GRADE 7 MATHEMATICS

Shape and Space

Shape and Space (Measurement) (7.SS.1)

Enduring Understanding(s):

Circle graphs show a comparison of each part to a whole using ratios.

Many geometric properties and attributes of shapes are related to measurement.

General Learning Outcome(s):

Use direct or indirect measurement to solve problems.

Specific Learning Outcome(s):	**Achievement Indicators:**
7.SS.1 Demonstrate an understanding of circles by ■ describing the relationships among radius, diameter, and circumference of circles ■ relating circumference to pi (π) ■ determining the sum of the central angles ■ constructing circles with a given radius or diameter ■ solving problems involving the radii, diameters, and circumferences of circles [C, CN, R, V]	➛ Illustrate and explain that the diameter is twice the radius in a circle. ➛ Illustrate and explain that the circumference is approximately three times the diameter in a circle. ➛ Explain that, for all circles, pi (π) is the ratio of the circumference to the diameter $\left(\frac{C}{d}\right)$, and its value is approximately 3.14. ➛ Explain, using an illustration, that the sum of the central angles of a circle is 360°. ➛ Draw a circle with a given radius or diameter with or without a compass. ➛ Solve a contextual problem involving circles.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of perimeter of regular and irregular shapes by
  - estimating perimeter using referents for centimetre or metre
  - measuring and recording perimeter (cm, m)
  - constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter
- Demonstrating an understanding of ratio, concretely, pictorially, and symbolically.

- Demonstrating an understanding of angles by
  - identifying examples of angles in the environment
  - classifying angles according to their measure
  - estimating the measure of angles using 45°, 90°, and 180° as reference angles
  - determining angle measures in degrees
  - drawing and labelling angles when the measure is specified
- Developing and applying a formula for determining the
  - perimeter of polygons
  - area of rectangles
  - volume of right rectangular prisms

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Constructing a table of values from a relation, graphing the table of values, and analyzing the graph to draw conclusions and solve problems.
- Evaluating an expression given the value of the variable(s).
- Modelling and solving problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers.
- Modelling and solving problems that can be represented by linear equations of the form
  - $ax + b = c$
  - $ax = b$
  - $\frac{x}{a} = b,\ a \neq 0$

  concretely, pictorially, and symbolically, where a, b, and c, are whole numbers.
- Developing and applying a formula for determining the area of
  - triangles
  - parallelograms
  - circles
- Constructing, labelling, and interpreting circle graphs to solve problems.

Background Information

Angles and Circles

Students come to Grade 7 with a background in classifying and measuring angles. *Angles* are formed from two rays emanating from a common point, and *circles* are all the points equidistant from a common point. Angles are measured as fractions of circles, each degree being $\frac{1}{360}$ of a circle. The number 360 is said to be related to Sumerian and Babylonian observations of tracking the movement of astronomical objects through the sky for the 360 days in their year. The number 360 is convenient because it has multiple factors.

In the learning experiences that follow, the close relation between angles and circles is used as a starting point to develop an understanding of circle concepts. A *circle* is the full rotation of an angle. The ability to look for and describe patterns with variables and equations is used to discover the relationships and ratios within circles, and these ratios are used to solve contextual problems. Learning experiences that involve describing the relationships in circles and solving problems involving circles correspond well with the Variables and Equations substrand of the Patterns and Relations strand.

The first concept developed in the following learning experiences is the sum of central angles. Angles are used to develop concepts related to radius, circle, and circumference. The concept of radius is used to construct circles of a given size, and students measure circumference in terms of radius in their first investigation about relationships in circles. The term *diameter* is built from connecting two radii, and further relationships are determined between diameter and circumference. Students develop accurate measuring skills, and discover the ratio between the circumference and the diameter of circles. The ratio is approximated as 3.14, and is commonly referred to as *pi* (π). The relation $\left(\frac{C}{d}\right)$ is a very important mathematical constant. There is evidence of its use in Ancient Egypt, Ancient Babylon, Ancient Israel, and Ancient India. The Ancient Greeks studied the relationship very carefully and represented it as $\frac{22}{7}$. For every circle, the circumference divided by the diameter is a non-terminating non-repeating decimal. In the 1700s, it was given a special name, *pi*. The name was chosen because pi is the first letter in the Greek phrase for perimeter/diameter. It is common to use the approximate value 3.14 for pi. Many people are fascinated with the number that represents pi. There is even a special Pi Day celebrated March 14 (3/14).

You may wish to have students research pi and share the information they find. It is important that students recognize that pi is not so much a special number as it is a special relationship (the relationship of the circumference of a circle divided by its diameter).

Teachers are encouraged to provide hands-on learning activities and group work as a means for students to develop skills and to explore and discover the concepts and relationships within circles. Guide students in their learning and provide vocabulary to describe the concepts, while allowing students to make discoveries. Based on the relationships they discover, students can develop contextual problems for one another to solve.

Mathematical Language

angle
arc
central angle
circle
circumference
classifications of angles (acute, obtuse, reflex, right, straight, complementary, and supplementary)
diameter
pi
radius

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 7.SS.1.1: Assorted Angle Cards (or diagrams of various angles of different sizes, including acute, obtuse, reflex, right, and straight angles)
- BLM 7.SS.1.2: Angle Classifications, Angle Estimations and Measures, and Perimeter
- display board
- tacks, tape, or magnets to hold angle cards on a display
- protractors

Organization: Whole class, individual

Procedure:

1. Advise students they will review what they already know about angles by sorting angle cards into the different classifications of angles. Give each student one of the cards from BLM 7.SS.1: Assorted Angle Cards. Invite students to post their angle cards (or sketch the represented angles) with the similar angles on the display board.

2. Students can critique the angle display and share any adjustments they would like to make to correct it. Review definitions of the different classifications of angles (*acute*, *obtuse*, *reflex*, *right*, and *straight angles*).
3. Return the cards to students. Invite students to use their estimation skills to identify an angle that matches an approximate measurement. They may show their response by displaying a card or by sketching an angle. Ask students how to verify the measurement. Review the use of protractors, and have students measure their angles using protractors.
4. Review the concept of perimeter by having students estimate the measure of the perimeter of designated surfaces (e.g., a tissue box, desk surface, classroom door, window, ceiling), and having them justify their responses. Tracing the edge of objects with a finger reinforces that the perimeter is the distance around the outside of the object.
5. Distribute copies of BLM 7.SS.1.2: Angle Classifications, Angle Estimations and Measures, and Perimeter, and have students respond to the questions provided.

Variations:

- Use the angle cards to play games in which students collect a set of one classification of angles or a set of each classification of angles (acute, obtuse, reflex, right, straight). Or use the cards to play other types of games (e.g., Concentration, Pit).
- Play an I Spy game to develop students' understanding of perimeter (e.g., "I spy a perimeter close to . . .").

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Classify angles of different sizes as acute, obtuse, reflex, right, or straight angles.
- ❒ Estimate the size of angles and confirm the estimate by measuring the angles with a protractor.
- ❒ Demonstrate an understanding of perimeter.

Suggestions for Instruction

- **Explain, using an illustration, that the sum of the central angles of a circle is 360°.**

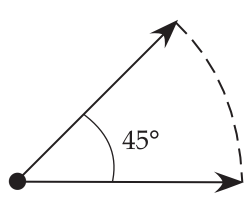
Materials:

- BLM 7.SS.1.3: Cut-outs for Angles of Different Measures (one of each of the six types per group, copied on card stock or on bond paper, plus extra copies if students will make posters)
- demonstration board
- large cut-outs of angles (two copies of 45° and 90° angles, one copy of 180° angle) (optional)
- card stock or bond paper
- scissors
- protractors
- math journals
- file cards
- resealable bags for storage (optional)
- poster paper (optional)
- glue (optional)
- computer software (optional)

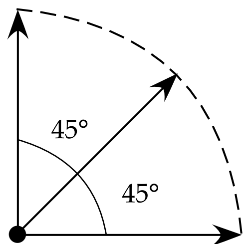
Organization: Whole class, small groups

Procedure:

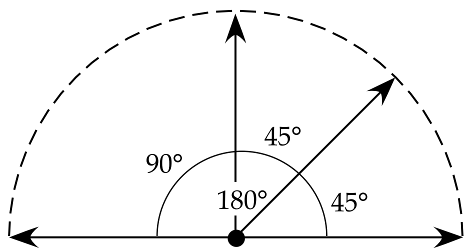
1. Guide students through a class discussion while building a circle using angles. A sample procedure follows:
   - Draw or use a large cut-out of a 45° angle, and have students estimate the measure of the angle. Ask what the diagram resembles if the ends of the rays are connected with an arc.



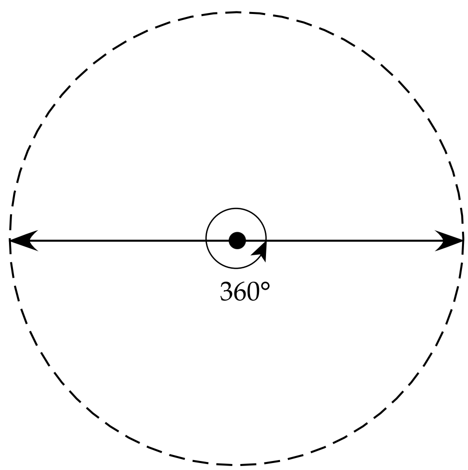
- Stack two angles, one next to the other, with the vertices meeting. Ask for an approximation of the angle represented (90°). Connect the rays with another arc, and ask what the image resembles now.



- Line up a reflection of another 90° angle alongside the image. Continue to ask for an estimation of the measure of the angle (180°), and what the angle resembles.



- Complete the task by adding a 180° angle below the image. Solicit an estimation of the measure and a description of the image (360° circle).



- Point out to students (either now or following some more investigation) that the vertices of all the angles meet at one point in the centre of the circle. The measure of each of the angles is taken from the centre of the circle. These angles are called *central angles*. Each central angle has its vertex at the centre of the circle, and each ray radiates to a different point on the edge of the circle or circumference.

2. Distribute copies of BLM 7.SS.1.3: Cut-outs for Angles of Different Measures, and have groups carry out the following investigation:
   - Each group will need one of each of the differently partitioned circles (halves, thirds, quarters, sixths, eighths, and twelfths). Students can share the circles within their group to even out the number of pieces each student will work with (e.g., thirds and quarters, halves and sixths).
   - Each student accurately measures the angles in the sections and neatly records the angle measures inside each section. Students then carefully cut out each section.
   - Have students combine pieces with angles of different sizes to form circles, and calculate the sum of the angles of the sections that complete the circles. Then repeat the process, constructing a number of circles with central angles of different sizes.
   - After an appropriate time, reassemble as a class, and ask students to share what they have discovered during this investigation. If you did not introduce the term *central angle* earlier, do so now. There are interesting relationships in circles. Ask students what they can conclude about the measures of central angles in a circle (e.g., the sum of the central angles is 360°). Note the connection between angles and circles. Angles are measured as fractions of circles. Each degree is $\frac{1}{360}$ of a circle. Have students add notes to their math journals.

Variations:

- If the technology and skills are available, use computer software to draw, measure, and calculate the sum of angles. Computer applets or games may also be used.

  Sample Website:

  Computer applets are available on the following websites:

  Math Open Reference. *Angles*. 2009. www.mathopenref.com/tocs/anglestoc.html.

  ______. *Circles*. 2009. www.mathopenref.com/tocs/circlestoc.html.

- Have students find the measure of angles between different points on marked circles used in everyday objects (e.g., degrees between points on a compass, minutes or hours on a clock, positions on dials to measure temperature or speed). Results could be displayed in posters, with the measures of the angles between the points, and the sum of all the angles.
- Play a game in which students find the missing angle. Present a circle with one or more central angles. Students determine the measure of the remaining angle. The questions can also be used as Entry Slips or Exit Slips.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Explain, using an illustration, that the sum of the central angles of a circle is 360°.
- ❒ Visualize central angles.
- ❒ Use mental mathematics and estimation strategies to solve problems.

Suggestions for Instruction

- **Explain, using an illustration, that the sum of the central angles of a circle is 360°.**
- **Solve a contextual problem involving circles.**

Materials:

- BLM 7.SS.1.4: Hinge Templates for Making Angles
- scissors
- push-pins (to fasten the hinges)
- corrugated cardboard (on which to pin the centre of the circle)
- protractors
- number cubes, multi-sided number cubes, or spinners
- blank paper
- pencils
- string (optional)
- masking tape or chalk (optional)
- computer software (optional)

Organization: Pairs, whole class

Procedure:

1. Have pairs of students take turns building a circle from joined angles and calculating the sum of the angles. A suggested procedure follows:
   a) Students throw a number cube to determine who makes the first angle, and then throw it again to indicate the numbers of angles (turns) to make. Students place blank paper on the cardboard, mark a centre point for the circle, and push a push-pin through the hinge to keep it in the centre of the circle.
   b) The first person marks the edge of the hinge, opens the angle to a desired size, marks the position of the angle, and uses a protractor to measure the angle formed.
   c) The second person records the angle and keeps a running sum of the angle measures.
   d) The partners then switch roles. The second person begins from the last mark, forms an angle, marks the edge point, and measures the angle. The first person records the angle measure and adds the measure to the sum of the angles.
   e) Partners continue switching roles until they have returned to the starting point or have formed the designated number of angles.
   f) Students record the sum of the angles and perform another round.
2. Meet as a class, and have students report on the sums of the central angles that were obtained during the investigation. Compare the sums of the angles for each circle. Ask why the sums are close to, but not exactly, 360°, and what changes could be made to the procedure to increase accuracy. Sources of error could include errors in lining up and marking the hinges, in using the protractor, or in making calculations, variations in the thickness of pencil lines or the position on the hinge used for marking, and so on. If there is sufficient interest, and time is available, challenge students to repeat the investigation and try to eliminate the sources of error.
3. Present the following problem for students to solve:

   A pizza was sitting on top of the stove. Jack cut out a piece of pizza and ate it. The central angle of the missing piece was 45°. Lisa came by, sliced some pizza, ate it, and left. The central angle of the remaining pizza was 90°. How much of the pizza did Lisa eat?

Variation:

- Investigate the sum of central angles using computer applets.

  Sample Website:

  Computer applets are available on the following website:

  Math Open Reference. "Central Angle." *Circles*. 2009. www.mathopenref.com/circlecentral.html.

  Drag a point on the circumference to make central angles of different sizes.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Explain, using an illustration, that the sum of the central angles of a circle is 360°.
- ❒ Solve a contextual problem involving circles.
- ❒ Reason mathematically to solve problems.

Suggestions for Instruction

- **Illustrate and explain that the diameter is twice the radius in a circle.**
- **Illustrate and explain that the circumference is approximately three times the diameter in a circle.**
- **Draw a circle with a given radius or diameter with or without a compass.**

Part A:

Materials:

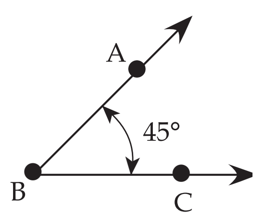
- math journals
- materials to mark a trail as a large circle is formed (e.g., sidewalk chalk for hard surfaces, a stick for gravel or soil, a bag of flour, puffed rice, or popcorn for lawns, books, self-stick notes, or scraps from a hole puncher for indoor floors)

Organization: Whole class, small groups (of five students)

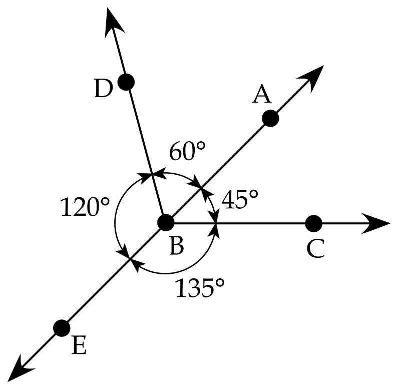
Procedure:

Review that the sum of central angles is 360°, and develop definitions of the terms *radius*, *circle*, and *circumference* by constructing circles, as described below.

1. Have students do the following:
   a) sketch an angle, making the rays fairly long. Label the angle ∠ABC. Write the approximate measure of the angle near the vertex.



b) Add two adjacent angles that each share an arm of the original angle and the vertex B. To label the new angles, add the points D and E. There are now four adjacent angles.



c) Estimate the measure of each angle. Add the measures of the central angles. Determine whether they are close to 360°, and if not, explain why not.

2. Have students draw a circle that passes through the arms of each angle. Then have them ask a partner to rate the roundness of the circle on a scale of 1 to 10.
3. Together with students, describe the criteria for a perfect circle. For example, the distance from the centre to the outside of the circle must always be same. Inform students that distance is called a *radius*. The measure for every radius of the same circle is identical. In fact, a *circle* is the set of points on a flat surface equal distances from a fixed point. The distance around those points (or the perimeter of the circle) is called the *circumference*.
4. Have each student make a math journal entry to define the terms *radius*, *circle*, and *circumference*.
5. Place students into groups of five. Have each group make a plan for creating a large nearly perfect circle (outdoors, in the gymnasium, or wherever space is available). Five students may make a circle with a diameter of 12 m or more, so ensure there is sufficient space for the class. Inform groups of the materials that will be available to them, and discuss the cleanup requirements. If students are having difficulty getting started, or think only of holding hands and spreading out, suggest they think of forming a radius. Students holding hands at arm's length can create quite a long radius. The centre person must be anchored securely to avoid being pulled out of position as the students forming the radius pivot around the centre. The last person can leave a trail for the circumference. Alternatively, make one large circle, have five or six students form the radius, and ask the other students to stand as markers around the circumference as the circle is formed.

6. When students have completed the circle, have them carefully step outside their circle and evaluate it. Recalling the power and beauty that can exist in discovering patterns, ask students to relate the radius of the circle to its circumference. Measuring the circumference in terms of the radius is a good way to do this. Have students do the following:
   - Clearly mark a starting point on the circle.
   - Line up the radius along the circumference, beginning at the starting point.
   - Fold the radius over on itself until it returns to the starting point, counting the number of radii at each fold.

   Have students share how many lengths of the radius the circumference of their circles is (approximately six). Have students make larger and smaller circles and test the relationship between the circumference and the radius.
7. Return indoors and have students make a math journal entry about what they learned in this learning activity.

Part B:

Materials:

- math journals
- corrugated cardboard, large paper, or other media on which to draw large circles
- string or light-gauge wire
- push-pins or nails with a large head
- compasses and pencils
- rulers, metre sticks, tape measures, and trundle wheels

Organization: Pairs or small groups

Procedure:

In Part B of this learning experience, students build on what they learned in Part A to enable them to draw circles of a given radius. They also learn about diameter and its relation to the radius and the circumference of a circle.

1. Review what students have written in their math journals about radius, circle, circumference, and how to build a perfect circle. Have them use their math journals to sketch a rough circle and draw two radii for that circle. Point out that they have drawn a central angle. By drawing more radii, they add more central angles. Ask students to draw a circle with two radii that are perfectly lined up with each other. They form an angle of 180°. This arrangement of radii is the *diameter* of the circle. Have students do the following:
   - Describe the diameter of a circle. It is a straight line passing through the centre of the circle.
   - Describe the relationship between the radius and the diameter. The diameter is twice as long as the radius, $d = 2r$. (In Part A, the relationship between the radius and the circumference was found to be $\sim 6r = C$.)

- Predict the relationship between the diameter and the circumference.
- Find a way to test the prediction.
- Make a math journal entry to define *diameter*.

