Grade 7 Mathematics

Patterns and Relations

Patterns and Relations (Patterns, and Variables and Equations) (7.PR.1, 7.PR.2, 7.PR.3, 7.PR.4, 7.PR.5, 7.PR.6, 7.PR.7)

Enduring Understanding(s):

Words, tables, graphs, and expressions are different representations of the same pattern.

Preservation of equality is used to solve equations.

The principles of operations used with whole numbers also apply to operations with decimals, fractions, and integers.

Number sense and mental mathematics strategies are used to estimate answers and lead to flexible thinking.

General Learning Outcome(s):

Use patterns to describe the world and solve problems.

Represent algebraic expressions in multiple ways.

Note:
Background Information for the Patterns and Relations strand is presented in a slightly different order than in other strands. This variation is intended to accommodate learning experiences that integrate achievement indicators from learning outcomes in both the Patterns substrand and the Variables and Equations substrand. Some achievement indicators related to Variables and Equations are developed using student experiences devoted to exploring Patterns.

Specific Learning Outcome(s):	**Achievement Indicators:**
7.PR.1 Demonstrate an understanding of oral and written patterns and their equivalent relations. [C, CN, R]	➛ Formulate a relation to represent the relationship in an oral or a written pattern. ➛ Provide a context for a relation that represents a pattern. ➛ Represent a pattern in the environment using a relation.

(continued)

Specific Learning Outcome(s):	**Achievement Indicators:**
7.PR.2 Construct a table of values from a relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, R, V]	➛ Create a table of values for a relation by substituting values for the variable. ➛ Create a table of values using a relation, and graph the table of values (limited to discrete elements). ➛ Sketch the graph from a table of values created for a relation, and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between n and $2n + 3$). ➛ Describe the relationship shown on a graph using everyday language in spoken or written form to solve problems. ➛ Match a set of relations to a set of graphs. ➛ Match a set of graphs to a set of relations.
7.PR.3 Demonstrate an understanding of preservation of equality by ▪ modelling preservation of equality, concretely, pictorially, and symbolically ▪ applying preservation of equality to solve equations [C, CN, PS, R, V]	➛ Model the preservation of equality for addition, subtraction, multiplication, or division using concrete materials or using pictorial representations, explain the process orally, and record it symbolically. ➛ Solve a problem by applying preservation of equality.
7.PR.4 Explain the difference between an expression and an equation. [C, CN]	➛ Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation. ➛ Explain what a variable is and how it is used in an expression. ➛ Provide an example of an expression and an equation, and explain how they are similar and different.
7.PR.5 Evaluate an expression given the value of the variable(s). [CN, R]	➛ Substitute a value for each unknown in an expression and evaluate the expression.

(continued)

Specific Learning Outcome(s):	**Achievement Indicators:**
7.PR.6 Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers. [CN, PS, R, V]	➛ Represent a problem with a linear equation and solve the equation using concrete models. ➛ Draw a visual representation of the steps required to solve a linear equation. ➛ Solve a problem using a linear equation. ➛ Verify the solution to a linear equation using concrete materials or diagrams. ➛ Substitute a possible solution for the variable in a linear equation to verify the equality.
7.PR.7 Model and solve problems that can be represented by linear equations of the form: ■ $ax + b = c$ ■ $ax = b$ ■ $\frac{x}{a} = b, a \neq 0$ concretely, pictorially, and symbolically, where a, b, and c, are whole numbers. [CN, PS, R, V]	➛ Model a problem with a linear equation and solve the equation using concrete models. ➛ Draw a visual representation of the steps used to solve a linear equation. ➛ Solve a problem using a linear equation and record the process. ➛ Verify the solution to a linear equation using concrete materials or diagrams. ➛ Substitute a possible solution for the variable in a linear equation to verify the equality.

Prior Knowledge

Students may have had experience with the following:

- Describing and applying mental mathematics strategies, such as
  - skip-counting from a known fact
  - using doubling or halving
  - using doubling and adding one more group
  - using patterns in the 9s facts
  - using repeated doubling

  to develop recall of basic multiplication facts to 9 × 9 and related division facts.
- Using charts, tables, graphs, and diagrams to
  - identify and describe patterns
  - reproduce a pattern using concrete materials
  - represent patterns and describe relationships to solve problems
  - identify and explain mathematical relationships to solve problems

- Expressing a problem as an equation in which a symbol is used to represent an unknown number.
- Solving one-step equations involving a symbol to represent an unknown number.
- Determining the pattern rule to make predictions about subsequent elements.
- Solving problems involving single-variable (expressed as symbols or letters), one-step equations with whole-number coefficients, and whole-number solutions.
- Demonstrating an understanding of factors and multiples by
  - determining multiples and factors of numbers less than 100
  - identifying prime and composite numbers
  - solving problems involving factors or multiples
- Demonstrating an understanding of ratio, concretely, pictorially, and symbolically.
- Demonstrating an understanding of integers, concretely, pictorially, and symbolically.
- Demonstrating an understanding of the relationships within tables of values to solve problems.
- Representing generalizations arising from number relationships using equations with letter variables.
- Demonstrating and explaining the meaning of preservation of equality concretely, pictorially, and symbolically.
- Identifying and plotting points in the first quadrant of a Cartesian plane using whole-number ordered pairs.
- Creating, labelling, and interpreting line graphs to draw conclusions.
- Graphing collected data and analyzing the graph to solve problems.

For more information on prior knowledge, refer to the following resource:

Manitoba Education and Advanced Learning. *Glance Across the Grades: Kindergarten to Grade 9 Mathematics*. Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/cur/math/glance_k-9/index.html.

Related Knowledge

Students should be introduced to the following:

- Determining and explaining why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0.
- Demonstrating an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences).

- Demonstrating an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically.
- Comparing and ordering fractions, decimals (to thousandths), and integers by using
  - benchmarks
  - place value
  - equivalent fractions and/or decimals
- Demonstrating an understanding of circles by
  - describing the relationships among radius, diameter, and circumference of circles
  - relating circumference to pi
  - determining the sum of the central angles
  - constructing circles with a given radius or diameter
  - solving problems involving the radii, diameters, and circumferences of circles
- Developing and applying a formula for determining the area of
  - triangles
  - parallelograms
  - circles
- Identifying and plotting points in the four quadrants of a Cartesian plane using ordered pairs.
- Performing and describing transformations of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral vertices).
- Demonstrating an understanding of central tendency and range by
  - determining the measures of central tendency (mean, median, mode) and range
  - determining the most appropriate measures of central tendency to report findings
- Constructing, labelling, and interpreting circle graphs to solve problems.

Background Information

Patterns

The world is full of patterns. They are found in multiple contexts both in nature and in the creations of people. Patterns are prevalent in plant and animal life, as well as in the physical world. They are evident in the arts, music, structures, movement, time, and space. Our number system is rooted in pattern, and an understanding of pattern is the basis of mathematical concepts in every strand of mathematics.

As students interpret patterns and generalize the relationships represented by them, they develop algebraic thinking and reasoning skills that enable them to apply mathematics in everyday situations. These generalizations and the equations and formulas derived from them are powerful tools for making predictions and solving problems. They make mathematics meaningful, and are also important for the mathematics that students will study in later grades.

Someone who possesses the ability to identify a new pattern and its symbolic relation can solve a problem that previously seemed insurmountable. That person may make a new relational discovery leading to a new advancement in science or technology. Such was the case with Dmitri Mendeleev's work on the periodic table, Albert Einstein's formulation of the $E = mc^2$ equation, and the more recent work on the relationships between prime numbers and quantum mechanics.

Of more interest to Middle Years students will be the schoolroom mathematics story commonly attributed to the German mathematician Carl Gauss. Around 1787, Carl Gauss's teacher directed his class to find the sum of the consecutive numbers 1 to 100. The 10-year-old Carl promptly submitted an answer of 5050. When questioned how he could possibly perform the calculation so quickly, Carl explained he did not add the numbers from 1 to 100. Rather, he saw a pattern. He paired the largest and the smallest numbers ($1 + 100 = 101$, $2 + 99 = 101$, $3 + 98 = 101$), and determined the set contained 50 such pairs, each totalling 101. Thus, the total sum was 50×101, or 5050. Presenting the same task to students in today's classroom will likely uncover some interesting discussion regarding patterns, and reveal that there are many ways to view a problem and its solution.

The process of making connections in patterns is developed in the learning outcomes of previous grades. It involves converting patterns to numeric values, extending the patterns, graphing the patterns, and explaining the mathematical relationship between the quantities. The relationship is expressed in mathematical symbols using the language of algebra and extended to an equation or formula. The Grade 7 learning outcomes focus on matching symbolic relations, in the form of algebraic expressions and equations, with pattern contexts and the representations of these relations as tables of values and graphs. The relations are then used to solve problems. The ability to extend patterns and represent them as tables, graphs, and equations is assessed in the Grade 7 Numeracy Assessment. Therefore, background information regarding patterns, how to represent them, and how to identify the relationships within them is provided, beginning with a discussion of the categories of patterns.

Categories of Patterns

For the purposes of this document, there are two main categories of patterns: repeating patterns and growing patterns.

- **Repeating patterns:** *Repeating patterns* consist of repeated sequences or arrangements of items about which predictions can be made. In a repeating pattern, a set of elements, referred to as a *core*, appears in a set order over and over again, or appears as a transformation over and over again. Colour patterns, shape patterns, rhyme patterns, and tessellations are examples of repeating patterns. Numerals may also be arranged in repeating patterns.

  Examples:

  ★★○★★○★★○

  121312131213

- **Growing patterns:** *Growing patterns* (also called *sequences* or *number patterns*) consist of a series of steps called *figures* or *terms*. Each term is related to the previous term according to a pattern. The terms are numbered according to their sequential order, and each term is assigned a corresponding numeric term value, which is determined by the number of items in that term. Growing patterns may grow or shrink, depending on whether the numeric values of the terms increase or decrease. The growth may occur at a constant or non-constant rate.

  Example:

Growing Patterns						
Pictorial representation of a pattern	•••	•• •••	•• •• •••	•• •• •• •••		
Term number (the position of the term in the sequence)	1	2	3	4	5	6
Term value (the number of items in the term)	3	5	7	9		

Growing patterns, which are emphasized in Grade 7, are further classified as arithmetic sequences or geometric sequences.

- **Arithmetic sequences:** In arithmetic sequences, the rate of change is constant.
  - Each term value changes by a fixed amount in relation to the previous term value.
  - A particular value is added to or subtracted from the previous term value.
  - An example is the pattern 2, 4, 6, . . . , where each successive term value increases by a value of 2.
  - Arithmetic sequences are generally easier to identify than geometric sequences.
- **Geometric sequences:** In geometric sequences, the next term value is a multiple of the previous term value. Pattern growth is not constant.
  - In the example 2, 4, 8, 16, . . . , each term value is twice the previous value.
  - In the example 900, 300, 100 . . . , each term value is $\frac{1}{3}$ the amount of the previous value.

In the Fibonacci (Leonardo Pisano) number series (1, 1, 2, 3, 5, 8, 13, . . .), each number is the sum of the two previous numbers. It is neither arithmetic nor geometric.

Interesting patterns occur in squares, cubic numbers, and triangular numbers, as well as in Fibonacci numbers and fractals. While the patterns are interesting to explore, keep in mind that Grade 7 learning outcomes are limited to linear relations, and do not include powers and exponents. Students may be able to describe recursive relationships in these patterns, but not the explicit rules with relationships that describe the equation of the patterns.

Patterns are all around us, ranging from four legs on a chair to the patterns found in geometry, art, architecture, and music. The lengths and diameters of pipes in a pipe organ provide an example of relationships in pitches and harmonics and different octaves.

Pleasant patterns often correspond to the ratio of the golden mean or rectangle. The patterns in sound waves and light waves are converted to numeric values, and the relationships are used in producing CDs, DVDs, and other digital technology.

Two Types of Relationships in Patterns

Two types of relationships in patterns are recursive and explicit relationships.

- **Recursive relationships:** These relationships explain how each term in the pattern compares or relates to the preceding term in the pattern. They are useful for extending patterns and for completing tables of values. In the pattern 2, 4, 6, . . . , each successive term value increases by a value of 2. The words, "begin at 2 and add 2 to each term value," or "the term value plus 2 equals the next term value," or the expression, "$t + 2$, where t is the previous term value" all describe the recursive relationship in the pattern. This relationship is limiting when the need arises to determine term values for terms that are not close to those known. It also represents a misuse of the variable t.
- **Explicit relationships (or rule):** These relationships relate the term number to the term value for each term. They are used to predict the nth value in a pattern, and to express the pattern as an equation or formula. The words, "the term number multiplied by 2" describes the explicit relationship in the pattern 2, 4, 6, ... Recall that multiplication is repeated addition; thus, the relationship is also described as, "the term number multiplied by 2," or "$2n$, where n is the term number."

  Placing the numbers in a table of values makes it easier to compare the term and term value numbers.

Term number	1	2	3	4
Term value	2	4	6	8

When assessing student performance, keep in mind that Grade 7 learning outcomes are limited to discrete elements and linear relations, and do not include powers and exponents. Students may be able to describe recursive relationships in some geometric patterns, but may not be able to articulate symbolic relations to describe the explicit rules in these patterns. When asking students to represent the pattern algebraically, the expectation is that students will be able to represent the explicit pattern, $2n$.

