GRADE 6 MATHEMATICS

Patterns and Relations

Grade 6: Patterns and Relations (Patterns) (6.PR.1, 6.PR.2)

Enduring Understanding(s):

Words, tables, graphs, and expressions are different representations of the same pattern.

General Learning Outcome(s):

Use patterns to describe the world and solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
6.PR.1 Demonstrate an understanding of the relationships within tables of values to solve problems. [C, CN, PS, R]	➛ Generate values in one column of a table of values, values in the other column, and a pattern rule. ➛ State, using mathematical language, the relationship in a table of values. ➛ Create a concrete or pictorial representation of the relationship shown in a table of values. ➛ Predict the value of an unknown term using the relationship in a table of values and verify the prediction. ➛ Formulate a rule to describe the relationship between two columns of numbers in a table of values. ➛ Identify missing elements in a table of values. ➛ Identify and correct errors in a table of values. ➛ Describe the pattern within each column of a table of values. ➛ Create a table of values to record and reveal a pattern to solve a problem.
6.PR.2 Represent and describe patterns and relationships using graphs and tables. [C, CN, ME, PS, R, V]	➛ Translate a pattern to a table of values and graph the table of values (limit to linear graphs with discrete elements). ➛ Create a table of values from a pattern or a graph. ➛ Describe, using everyday language, orally or in writing, the relationship shown on a graph.

Prior Knowledge

Students may have had experience with the following:

- Adding and subtracting whole numbers, each numeral not greater than four digits
- Multiplying whole numbers, using 2- or 3-digit numerals by 1-digit numerals
- Dividing whole numbers, using a 1-digit divisor and up to a 2-digit dividend
- Using concrete materials to reproduce a pattern shown in a table or chart
- Making predictions about subsequent elements based on the pattern rule
- Solving one-step single-variable equations, using whole numbers only

Related Knowledge

Students should be introduced to the following:

- Using concrete objects, pictures, and symbols to demonstrate an understanding of integers
- Drawing conclusions after they create, label, and interpret line graphs
- Solving problems by graphing collected data and analyzing the graph

Background Information

In earlier grades, students were developing their understanding of repeating patterns, increasing patterns, and decreasing patterns by using manipulatives, sounds, actions, diagrams, and numbers. Students later used tables and charts to identify, describe, and reproduce patterns, identify and explain mathematical relationships, and determine a pattern rule. Look at the following pattern:

□□ □□□□ □□□□□□ □□□□□□□□ □□□□□□□□□□

Mathematical patterns, such as the one above, can be used to give elaborate visual representations of changes. They show a step-by-step change in quantities. The step-by-step change in quantities can also be represented in a table, as shown here.

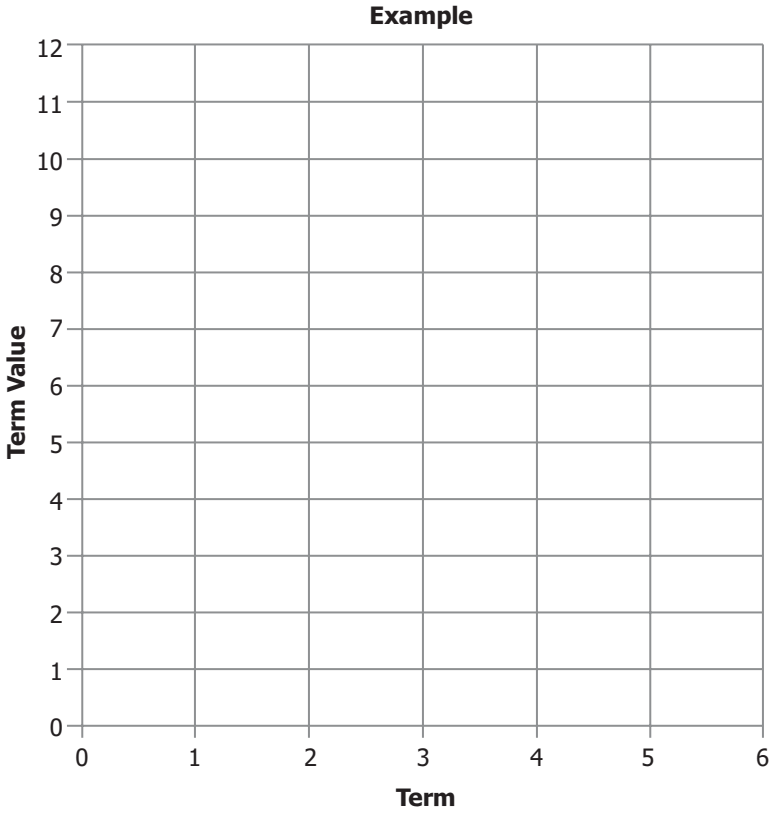
Term	1	2	3	4	5
Term value	2	4	6	8	10

This table shows that the value of each term is two more than the value of the previous term. This type of generalization (rule), by which you can find the value of a term based on the term value of the previous terms, is called a r**ecursive generalization**. Along with using pictorial representations, tables, and words, you can also use algebraic expressions to represent patterns or describe rules. The recursive generalization that describes the above pattern can be expressed by the algebraic expression $n + 2$. Using a recursive

