# *Unit A: Problem Analysis*

## *Half Course I*

# *HALF COURSE I*

**Unit A: Problem Analysis**

**Hours: 9**

**General Learning Outcome:**

**Develop and use mathematical strategies to solve problems in different situations.**

*The intent of this unit is to provide a range of interesting problems of a primarily non-algebraic nature. These problems augment the work of other units.*

**Specific Outcome**

A-1 Solve problems using a variety of non-algebraic approaches.

# *PROBLEM ANALYSIS*

**Instructional Materials**

- *Essentials of Mathematics 10*
- See Appendix I for possible activities.
- See Appendix II for additional resources.

**PRESCRIBED LEARNING OUTCOMES**

**General Outcome**

Develop and use mathematical strategies to solve problems in different situations.

**Specific Outcome(s)**

A-1 solve problems using a variety of non-algebraic approaches

**SUGGESTIONS FOR INSTRUCTION**

The problems contained in Appendix I are intended to provide material which is interesting in its own right and which complements the other units of the program. It is illustrative rather than exhaustive. Some activities have been chosen to illustrate a wide variety of job and consumer applications of mathematics that are largely non-algebraic. Others have been chosen because they are intrinsically interesting or because they challenge students to find and to use new ways of analyzing and thinking mathematically. All students do not need to engage in the same activities.

The activities in Appendix I are presented in **no** particular sequence. Teachers are encouraged to supplement this set of activities with material from other sources, such as the Internet. A preliminary list of possible resources is included in Appendix II.

It is suggested that these problems and activities be interspersed throughout the course as either extensions, enrichment, or a change of pace in the day-to-day work of the classroom. Some of them will link directly to particular units, but most are independent and **may** be used at any time. One approach would be to introduce problem analysis with a few days, possibly up to a week, of work on these activities. Intersperse the remainder throughout the course.

**Note:** Some learning experiences require the use of imperial units of measure. Teachers have the following options:

1. Introduce students to the necessary imperial units of measure or wait until the work in the Geometry Project has been done.
2. Change the problems to metric units of measure, where appropriate.
3. Substitute alternate learning experiences or problems.

## SUGGESTIONS FOR ASSESSMENT

Students' progress should be assessed over long periods of time. Look, for example, for an increasing use of a variety of problem-solving strategies and increasingly sophisticated explanations. Anecdotal records of how students work in pairs or groups on these activities is appropriate. Well-developed solutions and examples of reasoning could become part of a student's portfolio.

Problem-solving activities are generally not appropriate on pencil-and-paper timed tests.

## SUGGESTED LEARNING RESOURCES

**Print**

Austin, J.D. *Applications of Secondary School Mathematics.* Reston, VA: NCTM, 1991.

Giblin, P., and I. Porteous. *Challenging Mathematics.* Toronto/New York: Oxford University Press, 1990.

Hirsch, C.R., and R.A. Laing. *Activities of Learning and Teaching*. Reston, VA: NCTM, 1993.

Mathematical Association of America and National Council of Teachers of Mathematics. *A Sourcebook of Applications of School Mathematics*. Reston, VA: NCTM, 1980.

National Council of Teachers of Mathematics. *NCTM Student Math Notes*. Reston, VA: NCTM, n.d.

*Senior 2 Consumer Mathematics (25S) Part I: A Course for Distance Learning*. Winnipeg, MB: Manitoba Education and Training, 2000.
— Cover Assignments Modules 1-5

Swetz, F., and J.S. Hartzler. *Mathematical Modeling in the Secondary School Curriculum*. Reston, VA: NCTM, 1991.

Two journals which contain useful teaching ideas are:

*The Mathematics Teacher.* National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA: 22091-1593.

*Mathematics in School.* The Mathematical Association, 259 London Road, Leicester, UK: LE2 3BE.

See Appendix II for a list of additional resources.

# Appendix I

# Teacher Information: Networks I

### Skills Required

- pattern recognition

### When To Do

This learning experience can be used at any point in the course. If students have not seen the material on networking before, networks should be introduced before they do the Museum Guard problem.

### Teacher Information

1. A closed network of all even vertices can always be traversed without travelling any arc twice.
2. If a closed network contains exactly two odd vertices, it can be traversed without travelling any arc twice by starting at an odd vertex.
3. If a closed network contains more than two odd vertices, it cannot be traversed without retracing an arc.
4. This problem is extended in Half Course II.

