Unit A: Problem Analysis

Half Course II

HALF COURSE II

Unit A: Problem Analysis

Hours: 4

General Learning Outcome:

Develop and use mathematical strategies to solve problems in different situations.

The intent of this unit is to provide a range of interesting problems of a primarily non-algebraic nature. These problems augment the work of other units.

Specific Outcome

A-1 Solve problems using a variety of non-algebraic approaches.

PROBLEM ANALYSIS

Instructional Materials

- *Essentials of Mathematics 10*
- See Appendix I for possible activities.
- See Appendix II for additional resources.

PRESCRIBED LEARNING OUTCOMES

General Outcome

Develop and use mathematical strategies to solve problems in different situations.

Specific Outcome(s)

A-1 solve problems using a variety of non-algebraic approaches

SUGGESTIONS FOR INSTRUCTION

The problems contained in Appendix I are intended to provide material which is interesting in its own right and which complements the other units of the program. It is illustrative rather than exhaustive. Some activities have been chosen to illustrate a wide variety of job and consumer applications of mathematics that are largely non-algebraic. Others have been chosen because they are intrinsically interesting or because they challenge students to find and to use new ways of analyzing and thinking mathematically. All students do not need to engage in the same activities.

The activities in Appendix I are presented in **no** particular sequence. Teachers are encouraged to supplement this set of activities with material from other sources, such as the Internet. A preliminary list of possible resources is included in Appendix II.

It is suggested that these problems and activities be interspersed throughout the course as either extensions, enrichment, or a change of pace in the day-to-day work of the classroom. Some of them will link directly to particular units, but most are independent and **may** be used at any time. One approach would be to introduce problem analysis with a few days, possibly up to a week, of work on these activities. Intersperse the remainder throughout the course.

Note: Some learning experiences require the use of imperial units of measure. Teachers have the following options:

1. Introduce students to the necessary imperial units of measure or wait until the work in the Geometry Project has been done.
2. Change the problems to metric units of measure, where appropriate.
3. Substitute alternate learning experiences or problems.

SUGGESTIONS FOR ASSESSMENT

Students' progress should be assessed over long periods of time. Look, for example, for an increasing use of a variety of problem-solving strategies and increasingly sophisticated explanations. Anecdotal records of how students work in pairs or groups on these activities is appropriate. Well-developed solutions and examples of reasoning could become part of a student's portfolio.

Problem-solving activities are generally not appropriate on pencil-and-paper timed tests.

SUGGESTED LEARNING RESOURCES

Print

Austin, J.D. *Applications of Secondary School Mathematics*. Reston, VA: NCTM, 1991.

Giblin, P., and I. Porteous. *Challenging Mathematics.* Toronto/New York: Oxford University Press, 1990.

Hirsch, C.R., and R.A. Laing. *Activities of Learning and Teaching*. Reston, VA: NCTM, 1993.

Mathematical Association of America and National Council of Teachers of Mathematics. *A Sourcebook of Applications of School Mathematics*. Reston, VA: NCTM, 1980.

National Council of Teachers of Mathematics. *NCTM Student Math Notes*. Reston, VA: NCTM, n.d.

Senior 2 Consumer Mathematics (25S) Part I: A Course for Distance Learning. Winnipeg, MB: Manitoba Education and Training, 2000.
— Cover Assignments Modules 1-5

Swetz, F., and J.S. Hartzler. *Mathematical Modeling in the Secondary School Curriculum*. Reston, VA: NCTM, 1991.

Two journals which contain useful teaching ideas are:

The Mathematics Teacher. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA: 22091-1593.

Mathematics in School. The Mathematical Association, 259 London Road, Leicester, UK: LE2 3BE.

See Appendix II for a list of additional resources.

Appendix I

Teacher Information: Networks II

Skills Required

- pattern recognition
- systematic counting

When To Do

This learning experience can be completed any time after completing Networks I in Half Course I. This problem is best done prior to the Travelling Sales Rep activity.

Teacher Information

The problem is an extension of Networks I in Half Course I. In this problem the students are required to count the number of even vertices and odd vertices. After filling in the chart, the students determine whether the figure can be drawn without actually drawing it.

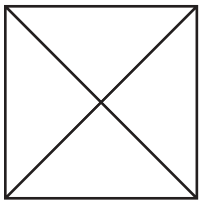
1. A closed network of all even vertices can always be traversed without travelling any arc twice.
2. If a closed network contains exactly two odd vertices, it can be traversed without travelling any arc twice by starting at an odd vertex.
3. If a closed network contains more than two odd vertices, it cannot be traversed without retracing an arc.

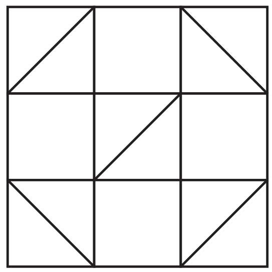
Chart

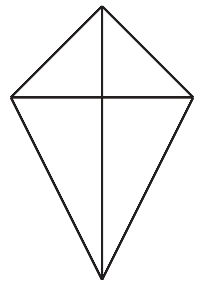
Figure	Number of Even Vertices	Number of Odd Vertices	Can the Figure Be Drawn?
a	1	4	no
b	14	2	yes
c	1	4	no
d	1	6	no
e	7	2	yes
f	16	0	yes
g	4	0	yes

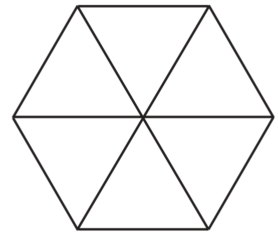
Blackline Master: Networks II

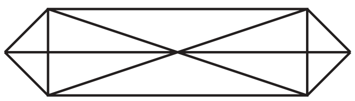
Try to draw each figure below using a continuous line without drawing any arc twice.