Representing Patterns and Identifying Relationships

Patterns can be represented in different ways. The recursive and explicit relationships between the elements exist in the different representations of a pattern. Generalizing explicit relationships can be challenging, and may require persistence. Each type of representation provides a different view and a different way to think about the relationships. Encourage students to work toward identifying the relationship in each type of representation. To increase students' ability to think symbolically, begin with more obvious relations and teach students to ask themselves increasingly complex questions about the relations (e.g., What remains the same? What changes? By how much does it change? Is this true for every term in the sequence? How can that idea be represented? What happens if . . . ?).

The following process for representing patterns flows from the concrete or pictorial to the symbolic. It is important to guide students through the process.

- **Concrete or pictorial representation:** The context of the pattern exists in the physical pattern itself, and can be represented concretely or pictorially. Students can examine the physical terms to determine what remained the same in each term and what changed. Playing with colour or arrangement of patterns can often help students to see the constant and the changing aspects of a pattern.

  Example:

Pictorial representation of a pattern	•••	•• •••	•• •• •••	•• •• •• •••		

  In this example, two dots are added to the top of each new term. The term value = the previous term value + 2.

- **Charts or tables of values:** Charts or tables of values display numeric representation of the pattern values, and may also be used to record the recursive changes between the terms. These representations, as illustrated in the following example, facilitate numeric comparisons. They can be presented in horizontal or vertical format.

  Example:

Term number (*n*) (the position of the term in the sequence)	1	2	3	4	5	6
Term value (*v*) (the number of items in the term)	3	5	7	9		

Read the table in one direction (across in the table above) to identify the *recursive relationship* (each term value increases by 2). Read in the opposite direction (down in the table above) to get information about the *explicit relationship.* Ask, "How can the term number be changed to get the term value?" Here, multiply the term number by 2 and add 1. Express the relationship symbolically as the expression,

$2n + 1$ or equation $2n + 1 = v$,

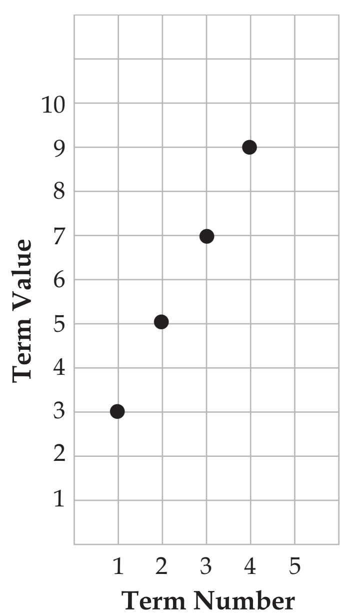
where n = the term number and v = the term value

If the relationships are not evident in the charts or tables, examine the other representations for clues. Students will have an easier time seeing numeric relationships if they have a good grasp of addition and multiplication facts.

- **Graphs:** A graph provides a concrete picture of the relationships in the pattern. It provides clear evidence of whether the values are increasing or decreasing, how quickly the change is happening, and the increment of change. Relationships can be described by articulating these changes. Each point on the graph represents an (x, y) pair. Try to express both numbers in terms of the x-value (e.g., $(x, x + 2)$). If the points line up along a 45° incline, the relation may have only a constant (e.g., it has a coefficient of 1). If the incline is steeper, the relation will have a coefficient greater than 1, and it may have a constant. If it is less steep, the relation will have a coefficient less than 1 and greater than zero, and it may have a constant.

  Note: In the previous equation, $v = 2n + 1$, and 2 is the coefficient and 1 is the constant.

  Example:



> **Note:**
> The Grade 7 learning outcomes deal with discrete data. Since *n* and *v* refer specifically to the term number and term value, they are represented by natural numbers (1, 2, 3, 4, . . .). As a result, no line should be drawn through the points.

  - In this example, the plotted points lie in a straight line. Therefore, this pattern is a *linear relation*.
  - As x increases, so does y. So the pattern is an *increasing linear relation*.

- Evaluating the recursive relationship shows that the value of the first term is 3, and the subsequent terms increase by 2.
- Going to the first term, and then going back one step from the first term, and then removing 2 from the first term value results in 1. Evaluating the explicit term could begin with thinking that something must be added to 1 to equal each term value. In the symbolic relation, + 1 is present in every term. It is referred to as a *constant*. Through experimentation, students can determine that the y-value, when x is zero, represents the constant—this is represented by the place where the relation crosses the y-axis on a graph.
- On the graph, for each increase in 1 horizontally (each increase in 1 by x, or each time the term number increases by 1) the vertical increase is 2 (y increases by 2, or the term value increases by 2). This can be expressed using the relation $2x$, where 2 is referred to as the *numerical coefficient.*

- **Mathematical language (words):** Mathematical language can be used to describe the pattern as it appears in the physical representation, chart, or graph. A clear description of a pattern can help students to recognize the explicit relationship. If students describe a pattern as "increasing by two dots each time," they may see the recursive relationship. If, however, they can be more descriptive and say, "the pattern starts with three dots, and two dots are added on top each time," they may begin to notice that there is an explicit relationship.

- **Mathematical symbols:** Mathematical symbols can be used to create expressions, formulas, and equations to represent the relationships. The pattern in the previous table can be represented symbolically by the

  expression $2n + 1$ or equation $2n + 1 = v$,

  where n = the term number and v = the term value

A blackline master for recording these representations is available in BLM 7.PR.1: Patterns: A Process. The process flows from the concrete to the pictorial to the symbolic, and it is important to guide students through it.

The Meaning of Variables

Variables are symbols used to take the place of numbers. Variables provide the ability to express generalizations without any attachment to a specific value. In writing relations, students will gain experience working with all three applications in which variables are used. Understanding the different applications can help reduce confusion over what a variable actually stands for.

Caution:
When using variables in relations, keep in mind the following:

- Any visual can serve as a variable, but it is conventional to use lowercase letters. For example, $2n + 1$ and $2x + 1$ represent the same situation. Both n and x are variables that take the place of whatever number. In earlier grades, students often use pictures or shapes to represent a variable. It is important that students understand mathematical conventions, and regularly use lower case letters for variables.
- Using a variety of variables with students is important. Be careful, however, not to select variables that may be confused with units to represent a scenario where units may be present.
- The variable x is often confused with the multiplication symbol $\times$. It is appropriate for students to begin representing multiplication using parentheses, $2(x)$, the multiplication dot $2 \cdot x$, or, in its simplified form, $2x$.

Applications in Which Variables Are Used

Variables can be used for the following purposes:

- **To represent a value that changes.** In the preceding example of the chart of values, the variable n represents the term number of the pattern. The term number is a value that changes for each term. The variable n can represent any particular term number, but only one term value for a given term number. Any variable can be chosen to represent the term number (e.g., for the expression $2n + 1$ and for a term number of 3, there can only be one term value, 7).
- **To solve for a specific unknown.** The explicit rule to determine a term value in the above example is $2n + 1$, where n is the term number. Substituting any term number for n and simplifying the expression will reveal the unknown term value for that particular term number n.
- **To simplify an expression.** In the above example, n represents the term number, and $2n + 1$ represents the term value. Together, $(n, 2n + 1)$ represent the x- and y-coordinates used for each term on a graph. The variable y represents the simplified expression $2n + 1$ for each term value ($y = 2n + 1$).

Equations and Expressions

- In the above example, $y = 2x + 1$, the expression, $2x + 1$ represents the term value. The variable y also represents the same term value. The expressions $2x + 1$ and y are different names for the same value. They are equivalent expressions.
- An equal sign (=) is used in an equation to show that both expressions represent the same value, $y = 2x + 1$. All equations contain an equal sign between two different representations of the same value.
- Expressions are relations that do not contain an equal sign. They provide only one description of the value referred to. The expression $2x + 1$ represents a value, and y is a separate expression that represents the same value.
- Watch for confusion in students about the use of equal signs. When students see an equal sign, they sometimes interpret it as a direction to do something with the numbers preceding it. They forget the equal sign's role as a symbol of equality. For example, if students see $3 \times 4 = n \cdot 6$, some might think they are being asked to multiply 3×4, which would result in $n = 12$. If n were 12, the statement would say $3 \times 4 = 12 \cdot 6$ or $12 = 72$, and that is not a true statement.

Variables and Equations

Recall that in the equation, $2x + 3 = y$, 2 is the numerical coefficient, x and y are variables, + 3 is a constant, and the equal sign indicates an equation. Altogether, they represent an equation made up of two equivalent expressions, $2x + 3$ and y.

In previous grades, students model the preservation of equality concretely, pictorially, and orally, and represent and verify equivalent forms of an equation. In Grade 7, students extend this ability to using preservation of equality to solve equations and problems.

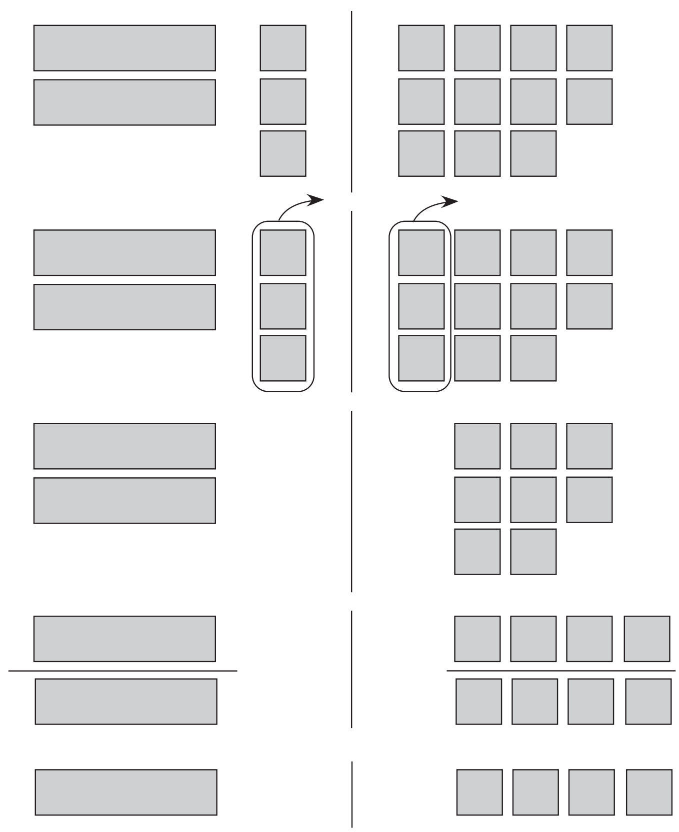
Strategies for Solving Linear Equations

Students may apply various strategies, such as the following, to solve linear equations:

- **Use intuition.** Students will be able to solve some linear equations intuitively, by recalling a related arithmetic fact or by recognizing a relation (e.g., doubling, 1 more or less).
- **Substitute a value for a variable.** The substitution is essentially a guess-and-check strategy that is verified through substitution. It is good practice to verify all solutions by substituting the solution for the variable and working through the equation.
- **Graph the equation.** Create a table of values and use a graphical representation of the equation to read the information required. (This strategy is outlined in relation to learning outcome 7.PR.2.)

- **Use counters to model the equation.** This strategy works well for whole numbers. For example, to model $3n = 24$, a student could distribute 24 counters into 3 equal groups and count 8 in each group. To model $3n + 4 = 22$, a student could distribute 22 counters so that there are an equal number in 3 groups and a group of 4 by themselves. There would be 6 in each group of 3, so $n = 6$ in this equation. Students may extend this strategy to working backwards.
- **Work backwards through the equation.** In the previous example, $3n + 4 = 22$, take 22 counters. The last direction in the equation is to add 4, so do the reverse and remove 4, which would leave 18 counters. Some number multiplied by 3 is 18. The opposite of multiplying is dividing, so divide 18 by 3. The result is 6. Take care that students do not misinterpret the equal sign as a direction to do some operation rather than as a symbol that separates equal quantities.
- **Use algebra tiles.** The rectangular tile is used to represent x. The small squares are used to represent units. One colour represents positive integers, and another colour represents negative integers. A vertical rod is used to represent the equal sign. When working with algebra tiles, it is important to be consistent about what each manipulative represents. Different sources sometimes have different representations.

  Example:



$$2n + 3 = 11$$

$$2n + 3 - 3 = 11 - 3$$

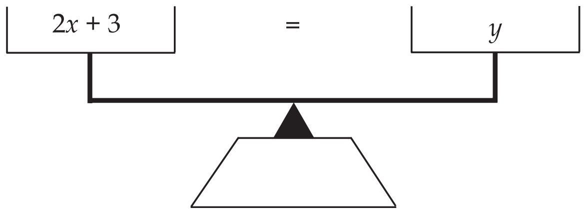
$$2n = 8$$

$$\frac{2n}{2} = \frac{8}{2}$$

$$1n = 4$$

- **Use a balance scale model.** This model is based on the principle that an equation represents two equal expressions separated by an equal sign. The equal sign represents the fulcrum or balance point of a scale, and the expressions on either side represent masses placed in either pan of the balance. The expressions are equal—both represent the same value and can symbolize equal masses.