generalization works well if you have a small number of terms, such as in the above example. Finding the value of the 86th term, though, would take a lengthy process, since you would need to find the value of each of the 85 terms preceding it. It is also possible to look at this table in a slightly different way.

To find the value of a particular term in the above shown table, we simply take the term number and double it. For example, we can see two squares in the first term and we say that the first term has a term value of 2; similarly, we can say that the third term has a term value of 6. The rule for the above shown table of values is $2n$. This means that if we extend the above shown pattern, term 6 would have a term value of 12, term 10 would have a term value of two times ten, which is twenty ($2 \times 10 = 20$), and term 86 would have a term value of two times eighty-six, which is one hundred seventy-two ($2 \times 86 = 172$). This type of generalization (rule), which expresses the relationship between the term number and its value, is called an **explicit generalization**. Students might also benefit from a discussion concerning the two types of generalizations as stated above (a recursive generalization and an explicit generalization).

Transferring information from tables to graphs will be easier for students if they are reminded to label the horizontal axis as the "term" and the vertical axis as the "term value."



It might be helpful for students if they are reminded that a table of values and a graph are two ways of representing the same information. Ask students to describe how the two representations are similar.

Mathematical Language

data
element
explicit generalization
graph
pattern
pattern rule
recursive generalization
table of values
term

Learning Experiences

Assessing Prior Knowledge

Materials:

- overhead projector
- transparencies
- paper and pencil

Organization: Groups of four (seated)

Procedure:

1. Tell students that the following few lessons will be spent on expanding their knowledge of patterns.
2. Tell them to put on their "thinking caps" and see what they can remember from Grade 5 or earlier grades.
3. Place the word "Patterns" on an overhead transparency and ask students: "What do you know about patterns? Discuss in your groups your ideas about patterns."
4. Listen to their discussions. Are these discussions revealing their knowledge as to the following questions: What is a pattern? What is the next term? How do I find the next term? What is the pattern rule?
5. Ask students to give an example of a pattern, and write down what makes their example a pattern.

Suggestions for Instruction

- **Generate values in one column of a table of values, values in the other column, and a pattern rule.**
- **Formulate a rule to describe the relationship between two columns of numbers in a table of values.**
- **Create a table of values to record and reveal a pattern to solve a problem.**

Materials:

- BLM 6.PR.1: Pattern Introduction

Procedure:

1. Using an overhead projector, show the class the following pattern, as found in BLM 6.PR.1.

2. Ask students to discuss the following questions:
   a) How many terms are there in this pattern?
   b) How many circles do you see in the first term?
   c) How many circles do you see in the second term?
3. Let them discuss with their group members what a table of values would look like for this pattern, and then, using a pencil and paper, let each of them draw the table of values.
4. Ask students to discuss what pattern rule they see, and write it down.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Generate values in one column of a table of values, values in the other column, and a pattern rule.
- ❒ Formulate a rule to describe the relationship between two columns of numbers in a table of values, as in the above example $n + 1$.
- ❒ Create a table of values to record and reveal a pattern to solve a problem.

Suggestions for Instruction

- **State, using mathematical language, the relationship in a table of values.**
- **Create a concrete or pictorial representation of the relationship shown in a table of values.**
- **Predict the value of an unknown term using the relationship in a table of values and verify the prediction.**
- **Formulate a rule to describe the relationship between two columns of numbers in a table of values.**

Materials:

- BLM 6.PR.2: Horizontal Table #1

Procedure:

1. Display the following table on a poster-size paper, or distribute to students copies of BLM 6.PR.2.

Term	1	2	3	4	5
Number of Tiles	3	5	7	9	11

2. In groups of two, have students discuss the pattern.
3. Have each pair of students create the pattern with building blocks.
4. Ask each student to draw the pattern on paper.
5. Ask students to predict how many tiles term 6 would have if the pattern were extended.
6. Ask students to predict how many tiles term 32 would have if the pattern were extended.
7. Ask students to formulate a rule using words, symbols, and diagrams (emphasize multiple representations).
8. Ask students to verify their predictions.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ State, using mathematical language, the relationship in a table of values.
- ❒ Create a concrete or pictorial representation of the relationship shown in a table of values.
- ❒ Predict the value of an unknown term using the relationship in a table of values, and verify the prediction, as 13 is the value of term 6.
- ❒ Formulate a rule to describe the relationship between two columns of numbers in a table of values (the pattern above could be described as 2 more than the number before, one more than twice the term number, or $2t + 1$).

