GRADE 6 MATHEMATICS

Number

Grade 6: Number (6.N.1, 6.N.2)

Enduring Understanding(s):

The position of a digit in a number determines its value.

Each place value position is 10 times greater than the place value position to its right.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.1 Demonstrate an understanding of place value for numbers ▪ greater than one million ▪ less than one-thousandth. [C, CN, R, T]	➛ Explain how the pattern of the place value system (e.g., the repetition of ones, tens, and hundreds) makes it possible to read and write numerals for numbers of any magnitude. ➛ Provide examples of where large numbers and small decimals are used (e.g., media, science, medicine, technology).
6.N.2 Solve problems involving large numbers, using technology. [ME, PS, T]	➛ Identify which operation is necessary to solve a problem and solve it. ➛ Determine the reasonableness of an answer. ➛ Estimate the answer and solve a problem. ➛ Identify and correct errors in a solution to a problem that involves large numbers.

Prior Knowledge

Students may have had experience with the following:

- Representing and describing whole numbers to 1 000 000
- Applying estimation strategies in problem-solving situations
- Describing and representing decimals (tenths, hundredths, thousandths)
- Comparing and ordering decimals (tenths, hundredths, thousandths)
- Comparing and ordering numbers to 10 000
- Demonstrating an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals)
- Describing and representing decimals (tenths and hundredths)

- Estimating quantities less than 1000
- Illustrating, concretely and pictorially, the meaning of place value of numerals to 1000

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of integers, concretely, pictorially, and symbolically

Background Information

Students extend their number pattern knowledge to numbers greater than a million and less than one-thousandth by knowing and understanding

- that each place represents ten times as much as the place to its right.
- that each place represents one-tenth as much as the place to its left.
- that places are organized into threes and that this pattern goes beyond millions into billions, trillions, and also into ten-thousandths, hundred-thousandths, etc.
- that when we write numbers, spaces, not commas, are used to show the place value of numbers, with the exception of 4-digit numbers (e.g., 5432).
- that when we read numbers greater than a million, we read 4 567 890 123 as four billion, five hundred sixty-seven million, eight hundred ninety thousand, one hundred twenty-three. The word "and" is used for decimal numbers (e.g., 654.78 is read as six hundred fifty-four **and** seventy-eight hundredths (not six hundred fifty four **point** seventy eight).
- that 0 is an important place holder. The number 30 indicates that there are 3 tens and 0 is in the ones place. In the number 285 027, the 0 indicates that there are no hundreds (or there are 50 hundreds) and in decimal numbers such as 1.09, the zero indicates that there are no tenths.

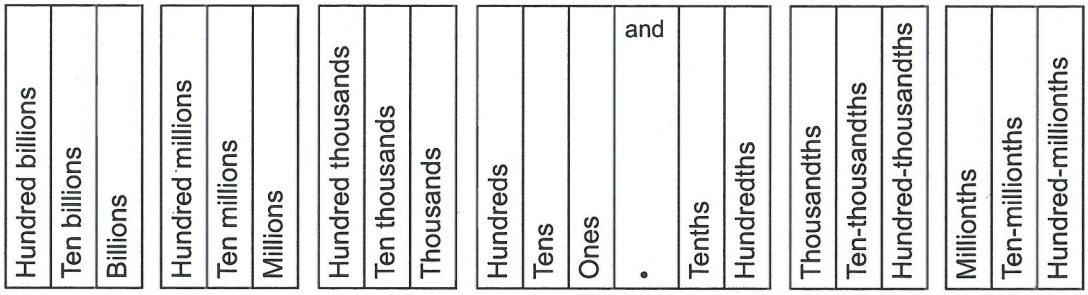
Teachers should have students

- explore and explain how the numbers system pattern works.
- investigate and be able to explain where and when large numbers are used and where or when we need to use the precision of small decimals.
- read numbers using the appropriate vocabulary and place value.

With this knowledge, students will develop flexibility with reading, identifying, and representing numbers, which will support the development of mental math and estimation skills.

Glossary terms can be found at <www.edu.gov.mb.ca/k12/curr/math/glossary_k-8/index.html>.

Base-Ten Number System: The system of numbers we use is called the **base-ten number system**. It is a place-value number system in which 10 digits, 0 through 9, are used to represent a number, and the value of each place is 10 times the value of the place to its right. The value of any digit in the number is the product of that digit and its place value.



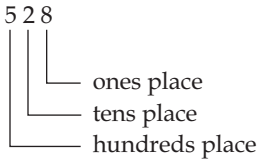
10 ones = 1 ten
10 tens = 1 hundred
10 hundreds = 1 thousand
10 thousands = 1 ten thousand
10 ten thousands = 1 hundred thousand
10 hundred thousands = 1 million

10 tenths = 1 one
10 hundredths = 1 tenth
10 thousandths = 1 hundredth
10 ten-thousandths = 1 thousandth
10 hundred-thousandths = 1 ten-thousandth
10 millionths = 1 hundred-thousandth

Place value is the value of a digit in a number based on its position.

Example

In the number 528, the 5 has a value of 5 hundreds (or 500), the 2 has a value of 2 tens (or 20), and the 8 has a value of 8 ones (or 8).



Numeral is the written symbol that represents a number.

Number is the concept of an amount, quantity, or how many items there are in a collection. Numbers play an important part in our lives. We use numbers in school, at the workplace, and in our daily living.

Decimal is a fractional number written in base-ten form; a mixed decimal number has a whole number part as well (e.g., 0.32 is a decimal number and 3.5 is a mixed decimal number). A period or dot (such as in the previous example) separating the ones place from the tenths place in decimal numbers or dollars from cents in money is called a **decimal point**. When numbers are spoken, the decimal point is read as "and" (e.g., 3.2 is read as three and two-tenths).

Suggested Activities

- Explore number patterns using models such as base-ten blocks, Cuisinaire rods, number lines, thousands grids, and decimal squares.
- Create an "A-B-C" book of real-work numbers (e.g., A is for an ant's antenna measured in small decimal points, C is for the population of Canada—32 000 000).
- Place the place-value game using playing cards with the face cards removed (tens can be zeros). Have students draw nine lines on a piece of paper in place-value format (___ ___ ___ ___ ___ ___ ___ ___ ___). Select one card at a time and ask the students to make the largest number that they can, using the numbers as they are drawn. Once the number is placed, it cannot be moved. **Variation:** Place a decimal in the visual and have students make the smallest number that could be made using the cards.
- Compare the populations of large and small cities and draw conclusions or create problems using these numbers (e.g., world population is 6 767 805 208, the population of China is 1 338 612 968, and the population of Canada is 32 440 970).

Mathematical Language

decimal number

decimal point

place value

Learning Experiences

Assessing Prior Knowledge

Organization: Whole class

Materials: BLM 5–8:17: Number Fan

Procedure:

1. Tell students you want to check what they remember about the decimal system.
2. Have a class discussion. Ask questions such as:
   a) "What are the 10 digits we use for representing all numbers?"
   b) "What makes it possible for us to represent large numbers, using only the 10 digits?"
   c) "What makes it possible for us to represent small numbers that are less than one, using only the 10 digits?"
3. Discuss place value and decimal place.

4. Provide students with a number fan and have them show you
   a) twenty-eight hundredths
   b) nineteen tenths
   c) one thousand nine hundred eight
   d) a number larger than one thousand twenty-three
   e) a number smaller than one-hundredth

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Demonstrate an understanding of place value related to whole numbers.
- ❒ Demonstrate an understanding of place value related to decimal numbers.

Suggestions for Instruction

- **Explain how the pattern of the place value system (e.g., the repetition of ones, tens, and hundreds) makes it possible to read and write numerals for numbers of any magnitude.**
- **Provide examples of where large numbers and small decimals are used (e.g., media, science, medicine, technology).**

Materials:

- pencil and paper

Organization: Whole class/small groups

Procedure:

1. Write a few three-digit numerals on the board, such as the following:
   a) 385
   b) 761
   c) 208
2. Ask a student volunteer to read each numeral.
3. Write a few six-digit numerals on the board, such as the following:
   a) 378 321
   b) 618 026
   c) 320 180

4. Ask a student volunteer to read each numeral.
5. Discuss what students noticed about this second set.
6. Write a few numerals on the board that are greater than one million, such as the following:
   a) 38 851 406
   b) 76 211 318
   c) 208 182 281
7. For each numeral, read the numeral to the students first, and then ask the students to read each numeral.
8. Tell students to discuss the following questions with their group members:
   a) How did you know how to read each numeral?
   b) Did you detect a pattern? Explain.
   c) Where would you find large numbers that are greater than one million?
9. Ask one student volunteer to explain orally the replies of his or her group.
10. Tell each group to
    a) think of a few large numbers and where those numbers could be found or used
    b) write the numerals on one side of their paper, and the examples of their use on the other side
11. Have members of one group go in front of the class and present their numbers.
12. Have the rest of the class read them out loud and provide examples of their use.
13. Have one of the group members write the examples on the board.
14. Repeat with the other groups.
15. At the end, check if the examples on the board covered all the examples provided on the backs of the papers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Read numerals greater than one million.
- ❒ Write numerals greater than one million.
- ❒ Provide examples of where large numbers are used.

Suggestions for Instruction

- **Explain how the pattern of the place value system (e.g., the repetition of ones, tens, and hundreds) makes it possible to read and write numerals for numbers of any magnitude.**
- **Provide examples of where large numbers and small decimals are used (e.g., media, science, medicine, technology).**

Materials:

- pencil and paper
- BLM 6.N.1.1: Small Decimals

Organization: Whole class/pairs

Procedure:

1. Place a transparent copy of BLM 6.N.1.1 on the overhead projector or display it electronically.
2. Read each numeral and have the students repeat each one right after you.
3. Discuss what students noticed about this set of numerals (e.g., decimal place, place value).
4. Discuss where such small decimals are to be used.
5. Tell each student to
   a) think of a few small decimals and where those numbers could be found or used
   b) write the numerals on one side of the paper, and the examples of their use on the other side
6. Have each student read his or her partner's numbers and provide examples of their use.
7. Check whether the examples the student provided cover all the examples provided on the backs of the papers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Read numerals smaller than one-thousandth.
- ❒ Write numerals smaller than one-thousandth.
- ❒ Provide examples of where small decimal numbers are used.

Suggestions for Instruction

- **Identify which operation is necessary to solve a problem and solve it.**

Materials:

- pencil and paper
- BLM 6.N.2.1: Nedy's Bike Ride

Organization: Whole class/individual

Procedure:

1. Place a transparent copy of BLM 6.N.2.1 on the overhead projector or display it electronically.
2. Have the student
   a) read the problem
   b) choose between "Today" and "Yesterday"
   c) identify which operation is necessary to solve the problem
   d) solve the problem
3. Discuss what students noticed about the problem (e.g., two units of measure are used: metres and kilometres).
4. Have each student design a problem.
5. Let students exchange their work with their neighbour.
6. Tell them to
   a) read the problem
   b) identify which operation is necessary to solve the problem
   c) solve the problem
7. Have a class discussion on these student-made problems.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Read the problem.
- ❒ Identify which operation is necessary to solve the problem.
- ❒ Solve the problem.
- ❒ Notice that two units of measure are used: metres and kilometres.
- ❒ Convert units.

Suggestions for Instruction

- **Determine the reasonableness of an answer.**

Materials:

- pencil
- BLM 6.N.2.2: Am I Reasonable?

Organization: Whole class/individual

Procedure:

1. Using large numbers, write on the board one question for each operation (addition, subtraction, multiplication, and division). For example:
   a) 29 000 000 + 63 000 000 =
   b) 48 000 000 – 29 000 000 =
   c) 3 000 000 × 18 =
   d) 86 000 000 ÷ 21 =
2. Discuss what is a reasonable answer for each of the examples. What strategies did students use to determine reasonable answers?
3. Distribute to each student a copy of BLM 6.N.2.2 (or display it for the whole class on an overhead/screen/interactive whiteboard).
4. Tell students to
   a) read each problem carefully
   b) determine if the answer is reasonable
5. Have a class discussion regarding the reasonableness of each problem. What makes an answer reasonable? Do you always need an exact answer? When is it okay to make an estimate?

Observation Checklist

☑ Listen to and observe students' responses to determine whether students can do the following:

- ❒ Read large numerals.
- ❒ Add large numbers.
- ❒ Subtract large numbers.
- ❒ Multiply large numbers.
- ❒ Divide large numbers.
- ❒ Determine the reasonableness of an answer given operations with large numbers.

Suggestions for Instruction

- **Estimate the answer and solve a problem.**

Materials:

- pencil
- calculator
- BLM 6.N.2.3: Estimate and Solve

Organization: Whole class/individual

Procedure:

1. Discuss estimation with the class.
2. Using large numbers, write on the board one question for each operation (addition, subtraction, multiplication, and division). For example:
   a) 38 000 000 + 23 000 500 =
   b) 42 006 000 – 17 895 000 =
   c) 4 000 000 × 19 =
   d) 64 030 000 ÷ 21 =
3. For each of the examples,
   a) have the class estimate the answer
   b) discuss how they estimated the answer
   c) have students use their calculator to solve the problem

4. Distribute to each student a copy of BLM 6.N.2.3 (or display for class).
5. Tell students to
   a) estimate the answer
   b) use their calculator to solve the problem
6. Have a class discussion regarding the closeness of their estimation to the solution of each problem.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Estimate large numerals.
- ❒ Use a calculator to add, subtract, multiply, and divide large numbers.

Suggestions for Instruction

- **Identify and correct errors in a solution to a problem that involves large numbers.**

Materials:

- pencil
- calculator
- BLM 6.N.2.4: Identify and Correct

Organization: Whole class/individual

Procedure:

1. For each operation (addition, subtraction, multiplication, and division), ask a different student to
   a) use large numbers and write one question on the board
   b) write an equation choosing either a correct or an incorrect solution

   Examples:

   a) $47\,000\,000 + 45\,000\,000 \stackrel{?}{=} 92\,000\,000$
   b) $128\,000\,000 - 29\,000\,000 \stackrel{?}{=} 101\,000\,000$
   c) $3\,000\,000 \times 18 \stackrel{?}{=} 54\,000\,000$
   d) $96\,000\,000 \div 32 \stackrel{?}{=} 30\,000\,000$

2. Discuss the correctness of each reply ((a) and (c) are correct, (b) and (d) are not correct).
3. Have the class identify the errors (incorrect subtraction for (b), and incorrect place value for (d)).
4. Correct errors.

   For the above example (b), the correct statement should read:

   128 000 000 – 29 000 000 = 99 000 000

   For the above example (d), the correct statement should read:

   96 000 000 ÷ 32 = 3 000 000
5. Distribute to each student a copy of BLM 6.N.2.4 (or display for class).
6. Tell students to
   a) read each problem carefully
   b) identify the errors
   c) correct the errors
7. Have a class discussion regarding the errors and corrections to each problem.
8. Discuss common computational errors with students.
9. Have students create their own questions with errors and switch with a partner.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify errors in a solution to a problem that involves large numbers.
- ❒ Correct errors in a solution to a problem that involves large numbers.

Grade 6: Number (6.N.3)

Enduring Understanding(s):

Some numbers have only two factors: the number itself, and one.

Some numbers have many factors; they can also be said to be composed of multiples of numbers other than one and itself.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.3 Demonstrate an understanding of factors and multiples by ▪ determining multiples and factors of numbers less than 100 ▪ identifying prime and composite numbers ▪ solving problems involving factors or multiples [PS, R, V]	➛ Identify multiples for a number and explain the strategy used to identify them. ➛ Determine all the whole-number factors of a number using arrays. ➛ Identify the factors for a number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers or factor trees). ➛ Identify common factors and common multiples for 2 or 3 numbers. ➛ Provide an example of a prime number and explain why it is a prime number. ➛ Provide an example of a composite number and explain why it is a composite number. ➛ Sort a set of numbers as prime and composite. ➛ Solve a problem involving factors, multiples, the largest common factor, or the lowest common multiple. ➛ Explain why 0 and 1 are neither prime nor composite.