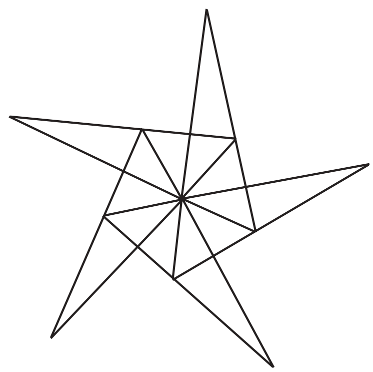
a)


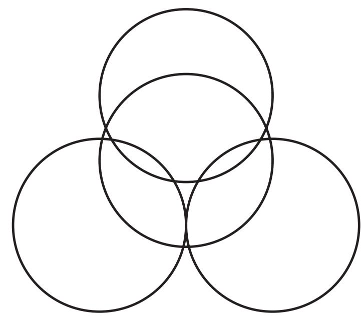
b)


c)


d)


e)


f)


g)


For each figure, count the number of lines which are around each point (vertex) in the figure. Count the number of vertices which have an even number of lines and those that have an odd number of lines. Record your data on the table provided.

Figure	Number of Even Vertices	Number of Odd Vertices	Can the Figure Be Drawn?
a			
b			
c			
d			
e			
f			
g			

1. From the data in the table, is it possible to determine whether the figure can be drawn without actually drawing it? (**Hint:** Look at the **odd** vertices for each picture.) Explain.
2. Create a figure that cannot be drawn.
3. Create a figure that can be drawn.

Teacher Information: Travelling Sales Representative

Skills Required

- basic arithmetic (addition) or use of a calculator
- organization

When to Do

This may be done at any time, since the prerequisite skills are minimal. For some students it may be "fun." This learning experience could provide a change of pace as students work on other units.

Teaching Suggestions

There is no known algorithm for solving the general travelling sales representative problem (finding a path which includes each vertex in a network exactly once). This may be of interest to students.

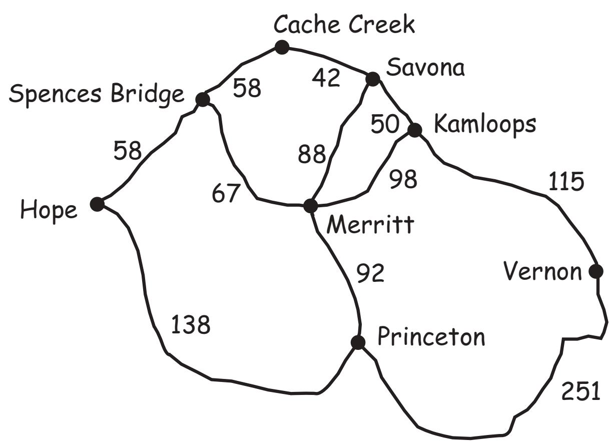
Solutions

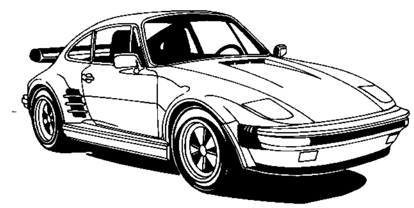
1. Spences Bridge — Hope — Princeton — Vernon — Kamloops — Merritt — Savona — Cache Creek — Spences Bridge, among others.
2. Spences Bridge — Hope — Princeton — Kamloops — Vernon — Kamloops — Savona — Cache Creek — Spences Bridge, or vice versa, 766 km.

Blackline Master: Travelling Sales Representative

A travelling sales representative whose territory includes a part of British Columbia wishes to travel to each of the towns on the map below that are designated by "•". The trip is to begin and end in Spences Bridge.

1. Find a route which satisfies the condition and which passes through each • town exactly once.
2. Find the shortest route which includes each town at least once.





Travelling Sales Representative: From Mathematical Association of America and National Council of Teachers of Mathematics, "Travelling Sales Representative." *A Sourcebook of Applications of School Mathematics*. Copyright © 1980 by National Council of Teachers of Mathematics.

Teacher Information: Designing a Quilt

Skills Required

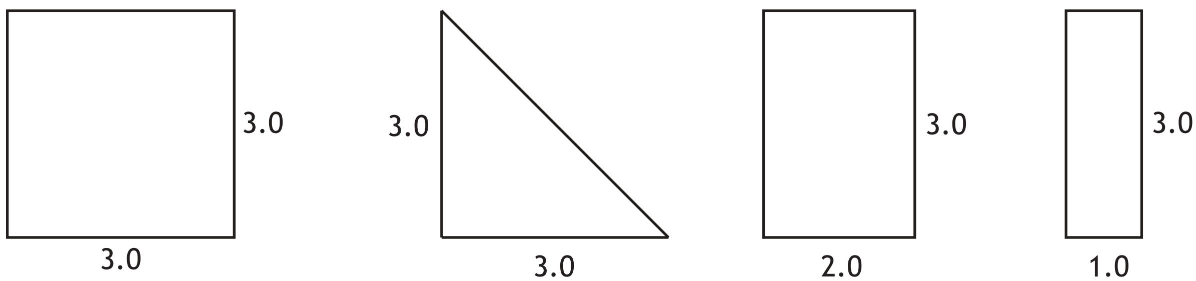
- finding area of rectangles and triangles
- visualization
- spatial design

When To Do

This activity may be introduced at any time and will provide a review of area concepts. It might provide a break during the Geometry Project.