2. Direct students to the materials available to make large circles. Explain that their task is to develop a method to draw circles of a given radius or diameter. The circle size can be determined by the teacher, by a partner, or by the students themselves. Have students make small and large circles. When students have had some time to explore, ask them to share their success stories and frustrations, so they can help each other and refine their methods. Some students may use string, and may need to learn to tie a slip knot. Some students may use a strip of cardboard with a hole for the centre and hole for a pencil. They may have creative ideas.
3. Continue the exploration. Aim for producing circles of a specific radius or diameter. Have students compare the size of the circle drawn to the intended size by measuring the radius and/or diameter with a ruler or a metre stick. If possible, also investigate the relationship between the length of the diameter and the circumference to test the predictions that were made.
4. After students have had sufficient time to explore, congratulate them on their success. They will have invented some effective methods to draw circles accurately, especially large ones.
5. Introduce students to the compass in their geometry kits, and show them how to use it. The metal point is held at the centre of the circle, and the pencil point will form marks along the circumference as it revolves around the centre point. The distance between the metal point and the pencil point will be the radius. Have them draw multiple circles of different sizes to perfect the technique.
6. To end the class, have students draw two circles in their math journals. Specify the radius for one of the circles, and the diameter for the other. Ask them to comment on anything they found out about the relationship between the diameter and the circumference.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Illustrate and explain that the diameter is twice the radius in a circle.
- ❒ Illustrate and explain that the circumference is approximately three times the diameter in a circle.
- ❒ Draw a circle with a given radius or diameter with or without a compass.

Suggestions for Instruction

- **Illustrate and explain that the circumference is approximately three times the diameter in a circle.**
- **Explain that, for all circles, pi (π) is the ratio of the circumference to the diameter $\left(\frac{C}{d}\right)$, and its value is approximately 3.14.**

Materials:

- compasses and pencils
- rulers, metre sticks, tape measures, and trundle wheels
- measuring tape
- string, ribbon, or light-gauge wire
- push-pins
- corrugated cardboard
- circular objects or cylinders
- tool for finding the centre of circles (optional)
- BLM 7.SS.1.5: A Table to Compare Measures of Circles (optional)
- computers and spreadsheets (optional)

Organization: Small groups or whole class

Procedure:

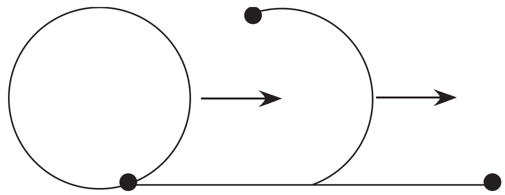
In this learning experience, students focus on comparing the radius, diameter, and circumference of circles in an effort to discover pi, the ratio of the circumference to the diameter $\left(\frac{C}{d}\right)$.

1. Have students quickly review what they have learned about central angles, radius, diameter, and circumference, and their relationships, and about how to draw circles.
2. Explain that students will now use a variety of circles and circular objects and accurately measure their radius, diameter, and circumference to find a famous and useful relationship between the measures.

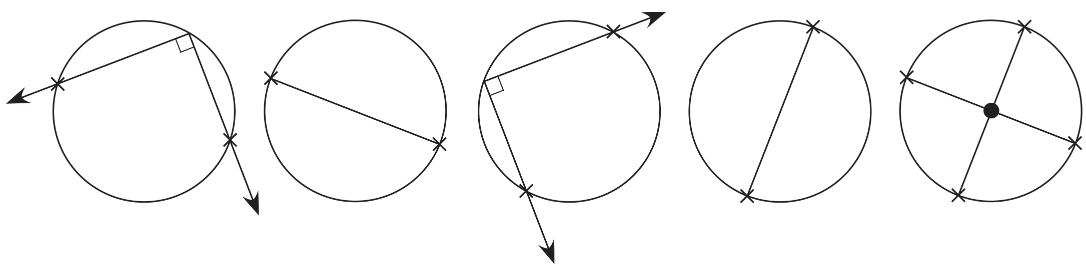
   Inform students there are several ways to measure circumference, such as the following:

   - Wrap a measuring tape around the circumference of a circle.
   - Wrap a string or a ribbon around the circumference, and then measure the string or ribbon.
   - Mark a starting point on the circle and roll the circle along a ruler, returning to the starting point.

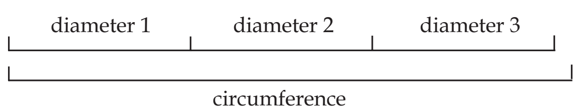
- Roll the circle along a paper and mark the starting and ending points. Connect the points with a line, and measure the line.



To find the centre of a circle, students can put the vertex of a right angle at a point along the circumference, and mark where the arms of the right angle cross the circumference. Joining these two marks creates a diameter of the circle. Repositioning the right angle approximately a quarter way around the circle and repeating the process will create another diameter. The point where the two diameters cross is the centre of the circle. Knowing the centre of the circle allows students to measure the radius and the diameter.



Note: Rolling out the line of the circumference is a useful strategy, as students can then measure the diameter and physically place the diameter over the line and see how many diameters long the line is. This requires no calculation.



3. Distribute copies of BLM 7.SS.1.5: A Table to Compare Measures of Circles, or have students create their own tables.
4. Ask students to find circular objects of various sizes in the classroom or school, accurately measure the radius, diameter, and circumference of the objects, and record the required data. Have them include large circles (e.g., the centre circle on the basketball court) available in the school. Have students record and calculate the ratios and look for any consistent relationships.

5. Reassemble as a class and discuss students' findings. Students' measurements will not be completely accurate, so the calculations to decimal places will not be consistent. Nevertheless, students should have found that, regardless of the size of the circle, the circumference is always a little longer than three diameters of that circle ($C = 3d$ and a little more). If students have measured very carefully, they should have calculated values between 3.1 and 3.2. Explain that this relation $\left(\frac{C}{d}\right)$ is a very important mathematical constant. It is commonly approximated as 3.14, and is termed *pi* (π). It is important that students recognize that pi is not so much a special number as it is a special relationship, the relationship of the circumference of a circle divided by its diameter. (For more information on pi, refer to the Background Information.) **Note:** Remember to celebrate Pi Day (March 14).

Variation:

- Use spreadsheets to enter required data and calculate the value of the relations.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Illustrate and explain that the circumference is approximately three times the diameter in a circle.
- ❐ Explain that, for all circles, pi is the ratio of the circumference to the diameter $\left(\frac{C}{d}\right)$, and its value is approximately 3.14.

- **Solve a contextual problem involving circles.**

Materials:

- coloured paper on which to print problems
- scissors
- tape
- math journals
- computers for publishing (optional)

Organization: Small groups, whole class

Procedure:

In this learning activity, students work in groups to review what they have discovered about the relationships in circles, what the relationships mean, and the different notations for recording those relationships.

1. Challenge students to record the relationships in circles in as many equivalent ways as they can. They can include words, diagrams, and mathematical symbols. Considering equivalent values using opposite operations will be helpful.
2. Have groups create small posters with ideas such as the following:
   - The sum of the central angles always equals 360°.
   - If I know the radius, I can know the diameter too, because $d = 2r$.
   - If you tell me the diameter, I can tell you the radius $\left(r = \frac{1}{2}d\right)$.
   - $\frac{C}{d}$ = a constant pi (π), and pi has an approximate value of 3.14.

     $\frac{C}{d} \approx 3.14$, so $3.14 \cdot d \approx C$ and $\frac{C}{3.14} \approx d$. If I know the circumference, I can find the diameter, and vice versa.
   - $2r$ can replace d in every relation, so $3.14 \cdot 2 \cdot r \approx C$ or $6.28r \approx C$. I can figure out the circumference of a circle if I know the radius of the circle.
3. After giving groups sufficient time to work on their posters, post their work and have a Gallery Walk, giving students an opportunity to compare the representations in the different posters. Reassemble as a class and share observations.

4. Supply students with the following sample problem, and ask them to work in their groups to solve it:

    Your sister is preparing for her wedding. You're at the store with your mom, who is picking up last-minute items that are needed to finish the decorating. The list includes a special lace border to go around the round table on which the desserts will be displayed. The measurement given for the table is 1 metre in diameter. "Oh no!" your mother sighs. "I've been given the wrong information. The border goes around the table, not across it. How am I supposed to know how much lace border to buy?" "Never fear, mother," you say. "The math constant pi holds the answer to your problem. Give me a minute, and I will tell you the length of the border you need to buy to go around a table with a diameter of 1 metre." What is the answer your mother needs?

5. Have students use the relationships on their posters to help them prepare contextual problems about finding the various measurements of circles when only one other measurement is given. Ask groups to be imaginative and creative in the problems they write and the ways in which they present the problems. Have them supply the solutions to their problems, concealed under a flap or using some other method. Groups may wish to create problems centred on a particular theme or event. Remind students that circles can be unravelled. They can roll a circle to measure its circumference. The circumference of the circle equals the distance the outside of a circle can travel in one revolution. So, students can compare the distance travelled in one revolution for circles of different sizes, or calculate the revolutions required to travel a certain distance using circles of different sizes. If a circle is peeled in layers, each successive strip will be a little smaller. If the thickness of the strips remains constant, students could solve problems such as figuring out the diameter of a rolled hose compared to its length.
6. Have students solve the problems created by their classmates, and verify their solutions. Have students write a math journal entry commenting on their success in solving the problems. Also have them comment on which types of problems they preferred, and which problems they found more difficult.

Variations:

- Provide scaffolding in the form of templates for students who may have trouble composing appropriate problems.
- Have students publish their problems using computer software.
- Serve a variety of pie at the pi celebration (e.g., pizza pie, spinach pie).

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Solve a contextual problem involving circles.

NOTES

Shape and Space (Measurement) (7.SS.2)

Enduring Understanding(s):

Many geometric properties and attributes of shapes are related to measurement.

The area of a rectangle can be used to develop the formula for the area of other shapes.

General Learning Outcome(s):

Use direct or indirect measurement to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
7.SS.2 Develop and apply a formula for determining the area of ■ triangles ■ parallelograms ■ circles. [CN, PS, R, V]	➛ Illustrate and explain how the area of a rectangle can be used to determine the area of a triangle. ➛ Generalize a rule to create a formula for determining the area of triangles. ➛ Illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram. ➛ Generalize a rule to create a formula for determining the area of parallelograms. ➛ Illustrate and explain how to estimate the area of a circle without the use of a formula. ➛ Apply a formula for determining the area of a circle. ➛ Solve a problem involving the area of triangles, parallelograms, or circles.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of the area of regular and irregular 2-D shapes by
  - recognizing that area is measured in square units
  - selecting and justifying referents for the units cm^2 or m^2
  - estimating area by using referents for cm^2 or m^2
  - determining and recording area (cm^2 or m^2)
  - constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area

- Solving problems involving single-variable (expressed as symbols or letters), one-step equations with whole-number coefficients, and whole-number solutions.
- Designing and constructing different rectangles given either perimeter or area, or both (whole numbers), and drawing conclusions.
- Demonstrating an understanding of measuring length (mm) by
  - selecting and justifying referents for the unit mm
  - modelling and describing the relationship between mm and cm units, and between mm and m units
- Describing and providing examples of edges and faces of 3-D objects, and sides of 2-D shapes, that are
  - parallel
  - intersecting
  - perpendicular
  - vertical
  - horizontal
- Identifying and sorting quadrilaterals, including
  - rectangles
  - squares
  - trapezoids
  - parallelograms
  - rhombuses

  according to their attributes.
- Demonstrating an understanding of the relationships within tables of values to solve problems.
- Representing and describing patterns and relationships using graphs and tables.
- Representing generalizations arising from number relationships using equations with letter variables.
- Demonstrating an understanding of angles by
  - identifying examples of angles in the environment
  - classifying angles according to their measure
  - estimating the measure of angles using 45°, 90°, and 180° as reference angles
  - determining angle measures in degrees
  - drawing and labelling angles when the measure is specified
- Developing and applying a formula for determining the
  - perimeter of polygons
  - area of rectangles
  - volume of right rectangular prisms

- Constructing and comparing triangles, including
  - scalene
  - isosceles
  - equilateral
  - right
  - obtuse
  - acute

  in different orientations.
- Describing and comparing the sides and angles of regular and irregular polygons.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of oral and written patterns and their corresponding relations.
- Constructing a table of values from a relation, graphing the table of values, and analyzing the graph to draw conclusions and solve problems.
- Demonstrating an understanding of preservation of equality by
  - modelling preservation of equality, concretely, pictorially, and symbolically
  - applying preservation of equality to solve equations
- Explaining the difference between an expression and an equation.
- Evaluating an expression given the value of the variable(s).
- Modelling and solving problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers.
- Modelling and solving problems that can be represented by linear equations of the form
  - $ax + b = c$
  - $ax = b$
  - $\frac{x}{a} - b$, $a \neq 0$

  concretely, pictorially, and symbolically, where a, b, and c, are whole numbers.

- Demonstrating an understanding of circles by
  - describing the relationships among radius, diameter, and circumference of circles
  - relating circumference to pi
  - determining the sum of the central angles
  - constructing circles with a given radius or diameter
  - solving problems involving the radii, diameters, and circumferences of circles
- Performing geometric constructions, including
  - perpendicular line segments
  - parallel line segments
  - perpendicular bisectors
  - angle bisectors

Background Information

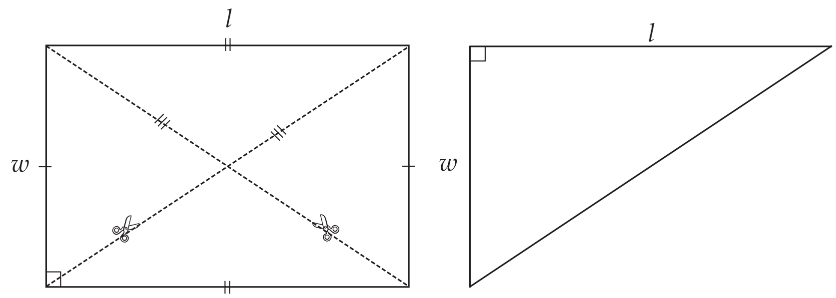
Calculating Area

Before entering Grade 7, students constructed rectangles of a given area, and generalized a rule for determining the area of a rectangle. In Grade 7, students build on their knowledge of rectangles to generalize rules for determining the area of a triangle and of a parallelogram. They also use their familiarity with rectangles to find the area of a circle. Determining area is a useful skill for identifying and comparing the sizes of objects of different shapes, whether they are circular, rectangular, or triangular. There are many practical applications for determining area (e.g., identifying which size or shape of pizza is a better buy, determining the cost of flooring). Determining area is also a prerequisite skill for determining the volume of prisms and cylinders in later grades.

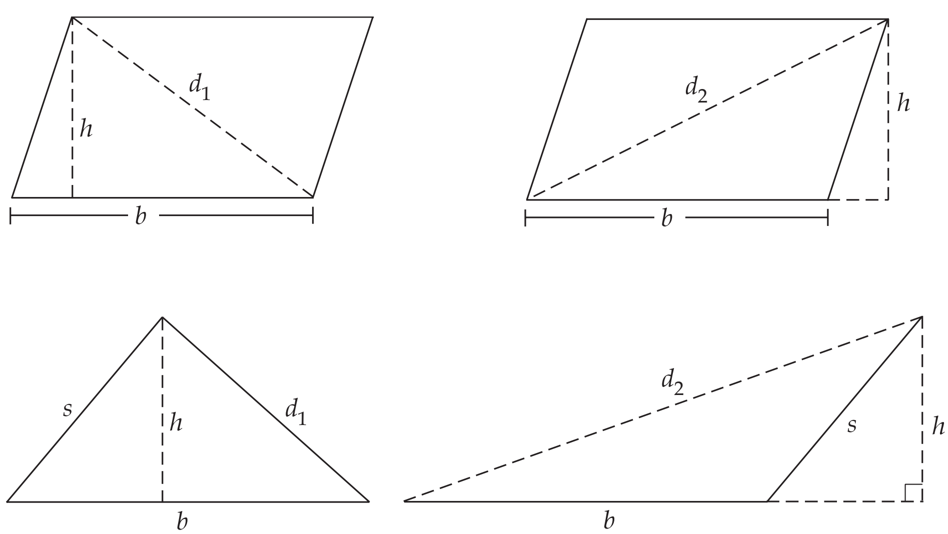
To make generalizations regarding area, students must have a good conceptual understanding of what area is, and of how to find the area of a rectangle. The formula $l \cdot w$ is a way of counting the squares contained in the area of a rectangle. It represents area as an array of squares, and provides a way of counting them. That is, the formula $l \cdot w$ represents the number of squares in each row multiplied by the number of rows. If necessary, rebuild these understandings before having students develop rules regarding the area of triangles and parallelograms.

To begin, review types of quadrilaterals and classifications of triangles, to ensure students have vocabulary to communicate clearly about their learning. The ability to identify the base and height of shapes is also important for clear communication about shapes. Identifying base and height is also required to develop and apply formulas. Any flat side of an object can serve as its base. The base in any situation depends on the orientation of the object. Height is measured in relation to the base; therefore, an object may have different heights, depending on its orientation. Height is the distance between the highest point of the object and its base. It is measured along a line that is perpendicular to the base.

Provide students with opportunities to explore methods of finding the areas of figures and to discover the relationships between the figures and rectangles. As students gain an understanding of the concepts and see the relationships, the generalization of $b \cdot w$ will become apparent as the generalized formula for finding the area of a parallelogram. As they observe that two identical triangles form a parallelogram, they will understand that $\frac{1}{2}b \cdot h$ identifies the area of a triangle. When a rectangle is halved along a diagonal, all the resulting triangles are right triangles. It is clear that their area is one-half of the area of the rectangle, or $\frac{1}{2}l \cdot w$. The length and width are obtained by measuring the sides of the rectangle. For a right triangle, the area can be calculated by measuring the sides of the triangle next to the right angle.

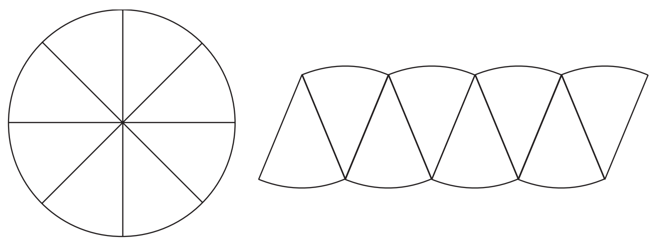


When a parallelogram is halved along a diagonal, two triangles of different sizes and shapes may be created, depending on the diagonal used to halve the parallelogram. It is still evident that the area of either triangle is one-half the area of the parallelogram, but neither the area of the parallelogram nor the triangle can be calculated by measuring the sides. The base and the height of the figures must be determined in order to calculate the area.



Relating the properties of parallelograms to those of rectangles is a way of establishing the concept of base × height as the way to determine the area of a parallelogram. Any two identical triangles can be arranged to form a parallelogram. The triangle is related to the parallelogram, and the parallelogram is related to the rectangle. There is an advantage to studying the area of parallelograms before studying the area of triangles, because the use of base and height in relation to area has already been established and there is no need to separate right triangles from other triangles.

When a circle is sectioned through the centre, and the pieces are rearranged so that the orientation of the centre alternates between pointing up and pointing down, the pieces will form an approximate rectangle. The approximate rectangle can be used to estimate the area of a circle, and to explain the formula for finding the area of a circle.



When students build their own generalizations from their own experiences, they will understand the connections in formulas. Building these connections provides the best opportunities for students to remember formulas and to apply them correctly. If students forget a formula, they are in a position to rebuild it, test it, and carry on using it.

The suggested learning experiences that follow are closely related. Students are sent on three missions to discover methods of calculating areas of parallelograms, triangles, and circles. Each endeavour is based on knowledge of the rectangle. The final learning activity engages students in a problem-solving party.