### What To Do

Have the students work independently or in pairs. Have the students try drawing each figure. You may have to encourage the students to be persistent and try again if they were unscussessful at drawing the figure the first time. Tell them to attempt the figure again by starting at a different vertex.

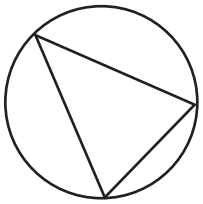
Have the students fill in the chart and answer the questions. After students complete this problem, you may want to have them compare their results with another group or with the whole class.

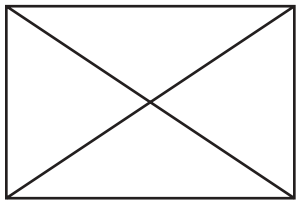
### Chart

| Figure | Can the Figure Be Drawn? |
|---|---|
| a | yes |
| b | no |
| c | yes |
| d | no |
| e | yes |
| f | yes |

# Blackline Master: Networks I

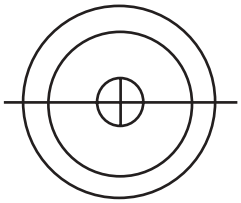
Try to draw each of the figures below using a continuous line without drawing any arc twice.

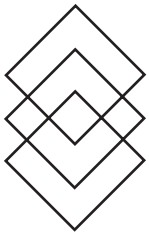
a) 

b) 

c) 

d) 

e) 

f) 

**Questions**

1.

| Figure | Can the Figure Be Drawn? |
|---|---|
| a<br>b<br>c<br>d<br>e<br>f | |

2. What difficulties did you encounter when trying to draw the figures?
3. What did you discover about the figures that you could draw?
4. What did you notice about the figures that you could not draw?

# Teacher Information: The Museum Guard

### Skills Required

- geometric visualization
- experience with networks

### When to Do

This can be undertaken at any time, but preferably after students have completed the previous activity on networks. The principles learned there can be applied to the Museum Guard.

### Teaching Suggestions

You may need to remind students of the conclusions drawn from the Network activity, particularly if some time has elapsed since it was completed. If students have difficulty, suggest that they draw each wing as a network — some latitude is required in doing this because it is a "real" situation.

### Solutions

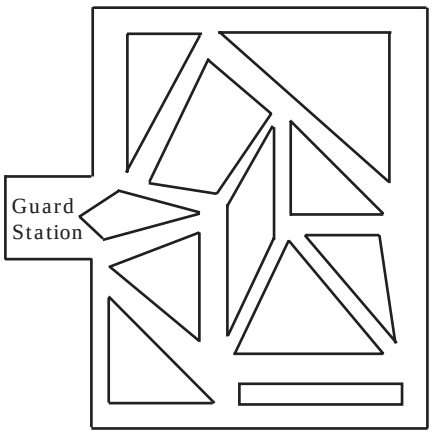
The East Wing (two odd vertices) and Central Wing (no odd vertices) are traceable, i.e., there is a desired path. This is not so for the West Wing, which has one odd vertex.

**The Museum Guard:** From Mathematical Association of America and National Council of Teachers of Mathematics, "The Museum Guard." *A Sourcebook of Applications of School Mathematics*. 

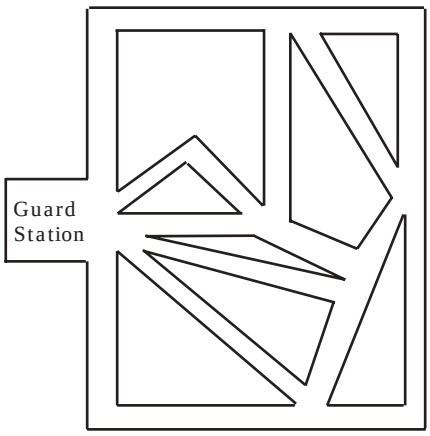
## Blackline Master: The Museum Guard

The guard in a museum is required to walk a path through the halls of the museum in such a way that he walks every hall at least once each hour. The guard is naturally interested in a path which passes through every hall exactly once. The maps of the three wings of the museum are shown below.