Note: It is important that students are able to see both the pattern in the number of tiles (recursive generalization) and the relationship between the term number and the number of tiles (explicit generalization). Both of these are valuable skills.

Suggestions for Instruction

- **Formulate a rule to describe the relationship between two columns of numbers in a table of values.**
- **Identify missing elements in a table of values.**
- **Describe the pattern within each column of a table of values.**
- **Identify and correct errors in a table of values.**

Materials:

- BLM 6.PR.3: Horizontal Table #2

Procedure:

1. Have students sit in groups of four. Let each of them have a copy of the pattern.
2. Using an overhead projector, show the following pattern from BLM 6.PR.3 to the class:

Term	1	2	3	4			7	8	9
Term Value	9	19	29		49	59		87	91

3. Ask students to discuss and fill in the missing elements.
4. Ask students to discuss what pattern rule they see and to write it down.
5. Ask them how the term values are related to each other.
6. Ask them if they can identify the two errors in the table of values and correct them.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Formulate a rule to describe the relationship between two rows of numbers in a table of values, such as the term value is one less than 10 times the term number or $10n - 1$ for the above table.
- ❒ Identify missing elements in a table of values.
- ❒ Describe the pattern within each row of a table of values. (In the second row, each number is 10 more than the number before.)
- ❒ Identify and correct errors in a table of values. (Errors: 87 and 91 corrections: 79 and 89).

Suggestions for Instruction

- **Formulate a rule to describe the relationship between two rows of numbers in a table of values.**
- **Identify missing elements in a table of values.**
- **Create a table of values to record and reveal a pattern to solve a problem.**

Procedure:

1. Hand out the following question to each student.

   The music teacher was happy to see a daily increase in students in the choir. On the first day of classes, eight students came to choir. On the second day, 11 students came to choir. The following day, three new students came to choir. How many students will show up to choir on day 10 if this pattern continues?

Day	1	2	3	4	5
Number of Students					

2. Ask students to enter all known information into the table of values, working individually.

3. Ask them to find the pattern rule (half of the class can use the recursive generalization and the other half of the class can use the explicit generalization), and complete the missing values.
4. Find the answer to the question.
5. How many students would show up to choir on day 25 if this pattern continued?
6. Have a class discussion on the different methods of completing the problem (e.g., ask students to list the positive aspects of each method).

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Create a table of values to record and reveal a pattern to solve a problem.
- ❒ Formulate a rule to describe the relationship between two rows of numbers in a table of values, such as the above rule $(n - 1)(3) + 8$.
- ❒ Identify missing elements in a table of values.

Suggestions for Instruction

- **Formulate a rule to describe the relationship between two columns of numbers in a table of values.**
- **Identify missing elements in a table of values.**
- **Describe the pattern within each column of a table of values.**

Materials:

- BLM 6.PR.4: Vertical Table #1

Procedure:

1. Tell students that today you want them to be little detectives.
2. Show on the overhead projector BLM 6.PR.4, and read the text part to them.
3. Ask students to look at the table carefully and write in their journals what these numbers could mean.
4. Ask students to discuss what pattern rule they see and write it down.
5. Ask them to fill in the missing numbers.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Formulate a rule to describe the relationship between two columns of numbers in a table of values, such as the term value is two less than three times the term number, or $3n - 2$ for the vertical table #1.
- ❒ Identify missing elements in a table of values.
- ❒ Describe the pattern within each column of a table of values, such as in the term value column each number is three more than the number before it.

Suggestions for Instruction

- **Create a table of values from a pattern or a graph.**
- **Describe, using everyday language, orally or in writing, the relationship shown on a graph.**

Materials:

- BLM 6.PR.4: Vertical Table
- BLM 6.PR.5: King Klonig's Graph

Procedure:

1. Distribute copies of BLM 6.PR.5 on coloured paper, and have students sit in groups with other students who received the same coloured paper (about four or five groups).
2. Ask students to discuss the following questions with their group members:
   a) What could King Klonig's graph represent?
   b) How many terms are shown on King Klonig's graph?
   c) What is the term value of the first term?
   d) What is the term value of the last term shown on this graph?
   e) How simple or difficult would it be to predict the term value of term 38?
3. Let each group choose one person to present the main ideas of their discussion to the class.
4. Ask "What would a table of values look like for this pattern?" Then, using a pencil and paper, let each of them draw the table of values.

