you buy 12 doughnuts—chocolate and glazed. The ratio of chocolate to glazed doughnuts is 5:7. But you could also compare the number of chocolate doughnuts to the total number of doughnuts (5:12). This is sometimes called a *part-to-whole ratio*.

In the ratio 5:7, the numbers 5 and 7 are called the *terms* of the ratio. The first term is 5 and the second term is 7. Ratios should be taught in the context of everyday situations and students should have opportunities to use ratios with concrete materials (e.g., making orange juice from concentrate requires 3 cans of water: 1 can concentrate).

It is common to name ratios using fractions. Doing so, however, may cause confusion for Middle Years students. To avoid any misconceptions, two important points need to be understood about ratio:

- Ratio is one of the meanings of fraction. When you say, $\frac{3}{4}$ of the flowers in your garden are annuals, you are comparing the annuals to all the flowers (a part-to-whole ratio), and this is as valid as saying 3 out of 4 flowers are annuals. However, some ratios can never be written as fractions. For example, the ratio 9:0 would result in the denominator of 0, which is mathematically incorrect
- If you have 1 taxi for 5 people, the ratio 1:5 is not a fraction. The ratio can be written as $\frac{1}{5}$, but the 5 people are not the whole and the taxi is not 1 part of that whole.
- Although part-to-part ratios are sometimes written as fractions, it is important not to change these fractions to percents. For example, you may see the ratio of boys to girls written as 4:5 or $\frac{4}{5}$, but it would be mathematically incorrect to say 80% are boys. Reinforce that fractions represent part-to-whole relationships.

To avoid confusion about the concept of naming ratios using fractions, use the traditional notation (e.g., 2:3) or the words (2 to 3) when expressing ratios until students have a clear understanding of ratio. Then extend the concept to include the link between ratios and fractions.

## Rates

Sometimes a ratio can also be a rate. A *rate* is a comparison that relates the measures for two different types of quantities. For each measure, the unit is different and is included when writing the ratio or rate.

*Examples:*

- All prices are rates and ratios (e.g., 49 cents each, 3 for a dollar, $1.99 per kilogram).
- The comparison of time to distance is a rate (e.g., driving at 65 kilometres per hour).
- Changes between two units of measure are also rates or ratios (e.g., centimetres per metre, millilitres per litre, map scales).
- Unit pricing is a common rate used to compare value when purchasing items (e.g., 35¢ per can, $1.24 per kilogram). It is, however, commonly misunderstood by both students and consumers.

### Proportion

A *proportion* is a statement of the equality between two ratios.

*Examples:*

- A rectangle is drawn on grid paper and you want to copy it, but triple its size.
- The florist has a special on bouquets. Customers receive two free carnations for every rose purchased. You want to buy one of these bouquets for your mother. If you want 6 roses in the bouquet, how many carnations will it include?

Each of these situations deals with proportion. To solve the question of the flowers, students would set up a proportion statement such as the following:

| Roses:Carnations | | Roses:Carnations |
|---|---|---|
| 6:? | = | 1:2 |

Students should be very secure in using this type of notation before they are introduced to the fraction symbol for proportion.

Although proportions and equivalent fractions appear to be the same thing, they are not. Equivalent fractions are different symbols for the same amount. If you colour $\frac{1}{2}$ a piece of paper and fold it so that it now shows $\frac{2}{4}$, you still have the same amount coloured ($\frac{1}{2} = \frac{2}{4}$). On the other hand, if you buy two bouquets of flowers and one has 1 rose and 2 carnations and the other has 2 roses and 4 carnations, the total number of flowers is different, but the ratio of roses to carnations is the same (1:2 = 2:4) and, therefore, proportional.

## Mathematical Language

equivalent fraction

part-to-part ratio

part-to-whole ratio

proportion

rate

three-term ratio

two-term ratio

unit price

unit rate

**Assessing Prior Knowledge**

**Materials**: Pattern blocks, BLM 8.N.4.1: Ratio Pre-Assessment

**Organization**: Individual

**Procedure**:

1. Tell students that in the next few lessons they will be learning about rates, ratios, and proportions; however, you first need to find out what they already know about ratios.
2. Ask students to complete BLM 8.N.4.1: Ratio Pre-Assessment.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:
   - ❒ Represent ratios concretely.
   - ❒ Represent ratios pictorially.
   - ❒ Represent ratios symbolically.
   - ❒ Solve problems involving ratios.

## Suggestions for Instruction

- **Express a two-term ratio from a context in the forms 3:5 or 3 to 5.**
- **Express a three-term ratio from a context in the forms 4:7:3 or 4 to 7 to 3.**

**Materials:** Red, white, and blue poker chips (or any other item that can be used to create a ratio), 6 glass beakers or jars, math journals

**Organization:** Small group/whole class

**Procedure:**

1. Tell students that in the next few lessons they will be learning about ratios.
2. Place a variety of the three colours of poker chips into beakers. Ensure that each beaker has 10 chips in total.

3. Divide students into small groups. Ask each group to answer the following questions and be prepared to share responses with the rest of the class:
   - How could you compare the number of blue chips to the number of red chips?
   - How could you compare the number of blue chips to the number of white chips?
   - How could you compare the number of red chips to the number of white chips?
   - How could you compare the number of blue chips to the total number of chips?
   - How could you compare the number of red chips to the total number of chips?
   - How could you compare the number of white chips to the total number of chips?
   - How could you compare the number of blue and red chips to the number of white chips?
   - How could you compare the number of red and white chips to the number of blue chips?
   - How could you compare the number of blue and red chips to the total number of chips?
   - How could you compare the number of blue chips to the number of red chips to the number of white chips?
4. After groups have shared their results with the class, define *ratio*. Put the definition on the bulletin board. Rewrite the ratio for the first statement in step 3 above. Tell students that this ratio is a two-term ratio. Rewrite the ratio for the last statement in step 3 above. Tell students that this ratio is a three-term ratio.
5. Have students return to their groups. Ask them to decide on a definition for *two-term ratios* and *three-term ratios* and share their definitions with the class.
6. Have students explain the following in their math journals, using words and diagrams: a ratio, a two-term ratio, and a three-term ratio.

**Observation Checklist**

☑ Use students' math journal responses to determine whether they can do the following:

- ❒ Explain two-term ratio using words and diagrams.
- ❒ Explain three-term ratio using words and diagrams.

Suggestions for Instruction

- **Express a part-to-part ratio as a part-to-whole ratio (e.g., Given the ratio of frozen juice to water is 1 can to 4 cans, this ratio can be written as $\frac{1}{4}$, or 1:4, or 1 to 4 [part-to-part ratio]. Related part-to-whole ratios are $\frac{1}{5}$, or 1:5, or 1 to 5, which is the ratio of juice to solution, or $\frac{4}{5}$, or 4:5, or 4 to 5, which is the ratio of water to solution.).**
- **Identify and describe ratios and rates from real-life examples and record them symbolically.**
- **Express a ratio as a percent and explain why a rate cannot be represented as a percent.**

**Materials:** 6 cans of a variety of frozen fruit juice concentrate, 6 pitchers, math journals

**Organization:** Small group/whole class

**Procedure:**

1. Divide students into small groups, and ask them to follow the directions to make fruit juice from frozen concentrate.
   - Write the ratio of cans of frozen fruit juice concentrate to the number of cans of water. (Usually 1:3 or 1:4)
   - Write the ratio of cans of frozen fruit juice concentrate to the total number of cans of juice made. (Either 1:4 or 1:5)
2. Ask students to answer the following questions:
   - How does the ratio of cans of frozen fruit juice concentrate to the number of cans of water reflect a part-to-part ratio?
   - How does the ratio of frozen fruit juice concentrate to the total number of cans of juice made reflect a part-to-whole ratio?
3. Explain that a part-to-whole ratio can also be written as a percent.
   - Ask students to express the part-to-whole ratio of cans of frozen juice concentrate to the total solution of juice made as a percent. (1:5 or 20%)
   - Ask students to express the ratio of water to the total solution. (4:5 or 80%)

**Note:** Although part-to-part ratios can be written as a fraction for proportional equivalency (e.g., one can of concentrate to four cans of water [1:4] can be increased 10-fold to 10:40 or $\frac{1}{4} = \frac{10}{40}$), reinforce with students that fractions represent part-to-whole relationships. Conversion of ratios to a percent makes sense only in the part-to-whole context.

4. Ask students to share their responses with the class. As a class, discuss that part-to-part ratios compare different parts of a group to each other and part-to-whole ratios compare one part of a group to the whole group. So, in this case, the two parts of the groups are the frozen concentrate and the water. The total is the 4 or 5 cans of total liquid the mixture would make.
5. Ask students to write the ratio of boys to girls in the class and the ratio of boys to the total student population.
6. Ask students: How does the ratio of boys to girls reflect a part-to-part ratio and the ratio of boys to the total student population reflect a part-to-whole ratio? Discuss responses as a class.
7. Have students discuss the following in their math journals:
   - Describe, using words and diagrams, part-to-part ratios and part-to-whole ratios.
   - Use real-life examples to enhance the description of ratios.

**Observation Checklist**

☑ Use students' math journal responses to determine whether they can do the following:

- ❒ Describe part-to-part ratios.
- ❒ Describe part-to-whole ratios.
- ❒ List real-life examples of ratios.

## Suggestions for Instruction

- **Identify and describe ratios and rates from real-life examples, and record them symbolically.**
- **Express a rate using words or symbols (e.g., 20 L per 100 km or 20 L/100 km).**
- **Express a ratio as a percent, and explain why a rate cannot be represented as a percent.**

**Materials:** Poster paper, flyers, the Internet, math journals

**Organization:** Whole class/individual

**Procedure:**

1. Write the following examples of rates on the whiteboard:
   100 km/h, 70 beats/min, $1.69/100 g, $9.50/h
2. Ask students whether they can tell you what the rates are in the examples provided. (They may say speed, heart rate, money, wages, and so on.)
3. Explain to students that all of them are examples of rates. Rates are special ratios. *Rates* compare two quantities measured in different units. Ask students what the different units are in the examples provided (i.e., distance and time, speed of heart rate and time, money and mass, and money and time). For this reason, rates can never be expressed as a percent.
4. Ask students where they may have seen the examples of rates in the real world.
5. Ask students whether they have seen any other rates. (Generate a list from student responses.)
6. Write the following examples or rates on the whiteboard:
   400 km per 4 h, 140 beats per 2 min, $16.90 per 1000 g, $28.50 per 3 h
7. Ask students whether they see any difference between the original list of rates and the new list of rates. (You want to end up with the understanding that both lists are rates, but the first list is unit rates. A *unit rate* is a rate in which the second term is one.)
8. Have students find examples of rates (e.g., in shops, in flyers, on the Internet) or develop their own posters displaying rates. Have students describe, in their math journals, the meaning of their sample rates using words and symbols.