Mathematical Language

area
base
formula
height
horizontal
intersecting
length
parallel
parallelogram
perpendicular
polygon
quadrilateral (square, rectangle, rhombus, trapezoid, parallelogram)
rectangle
square units
triangle (obtuse, right, acute, scalene, equilateral, isosceles)
vertex
vertical
width

Assessing Prior Knowledge

Materials:

- BLM 7.SS.2.1: The Area of Rectangles (Assessing Prior Knowledge)
- demonstration board
- paper for writing clues (one-quarter sheets)
- two long pencils, pens, or straws (for each student)
- BLM 5–8.21: Isometric Dot Paper (optional)
- BLM 5–8.22: Dot Paper (optional)
- geoboards (optional)

Organization: Groups of varying sizes, whole class, individual

Procedure:

1. Review the vocabulary terms *perpendicular*, *parallel*, *intersecting*, *vertical*, and *horizontal* by playing a Simon Says type of game.
   - The game can be played with a group of any size. Choose a group size that is best for your class situation.
   - As Simon gives directions, students use their arms or a pair of pencils, pens, or straws to demonstrate the formation.
   - The phrase "Simon says" must preface the direction. For example, "Simon says, show parallel lines." Anyone not showing the lines correctly is out for the round.
   - If the phrase "Simon says" does not preface the direction, anyone performing the demonstration is out.
   - The last person remaining in the game wins, or becomes the new Simon.
2. Review types of quadrilaterals and classifications of triangles by playing with riddles.
   - The game can be played with a group of any size. Choose a group size that is best for your class situation.
   - Include the following quadrilaterals and triangles:
     - quadrilaterals: rectangles, squares, trapezoids, parallelograms, and rhombuses
     - triangles: scalene, isosceles, equilateral, right, obtuse, and acute

- Assign each student one or more quadrilaterals and/or triangles (different ones for each student).
- Ask students to consider the attributes of a given shape, and then write four clues from which their classmates can identify the shape.
- Students fold a sheet of paper in half. They write the clues outside the fold, and the name of the shape, accompanied by an accurate diagram, inside the fold.
- Have students take turns offering their clues to the group.
- When individuals identify a shape correctly, they get possession of the card, or earn x number of points, and so on.

3. Through a whole-class discussion focused on quadrilaterals and triangles, review the concepts of base and height using objects and diagrams.
4. To review what students know about calculating the area of rectangles, distribute copies of BLM 7.SS.2.1: The Area of Rectangles (Assessing Prior Knowledge), and have students complete the tasks individually.

Variations:

- Have students review shapes with a Pictionary type of game. The designer draws the designated quadrilateral or triangle, and group members guess its name. The student who guesses correctly becomes the next designer. Add competition to the game by supplying a set of cards with the shape names, and having groups compete to be the first to complete designs of the set of shapes.
- Use geoboards, geoboard templates, or dot paper to demonstrate the different shapes and to explore various sizes of rectangles with the same area. (See BLM 5–8.21: Isometric Dot Paper and BLM 5–8.22: Dot Paper.)

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Identify different classifications of triangles and quadrilaterals according to their attributes.
- ❒ Identify parallel, perpendicular, intersecting, horizontal, and vertical lines.
- ❒ Identify the base and height of diagrams.
- ❒ Demonstrate understanding that area is measured in square units.
- ❒ Calculate the area of rectangles.

Suggestions for Instruction

- **Illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram.**
- **Generalize a rule to create a formula for determining the area of parallelograms.**
- **Solve a problem involving the area of triangles, parallelograms, or circles.**

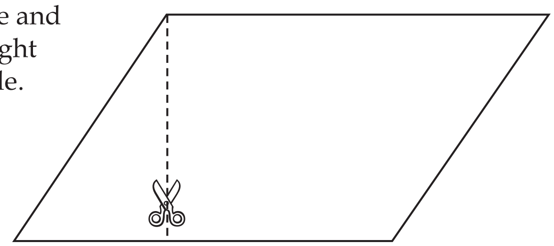
Materials:
- grid paper
- rulers
- scissors
- tape
- calculators
- math journals
- geoboards and elastics (optional)
- BLM 5–8.22: Dot Paper (optional)
- BLM 5–8.9: Centimetre Grid Paper (optional)

Organization: Investigative teams, whole class, individual or pairs

Procedure:

1. Assign students the mission of using their investigative and mathematical reasoning skills to discover and provide the world with a method that will quickly determine the area of any parallelogram, of any dimension, in any situation. Allow students to conduct their own investigation, but provide whatever hints are required to keep them on track.
2. The first step is to identify a strategic plan for the mission. Preliminary stages could include the following:
   a) Consider how to identify a parallelogram, varieties of parallelograms, and similarities and differences between parallelograms. Identify the attributes of the parallelograms for which to collect data.
   b) Examine the initial findings of areas for different parallelograms, including parallelograms with the same area but different shapes. Identify the attributes of the parallelograms to measure and investigate.
   c) Look for connections between the known and the unknown. How to determine the area of a rectangle is a known. Is there a similarity between rectangles and parallelograms that may help guide the investigation? (A parallelogram can be divided and reassembled as a rectangle. The reassembly does not change the length or height of the figure. The base is removed from one end of the figure and reattached at the opposite end. The height of the figure does not change, just the interior angles change.)

   Cut along the dotted line and tape the section to the right vertex to form a rectangle.

   d) Identify relationships between a parallelogram's attributes and its area, and search for a pattern. Look for a connection between the length of the base and the height that equals the count of the squares.
3. After identifying a method to determine the area of any parallelogram, of any dimension, in any situation, students test it for a variety of parallelograms. Encourage students to base their formulas on lengths of the base and the height. If their formula works consistently, the mission is accomplished.
4. Hold a debriefing session with the class, asking students to share their strategies and findings.
   a) In the discussion, include situations in which the area of a parallelogram is known and the length of the base or height of the parallelogram needs to be determined.

b) Discuss the value of knowing the length of the side, and whether the measurement is useful in finding the area.

c) Ask whether a parallelogram can be accurately reproduced given the area and the length of the base or height.

5. Have students work individually, or in pairs, to create one or two parallelogram problems for their partners to solve.
6. Give students a set of problems, with solutions concealed. Working out a solution verifies that the creator of the problem understands what he or she is doing, and that the problem “works.” Attempting to work out a solution may indicate the problem requires revision, and provides feedback for the problem solver.

   Examples of Problems:

   - Draw two different parallelograms that have an area of x square units, or a base of x units and an area of x units.
   - Provide drawings of parallelograms from which to calculate area.
   - Provide dimensions for two parallelograms and ask which parallelogram has the larger area.

Variations:

- If students become stuck in their investigation, provide them with a diagram of a parallelogram on centimetre grid paper and provide enough prompts for the discovery to be made. (See BLM 5–8.9: Centimetre Grid Paper.)
- Geoboards provide an easy way to investigate parallelograms of the same base and height, but with different interior angles. Investigate the change from a rectangle to many different parallelograms with the same area by systematically shifting the base one unit to the right of the top. This looks interesting on paper as well, and it is self-evident that the dimensions for the base and height have not changed. It also provides an illustration of circumstances in which the height of the parallelogram is not obvious, and must be measured outside the parallelogram.

Observation Checklist

☑ Listen to and observe students’ responses to determine whether students can do the following:

- ❒ Illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram.
- ❒ Generalize a rule to create a formula for determining the area of parallelograms.
- ❒ Solve a problem involving the area of triangles, parallelograms, or circles.

Suggestions for Instruction

- **Illustrate and explain how the area of a rectangle can be used to determine the area of a triangle.**
- **Generalize a rule to create a formula for determining the area of triangles.**
- **Solve a problem involving the area of triangles, parallelograms, or circles.**

Materials:

- file from the investigation of parallelograms
- grid paper
- rulers
- scissors
- tape
- calculators
- math journals
- geoboards and elastics (optional)
- BLM 5–8.22: Dot Paper (optional)

Organization: Investigative teams (from previous learning activity), whole class, individual or pairs

Procedure:

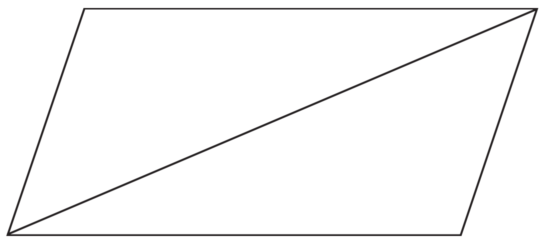
This learning experience on determining the area of a triangle is designed to follow the previous learning activity on determining the area of parallelograms.

1. Inform students they have been assigned a new mission that requires investigators with mathematical reasoning skills. This time, the world is in need of a method or formula that will quickly determine the area of any triangle of any classification, of any dimension, in any situation. Their mission is to discover the formula. Once again, allow students to conduct their own investigation, but provide whatever hints are required to keep them on track. A general suggested protocol is outlined below. For more detail, refer to the procedure for finding the area of parallelograms (in the previous learning experience).
2. The first step is to develop a strategic plan. This may include the following:
   a) Identify classifications of triangles, their similarities and differences, and the attributes for which to collect data.
   b) Collect some initial data about area for different triangles, including triangles with the same area, but different shapes. Identify how to measure the height of a triangle. Choose which attributes to measure for the investigation.

c) Look for connections between the known and the unknown. Look for similarities between triangles and parallelograms and rectangles. Similarities provide clues to help guide the investigation.

Example:

- Any two congruent triangles can be arranged to form a parallelogram. Therefore, their area must be $\frac{1}{2}$ the area of the parallelogram.



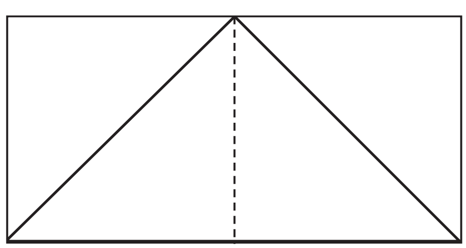
- Triangles also have bases and heights. A triangle can be enclosed within a rectangle of the same height and sharing the same base. A perpendicular line can be drawn from the highest vertex of the triangle to its base. The line divides both the triangle and the rectangle in two. The sides of the triangles become the diagonals of the two rectangles.
- Students can explore finding the area of different triangles, using computer applets.

  Sample Website:

  Computer applets are available on the following website:

  Bogomolny, A. "Area of Triangle." *Interactive Mathematics Miscellany and Puzzles*. 1996–2011. www.cut-the-knot.org/Curriculum/Geometry/AreaOfTriangle.shtml.

  Choose the colour option to make the relation more obvious.



- The area of each triangle is $\frac{1}{2}$ the area of each rectangle; therefore, the area of the original triangle is $\frac{1}{2}$ the area of the original rectangle. The squares in the area of the triangle equal $\frac{1}{2}$ the number of squares in a rectangle with the same base and height. Using an array model to count the squares leads to the formula $\frac{1}{2}b \cdot h$.

3. Look for a relationship between the base and height of triangles, and their areas. Look for a mathematical relationship that will equal the area for a given triangle.
4. Identify a formula for determining the area of a triangle, and test it on a variety of triangles. If the formula works consistently, another useful formula has been discovered, and is ready for use.
5. Debrief as a class to discuss findings and strategies. For example, dividing a parallelogram through the diagonal creates two congruent triangles, and dividing through the opposite diagonal creates two different congruent triangles. Students may wish to discuss why that is so. What value is there in measuring the length of the sides of the triangle? Can a triangle be reproduced by knowing only the base and the height? Can a triangle be reproduced by knowing only the area, or the area and the base?
6. Have students work individually or in pairs to create problems and solutions related to determining the area of a triangle, and then exchange problems and find the solutions. These problems and solutions may be used as Exit Slips.

Variations:

- Explore the effect of varying the height and base measurements of a triangle or the area of the triangles. Also explore creating different triangles with the same area. Geoboards, paper templates of geoboards, or dot paper may be used. (See BLM 5–8.22: Dot Paper.)
- Explore the relationships between the height, base, and area of triangles using computer applets.

  Sample Website:

  Computer applets are available on the following website:

  Math Open Reference. "Area of a Triangle." *Triangles*. 2009. www.mathopenref.com/trianglearea.html.

  This applet allows students to change the length of the base, the height of the triangle, or the measures of the angles, while the applet constantly measures the matching area.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Illustrate and explain how the area of a rectangle can be used to determine the area of a triangle.
- ❒ Generalize a rule to create a formula for determining the area of triangles.
- ❒ Solve a problem involving the area of triangles, parallelograms, or circles.

Suggestions for Instruction

- **Illustrate and explain how to estimate the area of a circle without the use of a formula.**
- **Apply a formula for determining the area of a circle.**
- **Solve a problem involving the area of triangles, parallelograms, or circles.**

Materials:

- BLM 7.SS.2.2: Circles for Estimating Area
- math journals
- grid paper
- rulers
- scissors
- glue or tape
- markers of different colours
- calculators

Organization: Individual or pairs, whole class, individual

Procedure:

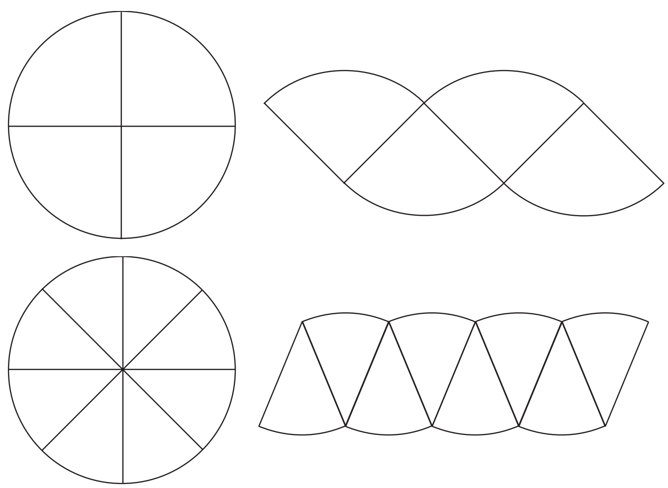
In this learning experience, students investigate the area of a circle. Be prepared to use more than one class for the investigation and application, depending on the amount of time the students use for exploring and the depth of their exploration.

1. As an introductory problem, or as a closing exercise, present round pizzas (cut into even wedges) and rectangular pizzas (close in area, but not too close) or cut-outs of these, and ask students to compare the two pizzas and find out which shape of pizza is larger. Have students record evidence for their decision in their math journals.
2. Inform students their current mission is to find effective methods to approximate the area of circles.
3. Present students with one or more circles and ask them to approximate the area.
4. When students have a good estimate, ask them to use their math journals to note the procedure they followed. Then ask them to attempt to find more methods that work for them. Encourage students to steer their own investigation, connecting the known with the unknown.
5. As students work on their investigation, circulate among the class and observe. If students experience difficulty in their work, ask guiding questions or suggest an action. Their investigative methods may include the following:
   a) Trace the circle onto grid paper and count the number of squares.

b) Draw the smallest square the circle could fit into, calculate the area of the square, and determine some amount to deduct to compensate for the area of the square that does not include part of the circle.

c) Draw the smallest square to contain the circle, and the largest square inside the circle (use the diagonals of the large square to indicate the position of the corners). Then think, the circle is larger than the small square and smaller than the large square, so the area of the circle must cover an area between the areas of both squares. The number midway between is a good approximation.

d) Consider whether the circle could be transformed into a rectangle. Section a circle into four pieces and try arranging the pieces to approximate a rectangle, as illustrated below. (Colouring the circumference of the circle before beginning helps identify the orientation of the pieces.) Alternating the centre points between pointing up and pointing down elongates the circle. Halve each of the four pieces, and arrange the pieces to approximate a rectangle again. It becomes evident that the more times the sections are halved, the more the elongation resembles a parallelogram. Measuring the base and the height of this parallelogram will approximate the area of the circle.



6. After students have had sufficient time to develop their investigative methods and have made some discoveries, reassemble as a class for a debriefing session. Discuss students' findings regarding the areas of the circles, and the methods they found most useful, or most accurate.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Illustrate and explain how to estimate the area of a circle without the use of a formula.
- ❐ Apply a formula for determining the area of a circle.
- ❐ Solve a problem involving the area of triangles, parallelograms, or circles.

Suggestions for Instruction

- **Solve a problem involving the area of triangles, parallelograms, or circles.**

Materials:

- rulers
- grid paper
- protractors
- paper for publishing problems
- calculators
- math journals or notebooks, or logs of completed problems
- art or craft supplies (optional)
- snacks for the party (optional)
- computers for publishing problems (optional)

Organization: Individual, pairs, or small groups

Procedure:

1. Planning

   Have students think of contexts where parallelograms, triangles, and circles may appear in everyday situations (decide whether or not to exclude rectangles). Examples for parallelograms may include logo designs, personal flags, geometric art designs, optical illusions, tangrams, one half of gable roofs, and architectural designs.

Example:

The following website has a photograph of a building in the shape of a parallelogram along the Elbe River in Hamburg, Germany.

Stich, Mike. "Parallelogram." 22 May 2006. *flikr.* www.flickr.com/photos/mdstich/357560359/.

2. Preparing the Problems

a) Give students an opportunity to create diagrams, designs of structures, logos, artwork, puzzles, and so on, that involve only parallelograms, triangles, or circles, or combinations of these shapes.

b) When the design projects are complete, have students use their designs as the subject to write problems whose solutions require calculating area and to provide a solution for each problem created.

Examples:

- Determine the square metres of glass required for windows in a building.
- Determine the square metres of material required for the different shapes in a flag, or in a piece of artwork.
- Create "three of these things belong" questions, where three parallelograms or triangles with different base and height combinations, or different interior angles, have the same area, and one has a different area.
- Create problems for combinations of all three shapes. The challenge is to find the shape with the different area.
- Include problems that require calculating area, and problems that supply area and require calculating bases, heights, radii, or circumference.

3. Solving the Problems

a) When the projects are complete, students can share them with one another at a party. The focus of the party becomes solving various problems.

b) Perhaps completed problems could be exchanged for drinks or snacks at the party (sign off each problem that has been exchanged).

c) Assign greater value to problems that are more challenging.

d) Agree on a definite number of problems to complete, or solve problems with an aim to accumulate a target number of square units, or have a contest to accumulate the greatest number of square units in problem answers.

e) Students will be busy calculating many areas. Have fun.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Solve a problem involving the area of triangles, parallelograms, or circles.

NOTES

Shape and Space (3-D Objects and 2-D Shapes) (7.SS.3)

Enduring Understanding(s):

Many geometric properties and attributes of shapes are related to measurement.

While geometric figures are constructed and transformed, their proportional attributes are maintained.

General Learning Outcome(s):

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Specific Learning Outcome(s):	Achievement Indicators:
7.SS.3 Perform geometric constructions, including ▪ perpendicular line segments ▪ parallel line segments ▪ perpendicular bisectors ▪ angle bisectors [CN, R, V]	➢ Describe examples of parallel line segments, perpendicular line segments, perpendicular bisectors, and angle bisectors in the environment. ➢ Identify line segments on a diagram that are parallel or perpendicular. ➢ Draw a line segment perpendicular to another line segment, and explain why they are perpendicular. ➢ Draw a line segment parallel to another line segment, and explain why they are parallel. ➢ Draw the bisector of an angle using more than one method, and verify that the resulting angles are equal. ➢ Draw the perpendicular bisector of a line segment using more than one method, and verify the construction.

Prior Knowledge

Students may have had experience with the following:

- Describing and constructing rectangular and triangular prisms.
- Demonstrating an understanding of measuring length (mm) by
  - selecting and justifying referents for the unit mm
  - modelling and describing the relationship between mm and cm units, and between mm and m units

- Describing and providing examples of edges and faces of 3-D objects, and sides of 2-D shapes, that are
  - parallel
  - intersecting
  - perpendicular
  - vertical
  - horizontal
- Identifying and sorting quadrilaterals, including
  - rectangles
  - squares
  - trapezoids
  - parallelograms
  - rhombuses

  according to their attributes.
- Demonstrating an understanding of angles by
  - identifying examples of angles in the environment
  - classifying angles according to their measure
  - estimating the measure of angles using 45°, 90°, and 180° as reference angles
  - determining angle measures in degrees
  - drawing and labelling angles when the measure is specified
- Demonstrating that the sum of interior angles is
  - 180° in a triangle
  - 360° in a quadrilateral
- Constructing and comparing triangles, including
  - scalene
  - isosceles
  - equilateral
  - right
  - obtuse
  - acute

  in different orientations.

- Describing and comparing the sides and angles of regular and irregular polygons.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Developing and applying a formula for determining the area of
  - triangles
  - parallelograms
  - circles

Background Information

The concepts of perpendicular and parallel surround us in everyday life. In Grade 5, in their study of 2-D shapes and 3-D objects, students identified examples of perpendicular and parallel sides, edges, and faces. They also identified examples of perpendicular and parallel line segments in the environment. Because Grade 5 students lack experience with measuring angles, the angle formed by perpendicular lines was identified as having "square" corners. In Grade 7, students will create parallel and perpendicular line segments and bisectors, as well as angle bisectors, using geometric constructions.