5. Ask students to discuss what pattern rule they see and, using everyday language, describe it in their journals.
6. Read out some answers orally.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Create a table of values from a pattern or a graph.
- ❒ Describe, using everyday language, orally or in writing, the relationship shown on a graph, as the term value is five more than two times the term number for this example.

Suggestions for Instruction

- **Translate a pattern to a table of values and graph the table of values (limit to linear graphs with discrete elements).**
- **Describe, using everyday language, orally or in writing, the relationship shown on a graph.**

Materials:

- BLM 6.PR.6: Lily's Pattern

Procedure:

1. Distribute copies of BLM PR.6.
2. Ask for four volunteers, each representing a term, to clap according to his or her term value.
3. Ask students to create a table of values for this pattern in their notebooks, and then extend the table to term 6.
4. Then ask "What would the value of term 10 be if the pattern were extended?" Let them give their answers orally.
5. Tell students to create a graph for their table of values.
6. Have a class discussion on what pattern rule they see, and then ask them to describe it in their journals using everyday language.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Translate a pattern to a table of values and graph the table of values (limit to linear graphs with discrete elements).
- ❒ Describe, using everyday language, orally or in writing, the relationship shown on a graph, as the term value is one less than two times the term number for the above-stated example.

PUTTING THE PIECES TOGETHER

How Many Sticks to Each Bottle Cap?

Purpose: The purpose of this activity is to have students refresh and use their knowledge of number concepts, patterns, and statistics. Specifically, this activity was designed for students to do the following:

- Count objects and record the data
- Compare and order numbers
- Arrange numbers in ascending order
- Add, subtract, multiply, and divide whole numbers
- Determine the rule for the pattern
- Predict larger and smaller values

As well, the following mathematical processes are demonstrated by this activity:

- Communication
- Connections
- Mental mathematics
- Problem solving
- Reasoning

Curricular Links: Science

Materials/Resources:

- several boxes
- a lot of bottle caps and sticks
- paper and pencil

Organization: Small Groups

Inquiry:

Scenario

Arrange desks in such a way that students can work in small groups. Choose your pattern rule. Based on your pattern rule, prepare a box for each group containing some bottle caps and some sticks. Let's say you choose the rule: the number of sticks = twice the number of bottle caps +3, and that you have six groups. Then, one box might contain eight bottle caps and 19 sticks; another box might contain 17 bottle caps and 37 sticks; still another box might contain 24 bottle caps and 51 sticks. You can choose larger or smaller amounts for the other three boxes as long as the pattern rule for each box is the same.

Procedure:

- Have a box with a predetermined number of bottle caps and sticks on a desk for each group.
- Tell students that each group has some treasures in their boxes and they need to figure out the similarities or differences.
- Ask them to discuss which mathematical processes they need to use, and write them down.
- Have each group post their findings on the board at the front of the classroom.
- Tell students to discuss in their groups what they need to do in order to find a pattern rule.
- Tell students to record their processes and rules.
- Have each group post their new work on the board under their previous work.

Assessment:

Check students' replies to determine whether they can do the following:

- ❑ Compare and order numbers
- ❑ Arrange numbers in ascending order
- ❑ Add, subtract, multiply, and divide whole numbers
- ❑ Determine the rule for the pattern
- ❑ Predict larger and smaller values and verify their predictions.

Grade 6: Patterns and Relations (Variables and Equations) (6.PR.3, 6.PR.4)

Enduring Understanding(s):

Preservation of equality is used to solve equations.

Number patterns and relationships can be represented using variables.

General Learning Outcome(s):

Represent algebraic expressions in multiple ways.

Specific Learning Outcome(s):	Achievement Indicators:
6.PR.3 Represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]	➛ Write and explain the formula for finding the perimeter of any rectangle. ➛ Write and explain the formula for finding the area of any rectangle. ➛ Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., $a + b = b + a$ or $a \times b = b \times a$). ➛ Describe the relationship in a table using a mathematical expression. ➛ Represent a pattern rule using a simple mathematical expression, such as $4d$ or $2n + 1$.

(continued)

Specific Learning Outcome(s):	Achievement Indicators:
6.PR.4 Demonstrate and explain the meaning of preservation of equality, concretely, pictorially, and symbolically. [C, CN, PS, R, V]	➛ Model the preservation of equality for addition using concrete materials, such as a balance or using pictorial representations, and orally explain the process. ➛ Model the preservation of equality for subtraction using concrete materials, such as a balance or using pictorial representations, and orally explain the process. ➛ Model the preservation of equality for multiplication using concrete materials, such as a balance or using pictorial representations, and orally explain the process. ➛ Model the preservation of equality for division using concrete materials, such as a balance or using pictorial representations, and orally explain the process. ➛ Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$].