Teacher Information

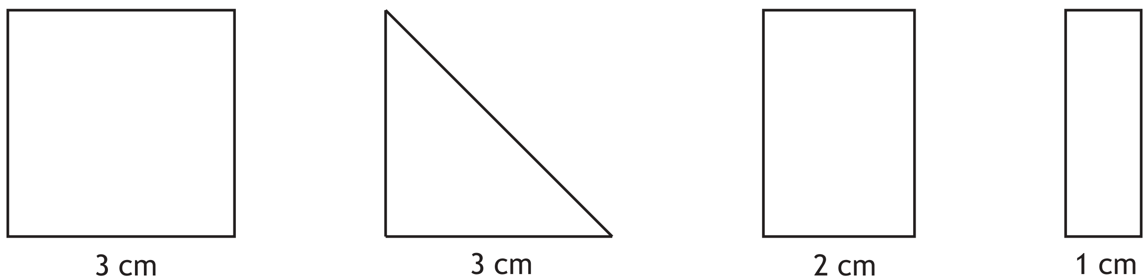
The measurement of each shape drawn at a scale of 1:10 should be:



1. Students can use any combination, but the result must show a square or rectangle.
2. Count how many of each shape in the block, and find the areas.
3. Students may choose to cut their pattern to fit the given dimensions, or they may alter the original block to fit. The assumptions listed should find the students explaining what they did and why.
4. Students should give a reasonable size for an adult-sized quilt which could vary (approximately 150 cm by 250 cm).
5. Depending on the block pattern, calculate cost of all four shapes.

Blackline Master: Designing a Quilt

Many quilts have block patterns which are repeated. You are given these four basic shapes drawn to a scale of 1:10. Each shape is a different colour.



1. Design a block pattern using any combination and number of the shapes. The block must end up as a square or a rectangle.
2. Calculate the area of material needed for each shape to make your block.

   A rectangle = b x h

   A triangle = $\frac{b \times h}{2}$

3. You are making a quilt for a baby's crib. The quilt must measure 120 cm by 120 cm. How many of your block patterns would be required to make this quilt? Find the total area of material required for each shape. State any assumptions you made.
4. You are making a quilt for an adult's bed. Decide on the dimensions needed. How many of your block patterns would be required to make this quilt? Find the total area of material required for each shape. State any assumptions made.
5. The material is sold at $4.80/m^2 plus taxes. Find the total cost for each quilt.

Teacher Information: Purchasing Lumber

Skills Required

- area and volume calculations, either by hand or calculator
- understanding of perimeter to build a fence
- some notions of design, style, and need for privacy
- cost calculations

When to Do

This activity may be introduced at any time and will provide review for many of the measurement concepts. It might provide a break during the Geometry Project or the Consumer Decisions unit.

Teaching Suggestions

In doing the problem of designing and estimating the cost of the fence, some discussion of the factors involved in the decision making would be appropriate. Such topics as design, the need for privacy, and costs are important to consider.

Solutions

1. a) 24

   b) 48

   c) 100

   d) $\frac{262}{3} = 26.67$

   e) 50

2. One board foot = 144 cubic inches. One can calculate what fraction of 12 square inches the cross-section of the given board is and then multiply by the length. Or if you let t = thickness, w = width, and l = length (all in inches), then the following formula applies:

$$\text{Board Feet} = \frac{t \bullet w \bullet l}{144}$$

Problem

There is no correct answer to this question. Students will need to make choices and design the fence (see Factors to Consider). To calculate the cost of material, students will need to research the cost of lumber, paint, and rails before calculations can be done. Students may want to use a spreadsheet for the calculations.

Purchasing Lumber: Adapted from Blocksma, M., *Reading the Numbers*. Copyright © 1989 Mary Blocksma. Reprinted by permission of Mary Blocksma.

Blackline Master: Purchasing Lumber

When purchasing lumber, a consumer should have an understanding of how lumber is graded and know the difference between the two systems by which dimensional lumber is sold — linear feet versus board feet.

Linear Feet

Planed or surfaced lumber is often sold by the linear foot. If a board is priced at $1.50 per linear foot, you will be charged $1.50 for every foot of that board you buy.

Board Feet

Rough or unplaned lumber is often sold by the board foot. A board foot actually refers to a specific volume of wood. A board foot represents a piece of wood one inch thick, 12 inches wide, and one foot long (i.e., a square foot that is one inch thick).

1. How many board feet are the following?

   a) 24 feet of 2" × 6"

   ____

   b) 36 feet of 2" × 8"

   ____

   c) 60 feet of 2" × 10"

   ____

   d) 80 feet of 1" × 4"

   ____

   e) 100 feet of 1" × 6"

2. Given the thickness and width of the board in inches, and the length of the board in feet, is there a formula which would make these calculations easy?

   ____

   ____

Blackline Master: Purchasing Lumber (continued)

Grading Lumber

Hardwoods such as oak or maple are used in furniture and finishing the interior of homes. Softwoods such as cedar, fir, or spruce are used for framing buildings or in general construction. Hardwoods are graded differently than softwoods. In softwoods, cedar and fir can be purchased as “clear” or “select.” “Clears” are boards with no knots and are the most expensive. “Select” boards are of lesser quality than clears, but defects are minor. Spruce and all other lumber that is below these two levels fall into the construction category.

No. 1 (construction).	many knots (diameter under two inches)
No. 2 (standard)	many knots (diameter up to 3½ inches)
No. 3 (utility).	allows open knots, splits, pitch
No. 4 (economy)	lowest grade

Rough vs Planed Lumber

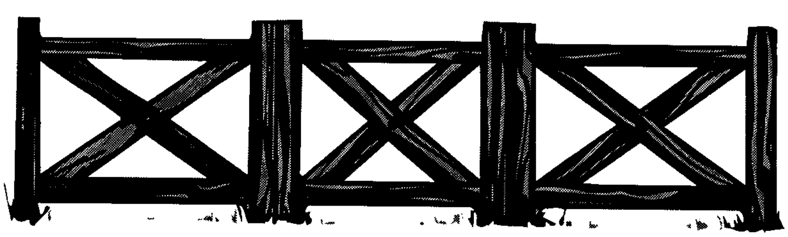
All lumber is measured in the rough. A 2" × 4" in rough lumber will measure 2 inches by 4 inches. If you buy a 2" × 4" that is planed it will lose some of the original wood in the finishing process. Its measurement once planed will be 1½" × 3½". (Typically ½ inch is removed from 2-inch boards and ¼ inch is removed from 1-inch boards.) A planed 1" × 6" will have measurements approximately ¾" × 5½".