  Example:

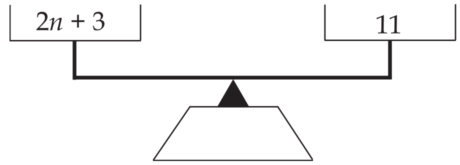
  - In the balance scale metaphor, changing the mass on one side of the fulcrum will tip the scale. Making an identical change on the opposite side of the fulcrum will rebalance the scale.
  - In the concrete model, a balance scale is used along with identical objects, such as blocks, cubes, or marbles, to represent numbers, and paper bags or polystyrene cups, to represent variables. Designated objects are added to the bags evenly, and identical changes are made to both sides of the scale until balance is achieved. The objects in the bag could be counted to obtain the value of the variable, or the items can be manipulated until one bag is isolated on one side of the scale. The quantity it represents is isolated on the opposite side, and the scale is at equilibrium.
  - In the symbolic representation of the model, the equation is solved by performing identical operations on either side of the equal sign, until a variable remains on one side and a value on the other. (This method is developed in the learning experiences suggested for learning outcome 7.PR.3.)

Example:

Represent $2n + 3 = 11$ as a balance.

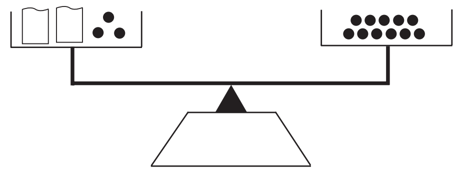
● represents a chip

 represents a bag containing an unknown number of chips



$2n + 3 = 11$

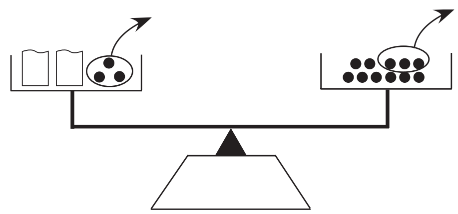
Show this concretely (or pictorially).



$2n + 3 = 11$

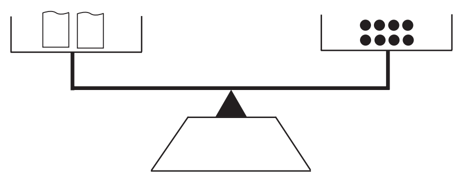
$\quad -3 \quad -3$

Maintaining balance, remove 3 chips from each side.



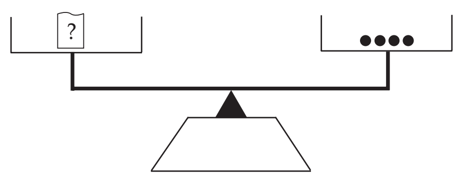
$2n = 8$

Simplify.



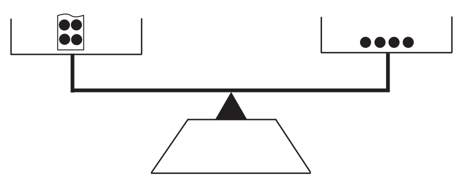
$\frac{2n}{2} = \frac{8}{2}$

Determine the number of chips that would be in each bag.



$n = 4$

Simplify.

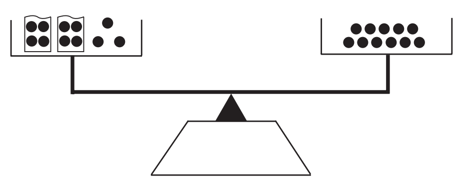


$2n + 3 = 11$ (?)

$8 + 3 = 11$ (?)

$11 = 11$ (✔)

Check.



Be sure to arrange learning experiences in such a way that students have ample opportunity to work with a variety of concrete materials when solving linear equations through preservation of equality, to explain the process orally, to represent it pictorially, and to record it symbolically.

The skills students develop in solving linear equations through preservation of equality, and the experience they gain representing patterns and contextual situations as relations and linear equations, can be combined to solve problems with ease.

Mathematical Language

algebraic expression

constant

coordinates

core

element

equation

equivalent

evaluate

explicit relationship

expression

graph

linear relation

numerical coefficient

pattern

recursive relationship

relation

solution

solve

substitution

table of values

term (step number, figure number)

value

variable

Assessing Prior Knowledge

Materials:

- BLM 7.PR.1: Patterns: A Process
- BLM 7.PR.2: Sample Patterns
- demonstration board
- manipulatives (e.g., cubes, pattern blocks)
- grid paper
- math journals or notebooks

Organization: Whole class, individual

Procedure:

1. Introduce the topic of patterns with a class discussion.
   a) Ask students to share patterns they have seen, or present them with samples of patterns. Ask them to describe the patterns. Ask whether there are other ways to represent the same pattern. Review pattern-related vocabulary as opportunity arises during the discussion.
   b) Select an example of a pattern from BLM 7.PR.2: Sample Patterns, or use a student example, including three or four terms of a basic growing pattern. Record the example on the demonstration board. Students may build the pattern and/or record it in their math journals or notebooks. (Caution: Avoid a triangular number pattern of adding an additional row one item longer for each term.)
   c) Have students extend the pattern another three terms. Ask them to share the rule they used for extending the pattern using words, and then using a mathematical expression. Record the recursive rules or relations, and have students do the same.
   d) Have students complete a chart of the term numbers and term values. Record the chart. Then complete a table of values, with one column being the term number (x), and the other being the term value (the relation to x) or (y).

Note:
This learning experience could be used within a body of evidence to report on the following competency on the Grade 7 Numeracy Assessment:

Student uses number patterns to solve mathematical problems.

Reference:
Manitoba Education and Advanced Learning. *Middle Years Assessment: Grade 7 Mathematics: Support Document for Teachers: English Program.* Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/assess/support/math7/.

e) Transfer the values in the table to a grid plot. Review the coordinate plane, and label the x-axis and y-axis as you do so. Include term number and term value in the labels. Ask students whether the points should be joined.

Note:
Learning outcomes in Grade 7 (and in previous grades) deal with discrete data. Since Grade 7 learning outcomes limit graphing to discrete data, the points should not be connected.

f) Ask students to describe the explicit relationship in words and as a symbolic relation. Discuss strategies used to determine the explicit relationship. For example, evaluating whether the pattern is increasing or decreasing, and by how much, informs students about the operation used in the expression. (For a discussion on Representing Patterns and Identifying Relationships, see the Background Information for learning outcomes 7.PR.1 and 7.PR.2.) Have students record strategy tips.

g) Connect the x- and y-variables to the term number and the term values and to the coordinate points. Write an equation to represent y in terms of x. In the example $(x, 2x + 1)$, $y = 2x + 1$. These terms can be added to the graph labels. (This topic is explored further in a later learning activity about connecting relations to oral and written patterns.)

2. Choose a pattern for students to work with independently. Distribute copies of BLM 7.PR.1: Patterns: A Process and BLM 7.PR.2: Sample Patterns. Ask students to complete the five representations of one or more pattern examples.

Variations:

- Students may choose or create their own pattern to represent in the five ways. Or they may choose or create a pattern for a classmate to represent.
- If students are having difficulty focusing on extending pattern variations, have them begin by using concrete materials to represent the patterns and to extend them.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Extend a pattern.
- ❒ Create a chart and a table of values representing the pattern.
- ❒ Represent the pattern as a graph, and label the graph.
- ❒ Describe a recursive relationship to represent the pattern with words and with a symbolic expression.
- ❒ Identify the explicit relationship in the pattern as words, and represent it as a symbolic expression and/or an equation.

Suggestions for Instruction

- **Formulate a relation to represent the relationship in an oral or a written pattern.**
- **Provide a context for a relation that represents a pattern.**
- **Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation.**
- **Explain what a variable is and how it is used in an expression.**
- **Provide an example of an expression and an equation, and explain how they are similar and different.**
- **Substitute a value for each unknown in an expression and evaluate the expression.**

Materials:

- BLM 7.PR.3: Directions for Playing a Relations Game
- math journals or notebooks

Organization: Whole class, small groups, individual

Procedure:

1. Introduce students to a relations game, such as that found on BLM 7.PR.3: Directions for Playing a Relations Game.
2. Demonstrate the game to the class.
3. Divide the class into small groups and have them play the game.

4. Provide students with the following problems and have them record their responses in their math journals or notebooks:
   a) Amanda puts a 3 into the function machine and gets out a 7. Use symbols or words to show three different rules the function machine could be following.
   b) The function machine continues to use the same rule, but this time, Amanda puts in a 6 and gets out a 13. Use symbols and words to show one rule that you think the function machine is following.
   c) The function machine continues to use the same rule. Predict what the output will be if the input is 5. Explain how you know this.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:
- ❒ Predict an element in a pattern based on a pattern rule.
- ❒ Describe a recursive relationship to represent the pattern with words and with a symbolic expression.
- ❒ Identify the explicit relationship in the pattern as words, and represent it as a symbolic expression and/or an equation.

Suggestions for Instruction

- **Formulate a relation to represent the relationship in an oral or a written pattern.**
- **Provide a context for a relation that represents a pattern.**
- **Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation.**
- **Explain what a variable is and how it is used in an expression.**
- **Provide an example of an expression and an equation, and explain how they are similar and different.**
- **Substitute a value for each unknown in an expression and evaluate the expression.**

Materials:
- BLM 7.PR.1: Patterns: A Process (completed—from Assessing Prior Knowledge learning activity)
- BLM 7.PR.4: Understanding Concepts in Patterns and Relations
- demonstration board
- Venn diagram (optional)
- textbook glossaries, mathematics dictionaries, and/or other references (optional)

Organization: Pairs, whole class, individual

Procedure:

Part A

1. Activate students' background knowledge about algebra and algebraic terms using a Think-Pair-Share strategy in which students think about a question individually, and then share their ideas, first with a partner and then with the whole class. The following questions and comments are offered as a guide.

   a) What is algebra?

      As students share responses with the class, include the concept that algebra is the language of symbols used to represent the relationships in patterns.

   b) What are some of the symbols used in algebra and what do they represent?

      As opportunity arises during the sharing and during the next steps of the learning activity, develop students' understanding of the following vocabulary terms: *relation, variable, numerical coefficient, constant term, expression,* and *equation*. Encourage students to use proper terminology as they explore patterns. Record terms on the demonstration board as they arise.

2. Use students' experiences with relations in patterns to develop the meaning and use of the vocabulary terms *relation* and *variable*. Record terms on the demonstration board as they arise. Have students examine their completed work from the Assessing Prior Knowledge learning activity (BLM 7.PR.1: Patterns: A Process). Some guiding questions and comments are suggested below. Also connect vocabulary terms to visible and familiar contexts.

   a) **Relation:** Examine the section (on BLM 7.PR.1: Patterns: A Process) where students describe the pattern in their own words, and compare it to the algebraic expression and the equation.

      - Ask students how these are related.

        The algebraic statements are symbolic representations of the words used to describe the patterns. The opposite is true as well. The equation gives directions to perform some operation on the term number to come up with the term value. It dictates what to do with the x-value to get the y-value. In a relation, one number in a pair is used to identify the other number, or the related number, in the pair. In the equation $2x = y$, the expression $2x$ relates x to y. The expression $2x$ is a relation. In everyday life, the relation could, for example, describe the number of chairs at each table: $2x$ = total # of chairs, where x = the # of tables.

      - Ask students to share their patterns (from BLM 7.PR.1: Patterns: A Process) and identify the matching symbolic relations, and vice versa. Challenge them to generate examples of relations to represent familiar contexts and to provide contexts to match some relations.

b) **Variable:** Students used variables in the expressions that represent the relationships between term numbers and term values and in the algebraic equations.

- Ask students to share which variables they used in their representations, and what the variables represent in each case. Increasing students' awareness of the three different uses for variables may help reduce some of the confusion they may encounter using algebra. This discussion also provides an opportunity to discuss conventions and cautions in choosing symbols. (See the Background Information.) You may wish to have students use a variety of variables to express contextual relationships or to identify possible contexts for a relation (e.g., $4g = s$. 4 students in a group, $4 \cdot$ # of groups = the # of students). Practise substituting values for the variables.
- Point out how the numbers represented by the variable vary or change, depending on which figure of the pattern is being referred to. This is one way variables are used. Different variables in one equation represent quantities of different items. The same variable in one equation always represents quantities of the same thing.
- Replacing a variable with a number in the expression or equation generates the value for the other number. Ask students to generate term values for specific term numbers, by substituting the term number for the variable in the relation. Another way variables are used is to find a particular number. Here, the particular number is represented by a relation.
- Have students examine the graph (on BLM 7.PR.1: Patterns: A Process). How are the x-values and the y-values for each point related? The y-values are expressed in terms of the x-values in the relationship rules. Both axes on the graphs may be labelled in terms of x. Consider the algebraic equations: y = (the relation in terms of x), and y is a simpler name for the change you made to x. A third use for variables is as a simplified name for a relation. Variables can be used to make generalized statements about mathematical relationships, such as $l \times w$ = Area.

Note:
This learning experience could be used within a body of evidence to report on the following competency on the Grade 7 Numeracy Assessment:

Student uses number patterns to solve mathematical problems.

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3. Have students work with their partners to practise generating some contextual relations using variables or to match a context to a relation. Practise substituting values in the relations. Reassemble as a class and share a few responses to verify students' understanding.
4. Provide students with copies of BLM 7.PR.4: Understanding Concepts in Patterns and Relations. Ask them to define and provide an example of the terms *variable* and *relation*.