Prior Knowledge

Students may have had experience with the following:

- Representing and describing whole numbers to 1 000 000
- Determining multiplication facts (to 81) and related division facts
- Applying mental mathematics strategies for multiplication

- Demonstrating an understanding of multiplication to solve problems
- Demonstrating an understanding of division with and without concrete materials, and interpreting remainders to solve problems

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of integers, concretely, pictorially, and symbolically
- Demonstrating an understanding of multiplication and division of decimals

Background Information

Number is the concept of an amount, quantity, or how many items there are in a collection. Numbers play an important part in our lives. We use numbers in school, at the workplace, and in our daily living. The written symbol that represents a number is called a **numeral**.

Numbers can be prime or composite. A **prime number** is a number greater than 1 that has exactly two different factors: 1 and itself (e.g., 3 is a prime number, as its only factors are 1 and 3).

Prime Numbers		Non-Prime Numbers	
Number	**Factors**	**Number**	**Factors**
2	1, 2	6	1, 2, 3, 6
7	1, 7	8	1, 2, 4, 8
11	1, 11	15	1, 3, 5, 15
17	1, 17	25	1, 5, 25

A **composite number** is a whole number that has more than two factors (e.g., 4 is a composite number because it has three factors: 1, 2, and 4).

Each number has multiples and factors.

A **multiple** is the product of a given whole number and any other whole number.

Examples

18 is a multiple of 6 (since $6 \times 3 = 18$).

18 is a multiple of 18 (since $18 \times 1 = 18$).

18 is NOT a multiple of 8.

To find the first few multiples of 3, multiply 3 by 1, 2, 3 to get 3, 6, 9. Multiples can also be found using skip counting.

A whole number that is a multiple of two or more given numbers is called a **common multiple** (e.g., common multiples of 2, 3, and 4 are 12, 24, 36, 48, . . .).

Factors are numbers that are multiplied to get a product (3 and 4 are factors of 12). Factors can be used in the following ways:

a) A number or expression that is multiplied by another to yield a product (e.g., a factor of 24 is 8 because $8 \times 3 = 24$, and a factor of $3n$ is n because $3 \cdot n = 3n$).

b) To express as a product of two or more factors (e.g., if the question is "factor 36," the answer could be 2×18 or $2 \times 3 \times 6$). A whole number that is a factor of two or more given numbers or expressions is called a **common factor** (e.g., 7 is a common factor of 14, $14x$, $14w$, . . . 21, $21x$, $21w$, . . . 28, $28x$, $28w$, . . .).

Factors of a number are never greater than the number. The greatest factor is always the number itself and the least factor is always 1. The second factor is always half of the number or less (unless the number is prime).

A **prime number** has only two factors: 1 and itself (e.g., 29 has only 2 factors, 1 and 29). The concept of prime numbers applies only to whole numbers.

Composite numbers have more than two factors and include all non-prime numbers other than 1 and 0 (e.g., 9 has the factors 1, 3, and 9). The numbers 0 and 1 are neither prime nor composite because 1 has only one factor (itself) and 0 has an infinite number of divisors and cannot be uniquely written as a product of 2 factors.

A **whole number** that is a factor of two or more given numbers or expressions is called a **common factor** (e.g., a common factor for 18 and 21 is 3, because $3 \times 6 = 18$ and $3 \times 7 = 21$).

Mathematical Language

composite number

factor

multiple

prime number

LEARNING EXPERIENCES

Assessing Prior Knowledge

Materials:

- demonstration board/small white boards for each student

Organization: Whole class

Procedure:

1. Tell students they will be learning about composites and primes, but first you want to check what they know about factors and multiples.
2. Tell them they will need to write their answers on the demonstration board and show their responses to the teacher. They can use numbers between 1 and 50 for their responses.
3. Ask the following questions:
   a) Which numbers are multiples of 6?
   b) Which numbers have a factor of 10?
   c) What are the factors of 12?
   d) Which numbers are multiples of 9?
   e) What are the factors of 30?
   f) Which numbers have a factor of 7?
4. Discuss what they know about factors and multiples.
5. Tell them that this time they need to examine a few special numbers, and then write on the board:

   2, 7, 13, 19, 31
6. Ask what students observe about these numbers.
7. Ask them to provide more examples of these numbers without factors.
8. How are they different from numbers such as: 8, 12, 20, 24, or 30?
9. Discuss prime and composite numbers and brainstorm other examples.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Name multiples of a given number between 1 and 50.
- ❒ Name factors of a given number.
- ❒ Relate prior knowledge of operations on whole numbers to reasoning about factors, multiples, primes, and composites.

Suggestions for Instruction

- **Identify multiples for a number and explain the strategy used to identify them.**
- **Provide an example of a prime number and explain why it is a prime number.**
- **Provide an example of a composite number and explain why it is a composite number.**
- **Sort a set of numbers as prime and composite.**
- **Explain why 0 and 1 are neither prime nor composite.**

Materials:

- coloured pencils (9 distinct colours)
- BLM 6.N.3.1: Multiples and Factors, Primes and Composites, or a hundred board

Organization: Pairs or individuals

Procedure:

1. Student directions:
   a) Place a yellow checkmark in the squares that are multiples of 2. Look at the pattern and describe it. (alternating squares, checkerboard, even numbers)
   b) Place a red checkmark in the squares that are multiples of 3. Look at the pattern and describe it. (Some numbers have two checkmarks or factors, some odd and some even numbers)
   c) Place a dark blue checkmark in the squares that are multiples of 4. Look at the pattern and describe it. (Even, some numbers have three factors)
   d) Continue filling in the hundreds chart using
      - purple for multiples of 5
      - green for multiples of 6
      - orange for multiples of 7
      - black for multiples of 8
      - pink for multiples of 9
      - light blue for multiples of 10
2. Have a discussion about multiples and factors. Have students identify what numbers are composite and what numbers are prime. Ask students to explain why 0 and 1 are neither prime nor composite.
3. Students should keep the chart and include a key so that they can check on factors in future activities.

Variation:

As an alternative activity, use BLM 6.N.3.1. Repeat the student directions above, but this time use cube links instead of pencil crayons and build factor towers over each number on the hundred board.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Name multiples of a given number.
- ❒ Identify the factors of a given number.
- ❒ Identify the prime numbers to 100.
- ❒ Explain the strategy used to identify the multiples of a given number.
- ❒ Explain what makes a number prime.
- ❒ Identify prime and composite numbers to 100.
- ❒ Explain why 0 and 1 are neither prime nor composite.

Suggestions for Instruction

- **Determine all the whole-number factors of a number using arrays.**

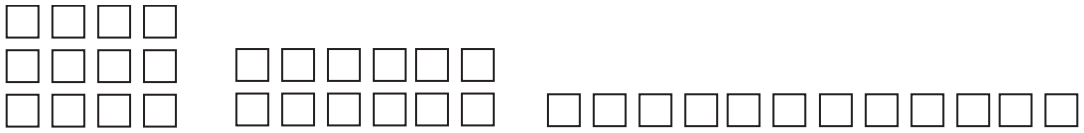
Materials:

- paper and pencil

Organization: Whole class/small group

Procedure:

1. Discuss factors with the class.
2. Ask students what are the whole-number factors of a few numbers of your choice. Choose numbers up to 20 first, and then you can increase numbers to 50.
3. Ask students what they know about arrays.
4. Ask a few students to come to the board and give an example of an array.
5. Tell students to
   a) work in small groups.
   b) use arrays to determine all the whole-number factors of 12 (see example).



3 x 4 2 x 6 1 x 12

6. Repeat this process using 64.

Extension:

1. Work as a class to build a display of all the possible arrays for every number from 1 to 100. They can use BLM 5–8.9: Centimetre Grid Paper to cut out arrays. Some arrays may have to be taped together (for example, a 21 × 1 array would need a whole column of 20 squares plus one more square taped to it). The display can stay on the classroom wall, as it will be useful when investigating area and perimeter.
2. For each array, label the perimeter and area of the rectangle.
3. For each number, label it as prime or composite.
4. Once the display is complete, have students write a reflection in their journal describing any patterns they notice or describing interesting observations they made about the display.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Explain what factors are.
- ❒ Name factors of a given number.
- ❒ Explain what an array is.
- ❒ Use arrays to determine all the whole-number factors of a number.

Suggestions for Instruction

- **Identify the factors for a number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers or factor trees).**

Materials:

- paper
- pencil
- overhead projector/smart board or projector

Organization: Pairs

Procedure:

1. Discuss the different strategies for identifying factors for a number.
2. Discuss what they like and what they dislike about each strategy.
3. Use a smart board, a projector, or an overhead projector to write "36." Show students how to identify factors using arrays, repeated division, number facts, or factor trees.
4. Tell students to
   a) identify the factors of various numbers
   b) explain to a partner the strategy they used to identify the factors of the number
5. Circulate to check if they are identifying the correct factors for the number.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Name different strategies for identifying factors for a number.
- ❒ Identify factors of a given number.
- ❒ Explain the strategy used to identify the factors of a given number.

Suggestions for Instruction

- **Identify common factors and common multiples for 2 or 3 numbers.**

Materials:

- paper
- pencil
- BLM 6.N.3.2: What's Common?

Organization: Small groups

Procedure:

1. Write two numerals on the board (e.g., 12 and 15). Discuss how to find the common factors of these numbers, and demonstrate various strategies.
2. Ask students to identify a common factor for 24 and 32. Have students share the strategy they used.
3. Discuss what students know about common factors for two numbers.
4. Repeat with three numbers (e.g., 64, 24, 32).
5. Place two other numerals on the board (e.g., 8 and 12).
6. Ask students to identify common multiples for the two numbers.
7. Discuss what students know about common multiples for the two numbers.
8. Repeat with three numbers (5, 15, 2).
9. Have students complete BLM 6.N.3.2.
10. Circulate to check if they are identifying the correct common factors and correct common multiples for each set of numbers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify common factors of two or three numbers.
- ❒ Identify common multiples of two or three numbers.

Suggestions for Instruction

- **Provide an example of a prime number and explain why it is a prime number.**
- **Provide an example of a composite number and explain why it is a composite number.**
- **Sort a set of numbers as prime and composite.**
- **Solve a problem involving factors, multiples, the largest common factor, or the lowest common multiple.**
- **Explain why 0 and 1 are neither prime nor composite.**

Materials:

- paper
- pencil
- BLM 6.N.3.3: Dilly's Dilemma

Organization: Pairs

Procedure:

1. Tell students they will work in pairs to solve a problem.
2. Ask students to tell you what they remember about the largest common factor.
3. Have students provide an example of the largest common factor of two numbers (e.g., 14 and 21).
4. Ask students the following:
   a) Are the factors always prime numbers?
   b) What is a prime number?
   c) Are 0 and 1 prime or composite? Why?
5. Ask students to tell you what they remember about the lowest common multiple.
6. Have students provide an example of the lowest common multiple of two numbers (e.g., 14 and 21).

7. Distribute to each student a copy of BLM 6.N.3.3, or display it on an interactive whiteboard.
8. Ask students to complete the following:
   a) Find the largest common factor for the three numbers.
   b) Find the lowest common multiple for the three numbers.
   c) Should Dilly give Bobby the largest common factor for the three numbers? Why?
   d) Should Dilly give Johnny the lowest common multiple for the three numbers? Why?
   e) How much money would Dilly have if she gave both twins what they asked for?
9. Have students share their answers and justify their solutions.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Explain what is a prime number.
- ❒ Explain what is the largest common factor.
- ❒ Provide an example of the largest common factor.
- ❒ Explain what is the lowest common multiple.
- ❒ Provide an example of the lowest common multiple.
- ❒ Explain why 0 and 1 are neither prime nor composite.
- ❒ Solve a problem involving the largest common factor.
- ❒ Solve a problem involving the lowest common multiple.

Factors and Multiples

Purpose: The purpose of this activity is for students to apply their knowledge of factors and multiples. Students will need some prior knowledge, such as how to multiply and divide numbers. The processes that are demonstrated by this task are communication, connections, problem solving, and reasoning.

Curricular Links:

Materials/Resources:

- paper and pencil
- poster-sized paper
- BLM 6.N.3.4: The Ten of Us

Organization:

Small groups

Inquiry:

Scenario

Students will be working in small groups. They will examine some numbers of their choice in order to find five pairs of numbers that fit the described criteria.

Procedure

Hand out to each group a copy of the following:

1. Instruction sheet
2. BLM 6.N.3.4: The Ten of Us

1. Students read BLM 6.N.3.4 carefully.
2. Students discuss with their group members how they will go about finding the five pairs of numbers as described in BLM 6.N.3.4, where
   - the second number is two times the first number
   - the second number has twice as many factors as the first one
3. Students find five pairs of numbers as described in BLM 6.N.3.4, where
   - the second number is two times the first number
   - the second number has twice as many factors as the first one
4. Students state which numbers are prime and which numbers are composite.
5. In their journals, students describe
   - how they found the five pairs of numbers
   - how they figured out which numbers are prime and which numbers are composite

6. On a poster-sized paper, students record the five pairs of numbers and their factors by making a chart similar to the one below:

First Number	A =	B =	C =	D =	E =
Factors of First Number					
Second Number	2 times A =	2 times B =	2 times C =	2 times D =	2 times E =
Factors of Second Number					

7. Then, students show on the (same or different) poster-sized paper the mathematical procedures they used to find the factors.

   Web link: www.ixl.com/math/grade/six/

Assessment:

Use the following observation checklist to assess student learning:

The student can do the following:	Yes	No	Comment
Solve a problem involving factors and multiples.			
Identify a prime number.			
Identify a composite number.			
Multiply numbers.			
Divide numbers.			
Factor numbers.			
Effectively communicate with peers.			
Cooperate with peers.			

Extension:

Taking it further

Students use their calculators this time to repeat the task using very large numbers.

Extension Activities:

Have students

- express 36 as the product of two factors in as many ways as possible
- find the number less than 50 (or 100) that has the most factors
- show all the factors of 48 by drawing or colouring arrays on square grid paper
- solve problems involving factors and multiples (e.g., Ms. Sherry has 34 students in her class. How many different-sized groups of students can she make so that all groups are the same size?—1, 2, 3, 4, 6, 8, 12, 24)
- use a computer or a calculator to help students determine the prime numbers up to 100
- list all the factors of 8 and the first ten multiples of 8

Ask students

- to explain, without dividing, why 2 cannot be a factor of 47
- to identify a number with five factors
- to find three pairs of prime numbers that differ by two (e.g., 5 and 7)
- to explain why it is easy to know that certain large numbers are not prime, even without factoring them (e.g., 4 283 495)
- to explain why 2 and 3 are consecutive prime numbers but why can there be no other examples of consecutive prime numbers

Grade 6: Number (6.N.4)

Enduring Understanding(s):

Improper fractions and mixed numbers are two ways of writing fractions greater than 1.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.4 Relate improper fractions to mixed numbers. [CN, ME, R, V]	➛ Demonstrate using models that an improper fraction represents a number greater than 1. ➛ Express improper fractions as mixed numbers. ➛ Express mixed numbers as improper fractions. ➛ Place a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain strategies used to determine position.

Prior Knowledge

Students may have had experience with the following:

- Determining multiplication facts (to 81) and related division facts
- Demonstrating an understanding of division (3-digit numerals by 1-digit numerals) with and without concrete materials, and interpreting remainders to solve problems
- Demonstrating an understanding of fractions

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of common factors and multiples
- Demonstrating and explaining the meaning of preservation of equality

Background Information

This section of the Grade 6 Number Strand deals with improper fractions and mixed numbers. Before students explore improper fractions and mixed numbers, they need to review the concept of fractions.