Problem

Mr. Smith owns a rectangular lot which is 60' wide by 100' deep. He wants to enclose his entire back yard on three sides with a wooden fence. The back yard measures 60' wide by 30' deep. The fence is to be 6' high. 4 × 4 fence posts are 8' long and are to be spaced no farther than 8 feet. Two 2 × 4s are to be placed between posts. The fence is to be covered with 1 × 6 boards. Calculate the amount of lumber required and the cost of material.

Factors to Consider

- How important is privacy? How much space do you want between boards? Do you want boards on both sides of the fence?
- How much time do you want to spend on maintenance? Do you use plain spruce and paint or stain the fence? Do you want to consider how long the fence should last? Is there an advantage to building with treated lumber or cedar?



Teacher Information: How Can I Get There?

Skills Required

- pattern recognition
- communication (in pairs)
- spatial visualization
- organization of data

When To Do

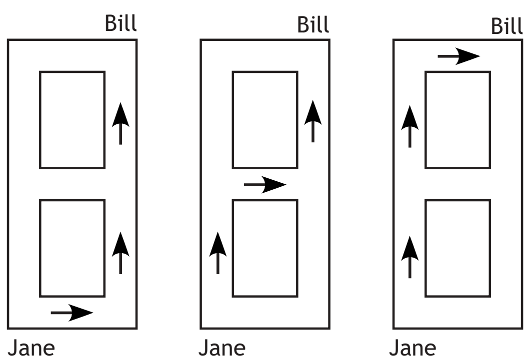
This can be used any time.

Teacher Information

May work well if students work in pairs. Depending on the class, this activity may work better as a teacher-directed activity.

Solutions

1.

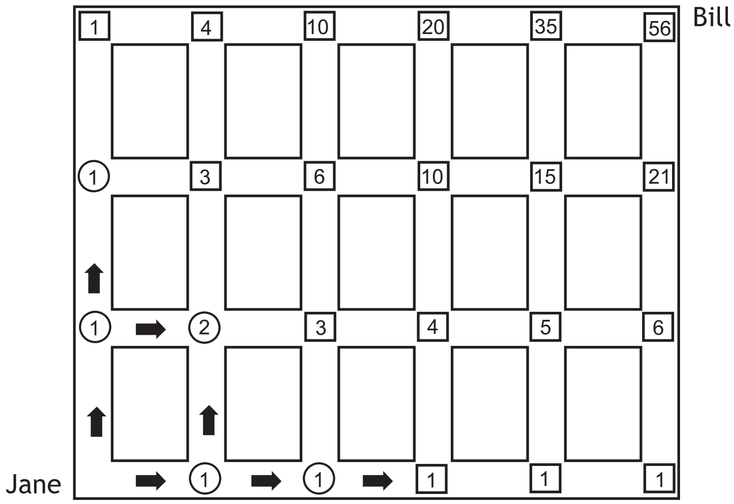


2. RRU, URU, UUR
3. RRR, 1
4. RRU, RUR, URR, 3
5. RUU, URU, UUR, 3
6. UUU, 1
7. B and C
8. 1 + 2 = 3
9. C
10. 1

How Can I Get There?: From Kring, B., "How Can I Get There?" *NCTM Student Math Notes* (Nov. 1996).

Teacher Information: How Can I Get There? (continued)

11.

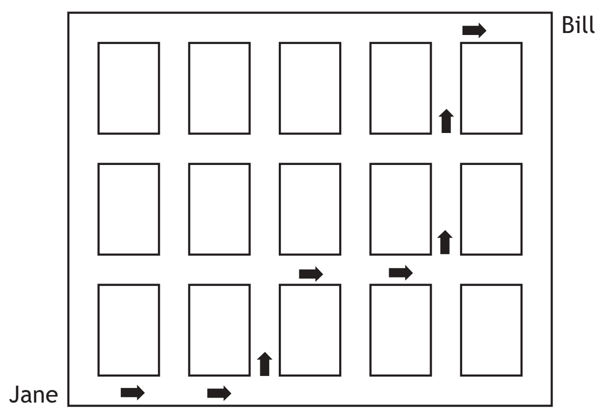


12. 56 days

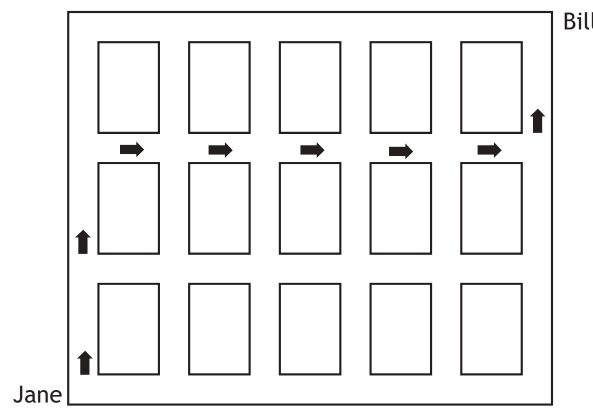
13. 3, 5

Blackline Master: How Can I Get There?