**Observation Checklist**

☑ Use students' math journal responses to determine whether they can do the following:

- ❒ Expresses rates using words or symbols.
- ❒ Make connections between rate and ratio.
- ❒ Make connections between rate and real-life examples.

## Suggestions for Instruction

- **Explain the meaning of $\frac{a}{b}$ within a context.**
- **Provide a context in which $\frac{a}{b}$ represents a**
  - **fraction**
  - **rate**
  - **ratio**
  - **quotient**
  - **probability**

**Materials:** BLM 8.N.4.2: Meaning of $\frac{a}{b}$ ?, chart paper, math journals

**Organization:** Whole class/small group

**Procedures:**

1. Explain to students that $\frac{a}{b}$ can represent a fraction, rate, ratio, quotient, or probability depending on the context.
2. Facilitate a short class discussion to ensure that students understand the meaning of each of these terms: *fraction, rate, ratio, quotient,* and *probability*.
3. Inform students that they will be playing an adapted version of the game *Four Corners*.
   - Explain that five spaces in the room are labelled *fraction, rate, ratio, quotient*, and *probability*.
   - Present students with one scenario from BLM 8.N.4.2: Meaning of $\frac{a}{b}$ ?
   - Allow students some thinking time, and then have them write on a piece of paper which of the five meanings of $\frac{a}{b}$ the scenario represents.
   - Ask students to go to the appropriate space in the room.
   - Have all students in a given space work together to come up with an explanation of why they made their selection in order to try to convince the other groups to change their minds.
   - All groups have a chance to share their reasons with the rest of the class. Students may move from group to group as many times as they like if they have been convinced by another group.
   - Afterward, reveal which is the correct meaning of the given context, and facilitate further discussion with the class.
4. Repeat the process with the remaining scenarios from BLM 8.N.4.2: Meaning of $\frac{a}{b}$ ?
5. In their math journals, students use the ratio 2 to 3 in a way that means fraction, rate, ratio, quotient, and probability.

**Observation Checklist**

☑ Observe students and use their math journals to determine whether students can do the following:

- ❒ Explain when $\frac{a}{b}$ represents a fraction.
- ❒ Explain when $\frac{a}{b}$ represents a rate.
- ❒ Explain when $\frac{a}{b}$ represents a ratio.
- ❒ Explain when $\frac{a}{b}$ represents a quotient.
- ❒ Explain when $\frac{a}{b}$ represents a probability.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a fraction.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a rate.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a ratio.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a quotient.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a probability.
- ❒ Communicate mathematically

## Suggestions for Instruction

- **Solve a problem involving rate, ratio, or percent.**

**Materials:** BLM 8.N.4.3: Problem Solving, chart paper, math journals

**Organization:** Individual/small group

**Procedure:**

1. Explain to students that they will be solving problems using their understanding of rate, ratio, fractions, and percent.
2. Have students form small groups, and provide each group with a copy of BLM 8.N.4.3: Problem Solving. Ask students to solve the problems presented, record the problem-solving process on chart paper, and be prepared to explain their results.
3. As groups present their results, identify the different strategies that were used to solve the problems.

4. Ask students to respond to the following in their math journals:

   There are two hundred students at Santa B Middle School in North Pole City. If 42% of the students are female, what is the ratio of female students to male students? Present your answer in a variety of forms. (42:58, 84:116, 21/29)

**Observation Checklist**

☑ Use students' math journal responses to determine whether students can do the following:

- ❒ Solve problems involving rate, ratio, or percent.
- ❒ Apply problem-solving strategies.
- ❒ Communicate solutions to problems using mathematical language.
- ❒ Use prior knowledge to reason mathematically.

## Suggestions for Instruction

- **Express a two-term ratio from a context in the forms 3:5 or 3 to 5.**
- **Express a three-term ratio from a context in the forms 4:7:3 or 4 to 7 to 3.**
- **Express a part-to-part ratio as a part-to-whole ratio (e.g., Given the ratio of frozen juice to water is 1 can to 4 cans, this ratio can be written as $\frac{1}{4}$, or 1:4, or 1 to 4 [part-to-part ratio]. Related part-to-whole ratios are $\frac{1}{5}$, or 1:5, or 1 to 5, which is the ratio of juice to solution, or $\frac{4}{5}$, or 4:5, or 4 to 5, which is the ratio of water to solution.).**
- **Solve a problem involving rate, ratio, or percent.**

**Materials:** BLM 5–8.9: Centimetre Grid Paper, pencil crayons of various colours, chart paper, poster paper

**Organization:** Pairs

**Procedure:**

1. Tell students that they will be creating designs on a 5 × 5 grid using three colours of pencil crayons.
2. Have students select a partner to work with.
3. Ask each pair to decide on three colours that both partners will use to make their designs.

4. Ask students to decide on a ratio of the three colours they will use in their designs. Both students in a pair should use the same ratio.
5. Once the designs are complete, have students predict the number of squares that would be in each colour in their designs if they were to use a 10 × 10 grid and follow the same design (essentially, scaling up their diagrams). Have students exchange designs with their partners and create each others' designs using a 10 × 10 grid.
6. Ask students to create a poster that includes
   - their designs
   - the colours expressed as fractions, as ratios, as percents, and in words

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:
- ❐ Solve problems involving rate, ratio, or percent.
- ❐ Apply problem-solving strategies.
- ❐ Correctly express a two-term ratio.
- ❐ Correctly express a three-term ratio.
- ❐ Correctly express a ratio as part-to-part and part-to-whole.
- ❐ Correctly express a ratio in fractional, ratio, and percent notation, or in words.

## Putting the Pieces Together

### Grade 8 Farewell

**Introduction:**

Schools that go up to Grade 8 often have a celebration at the end of the year before students head to high school. Some schools call it a Grade 8 graduation; others call it a Grade 8 farewell. This learning task allows students to participate in a real-life situation in which they volunteer to plan the Grade 8 farewell for a school.

**Purpose:**

Students will use skills in collecting data and solving proportions.

**Curricular Links:** LwICT, ELA

**Materials/Resources:** Access to *Microsoft* PowerPoint, the Internet

**Organization:** Individual or small group

**Scenario:**

- You are organizing the school's Grade 8 farewell. There will be 75 people attending.
- You will be planning everything related to the food.
  - You know that you will be having pizza for the main course. What types of pizza will you need to order, and how much?
  - How much dessert will you need?
  - How many drinks will you need?
  - How much cutlery will you need? how many glasses and plates? how many napkins?
  - Can you think of anything else you might need?
- You will be planning the balloon decorations.
  - How many balloons will you be using?
  - How many balloons are in a package?
- You will be arranging the DJ who will play the music for the farewell.
- You will need to determine the final cost for the entire farewell and then determine the price per student so that you know how much to charge each student for attending the farewell.

**Procedure:**

1. Create a survey to determine what types of pizzas to order. Then determine how many of each type need to be ordered based on the number of slices in the large pizza size.
2. Survey one class and base your decisions on that population.
3. If the desserts come in amounts of one dozen, how many will you need to order? (Is everyone getting one dessert or more than one?)
4. If the drinks come in cases of 24, how many will you need to order? (Is everyone getting one drink or more than one?)
5. If the cutlery, paper plates, glasses, and napkins come in packages of 24, how many of each will you need to purchase?
6. Research DJs in your area. Select one DJ and include the price. You may have to call for a price.
7. Once you have collected all the information you need, put it into a PowerPoint so that you can present your work to your peers.
8. All the math must be done showing ratios/proportions.
9. If you ordered more or less, you need to explain your reasoning behind it.
10. Include the final cost per student and the determined ticket price for students to attend the farewell.

11. Include an assumptions page, since you will have to make many assumptions as you work through this planning process.

**Assessment:**

The following rubric can be used to assess achievement of the mathematics learning outcomes.
**Note:** Other rubrics may be added to assess LwICT and ELA learning outcomes.

| Criteria | Meeting Expectations | Developing to Meet Expectations | Beginning to Meet Expectations | Incomplete |
|---|---|---|---|---|
| The student | | | | |
| ■ demonstrates an understanding of surveys | ❒ clearly states survey questions<br>❒ provides a clearly organized survey tracking sheet<br>❒ keeps a tally that is easy to read<br>❒ provides a conclusion that clearly explains why the types of pizzas and the number of each type were ordered | ❒ states survey questions<br>❒ provides a somewhat organized survey tracking sheet<br>❒ keeps a tally<br>❒ provides a conclusion that partially explains why the types of pizzas and the number of each type were ordered | ❒ vaguely states survey questions<br>❒ provides a disorganized survey tracking sheet<br>❒ keeps a vague tally<br>❒ provides a conclusion but no explanation, just states the types of pizzas and the number of each type ordered | ❒ does not conduct the survey |
| ■ solves problems that involve rates, ratios, and proportional reasoning | ❒ provides an explanation of all the rates, ratios, and proportional reasoning used | ❒ provides an explanation of some of the rates, ratios, and proportional reasoning used | ❒ provides an explanation of few of the rates, ratios, and proportional reasoning used | ❒ provides no explanation of the rates, ratios, and proportional reasoning used |
| ■ makes assumptions in order to solve problems | ❒ clearly explains rationalizations behind the assumptions made | ❒ partially explains rationalizations behind the assumptions made | ❒ vaguely explains rationalizations behind the assumptions made | ❒ does not explain rationalizations behind the assumptions made |

**Extension:**

Students can use a pie chart to show the allocation of expenses and use this pie chart to predict the expenses of a similar party with $x$ number of people attending.