Geometric Constructions

Geometric constructions are connected to the Ancient Greeks and Euclidean geometry. They are different than drawings in that the only tools used in creating geometric constructions are a straightedge, a compass, and a pencil. Interesting connections exist between geometric constructions, art, and architecture in many different cultures. Parallel and perpendicular lines are also important to surveyors, designers, engineers, contractors, and people building just about anything.

Students may enjoy recreating geometric constructions. Knowing about various applications may provide students with an increased purpose and motivation for using a straightedge and a compass to create lines and bisectors.

Lines, Rays, and Line Segments

Lines, rays, and line segments are made up of sets of points that are straight and one-dimensional. Their only dimension is length.

- A *line* is a set of points that extends indefinitely in opposite directions.

  Example:

  The line $\overleftrightarrow{AB}$

- A *ray* is a set of points that extends indefinitely in one direction.

  Example:

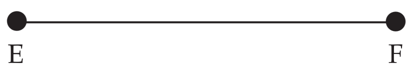
  The ray $\overrightarrow{CD}$

- A *line segment* is a set of points along a line with two finite endpoints.

  Example:

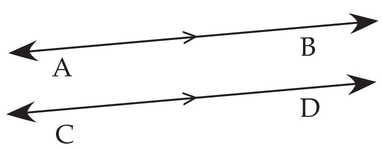
  The line segment $\overline{EF}$:

Lines, rays, and line segments can be parallel or intersecting. Lines that intersect at right angles are perpendicular lines.

- *Parallel lines* never meet; they are always the same distance apart. In diagrams, indicate parallel lines by marking an arrow on the line.

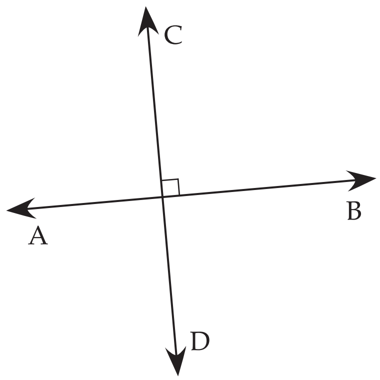
  Example:

  The symbol || indicates that the lines are parallel, as in $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

- *Perpendicular lines* intersect at 90° angles. In diagrams, indicate perpendicular lines by drawing a small square where the lines join.

  Example:

  The symbol $\perp$ indicates that the lines are perpendicular, as in $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

Lines and angles can be bisected. In the word *bisectors*, *bi* means two and *sect* means to cut. When a line or an angle is bisected, it is cut into two pieces of equal size. We could say it is divided in half or divided down the middle.

Students will perform geometric constructions, including constructions of perpendicular bisectors and angle bisectors:

- A *perpendicular bisector* is a line, ray, or line segment that divides a line segment into two equal segments and is perpendicular to the original line, ray, or segment.
- An *angle bisector* is a line or ray that divides an angle into two angles of equal size.

Methods for creating each of these constructions are outlined in the learning experiences suggested for learning outcome 7.SS.3.

Mathematical Language

angle	line segment
angle bisector	perpendicular
arc	perpendicular bisector
bisect	perpendicular lines
bisector	parallel
intersecting lines	parallel lines
line	ray

Assessing Prior Knowledge

Materials:

- BLM 7.SS.3.1: Parallel and Perpendicular Lines (Assessing Prior Knowledge)
- skipping ropes or other types of ropes (two ropes per group of six students)
- math journals or notebooks

Organization: Small groups (of six students), individual

Procedure:

1. Form students into groups of six students—two to hold one rope, two to hold the other rope, one to verify that the lines are parallel or perpendicular, and one to record the group's action.
2. Ask students to use two or more skipping ropes to demonstrate a variety of parallel lines and provide evidence to verify that the lines are parallel. Evidence may include sliding an object between the ropes to demonstrate the ropes are the same distance apart at each point along the ropes, or measuring the distance between the ropes at several intervals.
3. Once students have perfected making parallel lines in several positions, repeat the process. This time, have them demonstrate a variety of perpendicular lines and provide evidence that the lines are perpendicular. As evidence, they may place a right-angle object, such as a book, at the intersection of the lines, or they may form the lines against an object with right angles, such as the side and front edges of a desk or a table.
4. Challenge students to find the middle of a line, prove it is the middle, and record their demonstration in their math journals or notebooks.
5. Have students demonstrate an angle, using their ropes. Ask them to add another line that would divide the line in half and create two equal angles. Challenge them to verify that the angles are equal, and have them record the process.
6. When the groups have completed the demonstrations, provided evidence, and recorded their work, distribute copies of BLM 7.SS.3.1: Parallel and Perpendicular Lines (Assessing Prior Knowledge), and have students complete the tasks individually.

Variations:

- Play a version of Simon Says, as outlined in the Assessing Prior Knowledge learning experience suggested for learning outcome 7.SS.2.
- Ask students to create a simple drawing composed of a certain number of parallel and perpendicular lines and write directions for each line formed. When directions are complete, have students join a partner, and either exchange direction sheets or give oral directions to make the composition. When the drawings are finished, students compare them to the original drawings and analyze any discrepancies. Alternatively, give directions to the whole class, and compare the products with a projection of the original drawing. Compare students' drawings to the original and discuss any differences.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify parallel lines.
- ❒ Identify perpendicular lines by using a square corner.
- ❒ Draw and name angles, lines, rays, and line segments.
- ❒ Find the middle of a line.
- ❒ Divide an angle in half.

Suggestions for Instruction

- **Describe examples of parallel line segments, perpendicular line segments, perpendicular bisectors, and angle bisectors in the environment.**

Materials:

- magazines
- poster paper
- electronic or other display medium
- Internet access (optional)
- cameras (optional)
- projector (optional)

Organization: Whole class, pairs or individuals

Procedure:

1. Introduce the class to the term *bisector*. Verify students' understanding of the term by asking them to identify or illustrate examples of perpendicular lines that bisect another line, or lines that bisect an angle.
2. Tell students they will participate in a treasure hunt to find examples of parallel line segments, perpendicular line segments, perpendicular bisectors, and angle bisectors. They can work individually or in pairs.
3. Together with students, set criteria for the number of each line to find. Places to search could include the classroom, lockers or desks, the hallways, the gymnasium, the playground, and other areas of the school. Alternatively, have students search magazines or the Internet for examples. (Examples may consist of musical instruments, such as keyboards and string instruments, the fulcrum on a balance scale, window frames, door frames, the separation in a double door, hallways, sidewalk cracks, brick patterns, goalposts, court markings in the gymnasium or outdoors, lanes in a swimming pool, streets or paths, the path of tires or skis, mitred corners on a box, logos and emblems, the alphabet.)
4. After the search criteria are established, have students undertake their treasure hunt. They can record their findings as sketches and labels, or take photographs.
5. They may wish to continue their search for homework.
6. When students have collected a sufficient number of treasures, have them organize their examples according to the type of lines represented, and choose a format for presenting their findings. They may choose a collage, a poster, a large classroom display, or an electronic display or slide show.
7. Arrange for students to share their findings with each other.

Variations:

- Using a projector, show examples of the different lines in various contexts. Ask students to identify and describe the various lines they see in the examples.
- Play a version of I Spy, asking students to identify parallel line segments, perpendicular line segments, perpendicular bisectors, and angle bisectors in the classroom or in another environment.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Describe examples of parallel line segments, perpendicular line segments, perpendicular bisectors, and angle bisectors in the environment.

Suggestions for Instruction

- **Identify line segments on a diagram that are parallel or perpendicular.**

Materials:

- schematic diagrams in books, blueprints, or drawings
- rulers
- grid paper
- tracing paper (optional)
- highlighters or pencil crayons of different colours (optional)
- computer drawing program (optional)
- number cubes (optional)
- photographs with examples of parallel or perpendicular lines (optional)

Organization: Whole class, individual

Procedure:

1. Introduce students to schematic diagrams through books, blueprints, or drawings.
2. Introduce them to the symbols that indicate parallel and perpendicular lines.
3. Have students create a schematic diagram showing the interior structure of a wood-framed building. Ask them to include top plates, bottom plates, window and door frames, and headers. Have them mark arrows and square corners on their diagrams to indicate the parallel and perpendicular line segments.
4. Have students make some general statements, at the bottom of their diagrams, regarding perpendicular and parallel line segments that appear in their diagrams.

Variations:

- Instead of creating a schematic diagram of a wood-framed building, students could diagram the shell of a bus, plane, ship, sport court, sewing pattern, road map, or airport runway.
- Have students create artwork composed of coloured lines and angles (perhaps similar to the work of Piet Mondrian) and labelled points. Ask them to create a key to go with their artwork that lists the parallel and perpendicular lines. The pictures and keys can be posted for display.
- Before indicating the parallel and perpendicular line segments in the diagrams described above, students could exchange diagrams with their partners, who would mark indicator lines on their drawings, and then return them to the creators to verify, discussing any discrepancies.
- Have students build an origami figure or shape, unfold it, trace the lines, and indicate parallel and perpendicular lines in the fold lines.

- Have students use a computer drawing program to create diagrams of interconnected lines. They can identify the parallel and perpendicular lines on their own diagrams, and exchange work with partners.
- Students could use their diagrams (or a supplied diagram) to play a game with a partner. Students each choose a different colour of highlighter or light-coloured pencil crayon. They pick either odd or even numbers on a number cube to represent parallel or perpendicular lines. Students take turns shaking the number cube and marking either a set of parallel lines or a set of perpendicular lines with their selected colour. If students cannot find a set of lines, they forfeit their turn. The player with the greatest number of sets of lines wins.
- Provide students with photographs on which they can mark the parallel or perpendicular lines represented, or have them create a schematic diagram using symbols to indicate the parallel and perpendicular line segments.
- Have students identify parallel and perpendicular lines in capital letters of the alphabet.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify parallel or perpendicular line segments on a diagram.

Suggestions for Instruction

- **Draw a line segment perpendicular to another line segment, and explain why they are perpendicular.**

Materials:

- records from Assessing Prior Knowledge learning activity (optional)
- math journals or notebooks
- geometry sets with straightedges or rulers, right triangles, protractors, and compasses
- Miras
- tracing paper
- pens or markers of different colours
- spinners or number cubes (optional)
- BLM 7.SS.3.2: Creating Perpendicular Lines (optional)

Organization: Whole class, small groups, individual

Procedure:

1. Ask students to share the methods they used in the Assessing Prior Knowledge learning activity to create perpendicular lines. Students will likely have used square corners, some may have used protractors to measure 90° angles, and some may have carefully folded paper and used the folds as guidelines for perpendicular lines.
2. Challenge students, working in groups, to think of multiple methods that could be used to draw perpendicular lines. Encourage students to use the tools in their geometry kits, Miras, and tracing paper. Have each student use his or her math journal or notebook to record the different methods their group thinks of. For each method, students draw an example and explain their thinking. Encourage students to make geometric drawings carefully, labelling points and indicating perpendicular lines with a square insert in the corner. Have students include comments (e.g., "I know $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ because . . . . The reason this method works is because . . . . Suggestions for avoiding errors when using this method are . . . . Situations for which this method is recommended include . . . .).
3. When students have had sufficient time for their group work, reassemble as a class and have students share their ideas and explanations regarding the methods that could be used to draw perpendicular lines. Have students add any new ideas to their math journal entries, using another colour to highlight new learning.
4. Encourage students to think critically about and comment on the ideas that are shared in class. They may acknowledge ideas they agree with, express appreciation for the way ideas are explained, ask "how" or "why" questions, or offer further suggestions or support for an idea. If the class has not addressed a specific method when the sharing is finished, provide some guiding questions or hints, and send students back to work to develop another idea.

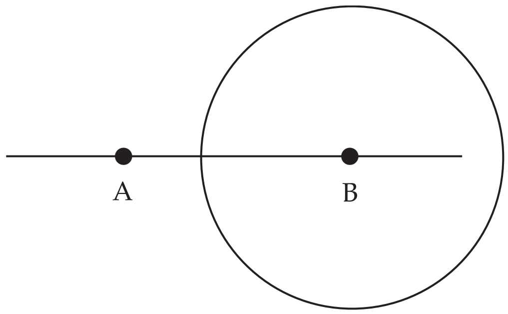
   Methods of drawing perpendicular lines may include the following:

   a) Use a square corner. Draw a line segment using a straightedge. Place the right-angle triangle with one side of the right-angle corner lined up with the line segment. Trace a vertical line segment along the adjoining side of the triangle. The line segment is perpendicular because both line segments intersect at 90º angles, or form a square corner. Following the same principle, artists and designers use T-squares to make perpendicular line segments. Carpenters use carpenter squares to do the same thing.

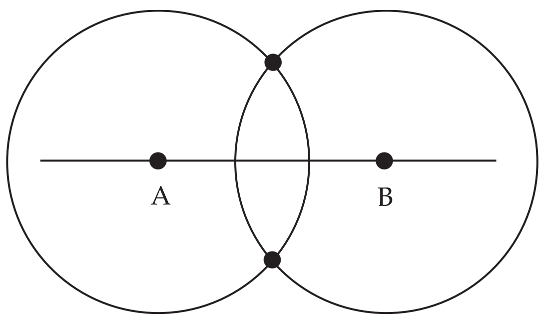
   b) Use a protractor. Draw a line segment using a straightedge. Mark a point on the line segment. Align the point and the base line segment with the cross lines on the protractor. Mark the 90º measure. Use a straightedge to connect the original point and the 90º mark. The resulting line segment is perpendicular because the angle between the two line segments is 90º.

c) Use a straightedge and a compass. Draw a line segment using a straightedge. Mark any two points along the line segment.

Use the compass to draw a circle around one of the points that has a radius greater than half the distance between the points. That radius is necessary for the circles to intersect.

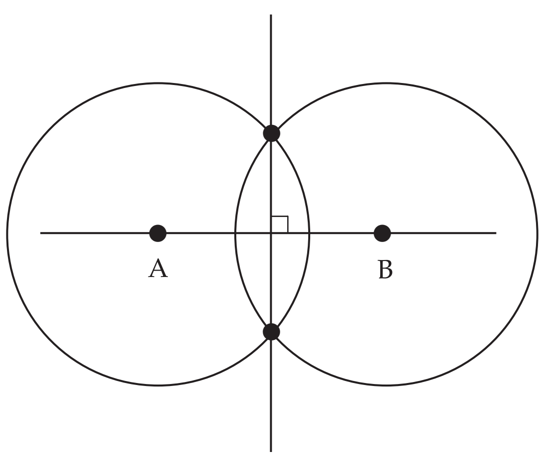


Ensuring that the radius is large enough to have the two circles intersect, draw a circle around the second point.



Note:
When constructing the perpendicular bisector, it is necessary for the radii of the two circles to be the same.

The circles will intersect at two points. Use a straightedge to connect those two points. The angle between the original line segment and the resulting line segment measures 90^{o}; therefore, the line segments are perpendicular to each other.



Note: With this method, it is not necessary to draw the entire circle; only an arc needs to be drawn. However, the circle emphasizes that the points joined to form the perpendicular line segments are radii of congruent circles, and, therefore, are the same length. Introduce students to the use of this method in art and medieval architecture, and how it can be used to determine large-scale perpendicular lines outdoors or in the sky.

d) Use a Mira. Use a straightedge to draw a line segment. Lay the Mira across the line segment and adjust its position until the reflection of the line segment in the Mira lines up on top of the line segment itself. Trace a line segment along the edge of the Mira. That line segment is perpendicular to the original line segment because the angles at the intersection of the line segments are right angles.

e) Use tracing paper. Use a straightedge to draw a line segment. Carefully fold the paper across the line segment so that the portion of the line on the top paper lies on top of the line segment under the folded paper. When the line segments are aligned, crease the fold in the paper. Open it up and use a straightedge to trace the line segment along the crease. The line segments are perpendicular because the angle formed at the intersection measures 90°.

5. Have students practise drawing line segments using each of the methods. Then ask them to make a math journal entry commenting on which method they prefer, which method they believe to be most accurate, and which applications each method is best suited for.

Variations:

- Provide students with directions to produce specific line segments using specific methods. For example, draw line segment YZ 6 cm long. Use a straightedge and a protractor to draw a perpendicular line segment that crosses $\overline{YZ}$ at point X, 4 cm away from Y.
- Create a spinner identifying the five different methods for drawing perpendicular lines, or assign the numbers on a number cube to the five different methods, with the sixth number representing a missed turn. Students take turns spinning or rolling, and then drawing perpendicular line segments using the designated method. The first person to draw perpendicular line segments successfully using all five methods wins. Students can use BLM 7.SS.3.2: Creating Perpendicular Lines to record progress.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Draw a line segment perpendicular to another line segment, and explain why they are perpendicular.

Suggestions for Instruction

- **Draw the perpendicular bisector of a line segment using more than one method, and verify the construction.**

Materials:

- math journals or notebooks
- geometry sets with straightedges or rulers, right triangles, protractors, and compasses
- tracing paper
- Miras
- Number cubes or spinners (optional)
- BLM 7.SS.3.3: Creating Perpendicular Bisectors (optional)

Organization: Small groups, whole class, individual or pairs

Procedure:

Be prepared to be flexible regarding the time required for this learning experience. Ensure students have ample opportunity to explore and develop their understanding of different methods of drawing perpendicular bisectors before beginning the presentations.

1. Ask students to define *perpendicular bisector* and to explain how a perpendicular bisector differs from a perpendicular line segment.
2. Discuss why and when people may use a perpendicular bisector. For example, a perpendicular bisector may be used to find the best place to put a single support under a beam, to find the division in a drawing, a design, or a building from which to build formal symmetry, to divide a piece of property into equal portions, to find a line that is the same distance from two points. The last application may be important in a variety of contexts, ranging from planning a city or a meeting spot to setting up a lemonade stand.
3. Remind students that in the previous learning experience, they worked with five different methods to draw perpendicular lines. Ask students to work in groups to review those methods and to determine whether or not each method could be used as is, or with some modification, to draw a perpendicular bisector. As in the previous learning experience, have students provide proof that the perpendicular bisectors are perpendicular and bisect the line segment. They should also consider hints to ensure success in using a given method, ideas on how changing the technique affects the outcome, and explanations for why a method works.
4. Have each student make a math journal entry discussing the methods for creating perpendicular bisectors, including explanations and pointers regarding techniques and applications.

5. When students have had sufficient time for their group exploration, reassemble as a class and call upon different groups to present their modifications for one of the methods to draw a perpendicular bisector. Discuss their presentations.
6. Encourage students to think critically about and comment on the ideas that are shared in class. They may acknowledge ideas they agree with, express appreciation for ideas that are explained, ask "how" or "why" questions, or offer further suggestions or support for an idea. If the class has not addressed a specific method when the sharing is finished, provide some guiding questions or hints, and send them back to work to develop another idea.

   The methods used for drawing perpendicular lines could also be used for drawing perpendicular bisectors, but with the following modifications:

   a) Use a square corner.

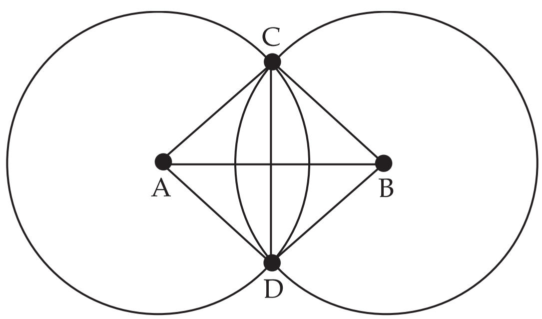
      Modification: Before beginning, find the midpoint of the line. Include ideas on how to find the midpoint. Place the right angle at the midpoint.

   b) Use a protractor.

      Modification: Once again, find the midpoint of the line and measure the 90º angle from the midpoint.

   c) Use a straightedge and a compass.