Jane visits her buddy Bill each day. Bill lives eight blocks away, and Jane likes to vary her routine to keep her interest, since the scenery is not especially exciting. Using a map of the neighbourhood, Jane decides to figure out how many different ways she can take if she always goes either to the right or up. She chooses to take a different eight-block route each day. She wonders how many different days will pass before she has taken every possible route. Solid arrows are used to show possible routes. Possible route 1 can be summarized by RRURRUUR, where R is "go right" and U is "go up." Possible route 2 can be summarized by UURRRRRU.

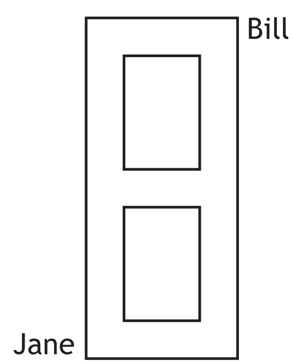


A possible route 1.



A possible route 2.

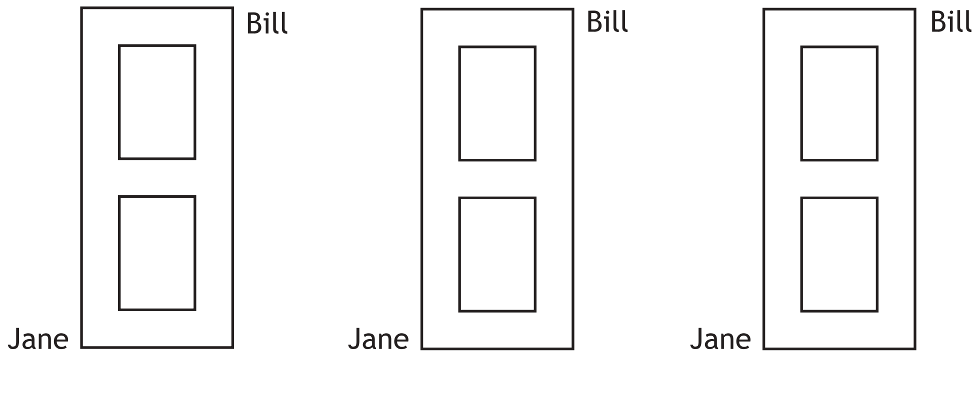
Many paths are available. Jane wants to get organized so that she does not repeat any routes to Bill's house. Let us help her by first looking at the simpler exercise at the right.



How Can I Get There?: From Kring, B., "How Can I Get There?" *NCTM Student Math Notes* (Nov. 1996).

Blackline Master: How Can I Get There? (continued)

1. Use solid arrows, one for each block travelled, as shown in possible routes 1 and 2, to draw a different possible route on each diagram below. Remember, Jane has decided that she can go only either to the right or up.



2. Symbolize your three different routes using U and R as shown in the previous examples.

In the original problem, when Jane leaves her house, she must travel either to the right or up. We can indicate this path by drawing solid arrows. Thus, Jane has only one way, indicated by ❶ in **Figure 1**, to get to either of the first two possible intersections by travelling either R or U.

If Jane first travels R, the two possible second blocks to travel are indicated by the clear arrows (➪). If Jane first travels U the two possible second blocks to travel are indicated by the solid arrows (➠). See **Figure 1**.

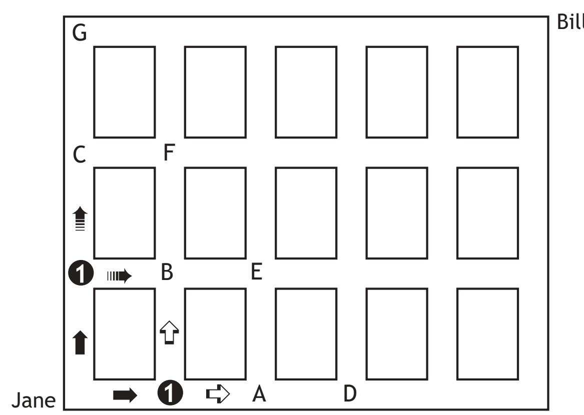


Figure 1

Blackline Master: How Can I Get There? (continued)

In **Figure 1**, Jane gets to intersection A by going RR; to get to intersection C, she must go UU. To get to intersection B, she could go either RU or UR. The total number of ways to get to B are indicated by the ❷ in **Figure 2**. Using Us and Rs, symbolize the paths that will get Jane from her house to these locations:

3. Intersection D:________________ How many paths are possible?_________
4. Intersection E:________________ How many paths are possible?_________
5. Intersection F:________________ How many paths are possible?_________
6. Intersection G:________________ How many paths are possible?_________

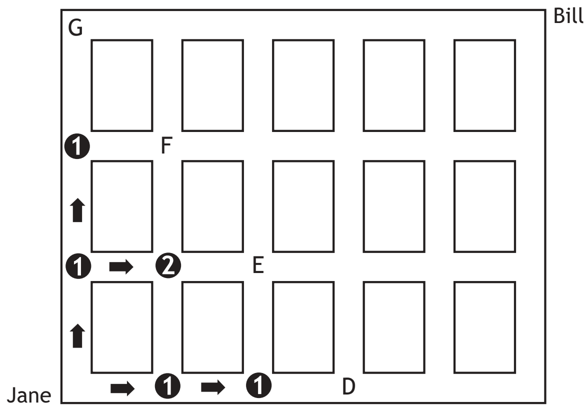


Figure 2

Because Jane can go only to the right or up, the only way she can get to intersection D is from intersection A. Since only one way exists to get to intersection A, only one way exists to get to intersection D. Jane can get to intersection E from either intersection A or intersection B. Since only one way exists to get to intersection A and two ways exist to get to intersection B, 2 + 1, or 3, ways are available to get to intersection E.