# NOTES

# **Number**—8.N.6, 8.N.8

**Enduring Understandings:**

Fractions represent parts of a whole or part of a group.

Percents, fractions, ratios, and decimals are different representations of the same quantity.

Fractions can represent division.

Multiplication does not always make a bigger group.

The principles of operations used with whole numbers also apply to operations with decimals, fractions, and integers.

**General Learning Outcome:**

Develop number sense.

| Specific Learning Outcome(s): | Achievement Indicators: |
|---|---|
| **8.N.6** Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. [C, CN, ME, PS] | ➛ Identify the operation(s) required to solve a problem involving positive fractions.<br>➛ Provide a context involving the multiplying of two positive fractions.<br>➛ Provide a context involving the dividing of two positive fractions.<br>➛ Express a positive mixed number as an improper fraction and a positive improper fraction as a mixed number.<br>➛ Model multiplication of a positive fraction by a whole number, concretely or pictorially, and record the process.<br>➛ Model multiplication of a positive fraction by a positive fraction, concretely or pictorially, and record the process.<br>➛ Model division of a positive fraction by a whole number, concretely or pictorially, and record the process.<br>➛ Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.<br>➛ Solve a problem involving positive fractions taking into consideration order of operations (limited to problems with positive solutions). |

*continued*

| Specific Learning Outcome(s): | Achievement Indicators: |
|---|---|
| **8.N.8** Solve problems involving positive rational numbers. [C, CN, ME, PS, R, T, V] | ➢ Identify the operations(s) required to solve a problem involving positive rational numbers.<br>➢ Determine the reasonableness of an answer to a problem involving positive rational numbers.<br>➢ Estimate the solution and solve a problem involving positive rational numbers.<br>➢ Identify and correct errors in the solution to a problem involving positive rational numbers. |

## Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of multiplication (2-digit numerals by 2-digit numerals) to solve problems
- Relating decimals to fractions (tenths, hundredths, thousandths)
- Demonstrating an understanding of factors and multiples by
  - determining multiples and factors of numbers less than 100
  - identifying prime and composite numbers
  - solving problems involving factors or multiples
- Relating improper fractions to mixed numbers
- Demonstrating an understanding of multiplication and division of decimals involving
  - 1-digit whole-number multipliers
  - 1-digit natural number divisors
  - multipliers and divisors that are multiples of 10
- Explaining and applying the order of operations, excluding exponents (limited to whole numbers)
- Demonstrating an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected)
- Demonstrating an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions
- Demonstrating an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences)

### Multiplying and Dividing Fractions

Having a concrete, pictorial, and symbolic understanding of what happens when multiplying and dividing fractions enables students to have a better conceptual understanding of how and why the various methods work. Students should apply their prior knowledge of fractions, and of performing operations on whole and decimal numbers, when learning about multiplying and dividing fractions.

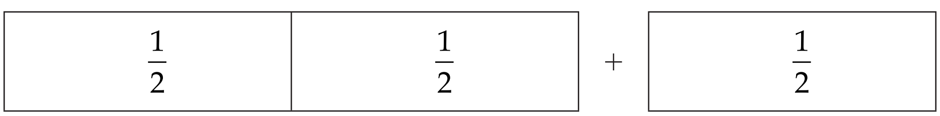
It is essential to give students time to develop their conceptual understanding of multiplying and dividing fractions. Once students understand what is happening using concrete and pictorial representations, they can develop and apply a symbolic method for multiplying and dividing fractions.

### Meanings of Multiplication

It is important that students are able to work flexibly with the various meanings of multiplication.

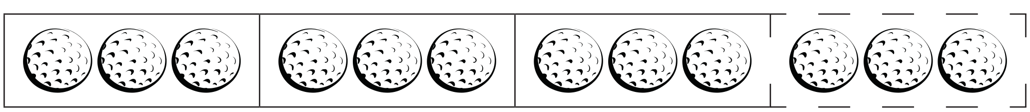
- **Multiplication as repeated addition:**

  $3 \bullet \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1\frac{1}{2}$



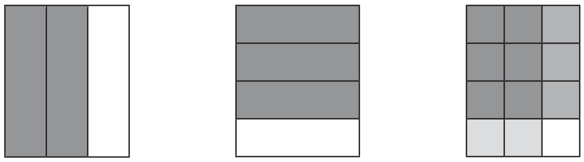

- **Multiplication as equal sets or groups:**

  $\frac{3}{4} \bullet 12$ (think of $\frac{3}{4}$ of a set of 12 objects) = 9



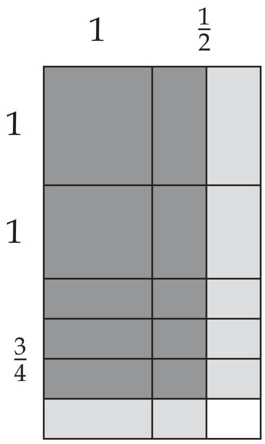
- **Multiplication as a rectangular area:**

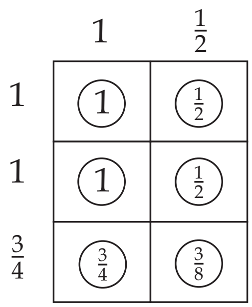
$$\frac{2}{3} \quad \bullet \quad \frac{3}{4} \quad = \quad \frac{6}{12} \text{ or } \frac{1}{2}$$



**Note:** At first, students can model this using concrete materials or on square grid paper. As students learn to trust the count and become more skilled, they may be able to draw the multiplications free-hand.

$$1\frac{1}{2} \bullet 2\frac{3}{4}$$



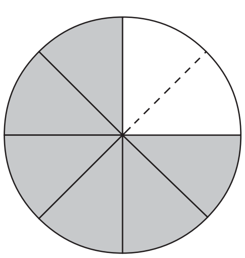

$$\begin{aligned} 1\frac{1}{2} \bullet 2\frac{3}{4} &= 1 + 1 + \frac{1}{2} + \frac{1}{2} + \frac{3}{4} + \frac{3}{8} \\ &= 3 + \frac{3}{4} + \frac{3}{8} \\ &= 3 + \frac{6}{8} + \frac{3}{8} \\ &= 3 + \frac{9}{8} \\ &= 3 + 1\frac{1}{8} \\ &= 4\frac{1}{8} \end{aligned}$$

## Meanings of Division

It is important that students are able to work flexibly with the various meanings of division.

- **Division as equal sharing:**

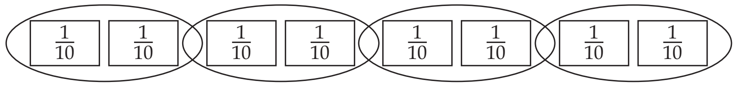
$\frac{3}{4}$ of a pizza shared among 6 people:



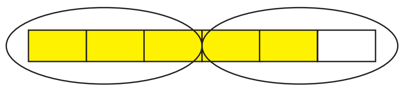
$$\frac{3}{4} \div 6 = \frac{1}{8}$$

- **Division as equal grouping:**

$$\frac{8}{10} \div 4 = \frac{2}{10} \text{ or } \frac{1}{5}$$

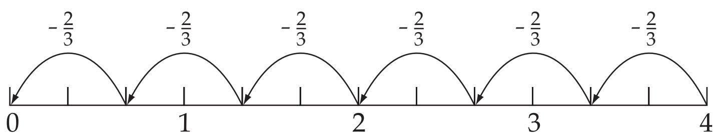



$$\frac{5}{6} \div \frac{1}{2} = 1\frac{2}{3}$$



- **Division as repeated subtraction:**

$$4 \div \frac{2}{3} = 6$$




## Mathematical Language

denominator

fraction

improper fraction

mixed number

numerator

order of operations

proper fraction

rational numbers

reciprocal

**Assessing Prior Knowledge**

A pre-assessment will help determine students' level of understanding of fractions and provide a baseline for instruction. It will direct teaching based on students' learning needs. Make sure the pre-assessment is similar to the post-assessment, thereby enabling students to focus on areas for improvement and then providing them with opportunities to show that they have developed an understanding of the concepts.

**Materials:** 7 pieces of chart paper per group, math journals

**Organization:** Small group/whole class/individual

**Procedure:**

1. Tell students that they will be learning to multiply and divide fractions over the next few lessons; however, you first need to determine what they already know about fractions.
2. Have students form small groups. Provide each group with the following list and one sheet of chart paper for each concept in the list: factors, multiples, prime numbers, composite numbers, improper fractions and mixed numbers, adding fractions, and subtracting fractions.
3. Have each group explain, to the best of their ability, each concept, using words, diagrams, symbols, or concrete items. Students need to record their work on the chart paper and be prepared to share their group's thoughts to the class.
4. Review students' responses, providing opportunities for questions and discussions. Encourage groups to add to their papers as needed when listening to other groups.
5. In their math journals, have students
   - write the following at the top of the page: factors, multiples, prime numbers, composite numbers, improper fractions and mixed numbers, adding fractions, and subtracting fractions
   - summarize their understanding of the concepts based on the learning activity in which they participated

**Observation Checklist**

☑ Use students' math journal responses to determine whether students can do the following:

- ❒ List factors of numbers less than 100.
- ❒ Identify multiples of different numbers.
- ❒ Explain what prime numbers are.
- ❒ Explain what composite numbers are.
- ❒ Explain the relationship between improper fractions and mixed numbers.
- ❒ Explain how to add fractions.
- ❒ Explain how to subtract fractions.

## Suggestions for Instruction

- **Express a positive mixed number as an improper fraction and a positive improper fraction as a mixed number.**

**Materials:** Pattern blocks or 4 or 5 copies of cut paper fraction bars per student (see BLM 5–8.12: Fraction Bars), BLM 8.N.6.1: Mixed Numbers and Improper Fractions

**Organization:** Whole class/individual

**Procedure:**

1. Begin by having students show various representations for one (e.g., $\frac{4}{4}, \frac{10}{10}$).
2. Provide students with 5 halves. Ask them:
   - What fraction do these pieces represent? ($\frac{5}{2}$)
   - What type of fraction is $\frac{5}{2}$? (Improper fraction)
   - Is there another way $\frac{5}{2}$s can be represented? (As a mixed number: $2\frac{1}{2}$)
   - Can you put the pieces together to show $\frac{5}{2}$ as $2\frac{1}{2}$?
3. Ask students to show you $\frac{8}{6}$. (They need to have 8 sixth pieces.) Walk around the classroom to ensure that students have the correct amount. Have students represent $\frac{8}{6}$ as a mixed number. ($1\frac{2}{6}$) Some may be able to simplify it to $1\frac{1}{3}$.
4. Ask students whether they can come up with a strategy for converting improper fractions to mixed numbers. Facilitate a class discussion about their strategies.
5. Write $1\frac{3}{4}$ on the whiteboard. Ask students to read the mixed number.