A **fraction** is a number that represents part of a whole, part of a set, or a quotient in the form $\left(\frac{a}{b}\right)$, which can be read as *a* divided by *b*.

A fraction whose numerator is smaller than its denominator is referred to as a proper fraction (e.g., $\frac{3}{5}$).

An **improper fraction** is a fraction whose numerator is greater than its denominator (i.e., a fraction with a value greater than 1).

A fraction (proper and improper) can be represented as

- a fraction of a region or whole
- a measurement, such as a name for a point on a number line
- parts of a group or set

Students need to recognize fractions as less than 1 (proper) and as greater than 1 (improper). This recognition is of greater importance than the terminology.

Improper fractions can be expressed as mixed numbers.

A **mixed number** is a number larger than 1 composed of a whole number and a proper fraction (e.g., $1\frac{1}{2}$).

Mathematical Language

denominator

improper fraction

mixed number

numerator

LEARNING EXPERIENCES

Assessing Prior Knowledge

Materials:

- paper and pencil

Organization: Whole class/individual

Procedure:

1. Tell students during the next few classes they will be learning about improper fractions and mixed numbers, but first you want to check what they remember about equivalent fractions.
2. Write down the following fractions:

   $\frac{3}{5}$ $\frac{12}{20}$ $\frac{18}{25}$ $\frac{27}{45}$

   a) State whether the fractions are equivalent.

   b) Give your reasons why you believe that they are or they are not equivalent fractions.
3. Discuss the student responses.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize equivalent fractions.
- ❒ Explain what makes equivalent fractions.

Suggestions for Instruction

- **Place a set of fractions, including mixed numbers and improper fractions, on a horizontal or vertical number line, and explain the strategies to determine position.**

Materials:

- Fraction Cards $\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{3}{8}, \frac{5}{8}, \frac{3}{16}, \frac{7}{16}, \frac{15}{16}, \frac{9}{16}, \frac{15}{24}, \frac{12}{32}, \frac{1}{1}, \text{and } \frac{2}{1}\right)$
  BLM 6.N.4.1: Fractions (Fraction cards can be printed on card stock paper. Set D [Fractions] and Set I [Mixed Fractions] can be downloaded from the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/my_games/index.html>.)
- Clothespins

Organization: Whole class/individual

Procedure:

1. Draw on the board a large horizontal number line from 0 to 2 or hang a clothesline across the front of the class.
2. Have students identify a few benchmarks by taping the cards on the board at $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \text{and } 2,$ or have students clothespin the numbers on the clothesline at these points.
3. Tell students to draw in their notebooks a number line just like the one on the board, making sure they also identify the benchmarks.
4. Then, hand out a few fraction cards and have students place them on the board or clothesline in order, such as :

   $\frac{7}{8}\ \frac{3}{8}\ \frac{5}{8}\ \frac{3}{16}\ \frac{7}{16}\ \frac{15}{16}\ \frac{9}{16}\ \frac{15}{24}\ \frac{12}{32}$

   a) Have students rationalize their choice of position and check for accuracy.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use the number line to order fractions.
- ❒ Order fractions with like and unlike denominators.
- ❒ Explain the strategy used to determine the order.
- ❒ Relate improper fractions to mixed numbers.
- ❒ Place fractions, mixed numbers, and improper fractions on a number line.

Suggestions for Instruction

- **Demonstrate using models that an improper fraction represents a number greater than 1.**

Materials:

- scissors
- envelopes
- small cards
- BLM 6.N.4.2 Fraction Circles or disk sectors (downloadable from the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/my_games/110_disc_sectors.pdf.>

Organization: Five groups

Procedure:

1. Before the students enter, arrange the desks so you have five groups.
2. Place one copy of a different **Fraction Circles** sheet for each group, an envelope identifying the fraction, and a card inside the envelope stating five improper fractions appropriate for the particular fraction circles. For example:
   - Group A will have a copy of Fraction Circles $\left(\frac{1}{2}\text{s}\right)$, an envelope with the word "halves" written on it, and a card inside the envelope stating the following fractions:

     $\frac{3}{2}$ $\frac{5}{2}$ $\frac{7}{2}$ $\frac{9}{2}$ $\frac{11}{2}$

   - Group B will have a copy of Fraction Circles $\left(\frac{1}{3}\text{s}\right)$, an envelope with the word "thirds" written on it, and a card inside the envelope stating the following fractions:

     $\frac{4}{3}$ $\frac{5}{3}$ $\frac{10}{3}$ $\frac{11}{3}$ $\frac{14}{3}$

   - Group C will have a copy of Fraction Circles $\left(\frac{1}{4}\text{s}\right)$, an envelope with the word "quarters" written on it, and a card inside the envelope stating the following fractions:

     $\frac{7}{4}$ $\frac{9}{4}$ $\frac{11}{4}$ $\frac{15}{4}$ $\frac{19}{4}$

   - Group D will have a copy of Fraction Circles $\left(\frac{1}{5}\text{s}\right)$, an envelope with the word "fifths" written on it, and a card inside the envelope stating the following fractions:

     $\frac{6}{5}$ $\frac{9}{5}$ $\frac{12}{5}$ $\frac{19}{5}$ $\frac{21}{5}$

- Group E will have a copy of Fraction Circles $\left(\frac{1}{6}\text{s}\right)$, an envelope with the word "sixths" written on it, and a card inside the envelope stating the following fractions:

  $\frac{8}{6}$ $\frac{13}{6}$ $\frac{17}{6}$ $\frac{20}{6}$ $\frac{25}{6}$

3. Give each group a BLM sheet with fraction circles and an envelope with a card inside.
4. Take the sheet with the fraction circles and cut out each circle carefully. Then cut each circle into its marked fractional parts. Place the fractional parts into your marked envelope.
5. Distribute to each group a copy of the blank circles from BLM 6.N.4.2.
6. Tell each group to do the following:
   a) Take the card from the envelope.
   b) Use the fractional parts and the sheet with the circles to produce the stated improper fractions.
   c) Take turns making each improper fraction marked on the envelope.
   d) When you have completed all the stated improper fractions, exchange with another group your envelope of fractional parts and card.
   e) Repeat the process with the new set of fractional parts and card.
   f) Each group should reproduce the improper fractions stated on all five cards.
7. Check that each group
   - understands the task
   - uses all five envelopes
8. Discuss what students observed about improper fractions.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

❒ Demonstrate using models that an improper fraction represents a number greater than 1.

Suggestions for Instruction

- **Express improper fractions as mixed numbers.**

Materials:

- pencil
- scissors
- BLM 6.N.4.2: Fraction Circles
- BLM 6.N.4.3: Improper Fractions and Mixed Numbers

Organization: Pairs/individual

Procedure:

1. Distribute to each student a copy of the blank circles and the $\frac{1}{5}$s from BLM 6.N.4.2 and BLM 6.N.4.3.

2 Have students do the following:

   a) Take the sheet with the fifths

   b) Cut out each circle carefully.

   c) Cut each circle into its marked fractional parts.

3 When all the cutting is completed, have students do the following:

   a) Complete the worksheet in BLM 6.N.4.3.

   b) Use the fractional parts and the sheet containing blank circles from BLM 6.N.4.2 to help you complete each task as marked.

   c) Discuss with your partner your work on how to record mixed numbers.

   d) In the last four blank rows, have students share two improper fractions with a partner using fifths.

   e) Write both partners' improper fractions on the worksheet.

   f) Again, use the fractional parts and the sheet containing blank circles from BLM 6.N.4.2 to help you make a pictorial representation, and write an appropriate mixed number.

4 Circulate to check that students have the correct mixed number for each improper fraction.

5 Have students record in their journals what they learned about improper fractions and mixed numbers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use concrete objects to represent improper fractions.
- ❒ Make pictorial representations of improper fractions.
- ❒ Provide a correct mixed number for an improper fraction.

Suggestions for Instruction

- **Express mixed numbers as improper fractions.**

Materials:

- pencil
- BLM 6.N.4.4: State My Fraction

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 6.N.4.4.
2. Student directions:
   a) One member of your group will state a mixed number.
   b) Everyone will record it.
   c) Discuss with your group members what kind of pictorial representation you can have.
   d) Draw the pictorial representation.
   e) State the improper fraction.
   f) Have another member of your group state a mixed number.
   g) Repeat the process.
   h) Take turns until the entire sheet is filled.
3. Circulate to check that students understand the task and are stating the correct improper fraction for each mixed number.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ State a mixed number.
- ❒ Give a pictorial representation of a mixed number.
- ❒ Express mixed numbers as improper fractions.

Suggestions for Instruction

- **Express mixed numbers as improper fractions.**

Materials:

- pencil
- BLM 6.N.4.6: Vertical Number Line

Organization: Groups of three

Procedure:

1. Distribute to each student a copy of BLM 6.N.4.6.
2. Tell students they will play a game called "Proper or Improper," and state the rules of the game:
   a) Name some proper and some improper fractions.
   b) Make sure all the fractions fall between 0 and 8.
   c) Make sure all the fractions are thirds or sixths.
   d) Use a sheet of paper for keeping a record of correct scores.
   e) Roll a die to determine who starts.
   f) The one who rolls the smallest number will be the first caller and will start the game.
   g) The caller will state a fraction and mark a dot on his/her vertical number line, making sure the group partners do not see where the dot is placed.
   h) The student on the left of the caller will
      - place a dot on the appropriate spot on his or her vertical number line to mark the fraction
      - say whether the fraction is proper or improper
      - explain to the group the strategy he or she used to determine the position of the fraction

i) The caller and the student on the left of the caller will compare the position of their dots.

j) The student on the left of the caller will receive one point if the dots match or no points if the dots do not match.

k) Repeat the process.

l) The student on the left of the first caller will be the new caller. No one loses a turn.

m) Play 12 rounds.

n) The student with the highest score is the winner.

Variation: Use BLM 6.N.4.5 and play the game again but use only quarters and halves.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ State a fraction.
- ❒ Recognize whether a fraction is proper or improper.
- ❒ Place a set of fractions, including mixed numbers and improper fractions, on a vertical number line, and explain strategies used to determine position.

Grade 6: Number (6.N.5)

Enduring Understanding(s):

Ratios are not numbers; rather, they are comparisons of numbers or like items.

Percents, fractions, decimals, and ratios are different representations of the same quantity.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.5 Demonstrate an understanding of ratio, concretely, pictorially, and symbolically. [C, CN, PS, R, V]	➛ Provide a concrete or pictorial representation for a ratio. ➛ Write a ratio from a concrete or pictorial representation. ➛ Express a ratio in multiple forms, such as 3:5, $\frac{3}{5}$, or 3 to 5. ➛ Identify and describe ratios from real-life contexts and record them symbolically. ➛ Explain the part/whole and part/part ratios of a set (e.g., for a group of 3 girls and 5 boys, explain the ratios 3:5, 3:8, and 5:8). ➛ Solve a problem involving ratio.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of division with and without concrete materials, and interpreting remainders to solve problems
- Demonstrating an understanding of fractions
- Demonstrating an understanding of decimals
- Relating decimals to fractions
- Comparing and ordering decimals

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of common factors and multiples
- Demonstrating an understanding of multiplication and division of decimals
- Demonstrating and explaining the meaning of preservation of equality

Background Information

This section of the Grade 6 Number Strand deals with ratios. Prior to teaching ratios, it would help students if they had a quick review of fractions. Although a ratio is not a fraction, it may be written in the form of a fraction.

Students may find ratios difficult to understand; therefore, it is imperative that students understand the similarities and differences between fractions and ratios.

A **fraction** is a number that represents part of a whole, part of a set, or a quotient in the form $\frac{a}{b}$, which can be read as *a* divided by *b*.

Ratio is a comparison of two numbers or two like quantities by division (e.g., the ratio of girls to boys is three to five $\left(\frac{3}{5}\right)$, 3 to 5, or 3:5).

Some ratios are comparisons of one part of a whole to another part of a whole. This is sometimes called a part-to-part ratio. For example:

- To make this recipe, you need 2 kg of white flour to every 3 kg of whole wheat flour.

In this example, the amount of white flour is compared to the whole wheat flour, and the ratio is 2:3.

- You buy 12 doughnuts: 5 chocolate and 7 glazed.

The ratio of chocolate to glazed is 5:7. But you could also compare the number of chocolate doughnuts to the total number of doughnuts. This ratio is 5:12, and is sometimes called a part-to-whole ratio.

When connecting fractions and ratios, it is important to maintain the meaning of the numerator and denominator. Students are likely more familiar with the fraction, meaning where the numerator is part of a whole (or a set). For example, $\frac{5}{12}$ of the doughnuts are chocolate. Thinking of the numerator as part of a part may be more difficult and confusing for students. For example, there are $\frac{7}{5}$ as many glazed doughnuts as chocolate.

Mathematical Language

part/part
part/whole
percent
ratio

Learning Experiences

Assessing Prior Knowledge

Materials:

- BLM 5–8.12: Fraction Bars (found in the complete package of *Grades 5 to 8 Blackline Masters*)

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 5-8.12, and tell them to
   a) discuss with their group members which fraction needs to be written in each space, and fill in each bar appropriately
   b) select one fraction and use it in a sentence that describes an everyday occurrence
2. Circulate to check that they are
   a) understanding the work
   b) recording the fractions correctly
3. Discuss fractions with the class.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand the meaning of fractions.
- ❒ Record fractions correctly.
- ❒ Use a fraction to describe a real-world situation.

Suggestions for Instruction

- **Provide a concrete or pictorial representation for a ratio.**
- **Solve a problem involving ratio.**

Materials:

- pencil and paper
- candies or small blocks

Organization: Pairs/whole class

Procedure:

1. Present students with the following scenario:

   It is Halloween, and your little brother was too sick to go out trick or treating. You decided that you would collect candies, and for every four that you kept, you would give three to your little brother.

2. Ask students
   a) to model this scenario concretely
   b) to predict the number of candies that you and your little brother will each receive
   c) who will receive more candies, and how you know this
3. As a class, discuss students' answers to the questions and derive a meaning for ratio.

Suggestions for Instruction

- **Provide a concrete or pictorial representation for a ratio.**

Materials:

- Cube A Links

Organization: Whole class/pairs

Procedure:

1. Prepare 10 cubes linked together in this formation: G (green) Y (yellow)

   G G G G Y G G G G Y
2. Ask students what colours are described by the following ratios:
   a) 2:8
   b) 4:1
   c) 8:10

3. Hand out a 10-by-10 grid. Colour the grid using the pattern above. How will the ratios used above change?
4. Have students create their own 10-cube link using two colours. Using the model above, create three ratios reflected in your pattern, and ask a partner to identify the colours.
5. What will the same ratios be in a 10-by-10 grid using the new pattern?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Produce a concrete representation by demonstrate a ratio.
- ❒ Produce a pictorial representation for a ratio.

Suggestions for Instruction

- **Identify and describe ratios from real-life contexts and record them symbolically.**
- **Write a ratio from a concrete or pictorial representation for a ratio.**

Materials:

- pencil and paper
- 10 blocks or cubes (two different colours)
- brown paper bag

Organization: Pairs

Procedure:

Using the coloured blocks, each partner will have a chance to guess the ratio of coloured cubes in a paper bag.

Student Directions:

1. Have partner "A" come up to the front and select 10 blocks in any combination of two colours (e.g., four pink and six black, two white, and eight blue). Place the cubes secretively in the paper bag.
2. Partner "B" will draw one cube at a time from the paper bag, and will record the colour of the cube drawn out of the bag, replacing the cube each time. Partner "B" will do this 10 times. After 10 pulls, partner "B" will guess the ratio of coloured cubes by writing down the ratio of cubes he or she believes is in the bag.

3. See how close your ratio is to the real ratio of coloured cubes. Reverse roles and repeat.

Note: This activity can also be used for Outcome 6.SP.4: Demonstrate an understanding of probability.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Represent a ratio using concrete material.