Part B

5. Continue using the work completed in Part A as a reference. Use students' experiences with relations in patterns to review the vocabulary terms *relation* and *variable*, and to develop the meaning and use of the vocabulary terms *expression*, *equation*, *constant term*, and *numerical coefficient*. Record each term on the demonstration board as it arises. Some guiding questions and comments follow.
   a) **Expressions and equations:** The previous discussion about *expressions* and *equations* in relations provided some background experience with these terms. Now, compare and differentiate the terms *expressions* and *equations*. Expressions contain variables and operations that represent one name for a value. There is no equal sign in an expression. An equation contains two expressions that are equal to each other. Students commonly misunderstand the equal sign as a directive. (Refer to the discussion regarding expressions and equations in relations in Part A of this learning experience and to the Background Information.)
   b) **Constant term:** A representation of the constant term is readily seen on a graph by examining the point at which the relation meets the y-axis ($x = 0$). It shows what quantity is at the base of each step in the pattern, and, therefore, must be added (or subtracted) each time you calculate a term value. Constant terms are separated from variables with an addition or a subtraction symbol. Ask students to identify constant terms in any of their relations.
   c) **Numerical coefficient:** In some patterns, the term number (variable) is multiplied by the same amount in each term. Ask students to identify a numerical coefficient for any of the variables in their relations. The slope of the line in the graphical representation of patterns provides a clue to the presence of a coefficient in a relation. Discuss conventions of notation.
6. Provide several examples of relations in words, expressions, and equations, or have students create their own examples. Have students work with their partners to identify relations, variables, numerical coefficients, constant terms, expressions, and equations in the examples, and substitute values for the variables. Verify the accuracy of students' responses.
7. Provide students with copies of BLM 7.PR.4: Understanding Concepts in Patterns and Relations or a Venn diagram. Ask them to define and provide an example of each of the following terms: *expression, equation, constant term,* and *numerical coefficient*. If you choose to use the Venn diagram, ask students to compare *expression* with *equation* and *constant term* with *numerical coefficient*.

Variations:

- Have students create and/or complete crossword puzzles of the vocabulary terms.
- Have students create a beginning algebra booklet, including the terms, definitions, and examples. This booklet could be used as an appendix for another booklet idea in a culminating learning activity.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❐ Formulate a relation to represent the relationship in an oral or written pattern.
- ❐ Provide a context for a relation that represents a pattern.
- ❐ Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation.
- ❐ Explain what a variable is and how it is used in an expression.
- ❐ Provide an example of an expression and an equation, and explain how they are similar and different.
- ❐ Substitute a value for each unknown in an expression and evaluate the expression.

Suggestions for Instruction

- **Formulate a relation to represent the relationship in an oral or a written pattern.**
- **Provide a context for a relation that represents a pattern.**
- **Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation.**
- **Explain what a variable is and how it is used in an expression.**
- **Provide an example of an expression and an equation, and explain how they are similar and different.**
- **Substitute a value for each unknown in an expression and evaluate the expression.**

Materials:

- BLM 7.PR.5: Possible Word Pattern Contexts to Match a Relation
- BLM 7.PR.6: Formulating Relations to Match Word Descriptions of Patterns
- demonstration board
- card stock (optional)

Organization: Whole class, individual

Procedure:

Remind students, as noted in the previous learning activity, that patterns are represented in many equivalent forms. The pattern can be recognized in each of the forms, and one form can be translated into another. Each different representation provides a different view of the same pattern. The more views students see, the greater their understanding of the pattern will be. A word description of the pattern can be used to make a physical representation of the pattern, as well as a chart, a table of values, and/or a graph. The x- and y-values of the graph are also represented in the word description of the pattern, and can be used to form an algebraic equation.

Note:
This learning experience could be used within a body of evidence to report on the following competency on the Grade 7 Numeracy Assessment:

Student uses number patterns to solve mathematical problems.

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Part A

1. Help students establish a process of writing a relation to match a word description of a pattern or context by working backwards from a relation.
   a) Begin by analyzing a relation such as $x + 1 = y$. Dissect the terms of the relation and identify their symbolism. There are two expressions: $x + 1$ and y. The equation tells us that the two expressions are equal to each other. There are two variables: x represents some number of things and y is a number of other things. There is one operation, a constant term of + 1. Together, the two expressions indicate that there is one more y thing than there are x things. To find the number of y things there are, use the quantity that x represents and add 1.

Note:
The variables x and y are used throughout this learning activity to reinforce the connection of variables to term number charts and/or term value charts and the graphical representations of patterns on a coordinate grid.
If you wish, emphasize that any symbol or name can be used to represent the variable, or develop that concept in a later learning activity.

b) Create a possible context or situation that this relation could represent. Imagine something of which x could be a quantity. The constant will be an unchanging quantity of that same thing, and y will be the combined quantity.

Example 1:

The relation could represent people: the number of guests attending your party (x), you, who will be there no matter what (+ 1), and the total number of people at your party (y). Substitute some values for x, and solve the equation to get values for y. Consider whether or not the values make sense in relation to each other. If not, something needs fixing. For example, if 3 people come + 1 (you), there will be 4 people at the party. That is reasonable. Reinforce that $x + 1$ is an expression that tells us how to find the value of y. In this example, $x + 1$ is one name for the number of people at the party. And the number y represents another name for the number of people at the party. So, $x + 1$ and y represent the same number. They are equal. The two expressions are combined as the equation $x + 1 = y$.

Example 2:

The relation could represent money: the number of dollars you decide to take from your piggy bank (x), the one-dollar coupon you have (+ 1), and the dollars you can spend at the restaurant (y). Verify the relation through substitution. If you withdraw $10 and add the $1, you will have $11. That is reasonable.

2. Complete a few examples of different relations together with students. Also include examples where $x = y$, and where y decreases as x increases. Have students independently create possible word situations to match a particular relation. BLM 7.PR.5: Possible Word Pattern Contexts to Match a Relation provides a framework for recording these. You may wish to assign the relations, or students may create their own. Remind students to substitute values to verify the reasonableness of their relations. After allowing sufficient time for individual work, have students share some context and relation matches with the class so you can verify their understanding. Assign more independent or partner practice if it seems beneficial.

Part B

3. Reassemble as a class, and establish a procedure to reverse the above process, enabling students to formulate a relation to match the word description of a pattern. Do this by analyzing the word description of the pattern to find parts to represent x, y, and constants or numerical coefficients. Combine the parts to write a matching symbolic relation. A chart such as the following can serve as an organizational tool.

Writing a Symbolic Relation		
One quantity that can be represented by a variable (similar to the term number) Represent with x	An operation that tells what to do to x (the term number) to get y (the term value) Record as a constant (+ or –) or numerical coefficient (× or ÷)	A quantity that will be represented by the y-variable (similar to the term value) Represent with y

4. Illustrate how to analyze the description and find the parts to represent x, y, and constants or numerical coefficients by using examples such as the following:

   Example 1:

   A girl owns three horses. She purchases more horses at an auction; consequently, she now has more horses.

   - Look for some quantity that acts like a term number, in that it can change in a step-by-step fashion. That number will be represented by the x-variable. The girl may buy 1, or 2, or 3, or 4, or . . . horses. In this case, the x-variable will be the number of horses she buys.
   - Next, identify which quantity will be represented by the y-variable. This is similar to a term value. The term value depends on the term number. The number of horses she ends up with depends on how many she buys, so the y-variable will be the number of horses she ends up with.
   - Then, consider the presence of a constant or a numerical coefficient. If there is a quantity to start with, or one to remove at the end, there will be a constant to add or subtract. If a variable is being multiplied or divided, there will be a numerical coefficient to connect to the variable. In this example, the girl starts with three horses. She has three horses no matter how many she buys. These three horses are represented by the constant + 3.
   - Put the pieces together in the relation $x + 3 = y$. Reinforce this is an equation. It contains two equal expressions. Note the convention to place the variable first and the constant after. Establish these conventions as examples arise.

Example 2:

A boy sells hats at the fair for \$5 each. He pays \$25 for a daily vendor licence. At the end, he has some money. The number of hats sold could be 1, or 2, or 3, or 4, or . . . . The number of hats sold will be represented by the x-variable (term number). In the end, the boy will have made some money. That amount of money is represented by the y-variable. He will receive \$5 for each hat he sells. To find out how much money he receives, multiply the number of hats sold by \$5. So, \$5 is the numerical coefficient. Five times the number of hats equals the money received ($5x$). The boy must pay the \$25 fee, no matter how many hats he sells. That will the constant (– \$25). The money he ends up with will be the y-value. The relation is $5x - 25 = y$.

5. As a class, complete a few examples of different relations. Include examples where $x = y$, where there are numerical coefficients, and where there are positive and negative constants.
6. Distribute copies of BLM 7.PR.6: Formulating Relations to Match Word Descriptions of Patterns. Have students individually analyze word descriptions of patterns to formulate relations. After giving students sufficient time to formulate relations, have them use their relations to represent the contexts. Verify their understanding, and assign more independent practice if it seems beneficial.

Variation:

- Use cards or a master sheet of pattern descriptions and relations to conduct a quiz game such as Relation Baseball. Read a pattern description or a relation, and have a student respond with either a matching relation or a pattern description. (For directions on, and variations of, playing a similar baseball game, refer to the Assessing Prior Knowledge suggestion for learning outcome 7.N.6.)

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Formulate a relation to represent the relationship in an oral or written pattern.
- ❒ Provide a context for a relation that represents a pattern.
- ❒ Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation.
- ❒ Explain what a variable is and how it is used in an expression.
- ❒ Provide an example of an expression and an equation, and explain how they are similar and different.
- ❒ Substitute a value for each unknown in an expression and evaluate the expression.

Suggestions for Instruction

- **Formulate a relation to represent the relationship in an oral or a written pattern.**
- **Provide a context for a relation that represents a pattern.**
- **Represent a pattern in the environment using a relation.**
- **Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation.**
- **Explain what a variable is and how it is used in an expression.**
- **Provide an example of an expression and an equation, and explain how they are similar and different.**
- **Substitute a value for each unknown in an expression and evaluate the expression.**

Materials:

- a list of word descriptions of patterns for which students can formulate relations or word cards from previous learning activity (optional)
- math journals or notebooks
- three-column charts for recording relations and descriptions

Organization: Whole class, individual, small group

Procedure:

For this learning activity, have students act as detectives, with the goal of uncovering the relations that represent word descriptions of patterns.

1. Together with students, develop a strategy for finding clues from which to form the relations. Working through examples may be helpful.

   Suggested Strategies:

   - Uncover clues that would indicate a term number or a term value.
     - Use the variable x to represent the numeric value of the term number.
     - Use y to represent the numeric value of the term value.

Note:
This learning experience could be used within a body of evidence to report on the following competency on the Grade 7 Numeracy Assessment:

Student uses number patterns to solve mathematical problems.

Reference:
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- Uncover directions about how to find the term value from the term number. Is the operation applied addition, subtraction, multiplication, or division, or a combination of operations?
    - Use a constant term to represent addition or subtraction.
    - Use a numerical coefficient to represent multiplication or division.

2. Work through some examples together with students. Include contexts with various constants and numerical coefficients, and combinations of them.

   Example 1:

   Determine the number of traffic lights if there is one red light for every green light.

   Clues:

   - There are two objects that can be quantified. Each can be represented by a variable.
     - Let r represent the number of red lights.
     - Let g represent the number of green lights.
   - The number of green lights is the same as the number of red lights.
     - There are no constants or numerical coefficients.

   Relation: $r = g$

   Example 2:

   Determine the number of girls and boys in a class if there are two more girls than boys.

   Clues:

   - There are two terms to quantify, boys and girls.
   - There are two more girls than boys. This clue is important to ensure the operation is performed on the correct variable.
   - There are more girls than boys, so the number of boys + 2 = the number of girls.
     - The number of boys acts as the term number. So b represents the number of boys.
     - The number of girls depends on the number of boys. The number of girls acts as the term value, so g represents the number of girls.
   - The expression $b + 2$ results in the number of girls when there are b number of boys.

   Relation: $b + 2 = g$

Example 3:

Determine the number of wheels present in a collection of cars if each car has four wheels.

Clues:

- There are two terms to quantify, the number of cars and the number of wheels.
    - Let x represent the number of cars.
    - Let y represent the number of wheels.
- Every car has four wheels.
    - Multiply the number of cars by 4.
    - 4 is the coefficient for x.

Relation: $4x = y$

Use the above examples as an opportunity to discuss conventions such as writing $4x$, rather than $x \times 4$ or $x \cdot 4$, to avoid confusing variables and multiplication signs.

3. After working through several examples as a group, have students describe, in their math journals or notebooks, a strategy for finding a relation to match the description of a pattern.
4. Have pairs of students take turns doing the following:
    - **Student A:** Describe a context in which patterns are found and record it in the middle column of a three-column chart. Then record the relation to the pattern context description in the first column, cover it (or fold it so that the relation is hidden), and pass the chart to Student B.

      Example:

Relation (Student A)	Pattern Context Description	Relation (Student B)
Find the total number of ties (t) in a section (s). $t = s + 1$	A railroad track leaves 1435 mm between railway ties. The first 1435 mm section has two ties, and each 1435 mm section after that adds one more tie.	

- **Student B:** Write the relation for the pattern context description in the third column of the chart.