6. Have students model this number using their fraction materials.
7. Ask whether they can express this number as an improper fraction.
8. Repeat, using $2\frac{1}{3}$.
9. Ask students whether they can come up with a strategy for converting improper fractions to mixed numbers. Facilitate a class discussion about their strategies.
10. Ask students to convert the following mixed numbers to improper fractions, using a method of their choosing: $2\frac{2}{3}$, $3\frac{1}{2}$, $4\frac{4}{5}$. Review one at a time. Have individual students demonstrate the solution on the whiteboard. Allow opportunities for students to give feedback and ask questions.
11. Provide students with BLM 8.N.6.1: Mixed Numbers and Improper Fractions. Have them respond to the questions individually.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand that a mixed number and an improper fraction are two equivalent representations.
- ❒ Convert a mixed number to an improper fraction.
- ❒ Convert an improper fraction to a mixed number.
- ❒ Use mental mathematics and estimation strategies.

## Suggestions for Instruction

- **Express a positive mixed number as an improper fraction and a positive improper fraction as a mixed number.**

**Materials:** BLM 5–8.5: Number Cards with the zeros removed, BLM 8.N.6.2: Mixed Number War

**Organization:** Pairs/whole class

**Procedure:**

**Note:** Since this is a game of speed, have students paired up equally in their ability to convert between mixed numbers and improper fractions.

1. Tell students that they will be learning a new game called *Mixed Number War.* Explain how the game is played:
    - Player A flips one card and, without looking at the card, places it in either the numerator or the denominator place.

- Player B flips a card and, without looking at the card, places it in the remaining space.
- If a proper fraction is made, the first one to slap a hand onto the fraction wins the cards.
- If an improper fraction is made, the first one to name it correctly as a mixed number wins the cards.
- If a player slaps the cards when the fraction is not a proper fraction or incorrectly names the mixed number, the player's partner wins the cards.

2. Demonstrate to the class how the game is played, and have students play.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand that a mixed number and an improper fraction are two equivalent representations.
- ❒ Recognize proper and improper fractions.
- ❒ Convert an improper fraction to a mixed number.

## Suggestions for Instruction

- **Determine the reasonableness of an answer to a problem involving positive rational numbers.**
- **Estimate the solution and solve a problem involving positive rational numbers.**

**Materials:** White paper, highlighters, pens of different colours, math journals

**Organization:** Individual/whole class

**Procedure:**

1. Begin the lesson by telling students that they will be reviewing the addition and subtraction of proper fractions, improper fractions, and mixed numbers.
2. Ask students what they remember about adding and subtracting proper fractions, improper fractions, and mixed numbers. Provide students with sheets of white paper and have them brainstorm.

**Note:** *Rational numbers* are any numbers that can be written in fraction form. The denominator cannot be zero. Rational numbers can be added, subtracted, multiplied, and divided. This is a perfect time to review adding and subtracting fractions from Grade 7.

3. Once students have done this individually, ask them to take out a highlighter and pens of different colours. Ask students to share their ideas with the class, and make a large class web of their suggestions. Have students highlight the information they had put down on their sheets that is correct and add new information they didn't have on their sheets.
4. Collect students' work to check for any misconceptions in students' thinking, and provide clarification as needed.
5. In their math journals, have students demonstrate their understanding of adding and subtracting proper fractions, improper fractions, and mixed numbers, using words, symbols, and/or diagrams.

**Observation Checklist**

☑ Use students' math journal responses to determine whether they can do the following:

- ❒ Add and subtract proper fractions.
- ❒ Add and subtract improper fractions and mixed numbers.
- ❒ Create equivalent fractions where needed.
- ❒ Communicate mathematically.

## Suggestions for Instruction

- **Identify the operation(s) required to solve a problem involving positive rational numbers.**
- **Determine the reasonableness of an answer to a problem involving positive rational numbers.**
- **Estimate the solution and solve a problem involving positive rational numbers.**

**Materials:** BLM 8.N.6.3: Decimal Addition Wild Card, BLM 5–8.5: Number Cards

**Organization:** Pairs

**Procedure:**

1. Before students begin this learning activity, prepare enough sets of number cards for the class (sets contain four of each digit) by copying them on paper or card stock. Randomly mark (with a marker or sticker) the corner of the number side of four cards from each set—these will be the wild cards.

2. Explain to students that they will be learning a new game called *Decimal Addition Wild Card*. Explain the following rules, and demonstrate a round for the class:
   - The object of the game is to have the greatest sum after each round of play.
   - Shuffle the deck (including the wild cards) and place the cards face-down in a pile or spread out.
   - Player A selects one card, and then decides where to place the card on the top of his or her recording sheet.
   - Player B does the same.
   - Play continues in this manner until all spaces are filled on the top of each player's recording sheet.
   - Both players fill out their chart at the bottom of the recording sheet accordingly, and circle the player with the highest sum after the round.
   - The player with the greatest number of highest sums after nine rounds of play wins.

   Similar games could be played with any of the operations.

**Note:** If players select a wild card, they have two options:
- They may play the number shown on the card as they would any regular card.
- They can choose to swap the position of two cards, and select a new card.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:
- ❒ Use mental mathematics and estimation skills.
- ❒ Reason mathematically in order to place numbers.
- ❒ Add (subtract, multiply, divide) positive decimal numbers.

## Suggestions for Instruction

- **Model multiplication of a positive fraction by a whole number, concretely or pictorially, and record the process.**
- **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**

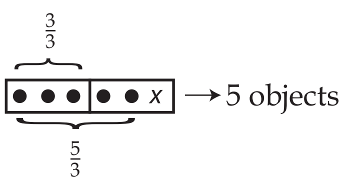
**Materials:** Pattern blocks, fraction bars, grid paper, square tiles, empty number lines, math journals

**Organization:** Whole class/individual

**Procedure:**

> **Note:** Whenever students are solving a problem, they are expected to explain what they did to get the answer, why they solved the problem that way, and why they think the solution makes sense.

1. Tell students that in the next few lessons they will be learning how to multiply and divide fractions concretely, pictorially, and symbolically.
2. Have one student neatly record onto a transparency all multiplication sentences and answers talked about in class.
3. Review multiplication by asking students the following question: What is *multiplication*? Encourage a variety of responses.
4. Provide students with a variety of manipulatives. If necessary, review with students the fractional representation of the fraction bars and pattern blocks.
5. Ask students to use the manipulatives to show $5 \bullet \frac{1}{6}$ and $3 \bullet \frac{5}{3}$. Some students may show manipulatives as groups (e.g., five groups of $\frac{1}{6}$), as repeated addition (e.g., $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$), or as equal sets (e.g., $\frac{5}{3}$ of a set of 3 objects).

   Discuss each of the three representations of multiplication.
6. Have students make up their own multiplication sentences that use a fraction and a whole number. Have them model this multiplication using all three meanings for multiplication.
7. Discuss the various models with the class, encouraging a variety of responses:
   - Which model is best?
   - Which model is most clear?
   - Which model is easiest to understand?
8. Ask the recorder to share the multiplication sentences and answers discussed in class.
9. Have the class discuss any patterns they see. Ask students whether they can come up with a rule for multiplying fractions and whole numbers based on what they have been doing. (You may have to demonstrate that a whole number is the same as any number over 1 in fraction form.)
10. Have students answer the following question in their math journals:
    - When multiplying a whole number by a proper fraction, what can you say about the size of the product in comparing it to the two factors? (The product would be less than the whole number factor and greater than the fraction factor.)
    - Use diagrams, words, and symbols to support your response.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❐ Connect the concept of multiplying whole numbers to that of fractions.
- ❐ Represent multiplication in a variety of ways.
- ❐ Multiply proper fractions by whole numbers concretely, pictorially, and symbolically.
- ❐ Communicate mathematically.
- ❐ Recognize a pattern in order to generalize a rule.

☑ Use students' math journal responses to determine whether they can do the following:

- ❐ Recognize that when multiplying a whole number by a fraction, the product would be less than the whole number and greater than the fraction factor.
- ❐ Demonstrate the multiplication of a proper fraction by a whole number in a variety of ways.
- ❐ Communicate mathematically.

## Suggestions for Instruction

- **Model division of a positive fraction by a whole number, concretely or pictorially, and record the process.**
- **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**

**Materials:** Pattern blocks, fraction bars, grid paper, square tiles, empty number lines, math journals

**Organization:** Whole class/individual

**Procedure:**

1. During the class discussion of division sentences, have one student neatly record onto a transparency all responses.
2. Review division by asking students the following question: What is *division*? Encourage a variety of responses.

3. Provide students with a variety of manipulatives. If necessary, review with students the fractional representation of the fraction bars and pattern blocks.
4. Ask students to use the manipulatives to show $\frac{5}{6} \div 5$ and $\frac{1}{5} \div 2$. (Some students may show equal sharing, equal grouping, or repeated subtraction. Discuss each of the three representations of division.)
5. Have students make up their own division sentences in which a whole number is divided by a fraction. Have them model this division using all three meanings of division.

   **Note:** Students may need help coming up with division sentences that will divide well by a whole number. To help students recognize the pattern and generalize a rule, you will want to help students select numbers that work well together.
6. Discuss the various models with the class, encouraging a variety of responses:
   - Which model is best?
   - Which model is most clear?
   - Which model is easiest to understand?
7. Ask the recorder to share the division sentences and answers that he or she has been recording.
8. Have the class discuss any patterns they see. Ask students whether they can come up with a rule for dividing fractions and whole numbers based on what they have been doing.
9. Ask students to answer the following problem in their math journals:
   Jared has $\frac{3}{4}$ of a bag of sunflower seeds. He is sharing the bag with four of his friends. Approximately what fraction of the bag of sunflower seeds will each person get, assuming they all receive the same amount? (Students should be able to approximate the solution using models or pictures.)