Prior Knowledge

Students may have had experience with the following:

- Using manipulatives and diagrams (0–100) to demonstrate and explain the meaning of equality and inequality
- Solving problems by identifying and explaining mathematical relationships using charts and diagrams
- Solving one-step equations that involve a symbol that represents an unknown number
- Making predictions about subsequent elements based on the pattern rule
- Solving one-step single-variable equations and problems involving these equations using whole numbers only

Related Knowledge

Students should be introduced to the following:

- Using whole numbers to explain and apply the order of operations, excluding exponents
- Solving problems by demonstrating an understanding of the relationships within tables of values

Background Information

In prior grades, students were exposed to translating word problems into equations and then solving them. Those equations consisted of whole number operations, such as $2 + 5 = 7$, $3 = 27 \div 9$, or $7 - 2 = 8 - 3$. Students were solving those equations by using a variety of methods, such as concrete objects, pictorial representations, and an empty square representing the unknown value.

Later, students were introduced to using letters as representations of unknown quantities. This change from using simple number operations to using letter operations is a big leap for students. Instead of having to solve something familiar, such as $28 + \square = 35$, they are looking at $28 + x = 35$. Students need to get comfortable using letters for representing unknown quantities. They need to be reminded that the letter representation means the same thing as the empty box; both can be used to represent an unknown quantity.

Using letters is simply one way to write a mathematical rule.

In a rectangle, the two opposite sides are equal in measure.

If we want to find a general rule that describes the perimeter of each of the above-shown rectangles, we could use words, such as the following:

1. the perimeter of each of these rectangles is equal to its length plus its width plus its length plus its width ($l + w + l + w$)
2. stated more concisely, the perimeter of each of these rectangles is equal to the sum of twice its length plus twice its width ($2l + 2w$)
3. the perimeter or each of these rectangles is equal to twice the sum of its length plus its width $2(l + w)$

We can also describe the perimeter of the above-shown rectangles by a mathematical equation using letters for variables, such as $p = 2l + 2w$, $p = 2w + 2l$, $2l + 2w = p$, $2(l + w) = p$, $p = 2(w + l)$, or $p = 2(l + w)$.

(Note: All of these equations are equivalent forms of the same equation.) The general formula tells us that **the perimeter of a specific rectangle is always equal to twice the sum of the length plus the width of that specific rectangle, no matter how small or how large that rectangle might be. If the dimensions of the rectangle increase, then the perimeter also increases; if the dimensions decrease, then the perimeter decreases as well. The equality is always preserved.**

Note: It might be beneficial to students if you review the commutative property of addition while discussing the perimeter of a rectangle (i.e., $l + w = w + l$).

In a rectangle, each of the four interior angles is a right angle.

If we want to find a general rule that describes the area of each of the above-shown rectangles, we could use words, such as: the area of each of these rectangles is equal to its length times its width or its width times its length. We can also describe the area of the above-shown rectangles by a mathematical equation using letters for variables, such as $a = l \cdot w$, $a = w \cdot l$, $w \cdot l = a$, or $l \cdot w = a$.

Note: All of the equations are equivalent forms of the same equation. The general formula tells us that the area of a specific rectangle is always equal to its length times its width regardless of the size of the rectangle. The equality is always preserved.

Note: It might be beneficial to students if you reviewed the commutative property of multiplication while discussing the area of a rectangle (i.e., $l \cdot w = w \cdot l$).

Mathematical Language

equal

equation

mathematical expression

unknown

variable

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 5–8.2: Concept Description Sheet #1
- pencil
- overhead projector

Organization: Small groups

Procedure:

1. Tell students that the following few lessons will be spent on expanding their knowledge of equations.
2. Ask them to try to remember what they learned in Grade 5 or earlier grades about equations.
3. Place the word "Equation" in the oval middle section on Concept Description Sheet #1 overhead transparency and ask them: "What do you know about equations?"
4. Tell students to write all they can about equations on their concept description sheet.
5. Walk around and observe student responses for each section of the concept description sheet. Are these responses revealing their knowledge as to the following questions: What is an equation? What is not an equation? What are the characteristics of an equation? What is a good picture or a diagram of an equation?
6. After students complete the concept sheet on equations, write on the board an arithmetic equation, such as $18 + 7 = 25$, and an algebraic equation, such as $6y + 4 = 22$.
7. Ask students what is different about these two equations, and have a discussion.
8. Tell students that variables are very important in mathematics. Then ask them what they remember about variables.
9. Write the word "Variable" on the board and tell students to do a concept sheet on variables using the same procedure as they did for the equation.
10. Again, walk around and observe student responses for each section of the concept description sheet. Consider whether these responses reveal their knowledge as to the following questions: What is a variable? What is not a variable? What are the characteristics of a variable? What is a good picture or a diagram of a variable?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Define what is an equation.
- ❒ Identify examples and non-examples of an equation.
- ❒ Define what is a variable.
- ❒ Identify examples and non-examples of a variable.
- ❒ Tell the difference between a simple (arithmetic) equation and an equation with a variable (algebraic equation).