Suggestions for Instruction

- **Express a ratio in multiple forms, such as 3:5, $\frac{3}{5}$, or 3 to 5.**

Materials:

- pencil
- BLM 6.N.5.1: Uncle Farley's Farm Animals
- BLM 6.N.5.2: Ratio Map for Uncle Farley's Farm Animals

Organization: Pairs

Procedure:

1. Distribute to each student BLM 6.N.5.1 and BLM 6.N.5.2.
2. Tell students to use the information in BLM 6.N.5.1 to make ratios using BLM 6.N.5.2.
3. Tell students to work in pairs to fill in BLM 6.N.5.2.
4. Circulate to check that students record the correct ratios.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Compare two items or numbers.
- ❒ Provide three different forms of representing a ratio.
- ❒ Understand the various forms that represent ratios.

Suggestions for Instruction

- **Identify and describe ratios from real-life contexts and record them symbolically.**

Materials:

- pencil and paper
- Internet
- magazines

Organization: Small groups (3 to 4 students)

Procedure:

1. Have students seated in groups of three or four.
2. Tell students that you want each group to
   a) identify 10 ratios from real-life contexts
   b) describe the ratios they identified from real-life contexts
   c) record the ratios symbolically
3. Circulate to check that students are identifying real-life ratios, describing them, and recording them correctly.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify ratios from real-life contexts.
- ❒ Describe the ratios they identified from real-life contexts.
- ❒ Record the ratios symbolically.

Suggestions for Instruction

- **Explain the part/whole and part/part ratios of a set (e.g., for a group of 3 girls and 5 boys, explain the ratios 3:5, 3:8, and 5:8).**

Materials:

- pencil

Organization: Whole class/small groups

Procedure:

1. Discuss part/whole and part/part ratios of a set. Ask students the following questions:
   a) How many girls are in this class?
   b) How many boys are in this class?
   c) How many students are in this class?
   d) What is the ratio of girls to boys?
   e) What is the ratio of boys to girls?
   f) What is the ratio of girls to the total number of students?
   g) What is the ratio of boys to the total number of students?
   h) What do you notice about the first two ratios?
   i) What do you notice about the last two ratios?
2. Distribute to each student a copy of BLM 6.N.5.1.
3. Tell students to write in their journals a few part/whole and part/part ratios of a set.
4. Circulate to check that students are providing correct examples of part/whole and part/part ratio.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Provide examples of part/whole and part/part ratios of a set.
- ❒ Explain the part/whole and part/part ratios of a set.

Suggestions for Instruction

- **Solve a problem involving ratio.**

Materials:

- pencil
- BLM 6.N.5.3: Ratio Problems

Organization: Individual/small groups

Procedure:

1. Distribute to each student a copy of BLM 6.N.5.3.
2. Tell students the following:
   a) Work individually to complete each problem.
   b) Compare your responses to the responses of the other group members.
   c) Discuss your work, explaining what you did to solve each problem.
   d) Write in your journal what you learned about solving problems involving ratio.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve problems involving ratio.
- ❒ Use the correct ratio to solve a problem.
- ❒ Explain their solution.

Suggestions for Instruction

- **Solve a problem involving ratio.**

Materials:

- pencil and paper
- BLM 6.N.5.4: Uncle Bert's Ratio Riddle

Organization: Individual

Procedure:

1. Place on an overhead projector a copy of BLM 6.N.5.4.
2. Say the following to the students:
   a) Danny wants to find out how far is the nearest store. Can you help him?
   b) Explain how you are going to solve the problem.
   c) Write down your solution.
   d) Explain why you chose to solve it that way.
3. Circulate to check that students are using the correct ratios.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Explain how they are going to solve a problem.
- ❒ Use the correct ratio to solve a problem.
- ❒ Write down the correct solution.
- ❒ Explain why they chose a certain way to solve a problem involving ratio.

Grade 6: Number (6.N.6)

Enduring Understanding(s):

Percents can be thought of as a ratio comparing to 100 or as a fraction out of 100.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.6 Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically. [C, CN, PS, R, V]	➛ Explain that "percent" means "out of 100." ➛ Explain that percent is the ratio of a certain number of units to 100 units. ➛ Use concrete materials and pictorial representations to illustrate a percent. ➛ Record the percent displayed in a concrete or pictorial representation. ➛ Express a percent as a fraction and a decimal. ➛ Identify and describe percents from real-life contexts and record them symbolically. ➛ Solve a problem involving percents.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of division with and without concrete materials, and interpreting remainders to solve problems
- Demonstrating an understanding of fractions
- Demonstrating an understanding of decimals
- Relating decimals to fractions
- Demonstrating an understanding of division

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of ratio

Background Information

Percent (%) is a number expressed in relation to 100, represented by the symbol % (e.g., 40 parts out of 100 is 40%).

Percent can be expressed as a ratio, fraction, or decimal. The above example, 40%, when expressed as a ratio, is 40:100; when expressed as a fraction, it is $\frac{40}{100}$, and when expressed as a decimal, it is 0.40 (or 0.4).

The tasks are designed to: (a) develop students' understanding of percents, and (b) have students relate percents to real-life contexts.

Mathematical Language

decimal

fraction

percent

ratio

LEARNING EXPERIENCES

Assessing Prior Knowledge

Materials:

- BLM 6.N.6.1: What Is My Equal?

Organization: Pairs

Procedure:

1. Tell students you want to check what they remember about fractions and decimals.
2. Have students complete BLM 6.N.6.1.
3. Have students discuss their results with their partner.
4. Circulate among the students to check that they know how to convert decimals to fractions and fractions to decimals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Convert decimals (hundredths) to fractions.
- ❒ Convert fractions $\left(\frac{x}{100}\right)$ to decimals.

Suggestions for Instruction

- **Explain that "percent" means "out of 100."**

Materials:

- pencil
- notebook
- BLM 6.N.6.2: Gizzy Saw These Birds

Organization: Small groups

Procedure:

1. Display BLM 6.N.6.2 on an overhead projector or interactive whiteboard.
2. Have students read the information on the BLM and discuss with their group members what Gizzy means when she says that
   a) 25% of the birds she saw were yellow
   b) 42% were black
   c) 10% were blue
   d) 15% were white
   e) 8% were red
3. Circulate among the students to check that they understand the meaning of percent.
4. Tell students to explain in their notebooks that "percent" means "out of 100."

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Relate examples stated in % form and numbers out of 100.
- ❒ Explain orally and in writing that "percent" means "out of 100."

Suggestions for Instruction

- **Explain that percent is the ratio of a certain number of units to 100 units.**

Organization: Whole class

Procedure:

1. Discuss ratios and percents.
2. Have students do the following:
   a) Provide examples of percents in real-life contexts.
   b) Write on the board the examples as provided.
   c) Write the example as a ratio of a certain number to 100 units.
3. Discuss what students noticed about the percent written as a ratio of a certain number to 100 units.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Provide an example of ratio.
- ❒ Provide an example of percent.
- ❒ Explain that percent is the ratio of a certain number of units to 100 units.

Suggestions for Instruction

- **Explain that percent is the ratio of a certain number of units to 100 units.**

Materials:

- scissors
- BLM 6.N.6.3: My Ratio Is... Who Has?

Organization: Two groups

Procedure:

1. Distribute to each group a copy of the two sheets of BLM 6.N.6.3.
   a) Students cut on the lines to create a set of 24 cards.
   b) Students then shuffle the cards and place them upside down on the desk.
   c) The shortest person in the group will be the dealer.
   d) The dealer distributes all the cards, one at a time, to each group member. Depending on the size of the group, some group members may have more cards than others.
   e) The person who was dealt the first card will start the game by reading his or her card (one of the cards) to the group.
   f) The person who has the match will read his or her match to the first question, and then read the next question stated on his or her card.
   g) The person who has the match to the second question will state that match and continue the game.
   h) The game is finished when all the cards are read. If done correctly, each card will be read once and the answer to the last card will be the statement on the first card.

2. Circulate to check that students are calling out the correct replies.
3. Discuss what students observed about percents and the ratio of a certain number of units to 100 units.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Match correctly a ratio and a percent.
- ❒ Explain that percent is the ratio of a certain number of units to 100 units.

Suggestions for Instruction

- **Use concrete materials and pictorial representations to illustrate a percent.**
- **Explain that percent is a ratio of a certain number of units to 100 units.**

Materials:

- pencil crayons
- BLM 6.N.6.4: 100-Square Grid Paper (or use a geoboard with different coloured elastics)

Organization: Individual

Procedure:

1. Distribute to each student a copy of BLM 6.N.6.4.
2. Tell students to colour
   a) 45% of the grid yellow
   b) 30% of the grid red
   c) 10% of the grid blue
   d) 15% of the grid black
3. Ask students what the ratio is for each coloured section, and include this in a legend or code linking colours of the grid to % and ratios.
4. Ask students this extension question: Shannon colours a shape on a 10-by-10 grid and says the ratio of her coloured part to the whole square is 3:5. On a blank 10-by-10 grid paper square, record three shapes that could represent this ratio. What percent is this shape of the entire grid?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use grid paper to represent percent.
- ❒ Illustrate percent correctly.
- ❒ Link ratios to percent.

Suggestions for Instruction

- **Use concrete materials and pictorial representations to illustrate a percent.**
- **Explain that percent is a ratio of a certain number of units to 100 units.**

Materials:

- rice (white and brown)

Organization: Pairs

Procedure:

1. Tell students:
   a) Bring 100 grains of rice to school. Make sure 30% of the grains are white and 70% are brown.
   b) If each grain of rice corresponds to one day, how many days are represented by the 100 grains of rice?
   c) If each white grain represents snowy days, how many snowy days are represented?
   d) If each brown grain of rice represents slushy days, how many slushy days are represented?
   e) If 50% of the days were windy, use your grains of rice to show one possibility.
   f) Describe your solution in your notebook.
   g) Are there other possibilities? Explain to your partner.
2. Circulate among the students to check that they understand the process.
3. Discuss what students observed about a percent. Extension: Read *The King's Chessboard* by David Birch.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use grains of rice to represent percent.
- ❒ Illustrate percent correctly.

Suggestions for Instruction

- **Record the percent displayed in a concrete or pictorial representation.**

Materials:

- pencil
- pencil crayons
- BLM 6.N.6.5: Percent Grids (Full Sheet)

Organization: Pairs

Procedure:

1. Distribute to each student a copy of BLM 6.N.6.5.
2. Tell students the following:
   a) They will be working in pairs.
   b) Each student will shade in an area of each grid to represent a different percent (e.g., the first grid may have 24% of the squares shaded, the second grid may have 90% of the squares shaded, and the third grid may have 51% of the squares shaded).
   c) Each student will shade the grids in the first row representing any percent of squares shaded that the student chooses. The three grids in a row should represent different percents.
   d) They will exchange papers with their partners.
   e) They will record the percent displayed above each grid.
   f) Each will then choose a different partner.
   g) They will repeat the process with their new partner by shading the grids in the second row. There should be no repeats of percents in any grid.
   h) Upon completing the process, they will choose a third partner.
   i) They will repeat the process with their third partner by shading the grids in the third row.

j) After completing the process, they will choose a fourth partner.

k) They will repeat the process with their fourth partner and by shading the grids in the fourth row.

3. Circulate to check that students are doing their task.
4. Have students record in their journals what they learned about displaying and recording a percent.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Create a visual representation of a percent.
- ❒ Record the percent displayed in a pictorial representation.

Suggestions for Instruction

- **Express a percent as a fraction and a decimal.**

Materials:

- pencil
- BLM 6.N.6.6: Percent, Fraction, and Decimal Sheet

Organization: Small groups

Procedure:

1. Distribute to each student BLM 6.N.6.6. Give students the following instructions:
   a) Have one student in your group call out a "percent."
   b) Have every student in the group write it down, including the person who called it.
   c) Have every student write down the fraction equivalent and the decimal equivalent.
   d) Repeat the process by having another student call out a "percent."
   e) Take turns calling out a "percent" until you have all 20 lines filled.
   f) Discuss your results.
   g) Correct your errors.
2. Circulate to check that students understand the intent of the task.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Express a percent as a fraction.
- ❒ Express a percent as a decimal.

Suggestions for Instruction

- **Express a percent as a fraction and a decimal.**

Materials:

- scissors
- numbered cubes
- BLM 6.N.6.7: Say My Equal Fraction, Say My Equal Decimal (both sheets)

Organization: Groups of six

Procedure:

1. Tell students the following instructions:
   a) Each group will receive two sheets of cards.
   b) Students cut the sheets on the lines to create a set of 24 cards.
   c) Students then shuffle the cards and place the set upside down in the centre of the desk.
   d) Each student will roll a numbered cube.
   e) The one with the smallest number starts.
   f) This student picks up the top card and reads it to the group.
   g) This student gives the card to the student on his or her left, who says the reply.
   h) If the reply is correct, the student puts it at the bottom of the pile and picks up a new card. Then the process repeats with the new person reading the card.
   i) If the reply is incorrect, the student who said the reply keeps the card and loses his or her turn to read a question. The student to his or her left picks up a card and reads it. Then the process repeats.
2. Circulate to check that the students understand the process.
3. Discuss what they observed about percents, fractions, and decimals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Express a percent as a fraction.
- ❒ Express a percent as a decimal.

Suggestions for Instruction

- **Identify and describe percents from real-life contexts and record them symbolically.**

Materials:

- newspapers
- paper and pencil

Organization: Small groups

Procedure:

1. Distribute a newspaper section to each group.
2. Provide students with the following instructions:
   a) Look through your newspaper.
   b) Identify percents that were taken from real-life contexts.
   c) Record the percents symbolically.
   d) Take turns identifying and describing other percents from real-life contexts not found in your newspaper. Share with the class.
3. Tell students to describe in their journals the percents from real-life contexts that the group members identified, and record them symbolically.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify percents from real-life contexts.
- ❒ Describe percents from real-life contexts.
- ❒ Record symbolically the percents they identified and described from real-life contexts.

Suggestions for Instruction

- **Solve a problem involving percents.**

Materials:

- pencil
- pencil crayons
- BLM 6.N.6.8: Percent Grids
- BLM 6.N.6.9: Grandpa's Berry Bushes

Organization: Individual

Procedure:

1. Display BLM 6.N.6.9.
2. Distribute to each student a copy of BLM 6.N.6.8.
3. Tell students to do the following:
   a) Work individually on this task.
   b) Read the information Grandpa provided.
   c) Discuss what each of those percents mean.
   d) Use your pencil crayons to create three different designs for Grandpa's berry bushes, using the percents Grandpa provided for you.
   e) Figure out, if Grandpa planted 50 bushes this year, how many would be
      i. raspberry?
      ii. gooseberry?
      iii. blackberry?
      iv. red current?

4. Circulate among the students to check that they are colouring the correct percentage for each type of bushes and performing the correct calculations

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ State the meaning of a given percent.
- ❒ Solve a problem involving percents.
- ❒ Create different designs involving percents.

Plan a School Party

Purpose: The purpose of this activity is to have students connect and apply their skills learned in the mathematics classroom to a real-life event. Students will need to apply their knowledge of fractions (proper and improper), ratios, and percents (N.4-6). They will also need to collect data, design a questionnaire, select a type of graph appropriate for displaying the data, and graph the data (SP.2&3). To complete this task, students will have to rely on some prior knowledge, such as comparison of numbers and knowledge of multiplication facts. The processes that are demonstrated by this task are communication, connection, mental mathematics and estimation, and problem solving. To make this task enjoyable, students will have to cooperate with their group members and with the students they survey.

Curricular Links: This task can be linked to health and social studies.

Materials/Resources:

- paper and pencil
- poster-sized paper

Students will need to find out what kind of food and drinks most students would select in order to create an afternoon of fun or a good school party

Organization:

- Put desks together for group work.
- Split class into two groups.