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:
- ❒ Connect the concept of dividing whole numbers to that of fractions.
- ❒ Represent division in a variety of ways.
- ❒ Divide proper fractions by whole numbers concretely, pictorially, and symbolically.
- ❒ Communicate mathematically.
- ❒ Recognize a pattern in order to generalize a rule.

## Suggestions for Instruction

- **Model multiplication of a positive fraction by a positive fraction, concretely or pictorially, and record the process.**
- **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**

**Materials:** Base-10 blocks, $8\frac{1}{2} \times 11$ paper, pencil crayons of different colours, BLM 5–8.5: Number Cards

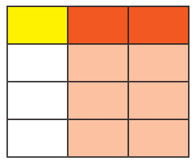
**Organization:** Whole class/individual/pairs

**Procedure:**

*Part A: Proper Fractions*

1. Have one student neatly record onto a transparency all multiplication sentences and answers talked about in class.
2. Tell students that one way to represent multiplication of whole numbers concretely or pictorially is to use the *area model*: length times width. Ask students to model $4 \bullet 6$ using base-10 blocks. Observe to ensure that the dimensions of the rectangle are $4 \times 6$ and that the area, or the space covered by the rectangle, is equal to the product of 4 and 6.
3. Provide students with paper, and ask them whether they can figure out a way to model $\frac{3}{4} \bullet \frac{1}{2}$, using the area model that was just reviewed. Give them the opportunity to explore their ideas before demonstrating how they can use paper folding to show multiplication of proper fractions.
4. Ask students to do the following:
   - Hold the paper in the landscape position and fold the paper to divide the paper into quarters.
   - Lightly shade in $\frac{3}{4}$ of the paper with a pencil crayon.
   - Turn the paper to the portrait position.
   - Fold the paper in half.
   - Lightly shade in $\frac{1}{2}$ of the paper with a pencil crayon of a different colour than the one used previously.
   - Into how many equal pieces is the paper folded now? (8)
   - Of those eight pieces, how many pieces have two colours? (3)
   - What fraction do the two coloured pieces represent? ($\frac{3}{4} \bullet \frac{1}{2}$ is $\frac{3}{8}$)
5. Ask students whether they can apply their understanding of how paper folding can be used to multiply fractions and solve the following using a diagram: $\frac{2}{3} \bullet \frac{1}{4}$.

6. Have students work through the following with the teacher:
   - Start by drawing a rectangle. Divide the rectangle into three equal parts and shade in two parts using a coloured pencil.



   - Divide the rectangle into four equal parts in the other direction (portrait). Shade in one of the four parts in a different colour.
   - Into how many equal parts is the rectangle divided? (12)
   - How many equal parts have two colours? (2)
   - What is that as a fraction? $\frac{2}{12}$ or $\frac{1}{6}$
   - So, $\frac{2}{3} \bullet \frac{1}{4} = \frac{2}{12}$ or $\frac{1}{6}$.
7. Discuss both the drawing method and the paper-folding method and have students decide which they like better.
8. Ask students to model $\frac{1}{5} \bullet \frac{3}{4}$ using paper folding or drawing.
9. Ask students to model $\frac{1}{2} \bullet \frac{7}{8}$ using paper folding or drawing.
10. Provide students with a set of number cards. Have them draw four cards to make two proper fractions. Have them multiply the fractions and then explain their method to a learning partner.
11. Ask the recorder to share the multiplication sentences and answers that he or she has been recording.
12. Have the class discuss any patterns they see. Ask students whether they can come up with a rule for multiplying proper fractions. Record this rule symbolically.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:
- ❒ Connect the concept of multiplying whole numbers to that of fractions.
- ❒ Represent multiplication in a variety of ways.
- ❒ Multiply proper fractions concretely, pictorially, and symbolically.
- ❒ Communicate mathematically.
- ❒ Recognize a pattern in order to generalize a rule.

*Part B: Mixed Numbers*

1. Use a series of steps similar to those outlined in Part A to have students develop strategies for multiplying mixed numbers.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Connect the concept of multiplying whole numbers to that of fractions.
- ❒ Represent multiplication in a variety of ways.
- ❒ Multiply mixed numbers concretely, pictorially, and symbolically.
- ❒ Communicate mathematically.
- ❒ Recognize a pattern in order to generalize a rule.

## Suggestions for Instruction

- **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**
- **Identify and correct errors in the solution to a problem involving positive rational numbers.**

**Materials:** BLM 5–8.12: Fraction Bars or BLM 5–8.19: Double Number Line, BLM 5–8.11: Multiplication Table, BLM 8.N.6.4: Fraction Multiplication and Division, math journals

**Organization:** Whole class/individual

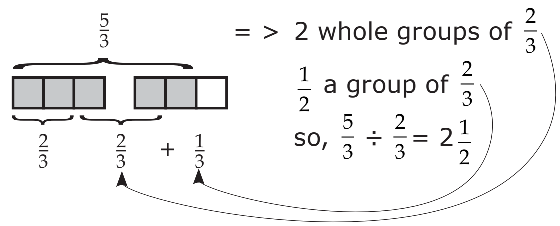
**Procedure:**

1. Tell students that by the end of this lesson, they will be able to determine and apply the rules for multiplying and dividing fractions, including mixed numbers.
2. Review division with students. Start with a whole number example (e.g., for 12 ÷ 3, think, how many groups of 3 go into 12?). Questions like this should put students in the mind frame that they are dividing a number into groups of a particular number.

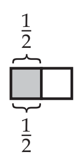
3. Students will need to use fraction bars to answer the following division questions:

| | Division |
|---|---|
| 1. | $\frac{1}{2} \div \frac{1}{2} =$ |
| 2. | $\frac{3}{4} \div \frac{2}{4} =$ |
| 3. | $\frac{4}{5} \div \frac{3}{5} =$ |
| 4. | $\frac{5}{6} \div \frac{4}{6} =$ |
| 5. | $\frac{5}{8} \div \frac{3}{8} =$ |
| 6. | $\frac{7}{10} \div \frac{2}{10} =$ |

**Note:** This concept may be very difficult for students to understand, so go through it slowly, one question at a time. The key is that the fraction becomes the portion of the divisor that is left over. (For example, in $\frac{5}{3} \div \frac{2}{3}$, $\frac{2}{3}$ can go into $\frac{5}{3}$ fully twice, $\frac{1}{2}$ of the $\frac{2}{3}$ is left over.)

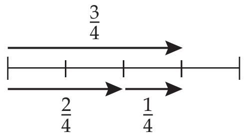



- Question #1 (need to have $\frac{1}{2}$ fraction bars):
  - Students will start out with $\frac{1}{2}$.
  - How many full groups of $\frac{1}{2}$ can you make? (1)
  - How many pieces are left over? (0)
  - So, the quotient is 1.




so, $\frac{1}{2} \div \frac{1}{2} = 1$

- Question #2 (need to have $\frac{1}{4}$ fraction bars):
  - Students will start out with $\frac{3}{4}$.
  - How many full groups of $\frac{2}{4}$ can be made from $\frac{3}{4}$? (1)
  - How many pieces are left over? (1)
  - So, one out of the two fraction bars that make $\frac{2}{4}$ is left over, or $\frac{1}{2}$ of the two fraction bars is left over. Therefore, the quotient is $1\frac{1}{2}$.

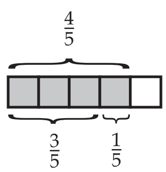



=> 1 whole group of $\frac{2}{4}$

$\frac{1}{2}$ of a group of $\frac{2}{4}$

so, $\frac{3}{4} \div \frac{2}{4} = 1\frac{1}{2}$

- Question #3 (need to have $\frac{1}{5}$ fraction bars):
  - Students will start out with $\frac{4}{5}$.
  - How many full groups of $\frac{3}{5}$ can you make? (1)
  - How many pieces are left over? (1)
  - So, one out of the three fractions bars that make $\frac{3}{5}$ is left over, or $\frac{1}{3}$ of the three fraction bars is left over. Therefore, the quotient is $1\frac{1}{3}$.

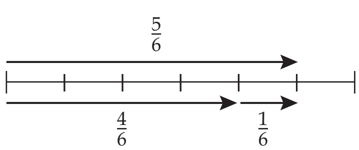



=> 1 whole group of $\frac{3}{5}$

$\frac{1}{3}$ of a group of $\frac{3}{5}$

so, $\frac{4}{5} \div \frac{3}{5} = 1\frac{1}{3}$

- Question #4 (need to have $\frac{1}{6}$ fraction bars):
  - Students will start out with $\frac{5}{6}$.
  - How many full groups of $\frac{4}{6}$ can you make? (1)
  - How many pieces are left over? (1)
  - So, one out of the four fraction bars that make $\frac{4}{6}$ is left over, or $\frac{1}{4}$ of the four fraction bars is left over. Therefore, the quotient is $1\frac{1}{4}$.

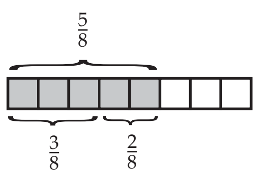



=> 1 whole group of $\frac{4}{6}$

$\frac{1}{4}$ of a group of $\frac{4}{6}$

so, $\frac{5}{6} \div \frac{4}{6} = 1\frac{1}{4}$

- Question #5 (need to have $\frac{1}{8}$ fraction bars):
  - Students will start out with $\frac{5}{8}$.
  - How many full groups of $\frac{3}{8}$ can you make? (1)
  - How many pieces are left over? (2)
  - So, two out of the three fraction bars that make $\frac{3}{8}$ are left over, or $\frac{2}{3}$ of the three fraction bars is left over. Therefore, the quotient is $1\frac{2}{3}$.