      Modification: When drawing the circle, use the endpoints of the line segment as the centres for the circles. Connect the intersections of the circles to create the perpendicular bisector.



   d) Use a Mira.

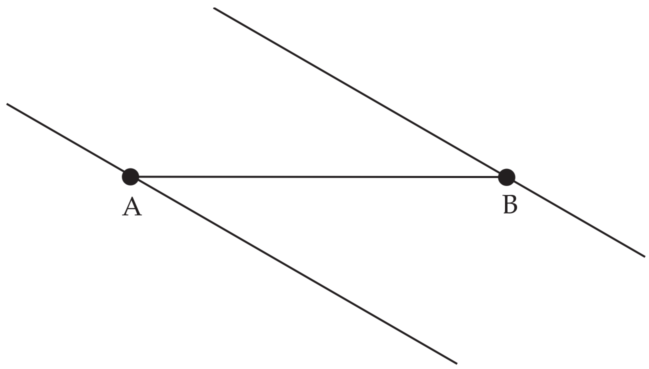
      Modification: When adjusting the Mira across the line segment, adjust it so the reflection of one endpoint of the line segment in the Mira lines up on top of the other endpoint of the line segment. Include hints for using the Mira successfully.

   e) Use tracing paper.

      Modification: When folding the paper across the line, fold it carefully to ensure the half of the line on the top paper lies on top of the line under it, and that the endpoints of the line segment lie exactly on top of each other.

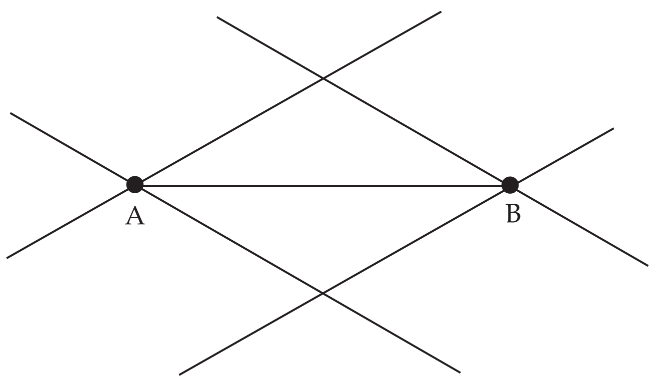
f) Use a ruler to create a rhombus. Creating a rhombus around the line segment will create the perpendicular bisector of the segment because the diagonals of a rhombus are perpendicular bisectors of each other.

- Lay a straightedge at an angle across the line segment. Adjust the angle of the straightedge until each endpoint just touches the straightedge. The endpoints will be on opposite sides of the straightedge. Trace a line along both the top and the bottom of the straightedge.

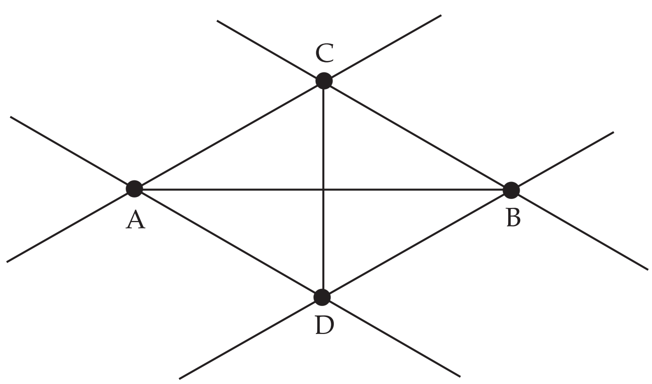


Note: For this method to work, the width of the straightedge needs to be less than the length of the line segment.

- Rotate the straightedge one-quarter turn until the end that was above the line is now below the line, and the end of the straightedge that was below is now above. Once again, adjust the straightedge so one point is above it and one point is below it, and trace both edges of the straightedge.



- Remove the straightedge. The intersecting lines create the vertices of the perpendicular bisector.



7. Have students practise using each of the six methods to create perpendicular bisectors. Individuals can create their own line segments or create line segments for their partners, or the teacher can assign line segments. After students have had sufficient practice in using the methods, ask them to make a math journal entry commenting on their preferred method, the method they think is most useful or most accurate, applications for the different methods, and so on.

Variations:

- Play a game in which students practise drawing perpendicular line segments. Students roll two number cubes to determine the length of a line segment. They spin a spinner, or roll a number cube, to determine the method to use to draw the bisector. They draw the perpendicular bisector or the line segment, and then verify that their drawing is correct by checking that the angles created measure 90º and that the line segments on either side of the bisector are equal in length. Partners work independently to be the first to create a bisector using all six methods. They can use BLM 7.SS.3.3: Creating Perpendicular Bisectors to record their progress.

- Investigate drawing perpendicular bisectors for each side of a triangle. Draw a circle using the point of intersection of the perpendicular bisectors as the centre of the circle and the radius as the distance from the centre to one of the vertices of the triangle. Continue this investigation for a variety of triangles, parallelograms, or other polygons.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Draw the perpendicular bisector of a line segment using more than one method, and verify the construction.

Suggestions for Instruction

- **Draw a line segment parallel to another line segment, and explain why they are parallel.**

Materials:

- math journals or notebooks
- geometry sets with straightedges or rulers, right triangles, protractors, and compasses
- tracing paper
- Miras
- straws, cardboard strips, stir sticks
- push-pins

Organization: Whole class, small groups

Procedure:

1. Ask students to recall the Assessing Prior Knowledge learning activity in which they created parallel lines using ropes. Review what parallel lines are and how to test whether a set of lines are parallel. Effective testing methods include the following:
   - Reflect lines in a Mira. If the Mira is placed perpendicular to parallel lines, both the lines will reflect on themselves.
   - Fold paper. If the paper is folded along a line perpendicular to the line segments, the folded lines will lie on top of each other.
   - Identify perpendicular lines between the two parallel lines, and measure to verify that the lines are the same distance apart.
2. Ask groups of students to identify as many methods as they can for creating parallel line segments. Have them test each of their methods to verify that the line segments created are parallel. Have students use their math journals to record the methods that work. If students need a hint, ask them whether lines that are perpendicular to the same line are parallel to each other.
3. Reassemble as a class, and have students share the methods they found. Discuss the advantages of each method and under what circumstances each method may be best to use. Have students add any new methods to their math journals.

   Methods for creating parallel line segments may include the following:

   a) Trace both edges of a straightedge, such as a ruler.

b) Diagonals of a rectangle are the same length and they bisect each other. If the diagonals are connected in the centre to form an X, all four arms of the X are equal to each other. Use two straws (or cardboard strips, stir sticks, and so on) to represent the diagonals, mark the midpoint of each, and connect them with a push-pin. Lay the X on a paper and connect two endpoints of adjacent arms with a straightedge. Trace a line segment. Mark the endpoints of the two remaining arms of the X and connect the points with a straightedge. The result is two parallel line segments. Stretch or collapse the X to adjust the distance between the parallel line segments.

c) Use a right-angle triangle, or some other square corner, to draw a line segment. Place a straightedge along the line. Set the base of the right angle on the straightedge, and trace the side. This creates a line segment perpendicular to the original line segment. Slide the right angle along the straightedge to any desired position, and trace the side of the right angle. All the line segments perpendicular to the original line segment are parallel to one another.

d) Connect points that are 90º and equidistant from the line segment. Use a right triangle or a protractor to draw two lines that are perpendicular to the original line segment. Measure and mark the same distance up each perpendicular line. Connect the marks to create a line segment parallel to the original line segment. The perpendicular lines are also parallel.

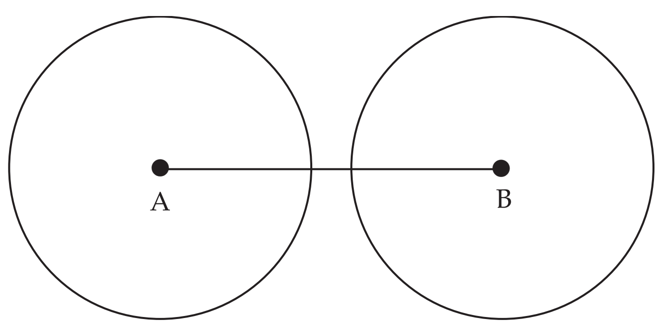
e) Use the edge of a Mira to draw a line segment. Then use the Mira to draw perpendicular lines, whose reflections are in line with the original line segment. The lines perpendicular to the original line segment are all parallel to each other.

f) Fold a piece of paper carefully with the corners matching. Fold the paper again in the same direction. Crease the folds well. Open the paper and, using a straightedge, trace line segments along the creases. The resulting line segments are parallel.

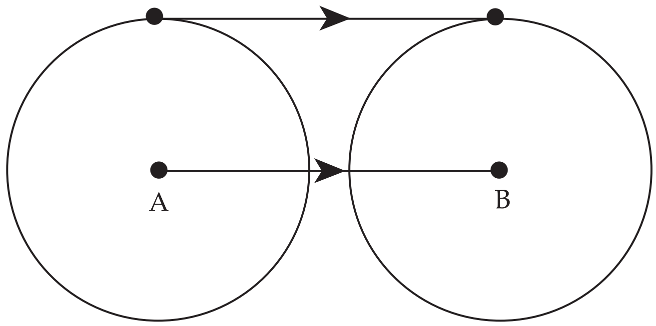
g) Use a compass and a straightedge.

h) Draw a line segment $\overline{AB}$.

- Use a compass to draw a circle around point A and a circle around point B with the same radius.



- Mark the highest (or lowest) points of each circle and connect them with a line. This line will be parallel to the original line segment $\overline{AB}$.



4. Have students practise drawing parallel line segments of different lengths and different distances apart. Partners, teachers, or the toss of a number cube can determine the length or distance, or students can choose their own measurements. After trying several methods to draw designated parallel line segments, students can choose the method they prefer and write about it in their math journals.

Variations:

- Create optical illusions by generating a set of parallel line segments, and then decorating each line with different colours or markings at different angles or lengths or thicknesses. Alternatively, divide the space between the parallel line segments with perpendicular lines of various widths and colours. Create a display of the illusions with a title (e.g., Find the Parallel Lines).

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Draw a line segment parallel to another line segment, and explain why they are parallel.

Suggestions for Instruction

- **Draw the bisector of an angle using more than one method, and verify that the resulting angles are equal.**

Materials:

- math journals or notebooks
- geometry sets with straightedges, protractors, and compasses
- Miras
- tracing paper
- geometry software (optional)

Organization: Small groups, pairs or individual, whole class

Procedure:

1. Ask students to explain what an angle bisector is, and how they could test to see whether or not a line actually bisects an angle.
2. Have small groups work together to devise multiple ways of generating angle bisectors for a particular angle, and prove that the method generates a true bisector. Ask students to enter successful methods in their math journals.
3. Reassemble as a class to share students' methods and discuss the benefits and applications or use of the suggested methods. Have students make additional notes in their math journals as needed.

   Methods for generating angle bisectors may include the following:

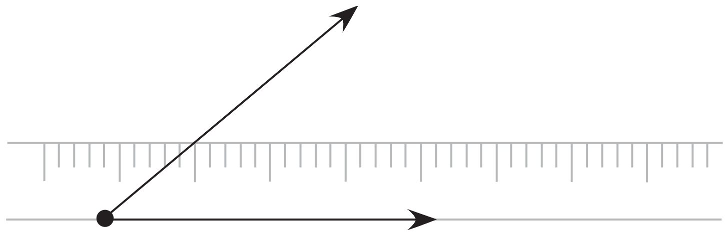
   a) Use a protractor to measure the original angle.
      - Divide the measurement in half.
      - Use the protractor to mark the new measure, and draw the bisector.

   b) Use tracing paper.
      - Copy the angle onto tracing paper.
      - Fold the paper so that the two original rays lie on top of each other.
      - Crease the paper along the fold.
      - Open the paper and use a straightedge to trace the crease.

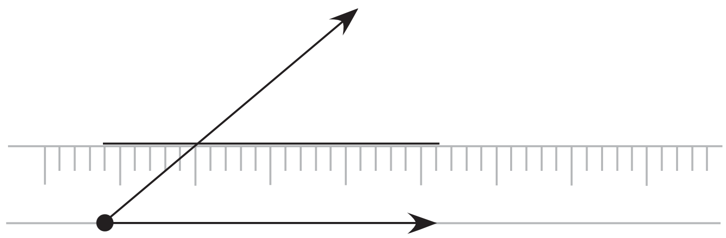
   c) Use a Mira.
      - Place the Mira so that part of the length lies over the vertex of the angle.
      - Adjust the angle of the Mira until each of the angle rays are reflected on top of each other.
      - Trace the edge of the Mira.

d) Use a ruler.

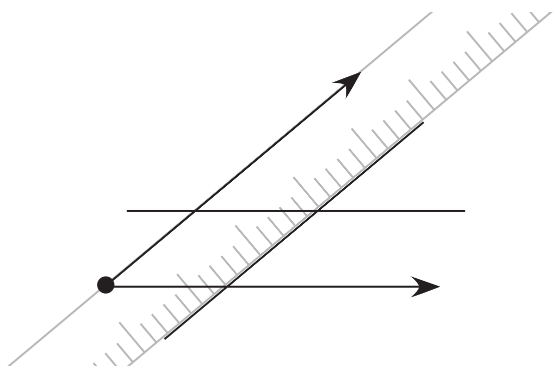
- Line up the edge of the ruler along one of the rays in the angle so that the ruler lies "inside" the angle.



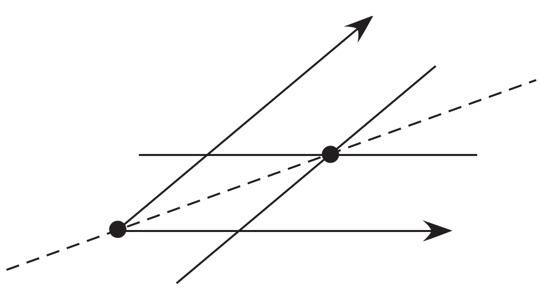
- Trace the side of the ruler not on the ray.



- Trace the side of the ruler for the other ray.

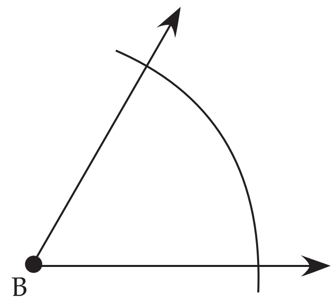


- Connect the vertex of the angle with the intersection of the lines just drawn.

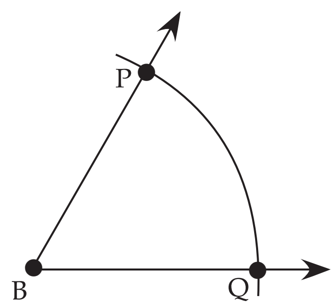


e) Use a compass and a straightedge.

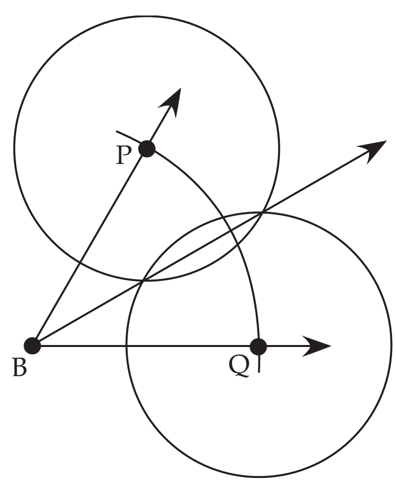
- Place the compass point on the vertex of the angle and draw an arc across the two rays.



- Place the compass point on one of the intersecting points, and draw a circle or an arc around the centre area of the angle. Keep the same radius setting, and draw a circle around the other point where the arc intersects the other ray. It is not necessary to draw the entire circles, just the intersection of the arcs of the circle approximately where the bisector will be.



- Use the straightedge to connect the intersection of the two circles with the vertex of the ray.



The arc across the rays creates two points equidistant from the vertex. Connect the two points with a straight line. Intersecting two circles with centre points on each of the ray points creates the perpendicular bisector of the line connecting them.

4. Have students practise drawing angle bisectors. Supply students with angles or angle measures for which students can create bisectors, or have students generate their own measures. Ask students to identify and justify a preferred technique for bisecting angles and write about it in their math journals.

Variations:

- Provide students with a handout outlining and illustrating the methods for creating an angle bisector, along with a compass and templates for those students who may need them.
- Have students create the angle bisectors of various triangles, and compare the results. Have them draw circles with the centre at the intersection of the angle bisectors, and the radii just touching one side of the triangle. Ask students to investigate what happens when they bisect the angles of parallelograms and other polygons.
- Have students use geometry software to practise and investigate angle bisectors of different shapes.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Draw the bisector of an angle using more than one method, and verify that the resulting angles are equal.
- ❐ Draw a line segment perpendicular to another line segment, and explain why they are perpendicular.
- ❐ Draw a line segment parallel to another line segment, and explain why they are parallel.
- ❐ Draw the bisector of an angle using more than one method, and verify that the resulting angles are equal.
- ❐ Draw the perpendicular bisector of a line segment using more than one method, and verify the construction.

Putting the Pieces Together

Maps, Floor Plans, or Design Projects

Introduction:

Students use geometric constructions to replicate a floor plan, create a map, or design a project.

Purpose:

In this investigation, students will demonstrate the ability to do the following (learning outcome connections are identified in parentheses):

- Construct circles and solve problems involving radius, diameter, and circumference of circles. (7.SS.1)
- Perform geometric constructions, including perpendicular and parallel line segments and bisectors. (7.SS.3)

Students will also demonstrate the following mathematical processes:

- Communication
- Connections
- Problem Solving
- Reasoning

Materials/Resources:

- geometry kits containing protractors, rulers, compasses, right triangles
- Miras
- tracing paper
- individual project supplies

Organization: Individual, whole class

Procedure:

Student Directions

1. Select a project, such as the following:
    a) Replicate the floor plan for a sport facility (e.g., court, field, rink).
    b) Design a map for a community, a fairground, a school campus, or a campground. Design major services to be equidistant from strategic points in the area.
    c) Create a design for a fence, lattice, fabric pattern, or piece of artwork.

2. As a class, set criteria for the following:
   a) the type and number of lines that must be included in the project (e.g., circles or half circles, parallel lines, perpendicular lines, perpendicular and angle bisectors)
   b) the equipment that may be used to create the lines
   c) a scoring rubric based on the criteria identified in (a) and (b)
3. Use what you have learned about angles, parallel and perpendicular lines, and bisectors to create your project.
4. Prepare a report on the types of lines included in your project, how you created the lines, methods you used to overcome challenges you faced creating the lines, and points you are proud of.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Construct circles and solve problems involving radius, diameter, and circumference of circles.
- ❒ Draw a line segment perpendicular to another line segment, and explain why they are perpendicular.
- ❒ Draw a line segment parallel to another line segment, and explain why they are parallel.
- ❒ Draw the bisector of an angle using more than one method, and verify that the resulting angles are equal.
- ❒ Draw the perpendicular bisector of a line segment using more than one method, and verify the construction.

Putting the Pieces Together

Design a Protractor

Introduction:

Students construct a protractor, using only a compass and a straightedge.

Purpose:

In this investigation, students will demonstrate the ability to do the following (learning outcome connections are identified in parentheses):

- Demonstrate an understanding of circles by determining the sum of the central angles. (7.SS.1)
- Perform geometric constructions, including perpendicular line segments, perpendicular bisectors, and angle bisectors. (7.SS.3)

Students will also demonstrate the following mathematical processes:

- Communication
- Connections
- Reasoning
- Technology
- Visualization

Materials/Resources:

- straightedge
- compass
- a long strip of paper, approximately 50 cm by 6 cm
- the Internet

Organization: Individual, small groups

Procedure:

Students fill in the grey areas of the following chart by reflecting on prior knowledge (from Grade 6) of sums of interior angles for polygons (6.SS.2), as well as by researching methods of polygon construction online.