Suggestions for Instruction

- **Write and explain the formula for finding the perimeter of any rectangle.**

Materials:

- large brown envelopes
- white envelopes
- rectangular sheets of paper
- recipe cards
- blank business cards
- rulers

Organization: Individual

Procedure:

1. Tell students that you will hand out five items, and that each item will have a rectangular shape.
2. Hand out a large brown envelope containing a rectangular sheet of paper, a recipe card, a blank business card, and a white envelope. Make sure none of the items are identical in size to any other item.
3. Tell students that each item will need a ruler, and then provide the following instructions:
   a) One item at a time, measure the perimeter and record your measurements on the item.
   b) When all five items are measured and recorded, write your observations in your journal.

c) Write a formula for finding the perimeter of any rectangle.

d) Explain your formula.

4. Have a class discussion on what the students noticed about all these objects. Ask them what was common to all these perimeters, and then have a few (e.g., three or four) students write their formula on the chalkboard and explain their formula to the class.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Tell that the opposite sides of a rectangle are equal in measure.
- ❒ Write the formula for finding the perimeter of any rectangle.
- ❒ Explain the formula for finding the perimeter of any rectangle.

Suggestions for Instruction

- **Write and explain the formula for finding the area of any rectangle.**

Materials:

- BLM 6.PR.7: Mrs. Dean's Carpet

Organization: Groups of two

Procedure:

1. Have students work in groups of two.
2. Distribute to each student a copy of BLM 6.PR.7.
3. Ask students to read the question and discuss it with their partner.
4. Ask students to create a table in their notebooks. (Observe that they write the correct numerals for the words.)
5. Have students discuss with their partner how Mrs. Dean figured out how much carpet she needed of each width.
6. Ask students to write a formula for finding the area of any rectangle, and to explain in their journals how they figured out their formula.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Multiply a double-digit by a single-digit whole number.
- ❒ Multiply a fraction by a single-digit whole number.
- ❒ Write the formula for finding the area of any rectangle.
- ❒ Explain the formula for finding the area of any rectangle.

Suggestions for Instruction

- **Develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., *a* + *b* = *b* + *a* or *a* x *b* = *b* x *a*).**

Materials:

- BLM 6.PR.8: Poff and Gloff's Math Homework

Organization: Individual

Procedure:

1. Distribute a copy of BLM 6.PR.8.
2. Tell students to read the question and do the work individually.
3. Check that students are on the right track.
4. Ask a student to write his or her equation on the chalkboard and explain it.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Write a simple mathematical expression.
- ❒ Develop equations using letter variables that illustrate the commutative property of addition (e.g., $a + b = b + a$).
- ❒ Justify equations using letter variables that illustrate the commutative property of addition (e.g., $a + b = b + a$).

Suggestions for Instruction

- **Develop and justify equations using letter variables that illustrate the commutative property of multiplication (e.g., *a* x *b* = *b* x *a*).**

Materials:

- BLM 6.PR.9: Equation Pairs

Organization: Individual/whole class

Procedure:

1. Distribute a copy of BLM 6.PR.9 to each student.
2. Ask students to write on their sheets the two appropriate equations for each pair of measurements.
3. Ask a student to write on the chalkboard a pair of numbers of his or her choice and the two appropriate multiplication equations.
4. Have a class discussion based on their work. Discuss the commutative property of multiplication.
5. Tell students to write an equation using letter variables, and to explain in their journals why their equation works.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Multiply simple mathematical expressions.
- ❒ Develop equations using letter variables that illustrate the commutative property of multiplication (e.g., $a \times b = b \times a$).
- ❒ Justify equations using letter variables that illustrate the commutative property of multiplication (e.g., $a \times b = b \times a$).

Suggestions for Instruction

- **Describe the relationship in a table using a mathematical expression.**

Materials:

- BLM 6.PR.10: Baskets and Oranges
- a large sheet of chart paper

Organization: Whole class

Procedure:

1. Copy the table from BLM 6.PR.10 on a large sheet of chart paper and post it on the board.
2. Ask students to analyze the table and write their observations in their math notebooks.
3. Ask a student to provide orally his or her observations to the class.
4. Ask the class to use a mathematical expression to describe the relationship in the table "Baskets and Oranges."
5. As a class, discuss the observations students made and the mathematical expressions they used to describe the relationship.
6. Have students analyze another pattern, such as $3n$, $5n$, or $2n + 1$. (Fit the level of difficulty according to your students' needs).