East Wing

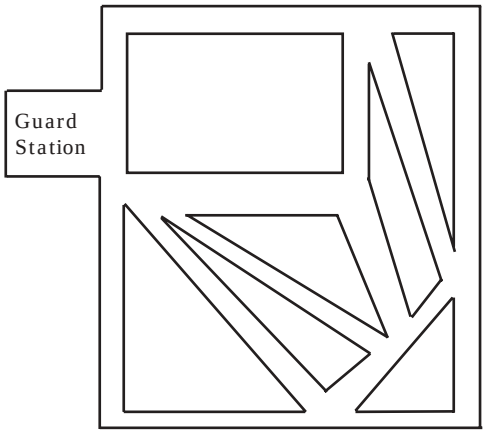



Central Wing




Which wings have the property that the guard can begin at the Guard Station and walk a path through the halls which passes through each hall only once and winds up back at the Guard Station? Find such a path if there is one.

West Wing




**The Museum Guard:** From Mathematical Association of America and National Council of Teachers of Mathematics, "The Museum Guard." *A Sourcebook of Applications of School Mathematics*. 

# Teacher Information: Magic Shapes I

**Skills Required**

- basic arithmetic

**When To Do**

Anytime

**Teacher Information**

*Definition:* A magic square is a square array of numbers arranged so that the sum of numbers in each row, column, and diagonal is the same.

*History:* Magic squares have been known since ancient times. The Lo Shu is one of the earliest known magic squares. There are several sources to find out more about the history of magic squares. Some of the websites listed below may be helpful.

There are many websites where you can learn about magic squares. In a search engine, type "math magic squares." Here are a few that may show up in the search.

1. <http://www.grogono.com/magic/>
   Grog's magic squares.
2. <http://www.magic-squares/de/magic.html>
   Magic Squares
3. <http://www.geocities.com/~harveyh/>
   Magic Squares, Magic Stars, and Other Patterns
4. <http://mathworld.wolfram.com/MagicSquare.html>
   Eric Weisstein's World of Mathematics
5. <http://www.mathforum.org/alejandre/magic.square.html>
   A Math Forum Web Unit on Magic Squares

**Answers**

*Magic Square*

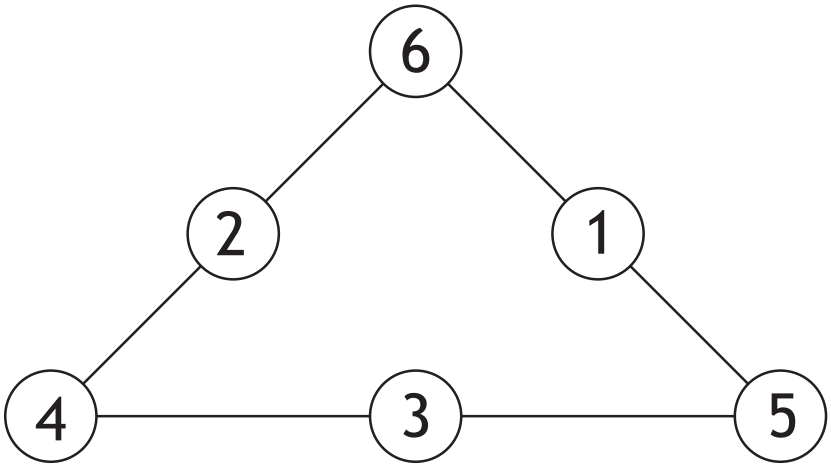
One answer:

| | | |
|---|---|---|
| 8 | 3 | 4 |
| 1 | 5 | 9 |
| 6 | 7 | 2 |

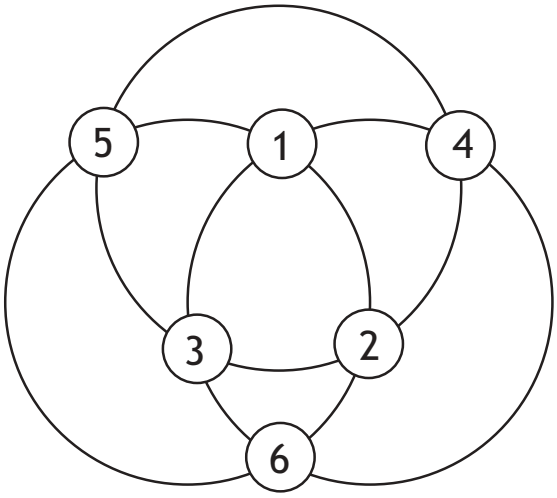
There are other possible answers. Have students come up with several answers and compare them. Are there any similarities? See the websites above for more information.