Polygon Name	Number of Sides	Regular Polygon Sum of Interior Angles	Regular Polygon Interior Angles	Constructible?
Triangle	3	180°	60°	Yes
Quadrilateral (Square)	4	360°	90°	Yes
Pentagon	5	540°	108°	Yes
Hexagon	6	720°	120°	Yes
Heptagon	7	900°	900/7 or 128.57° . . .	No
Octagon	8	1080°	135°	Yes
Nonagon	9	1260°	140°	No
Decagon	10	1440°	144°	Yes
Hendecagon	11	1620°	147.27° . . .	No
Dodecagon	12	1800°	150°	Yes

Note: Some polygon names and numbers of sides could be removed for students to find, if desired.

Student Directions

1. On a sheet of paper, draw a line segment of any length, large enough to be the baseline of a protractor, or the diameter of a circular protractor.
2. Using a compass and geometric constructions, draw the perpendicular bisector of this line segment (see page 54 of Shape and Space). This will form a 90° angle and will also mark the centre of this newly constructed protractor. To construct a standard protractor, extend the bisector upward only. To construct a circular protractor, extend this bisector both upward and downward.
3. Using a compass, draw a semicircle (or full circle) using the intersection of your two line segments as the centre, and the endpoints of your line segments as the outside of the circle. Extend all line segments so that they extend to the circumference of this circle.
4. Label the horizontal line segment as 180° on the far left, and 0° on the far right. Label the top of the vertical line segment as 90°, and if you are constructing a circular protractor, label the bottom of this line segment as 270°.

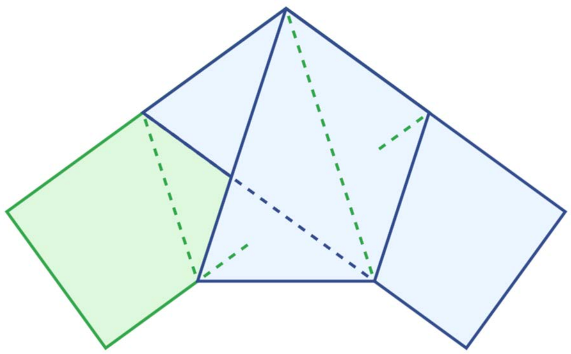
4. Using geometric constructions only, bisect the 90° angle to find the 45° angle (see page 65 of Shape and Space). Repeat this process in each quadrant, labelling with the appropriate angles of 45° and 135° (as well as 225° and 315° for circular protractors).
5. Construct an equilateral triangle and cut it out (or create an equilateral triangle using the centre and endpoint of the semicircle or circle as one side). This will give a 60° angle. Trace or mark this angle onto the newly formed protractor, and repeat this process wherever possible on the protractor to add new benchmarks.

6. Bisect angles on this protractor to find and mark more benchmarks. If done carefully and accurately, students could find and mark measurements along the whole protractor for every 15°.

Extension:

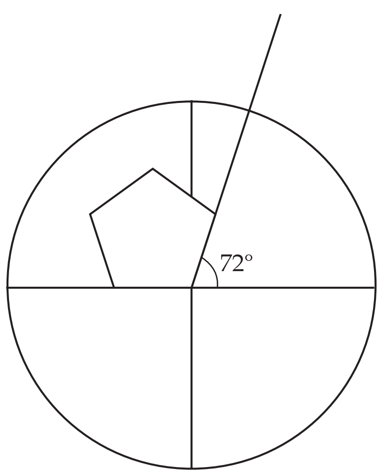
Student Directions

1. Construct a pentagon by folding a long strip of paper. By tying a regular overhand knot (a shoelace knot) with the paper, and flattening carefully as you tighten, you will form a regular pentagon with equal sides and angles.



Source: "File:Overhand-folded-ribbon-pentagon.svg." *Wikipedia: The Free Encyclopedia*. Wikimedia Foundation, Inc. https://commons.wikimedia.org/wiki/File:Overhand-folded-ribbon-pentagon.svg?uselang=en-ca (26 May 2016). [Public domain], via Wikimedia Commons.

2. Referring to the chart on the previous page, use the measurement of the pentagon's interior angles of 108°, and the supplementary angle of 72°, to add more measurements. To label an angle of 72°, place a vertex of the pentagon on the centre of the protractor, and the base of the protractor along the horizontal line to the left of the centre. Continue rotating the pentagon on the centre of the protractor, using other existing benchmarks to add measurements.



If this step is done carefully and accurately, students could find and mark measurements along the whole protractor for every 3°.

3. If you find you have missed any angles along the way, you can also use angle bisectors to find these missing measurements.

Further Extension:

1. Students research methods to construct other polygons, and find polygons that would allow for the completion of this learning activity to 1° of accuracy.

   See the following websites:

   Broderick, Shawn D. "Essay 2: Constructing Regular Polygons." *Jim Wilson, University of Georgia*. http://jwilson.coe.uga.edu/EMAT6680Fa08/Broderick/essay2/essay2.html (26 May 2016).

   "How to Construct Regular Polygons Using a Circle." *wikiHow*. www.wikihow.com/Construct-Regular-Polygons-Using-a-Circle (26 May 2016).

   "Nonagon." *Wikipedia: The Free Encyclopedia*. Wikimedia Foundation, Inc. https://en.wikipedia.org/wiki/Nonagon (26 May 2016).

2. Students research Gauss's proof that regular polygons can be constructed using a straightedge and compass.

3. Students can use the protractor to measure angles around the classroom, comparing their measurements with those of other students.

Shape and Space (Transformations) (7.SS.4)

Enduring Understanding(s):

The coordinate grid is used for plotting and locating points on a plane.

General Learning Outcome(s):

Describe and analyze the position and motion of objects and shapes.

Specific Learning Outcome(s):	**Achievement Indicators:**
7.SS.4 Identify and plot points in the four quadrants of a Cartesian plane using ordered pairs. [C, CN, V]	➛ Label the axes of a Cartesian plane and identify the origin. ➛ Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair. ➛ Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes. ➛ Draw shapes and designs, using ordered pairs, on a Cartesian plane. ➛ Create shapes and designs on a Cartesian plane and identify the points used.

Prior Knowledge

Students may have had experience with the following:

- Representing and describing patterns and relationships using graphs and tables.
- Identifying and plotting points in the first quadrant of a Cartesian plane using whole-number ordered pairs.
- Performing and describing single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole-number vertices).
- Creating, labelling, and interpreting line graphs to draw conclusions.
- Graphing collected data and analyzing the graph to solve problems.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of oral and written patterns and their corresponding relations.
- Constructing a table of values from a relation, graphing the table of values, and analyzing the graph to draw conclusions and solve problems.
- Modelling and solving problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers.
- Modelling and solving problems that can be represented by linear equations of the form
  - $ax + b = c$
  - $ax = b$
  - $\frac{x}{a} = b,\ a \neq 0$

  concretely, pictorially, and symbolically, where a, b, and c are whole numbers.
- Performing and describing transformations of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral vertices).

Background Information

In Grade 6, students graphed data, plotted points from ordered pairs, and drew shapes and designs in the first quadrant of a Cartesian plane. Experience with horizontal and vertical integral number lines in both Grades 6 and 7 prepared students to extend plotting skills to work in all four quadrants of a Cartesian plane. Plotting ordered pairs accurately is an important skill for performing and describing transformations in learning outcome 7.SS.5, and for graphing equations in Patterns and Relations.

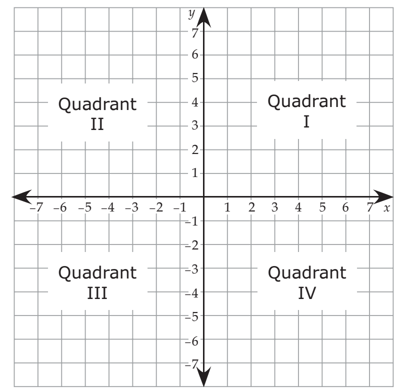
The Cartesian Plane

René Descartes, a French mathematician, philosopher, physicist, and writer who lived in the first part of the seventeenth century, developed the Cartesian plane. The Cartesian coordinate system allows geometric shapes to be expressed in algebraic equations.

To teach about the Cartesian plane, start with a number line and extend it to the left to include negative integers. This representation is one-dimensional (1-D). To make it two-dimensional (2-D), take a second number line and make it perpendicular to the first, running it through 0, with positive numbers extending above the 0 and negative numbers below. You now have a Cartesian plane.

Cartesian Plane

The Cartesian plane is formed by a horizontal axis and a vertical axis, labelled the *x*-axis and the *y*-axis respectively. It contains quadrants 1 to 4 (the quadrants are often labelled using Roman numerals I to IV).



Patterns can be drawn on a Cartesian plane. Therefore, they can be also described by an algebraic equation. To help students conceptualize this concept, show them a pattern on some material (e.g., on a piece of cloth, wrapping paper, wallpaper), and tell them that this pattern can be described by an algebraic equation, and then plotted on a Cartesian plane. The equation of the selected pattern might be too complex for Grade 7, but it is an equation, nevertheless.

A practical application of this concept can be found in computer-aided design (CAD), where equations describing 2-D (as well as 3-D) Cartesian planes are entered into a computer. The computer then instructs a machine to draw, cut, or stitch lines or designs onto wood, metal, textiles, and so on. Among its many other uses, CAD is used for embroidery and interior design. For example, the Department of Textile Sciences at the University of Manitoba is equipped with a CAD laboratory.

In the future, people might go to special boutiques and enter body-scanning booths in order to take their measurements. They could later send their measurements to an online store, which would create custom-made clothing for them. The booths would use Cartesian planes to calculate the person's measurements.

You can integrate the Cartesian plane with the study of geography by using the coordinates on a map of the world. The equator could be represented as the *x*-axis, and 0° longitude could be represented as the *y*-axis. Using the map, ask students to determine the number of degrees (in relation to both the *x*-axis and the *y*-axis) between two cities. Coordinates can be determined with any type of map. You could, for example, use a highway map or a topological map of the area around your school or community.

Since Grade 7 students are often interested in expressing themselves, you could have them create their own flag or symbol on a Cartesian plane, including the coordinates, and have them explain how that shape represents them. Begin by showing them a flag with a simple design (e.g., Switzerland's flag), and ask them to determine its coordinates on a Cartesian plane.

Mathematical Language

axes

Cartesian plane

coordinates

ordered pair

origin

x-axis

y-axis

Learning Experiences

Assessing Prior Knowledge

Materials:

- grid paper
- magnetic surface and magnetic tape (optional)

Organization: Pairs

Procedure:

1. Review the concept of ordered pairs and how to plot coordinates in the first quadrant of a Cartesian plane by playing a version of Battleship, Private Detective, hide-and-seek, or whatever title seems appropriate.

   Guidelines for the game follow:

   a) Divide the class into pairs for this game.

   b) Each player prepares two grids (about 10 × 10) by labelling the axes and numbering the grids according to the agreed-upon scale. Remind students to include the origin and to number the lines, but not the spaces. All the grids used in the same game must be identical.

c) Each player secretly hides the predetermined number of items in one of his or her grids (e.g., vessels for Battleship, a crook and clues for Private Detective, a number of hiding people for hide-and-seek). Plot each item as either a vertical line or a horizontal line of three adjacent points.

d) Students take turns naming ordered pairs in an attempt to uncover their partners' hidden items. As they name an ordered pair, they mark it on their own guessing grid, and the student who has hidden the objects marks the guessed point on the grid with the hidden items. If the guessing grid is visible to both students, they can limit errors in naming points. If the point that is named is part of an item, then the hider says, "hit," and the guesser receives another turn. When all points have been guessed, the identity of the object is revealed. The first player to uncover all the hidden objects is the winner.

Variation:

- Create a reusable display consisting of a coordinate grid on a magnetic surface, with magnetic tape on the back for use as hiding spots for various items (e.g., bushes, steps, shed, barrel, crate, tree, rock, car). Scatter the hiding spots at random intersections on the grid. In secret, hide the items behind hiding spots. Students take turns naming ordered pairs to find the hiding spots. The student who discovers the hiding spot wins a round. Play as a class, or allow small groups of students to take turns using the display.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify and plot points in the first quadrant of a Cartesian plane using whole-number ordered pairs.
- ❒ Reason mathematically in order to guess strategically.

Suggestions for Instruction

- **Label the axes of a Cartesian plane and identify the origin.**
- **Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.**
- **Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.**
- **Draw shapes and designs, using ordered pairs, on a Cartesian plane.**
- **Create shapes and designs on a Cartesian plane and identify the points used.**

Materials:

- grid paper
- lists of ordered pairs that, when plotted and connected together, form a simple shape or a picture in quadrant I (optional)

Organization: Individual, pairs or small groups

Procedure:

1. Have each student create a simple shape, such as a polygon, on quadrant I of a coordinate grid. The student then uses the plot to generate a list of ordered pairs.
2. Have students exchange lists with a partner, plot the points on the partner's list, and connect the points in the order given to create the polygon. Students verify each other's work.

Variations:

- Have one student share his or her list orally with a larger group or with the whole class. As the student reads the list, the other students plot and connect the points. When the list is complete, the reader shows what the finished product looks like, and the group or class discusses any discrepancies.
- Provide students with a handout of ordered pairs that create designs or pictures. Include a plot of a design or picture, and have students list the ordered pairs.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify and plot points in the first quadrant of a Cartesian plane using whole-number ordered pairs.

Suggestions for Instruction

- **Label the axes of a Cartesian plane and identify the origin.**
- **Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.**

Materials:

- demonstration board
- grid paper
- pencils and highlighters
- lists of ordered pairs that, when plotted and connected together, form a simple shape or a picture in quadrant I (optional)
- Neuschwander, Cindy. *Sir Cumference and the Viking Map: A Math Adventure.* Illus. Wayne Geehan. Watertown, MA: Charlesbridge Publishing Inc., 2012.

Organization: Whole class

Procedure:

1. Inform students they will create a math story. The story will require a main character who is able to move about freely (e.g., a dog in a field, a sightseeing tourist, a basketball player on a court, a ballerina on a stage, a taxi driver in New York city, a fly on the wall).
   a) Ask students to begin planning the story by making a mark on a point near the centre of a grid paper. Ask them to imagine that the mark represents the main character, and the grid paper represents the area where the character moves about. Have students label the original mark with the letter O (for original mark).
   b) Next, tell students to point their pencils to the O point and prepare to mark a trail on the grid as they follow their imaginary character on an adventure. Each time you call out a letter name, students form a point at the nearest intersection on the grid and label the point with the letter that was called.
   c) As you call out the letters of the alphabet, students track the movements of the character. Wait about five seconds between letters. Stop around letter J, and have students go back and make their points and letters obvious by highlighting them.
   d) Provide a sample copy of the trail on the demonstration board. Use either a student's copy or a copy that has been prepared ahead of time. The trail represents the path the character followed. The letters were written at regular intervals, so each point represents where the character was at a given time.
2. Tell students that a method is needed to describe the location of the character at any given time during the story. The location is to be given in relation to the starting point. Provide sufficient time for students to think, and perhaps talk with a partner, about a method.

a) If students experience difficulty in their work, ask them to concentrate just on the first points, and describe where point A is in relation to point O. Responses may include descriptions such as these: to the left, to the right, above, or below.

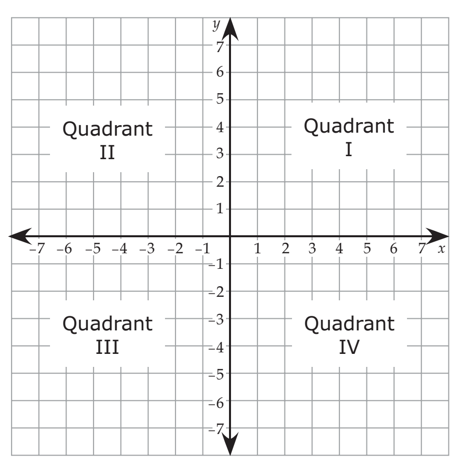
b) If students do not quantify the direction, ask them to make the descriptions more specific by indicating how far above or below the starting point a given point is, or how far to the left or to the right.

c) If students do not think of number lines, ask whether there are reference marks they could add to the grid paper to make the descriptions easier, such as a line to delineate left and right, and a line to separate points above and below the initial position. Adding numbers to the line would indicate how far right, and how far above. Negative numbers would indicate how far left and how far below. This amounts to adding both a horizontal and a vertical number line through the starting point of O.

3. Tell students about the history of the Cartesian plane. René Descartes, who lived in France in the early 1600s, is credited as being the first person to think of coordinating two intersecting number lines. This type of grid is very important to mathematicians, and is named for its inventor. The name Descartes means "from Cartes," so the adjective that describes his last name would be Cartesian, just as something from Canada is Canadian. The Cartesian plane mixes algebra and geometry and allows mathematicians to graph equations (which Grade 7 students do in the Patterns and Relations strand). The positions of the character in the story created in this learning activity could be described with an equation, although it may require a rather complicated equation to describe most of the patterns. There is a story that René Descartes thought of the Cartesian plane to describe the movements of a fly on the ceiling; however, this story is not verified.

4. Explain the features of the Cartesian plane. The horizontal number line in the Cartesian number line is termed the *x*-axis, and the vertical number line the *y*-axis. The point found at (0, 0) is called the *origin*. The scale is chosen based on the numbers in the situation. The four main areas are named *quadrants*. They are numbered 1 to 4 (often expressed as Roman numerals I to IV) in a counter-clockwise direction beginning with the positive coordinates with which we are most familiar.

Example:



5. If students have not already done so, have them include the following on their own grid papers: add the horizontal and vertical number lines through the origin, label the *x*-axis and the *y*-axis, and label the quadrants.
6. Returning to the adventures of the character in the story, have students sort the points where the character was in each quadrant and make a list. Ask them to include the quadrant, the name of the point, and the ordered pair that describes each point.
7. To complete the story about the character's adventures (where the character went), have students turn the grid into a map. If students are interested in writing the story, perhaps the project can be integrated with English language arts.

Variations:

- Begin this learning activity by reading *Sir Cumference and the Viking Map* by Cindy Neuschwander, as an example of a story that uses coordinates to find a buried treasure.
- Provide scaffolding (e.g., pre-plotted grid paper, a pre-numbered grid, a list of questions to guide conclusions) for those requiring it.
- Eliminate the "math story" and the student-created points. Provide a piece of grid paper with one point labelled as origin (O), and 10 points labelled A to J. Explain the quadrant system, and have students add the axes, scale, and quadrants. Have them list the ordered pairs of the points in each quadrant.
- Photocopy any type of map and add a grid system that aligns with some significant points, including an origin and an *x*-axis and a *y*-axis. Distribute copies of the map to students, and have them trace the axes and add labels. Then have students use ordered pairs to describe the position of significant points in relation to the origin, or identify the location for a point of ordered pairs. The link between coordinate grids and mapping the Earth with lines of longitude and latitude provides an integration point for mathematics and social studies.

  Sample Website:

  The following online resource, written for children, provides some history on mapping the Earth with lines of longitude and latitude:

  The Math Forum @ Drexel. "Ancient Greek Maps." *Chameleon Graphing.* 1994–2011. http://mathforum.org/cgraph/history/greekmaps.html.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Label the axes of a Cartesian plane and identify the origin.
- ❒ Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.

Suggestions for Instruction

- **Label the axes of a Cartesian plane and identify the origin.**
- **Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.**
- **Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.**

Materials:

- Cartesian plane
- grid paper
- rulers
- stickers (optional)
- magnetic board and magnetic tape (optional)

Organization: Whole class (divided into two teams), small groups (of three students)

Procedure:

1. Review the features of the Cartesian plane and the ways in which the coordinate grid can be labelled with different scales, depending on the numbers being worked with.
2. Using a Cartesian plane, plot several points and label them with letter names. Ask students to name the ordered pair that identifies the location of a point for a particular letter, or ask students to name a letter in a quadrant, or the quadrant that matches a letter. Divide the class into two teams, and have students complete this exercise as a competitive game.
3. When students have demonstrated a level of competency, ask them to prepare a similar plot of eight letter points and a key that identifies the ordered pairs that represent the points of each letter. Inform students that the plot will be used for a group game, so they need to make it large enough to be seen and keep the key separate for quick reference.
4. Have students work in groups of three. One student takes the role of leader. The leader displays the plot students made, and asks the other group members questions related to reading the plot (e.g., What ordered pair names the location of X? What letter is at the point of a particular ordered pair?). The group must decide whether the leader will alternate questions between the contestants or whether the contestants will compete to be the first to answer. The round is over when the points have all been identified. At the end of the round, the winner becomes the next leader.