  Example:

	Pattern Context Description	**Relation (Student B)**
	A railroad track leaves 1435 mm between railway ties. The first 1435 mm section has two ties, and each 1435 mm section after that adds one more tie.	Let y represent each 1435 mm section. $y + 1$

5. Students compare the relations they formulated to the answer keys, and discuss and resolve any discrepancies. They test the relations by substituting the variable and evaluating the expression. They verify that the relations make sense when compared to the descriptions.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Formulate a relation to represent the relationship in an oral or a written pattern.
- ❒ Provide a context for a relation that represents a pattern.
- ❒ Represent a pattern in the environment using a relation.
- ❒ Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation.
- ❒ Explain what a variable is and how it is used in an expression.
- ❒ Provide an example of an expression and an equation, and explain how they are similar and different.
- ❒ Substitute a value for each unknown in an expression and evaluate the expression.

Suggestions for Instruction

- **Formulate a relation to represent the relationship in an oral or a written pattern.**
- **Provide a context for a relation that represents a pattern.**
- **Represent a pattern in the environment using a relation.**
- **Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation.**
- **Substitute a value for each unknown in an expression and evaluate the expression.**

Materials:

- BLM 7.PR.7: Creating Word Descriptions of Patterns and Matching Relations

Organization: Individuals or pairs

Procedure:

1. As in the previous learning activities, review the concept that patterns have multiple representations and one representation can be used to formulate another. In this learning activity, students will find patterns in their surroundings and then represent the patterns as relations to play an I Spy game. They will need to substitute values and evaluate expressions to find the patterns represented by the relations.
2. Distribute copies of BLM 7.PR.7: Creating Word Descriptions of Patterns and Matching Relations. Have students survey the classroom to identify patterns they could represent using relations, and complete the chart provided on the BLM. Examples could include patterns in furniture, brick, tile, decoration, clothing, supplies, and so on.

Note:
This learning experience could be used within a body of evidence to report on the following competency on the Grade 7 Numeracy Assessment:

Student uses number patterns to solve mathematical problems.

Reference:
Manitoba Education and Advanced Learning. *Middle Years Assessment: Grade 7 Mathematics: Support Document for Teachers: English Program.* Winnipeg, MB: Manitoba Education and Advanced Learning, 2015. Available online at www.edu.gov.mb.ca/k12/assess/support/math7/.

3. Decide who will be the first student to offer a relation. That student says, "I spy a pattern that is represented with the relation ________. Can you guess what I see?" Other students will need to look for a pattern they think matches the relation. Then substitute a value for the variable, and evaluate the expression to ensure students' suggested pattern matches the relation. If individuals have a match, they raise a hand, and, when called upon, offer their suggestion to the one who spies. Students substantiate their proposal by substituting a value, evaluating the expression to verify x and y. If someone is correct, she or he becomes the one who spies, and offers a relation to the group. Choosing variables that begin with the first letter of the objects in the pattern makes the game easier.

Variations:

- To increase participation, play in small groups rather than in a large group.
- Change the environment by taking students to a new area indoors or outdoors to play the game.
- To limit the options, prepare a list of scenarios, and have students spy from the list.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Represent a pattern in the environment using a relation.
- ❒ Formulate a relation to represent the relationship in an oral or a written pattern.
- ❒ Provide a context for a relation that represents a pattern.
- ❒ Identify and provide an example of a constant term, a numerical coefficient, and a variable in an expression and an equation.
- ❒ Substitute a value for each unknown in an expression and evaluate the expression.
- ❒ Communicate mathematically.

Suggestions for Instruction

- **Create a table of values for a relation by substituting values for the variable.**
- **Create a table of values using a relation, and graph the table of values (limited to discrete elements).**
- **Match a set of relations to a set of graphs.**
- **Match a set of graphs to a set of relations.**
- **Substitute a value for each unknown in an expression and evaluate the expression.**

Materials:

- math journals or notebooks
- pens or markers of different colours
- demonstration board
- examples of relations (These can be teacher-generated or created in previous learning activities, such as completed BLMs, cards, or lists.)
- a selection of the following:
  - grid paper
  - templates for tables of values
  - cards with grids printed on them
  - cards with blank tables of values
  - blank cards
  - scissors and glue or tape
- display board
- graphing software or graphing calculators (optional)

Organization: Individuals or pairs, whole class

Procedure:

The procedure suggested for this learning activity (Part A) will be continued in the next two learning activities (Part B and Part C).

Part A

Present a progression of relations for students to work with, beginning with $x - 2$, followed by $2x$, then $2x - 2$, and then $2(x - 1)$. The following procedure uses $2(x - 1)$ as an example. You may prefer to use relations with addition, such as $(x + 2)$, $2x$, $2x + 2$, and $2(x + 1)$.

1. Present students with a sample relation such as the coordinate pair $(x, 2(x - 1))$ or the equation $2(x - 1) = y$. Challenge students to represent the relation as a graph in their math journals or notebooks, and to record the steps they followed to do so. If or when it seems appropriate, note that the relation is equivalent to $2x - 2$.
2. When students have had sufficient time to work individually, reassemble as a class and ask students to share their ideas about how to go about representing a relation as a graph. As students offer suggestions, record the process on the demonstration board. Ask guiding questions, and supply prompts to ensure the process is complete and understood. Suggest students make adjustments to their math journal entries as the discussion reveals steps they had not considered previously. Making additions in a different colour highlights for you and for students what students are learning during the sharing.

   Steps to Include:

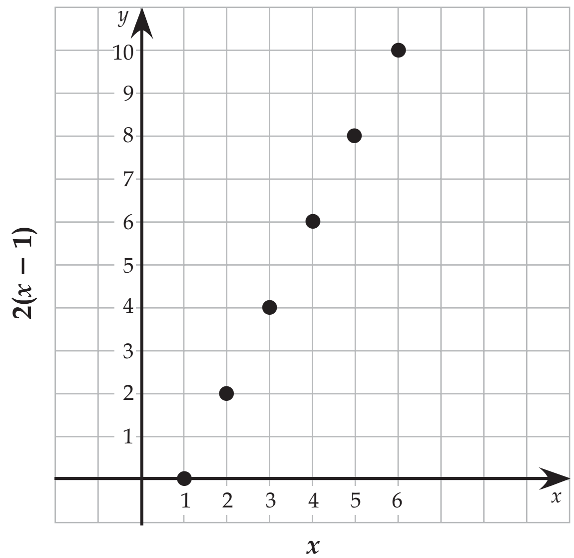
   a) Create a table of values.

   b) The x-value is similar to a term number, as shown in previous work with representing patterns. Supply data for the table by substituting values for the variable. Any number may be used to represent x, but beginning with a small number, and increasing in a consecutive fashion by even increments, will generate numbers that are most helpful for viewing relationships. If your class is familiar with adding and subtracting negative numbers (see learning outcome 7.N.6), include negative values, because integers appear in everyday situations involving money, depth below sea level or underground, lost time, and so on. Avoid using negative numbers with numerical coefficients, as multiplying integers is a Grade 8 learning outcome. Solving the expression with a particular x-value will generate a value equivalent to y. This is the value referred to as the term value in previous work. The relation $2(x - 1)$ explains the relation between x and y. Evaluating the relation generates the y-value.

$2(x - 1) = y$	
x-value	y-value $2(x - 1)$
1	0
2	2
3	4
4	6
5	8

c) Examine the range of numbers in the table of values and select an appropriate scale for the *x*- and *y*-axes. Draw and label the axes with *x* below the grid, and the relation that names *y* along the *y*-axis. Write numbers to indicate the scale of each axis. Remind students to align the centre of the numbers with the grid lines. If students are familiar with the four quadrants of the Cartesian plane (learning outcome 7.SS.4), they will be able to represent values with negative integers. Include a title for the graph. In this case, the graph represents the relation between *x* and 2(*x* – 1), so that is an appropriate title.

The Relation between
x and $2(x - 1)$



d) Plot the coordinate pairs on the graph. Since Grade 7 learning outcomes limit graphing to discrete data, the points should not be connected.

3. Distribute grid paper, templates for tables of values, and blank cards for writing the titles of the graphs. Students will use these supplies to create an interactive display. Assign relations to students, or have them create their own, or use previously generated relations. Ask students to create tables of values, as well as graphs to represent the relations. Remind students to choose appropriate scales and to label the axes.

4. Have students share their completed graphs. Verify their correctness. Then mount the graphs on the display board and distribute the table of values and the labels. Have students attach the pieces to the matching graphs on the display board. Students may have difficulty matching some labels. The next learning activity (Part B) provides strategies to make matching easier. Alternately, the tables and titles can be mounted, and students can match the corresponding graph. Leave room in the display for descriptions that will be made in the following learning activities (Part B and Part C).

Variations:

- Control the complexity and variety of relations by assigning specific relations to students. Work with one-step linear equations (e.g., $3x$ or $x + 1$), before moving to two-step linear equations $\left(\text{e.g., } 2x-2 \text{ or } \frac{1}{3}x+1\right)$.
- If students made cards with matching relations and word descriptions of patterns in a previous learning activity, they could add to their card sets by creating matching tables of values cards and graphs.
- The teacher or students can use computer software to generate the tables and graphs to represent particular relations. Have students find matches either in hard copies or electronically, if the technical skills and software are available.
- If the technology and know-how are available, students can use graphing calculators to investigate relations and see what types of graphs the relations produce.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Create a table of values for a relation by substituting values for the variable.
- ❒ Create a table of values using a relation and graph the table of values (limited to discrete elements).
- ❒ Match a set of relations to a set of graphs.
- ❒ Match a set of graphs to a set of relations.
- ❒ Substitute a value for each unknown in an expression and evaluate the expression.
- ❒ Reason mathematically in order to make mathematical connections.

Suggestions for Instruction

- **Sketch the graph from a table of values created for a relation, and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between *n* and 2*n* + 3).**
- **Describe the relationship shown on a graph using everyday language in spoken or written form to solve problems.**

Materials:

- math journals or notebooks
- pens or markers of different colours
- demonstration board
- tables of values, graphs, labels, and relations from the previous learning experience
- cards for recording descriptions of graphs (blank file cards or the type of cards used in previous learning experiences)
- display board

Organization: Individual or pairs, whole class

Procedure:

Part B

This is a continuation of the previous learning activity. Students use the products created in Part A and extend their prior knowledge of graphical representations of patterns to describe the relationships shown in a graph and in expressions and equations. Describe the relationships in each of the graphs from Part A: (x - 2), (2x), (2x – 2), and 2(x – 1). The following procedure uses the graph for 2(x – 1) as an example.

1. Have students examine the sample graphs they created in the previous learning activity (Part A). Challenge students to decipher the coded information the graphs contain about the relationship between x and 2(x – 1), and to write their discoveries in their math journals or notebooks.
2. After students have had sufficient time to work on their own or with a partner, reassemble as a class to debrief. Have students share their interpretations of their graphs, and add pertinent information to their math journals. Making additions in a different colour highlights what students are learning during discussion with others. Below are some comments you may wish to include in the discussion. (For additional information about graphs, see Representing Patterns and Identifying Relationships in the Background Information for learning outcomes 7.PR.1 and 7.PR.2.)

   Discussion Ideas:

   - **Linear relation:** All the points on the graph lie in a straight line. Verify this by placing a ruler along the points. When all the coordinate pairs of a relation lie in a straight line, the relation is called a *linear relation*.

- **Increasing linear relation:** The line goes up to the right. As the x-value increases, the y-value increases as well. The graph tells us that as we get more of whatever x represents, we will also get more of whatever y represents. The relation is described as an *increasing linear relation*.
- **Recursive relation in words and symbols:** A graph tells us how much the increase will be. The incremental steps from one point in the graph to the next may be described as "move one to the right and move up two." In this relation, for every additional x, there will be two additional ys. Each time x increases by 1, $2(x - 1)$ or y increases by 2.
- **Explicit relation:** The explicit relation is also shown in the graph if we wish to decipher it.
  - **Constant in the explicit relation:** If the line on which the points lie is followed backwards to the y-axis, we will notice that the y-value, when the relation hits the y-axis, is –2. When we use the distributive property to expand $2(x - 1)$ to $2x - \mathbf{2}$, we can see where the –2 is present in the linear relation. This –2 is referred to as the *constant*.
  - **Numerical coefficient in the explicit relation:** The slope, or incline, of the line upon which the points lie indicates there is also a small numerical coefficient in this term. It can be discovered in a variety of ways. For example, by examining the graph, we can see that for every increase of 1 in the x direction, there is an increase of 2 in the y-direction. This is represented by the coefficient in the relation, 2 ($\mathbf{2}(x - 1)$).
  - **Complete explicit relation:** The explicit relation contains $2x$ and –2. The combined explicit relation for the pattern is $2x - 2$, or the equation $2x - 2 = y$.
  - **Equivalent expressions:** The graph shows $2(x - 1)$ and $2x - 2$ are equivalent expressions. This provides an opportunity to talk about equations being two expressions for the same value, and to review the distributive property of multiplication.
- **Obtaining values that are not plotted:** Extending the line upon which the points lie, or looking at points between the ones that are plotted, and then reading the coordinate pairs, provides information about values that are not listed in a table of values, and provides answers to questions based on the relation.