## Teacher Information: Magic Shapes I (continued)

*Magic Pyramid*




*Sum Circle*




*Extension*

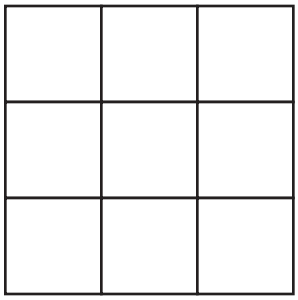
Have the students create an anti-magic square. An anti-magic square is a square where all the sums are different. Some of the websites mentioned above talk about anti-magic squares.

| | | |
|---|---|---|
| 5 | 1 | 3 |
| 4 | 2 | 6 |
| 8 | 7 | 9 |

# Blackline Master: Magic Shapes I

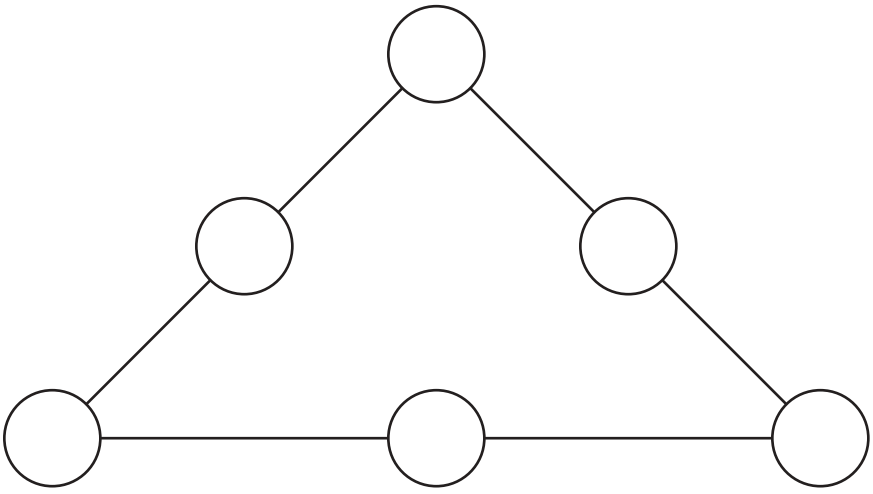
**Magic Square**

Place the numbers from 1 to 9 in each of the boxes. Do not repeat any of the numbers. Each vertical, horizontal, and diagonal line should add to 15.



**Magic Pyramid**

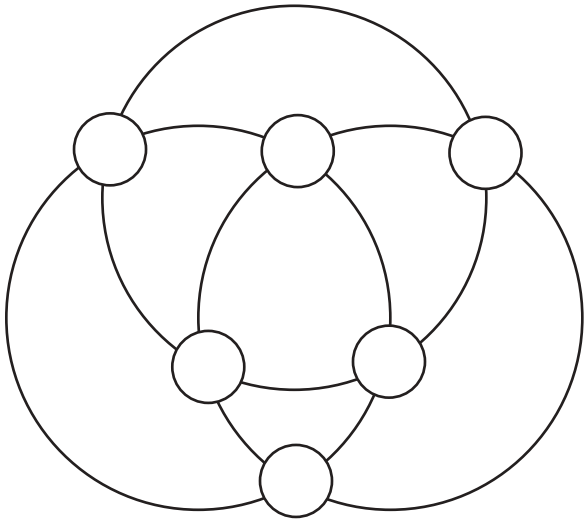
Place the numbers 1, 2, 3, 4, 5, 6 in the circles shown above. Only one number may be placed in each circle. When all six numbers have been placed, the sum of the three numbers for each side must be the same.



## Blackline Master: Magic Shapes I (continued)

**Sum Circle**

Place the numbers from 1 to 6 within the six smaller circles shown below. Each number must be used only once. The numbers must be placed so that the sum of the four numbers that fall on the circumference of a circle is equal to the sum of the numbers on the circumference of any other circle.



# Teacher Information: Number Blocks

**Skills Required**

- basic arithmetic

**When To Do**

Anytime

**Teacher Information**

*Hint:* If students are having trouble getting started, suggest that they find the sum of each column and the average of the three columns.

**Solution**

One of the 3s in the 2nd column moves to the 1st column.