Variations:

- Students can play online games identifying points on a Cartesian plane. Two examples are available on the following websites:

  FunBased Learning. "Medium Version of Graph Mole." *Graphing. 2007.* http://funbasedlearning.com/algebra/graphing/points2/.

  In this game, a mole pops up at random points and the player must select the ordered pair that identifies the spot. The game has three levels, each successive level requiring a faster response.

  Math-Play.com. "The Coordinate Plane." *Coordinate Plane Game.* www.math-play.com/Coordinate%20Plane%20Game/Coordinate%20Plane%20Game.html.

  In this game, players match a point on a Cartesian plane with *x*- or *y*-coordinates, ordered pairs, or the quadrant in which the point is located.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Label the axes of a Cartesian plane and identify the origin.
- ❒ Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.
- ❒ Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.

Suggestions for Instruction

- **Label the axes of a Cartesian plane and identify the origin.**
- **Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.**
- **Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.**

Materials:

- Cartesian plane
- grid paper
- rulers

Organization: Small groups (of five students)

Procedure:

1. Students play a modified group version of the game Connect Four or Four in a Row. A group can consist of five students—one host and four participants.
2. Using grid paper (1 cm or larger), each student creates a game sheet consisting of a Cartesian plane with an *x*-axis and a *y*-axis, each having five negative and five positive divisions. Do not label the scale.
3. Each participant chooses an easy-to-draw symbol (e.g., #, ▲, ●, ✔), and the host chooses the scale to be used for the round and records it at the top of the game sheet.
4. The participants take turns naming an ordered pair that matches the scale, and the host writes that participant's symbol on the matching point the participant names. The participants practise identifying the ordered pairs, and the host practises plotting the points.
5. The participants try to get four of their symbols in a row. When they achieve this, they become the host.
6. Each participant will need to monitor that the host is plotting the ordered pairs correctly. If participants name a point already marked by a symbol, they lose their turn.

Variations:

- Increase the size of the grid and the number of symbols to align.
- Use stickers instead of drawing symbols. Or play on a magnetic board and have students affix magnetic tape to the back of their symbol pieces.
- Play the game in larger groups and have students cooperate to plan strategies.
- Hold a tournament.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Label the axes of a Cartesian plane and identify the origin.
- ❒ Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.
- ❒ Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.

Suggestions for Instruction

- **Label the axes of a Cartesian plane and identify the origin.**
- **Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.**
- **Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.**
- **Draw shapes and designs, using ordered pairs, on a Cartesian plane.**

Materials:

- BLM 7.SS.4.1: Plotting Points on a Cartesian Plane
- rulers

Organization: Whole class, individual, pairs

Procedure:

1. As a class, review the axes, origin, and quadrants of a Cartesian plane.
2. Distribute copies of BLM 7.SS.4.1: Plotting Points on a Cartesian Plane, and have students individually label the axes with the appropriate scales, plot the points, and identify the quadrants in which the figures are located.
3. Have students, working in pairs, exchange their completed plots with their partners. They compare plots and quadrants and discuss any discrepancies.

Variations:

- Have students choose a scale, and label the axes of a Cartesian plane accordingly. Next, they draw a simple shape, such as a polygon or some other figure, in any quadrant or combination of quadrants, and use the plot to generate a list of ordered pairs. Have students exchange lists with a partner, plot the points on the partner's list, and connect the points in the order given to create the figure. Students can verify each other's work.
- Provide students with designs plotted on a Cartesian plane, and ask them to create a list of the ordered pairs to create the design.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Label the axes of a Cartesian plane and identify the origin.
- ❒ Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.
- ❒ Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.
- ❒ Draw shapes and designs, using ordered pairs, on a Cartesian plane.

Suggestions for Instruction

- **Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.**

Materials:

- BLM 7.SS.4.2: Cartesian Plane Quadrant Cards
- scissors
- grid paper
- rulers
- blank card templates (optional)

Organization: Small groups (of two to four students)

Procedure:

1. Have students form small groups.
2. Distribute one set of card sheets to each group of students and have them cut the sheets to separate the cards. (See BLM 7.SS.4.2: Cartesian Plane Quadrant Cards.)
3. After giving students sufficient time to separate the cards, call the groups together, and review Cartesian planes, axes, origin, labelling scales, and quadrants.
4. Give the following directions to students:
   a) Mix up the quadrant cards and put them in a stack.
   b) Lay four ordered pair cards face up on the table.
   c) The first player draws a quadrant card and matches it to the set of ordered pairs that would create a triangle in that quadrant or set of quadrants.

d) If students agree the match is correct, the player keeps the set, and another ordered pair card is turned face up. If it is not a match, the quadrant card goes back in the deck. Some plotting on grid paper may be necessary to verify responses.

e) Play continues to pass to the next player.

f) The student who makes the most sets wins.

Variations:

- Students can create additional ordered pair cards that create other figures.
- Students or the teacher can create additional ordered pair cards. The cards can be used to play "make a set" card games fashioned after Go Fish!, Rummy, Pit, and so on.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.
- ❒ Use reasoning and visualization to help determine the placement of points.

Suggestions for Instruction

- **Label the axes of a Cartesian plane and identify the origin.**
- **Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.**
- **Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.**
- **Draw shapes and designs, using ordered pairs, on a Cartesian plane.**
- **Create shapes and designs on a Cartesian plane and identify the points used.**

Materials:

- BLM 7.SS.4.3: Plot This Picture
- grid paper
- ruler
- pattern blocks (optional)

Organization: Individual

Procedure:

1. Provide students with grid paper and copies of BLM 7.SS.4.3: Plot This Picture, and ask them to plot and connect the specified points to create designs.
2. Ask students to create their own designs on grid paper and list the ordered pairs. If they need help, suggest using pattern blocks to build a design, and then tracing it onto grid paper.
3. Display students' creations.

Variation:

- Provide students with the finished plot and ask them to create a list of ordered pairs that match the picture.

  Sample Website:

  For a display of coordinate picture designs and a student gallery of completed pictures, refer to the following website:

  Plotting Coordinates.com. *CoordinArt News.* 2008–2010.
  www.plottingcoordinates.com/coordinartnews.html.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Label the axes of a Cartesian plane and identify the origin.
- ❒ Identify the location of a point in any quadrant of a Cartesian plane using an ordered pair.
- ❒ Plot the point corresponding to an ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.
- ❒ Draw shapes and designs, using ordered pairs, on a Cartesian plane.
- ❒ Create shapes and designs on a Cartesian plane and identify the points used.

Shape and Space (Transformations) (7.SS.5)

Enduring Understanding(s):

While geometric figures are constructed and transformed, their proportional attributes are maintained.

General Learning Outcome(s):

Describe and analyze the position and motion of objects and shapes.

Specific Learning Outcome(s):	Achievement Indicators:
7.SS.5 Perform and describe transformations of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral vertices). [C, CN, PS, T, V]	(It is intended that the original shape and its image have vertices with integral coordinates.) ➛ Identify the coordinates of the vertices of a 2-D shape on a Cartesian plane. ➛ Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane. ➛ Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or successive transformations on a Cartesian plane. ➛ Perform a transformation or consecutive transformations on a 2-D shape, and identify the coordinates of the vertices of the image. ➛ Describe the image resulting from the transformation of a 2-D shape on a Cartesian plane by comparing the coordinates of the vertices of the image.

Prior Knowledge

Students may have had experience with the following:

- Performing and identifying a single transformation (translation, rotation, or reflection) of a 2-D shape, and drawing and describing the image.
- Performing a combination of successive transformations of 2-D shapes to create a design, and identifying and describing the transformations in the first quadrant of a Cartesian plane (limited to whole-number vertices).

- Identifying and plotting points in the first quadrant of a Cartesian plane using whole-number ordered pairs.
- Graphing collected data and analyzing the graph to solve problems.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Performing geometric constructions, including
  - perpendicular line segments
  - parallel line segments
  - perpendicular bisectors
  - angle bisectors
- Identifying and plotting points in the four quadrants of a Cartesian plane using ordered pairs.

Background Information

Transformations

Movement is ubiquitous in our world; even the Earth itself is in constant motion. Movements occur as changes in size, shape, or position. The area of mathematics that brings geometry and algebra together to describe these changes is the study of transformations.

The three transformations that Middle Years students study are translations, reflections, and rotations. Informally, these transformations are referred to as slides, flips, and turns, and each relates to changes in position and/or orientations in a 2-D plane. In previous grades, students performed and described single transformations of 2-D shapes. In Grade 7, they extend their skills to work with successive transformations in all four quadrants of the plane.

The learning experiences suggested for learning outcome 7.SS.5 will help students to develop their understanding and appreciation of the transformations existing around them, enhance their problem-solving skills and spatial sense, and prepare them for further studies in algebra and geometry.

Students regularly encounter 2-D transformations represented in design patterns and computer graphics. They are evident on logos, fabric patterns, frieze patterns, wallpaper, architectural design, landscape design, and so on. Transformations can be used to create interesting symmetrical patterns. In addition, 2-D transformations on Cartesian planes can be used to represent physical movements in a single plane (e.g., sports plays, rides at a fair, traffic routes). Movie animation is created using motion geometry.

Many designs are symmetrical transformations of a core. Examples can be viewed and created online.

Sample Websites:

For a range of samples of rotated shapes, refer to the following website:

Wolfram Demonstrations Projects. *Symbol Rotation Patterns.* Contributed by Danielle Nogle. Based on a program by Chris Carlson. 2011. http://demonstrations.wolfram.com/SymbolRotationPatterns/.

The following website allows students to create designs with up to eight lines of symmetry:

MathIsFun. "Symmetry Artist." *Geometry*. 2011. www.mathsisfun.com/geometry/symmetry-artist.html.

Transformations are studied using 2-D shapes, or pre-images, and their images. The shapes are named by their vertices (ABC) and the images are labelled (A′B′C′), read as A prime, B prime, C prime, and so on. Successive images are labelled with additional prime marks (A″B″C″), and so on.

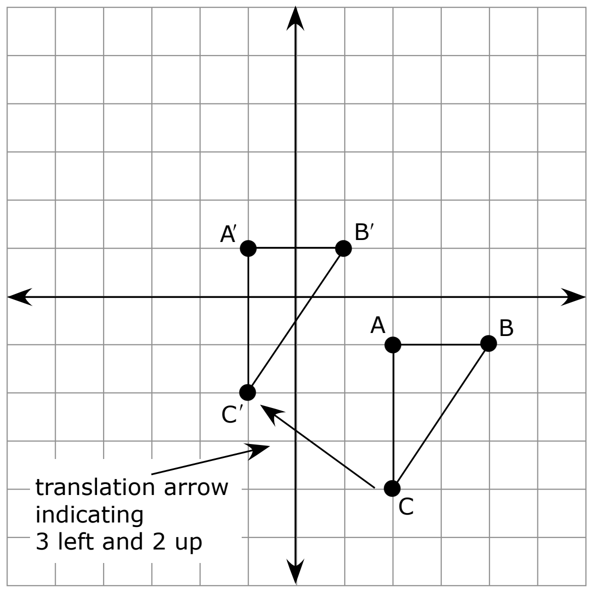
Translations, Reflections, and Rotations

In translations, reflections, and rotations, the shapes and their images are congruent, but their orientation on the plane and/or their location on the plane may change, depending on the shape and the type of transformation.

- **Translations (slides):** Each point in the shape moves the same distance and the same direction to create the image. The orientation of the shape and its image remain the same; only the location on the plane changes. Demonstrate translations on a coordinate grid by copying the shape onto grid paper of the same size, cutting it out, and physically sliding the copy on the grid. Another method is to count the horizontal and vertical moves of each vertex. A slide arrow indicates the direction of a translation.

Note:
As students advance in grades, it is important for them to be familiar with and use mathematical language. Encourage students to use the terms *translation*, *reflection*, and *rotation*, rather than *slide*, *flip*, and *turn*.

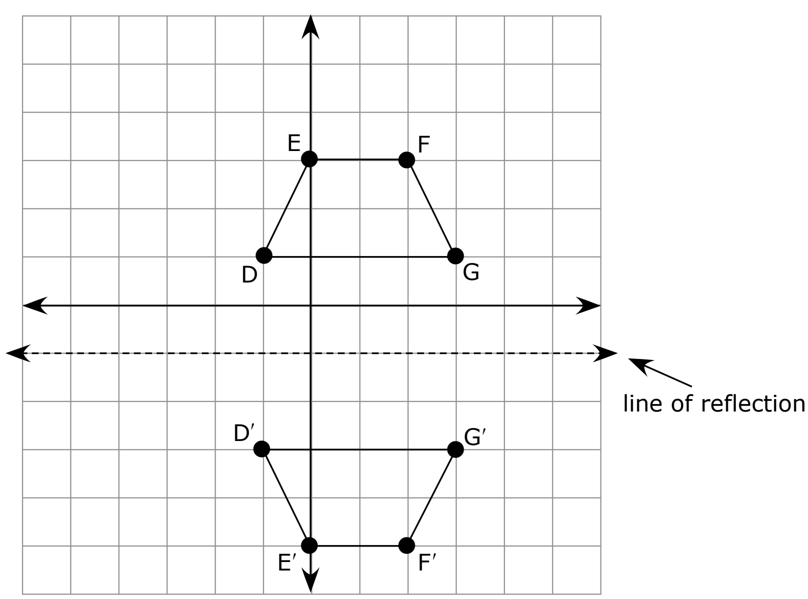
Example:



Translations are also commonly described using coordinate notation with square brackets (e.g., 3 left, 2 up is [–3, 2]).

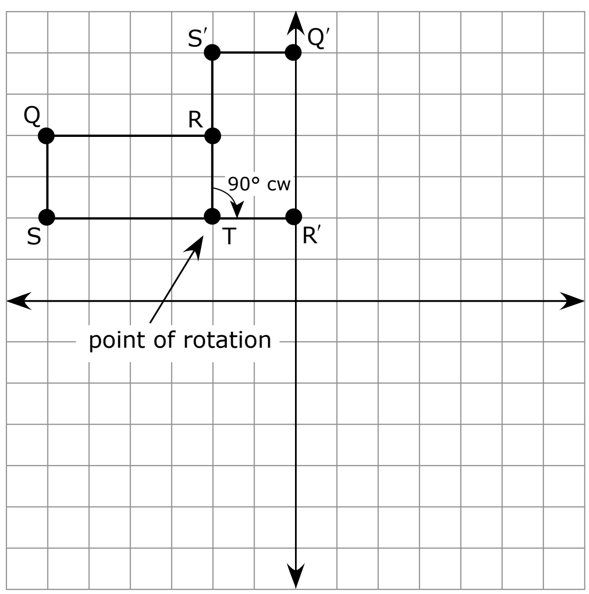
- **Reflections (flips):** The points in the shape and the matching points in the image are equal distances from a line of reflection. The line of reflection may be inside or outside the shape. The orientation of the figure flips over the line of reflection to create a mirror image in a new location. Demonstrate reflections by physically flipping a copy of the shape over the line of reflection, by placing a Mira along the reflection line, or by counting the perpendicular distance of each point from the mirror line. The line of reflection is indicated by marking a mirror line on the grid.

Example:



- **Rotations (turns):** The points of the shape are rotated the same number of degrees or fraction of a turn clockwise (cw) or counter-clockwise (ccw) around a point termed the centre (or point) of rotation. The centre of rotation may be any point inside or outside the figure. The orientation of the image will depend on the direction and the angle of rotation. The change in location of the image varies greatly, depending on the location of the centre of rotation and the direction and angle of rotation. Demonstrate rotations by physically rotating the shape the specified number of degrees around the centre of rotation, or by tracing the shape and the centre of rotation onto tracing paper, and then matching the centre of rotation and physically rotating the tracing paper. Rotating the side of the shape rather than just one point may help reduce accidental sliding during the rotation. Another technique to locate a rotation requires the use of a protractor to measure the angle and a compass to copy the line length. Specify rotations with a curved arrow that indicates the direction of the rotation, and write the number of degrees or the fraction of a turn for the rotation. Rotations are commonly described using a degree measure and direction (e.g., 90° ccw).

  Example:

Transformational changes can be described by identifying the type of transformation, the changes in orientation or position of the vertices, the horizontal and vertical movement, or the new x- and y-coordinates of the vertices of the image, or by stating the change in x- and y-coordinates between the shape and its image.

Mathematical Language

Cartesian plane
centre of rotation
clockwise
congruent
coordinates
counter-clockwise
image
line of reflection
quadrant
reflection
rotation
shape
transformation
translation
vertex
vertices

Learning Experiences

Assessing Prior Knowledge

Materials:

- large space (gymnasium or outdoors)
- one skipping rope (or another line) for each group
- math journals (optional)

Organization: Small groups (of six students), whole class

Procedure:

1. Form groups, with six students in each group.
2. Explain to students that they will be working together to demonstrate three types of transformations.

3. Review the concept that transformations are about moving and changing position.
   a) The three types of transformations students will demonstrate are translations (slides), reflections (flips), and rotations (turns).
      - Remember to include a mirror line for the reflection and reflect the entire shape.
      - Rotations require a centre of rotation, a direction, and an angle of rotation (e.g., 90º, 180º, 270º clockwise or counter-clockwise).
      - Centres of rotation and mirror lines can be inside or outside the shape.
   b) To show the pre-image and the resulting image, students can pair up as one point of the shape. To transform, they can divide, leaving one point (student) in the original location and one point (student) transformed to the new location. If this will confuse students, leave out this aspect and move the entire pre-image to the image.
4. Have group members work together to come up with a way to demonstrate the transformations using their bodies and a skipping rope. Later they will regroup to show their demonstration to their classmates and explain the transformation and its important features, including the pre-image, image orientation, and location.
5. As students are practising, circulate among them and provide hints and encouragement where needed.
6. Reassemble as a class and have students share their learning. Below are some ideas that may be included in the presentations:
   a) Translations may include triangle or rectangle arrangements.
      - Directions are given to slide so many steps forward, backward, right, or left.
      - The image has changed location only.
      - The points are still facing the same direction.
      - The orientation remains the same.
      - The image is still the same size or congruent to the pre-image.

b) Reflections may include students lying along a mirror line, and mirroring one another at any distance.
   - Each point and its partner are equal perpendicular distances to the mirror line.
   - The orientation of the pre-image has been flipped from the original.

c) Rotations require students to select a centre of rotation, a direction of rotation, and the amount or angle of rotation.
   - Students may be in a line with the group rotating around one end, the middle, or any point within the line, or they may rotate around another point away from the line.
   - They may also rotate in the formation of a triangle or a rectangle.

Variations:

- Assign one transformation to each group. Each group prepares a thorough presentation, complete with multiple examples.
- Have students work inside the classroom in pairs, performing the demonstrations with multiple objects rather than with their bodies. In place of presentations, students individually draw their example(s) and write an explanation in their math journals.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Perform and describe translations.
- ❒ Perform and describe reflections.
- ❒ Perform and describe rotations.

Assessing Prior Knowledge

Materials:

- grid paper (three for each student)
- polygon cut-outs from grid paper or pattern blocks, or some other small angular flat objects
- math journals
- Miras (optional)
- tracing paper (optional)
- push-pins (optional)

Organization: Whole class, individual or pairs

Procedure:

1. Have students practise demonstrating transformations. Direct students to place an object on the grid paper and demonstrate different types of transformations.
   a) Translations (slides up, down, right, or left, and diagonally)
   b) Reflections (flips over different lines of reflections in different directions)
   Use Miras to perform or confirm the reflections.
   c) Rotations about a point of rotation
   Vary the points of rotation to include vertices and points inside and outside the object. Tracing paper and push-pins may be used to perform or confirm the rotations.
2. Review notations. Model recording the various transformations, and review correct labelling.
   a) Label the vertices of the pre-image with capital letters (ABC) and label the corresponding vertices of the image with the letter, followed by a prime mark ($A'B'C'$). Successive images are labelled with additional prime marks ($A''B''C''$), and so on.
   b) Include symbols for slide arrows, lines of reflection or mirror lines, the centre of rotation, and the direction and amount of rotation.
   c) Translations are commonly written using slide arrows or using coordinate notation with square brackets (e.g., 3 left, 2 up is written as [–3, 2]).
   d) Reflections are written as reflected in the line (e.g., reflected in the line $x = 3$).
   e) Rotations are commonly written using a degree measure and direction (e.g., 90° ccw).