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Multiply two single-digit numbers.
- ❒ Write a simple mathematical expression.
- ❒ Describe the relationship in a table using a mathematical expression.

Suggestions for Instruction

- **Represent a pattern rule using a simple mathematical expression, such as 4*d* or 2*n* + 1.**

Organization: Whole class

Procedure:

Tell students that today's math class will be a little different. Then say the following:

1. "Look at my hands when I wave them and tell me how many fingers you see." (Wave both of your hands in the same direction and slowly, showing all 10 fingers.)
2. "How many fingers do you see waving at you?"
3. "We need one person to come to the front of the class and wave his or her hands with me." (Stand side by side and both of you wave slowly showing all your fingers.)
4. "How many fingers do you see waving now?"
5. "We need one more person to come to the front of the class and wave his or her hands with us." (Stand side-by-side and all three of you wave slowly showing all your fingers.)
6. "How many fingers do you see waving now?"
7. "If we had one more person with us at the front, how many fingers would you see then?"
8. "How many fingers would you see if nine of us were standing and waving?"
9. "Thanks to all of you for participating. And now, write in your journals a simple mathematical expression to describe the pattern rule for our little game, and describe how it works."

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Multiply by 10 orally.
- ❒ Write a simple mathematical expression.
- ❒ Represent a pattern rule using a simple mathematical expression.

Suggestions for Instruction

- **Model the preservation of equality for addition using concrete materials, such as a balance or using pictorial representations, and orally explain the process.**
- **Model the preservation of equality for subtraction using concrete materials, such as a balance or using pictorial representations, and orally explain the process.**

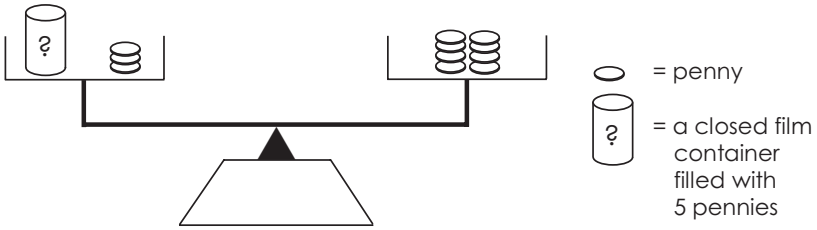
Materials:

- film containers
- pennies
- balance scales

Organization: Small groups

Procedure:

1. Have students sit in small groups (three to five students per group).
2. Distribute the following to each group:
   a) a closed film container containing five pennies
   b) a balance scale
   c) an empty film container identical to the filled container
   d) 11 pennies
3. Tell students to
   a) balance the scale with the empty film container on the left pan, and then remove the container
   b) put the filled film container and three pennies on the left pan of the scale and eight pennies on the right, as shown in the following diagram



4. Tell students to answer the following:
   a) "How many pennies are in the container?"
   b) "If there are eight pennies on one side, how many pennies are on the other side?"
   c) "What will happen if you remove three pennies from each side?" Remove the pennies. Empty the container to verify the number of hidden pennies.

5. Ask students to discuss in their groups what would happen if they added five more pennies to each side, and to make a pictorial representation in their journals.
6. Have students create their own equations using the containers and pennies, and have their partners solve them.

Note: Another way to account for the mass of the container is to omit the empty container in step 3 and to add the open empty container and lid to the right side of the balance.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Balance an equation (addition and subtraction).
- ❒ Model the preservation of equality for addition.
- ❒ Model the preservation of equality for subtraction.

Suggestions for Instruction

- **Model the preservation of equality for multiplication using concrete materials, such as a balance or using pictorial representations, and orally explain the process.**
- **Model the preservation of equality for division using concrete materials, such as a balance or using pictorial representations, and orally explain the process.**

Organization: Whole class

Procedure:

1. Draw two large circles on the chalkboard and put an equals sign between them.
2. Draw six little triangles in each large circle.
3. Have students discuss as one group the meaning of your diagram.
4. Draw two new circles underneath the first two circles and also place an equals sign between the new circles.

5. Ask the students what would happen if you doubled each side. Discuss.
6. Ask one student to place the correct amount of little triangles into each new circle.
7. Tell students to draw two circles in their journals and show what would happen if they divided each side by three.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Multiply simple numbers orally.
- ❒ Model the preservation of equality for multiplication
- ❒ Model the preservation of equality for division.