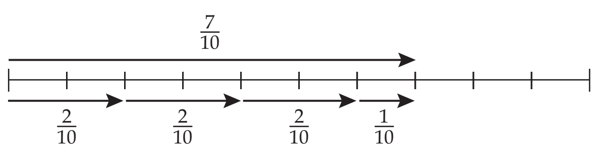



=> 1 whole group of $\frac{3}{8}$

$\frac{2}{3}$ of a group of $\frac{3}{8}$

so, $\frac{5}{8} \div \frac{3}{8} = 1\frac{2}{3}$

- Question #6 (need to have $\frac{1}{10}$ fraction bars):
  - Students will start out with $\frac{7}{10}$.
  - How many full groups of $\frac{2}{10}$ can you make? (3)
  - How many pieces are left over? (1)
  - So, one out of the two fraction bars that make $\frac{2}{10}$ is left over, or $\frac{1}{2}$ of the two fraction bars is left over. Therefore, the quotient is $3\frac{1}{2}$.



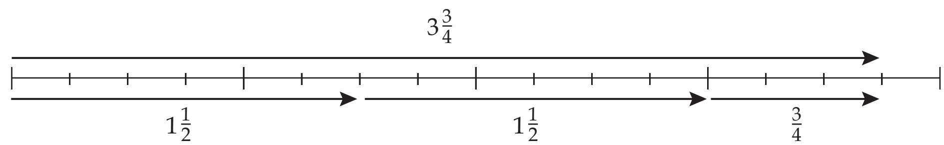

=> 3 whole groups of $\frac{2}{10}$

$\frac{1}{2}$ of a group of $\frac{2}{10}$

so, $\frac{7}{10} \div \frac{2}{10} = 3\frac{1}{2}$

4. Once students have solved the division questions, look at the multiplication questions. Students should already be familiar with a method for multiplication of fractions—numerator times numerator, denominator times denominator. Find the products from BLM 8.N.6.4: Fraction Multiplication and Division and have students draw a conclusion.
5. Ask students whether they can see a connection between the division statements and the multiplication statements. Hopefully, they will notice the method that if you multiply by the reciprocal, you end up with the same result as when you divide.

6. Ask students how that method can be applied to mixed numbers. (You need to change the mixed numbers to improper fractions and multiply by the reciprocal.)
7. Ask students to respond to the following question in their math journals: Jenna showed the following when calculating $3\frac{3}{4} \div 1\frac{1}{2}$.




$$3\frac{3}{4} \div 1\frac{1}{2} = 2\frac{3}{4}$$

Do you agree with Jenna's thought process? Explain your response using words, symbols, and diagrams.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Multiply proper fractions concretely.
- ❒ Multiply proper fractions pictorially.
- ❒ Multiply proper fractions symbolically.
- ❒ Multiply mixed numbers/improper fractions symbolically.
- ❒ Divide proper fractions concretely.
- ❒ Divide proper fractions pictorially.
- ❒ Divide proper fractions symbolically.
- ❒ Divide mixed numbers/improper fractions symbolically.

## Suggestions for Instruction

- **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**

**Materials:** BLM 8.N.6.5: Multiplying and Dividing Proper Fractions, Improper Fractions, and Mixed Numbers

**Organization:** Individual

**Procedure:**

1. Have students, individually, complete BLM 8.N.6.5: Multiplying and Dividing Proper Fractions, Improper Fractions, and Mixed Numbers.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Multiply proper fractions symbolically.
- ❒ Multiply improper fractions symbolically.
- ❒ Multiply mixed numbers symbolically.
- ❒ Divide proper fractions symbolically.
- ❒ Divide improper fractions symbolically.
- ❒ Divide mixed numbers symbolically.

## Suggestions for Instruction

- **Identify the operation(s) required to solve a problem involving positive fractions.**
- **Provide a context involving the multiplying of two positive fractions.**
- **Provide a context involving the dividing of two positive fractions.**
- **Identify the operation(s) required to solve a problem involving positive rational numbers.**
- **Determine the reasonableness of an answer to a problem involving positive rational numbers.**

**Materials:** BLM 8.N.6.6: Fraction Operations, chart paper, markers, math journals

**Organization:** Small group/whole class/individual

**Procedure:**

1. Have students form small groups, and provide each group with a copy of BLM 8.N.6.6: Fraction Operations. Explain that each group must
   - decide, as a group, what operation would be required for each problem and explain their reasoning
   - solve each problem
   - determine whether the solution seems reasonable and explain why they think their answer is reasonable
   - record their work on chart paper
2. Have groups present their work to the class, identifying various strategies they used to solve the problems. Observe whether there is a consensus as to which operation was used for each problem.
3. Provide opportunities for other students to add to and ask questions of the presenting groups.
4. Ask groups to create their own scenarios in which the multiplication and/or division of fractions is needed.
5. Ask each student to select two problems to solve, one that requires multiplication and one that requires division.
6. Have students solve these questions in their math journals, showing their work and discussing the reasonableness of their answers.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve problems involving multiplication of fractions.
- ❒ Solve problems involving the division of fractions.
- ❒ Correctly identify the operation needed to solve a question involving fractions.
- ❒ Use reasoning to determine the reasonableness of an answer.
- ❒ Use mental mathematics and estimation during calculations.
- ❒ Provide a context involving the multiplication of two positive fractions.
- ❒ Provide a context involving the division of two positive fractions.

# NOTES

# **Number**—8.N.7

**Enduring Understandings:**

The principles of operations used with whole numbers also apply to operations with decimals, fractions, and integers.

A positive integer and a negative integer are opposites when they are the same distance from zero on a number line.

The sum of two opposite numbers is 0.

**General Learning Outcome:**

Develop number sense.

| Specific Learning Outcome(s): | Achievement Indicators: |
|---|---|
| **8.N.7** Demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically. [C, CN, PS, R, V] | ➢ Identify the operation(s) required to solve a problem involving integers.<br>➢ Provide a context that requires multiplying two integers.<br>➢ Provide a context that requires dividing two integers.<br>➢ Model the process of multiplying two integers using concrete materials or pictorial representations, and record the process.<br>➢ Model the process of dividing an integer by an integer using concrete materials or pictorial representations, and record the process.<br>➢ Generalize and apply a rule for determining the sign of the product or quotient of integers.<br>➢ Solve a problem involving integers, taking into consideration order of operations. |

## PRIOR KNOWLEDGE

Students may have had experience with the following:

- Demonstrating an understanding of integers, concretely, pictorially, and symbolically
- Explaining and applying the order of operations, excluding exponents (limited to whole numbers)
- Demonstrating an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically

## BACKGROUND INFORMATION

### Multiplication and Division of Integers

When multiplying two numbers to get a product, the numbers being multiplied are called *factors*. For example, in 4 • 2 = 8, 4 and 2 are factors. Regardless of what order the factors are written, the product is the same (4 • 2 = 8 or 2 • 4 = 8). This is called the *commutative property*.

**commutative property**

A number property that states that an operation (addition or multiplication) is unaffected by the order in which the terms are added or multiplied.

*Examples:*

**Addition**

The sum remains the same (e.g., 2 + 3.5 = 3.5 + 2).

**Multiplication**

The product remains the same (e.g., 3 • 5 = 5 • 3).

Applying the commutative property to integers:

- (+4) • (–2) = (–8)

  This can be described as having four groups of –2, or (–2) + (–2) + (–2) + (–2).
- (–2) • (+4) = (–8)

  This can be described as having the negative of two groups of 4, or –[(+4) + (+4)].

Both (+4) • (–2) and (–2) • (+4) have the same value.

An *opposite integer* is an integer that, when added to another, creates a sum of zero. This is called the *zero principle*. *Opposite integers* are integers that are of equal distance from zero on a number line. Opposite integers are sometimes referred to as *zero pairs*. For example, –3 and +3 are opposite integers. You can also think of the multiplication scenario of –2 • 4 as the opposite of 2 times 4, and since 2 times 4 is 8, the opposite of that must be –8.

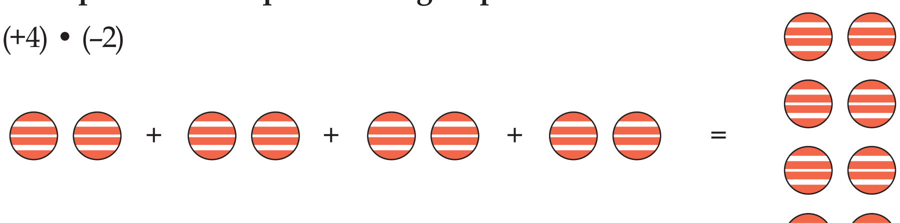
Integer disks or number lines can be used to explore and model the product or quotient of integers.

## Multiplication and Division Examples

In the following examples, the blue (solid) bingo chips represent positive integers and the red (striped) bingo chips represent negative integers. A zero pair is the pair made by –1 and +1
( and ) since its sum is zero.

- **Multiplication as equal sets or groups:**

(+4) • (–2)

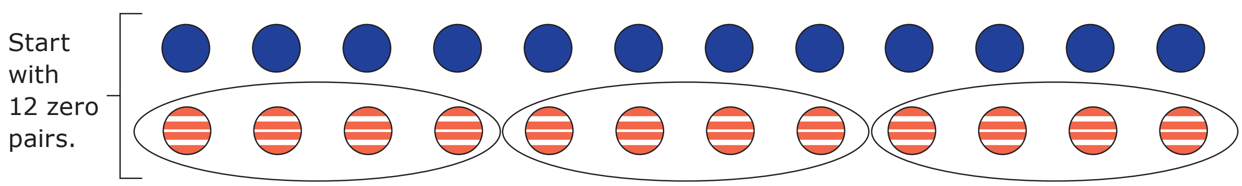



(+4) • (–2) = (–8)

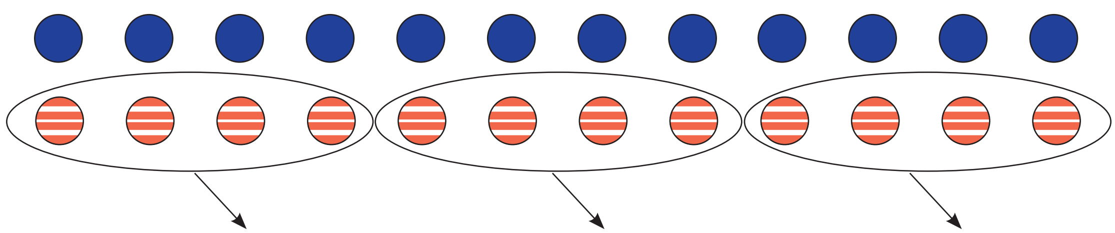
(–3) • (–4)

This is like saying three groups of –4 are being removed.