7. From which two intersections can Jane get to intersection F? ________________
8. How many paths can she take to get to intersection F? ___________________
9. From which intersections can Jane get to intersection G? ___________________
10. How many paths can she take to get to intersection G? ___________________

Blackline Master: How Can I Get There? (continued)

11. Look for a pattern to find the number of paths to get to each intersection along the route from Jane's house to Bill's house. Record these numbers at the appropriate intersections in **Figure 3**.

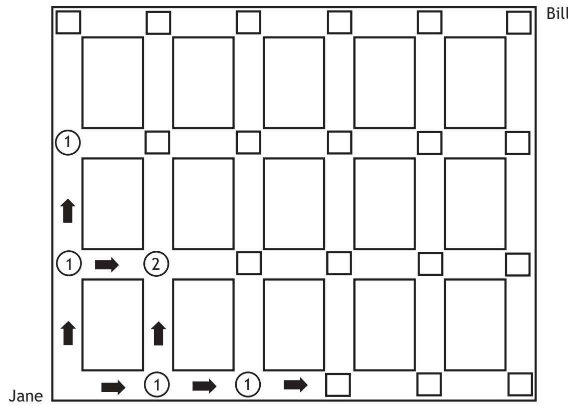


Figure 3

12. Because Jane will travel a different path each day, how many days will pass before she has taken every possible path? ________________

13. If each possible path is symbolized with Us and Rs, how many Us are in the path? _______ Rs? _______

Teacher Information: Magic Shapes II

Skills Required

- basic arithmetic

When To Do

Anytime

Teacher Information

Big Magic: Have students cut out and arrange the pieces. This could be done in small groups or as a whole class. If done with the whole class, have the cutouts on an overhead sheet.

This problem could be done as a game:

Arrange students in groups of 3 to 6

Place each set in an envelope.

Students distribute pieces as evenly as possible.

Without talking, students work together to assemble the square.

If students are frustrated, tell them the sum is 34. For the other puzzles, tell students the sum needed if they are frustrated.

Solutions

Big Magic

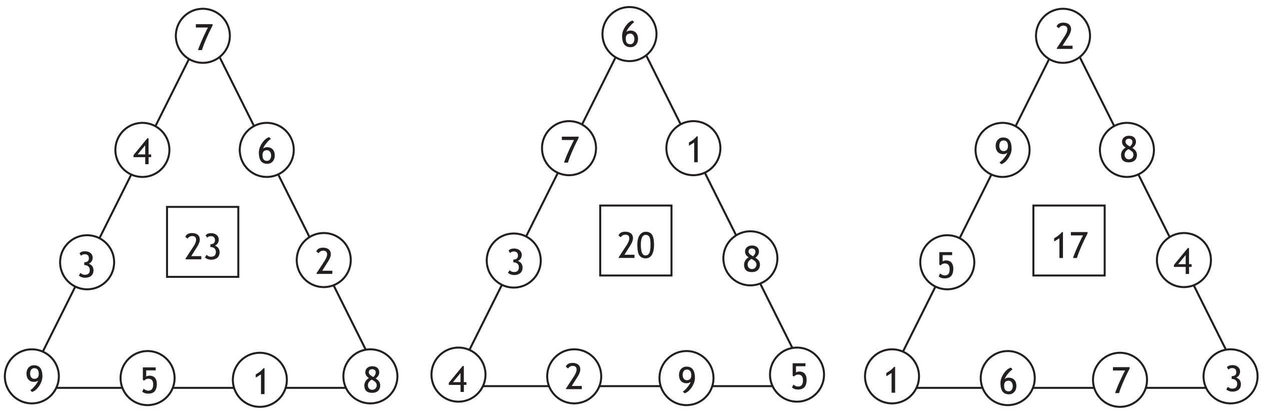
One solution is:

1	11	6	16
8	14	3	9
15	5	12	2
10	4	13	7

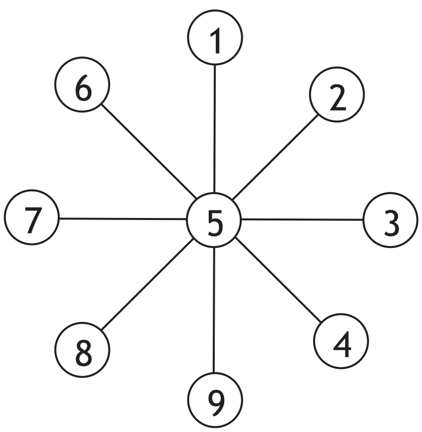
Teacher Information: Magic Shapes II (continued)

Solutions (continued)

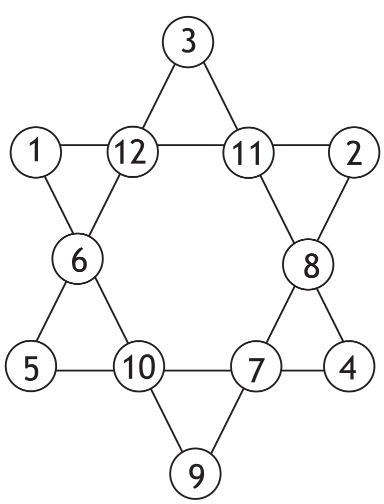
Magic Triangle



Circle Game



Magic Star



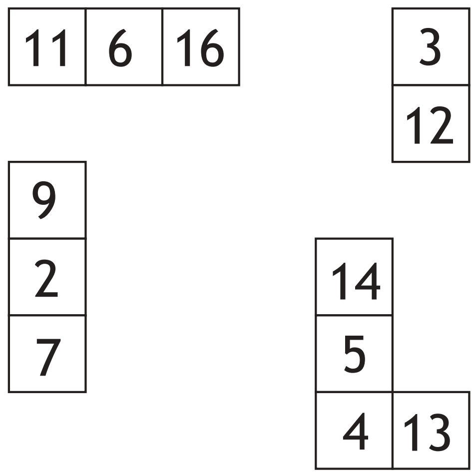
Extension:

Have students create their own magic figure.