Suggestions for Instruction

- **Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., 3*b* = 12 is the same as 3*b* + 5 = 12 + 5 or 2*r* = 7 is the same as 3(2*r*) = 3(7)].**

Materials:

- BLM 6.PR.11: Equivalent Forms of an Equation

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 6.PR.11.
2. Have students work in small groups (three to five students).
3. Distribute to each group a small container containing 40 buttons.
4. Tell students to discuss the work with their group members, and then write the equations and verify their work.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Add, subtract, multiply, and divide small numbers (under 100).
- ❒ Write equivalent forms of an equation by applying the preservation of equality.
- ❒ Verify equivalent forms of an equation using concrete materials.

Suggestions for Instruction

- **Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$].**

Materials:

- BLM 6.PR.12: I Have, Who Has…?

Organization: Whole class

Procedure:

1. Tell students that they will be playing an "equivalent forms of an equation" version of the game "I have, who has…?". Cut BLM 6.PR.12 into card-size pieces. Explain that each student will get one card (some students may get two cards if there are fewer than 36 students in the class).
2. Have one student start the game by reading his or her card. (Help them play the game once.)
3. Have the student who has the answer to the question read his or her card.
4. Continue the game in this fashion until it gets back to the person who started the game.
5. Play the game several times, each time asking a different student to start it.
6. Divide the class into four groups.
7. Have each student make his or her equivalent forms of an equation version of the game "I have, who has…?" and play it with the other members of the group.

Extension:

When a student answers, have the student explain what operation was done to both sides to make it equivalent.

For example: Question: Who has $6t = 0$?

Answer: I have $6t + 3 = 3$ because 3 was added to both sides of the equation.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Add, subtract, multiply, and divide small numbers (under 100).
- ❒ Recognize equivalent forms of an equation.
- ❒ Write equivalent forms of an equation by applying the preservation of equality.

Suggestions for Instruction

- **Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., 3*b* = 12 is the same as 3*b* + 5 = 12 + 5 or 2*r* = 7 is the same as 3(2*r*) = 3(7)].**

Materials:

- BLM 6.PR.13: Same As Cards
- BLM 6.PR.14: Same As Reply Sheet A
- BLM 6.PR.15: Same As Reply Sheet B

Organization: Groups of four

Procedure:

1. Have students work in groups of four.
2. Distribute to each group one set of BLM 6.PR.13: Same As Cards (64 cards) and two copies of Same As Reply Sheets A and B.
3. Tell students the following:
   a) Students sitting opposite each other should have the same reply sheets.
   b) The dealer will deal three cards to each student and place the rest of the cards face down in the centre.
   c) The person on the right side of the dealer will start by picking up one card from the top of the pile. If the new card has an equation equivalent to any of the four equations on his or her reply sheet, then he or she will read the two equivalent equations out loud (e.g., $3x + 7 = 3 + 7$ is the same as $3x = 3$) and place them on the reply sheet. If the new card does not match any of the equations on his or her reply sheet, then the student keeps the card with the first three cards.
   d) The next person (opposite the dealer) repeats the same procedure. This process continues until all the cards are picked up.
   e) When there are no more cards in the centre, the next student to pick a card will ask the person to the right to give him or her a card, and the process continues until someone has all eight equivalent forms of one of the equations on his or her reply sheet.
   f) The winner (the student with all eight equivalent forms of one of the equations on his or her reply sheet) will display and read all eight forms of the equation to the group.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Add, subtract, multiply, and divide small numbers (under 100).
- ❒ Recognize equivalent forms of an equation.

Suggestions for Instruction

- **Write equivalent forms of an equation by applying the preservation of equality, and verify using concrete materials [e.g., 3*b* = 12 is the same as 3*b* + 5 = 12 + 5 or 2*r* = 7 is the same as 3(2*r*) = 3(7)].**

Materials:

- BLM 6.PR.13: Same As Cards
- BLM 6.PR.16: Same As Record Sheet

Organization: Small group

Procedure:

1. Have students work in small groups (three to five students).
2. Distribute a set of Same As Cards to each group and a copy of BLM 6.PR.16: Same As Record Sheet to each student.
3. Have one student start by picking a card and reading the equation to the group.
4. Have everyone in the group write on the record sheet next to the simple form an equivalent equation to the one read and using the same operation (e.g., card reads: $8x + 13 = 25$; under the addition column, next to $8x = 12$, the students write an equation such as $8x + 5 = 12 + 5$, $8x + 5 = 17$, $8x + 2 = 12 + 2$, $8x + 2 = 14$, or $8x + 100 = 12 + 100$).
5. Have the next student pick a card, read the equation, and follow in this fashion until the record sheet is filled.
6. Have a class discussion on the preservation of equality.
7. Tell students to write in their journals what they learned from using the Same As Cards.

Observation Checklist

☑ Check students' replies to determine whether they can do the following:

- ❒ Add, subtract, multiply, and divide small numbers.
- ❒ Recognize equivalent forms of an equation.
- ❒ Write equivalent forms of an equation by applying the preservation of equality.