The 10 in the 3rd column moves to the 2nd column

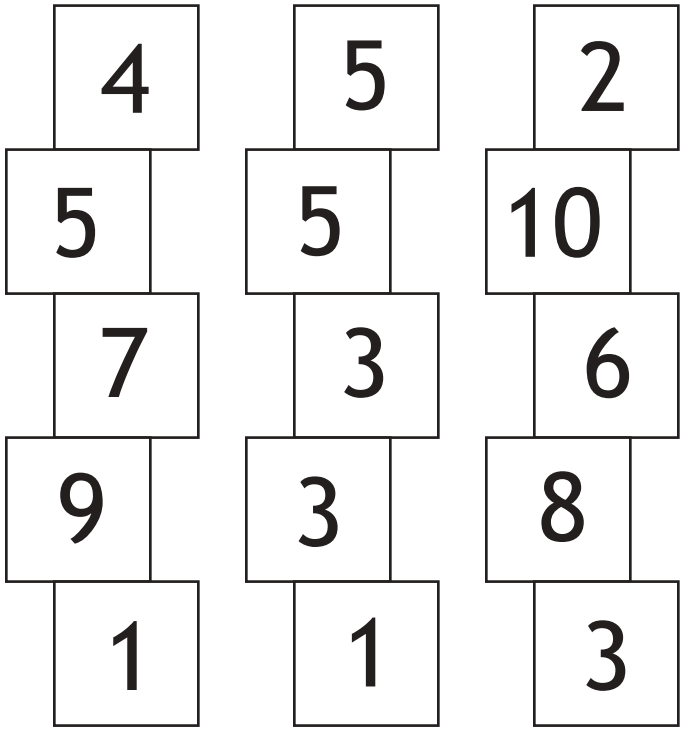
The 5 in the 1st column moves to the 3rd column

**Extension**

Have students create a similar problem.

## Blackline Master: Number Blocks

Take a look at the three stacks of numbered blocks below. Can you rearrange the blocks by exchanging one (and ***only one***) from each of the three stacks so that the sum of the numbers in each stack is equal to the sum of the numbers in the other two stacks?



# Teacher Information: Facelift

### Skills Required

- spatial visualization
- drawing 3-D figures on isometric paper (extension)

### When To Do

This may be done at any time. If students do the extension, they will need the skills developed in the Spatial Geometry unit.

### Teacher Information

If students are having difficulty, they could build it with interlocking cubes.

Have students consider the six faces of each cube and how many faces are joined to other cubes.

### Solutions

Figure 1: 18

Figure 2: 26

Figure 3: 22

### Extension

Have students design their own 3-D figure and draw the design on isometric paper.

## Blackline Master: Face Lift

Take a look at the shape below. You can see three of the four cubes it is made up of. Imagine picking the shape up and examining it from all angles. How many different cube faces can you count?

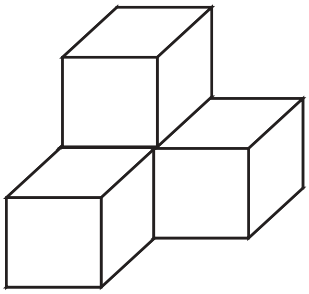


Figure 1

The "double L" shape is made up of six cubes. The sixth cube is hidden in the back of the middle layer. By examining the stack from all angles, how many faces could you see?

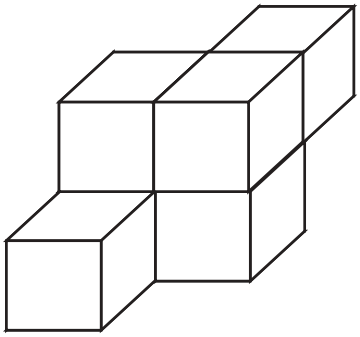


Figure 2

Try this one: If there are only five cubes, how many faces can you see by examining the shape from all angles?

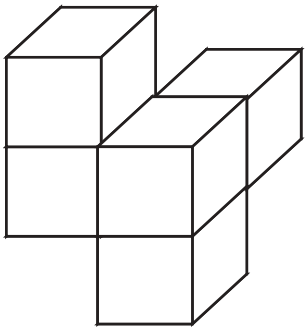


Figure 3

# Teacher Information: Chessboard

**Skills Required**

- basic arithmetic

**When To Do**

Anytime

**Teacher Information**

1. Have students do only the first row and look for a pattern.
2. Ask students when the girl would earn $1000.00 in total. When would the girl earn $1 000 000.00 in total?
3. Students don't need to find the total amount of money given in order to answer the question. They should justify their answer.
4. A spreadsheet could help the students.