Inquiry:

Scenario

Students will be working in two groups in their classroom as they design a survey questionnaire. Each of the four groups will be responsible for designing a specific section of the questionnaire (food and drinks). 100 copies of the survey questionnaire will be made, and the completed questionnaires will be analyzed and the results will be displayed.

Procedure

This task will require a few days.

Teacher:

i) Divide the class into two groups. One group will work on the food items and the other group will work on the drink items.

ii) Hand out a copy of the instruction sheet to each group.

Students' Directions:

Day 1: Let's design a survey questionnaire!

- Each group (food and drinks) working separately:
  a) Brainstorm and list items for your section of the survey questionnaire.
  b) Discuss which items your peers would most likely be interested to choose from.
  c) Choose seven items your peers would most likely be interested to choose from.
  d) Design a questionnaire similar to the one provided here.

Items	My Favourite One	I Also Like
1		
2		
3		
4		
5		
6		
7		

  e) List the seven chosen items under the "Items" column.
  f) Type up the questionnaire on the computer.
  g) Print 100 copies.

Day 2: Let's design a food and drinks chart!

- Make a specific chart for your group, and then do the following:
  a) List each of the seven items on the survey questionnaire.
  b) Put next to each item the quantity in which the item can be purchased (e.g., sets of 2, sets of 3, a dozen).
  c) Have a column for stating the number of students who chose the item as their favourite.
  d) Have a column for stating the fraction (the number of students who chose the item as their favourite/the quantity in which the item can be purchased).
  e) Have a column for stating the mixed number (the number of sets of 2s or dozens or whatever quantity in which the item can be purchased).

Sample Chart:

Item	Set in Which Item Can Be Purchased	Number of Favourites	Fraction	Mixed Number
1				
2				
3				
4				
5				
6				
7				

Day 3: Let's do the survey!

- Work in pairs (one member of the food group and one member of the drink group).
- Each pair will
  a) visit a designated class
  b) explain the purpose of the task
  c) ask for volunteers to fill out the survey questionnaires
  d) hand out the questionnaires to the volunteers (Each student volunteer will fill out a food and a drink questionnaire.)
  e) collect all survey questionnaires
  f) take all survey questionnaires back to the classroom

Day 4: Let's record the data!

- Separate the food survey questionnaires from the drink survey questionnaires.
- Work with members of your own group:
  a) Check the replies for each questionnaire.
  b) Record how many times each item was chosen as
     i) favourite
     ii) also liked
  c) Make a chart similar to the one below (like the questionnaire) on a poster-sized paper and fill in the data.

Items	Total Number For "My Favourite One"	Total Number For "I Also Like"
1		
2		
3		
4		
5		
6		
7		

d) Place the completed poster on the board.

e) Use the data on the poster to calculate the following for each item:

i) The % of students who chose it as "My favourite."

ii) The % of students who chose it as "I also like it."

iii) The ratio of "My favourite" to "I also like it."

Day 5: Let's display the results!

- Display each group (food and drinks) separately:

a) Use the data to fill in the five-column chart made on Day 2.

b) Estimate, using the last column of the five-column chart, how many sets of each item you would need to purchase if you had a party and wanted to serve the most favourite foods and drinks (e.g., How many sets of 2s of an item, or how many dozens of another item).

c) Discuss what is the best graph to display the "My favourite" items.

d) Create a graph to display the "My favourite" items.

Literature Link

Read: *The Grizzly Gazette* by Stuart J. Murphy and illustrated by Steve Bjorkman. This is a story about a summer camp where children have a club, a newsletter, a parade, and where everyone can vote for a mascot. It includes an interesting investigation into percentage as the vote goes on.

Assessment:

Use the following observation checklist to assess students' learning.

The student can do the following:	Yes	No	Comment
Design a questionnaire.			
Use fractions (proper and improper) correctly.			
Use percents.			
Use mixed numbers.			
Use ratios.			
Select an appropriate graph to display data.			
Design a graph.			
Correctly estimate quantities.			
Effectively communicate with peers.			
Cooperate with their peers.			

Extension:

Taking it further

Based on the ideas in this task or the ideas in *The Grizzly Gazette*, create your own story using percent, ratio, improper fractions, or mixed numbers.

Proportions and Giants

Purpose: The purpose of this activity is to have students connect and apply the skills they learned in the mathematics classroom to a real-life event. Students will need to apply their knowledge of fractions (proper and improper), ratios, and percents (N.4–6). They will also need to collect data, design a questionnaire, select a type of graph appropriate for displaying the data, and graph the data (SP.2&3). To complete this task, students will have to rely on some prior knowledge, such as comparing numbers and knowledge of multiplication facts. The processes that are demonstrated by this task are communication, connection, mental mathematics and estimation, and problem solving. To make this task enjoyable, students will have to cooperate with their group members and with the students they survey.

Curricular Links: This task can be linked to health, social studies, and English language arts.

Materials/Resources:

- *Jim and the Beanstalk* by Raymond Briggs
- Measurement tools, such as tape measures, rulers, metre sticks
- Calculators

Organization:

- Put desks together for group work.
- Split class into two groups.

Inquiry Procedure:

- Read *Jim and the Beanstalk*.
- Students will be working in groups to determine ratios and measurements. Question: At one point in the story, the giant says that his favourite food is "Three fried boys on toast." Using this fact, determine the size of the giant.
- Have students determine the steps they would need to take to accomplish this task.

Suggested Procedure

1. Have three boys lie on paper or on the ground. Draw a large piece of toast around their bodies or use tape to outline the size of the toast.
2. Have students measure this large toast and then a regular-sized piece of toast to get the correct ratio.
3. Using this ratio, measure an average child in class and determine how big the giant would be.

Extensions

Have students outline the giant in the classroom to visualize his size.

Have students suggest other literature that includes giants or little people to determine sizes and proportions. Compare them to Jim's giant. Make a classroom chart or make outlines of each character for the wall. Use visuals and descriptors of people and objects in each story for the ratios.

Examples

Haggar in *Harry Potter* by J.K. Rowling

The Hobbits in *The Lord of the Rings* by J.R.R. Tolkien

The Lilliputians in *Gulliver's Travels* by Jonathan Swift

The BFG by Roald Dahl

Spiderwick: A Giant Problem by Tony DiTerlizzi

The children in *Honey, I Shrunk the Kids* by Nancy Krulik

The Borrowers by Mary Norton

Also:

Jim spends a lot of time measuring the giant. Measure the same body parts on your own body. What is the ratio of your head circumference to the giant's? Is this same ratio consistent throughout? What is the ratio of your arm to your leg? Your head circumference to your wrist? Are these ratios the same for the students in your class?

Grade 6: Number (6.N.7)

Enduring Understanding(s):

Numbers can be positive or negative. Positive numbers are greater than zero. Negative numbers are less than zero.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.7 Demonstrate an understanding of integers, concretely, pictorially, and symbolically. [C, CN, R, V]	➛ Extend a horizontal or vertical number line by adding numbers less than zero and explain the pattern on each side of zero. ➛ Place a set of integers on a horizontal or vertical number line and explain how integers are ordered. ➛ Describe contexts in which integers are used (e.g., on a thermometer). ➛ Compare two integers, represent their relationship using the symbols <, >, and =, and verify using a horizontal or vertical number line. ➛ Order a set of integers in ascending or descending order.

Prior Knowledge

Students may have had experience with the following:

- Representing and describing whole numbers to 1 000 000
- Comparing and ordering numbers to 10 000
- Demonstrating an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals)

Background Information

Numbers, whether large or small, can be positive or negative.

Positive number is any number greater than 0, located to the right of 0 on a horizontal number line or above 0 on a vertical number line.

Negative number is a number that is less than 0, located to the left of 0 on a horizontal number line or below 0 on a vertical number line.

Not all numbers are large or small, positive or negative. The number that indicates no quantity, size, or magnitude is called zero. Zero is neither negative nor positive; zero is the additive identity. The zero is called the additive identity because, when any number and zero are added, the sum will be the same number (e.g., 8 + 0 = 8, and [–5] + 0 = [–5]).

Numbers can be grouped in sets. A **set** is any collection of things, without regard to their order. The members (or elements) of a set could be numbers, names, shapes, and so on.

The set of numbers consisting of the whole numbers (e.g., 1, 2, 3, 4, . . .), their opposites (e.g., –1, –2, –3, –4, . . .), and 0 is called integers.

The concept of integers is often difficult for students to understand, so we often use a number line when we introduce integers.

A **number line** is a line (vertical or horizontal) on which each point represents a number (for an example, see BLM 6.N.4.4).

It is a good idea to stress the use of correct terminology (i.e., when using integers, it is important that students say "negative" five or "negative" two rather than "minus" five or two).

It might be helpful to students if the teacher pointed out that

a) zero is neither positive nor negative and is always in the middle of the number line (horizontal or vertical)
b) on a horizontal number line, the positive numbers are always on the right-hand side and the negative numbers are always on the left-hand side of zero
c) on a vertical number line, the positive numbers are always above and the negative numbers are always below zero
d) the numbers immediately next to zero are 1 and –1
e) the larger the number is (positive or negative), the further it is located from zero.

Students also need to know, and be able to use correctly, the relational symbols: =, >, and <.

- = means both "equal to" and "balanced with" and is used to show that the left and the right side of a mathematical statement are equal, such as: 6 = 6, (–7) = (–7), (–9) + 9 = 0, 18 + 0 = 18, 23 – 9 = 16 – 2, 56 = 8 × 7, and 54 ÷ 6 = 9.
- > means "greater than" and is used to show that the left side of a mathematical statement is greater than the right side, such as: 296 > 184, 2 > 0, 0 > (–8), and (–2) > (–95) (conversely, the right-hand side is less than the left-hand side).

  296 + 100 > 184

  175 – 171 > –4

  16 + 18 > 15 + 18

  95 > 9 × 10
- < means "less than" and is used to show that the left side of a mathematical statement is less than the right side, such as: 28 < 31, 0 < 7, (–8) < 0, and (–836) < (–5) (conversely, the right-hand side is greater than the left-hand side).

  8 x 7 < 60

  21 – 18 < 18 + 21

  4 + 7 < 5 × 3

Mathematical Language

integer

negative number

number line

positive number

set

zero

Assessing Prior Knowledge

Organization: Whole class

Procedure:

1. Tell students they will be learning about integers, but first you want to check what they remember about the number line.
2. Draw a number line in the positive direction only.
3. Mark 0 and 10 on the number line. Mark every whole number increment between 0 and 10, but do not write the numerals under the markings.
4. Ask one student at a time to come to the board and do the following (keeping in mind that the number line is not limited to the numbers or increments marked on it):
   a) Place 5 on your number line.
   b) Place a number on the number line that is less than 5.
   c) Place 8 on your number line.
   d) Place 12 on your number line.
   e) Place a number on the number line that is less than 10.
   f) Place a number on the number line that is greater than 2.
   g) Place 7 on your number line.
   h) Place 14 on your number line.
5. Discuss with students how they decided where to place each numeral. Since 12 and 14 do not fall between 0 and 10, where would you place them?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Place a positive number on a number line
- ❒ Determine if a number is less than or greater than another number.

Suggestions for Instruction

- **Extend a horizontal or vertical number line by adding numbers less than zero and explain the pattern on each side of zero.**

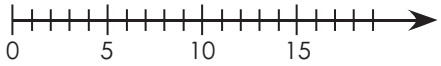
Materials:

- pencil and paper

Organization: Whole class/individual

Procedure:

1. Draw a horizontal number line on the board.
2. Place 0 in the middle of the number line.
3. Show notches for each increment to the right of zero (i.e., in the positive direction only). (Make all increments be of equal size.)
4. Mark every fifth increment (i.e., 5, 10, 15), like in the example below.



5. Write the following numerals on the board:
   a) 2
   b) 6
   c) 9
   d) 11
   e) 13
6. Ask a different student volunteer to come to the board each time (for a, b, c, d, and e) to place a dot in the correct spot on the number line, and write the numeral next to the dot.
7. Ask students what they noticed about the pattern on the right side of zero.
8. Show notches for each increment to the left of zero.
9. Mark every fifth increment, but leave out the negative sign for the sake of discussion (i.e., mark them incorrectly as 5, 10, and 15).
10. Have a discussion on this "new" number line that extends in two directions (i.e., to both sides of zero). (How would they know whether to place a dot on the right side or left side of zero?)
11. After the discussion, tell students: "Mathematicians solved this problem by putting a negative sign in front of each numeral on the left-hand side of zero (like this: –5, –10, –15)." Place a negative sign in front of the increments on the left-hand side of zero incorrectly marked as 5, 10, 15.

12. Write the following numerals on the board:
    a) –1
    b) 3
    c) –4
    d) –7
    e) 8
    f) –12
    g) –14
13. Have a discussion on which side of the zero they would place these numerals, and why.
14. Ask students to explain in their journal the pattern on the
    a) right-hand side of zero
    b) left-hand side of zero

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Explain the pattern on the right-hand side of zero.
- ❒ Explain the pattern on the left-hand side of zero.

Suggestions for Instruction

- **Place a set of integers on a horizontal or vertical number line and explain how integers are ordered.**

Materials:

- pencil
- BLM 5–8.19: Number Line

Organization: Individual/pairs

Procedure:

1. Distribute to each student a copy of BLM 5-8.19.
2. Write on the board: 2, –3, –6, 7, –11, 14.
3. Tell students to
   a) work individually
   b) label their number line (either vertically or horizontally)
   c) mark a dot for each numeral in the correct spot on the number line
   d) place the numeral next to each dot
4. Circulate to check that students are placing the dots in the correct spots on the number line.
5. Tell students to do the next part with a partner.
6. Provide students with the following instructions:
   a) One student will mark one dot on each side of zero.
   b) Tell the numeral to your partner.
   c) Compare your results.
   d) Repeat the game with the other student marking one dot on each side of zero.
   e) Do four rounds.
7. Discuss with your partner how integers are ordered, and write it in your journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Place integers correctly on a vertical number line.
- ❒ Explain how integers are ordered.

Suggestions for Instruction

- **Place a set of integers on a horizontal or vertical number line and explain how integers are ordered.**

Materials:

- pencil
- 6-sided die
- BLM 5–8.19: Number Line

Organization: Groups of three/whole class

Procedure:

1. Distribute to each student a copy of BLM 5-8.19.
2 Distribute to each group a 6-sided die.
3 Provide students with the following instructions:
   a) Each member of the group roles the die once.
   b) The one with the lowest number starts the game and will be the caller.
   c) The caller will place a dot on his/her number line and say the numeral.
   d) The other two group members will place a dot in the correct spot on their number line and mark the numeral next to it.
   e) The two will compare their work with the caller.
   f) Repeat. The student to the left of the caller will be the next caller.
   g) Repeat until everyone has had four turns.
4 Circulate to check that students are placing the dots in the correct spots on the number line.
5 Discuss with the whole class how integers are ordered.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Place integers correctly on a vertical number line.
- ❒ Explain how integers are ordered.

Suggestions for Instruction

- **Describe contexts in which integers are used (e.g., on a thermometer).**

Materials:

- pencil
- BLM 5–8.19: Number Line

Organization: Whole class

Procedure:

1. Place BLM 5-8.19 on the overhead projector or an interactive whiteboard, and label the appropriate increments.
2. Ask for individual volunteers to place on the number line dots representing –16, –8, –4, –1, 2, 6, and 11.
3. Ask students:
   a) What does the vertical number line remind you of? Why?
   b) How is a thermometer similar to a vertical number line?
   c) Where do you think integers are used?
   d) Describe contexts in which integers are used.
4. Have students use their journals to describe contexts in which integers are used.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Place a dot correctly on a vertical number line.
- ❒ Explain how integers are used.
- ❒ Describe contexts in which integers are used.