3. Ask students to identify some contexts that may be represented by linear relations that increase in value. Examples may include the purchase of multiple single-priced items (e.g., bottles of water at $2 per bottle), a quantity discount (e.g., the first item costs $3 and each additional item costs $2), money earned for hours of babysitting, distance travelled in relation to time, volume of drink required in relation to number of people being served, and so on.
4. Illustrate, or have students graph, a decreasing linear relation where the points lie in a line that goes down as you move to the right. Each time x increases in this relation, the y-value decreases (e.g., $6 - x$, $20 - 2x$). This time, the graph indicates there is an initial quantity that decreases in relation to the x-value.

5. Generate some contexts to represent decreasing linear relations. Examples could include a certain amount of savings in relation to the amount spent at a regular rate, a quantity of items in relation to the quantity used at a regular rate (e.g., There are 20 cans of cat food in the cupboard. The cat eats 2 cans every day.), and so on.

Variations:

- Have students work independently or in pairs to write descriptions of the graphs they created in the previous learning activity (Part A). Record descriptions on cards, using both everyday language and algebraic language. Circulate within the class to verify students are on the right track. When the cards are complete, post the relations, the tables of values, the graphs, and the descriptions on the display board. Having students post complete sets of their own work provides an assessment opportunity. Later, one or more individual parts of the display may be distributed among students, and reassembled with the correct matches.
- Play a version of the game Pictionary. One student illustrates a graph and the others guess the matching relation or description.
- Have students explore the relationships between relations and their representative graphs through an inquiry activity. Students create a list of relations that differ in a systematic way (e.g., increasing coefficients, constants, or negative constants). They make graphical representations of the relations, compare the two, and describe the changes. The generalized descriptions may be used to draw conclusions about the relationship between a relation and its representative graph.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Sketch the graph from a table of values created for a relation, and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between n and $2n + 3$).
- ❒ Describe the relationship shown on a graph using everyday language in spoken or written form to solve problems.
- ❒ Match a set of relations to a set of graphs.
- ❒ Match a set of graphs to a set of relations.
- ❒ Reason mathematically in order to make connections.

- **Sketch the graph from a table of values created for a relation, and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between *n* and 2*n* + 3).**
- **Describe the relationship shown on a graph using everyday language in spoken or written form to solve problems.**

Materials:

- BLM 7.PR.8: Template for Creating and Solving Problems Using Information from a Graph
- index cards or card stock
- a set of contextual problems and sketches of graphs that represent relations portrayed in the problems (optional)
- student-generated graphs from the previous two learning activities (Part A and Part B) (optional)
- graphing technology (optional)

Organization: Individual or pairs, whole class, individual

Procedure:

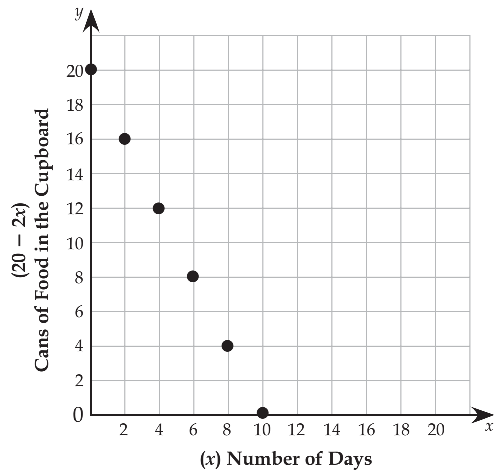
Part C

This learning activity may be used as a continuation of the previous two learning activities (Part A and Part B), or conducted as a single learning experience. Students may use the graphs they made previously or create new graphs.

1. Review descriptions of graphs by having students work independently or in pairs to generate contexts or situations that result in increasing or decreasing linear relations. Having students make rough graphical sketches of the relations will reinforce the connection between the descriptions and the graphs. After giving students sufficient time to work on their own or with a partner, have students share examples with the class. Verify students' understanding by evaluating their examples or by conducting a quick matching game. Display a number of sketched graphs. Read a problem and have students select the graph that represents the problem.

2. Model how a graph that illustrates a contextual situation can be used to solve problems. Present a graph such as the following.

   Example:



   Say that this graph represents the story of a cat and her food. The title indicates that the graph tells the part of the story related to how much cat food is in the cupboard each day. The labels on the axes of the graph tell us that x represents the number of days, and y represents the numbers of cans of cat food. The story begins with a number of cans of cat food in the cupboard. The points lie on a line, so the relation is linear. Therefore, the change will be constant. The line is going down to the right, so the relation is decreasing. We can, therefore, conclude there are fewer cans each day. Likely this is because the cat is eating the food.

   We can find out exactly how many cans the cat is eating each day by looking for the recursive relationship in the graph. Each day, there are 2 fewer cans, so the recursive relation is –2. We can conclude that the cat eats 2 cans of food each day ($-2x$). We can find out how many cans are in the cupboard on any given day by reading the coordinate pairs. For example, on day eight, the corresponding y-value is 4. There are 4 cans of food in the cupboard on the eighth day.

   Have students generate a list of questions that could be answered using this graph as a source of information (e.g., How many cans are in the cupboard to begin with? How many cans are left on the __th day? On which day will the cat run out of food if no more is added to the cupboard?).

   We can also find the explicit relationship in the graph. The graph begins at 20 and decreases by 2 each day. The equation $20 - 2x = y$ can be used to answer any of the questions as well, by substituting values for a variable and solving the equation. (This will be the focus of subsequent learning activities.)

3. Next, have students use their own graphs to create word problems to share with classmates. BLM 7.PR.8: Template for Creating and Solving Problems Using Information from a Graph may be used as a template for this purpose, and for assessing students' work. Have students choose a graph from the ones they created in the previous two learning activities (Part A and Part B), or have them create a new graph. The new graph may be generated from a table of values that was obtained by substituting values for x in a given relation.

The following steps are recommended:

a) Identify a story context.

b) Become specific about the story and write a title for the graph.

c) Label the x- and y-axes.

d) Describe the graph.

e) Identify the recursive relationship.

f) Identify the explicit relation.

g) List questions that could be answered by using the graph as a source of information, and state the answers.

4. Ask students to choose one or more of the questions from their list and write interesting word problems that can be solved using their graph as a source of information. Have them write a good copy of the problems on one side of index cards, and the solutions to the problems on the back of the cards, or under a flap, or on whatever medium has been chosen.
5. Students may wish to verify their work before displaying the problems.
6. Have students share their problems, and the matching graphs, title, and labels, for their classmates to practise answering. The questions may be presented to the entire class, distributed among small groups, or posted with the matching graphs.

Variations:

- Use technology to create graphs and matching word problems, for presentation or as interactive questions.
- Problem cards may be combined with the other sets to play games (e.g., matching games or quizzes).

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Sketch the graph from a table of values created for a relation, and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between n and $2n + 3$).
- ❒ Describe the relationship shown on a graph using everyday language in spoken or written form to solve problems.
- ❒ Communicate mathematically.

Assessing Prior Knowledge

Materials:

- BLM 7.PR.9: Associating Clue Words with Operations and Expressions
- demonstration board

Organization: Pairs or small groups, whole class

Procedure:

The intent of this learning activity is to have students make generalizations that will help them interpret mathematical scenarios. Students should not be required to memorize the associations made, but rather should gain confidence in recognizing associations.

1. Divide students into pairs or groups. Inform students they will be working in pairs or in small groups to complete a chart (e.g., BLM 7.PR.9: Associating Clue Words with Operations and Expressions). The chart will show whether students know some clue words that may indicate which operation to use and whether they know how to represent a given problem as an expression or as an equation (e.g., older than, Geri is 4 years older than Kasha, $k + 4$ or $k + 4 = g$).
2. Distribute copies of BLM 7.PR.9: Associating Clue Words with Operations and Expressions, and have students work together in pairs or in small groups to complete the charts.
3. When students have had sufficient time to complete the charts, reassemble as a class. Verify students' understanding by having some students share a phrase, while others identify the clue word, the operation, and the expression or the equation. Take time to verify responses by substituting values and checking for reasonableness.

Variation:

- After the charts are complete, divide the class into two teams, and have a contest to see which team matches more phrases.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Associate clue words with correct operations.
- ❒ Represent phrases with expressions or equations correctly.
- ❒ Substitute values in the phrases and test for reasonableness.

Suggestions for Instruction

- **Solve a problem using a linear equation.**
- **Solve a problem using a linear equation and record the process.**

Materials:

- BLM 7.PR.10: Solving Single-Variable One-Step Equations

Organization: Individual, whole class

Procedure:

This learning activity invites students to use and develop their current strategies for solving equations. Later learning experiences will be devoted to solving equations through the preservation of equality.

1. Activate students' background knowledge by presenting a single-variable one-step linear equation, such as $d + 9 = 15$, and asking students to solve it. Point out that here a variable is being used to represent an unknown quantity. Ask students to describe how they arrived at the answer. Reinforce that there are multiple ways to solve problems. Consult the Background Information for more information. Provide or solicit a question for each operation.
2. Distribute copies of BLM 7.PR.10: Solving Single-Variable One-Step Equations, and have students solve the equations individually.
3. When students have had sufficient time to solve the equations, reassemble as a class, and have students share their solutions and strategies. This is a good opportunity to assess students' repertoire of strategies, and to have students hear alternative strategies from their classmates. Ask students how they would solve the equations if the values were larger, and less mental mathematics friendly. (This learning activity provides background for BLM 7.PR.11: Writing Expressions and Solving Equations That Match Word Descriptions, which will be used in the next learning activity.)

Variations:

- Students could write contextual problems to match each expression. Or, they could use the equations as models to write additional equations. The problems/equations could be added to a classroom problem/question bank for games, Entry Slips or Exit Slips, and so on.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Solve problems using single-variable one-step equations.
- ❒ Explain a strategy used to solve the problems.
- ❒ Apply mental mathematics strategies to solve problems.

- **Solve a problem using a linear equation and record the process.**
- **Verify the solution to a linear equation using concrete materials or diagrams.**
- **Substitute a possible solution for the variable in a linear equation to verify the equality.**

Materials:

- BLM 7.PR.11: Writing Expressions and Solving Equations That Match Word Descriptions

Organization: Individual, whole class

Procedure:

This learning activity invites students to use and develop their current strategies for solving equations. Later learning experiences will be devoted to solving equations through the preservation of equality.

1. Distribute copies of BLM 7.PR.11: Writing Expressions and Solving Equations That Match Word Descriptions, and have students complete the tasks individually.
2. When students have had sufficient time to complete their work, have them reassemble as a class. Discuss students' responses. Test the reasonableness of the expressions, and substitute the solutions in the equations to verify their correctness. **Note:** Question 2(c) in the BLM requires applying the order of operations.

Variations:

- Students write word problems that could be represented by the descriptions in the problems.
- Students add cards with descriptions representing expressions or equations, or word problems that can be represented with linear equations, to the classroom problem/question bank for games, Entry Slips or Exit Slips, and so on.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Represent word descriptions with correct expressions.
- ❒ Write an equation to match a word description and solve the equation correctly.

Suggestions for Instruction

- **Represent a problem with a linear equation and solve the equation using concrete models.**
- **Model a problem with a linear equation and solve the equation using concrete models.**

Materials:

- demonstration board
- balance scales for each group of students (or student-made balances)
- blocks or cubes (interlocking optional)
- small paper bags, weigh boats, or polystyrene cups
- math journals or notebooks
- BLM 7.PR.12A: Representing Equivalent Expressions on a Balance Scale (Sample) (multiple copies optional)
- BLM 7.PR.12B: Representing Equivalent Expressions on a Balance Scale (Template)
- BLM 7.PR.12C: Representing Equivalent Expressions on a Balance Scale Using Variables for Unknowns (Sample)
- BLM 7.PR.12D: Representing Equivalent Expressions on a Balance Scale Using Variables for Unknowns (Template)
- BLM 7.PR.12E: Representing Equivalent Expressions (Template)

Organization: Whole class, small groups (of four)

Procedure:

1. Activate students' background knowledge by demonstrating an example to the class before students work in their small groups. This process is illustrated on BLM 7.PR.12A: Representing Equivalent Expressions on a Balance Scale (Sample). Record the demonstrated process on the demonstration board.
   a) Present the balance scale to students, and ask them to describe the technology and its purpose. Sketch a schematic balance scale on the demonstration board.
   b) Introduce the individual blocks as representing a value of 1. Blocks can be linked together to represent other quantities. Add a given number of blocks to one side of the scale. Count the blocks out loud, and arrange them neatly on the platform (e.g., 8). The pan will tip. Record the scale and the quantity pictorially and symbolically (8).
   c) Invite a student to rebalance the scale. Stipulate that the student must use blocks, and cannot duplicate what is already on the opposite pan (e.g., use 7 joined blocks and 1 single block). Record this addition and the change pictorially and symbolically.

d) Note that the two expressions are equivalent. The scale is balanced. Record the equation ($8 = 7 + 1$). Point out how the different parts of the equation symbolize the different parts of the scale.

e) Invite someone else to balance the scale in a different way, and record that equality. Include a third alternative.

f) All the expressions are equivalent to 8. Therefore, they must be equivalent to each other. Test the equivalency of the expressions by rearranging the 8 blocks on the balance scale into the different expressions. Record the equivalent expressions as sets or as a string (e.g., $8 = 7 + 1 = 4 + 4 = 3 + 5$). Note that all expressions between the equal signs are different names for the same quantity. This is also an opportunity to revisit the commutative property of addition.