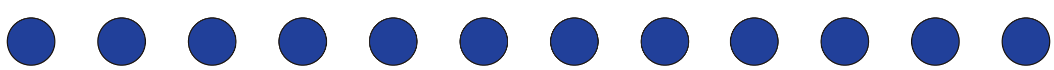
In order to do this, you need to start with zero, yet you must have enough zeros (or zero pairs) so that you can take away three groups of –4.




Take away three groups of –4.



You are left with +12.



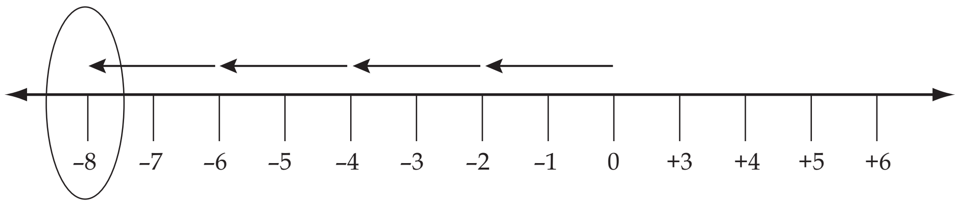
(–3) • (–4) = (+12)

(–3) • (–4) could also be thought of as the opposite of three groups of –4.

- **Multiplication as repeated addition:**

  $(+4) \bullet (-2)$

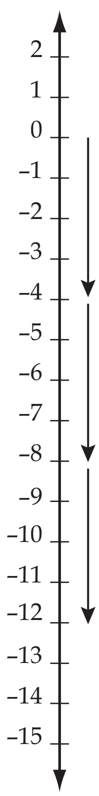
  $-2 \quad + \quad -2 \quad + \quad -2 \quad + \quad -2$




  $(+4) \bullet (-2) = (-8)$

  $(-3) \bullet (-4)$

This is like saying, the opposite of three groups of –4, and since three groups of –4 is –12, the opposite of that would be +12.



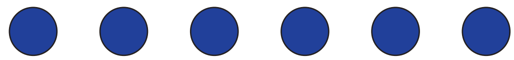

$3 \bullet -4$ is $-12$

so, $-3 \bullet -4$ is $+12$

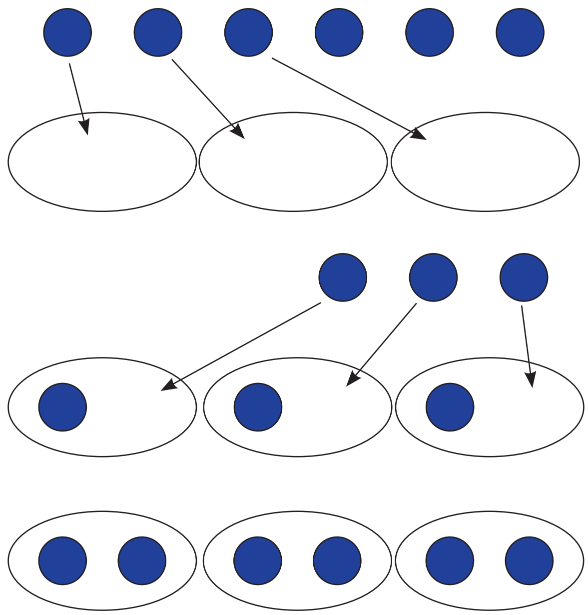
$(-3) \bullet (-4) = (+12)$

- **Division as equal sharing:**

  (+6) ÷ (+3)



  This can be interpreted as +6 shared equally among +3 groups.

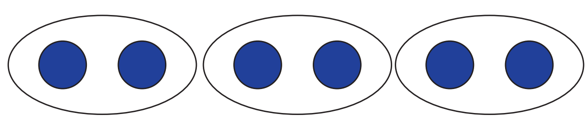


  (+6) ÷ (+3) = (+2)

- **Division as equal grouping:**

  (+6) ÷ (+3)

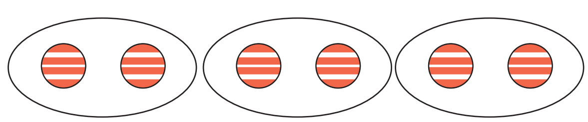
  This can be interpreted as 6 divided into three equal groups. How much is in each group?



  (+6) ÷ (+3) = (+2)

  (–6) ÷ (+3)

  This can be interpreted as –6 divided into three equal groups. How much is in each group?



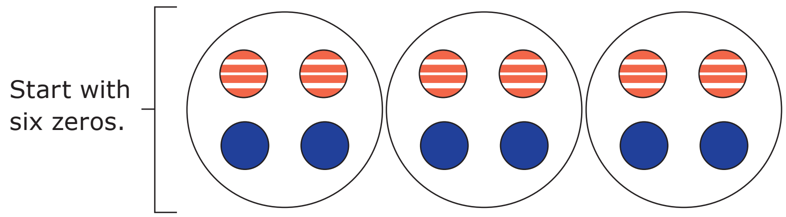
  (–6) ÷ (+3) = (–2)

- **Division as repeated subtraction:**

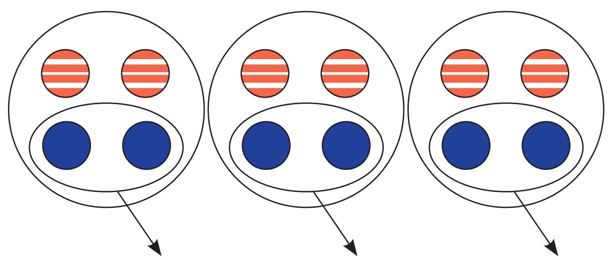
  (+6) ÷ (–3)

  Remember that when you multiplied a negative number by a negative number, it was the same as repeated subtraction. Division of negative numbers is similar. When you are dividing a positive number by a negative number, you will be removing the opposite number from the group, and determining what is left. Once again, you must start with zero before you can remove any integers, and then determine what is left in each group.

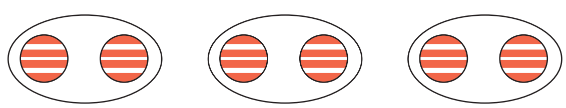
  For example, with +6 ÷ –3, you are dividing the six zeros into three groups.




  Each group has two zeros. From each zero group, you are removing groups of positive numbers.



  The result is that each group has –2 left.

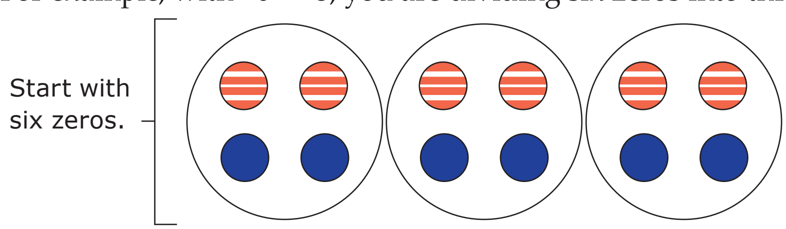


  (+6) ÷ (–3) = (–2)

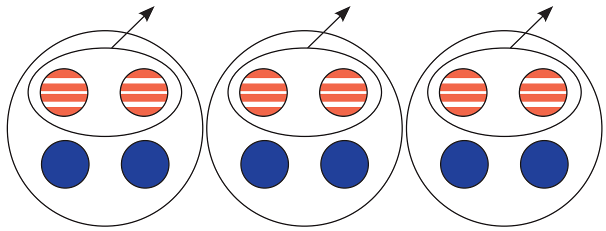
  (–6) ÷ (–3)

  When you are dividing a negative number by a negative number, you are removing groups of negative numbers from the zeros and will end up with positive groups of numbers.

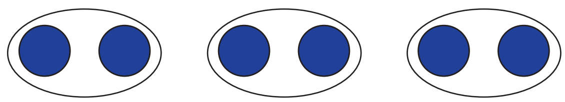
  For example, with –6 ÷ –3, you are dividing six zeros into three groups.



Each group has two zeros. From each zero group, you are removing groups of negative numbers.



The result is that each group has +2 left.



(–6) ÷ (–3) = (+2)

- **Division using fact families:**

  You can also think of division with integers with respect to their fact families. A *fact family* is anything that is true of a set of numbers.

  *Example:*

  3 • 5 = 15; 5 • 3 = 15; 15 ÷ 5 = 3; 15 ÷ 3 = 5

  *Example:*

  (–6) ÷ (–3)

  So, if students have an easier time understanding why multiplying integers makes sense, work with that knowledge and students' knowledge of fact families to build a generalization.

  (–6) ÷ (–3) = ?

  Students may think: (–3) • ? = (–6)

  (–3) • (2) = –6; (2) • (–3) = –6; (–6) ÷ (2) = –3; (–6) ÷ (–3) = (2)

  Therefore, (–6) ÷ (–3) = 2

## Mathematical Language

commutative property
integer
opposite integer
order of operations
zero pair
zero principle

**Assessing Prior Knowledge**

**Materials:** Positive and negative integer disks or bingo chips, BLM 8.N.7.1: Integer Pre-Assessment

**Organization:** Individual

**Procedure:**

1. Tell students that they will be extending their understanding of integers; however, you first need to determine what they already know about integers.
2. Provide each student with a group of positive and negative integer disks, as well as a copy of BLM 8.N.7.1: Integer Pre-Assessment. Ask students to complete this BLM.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Demonstrate that zeros do not change the value of integers.
- ❒ Add integers with like signs.
- ❒ Add integers with opposite signs.
- ❒ Subtract integers with like signs when the first term is larger than the second term.
- ❒ Subtract integers with like signs when the first term is smaller than the second term.
- ❒ Subtract integers with opposite signs.