Suggestions for Instruction

- **Compare two integers, represent their relationship using the symbols <, >, and =, and verify using a horizontal or vertical number line.**

Materials:

- pencil
- BLM 6.N.7.1: Integers

Organization: Small groups

Procedure:

1. Place on the overhead projector a transparency copy of BLM 6.N.7.1 or display it electronically.
2 Provide students with the following instructions:
   a) Compare each set of integers.
   b) Represent the relationship of each set of integers using the symbols <, >, and =.
   c) Draw a number line.
   d) Verify the relationship of each set of integers using a number line.
3 Discuss your results with your group members.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Compare two integers.
- ❒ Represent the relationship of two integers using the symbols <, >, and =.
- ❒ Draw a number line.
- ❒ Verify the relationship of two integers using a number line.

Suggestions for Instruction

- **Compare two integers, represent their relationship using the symbols <, >, and =, and verify using a horizontal or vertical number line.**

Materials:

- pencil
- BLM 5–8.19: Number Line
- BLM 6.N.7.2: Compare Integers

Organization: Groups of four

Procedure:

1. Distribute to each group a 6-sided die, a copy of BLM 5-8.19, and a copy of BLM 6.N.7.2.

2 Provide students with the following instructions:

   a) The object of the game is to tell if integer A <, >, or = integer B.

   b) Let every member of the group roll a die to see who will start the game.

   c) The person with the smallest number starts the game. He or she will choose integer A, record it on the Compare Integers sheet, and state integer A to the group.

   d) The person next to the game starter will choose integer B, record it on the Compare Integers sheet, and state integer B to the group.

   e) The next left person will compare the two integers, on the Compare Integers sheet represent the relationship using the symbols <, >, or =; and state to the group whether integer A is <, >, or = integer B.

   f) The fourth person will plot a dot for each integer on the number line to verify the relationship.

   g) For the next round, let the person who called integer B start the game.

   h) Repeat the game until your Compare Integers sheet is filled, making sure that the starter of the game for each new round is the person who called integer B.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Compare two integers.
- ❒ Represent the relationship of two integers using the symbols <, >, and =.
- ❒ Use a number line.
- ❒ Verify the relationship of two integers using a number line.

Suggestions for Instruction

- **Order a set of integers in ascending or descending order.**

Materials:

- pencil and paper

Organization: Whole class/small groups

Procedure:

1. Provide each group with the following instructions:
   a) Choose 12 integers.
   b) Write each one down as you are choosing it.
   c) Decide how you want to order them (ascending or descending order).
   d) Order your integers
2. Circulate to make sure students choose negative numbers as well as positive.
3. Have one member of each group write the 12 chosen numerals on the board in the order the group chose.
4. Discuss with the class the different sets of integers and their order.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Choose integers.
- ❒ Write numerals for each integer.
- ❒ Order a set of integers in ascending or descending order.

How Cold Is It?

Purpose: The purpose of this activity is for students to connect integers to real life. Students will need to apply their knowledge of integers and data collection and analysis. Students will need some prior knowledge such as comparing numbers. The processes that are demonstrated by this task are communication and connection. Since the purpose of this activity is to work with integers, have students select a Canadian city that has positive and negative temperatures. Doing this activity between November and March might ensure more variability in temperature.

Curricular Links: This task can be linked to science and social studies.

Materials/Resources:

- paper and pencil

Students will need to compare temperature in different cities.

Organization:

- Small groups

Inquiry:

Scenario

Students will be working in small groups. Each group will choose a city and follow the temperature changes during a five-day period. They will record the temperature each morning and afternoon at as close to the same time of day as possible (for example, 8:00 a.m. and 3:00 p.m. or midnight and noon each day).

Procedure

Teacher:

1. Divide the students into small groups.
2. Distribute the directions to each group.
3. Tell students to follow the directions.

Students:

1. Each group needs to choose a Canadian city and monitor its daily temperature.
2. Watch the weather station on a television set to monitor the temperature or go to the StatsCanada website at <http://climate.weather.gc.ca/>.

3. Make a chart like the one provided here:

Day	Morning Temperature	Afternoon Temperature
Monday		
Tuesday		
Wednesday		
Thursday		
Friday		

4. Record the temperature of your chosen city on the chart twice a day for five days in a row: in the morning and in the afternoon.
5. Make three number lines:
   - morning temperature
   - afternoon temperature
   - all temperatures
6. Compare the five morning temperatures and write up your observations.
7. Compare the five afternoon temperatures and write up your observations.
8. Get together with another group.
9. Compare your three number lines with the three number lines of the other group.
10. Write up your observations. What were the differences between the morning and afternoon temperatures each day?

Assessment:

Use the following observation checklist to assess students' learning.

The student can do the following:	Yes	No	Comment
Design a number line.			
Use a number line correctly.			
Use integers.			
Compare integers.			
Record data.			
Analyze data.			

Extension:

Record the daily high and low temperature for several days. Have the whole class do it together. You can use the daily paper for your information. Then:

- Ask students to predict the daily high and low temperature for the next two days based on their data.
- Ask them to compare their predictions with the actual temperatures.

Literature link:

The daily newspaper is an excellent source for data collection. The *Winnipeg Free Press* has a weather report where students can check for information, such as the daily temperature, the UV index, sunrise, sunset, moonrise, and moonset.

Grade 6: Number (6.N.8)

Enduring Understanding(s):

The position of a digit in a number determines its value.

Each place value position is 10 times greater than the place value position to its right.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.8 Demonstrate an understanding of multiplication and division of decimals (involving 1-digit whole-number multipliers, 1-digit natural number divisors, and multipliers and divisors that are multiples of 10), concretely, pictorially, and symbolically, by ▪ using personal strategies ▪ using the standard algorithms ▪ using estimation ▪ solving problems [C, CN, ME, PS, R, V]	➛ Estimate a product using front-end estimation (e.g., for 15.205 m × 4, think 15 m × 4, so the product is greater than 60 m), and place the decimal in the appropriate place. ➛ Estimate a quotient using front-end estimation (e.g., for \$26.83 ÷ 4, think 24 ÷ 4, so the quotient is greater than \$6), and place the decimal in the appropriate place. ➛ Predict products and quotients of decimals using estimation strategies. ➛ Identify and correct errors of decimal point placement in a product or quotient by estimating. ➛ Solve a problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9. ➛ Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10 (e.g., 2.47 × 10 = 24.7; 31.9 ÷ 100 = 0.319). ➛ Model and explain the relationship that exists among an algorithm, place value, and number properties. *continued*

Specific Learning Outcome(s):	Achievement Indicators:
	➛ Determine products and quotients using the standard algorithms of vertical multiplication (numbers arranged vertically and multiplied using single digits which are added to form a final product) and long division (the multiples of the divisor are subtracted from the dividend). ➛ Solve multiplication and division problems in context using personal strategies, and record the process. ➛ Refine personal strategies, such as mental math, to increase their efficiency when appropriate (e.g., 4.46 ÷ 2 think 446 ÷ 2 = 223, and then use front-end estimation to determine the placement of the decimal 2.23).

Prior Knowledge

Students may have had experience with the following:

- Applying estimation strategies in problem-solving situations
- Determining multiplication facts (to 81) and related division facts
- Applying mental mathematics strategies for multiplication
- Demonstrating an understanding of multiplication to solve problems
- Demonstrating an understanding of division with and without concrete materials, and interpreting remainders to solve problems
- Describing and representing decimals
- Comparing and ordering decimals

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of place value

Background Information

In previous years, students learned to describe and represent decimals. In Grade 6, students will learn to multiply and divide decimals, and will be required to demonstrate an understanding of these two operations with decimals.

The concept of decimals may present a difficulty for some students. Teachers need to place special emphasis on the decimal point (i.e., its meaning and placement—see definition below).

One of the difficulties a teacher might encounter occurs while teaching multiplication and division of decimals involving 1-digit whole number multipliers and 1-digit natural number divisors. A student, for example, may not understand whether he or she is supposed to keep track of the number of digits in front or behind the decimal point. When multiplying 8.23×3, a student may incorrectly write the answer as 2.469, saying that since there is only one numeral in front of the decimal point in the question, then there should only be one numeral before the decimal point in the reply. Or when dividing $397.26 \div 6$, a student may incorrectly write 662.1 for the same reason. With these types of examples, the teacher needs to get the student used to looking at the whole number part of the numeral. For the multiplication example, ask the student "What is the answer to 8×3?" After the student replies that it is 24, continue saying "If the answer to 8×3 is 24, then the answer to 8.23×3 is greater than 24 (in this example, 24.69)." For the division example, ask the student "What is the answer to $397 \div 6$?" After the student replies that it is 66 and the remainder is 1, continue saying "If the answer to $397 \div 6$ is 66 and the remainder is 1, then the answer to $397.26 \div 6$ is greater than 66."

When teaching multiplication and division of decimals involving multipliers and divisors that are multiples of 10, there may be a difficulty when assigning questions such as: 37.296×100, 42.36×10, $58.79 \div 10$, and $394.32 \div 100$. The student may express confusion about which way to move the decimal point. With these types of examples, try to get the student used to thinking: 37×100 is 3700, so 37.296×100 is greater than 3700; 42×10 is 420, so 42.36×10 is greater than 420; $58 \div 10$ is 5.8, so $58.79 \div 10$ is greater than 5.8; and $394 \div 100$ is 3.94, so $394.32 \div 100$ is greater than 3.94.

Definitions may also be beneficial to students. If they are exposed to the definitions of the required terminology in each lesson, they may understand them better. Terminology and definitions needed for 6.N.8 are listed below:

Decimal is a fractional number written in base-10 form; a mixed decimal number has a whole number part as well (e.g., 0.32 is a decimal number and 3.5 is a mixed decimal number).

Decimal point is a period or dot separating the ones place from the tenths place in decimal numbers, or dollars from cents in money. When numbers are spoken, the decimal point is read as "and" (e.g., 3.2 is read as three and two-tenths).

Divisor is the number by which the dividend is divided (e.g., in $12 \div 3 = 4$, 3 is the divisor).

Estimate is an answer that is an approximation.

Front-end estimation (also called front-end rounding) is a method for estimating an answer to a calculation problem by focusing on the front-end or left-most digits of a number.

Example:

Question

You buy a hamburger for $4.59, a drink for $1.96, and an ice cream cone for $0.95. Will a five-dollar bill cover the cost?

Front-end Strategy Solution Process

Total the front-end (dollar) amounts: $4 + $1 + $0 = $5.

Solution

A five-dollar bill will not cover the cost because the front-end estimate, which is always an underestimate, is $5.

Mental mathematics is a mathematical process by which computation is done "in the head," either in whole or in part.

Multiplier is the number by which the multiplicand is multiplied in a multiplication problem (e.g., in $1.2 \times 3 = 3.6$, 3 is the multiplier).

An **algorithm** is a system of finite procedures for solving a particular class of problems. The best known algorithms are the traditional paper-and-pencil procedures for adding, subtracting, multiplying, and dividing. Along with these standard algorithms, mathematics instruction should include an emphasis on understanding through mental mathematics, estimation, the use of technology, the development of invented procedures, and the use of alternative algorithms, such as area model multiplication, and adding up to solve subtraction problems.

By encouraging students to develop their own computation strategies and allowing them to use alternative algorithms, the emphasis in mathematics instruction is shifted to reasoning, problem solving, and conceptual understanding. Providing students with opportunities to invent their own strategies and use alternative algorithms enhances their number and operation sense. Students become more flexible in their thinking, more aware of the different ways to solve a problem, and more adept at selecting the most appropriate procedure for solving a problem. Discussion of the algorithms or strategies and their relationship to place value and number properties can also help students develop better reasoning and communication skills.

The standard algorithm for multiplication is where numbers are arranged vertically and multiplied using single digits, which are added to form a final product. Students are expected to use the standard algorithm as one of the tools for computation.

When teaching the traditional algorithm for multiplication, it is important to follow the concrete, pictorial, and symbolic sequence of teaching. The important idea is to allow students to construct meaning, not memorize procedures without understanding. Students' misconceptions (or their fuzzy understandings) can be reinforced by a poorly understood algorithm. Students should be able to explain the relationship that exists among an algorithm, place value, and number properties.

Teachers can facilitate students' understanding and use of a variety of computational strategies by

- providing a supporting and accepting environment
- allowing time for exploration and experimentation
- embedding computational tasks in real-life situations
- allowing students to discuss, analyze, and compare their solution strategies
- encouraging discussion that focuses on place value and number properties when defending the choice of a particular algorithm or strategy
- understanding that a child needs to be efficient at computation and that this looks different for each student

Models for Dividing Decimals

The focus is on dividing 2 numbers by building understanding through the use of models, non-traditional algorithms, and traditional algorithms.

Note: The term *traditional algorithm* is used to indicate the symbolic algorithm traditionally taught in North America. Throughout the world, many other algorithms are traditionally used.

Base-10 Blocks

Base-10 blocks can be used to represent the operations of addition, subtraction, multiplication, and division of decimals.

Students must have a fluent understanding of the numeric values for the model. If they lack this understanding, their attention will be focused on trying to make sense of the model instead of learning to compute with whole numbers. Have students physically separate the blocks, or cut paper grids, to help them attach meaningful values to the representations. Spend time naming various combinations of blocks and creating representations of various whole numbers to develop fluency.

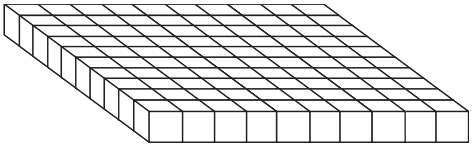
Base-10 grid paper serves as a two-dimensional representation of the base-10 blocks (see BLM 5-8.10: Base-Ten Grid Paper).

Note: It is important that students work flexibly with various representations to develop their understanding of operations with whole numbers, rather than memorizing the steps without understanding their meaning.

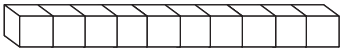
Representations:

If the flat represents one whole,

- its value is 1.0



then the rod represents a tenth,



- its value is 0.1

and the cube represents a hundredth.

- its value is 0.01

Note: Stress to students that although they may have used these materials before using different names, the materials can represent whatever they want them to as long as the relationship among the materials is mathematically correct.

Example:

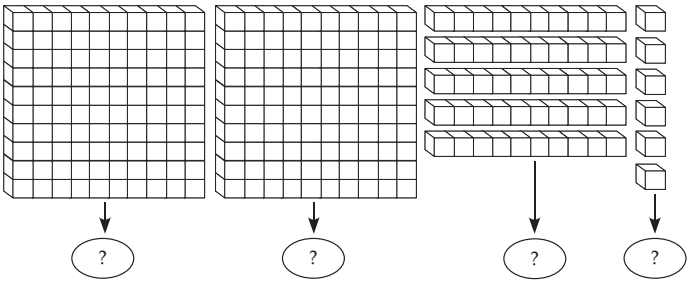
2.56 ÷ 4

Estimate:

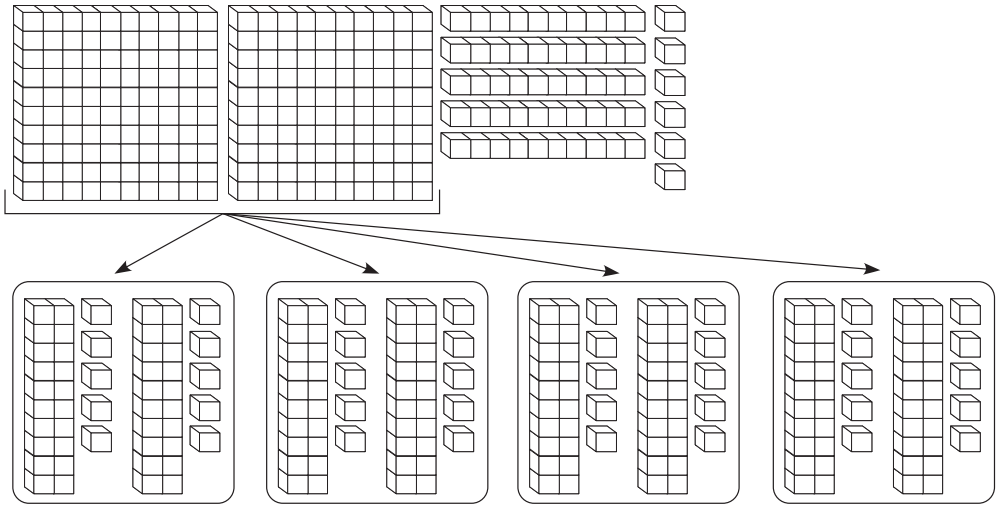
Think: 2 ÷ 4 = 0.5 (or 2 is half of 4). The answer will be less than 1 but greater than one-half.