2. Have students, working in groups, repeat the demonstrated process using their own values. Each student will record actions pictorially and symbolically. A suggested procedure and template are found on BLM 7.PR.12B: Representing Equivalent Expressions on a Balance Scale (Template). Templates are supplied for use with and without variables. Students may also use their math journals or notebooks.

   a) One student in each group has the responsibility of setting a quantity on either side of the scale, and then announces the quantity, draws a scale, and records the quantity.

   b) Other students in the group take turns representing equal quantities on the opposite side of the scale, or rearranging the quantities in a pan. They verbalize their actions as they perform them, and record the process pictorially and symbolically.

3. As soon as students are ready, reassemble as a class, and demonstrate balancing the scale using blocks or cubes in paper bags or cups. The concealed quantities represent the unknown meaning of a variable. It may be necessary to tare the scales to compensate for the mass of the empty bag or cup.

   a) Secretly add a number of blocks to the bag (e.g., 3). Add the bag and a number of cubes to one pan of the scale. Record the action pictorially and symbolically (($b + 4$), where b represents the quantity in the bag).

   b) Invite a student to balance the scale with a number of blocks, counting them in the process. Record the pictorial representation and the linear relation ($b + 4 = 7$).

   c) Invite a student to rename 7 in terms of b. (Use two bags with 3 blocks in each bag and 1 single block.)

   d) Another student can rearrange the blocks again, or identify the number of blocks in the bag.

   e) Demonstrate an unbalanced solution with an empty bag and some blocks on one pan and some blocks on the other pan. Ask students how to find the value of b and rebalance the scale. (Count blocks into the bag until the scale balances.)

4. Have students work in groups to repeat the process with variables. Once again, have students share roles to model, represent, and record equality in their math journals or on the BLM template(s).

This learning activity prepares students for demonstrating preservation of equality and using preservation of equality to solve equations.

Variation:

- Arrange blocks without a scale. Count to verify equivalency.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Use a balance scale to model equivalent expressions.
- ❒ Write equivalent expressions without variables as equations.
- ❒ Write equivalent expressions with variables as equations.
- ❒ Model and record the commutative property of addition.
- ❒ Reason mathematically.

Suggestions for Instruction

- **Substitute a value for each unknown in an expression and evaluate the expression.**

Materials:

- BLM 7.PR.13: Evaluating Expressions, Given a Value for the Variable
- marking pens

Organization: Whole class, individual

Procedure:

1. As a class, activate students' background knowledge by asking individual students to
   a) define a *variable* and provide an example of one
   b) define an *expression* and provide an example of an expression using the variable provided
   c) suggest a value for the variable
   d) substitute that value for the variable and evaluate the expression
   e) offer a different expression using the same variable
   f) evaluate the expression using the same value for the variable
   g) provide a different value and evaluate the expression using the new value

2. When you are satisfied that students are able to complete this task individually, distribute copies of BLM 7.PR.13: Evaluating Expressions, Given a Value for the Variable.
3. When students have had sufficient time to evaluate the expressions, ask them to reassemble as a class, share their answers, discuss any discrepancies, and use a marking pen to make any notes, corrections, or additions to their sheets. The sheets can be used for study notes at a later date.

Variation:

- Have students create some additional expressions of their own, using a new variable and/or values to substitute for the new variable in their expressions. Students could exchange their expressions and have a classmate assess them.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Substitute a value for each unknown in an expression and evaluate the expression.
- ❒ Apply mental mathematics strategies to solve problems.

Suggestions for Instruction

- **Model the preservation of equality for addition, subtraction, multiplication, or division using concrete materials or using pictorial representations, explain the process orally, and record it symbolically.**
- **Provide an example of an expression and an equation, and explain how they are similar and different.**

Materials:

- demonstration board
- balance scales for each group of students (or student-made balances)
- blocks or cubes (interlocking optional)
- small paper bags or polystyrene cups
- poster paper
- math journals or notebooks
- algebra tiles (optional)
- pens or markers of different colours (optional)

Organization: Whole class (with three recorders and one "voice"), small groups, individual

Procedure:

1. Activate students' background knowledge by having a group of students use a balance scale to model and record changes to an equation. One student may provide the concrete model, while two others record the pictorial and symbolic representations of the equation on the demonstration board. One student can act as "the voice," modelling self-talk during the investigation.
   a) Model an equation with no variables.
   b) Have students predict the outcome of adding 1 to the pan on one side of the balance.
   c) Perform the action. (The equation is unbalanced.) Record the action symbolically with less than (<) or greater than (>) symbols.
   d) Ask what will happen if 1 is added to the expression on the other pan. (Balance is restored.)
   e) Play a game, asking students to predict balance or tilt, and the direction of tilt, if different quantities are added to either or both of the pans. Have students model an equation, record the action, and comment on what is happening for each scenario as it is performed. Include equations with variables in the form of bags or cups.
   f) Ask students to formulate a conclusion about preserving equality in an equation when using addition. (Adding the same amount to each side of the equation preserves equality.)
2. Have students work in small groups to conduct the same investigation and draw a conclusion for each of the other operations: subtraction, multiplication, and division. Be sure to have them model concretely, represent pictorially, record symbolically, talk through and explain the process, and formulate conclusions.
3. Students can record conclusions in their math journals, or prepare personal posters about preserving equality in an equation. Ask them to include pictorial and symbolic representations, as well as an explanation of why the same operation must be applied to each expression to maintain equality. In this investigation, students have repeatedly used the terms *expression* and *equation*. Have them include a statement in their math journals that explains how expressions and equations are similar and how they are different.

This learning activity prepares students for solving a problem using preservation of equality in the next learning experience.

Variations:

- Use algebra tile models in place of, or in addition to, balance scale models.
- Investigate graphs as concrete representations of equivalent expressions. Graph the equivalent expressions in different colours on the same axes.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Model the preservation of equality for addition, subtraction, multiplication, or division using concrete materials or using pictorial representations, explain the process orally, and record it symbolically.
- ❒ Provide an example of an expression and an equation, and explain how they are similar and different.

Suggestions for Instruction

- **Solve a problem by applying preservation of equality.**
- **Draw a visual representation of the steps required to solve a linear equation.**
- **Verify the solution to a linear equation using concrete materials or diagrams.**
- **Substitute a possible solution for the variable in a linear equation to verify the equality.**

Materials:

- BLM 7.PR.14A: Solving Linear Equations: Pictorial and Symbolic Representations
- BLM 7.PR.14B: Solving Linear Equations with Constants: Applying the Preservation of Equality
- BLM 7.PR.14C: Solving Equations with Numerical Coefficients: Applying the Preservation of Equality
- BLM 7.PR.14D: Solving Linear Equations with Constants and Numerical Coefficients: Applying the Preservation of Equality
- demonstration board
- balance scales for each group of students (or student-made balances)
- blocks or cubes (interlocking optional)
- small paper bags or polystyrene cups
- math journals or notebooks
- counters and algebra tiles (optional)

Organization: Whole class, pairs or small groups

Procedure:

The previous learning experience, in which students modelled and explained the preservation of equality for each of the four operations, provides a foundation for this learning experience. The emphasis here is on having students develop a strategy to solve a linear equation.

Part A

1. **Represent a linear equation with a constant.**
   - Begin with a simple concrete representation of a one-step equation with a single variable.
   - Revisit the example of 3 blocks or cubes in a bag, combined with 4 single blocks, on one pan of the scale, balanced by 7 cubes on the other pan. Record the pictorial representation of the balance and the linear relation $b + 4 = 7$, where b represents the quantity in the bag.
2. **Represent the solution using the preservation of equality.**
   - Students know there are 3 blocks in the bag. Knowing basic number facts makes this an easy equation to solve. Propose that sometimes equations are not easy to solve, and it would be helpful to have a strategy to find the solution. Easy questions assist with developing strategies, because ideas and errors are more obvious in easy questions. Ask students to suggest some strategies.
   - Empty the bag. Students will observe that the balance tilts. Ask students how to find the value of b and rebalance the scale. Add blocks to the bag until the scale balances, counting the blocks in the process. Counting the blocks that balance the scale provides a concrete model to verify a solution.
   - Challenge students to prove there are 3 blocks in the bag by applying the principles they learned about the preservation of equality. Applying identical operations to both expressions will preserve the equality of the relation. Remove 4 from each side of the balance. Ending up with the bag on one pan and 3 blocks on the other pan indicates there are 3 blocks in the bag. When the equation is solved, verify the solution symbolically by substituting 3 in the original equation. Alternatively, verify the solution concretely by opening the bag and counting the blocks. As students make suggestions, record their suggestions pictorially and symbolically.
   - The strategy is outlined with both a balance scale and algebra tiles in the Background Information.
3. **Develop and test a strategy.**
   - When students are sufficiently prepared, suggest that they work with partners or in small groups to test more equations containing a variable and a constant. For each equation they test, have students talk through steps as they proceed, and verify the solution.

- Apply the strategy to equations in which the arithmetic is not so easy (e.g., $b + 17 = 42$). Include negative constants (e.g., $b - 9 = 7$). It may be necessary to review the principles of adding and subtracting integers.

4. **Record the process.**
   - When students have a process that works, ask them to record it on BLM 7.PR.14A: Solving Linear Equations: Pictorial and Symbolic Representations or in their math journals. Students include the linear equation and the pictorial and symbolic representations of the steps used to solve the equation. They also verify the solution, and articulate a streamlined process for solving a linear relation with a constant.
   - The process of solving a linear equation may include the following:
     - Aim to isolate the variable on one side of the equation and a quantity on the other side.
     - Remove the constant from the variable by adding its opposite to each side of the equation (similar to zero pairs—refer to Background Information for learning outcome 7.N.6).
     - Equate the variable with a quantity in the final equation.
     - Verify the solution by substituting the quantity for the variable in the equation.
5. **Apply the strategy.**
   - Ask students to test their process by applying it to linear equations with constants, such as those included on BLM 7.PR.14B: Solving Linear Equations with Constants: Applying the Preservation of Equality.

Part B

1. Review the process for solving a linear equation that students created in Part A. Have four volunteers work together to model and solve a linear equation with one variable and a constant. One volunteer talks through the steps, one models the concrete representation, one models the pictorial representation, and one models the symbolic representation.
2. Have students work with their partners or small groups to continue the investigation of Part A. In Part B, challenge students to outline a process to solve linear equations with a numerical coefficient (e.g., $4c = 36$), and then progress to a process to solve linear equations with a combination of coefficients and constants (e.g., $2b + 6 = 14$).
3. For each equation students work through, have them articulate the action they take, and verify their solution concretely or with substitution. Ask them to record both pictorial and symbolic steps to solve the equations, using either their math journals or BLM 7.PR.14A: Solving Linear Equations: Pictorial and Symbolic Representations.
4. Have students apply the process for solving linear equations by completing BLM 7.PR.14C: Solving Equations with Numerical Coefficients: Applying the Preservation of Equality and BLM 7.PR.14D: Solving Linear Equations with Constants and Numerical Coefficients: Applying the Preservation of Equality.

5. When students have completed their exploration, have individual students create a personal or class poster outlining the steps to solve linear equations using the preservation of equality. Remember to include a verification step.

Variations:

- Use a variety of concrete materials. Model solutions for the same or different problems using counters and/or commercial or student-made algebra tiles. Students benefit from being familiar with multiple representations.
- Extend the learning activity by having students create contextual problems to match the linear equations they work with.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Solve a problem by applying the preservation of equality.
- ❒ Draw a visual representation of the steps required to solve a linear equation.
- ❒ Verify the solution to a linear equation using concrete materials or diagrams.
- ❒ Substitute a possible solution for the variable in a linear equation to verify the equality.

Suggestions for Instruction

- **Represent a problem with a linear equation and solve the equation using concrete models.**
- **Model a problem with a linear equation and solve the equation using concrete models.**
- **Draw a visual representation of the steps required to solve a linear equation.**
- **Solve a problem using a linear equation.**
- **Solve a problem using a linear equation and record the process.**
- **Verify the solution to a linear equation using concrete materials or diagrams.**
- **Substitute a possible solution for the variable in a linear equation to verify the equality.**

Materials:

- demonstration board
- a collection of problems that can be represented by linear equations (use those completed in previous learning activities, those completed for the classroom question box, a collection of teacher-prepared problems, or BLM 7.PR.15: Problems to Represent with Linear Equations and with Concrete Materials)
- concrete materials to represent the preservation of equality (balance scales, tiles, counters)—one set of materials at each learning station
- recording booklets (two sheets of paper folded in half and stapled)—one booklet for each group

Organization: Learning stations with one or two problems at each station (one more station than the number of groups in the class), groups of four students

Procedure:

1. Set up learning stations in the classroom with one set of concrete materials and one or two problems at each station. Decide how many stations students must visit and how many problems students must complete.
2. Record the following four student roles on the demonstration board:
   a) Read the problem aloud. Record the linear equation that matches the problem.
   b) Explain the steps to follow in solving the problem. Model the solution to the problem using the concrete materials at the station.
   c) Record a diagram of the steps followed to solve the problem. Write the symbolic representation of the solution.
   d) Verify the solution, first by setting up the concrete materials in a balanced fashion, and then by substitution.

3. Students work together to find solutions to the problems, and take turns performing each of the four roles. They use the group's booklet to record equations, steps, solutions, and verification. Have students initial their entries in the booklet.