Blackline Master: Magic Shapes II

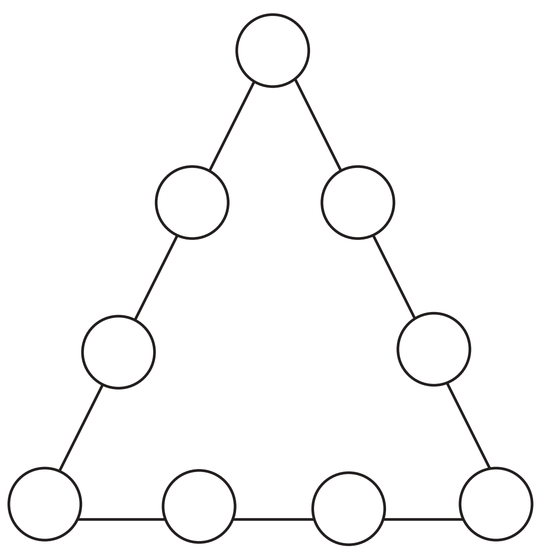
Big Magic

The sections below belong to a 4 x 4 magic square (the sum of each row, column, and diagonal are equal). Can you reassemble the magic square?



Magic Triangle

Place the numbers 1 through 9 in the circles. When you are finished, the sums of all three sides must be equal.

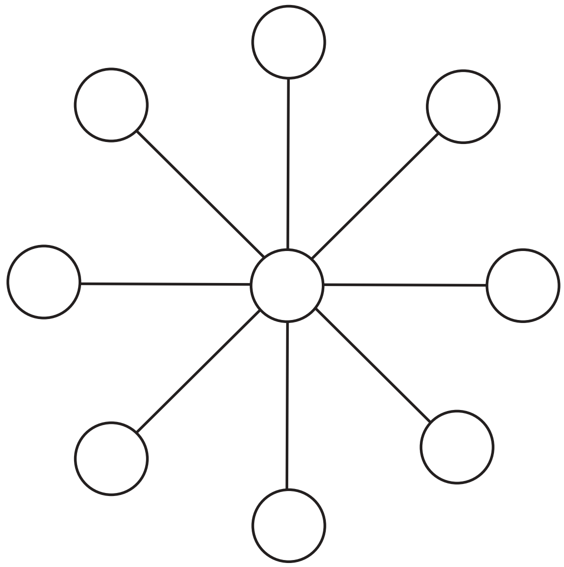


Can you find all three solutions?

Blackline Master: Magic Shapes II (continued)

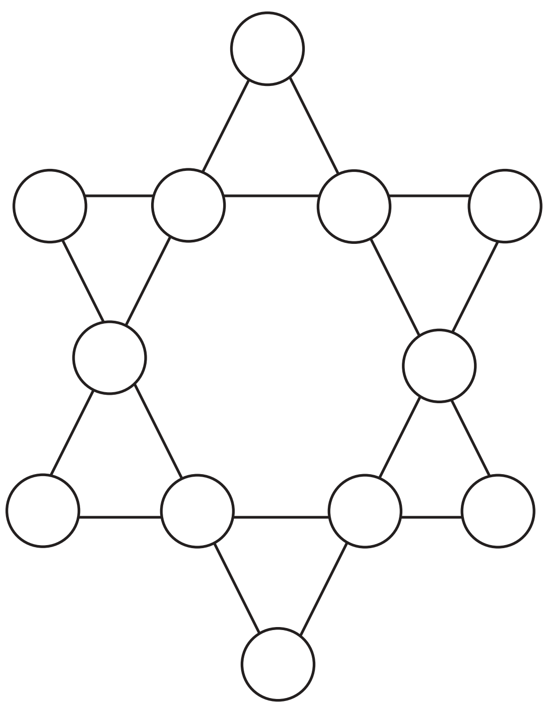
Circle Game

Can you place the numbers 1 through 9 in these circles so that the sum of the three circles connected vertically, horizontally, or diagonally is equal to 15?



Magic Star

Using the numbers 1 through 12, can you create a magic star where the sum of all rows of four equals 26?



Teacher Information: Check It Out

Skills Required

- spatial visualization

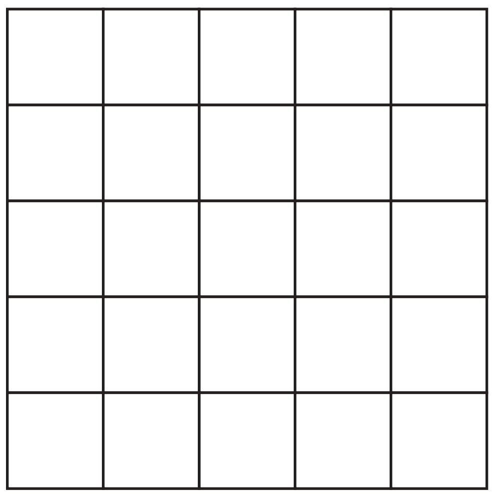
When To Do

This activity may be introduced at any time. It may provide a new context for the rotations and reflections section in the Spatial Geometry unit.