**Solution**

| Square # | 1 | 2 | 3 | 4 | 5 | 6 | • • • | 64 | |
|---|---|---|---|---|---|---|---|---|---|
| # of Pennies | 1 | 2 | 4 | 8 | 16 | 32 | | | $1.84 \times 10^{19}$ |
| Total | 1 | 3 | 7 | 15 | 31 | 63 | | | |
| or | 2–1 | 4–1 | 8–1 | 16–1 | 32–1 | 64–1 | | | |
| or | $2^1-1$ | $2^2-1$ | $2^3-1$ | $2^4-1$ | $2^5-1$ | $2^6-1$ | | $2^{64}-1$ | |

## Blackline Master: Chessboard

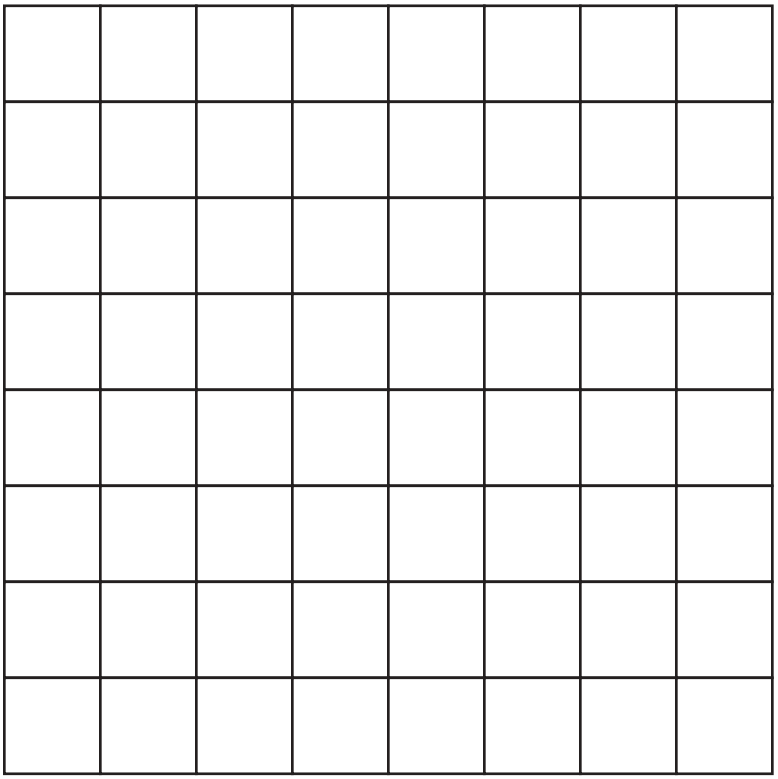
A long time ago in a galaxy far away there was a kingdom which was plagued by vermin. No matter how the king tried to get rid of the vermin, he was not successful. He put a message on his website stating the problem and sought adventurers to solve the problem.

There were no takers except one. She was a lowly peasant girl. She came and brought various weeds and seeds and made a brew and spread the brew liberally across the land. The vermin became sterile and the problem was solved.

The king offered her a reward — the handsome prince's hand in marriage, half the kingdom, free entry to Canada's Wonderland, anything. The girl refused all.

She said that her needs were simple. She asked the king to bring a chessboard and a pile of pennies. This was done. On the first square, she asked that one penny be placed, on the second, two pennies, and on the third square, four pennies. As each square was covered the number of pennies was twice the previous number until all 64 squares had been covered.

Did the peasant girl receive a fair reward? Explain and defend your answer.



# Teacher Information: Bridge Puzzle

### Skills Required

- critical thinking
- basic arithmetic

### When To Do

Anytime

### Teacher Information

1. Have students act out the puzzle to help them visualize what is happening.
2. There are no tricks to solving the puzzle. It can be done without throwing the flashlight, walking in the dark, or any other tricks.
3. Once students find one solution, encourage them to find the second solution.
4. There are similar puzzles to this one. One is used by a large corporation to test prospective employees. Employee applicants are given five minutes to solve the problem.

### Solution

| | | **Time Taken** |
|---|---|---|
| *Solution 1:* | Send 5, 10 across | 10 minutes |
| | Send 10 back | 10 minutes |
| | Send 20, 25 across | 25 minutes |
| | Send 5 back | 5 minutes |
| | Send 5, 10 across | 10 minutes |
| | | Total: 60 minutes |

| | | **Time Taken** |
|---|---|---|
| *Solution 2:* | Send 5, 10 across | 10 minutes |
| | Send 5 back | 5 minutes |
| | Send 20, 25 across | 25 minutes |
| | Send 10 back | 10 minutes |
| | Send 5, 10 across | 10 minutes |
| | | Total: 60 minutes |

## Blackline Master: Bridge Puzzle

Four explorers came to a hanging bridge one night. Only two people could cross the bridge at one time and they needed a flashlight.