3. Model descriptions of movement. Have students describe the change between the pre-image and the image.

   Examples:

   - The triangle was translated 3 to the right, and so the pre-image and image hold the same orientation.
   - The square was reflected through the line $x = 1$, and so the image looks as though a mirror was held on the right-hand side of it. The image looks inverted.
   - The rectangle was rotated 90° counter-clockwise around point B, and so the image has B in the same place, but now A is below B.

4. Have students record some transformations. Ask them to use one grid paper for each type of transformation and include a few examples of each.

   a) Remind students to include mirror lines and centres of rotation.

   b) They may record combinations or successive transformations using the same pre-image, or use various pre-images and separate transformations.

   c) Have students include descriptions of the transformations they record.

   d) Some students may repeat successive transformations of one pre-image to create design patterns, as demonstrated on the following website.

      Sample Website:

      Wolfram Demonstrations Projects. *Symbol Rotation Patterns*. http://demonstrations.wolfram.com/SymbolRotationPatterns/.

5. Post students' displays when they are complete.

6. Meet as a class for a quick debriefing.

   a) Discuss what students learned or were reminded of during this learning experience, as well as any difficulties they faced and how they overcame them.

   b) Have students make a record of their learning in their math journals.

Variations:

- Provide grid paper with some shapes already sketched on the paper, and provide specifications for some transformations. Have students perform the transformations and label the images. In place of the sketches, provide the ordered pairs for the vertices of the pre-images and have students plot the pre-images and images.
- Have students sketch pre-images, specify transformations, prepare a key, and exchange papers with a partner, who will create the images. Then students return the papers to each other, verify responses, and discuss any discrepancies.
- Various games and exercises involving transformations are available online.

  Sample Website:

  To explore reflections in different mirror lines or rotations with lines of symmetry, refer to the following website:

  MathIsFun.com. "Symmetry Artist." *Geometry*. 2011. www.mathsisfun.com/geometry/symmetry-artist.html.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Plot points in the first quadrant of a Cartesian plane.
- ❐ Perform transformations of translations.
- ❐ Perform transformations of reflections.
- ❐ Perform transformations of rotations.
- ❐ Record transformations.
- ❐ Describe transformations.

Suggestions for Instruction

- **Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.**
- **Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or successive transformations on a Cartesian plane.**
- **Perform a transformation or consecutive transformations on a 2-D shape, and identify the coordinates of the vertices of the image.**

Materials:

- grid paper labelled as a Cartesian plane
- one number cube and one coin (or spinners with integers) for each pair of students
- BLM 7.SS.5.1: Comparing Points

Organization: Whole class, pairs

Procedure:

1. Place students in pairs.
2. Distribute copies of BLM 7.SS.5.1: Comparing Points, along with the number cubes and coins.
3. Demonstrate the process of generating points.
   a) Assign meaning to the manipulatives (e.g., heads represent positive, tails represent negative, six represents 0).
   b) Roll the number cube and toss the coin to determine an x-coordinate, and then repeat the process for a y-coordinate. (Try to work without plotting the points). Call the point A, and record the point and coordinates in the chart.
   c) The partner rolls and tosses to identify the next coordinate, labels it B, and records the coordinates. Compare the position of the new point to the old point. Discuss responses and justifications.
4. Ask students to follow the above method for generating points as they complete BLM 7.SS.5.1: Comparing Points.
5. Circulate among students as they work with their partners, and monitor whether they are on track.
6. After students have had sufficient time to work in pairs, reassemble as a class and discuss what students learned.

Variations:

- Have students continue the learning activity and generate a new set of three to five points, connect them into a polygon, and follow the pattern of question 3 (on BLM 7.SS.5.1) with the new shape. Have students choose any transformation, or successive or combined transformations. Ask them to try predicting the location of the images and then confirm the location by plotting them. Students can create their own Cartesian planes on grid paper and charts.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.
- ❒ Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or successive transformations on a Cartesian plane.
- ❒ Perform a transformation or consecutive transformations on a 2-D shape, and identify the coordinates of the vertices of the image.

Suggestions for Instruction

- **Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.**

Materials:

- BLM 7.SS.5.2: A Coordinate Map and/or blank grid paper

Organization: Individual or pairs

Procedure:

1. Decide whether or not you will need to discuss and demonstrate notations, describing movements as units left, right, up, and down, and describing changes in terms of change in each coordinate, or if students have sufficient background knowledge to proceed from the examples.
2. Introduce BLM 7.SS.5.2: A Coordinate Map.
   - Maps provide small visuals of much larger spaces. They show how locations are related to one another and they provide a way to communicate about the locations of different places.

- Encourage students to be creative with their maps. They may represent any place, ranging from their desks to anywhere in the world, actual or imagined.

3. Have each student plot and label several locations on the grid and create a key, following the directions in step 1 of BLM 7.SS.5.2: A Coordinate Map.
4. In step 2, when making trips around the community, students may work individually or with a partner.
   - Students look at one map at a time.
   - The partner tells the mapmaker which trip to make.
   - The mapmaker records the trip on the chart, determines the movement, and records it.
   - The partner verifies whether the information is correct.
5. Students draw a Cartesian plane on their grid.
   - Students title the final column of the key and the fourth column of the grid Coordinates of Trip.
   - They record the coordinates of the places on the map and the trips.
   - Partners verify each other's work.
6. Have students complete the final column, describing the movement between coordinates in terms of change in the x- and y-coordinates.
   - A positive change in y is an upward movement, and a negative change represents a downward movement.
   - For the x-coordinate, a positive change is right and a negative change is left.

Variations:

- Students can prepare directions for a partner to plot. First they plot the points, specify the trip, or describe movements. Partners exchange papers and complete the missing information, and then return papers to each other and verify each other's work.
- The teacher plots and labels the points on the grid, and fills in some coordinates and descriptions of movements on the trip chart before distributing it to students. Students determine the trip destinations. This allows the teacher a little more control over the learning activity, and provides experiences for students to work the moves backwards.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.

Suggestions for Instruction

- **Identify the coordinates of the vertices of a 2-D shape on a Cartesian plane.**
- **Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or successive transformations on a Cartesian plane.**
- **Perform a transformation or consecutive transformations on a 2-D shape, and identify the coordinates of the vertices of the image.**
- **Describe the image resulting from the transformation of a given 2-D shape on a Cartesian plane by comparing the coordinates of the vertices of the image.**

Materials:

- BLM 7.SS.5.3: Cartesian Plane Map and UFO Templates
- BLM 7.SS.5.4: Exploring Transformations: UFO Pilot Training
- BLM 7.SS.5.5: Recording Transformations: Travel Logbook
- Cartesian plane and triangle shape to fit the grid (for demonstration)
- Miras
- tracing paper
- push-pins (optional)
- scissors (optional)

Organization: Whole class, pairs or small groups, individual

Procedure:

Preparation

1. Tell students they will be exploring transformations to gain experience to pilot an unidentified flying object (UFO) accurately on a Cartesian plane.
2. Distribute copies of BLM 7.SS.5.3: Cartesian Plane Map and UFO Templates and two triangles per student, or have students prepare their own planes and triangles.

Part A: Demonstration (whole class)

3. Demonstrate, or have a student demonstrate, various transformations of the UFO on the grid provided. The purpose of the demonstration is to ensure that students have the skill to explore productively on their own.
   - Students can mimic the demonstration using their own grid.
   - Ask a student to identify the coordinates of the shape.
   - Tell students what transformation to make. Remember to include lines of reflection, and points, degree, and direction of rotation.

- Transform, or have a student transform, the UFO and identify the coordinates of the image.
- Discuss the movements and descriptions that would compare the position of the image to the position of the shape or pre-image.
- Record the coordinates and various descriptions as a model for later reference.
- Try varying the information given and required.

  Examples:

  - State coordinates of the shape, supply a description of the movement, and ask students to identify the coordinates of the image and/or a possible transformation.
  - Have students identify the coordinates of a shape, given the coordinates of the image and a description of the transformation.
  - Supply students with coordinates of the shape and the image, and ask them to identify a possible transformation.
- Include demonstrations of successive transformations and combinations of transformations. More than one transformation can match the same description and result in the same image.

4. When students are ready to explore piloting the UFO, move to Part B of this learning activity.

Part B: Exploration (pairs or small groups)

5. Distribute copies of BLM 7.SS.5.4: Exploring Transformations: UFO Pilot Training.
6. Tell students they will explore transformations, with the aim of being able to control the UFO and pilot it to and from specific locations.
7. Have students work in pairs or in small groups to explore describing and predicting the images created by transforming the UFO. The BLM provides some reminders and suggestions, as well as a chart for recording observations.
8. When students feel competent, have them try some intentional trips, using different transformations to move between two points on the Cartesian plane map. Use the vertex O as the on/off point that must touch the points found on the Cartesian plane map (BLM 7.SS.5.3: Cartesian Plane Map and UFO Templates).
9. Students can challenge each other to find the most efficient transformations, or alternate transformations, to travel between points.
10. When they are successful, they are ready for Part C of this learning activity.
11. Issue UFO licences if you wish.

Part C: Transformation Travel (individual)

12. Distribute copies of BLM 7.SS.5.5: Recording Transformations: Travel Logbook. Students will need their copies of BLM 7.SS.5.3: Cartesian Plane Map and UFO Templates and their UFO triangles.
13. Decide on the number of trips to be completed and any specific transformations, or alternate routes, the pilots must make to complete their mission (e.g., make it to point F in less than five trips using at least two different transformations).
14. Have students label the plotted points to represent location destinations on their maps.
15. Have students pilot their UFO from one location to another using transformations, and ask them to complete BLM 7.SS.5.5: Recording Transformations: Travel Logbook. A trip takes vertex O (on/off) from one location point to the other.

Variations:

- Hold a UFO derby. Students compete to see who can travel from one destination to another using the fewest transformations. (Translations count as one transformation per unit.)
- Students can explore and practise transformations using computer transformation software, applets, or games.

  Sample Websites:

  The following websites allow students to transform squares, parallelograms, and triangles.

  BBC. "Shape, Space and Measures." *KS2 Bitesize.* 2011. www.bbc.co.uk/schools/ks2bitesize/maths/shape_space/transformation/play.shtml.

  In this computer game, students choose a mirror line or rotation points to reflect a pentagon house onto its shadow.

  Reed, Jim. "33.01 The Coordinate Plane." *Grade 7: The Learning Equation Math.* 1999. http://staff.argyll.epsb.ca/jreed/math7/strand3/3301.htm.

  Directions are displayed with the applet.

  Shodor. "Transmographer." *Interactive*. 1994–2011. www.shodor.org/interactivate/activities/Transmographer/.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Identify the coordinates of the vertices of a 2-D shape.
- ❐ Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.
- ❐ Describe the positional change of the vertices that result from a transformation or from successive transformations.
- ❐ Perform a transformation or consecutive transformations on a 2-D shape, and identify the coordinates of the vertices of the image.
- ❐ Describe the image resulting from the transformation of a shape by comparing the coordinates of the vertices of the image to the vertices of the pre-image.

Suggestions for Instruction

- **Identify the coordinates of the vertices of a 2-D shape on a Cartesian plane.**
- **Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or successive transformations on a Cartesian plane.**
- **Perform a transformation or consecutive transformations on a 2-D shape, and identify the coordinates of the vertices of the image.**
- **Describe the image resulting from the transformation of a given 2-D shape on a Cartesian plane by comparing the coordinates of the vertices of the image.**

Materials:

- BLM 7.SS.5.6: Creating a Design Using Reflections
- grid paper
- rulers
- art supplies (for designs)
- display board
- file cards (optional)
- transformation software, draw programs, computer applets (optional)

Organization: Individual, whole class

Procedure:

1. Distribute copies of BLM 7.SS.5.6: Creating a Design Using Reflections. Have students follow instructions to
   - plot a five-sided shape
   - reflect the shape on the *y*-axis
   - reflect the expanded shape on the *x*-axis to create the design
   - describe the changes between the initial shape and the images
2. Have students create their own symmetrical design by creating a basic shape and one or more transformations. Ask them to include
   - a plot of the shape and its images on a Cartesian plane
   - a chart of the coordinates of the shape and its images
   - directions for creating the image (written on a file card)
   - a description of the changes between the shape and its images
3. Have students colour and frame their designs. Post the designs and the directions on a display board.
4. Hold a class debriefing session in which students share their designs, and discuss difficulties and solutions they encountered while creating the designs.

Variations:

- Explore creating designs using transformation software, draw programs, or computer applets.
- Add the file cards with design directions to a box from which students can draw cards and follow the directions to create designs.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify the coordinates of the vertices of a 2-D shape.
- ❒ Describe the positional change of the vertices that result from a transformation or from successive transformations.
- ❒ Perform a transformation or consecutive transformations on a 2-D shape, and identify the coordinates of the vertices of the image.
- ❒ Describe the image resulting from the transformation of a shape by comparing the coordinates of the vertices of the image to the vertices of the pre-image.

Suggestions for Instruction

- **Identify the coordinates of the vertices of a 2-D shape on a Cartesian plane.**
- **Perform a transformation or consecutive transformations on a 2-D shape, and identify the coordinates of the vertices of the image.**

Materials:

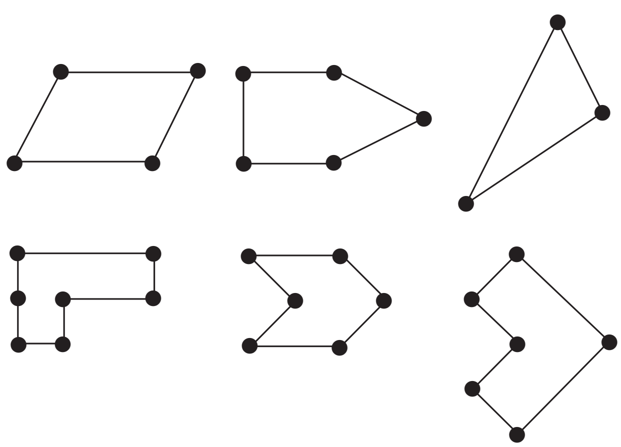
- BLM 7.SS.5.7: Which Plot Is Correct?
- number cubes
- rulers
- grid paper
- Miras (optional)
- tracing paper (optional)
- computer software or applets (optional)

Organization: Individual, whole class

Procedure:

1. Inform students that in this learning activity they will perform transformations on designated shapes.
2. Randomly assign shapes and transformations to each student. A suggested method follows:
   - Present six possible shapes and assign each a number from 1 to 6.

     Examples:



- List six types of transformations you would like students to practise, and assign each a number from 1 to 6.

  Suggestion:

  1—translation

  2—rotation inside shape

  3—reflection inside the shape

  4—rotation outside the shape

  5—reflection outside the shape

  6—combination

- Students roll a number cube to determine the shape and transformation(s) with which they will work.

3. Students individually plot their assigned shape on grid paper, choosing their own scale and coordinates to create the vertices of their shape.
4. Students choose details for their assigned transformation, and perform it on the shape.
5. Students write their name, draw a small diagram of their shape, provide a list of points and coordinates for the shape and image, and write directions for the transformation on the grid paper. This product will be used later in this learning activity.
6. Provide each student with a copy of BLM 7.SS.5.7: Which Plot Is Correct? Have students use the shape and transformation they made to create a multiple-choice puzzle on BLM 7.SS.5.7: Which Plot Is Correct?
   - Plot the correct shape in one of the locations A to D.
   - Plot the correct image in another box A to D.
   - Plot the shape and image incorrectly in the two remaining boxes. Suggest that students try not to be obvious as they create their errors.
7. Chronologically assign each student a puzzle number to write on his or her sheet, and on the original grid paper plot, which is the solution key.
8. Have students exchange papers with 10 or so people, and answer the puzzles in a chart like the one on page 2 of BLM 7.SS.5.7: Which Plot Is Correct?
9. Students return all puzzles to their creators.
10. Ask students to share the correct solutions to the puzzles by taking turns reading their puzzle number, and the correct letters for the shape and the image.
11. As a class, discuss any discrepancies identified, and refer to the original grid paper to correct any errors.
12. Store the puzzles and original plots in a folder for students to solve at other times.

Variations:

- Prepare several puzzles, copy them, and give the same sheet to each student to solve.
- Provide students with a list of coordinate points and transformation directions, and have them produce the coordinates for the image.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Identify the coordinates of the vertices of a 2-D shape.
- ❒ Perform a transformation or consecutive transformations on a 2-D shape, and identify the coordinates of the vertices of the image.

PUTTING THE PIECES TOGETHER

Golf Tournament or Animations

Introduction:

On easel pad grid paper, students will create a plan for a golf course that includes nine holes. The goal will be for a partner to complete the course with the fewest possible transformation "strokes."

Purpose:

In this investigation, students will demonstrate the ability to do the following (learning outcome connections are identified in parentheses):

- Construct circles with a given radius. (7.SS.1)
- Construct perpendicular and parallel line segments and bisectors. (7.SS.3)
- Perform and describe transformations of 2-D shapes in the four quadrants of a Cartesian plane. (7.SS.5)

Students will also demonstrate the following mathematical processes:

- Communication
- Connections
- Mental Mathematics and Estimation
- Problem Solving
- Reasoning

Materials/Resources:

- BLM 5–8.25: My Success with Mathematical Processes
- grid paper, easel pad grid paper, and poster paper
- ruler, compass, and protractor
- calculator
- art supplies
- animation software (optional)

Organization: Individual or pairs

Procedure:

Student Directions

1. Draw an x-axis and a y-axis centred on a large piece of easel pad grid paper.
2. On this Cartesian plane, begin drawing a golf course. Start by placing nine numbered tees on the plane. Then draw numbered holes for each tee, making sure you put the hole in a different quadrant than its corresponding tee. Paths from tees to holes may overlap or cross each other. Holes and tees should be placed on whole-number ordered pair coordinates, not between them.

3. Add the following eight obstacles on your nine-hole golf course, making sure you do not overlap these with any holes or tees:
   a) two water traps, with a 10 cm diameter
   b) two sand traps, with a radius of 7 cm and 5 cm
   c) two treed areas, with a radius of 3 cm and 4 cm
   d) two parallel rows of trees 20 cm by 2 cm
4. Play each other's golf courses by using transformations to move an imaginary ball from its tee to its hole. The goal is to complete the course with the fewest possible transformations, but only the following transformations are allowed:
   a) a translation directly left or right by up to 4 units
   b) a translation directly up or down by up to 4 units
   c) a clockwise or counter-clockwise rotation of 90°, 180°, or 270°, with a whole-number coordinate pair as the centre of the rotation
   d) reflections across the horizontal and vertical lines

   Balls cannot rest on obstacles, but can move over or around them.
5. Complete BLM 5–8.25: My Success with Mathematical Processes.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Construct circles of a given radius.
- ❒ Construct parallel lines.
- ❒ Describe transformations in the four quadrants of a Cartesian plane.

Extension:

Use animation software to create an animation based on transformations. Animations may include the following:

- a nature scene in which the sun comes up, travels through the sky, and sets, while a creature visits different places, finding food, drinking water, or hiding. Include a combination of jumping, sliding, turning, and spinning movements.
- a park scene with playground equipment, a play area, a garden area, a picnic area, entrances, and so on
- an amusement park with rides and concessions
- a city scene with a bank, a theatre, stores, restaurants, bus stops, medical offices, and so on
- a basketball or hockey game with a court or a rink drawn to scale