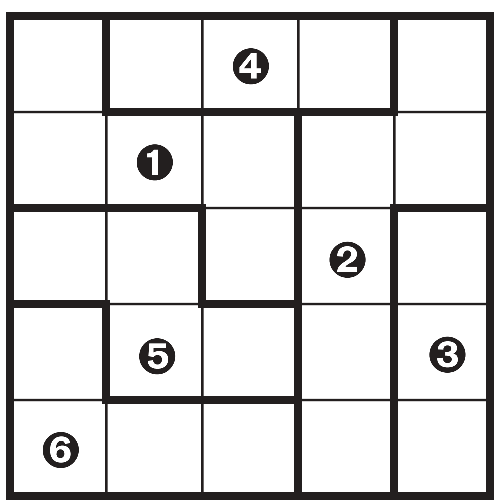
Teacher Information

Talk about a 5 x 5 checkerboard grid.

Students could cut out and manipulate the pieces on the template (optional).



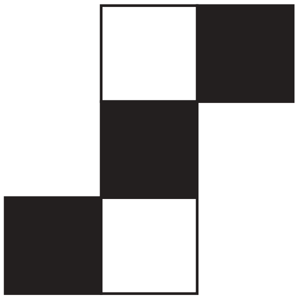
Solution



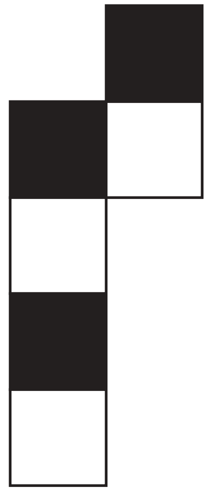
Blackline Master: Check It Out

The six sections below are parts of a 5 x 5 checkerboard grid. Can you piece them back together to form the original pattern?

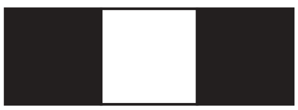
1.



2.

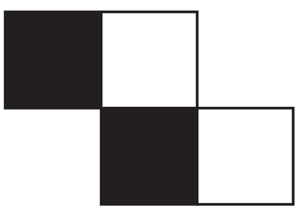


3.

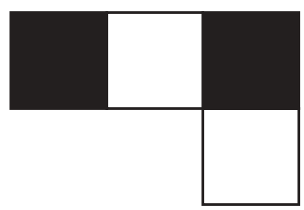


4.

5.



6.



Appendix II

Additional Resources

Print

The Association of Teachers of Mathematics. *Eight Days a Week: Puzzles, Problems and Questions to Activate the Mind*. The Association of Teachers of Mathematics. ISBN 1-898-611-09-2.

Brecker, Erwin. *Lateral Logic Puzzles*. Sterling Publishing Company, Inc. ISBN 0-8069-0618-9.

Bremner, John. *Mensa Maths Wizards for Kids*. Carleton Books Limited. ISBN 1-85868-555-9.

Carter, Philip, Ken Russell, and John Bremner. *The Ultimate Puzzle Challenge*. Carlton Books Ltd. ISBN 1-85868-716-0.

DeSpezio, Michael A. *Giant Book of Challenging Thinking Puzzles*. Sterling Publishing Company, Inc. ISBN 0-8069-2087-4.

Forte, Imogene, and Sandra Schur. *180 Icebreakers to Strengthen Critical Thinking and Problem-Solving Skills*. Incentive Publications, Inc. ISBN 0-86530-345-2.

Graham, Evelyne M. *Think-A-Grams*. Critical Thinking Press and Software.
ISBN Numbers: Book A1: 0-89455-329-1
Book A2: 0-89455-430-1
Book B1: 0-89455-330-5
Book B2: 0-89455-431-X
Book C1: 0-89455-331-3
Book C2: 0-89455-432-8

Hunter, J.A.H. *Entertaining Mathematical Teasers and How to Solve Them*. Dover Publications, Inc. ISBN 0-486-24500-4.

Matt-Smith, Geoffrey. *Mathematical Puzzles for Beginners and Enthusiasts*. Dover Publications, Inc. ISBN 0-486-20198-8.

Nash, Helen, and Dorothy Masterson. *Humorous Cryptograms*. Sterling Publishing Company, Inc. ISBN 0-8069-3982-6.

National Council of Teachers of Mathematics. *How to Evaluate Progress in Problem Solving*. National Council of Teachers of Mathematics. ISBN 0-87353-241-4.

Sloane, Paul, and Des MacHale. *Improve Your Lateral Thinking*. Sterling Publishing Company, Inc. ISBN 0-8069-1374-6.

Weber, Ken. *Five Minute Mysteries for the Armchair Detective*. Stoddart Publishing Co., Ltd. ISBN 0-7737-5210-2.

Internet

There are many sites on the Internet with problems and puzzles. If you are using a search engine to find these sites, search using the words “Mathematics Puzzles Problems.”

As of February 2002, the following sites were available:

AAA Math

<http://www.aaamath.com>

This site has games and practice sheets for various grade levels and topics. There are links to other sites on the web with games and puzzles.

Algebra Story and Word Problems

<http://www2.hawaii.edu/suremath/intro_algebra.html>

There are word problems for various subjects on this site. As well, there are helpful hints to assist in problem solving. Some of the problems may be too algebraic for Senior 2 Consumer Mathematics students.

Breaking Away from the Mathbook

<http://www.math.nmsu.edu/breakingaway/main.html>

Although the site is subtitled **Creative Projects for K-8**, some of the projects may be suitable for Senior 2 Consumer Mathematics students. One that might be interesting for students is *Creasing Paper Along Curves*.

Math Forum

<http://mathforum.org>

This is a good site to begin searching for problems and puzzles. One feature is **Problems of the Week**. New problems are available as well as a library of previous problems. Students can submit their answers and get some feedback. There are links to other math sites and several departments that are useful.

Word Problems for Kids

<http://www.stfx.ca/special/mathproblems/welcome.html>

This is a Canadian site with word problems, hints, and solutions from previous mathematics competitions. The problems are sorted by grade level. Choosing problems from grades 5 through 9 will lead to a wealth of non-algebraic problems.