Build 2.56 with the materials.

Use models to help find an exact number. What is 2.56 divided into 4 groups?

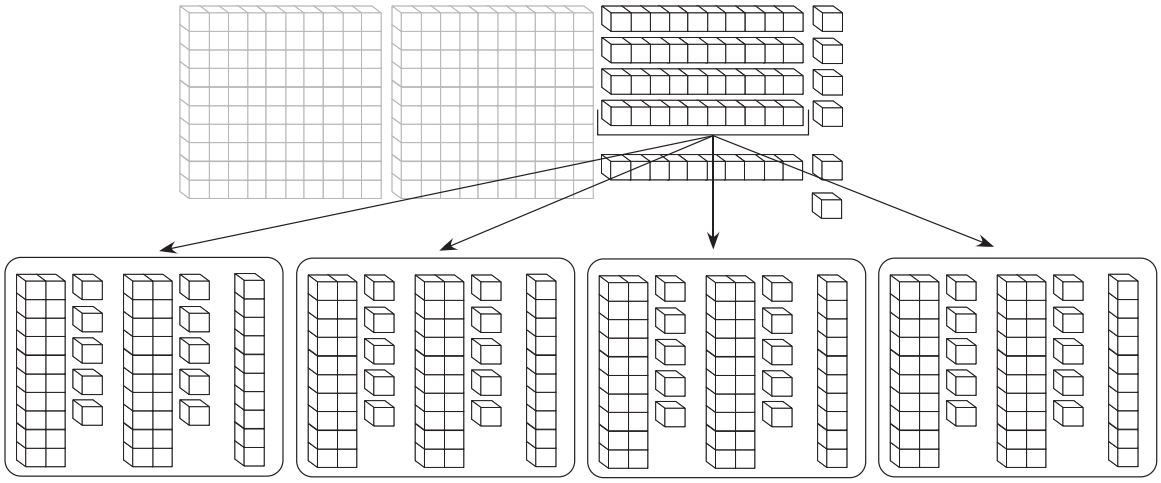


Think: "I can break each whole into 4 groups. There are 2 tenths and 5 hundredths in each group for every hundred I divide."

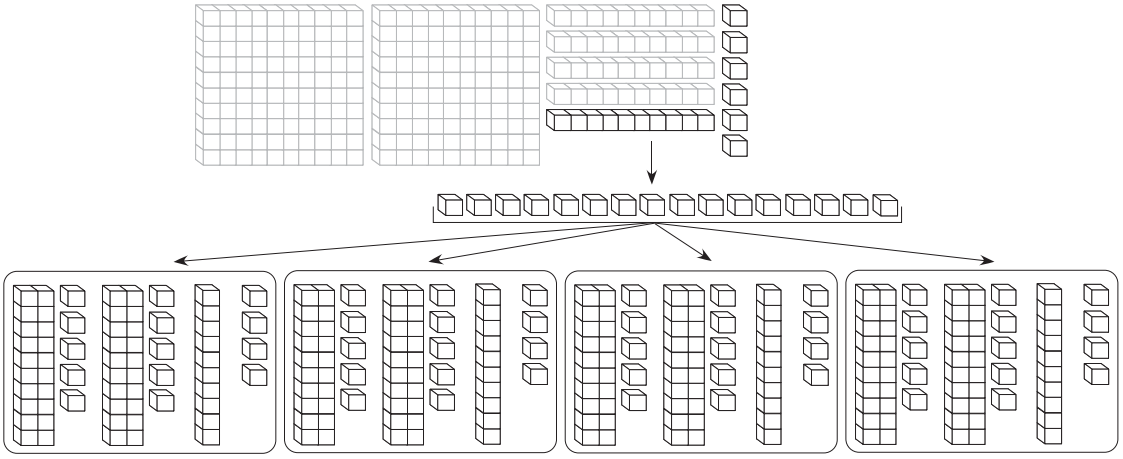


Think: "After I divide the 2.0 into 4 groups, there is nothing left over."

Think: "I can move one tenth to each group."



Think: "That leaves one tenth (or an additional 10 hundredths). There are 16 hundredths left, so I can move 4 more to each group."



Think: In each group of 4, there are 6 tenths and 4 hundredths or 0.64. So, 2.56 ÷ 4 is 0.64.

Long Division

Long division is a more compact algorithm used to show the division of multi-digit numbers. Once students have an understanding of the above models and methods, and they are confident with their understanding of division and place value, the "traditional algorithm" for division can be a quick and precise method of dividing. Algorithm 1 shows one way to divide decimals while the traditional algorithm shows the same process in a different way.

Students follow the same steps as they do when dividing whole numbers but should be aware of the correct placement of the decimal. Although 2.56 ÷ 4 and 25.6 ÷ 4 produce a quotient with the digits 64, students should estimate that 2.56 ÷ 4 will be less than 1.0 and 25.6 ÷ 4 will be close to 6.0.

Example:

Algorithm 1

$4\overline{)256}$	6 groups of 4
-24	
16	4 groups of 4
-16	
0	64

Traditional Algorithm

$$\begin{array}{r} 0.64 \\ 4\overline{)2.56} \\ -2.4 \\ \hline 0.16 \\ -0.16 \\ \hline 0 \end{array}$$

Models for Multiplying Decimals

The focus is on multiplying numbers by building understanding through the use of models, non-traditional algorithms, and traditional algorithms.

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Base-10 blocks can be used to represent the operations of addition, subtraction, multiplication, and division of decimals.

Students must have a fluent understanding of the numeric values for the model. If they lack this understanding, their attention will be focused on trying to make sense of the model instead of learning to compute with whole numbers. Have students physically separate the blocks, or cut paper grids, to help them attach meaningful values to the representations. Spend time naming various combinations of blocks and creating representations of various whole numbers to develop fluency.

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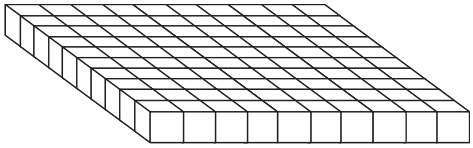
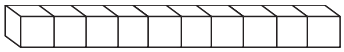


then the rod represents a tenth,

- its value is 0.1



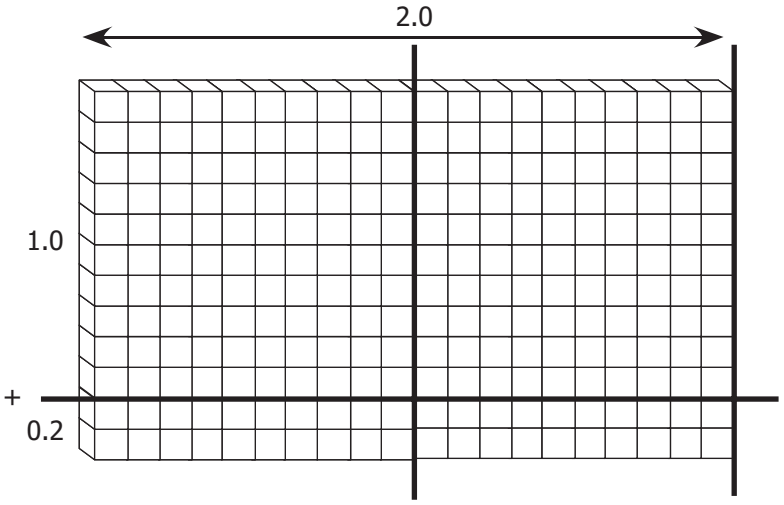
and the cube represents a hundredth.

- its value is 0.01

An area model can be used to multiply decimals. As with whole number multiplication, the algorithm can be modelled using base-10 blocks or pictorially.

Example:

2.0 × 1.2



To determine the partial products, think of numbers that are easier to multiply. For example, represent numbers according to place value and think:

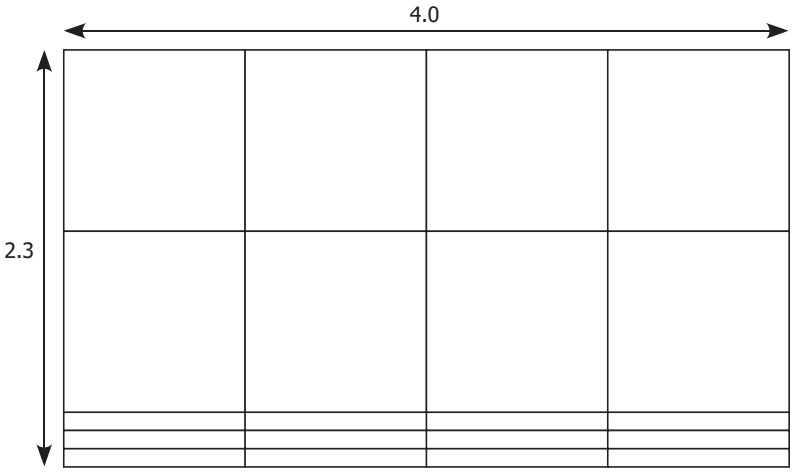
- "What is 2.0 × 1.0?" "2.0"
- "What is 2.0 × 0.2?" "0.4"

"So then, 2 × 1.2 must be 2.0 + 0.4 or 2.4."

Example:

4.0 × 2.3

$$\begin{array}{r} 4.0 \\ \times\ 2.3 \\ \hline 1.2 \\ +\ 8.0 \\ \hline 9.2 \end{array}$$



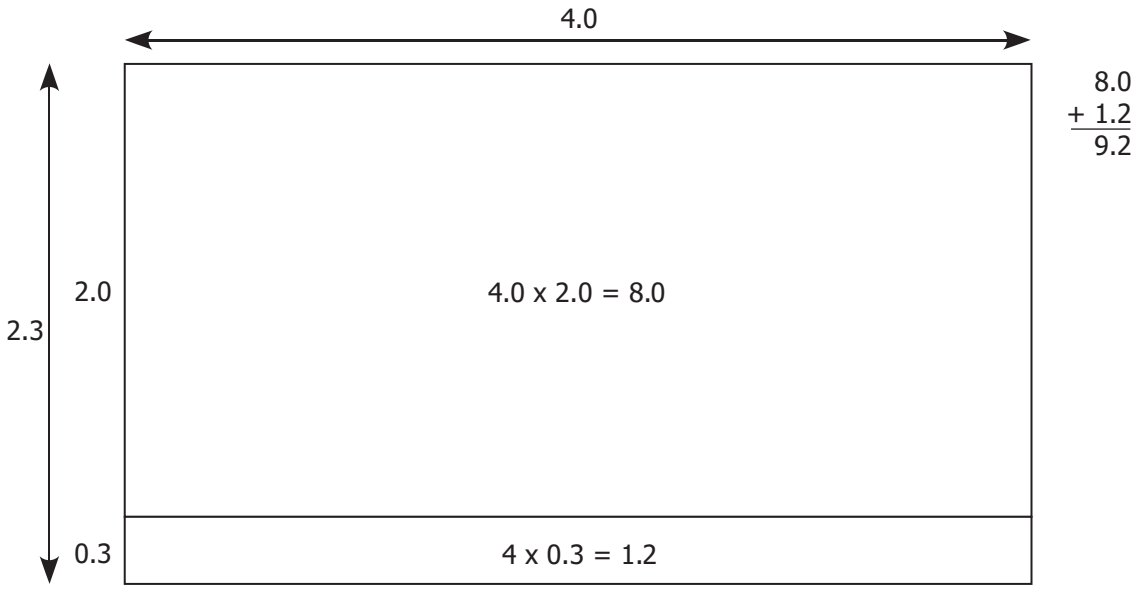
Think:

- 4.0 × 2.0 = 8.0
- 4.0 × 0.3 = 1.2

Then, 8.0 + 1.2 = 9.2.

Note: Because of the commutative property of multiplication, area models can be drawn in either orientation.

4.0 × 2.3 or 2.3 × 4.0



The partial products are determined as

$4.0 \times 2.0 = 8.0$

$4.0 \times 0.3 = 1.2$

So, $8.0 + 1.2 = 9.2$.

It is important to establish a convention of keeping the blocks organized, as it will help with developing future representations.

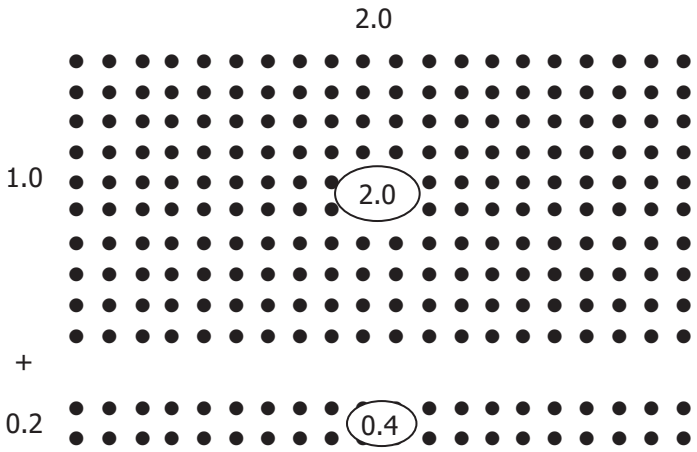
Base-10 blocks can be used to represent the active understanding of multiplication as a specific number of groups of a specific size, or the non-active array representation of a quantity.

When the blocks are arranged as a rectangle, the rectangle may be rotated and the quantity does not change. This is a verification of the commutative property of multiplication. The orientation of the array has no effect on the result, but in some places a convention has been established of representing the first number horizontally and the second vertically.

The array also serves as a model for area. It represents the area covered by a rectangle with a length of the multiplicand and a width of the multiplier, or vice versa.

Example:

2.0 × 1.2



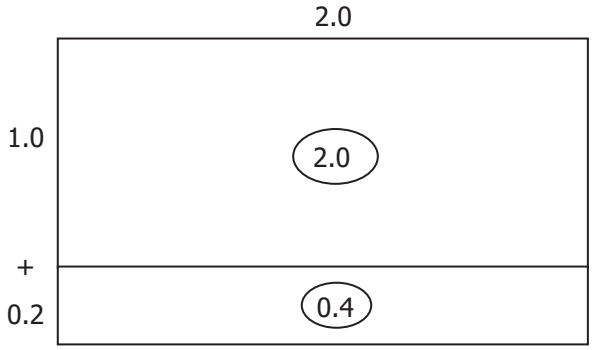
The partial products are determined in the same way as they are in the previous example.

The Area Model

Once students have an understanding of multiplication using base-10 blocks and/or base-10 paper, and they have a thorough understanding of the numerical value they represent, they can simply move to an area model. In this model, the length represents the multiplicand and the width represents the multiplier, or vice versa; however, the lengths and widths do not have to represent an exact measurement.

Example:

2.0 × 1.2



The partial products are determined in the same way as they are in the previous example.

The Traditional Algorithm

Once students have an understanding of the above models and methods and are confident with multiplication, the traditional algorithm can be quick and useful. Students will multiply as if the decimal is not there. This works because the digits are the same. It is important, however, to estimate to make sure the decimal is placed correctly.