They had one flashlight with enough power to last 60 minutes.

One of the explorers could cross the bridge in five minutes, a second explorer could cross in 10 minutes, and the other two explorers would need 20 and 25 minutes respectively.

How could all four explorers cross the bridge before the batteries went dead?

# Appendix II

## Additional Resources

### Print

The Association of Teachers of Mathematics. *Eight Days a Week: Puzzles, Problems and Questions to Activate the Mind*. The Association of Teachers of Mathematics. ISBN 1-898-611-09-2.

Brecker, Erwin. *Lateral Logic Puzzles*. Sterling Publishing Company, Inc. ISBN 0-8069-0618-9.

Bremner, John. *Mensa Maths Wizards for Kids*. Carleton Books Limited. ISBN 1-85868-555-9.

Carter, Philip, Ken Russell, and John Bremner. *The Ultimate Puzzle Challenge*. Carlton Books Ltd. ISBN 1-85868-716-0.

DeSpezio, Michael A. *Giant Book of Challenging Thinking Puzzles*. Sterling Publishing Company, Inc. ISBN 0-8069-2087-4.

Forte, Imogene, and Sandra Schur. *180 Icebreakers to Strengthen Critical Thinking and Problem-Solving Skills*. Incentive Publications, Inc. ISBN 0-86530-345-2.

Graham, Evelyne M. *Think-A-Grams*. Critical Thinking Press and Software.
ISBN Numbers: Book A1: 0-89455-329-1
Book A2: 0-89455-430-1
Book B1: 0-89455-330-5
Book B2: 0-89455-431-X
Book C1: 0-89455-331-3
Book C2: 0-89455-432-8

Hunter, J.A.H. *Entertaining Mathematical Teasers and How to Solve Them*. Dover Publications, Inc. ISBN 0-486-24500-4.

Matt-Smith, Geoffrey. *Mathematical Puzzles for Beginners and Enthusiasts*. Dover Publications, Inc. ISBN 0-486-20198-8.

Nash, Helen, and Dorothy Masterson. *Humorous Cryptograms*. Sterling Publishing Company, Inc. ISBN 0-8069-3982-6.

National Council of Teachers of Mathematics. *How to Evaluate Progress in Problem Solving*. National Council of Teachers of Mathematics. ISBN 0-87353-241-4.

Sloane, Paul, and Des MacHale. *Improve Your Lateral Thinking*. Sterling Publishing Company, Inc. ISBN 0-8069-1374-6.

Weber, Ken. *Five Minute Mysteries for the Armchair Detective*. Stoddart Publishing Co., Ltd. ISBN 0-7737-5210-2.

## Internet

There are many sites on the Internet with problems and puzzles. If you are using a search engine to find these sites, search using the words "Mathematics Puzzles Problems."

As of February 2002, the following sites were available:

*AAA Math*

<http://www.aaamath.com>

This site has games and practice sheets for various grade levels and topics. There are links to other sites on the web with games and puzzles.

*Algebra Story and Word Problems*

<http://www2.hawaii.edu/suremath/intro_algebra.html>

There are word problems for various subjects on this site. As well, there are helpful hints to assist in problem solving. Some of the problems may be too algebraic for Senior 2 Consumer Mathematics students.

*Breaking Away from the Mathbook*

<http://www.math.nmsu.edu/breakingaway/main.html>

Although the site is subtitled **Creative Projects for K-8**, some of the projects may be suitable for Senior 2 Consumer Mathematics students. One that might be interesting for students is *Creasing Paper Along Curves*.

*Math Forum*

<http://mathforum.org>

This is a good site to begin searching for problems and puzzles. One feature is **Problems of the Week**. New problems are available as well as a library of previous problems. Students can submit their answers and get some feedback. There are links to other math sites and several departments that are useful.

*Word Problems for Kids*

<http://www.stfx.ca/special/mathproblems/welcome.html>

This is a Canadian site with word problems, hints, and solutions from previous mathematics competitions. The problems are sorted by grade level. Choosing problems from Grades 5 through 9 will lead to a wealth of non-algebraic problems.