Suggestions for Instruction

- **Model the process of multiplying two integers using concrete materials or pictorial representations, and record the process.**
- **Generalize and apply a rule for determining the sign of the product or quotient of integers.**

**Materials:** Positive and negative integer disks, math journals

**Organization:** Whole class/individual/pairs

**Procedure:**

1. Review the concept of multiplication by asking students how they can represent 4 • 2. Include a discussion of commutative property, multiplication as repeated addition, and multiplication as groups/sets of. Include representations using integer disks and number lines.
2. Ask students what kind of numbers 4 and 2 are from the previous question. Discuss that they are positive integers.
3. Ask students whether they can apply their understanding of multiplication of positive integers to model (+5) • ( –3). Think five groups of –3 or (–3) + (–3) + (–3) + (–3) + (–3).
4. Ask students whether they can apply their understanding of multiplication of positive integers to model (–5) • (+3). Discuss their thinking and bring in the definition of commutative property, so that the question can be understood as (+3) • (–5) or three groups of –5.
5. Have students use their integer disks to model the following: (+5) • (+2), (+3) • (–4), (–2) • (+6). Ask students to look at the signs of the terms and then look at the sign of the products. What observations can students make?
6. Ask students to use integer disks to model the following: (–4) • (–3). Discuss their thinking. Record students’ responses on the whiteboard. With each response, discuss the pros and cons to determine ways to model (–4) • (–3).
7. Ask students to model the following: (+6) • (+2), (–4) • (+3), (+2) • (–2), (–3) • (–5). Have them record their process in their math journals using pictures, words, symbols, and numbers. Ask them to generalize a rule for determining the sign of the product of two integers.

**Note:** A possible method is to look at multiplying negative integers as repeated subtraction. Start with 12 zeros to model this statement and remove three groups of −4 from the zeros. What is left? Have students look at the signs of the terms and then look at the sign of the quotient. What do they notice?

8. Have students create four new multiplication sentences similar to those above, and exchange sentences with a partner. Have students determine the products symbolically.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use a model to show the multiplication of integers.
- ❒ Record the process of multiplication of integers.
- ❒ Generalize a rule for determining the sign of the product of integers.
- ❒ Apply a rule to determine the sign of the product of integers.

## Suggestions for Instruction

- **Provide a context that requires multiplying two integers.**
- **Solve a problem involving integers, taking into consideration order of operations.**

**Materials:** BLM 8.N.7.2: Solving Problems with Integers (A), chart paper, math journals

**Organization:** Small group/whole class/individual

**Procedure:**

1. Have students form small groups, and provide each group with a copy of BLM 8.N.7.2: Solving Problems with Integers (A).
2. Ask the groups to record their responses to the problems on chart paper and be prepared to share their solutions with the rest of the class. In their presentations, they must be able to explain why they chose to solve the problems the way they did.
3. As a class, discuss the solutions that the groups present. Allow opportunities to discuss different ways of solving the problems.
4. Ask students to create and solve their own problem requiring multiplication of integers. Have them record their work in their math journals.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a real-world scenario in which the multiplication of integers is required.
- ❒ Solve problems requiring the multiplication of integers.
- ❒ Communicate problem-solving strategies.

## Suggestions for Instruction

- **Model the process of dividing an integer by an integer using concrete materials or pictorial representations, and record the process.**
- **Generalize and apply a rule for determining the sign of the product or quotient of integers.**

**Materials:** Positive and negative integer disks, math journals

**Organization:** Whole class/individual

**Procedure:**

1. Review the concept of division by asking students how they can represent (+24) ÷ (+4).
2. Record all the different ways students represent 24 ÷ 4. Make sure that using integer disks is one way in which students model the division.
3. Ask students what kind of numbers 24 and 4 are from the previous question. In the discussion, indicate that they are positive integers.
4. Ask students to predict, using their knowledge of the multiplication of integers, what some rules might be for division of integers. Tell students that you will work with them to see whether their rules work.
5. Ask students whether they can apply their understanding of the division of positive integers to model (–15) ÷ (+3) using integer disks. Discuss their thinking, and explain that the model can be interpreted as the value of each group when –15 is divided into three groups.

6. Ask students how the following can be interpreted: (+15) ÷ (–3). In the discussion, explain that dividing by a negative number is like subtracting from a group. (Refer to the explanation in the Background Information.) Demonstrate how integer disks can be used to model division by starting with zero, removing a number from the group, and examining the result. Ask students to look at the signs of the terms and then look at the sign of the quotients. What observations can students make?
7. Set up the following and ask students to make connections.

| Multiplication | Division |
|---|---|
| (+4) • (+2) = (+8) | (+8) ÷ (+2) = (+4) |
| (+4) • (–2) = (–8) | (+8) ÷ (–2) = (–4) |
| (–4) • (+2) = (–8) | (–8) ÷ (+2) = (–4) |
| (–4) • (–2) = (+8) | (–8) ÷ (–2) = (+4) |

See whether students can come up with the following generalizations:

| Multiplication | Division |
|---|---|
| Positive times a positive = positive | Positive divided by a positive = positive |
| Positive times a negative = negative | Positive divided by a negative = negative |
| Negative times a positive = negative | Negative divided by a positive = negative |
| Negative times a negative = positive | Negative divided by a negative = positive |

8. Ask students to model the following: (+9) ÷ (+3), (+12) ÷ (–4), (–10) ÷ (+2), (–15) ÷ (–5). Have them record their process in their math journals. Ask them to write a rule for determining the sign of the product and quotient of integers.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:
- ❒ Use integer disks to model division of integers.
- ❒ Record the process of division of integers pictorially and symbolically.
- ❒ Generalize a rule for determining the sign of the quotient of integers.
- ❒ Apply a rule to determine the sign of the quotient of integers.

## Suggestions for Instruction

- **Provide a context that requires dividing two integers.**
- **Solve a problem involving integers, taking into consideration order of operations**

**Materials:** BLM 8.N.7.3: Solving Problems with Integers (B), chart paper, math journals

**Organization:** Small group/whole class/individual

**Procedure:**

1. Have students form small groups, and provide each group with a copy of BLM 8.N.7.3: Solving Problems with Integers (B).
2. Ask the groups to record their responses to the problems on chart paper and be prepared to share their solutions with the rest of the class. In their presentations, they must be able to explain why they chose to solve the problems the way they did.
3. As a class, discuss the solutions that the groups present. Allow opportunities to discuss different ways of solving the problems.
4. Ask students to create and solve their own problem requiring the division of integers. Have them record their work in their math journals.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a real-world scenario in which the division of integers is required.
- ❒ Solve problems requiring the division of integers.
- ❒ Communicate problem-solving strategies.

- **Solve a problem involving integers, taking into consideration order of operations.**
- **Identify the operation(s) required to solve a problem involving integers.**

**Materials:** BLM 8.N.7.4: Solving Problems with Integers (C), chart paper, math journals

**Organization:** Small group/whole class/individual

**Procedure:**

1. Have students form small groups, and provide each group with a copy of BLM 8.N.7.4: Solving Problems with Integers (C).
2. Ask the groups to record their responses to the problems on chart paper and be prepared to share their solutions with the rest of the class. In their presentations, they must be able to explain why they chose to solve the problems the way they did.
3. As a class, discuss the solutions that the groups present. Allow opportunities to discuss different ways of solving the problems.
4. Ask students to create and solve their own problem requiring the order of operations with integers. Have them record their work in their math journals.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a real-world scenario in which the order of operations with integers is required.
- ❒ Solve problems requiring the order of operations with integers.
- ❒ Identify the operation(s) needed to solve an integer problem.
- ❒ Communicate mathematically to solve problems.

- **Generalize and apply a rule for determining the sign of the product or quotient of integers.**

**Materials:** Deck of cards—Ace = 1, Jack = 11, Queen = 12, King = 0, other cards at face value (assign black cards as positive and red cards as negative)

**Organization:** Pairs

**Procedure:**

1. Pair up students in the class, with one student on each side of a desk. Arrange desks in a circle with one student on the inside of each desk and one student on the outside. Tell students they will play an integer multiplication game.
2. Demonstrate how to play the integer multiplication game.
   - Two players divide cards evenly between themselves.
   - The two players each turn over a card simultaneously and multiply the face value of the two cards. The fastest responder with the correct answer wins the hand. In the event of a tie, students turn over two more cards until one player says the correct answer out loud.
   - After about five minutes of play, the students in the inner circle rotate to the right to challenge a new set of students. Students can also challenge individuals in the room who they believe are at the same level as they are.

**Variation:** Students could play a variation of the game.

- Two players divide cards evenly between themselves.
- Students each turn over two cards simultaneously. Each student multiplies his or her own cards and the person with the greatest product collects all four cards. In the event of a tie (same answer), players turn over two more cards and multiply them.

**Observation Checklist**

☑ Observe students’ responses to determine whether they can do the following:

- ❐ Multiply positive and negative integers.
- ❐ Determine the greatest value of products.

- **Generalize and apply a rule for determining the sign of the product or quotient of integers.**
- **Identify the operation(s) required to solve a problem involving integers.**

**Materials:** Ten-sided number cube and integer disks (1 each per group), BLM 8.N.7.5: Number Line Race

**Organization:** Whole class/pairs

**Procedure:**

1. Have students form pairs, and provide each pair with a number cube and integer disk, as well as a copy of BLM 8.N.7.5: Number Line Race.
2. Tell students that they will be playing a game called *Number Line Race*. The object of the game is to cross out all numbers on a number line before their opponent does.
3. Demonstrate to the class how to play the game.
   - Player A rolls the number cube and flips the integer disk, and player B records the number in the Numbers Rolled column of his or her own chart (e.g., a 7 on the number cube and the chip lying red face up would be –7).
   - Repeat two more times, until the pair has recorded three integers.
   - Players A and B work individually for one minute using two or three of the integers and the order of operations to make as many integers as they can, recording all work in the Numbers Found column of their own charts.
   - Each player will cross out the numbers found on his or her own number line.
   - Play continues until one player has crossed out all numbers on the number line.
4. Have students play the game.

**Observation Checklist**

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply a rule for determining the sign of the product or quotient of integers.
- ❒ Identify the order of operations needed to make the needed numbers on their number line.
- ❒ Apply mental mathematics and reasoning skills while playing the game.