Variations:

- Offer students choice regarding the problems to be solved, or control the questions or type of questions students must answer.
- The problems above contain small quantities to facilitate modelling with concrete materials. For a given number of problems, have students write similar problems using larger numbers, and solve them using diagrams and/or symbolic representations.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Represent a problem with a linear equation and solve the equation using concrete models.
- ❒ Model a problem with a linear equation and solve the equation using concrete models.
- ❒ Draw a visual representation of the steps required to solve a linear equation.
- ❒ Solve a problem using a linear equation.
- ❒ Solve a problem using a linear equation and record the process.
- ❒ Verify the solution to a linear equation using concrete materials or diagrams.
- ❒ Substitute a possible solution for the variable in a linear equation to verify the equality.

Suggestions for Instruction

- **Represent a problem with a linear equation and solve the equation using concrete models.**
- **Model a problem with a linear equation and solve the equation using concrete models.**
- **Draw a visual representation of the steps required to solve a linear equation.**
- **Solve a problem using a linear equation.**
- **Solve a problem using a linear equation and record the process.**
- **Verify the solution to a linear equation using concrete materials or diagrams.**
- **Substitute a possible solution for the variable in a linear equation to verify the equality.**

Materials:

- access to research materials
- assorted materials to create questions and answers, game boards, and support materials
- booklets (for recording solutions)
- concrete materials for representing the preservation of equality (balance scales, blocks and bags, counters, algebra tiles)

Organization: Small groups, individual

Procedure:

Inform the class that groups of students will develop events or games, such as the following.

- **Around the World in 10 Equations**

  Students collect passport stamps as they move from one city to another on a regional or world map. To obtain transportation from one destination to the next, travellers must use a linear equation to solve a problem about information related to the region. Each traveller records equations and solutions in his or her passport book. Correct solutions earn travellers a passport stamp and a ticket to the next destination. Students receive a souvenir upon completing the journey.

- **Relations Regatta**

  Students enter a boat race. They collect strips to represent the distance completed as they progress through the course. To cover distance in the course, competitors must use linear equations to solve problems related to marine life, nautical vessels, and so on. Each competitor records the equation and solution in his or her logbook. Correct solutions earn participants a distance strip and an event pass to the next section of the course. Students receive a trophy for completing the race.

Part A

Students develop an event or a game.

1. Form groups and choose which event to host.
2. Select the topics on which to base questions.
3. Assign topics to individuals.
4. Individuals research their respective topics to collect information from which to create problems.
5. Each student creates two or three problems that can be represented by a linear equation, and tests the solution to the problems based on the preservation of equality.
6. Include one question with a constant, one with a numerical coefficient, and one with both a numerical coefficient and a constant.
7. Groups decide on the format for presenting the problems and concealing the solutions.
8. Individuals prepare good copies of the problems and concealed solutions. Solutions must include pictorial and symbolic representations of the solutions, and verification of the solutions.
9. Decide on the details required for the physical presentation to play the game and receive tokens.
10. Assign responsibilities, and create the product.
11. Test the game, and address any problem areas.

Part B

Students play the games created by others and collect the rewards.

1. Students decide which event they will participate in, and form groups to play each game.
2. Students progress through the game independently. Each individual reads the problems, and uses his or her booklet to represent each problem as a linear equation, and to record the steps used to solve the problem using the preservation of equality.
3. Students collect the rewards at the end of the game.

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Draw a visual representation of the steps required to solve a linear equation.
- ❒ Solve a problem using a linear equation.
- ❒ Solve a problem using a linear equation and record the process.
- ❒ Verify the solution to a linear equation using concrete materials or diagrams.
- ❒ Substitute a possible solution for the variable in a linear equation to verify the equality.

Relations Stories

Introduction:

Students create a book, graphic novel, or cartoon strip emphasizing patterns, their relations, and related vocabulary terms.

Purpose:

In this investigation, students have the opportunity to demonstrate any or all of the following abilities (connections to learning outcomes are identified in parentheses):

- Correlate oral and written patterns and linear relations. (7.PR.1)
- Construct and analyze a table of values and graphs to solve problems based on linear relations. (7.PR.2)
- Apply the preservation of equality to solve equations. (7.PR.3)
- Differentiate between expressions and equations. (7.PR.4)
- Evaluate expressions, given the value of the variable(s). (7.PR.5)
- Solve one-step linear equations. (7.PR.6)
- Solve problems represented by linear equations. (7.PR.7)
- Relate radii, diameters, and circumferences of circles and solve problems involving the measurement of circles. (7.SS.1)
- Apply formulas to determine the area of triangles, parallelograms, and circles. (7.SS.2)

Students will also demonstrate some or all of the following mathematical processes:

- Communication
- Connections
- Mental Mathematics and Estimation
- Problem Solving
- Reasoning
- Technology
- Visualization

Materials/Resources:

- BLM 5–8.25: My Success with Mathematical Processes
- books based on problems related to patterns, relations, and linear equations, such as the following:

  Gravett, Emily. *The Rabbit Problem.* London, UK: Macmillan Children's Books, 2009.

  McCallum, Ann. *Rabbits Rabbits Everywhere: A Fibonnaci Tale*. Illus. Gideon Kendall. Watertown, MA: Charlesbridge Publishing Inc., 2007.

  Neuschwander, Cindy. *Sir Cumference and the Isle of Immeter: A Math Adventure.* Illus. Wayne Geehan. Watertown, MA: Charlesbridge Publishing Inc., 2006.
- book-making supplies
- computer access (optional)

Organization: Individual, pairs

Procedure:

Student Directions

1. The world around us abounds with patterns and relations. Patterns can be described, can provide an interesting source of information and investigation, and can be used to create and solve mysteries.

   Example:

   Mathematician Leonardo Fibonacci lived in Italy around the year 1200. He introduced Hindu-Arabic numbers to Europe, and revealed an interesting number pattern in an investigation of the rate at which a single pair of rabbits multiplies. Interesting number patterns are prevalent in nature.

   Sample Website:

   Examples of patterns and relations (e.g., Fibonacci numbers, Pascal's triangle, fractals) can be viewed on websites such as the following:

   World-Mysteries.com. "Fibonacci Numbers in Nature and the Golden Ratio." *Science Mysteries*. 2002–2011. http://old.world-mysteries.com/sci_17.htm.
2. Listen to your teacher read *The Rabbit Problem* by Emily Gravett. Note references to Fibonacci and to patterns and relations.
3. Note the book's organization around Fibonacci's question, and how its presentation as a calendar matches the investigation period of one year. Note the author's subtle references.
4. Listen to your teacher read *Sir Cumference and the Isle of Immeter* by Cindy Neuschwander. This book solves area problems using relationships between area and the sides of rectangles and relationships between radius and circumference.

5. Create your own Adventures in Algebra series of stories involving patterns, relations, variables, and equations.
   - Stories may be patterned after *The Rabbit Problem* and presented as a calendar using a scene for each month of the year, or stories may be presented as books, graphic novels, or cartoon strips.
   - Stories may include main characters or a hero such as the Master of Relations and his sidekick the Vari Able Generator.
   - Stories should present patterns, equations, or mysteries to solve, which may be design-oriented, or focus on music, or involve quantity or measurements such as time, distance, or area. The story components and assessment criteria are outlined in My Planning Sheet for Relations Stories (see next page).
6. Once you have decided on a plan, share your ideas using My Planning Sheet for Relations Stories.
7. Begin work on your project.
8. Share your stories with peers, with younger students, as part of an authors' night, or in a library display.
9. Record your success using BLM 5–8.25: My Success with Mathematical Processes.

Assessment:

1. Students will demonstrate their learning in the different categories identified in Assessment of Relations Stories (see last two pages of Patterns and Relations), based on how they choose to complete the project. Have a conversation with each student about which learning he or she will demonstrate to you through the process of designing the product.
2. Work with students to develop assessment criteria in each of the identified categories.
3. The final assessment of each category should be based on a student's recent consistent demonstration of learning.
4. Distribute copies of BLM 5–8.25: My Success with Mathematical Processes, and have each student record his or her success with the mathematical processes.

My Planning Sheet for Relations Stories

Name ______________________________ Date ____________

What will I show? (Check at least four boxes in Section 1 and two boxes in Section 2)	How do I know that I have been successful?	How will I show it?
Section 1: Knowledge and Understanding of Mathematical Concepts		
❒ I can give examples of patterns and I can explain those patterns using math equations (like $d = 2c + 1$). (7.PR.1)	■ I gave one or more examples of patterns and I used math equations to show another way to explain the patterns.	
❒ I can look at a pattern and show the steps of a pattern using a T-chart and a graph. (7.PR.2)	■ I used a T-chart and a graph to show how a pattern changes.	
❒ I know that both sides of an equation are equal, and this helps me to solve equations. (7.PR.3)	■ I solved one or more equations and showed that I remembered to keep both sides of the equation equal when solving it.	
❒ I know the difference between an equation and an expression. (7.PR.4)	■ I showed an expression and an equation and I showed how they are different.	
❒ I can replace a variable with a number to solve an expression. (7.PR.5)	■ I replaced a variable with a number to solve an expression.	
❒ I know how to solve equations that can be solved in only one step (like $2y = 8$ or $p + 2 = -1$). (7.PR.6)	■ I solved equations that needed only one step to find the answer.	
❒ I can solve circle problems (like radius, circumference, and diameter). (7.SS.1)	■ I showed that I understand the different measurements in circles.	

(continued)

My Planning Sheet for Relations Stories (continued)

Name ______________________ Date ______________

What will I show? (Check at least four boxes in Section 1 and two boxes in Section 2)	**How do I know that I have been successful?**	**How will I show it?**
Section 1: Knowledge and Understanding of Mathematical Concepts (continued)		
❐ I can figure out the area of triangles, parallelograms, and circles using formulas. (7.SS.2)	■ I figured out the area of triangles, parallelograms, and circles using a formula I know or a formula that I figured out.	
❐ I can use math language when describing patterns.	■ I used math language that I already knew and math language that I was learning while showing what I knew about patterns.	
Section 2: Mental Mathematics and Estimation		
❐ I can use mental math and estimation to help me solve expressions (like $b + 5$). (7.PR.5)	■ I used mental math to solve expressions and to check that my answers were correct. ■ I estimated to make sure my answers made sense.	
❐ I can use mental math and estimation to help me solve and check equations that can be solved in only one step (like $2y = 8$ or $p + 2 = -1$). (7.PR.6)	■ I used mental math to solve equations and to check that my answers were correct. ■ I estimated to make sure my answers made sense.	
❐ I can use estimation when solving circle problems (radius, circumference, or diameter). (7.SS.1)	■ I estimated to make sure my answers made sense.	
❐ I can figure out the area of triangles, parallelograms, and circles using formulas. (7.SS.2)	■ I used mental math to figure out the area of triangles, parallelograms, and circles. ■ I estimated to make sure my answers made sense.	

Assessment of Relations Stories

Criteria	4	3	3	1	Not Demonstrated (ND)
Knowledge and Understanding of Mathematical Concepts					
❐ Provide examples or scenarios of patterns and linear relations. (7.PR.1)	■ makes connections among pattern examples/scenarios and their linear relations	■ demonstrates a good understanding of how to connect pattern examples/scenarios and their linear relations	■ connects basic pattern examples/scenarios and their linear relations	■ requires support to connect basic pattern examples/scenarios and their linear relations	■ does not connect pattern examples/scenarios and their linear relations
❐ Create tables of values and graphs based on linear relations. (7.PR.2)	■ accurately represents linear relations as graphs and tables of values				
❐ Use preservation of equality to solve equations. (7.PR.3)	■ demonstrates an understanding of preservation of equality when solving equations				
❐ Differentiate expressions from equations. (7.PR.4)	■ differentiates between expressions and equations				
❐ Evaluate expressions, given the value of the variable(s). (7.PR.5)	■ substitutes a value for the variable in order to solve an expression				
❐ Solve one-step linear equations. (7.PR.6)	■ solves one-step linear equations concretely, pictorially, or symbolically				
❐ Solve problems involving measurements of circles. (7.SS.1)	■ demonstrates an understanding of measurement related to circles (radius, circumference, and diameter)				

(continued)

Assessment of Relations Stories (continued)

Criteria	4	3	3	1	Not Demonstrated (ND)
Knowledge and Understanding of Mathematical Concepts (continued)					
❐ Determine the area of triangles, parallelograms, and circles by applying formulas. (7.SS.2)	■ develops/applies a formula for determining the area of triangles, parallelograms, and/or circles				
❐ Use related vocabulary.	■ demonstrates an understanding and application of mathematics vocabulary				
Mental Mathematics and Estimation					
❐ Evaluate expressions, given the value of the variable(s). (7.PR.5)	■ applies mental mathematics strategies to solve expressions				
❐ Solve one-step linear equations. (7.PR.6)	■ applies mental mathematics strategies to solve one-step linear equations and to check the accuracy of the solutions				
❐ Solve problems involving measurements of circles. (7.SS.1)	■ makes reasonable estimates when solving problems involving the measurements of circles				
❐ Determine the area of triangles, parallelograms, and circles by applying formulas. (7.SS.2)	■ applies mental mathematics strategies to determine the area of triangles, parallelograms, and circles, and makes reasonable estimates to determine the accuracy of the solutions				