Example:

$$\begin{array}{r} 2.33 \\ \times\ 4 \\ \hline \end{array}$$

Think:

$4 \times 0.03 = 0.12$

$4 \times 0.3 = 1.2$

$4 \times 2.0 = 8.0$

9.32

Traditional Algorithm:

$$\begin{array}{r} {\scriptstyle 1\ \ 1} \\ 2.33 \\ \times\ 4 \\ \hline 9.32 \end{array}$$

Even though students have learned the standard algorithm for multiplication, it is important to understand the rationale for place value placement of the decimal.

How are 2.33×4 and 23.3×4 both similar and different? They have the same digits and, when multiplied, will produce the same digits, 932. Discuss how 2.33×4 produces a product near to 8 while 23.3×4 produces a product near to 80.

Note: For further information regarding using models and methods of division to build understanding, see resources such as the following:

- "Big Ideas for Teaching Mathematics Grades 4–8" by Marian Small (found in Chapter 2 of *Varied Approaches for Multiplication and Division*)
- "Making Math Meaningful to Canadian Students K–8" by Marian Small (found in Chapter 10 of *Computational Strategies: Operations with Whole Numbers*)
- *Teaching Student Centered Mathematics* by John A. Van de Walle

Mathematical Language

decimal
decimal point
divisor
estimate
mixed decimal number
mental mathematics
multiples

Learning Experiences

Assessing Prior Knowledge

Organization: Whole class

Procedure:

1. Tell students that they will learn how to multiply and divide decimals, but first you want to check what they remember about decimals.
2. Write a few decimals on the board and ask students to read them.

   Examples:

   0.15

   0.398

   0.0317

   0.207

   0.004
3. Have a class discussion. Ask questions such as the following:
   a) Where are decimals used?
   b) What does the dot mean?
   c) If I want to write four-hundredths,
      - does it matter if I place zeros after the 4? Why?
      - does it matter how many zeros I place before the four? Why?
4. Discuss place value and decimal place.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand place value.
- ❒ Read decimals correctly.
- ❒ Use the decimal point correctly.

Suggestions for Instruction

- **Estimate a product using front-end estimation (e.g., for 15.205 m x 4, think 15 m x 4, so the product is greater than 60 m), and place the decimal in the appropriate place.**

Materials:

- pencil and paper

Organization: Whole class

Procedure:

1. Write a few decimals on the board, such as the following:
   a) 3.85
   b) 7.69
   c) 2.08
   d) 8.321
   e) 6.18 026
   f) 3.20 18
   g) 9.341
2. Ask a few student volunteers to do the following:
   a) Read each decimal.
   b) Say what each number will be after using front-end rounding.
3. Discuss with the class
   a) what the estimated product of each decimal × 3 will be after using front-end rounding
   b) where the decimal point will be placed in each product

4. Have students explain the following:
   a) 8.216 × 3 is greater than 24
   b) 21.08 × 4 is greater than 84
5. Have students place the decimal point in each product using front-end estimation:
   a) 25.024 × 3 = 75072
   b) 19.128 × 5 = 9564
   c) 84.301 × 2 = 168602
   d) 7.932 × 10 = 7932
   e) 3 869.54 × 100 = 386954
6. Discuss any other strategies students may have used to determine where to place the decimal in the product.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Read decimal numbers
- ❒ Apply front-end rounding.
- ❒ Place the decimal point correctly.

Suggestions for Instruction

- **Estimate a quotient using front-end estimation (e.g., for $26.83 ÷ 4, think $24 ÷ 4, so the quotient is greater than $6), and place the decimal in the appropriate place.**

Materials:

- pencil and paper
- BLM 6.N.8.1: Izabella's Teacher

Organization: Whole class/pairs

Procedure:

1. Display a copy of BLM 6.N.8.1.
2. Tell students to read the question carefully.

3. Discuss with the class
   a) what Izabella's teacher means by using front-end estimation to solve the decimal questions
   b) what Izabella's teacher means by wanting the class to carefully consider where they place the decimal point in the quotient
   c) how you solve the decimal questions presented on the BLM
4. Provide students with the following instructions:
   a) Place the decimal in each quotient.
   b) Discuss your work with your partner.
5. Circulate to make sure students understand how to divide decimals using front-end estimation and that they place the decimal point in the correct position.
6. Discuss any other strategies students may have used to determine where to place the decimal in the quotient.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Apply front-end rounding.
- ❒ Use the decimal point correctly.

Suggestions for Instruction

- **Predict products and quotients of decimals using estimation strategies.**

Materials:
- pencil and paper
- BLM 6.N.8.2: Decimal Products and Quotients

Organization: Small groups

Procedure:
1. Hand out to each student a copy of BLM 6.N.8.2.
2. Provide students with the following instructions:
   a) Read the questions carefully.
   b) Do not calculate.

c) Predict the products of decimal using estimation strategies.
d) Write down your prediction for each product.
e) Discuss your predictions with your group members.
f) Next, predict the quotients of decimal using estimation strategies.
g) Write down your prediction for each quotient.
h) Discuss your predictions with your group members.

3. Circulate to see that the students understand the work and are doing it correctly.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Multiply and divide by single digits.
- ❒ Multiply and divide by multiples of 10.
- ❒ Use estimation strategies.
- ❒ Predict the products of decimal using estimation strategies.
- ❒ Predict the quotients of decimal using estimation strategies.

Suggestions for Instruction

- **Identify and correct errors of decimal point placement in a product or quotient by estimating.**

Materials:

- pencil and paper
- BLM 6.N.8.3: Marie's Cell Phone Bill

Organization: Individual

Procedure:

1. Display BLM 6.N.8.3.
2. Provide students with the following instructions:
   a) Use the information on Marie's cell phone bill to calculate her payment requirement.
   b) Does your calculation of Marie's payment requirement match with the payment requirement on Marie's cell phone bill?

c) Use estimation and check how your estimation results of the payment requirement match your calculations and Marie's bill.

d) In your journal, describe your calculations and why your payment requirement does or does not match with the payment requirement on Marie's cell phone bill.

3. Circulate to see that students understand the problem and are solving it correctly.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve a problem involving decimals.
- ❒ Use estimation strategies to identify errors of decimal point placement in a product and a quotient.

Suggestions for Instruction

- **Identify and correct errors of decimal point placement in a product or quotient by estimating.**

Materials:

- pencil and paper
- BLM 6.N.8.4: Errors of Decimal Point Placement

Organization: Small groups

Procedure:

1. Distribute to each student a copy of BLM 6.N.8.4.
2. Provide students with the following instructions:
   a) Read the questions carefully.
   b) Use estimation strategies to
      - identify errors of decimal point placement
      - correct errors of decimal point placement

      Note: Some products and quotients may be correct.
   c) Write down your correction for each product and quotient.
   d) Discuss your errors and corrections with your group members.
3. Circulate to see that students understand the placement of the decimal point and are doing their work correctly.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use estimation strategies to identify errors of decimal point placement in a product and quotient.
- ❒ Use estimation strategies to correct errors of decimal point placement in a product and quotient.

Suggestions for Instruction

- **Solve a problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9.**

Materials:

- pencil and paper
- BLM 6.N.8.5: Multiplication and Division Problems involving Decimals

Organization: Pairs/whole class

Procedure:

1. Distribute to each student a copy of BLM 6.N.8.5.
2. Provide students with the following instructions:
   a) Read each problem carefully.
   b) Decide with your partner which problem set you will do (A or B).
   c) Discuss with your partner how you will solve each problem.
   d) Solve each problem on your own.
   e) Use estimation strategies to verify your placement of the decimal point in each reply.
   f) Compare your final answer and discuss your method of solution with your partner.
3. Discuss with the whole class the method used for solving the problems and the errors and difficulties with each problem.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve problems that involve multiplication and division of decimals.
- ❒ Use estimation strategies to verify placement of the decimal point.

Suggestions for Instruction

- **Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10 (e.g., 2.47 X 10 = 24.7; 31.9 ÷ 100 = 0.319).**

Materials:

- calculator
- BLM 6.N.8.6: Complete the Charts

Organization: Individual/partner/whole class

Procedure:

1. Distribute a copy of BLM 6.N.8.6.
2. Provide students with the following instructions:
   a) Use your calculators to complete individually the multiplication charts on the BLM 6.N.8.6 sheet.
   b) Discuss with your partner the patterns you see for multiplication.
   c) Then, use your calculators to complete individually the division charts on the BLM 6.N.8.6 sheet.
   d) Discuss with your partner the patterns you see for division.
3. Discuss with the whole class the patterns students see.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Discover a pattern.

Suggestions for Instruction

- **Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10 (e.g., 2.47 X 10 = 24.7; 31.9 ÷ 100 = 0.319).**

Materials:

- BLM 6.N.8.7: Use Mental Math

Organization: Whole class/individual

Procedure:

1. Display BLM 6.N.8.7.
2. Have students look at the questions.
3. Discuss how to determine the product or the quotient using mental math.
4. Ask different student volunteers to state the product or the quotient using mental math.
5. Tell students to explain in their journals how to use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understanding how to use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10.
- ❒ Use mental math to determine products or quotients involving decimals when the multiplier or divisor is a multiple of 10.

Suggestions for Instruction

- **Model and explain the relationship that exists between an algorithm place value, and number properties.**
- **Determine products and quotients using the standard algorithms of vertical multiplication (numbers arranged vertically and multiplied using single digits, which are added to form a final product) and long division (the multiples of the divisor are subtracted from the dividend).**

Materials:

- Base-10 blocks

Organization: Small groups of 3 or 4

Procedure:

1. Explain that the base-10 blocks will represent decimals.

   1 flat = 1 whole 1.0

   1 rod = 1 tenth 0.1

   1 cube = 1 hundredth 0.01

2. Students will be given 3 multiplication of decimal questions and 3 division of decimal questions. For each multiplication question, students will

   a) estimate the answer

   b) solve using an area model

   c) solve using repeated addition

   d) solve using the traditional algorithm

   For each division question, students will

   a) estimate the answer

   b) solve using the base-10 model

   c) solve using algorithm 1 (see Background Information)

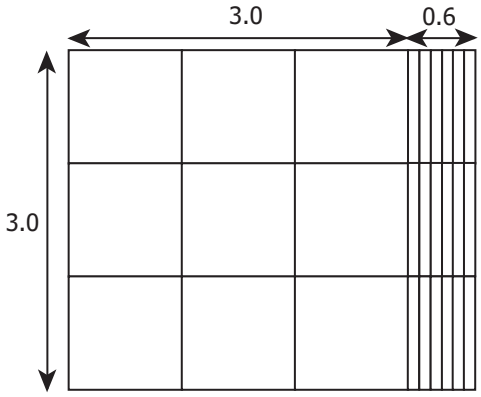
   d) solve using the traditional algorithm

Examples: Multiplication

1. 3.6×3.0

   a) Group estimate: $3 \times 3 = 9.0$

   b) Person #1: Draws the area model

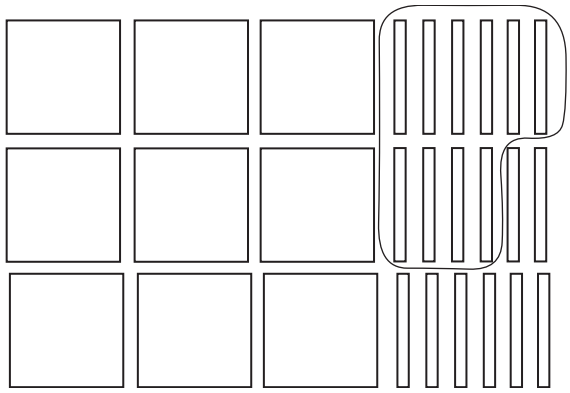


$3.0 \times 3.0 = 9.0$

$3.0 \times 0.6 = 1.8$

Total = 10.8

   c) Person #2: Models repeated addition



Regrouping into 10 and 8 tenths

10.8

   d) Person #3: Uses the traditional algorithm

$$\begin{array}{r} 3.6 \\ \times\ 3 \\ \hline 1.8 \\ 9.0 \\ \hline 10.8 \end{array}$$

Think 3×0.6

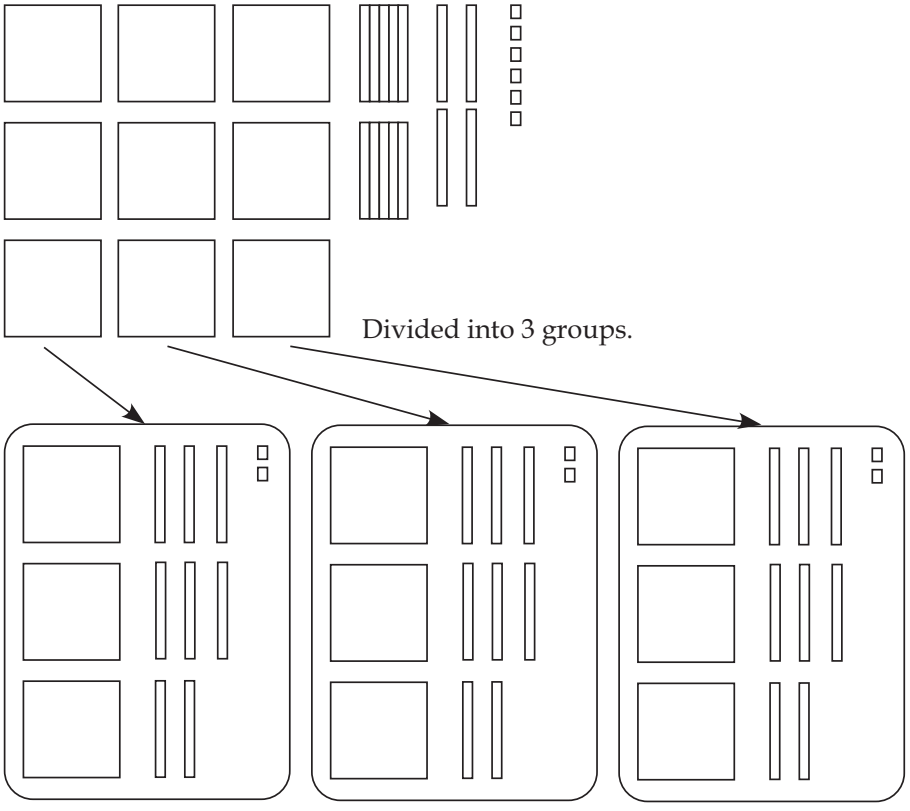
Think 3×3.0

$$\begin{array}{r} {\scriptstyle 1} \\ 3.6 \\ \times\ 3 \\ \hline 10.8 \end{array}$$

2. 6.2 × 5
   a) Estimate the answer
   b) Person #1: Shows repeated addition
   c) Person #2: Shows the traditional algorithm
   d) Person #3: Shows the area model
3. 9.1 × 2
   a) Estimate the answer
   b) Person #1: Shows the traditional algorithm
   c) Person #2: Shows the area model
   d) Person #3: Shows repeated addition

Examples: Division

1. 11.46 ÷ 3
   a) Group estimate: 11 ÷ 3 = answer between 3 and 4
   b) Person #1: Models using base-10 blocks



Each group has: 3 wholes
8 tenths = 3.82
3 hundredths

c) Person #2: Uses algorithm 1

$3\overline{)11.46}$

-9.0	3.0 groups of 3
2.46	0.8 groups of 3
-2.4	
0.06	0.02 groups of 3
-0.06	
0	3.82

d) Person #3: Uses the traditional algorithm

$$\begin{array}{r} 3.82 \\ 3\overline{)11.46} \\ \underline{-9}\downarrow \\ 2.4 \\ \underline{-2.4}\downarrow \\ 0.06 \\ \underline{-0.06} \\ 0 \end{array}$$

2. 16.8 ÷ 4
   a) Estimate the answer (16.8 ÷ 4 = 4)
   b) Person #1: Uses algorithm 1
   c) Person #2: Uses traditional algorithm
   d) Person #3: Uses base-10 blocks
3. 9.12 ÷ 2
   a) Estimate the answer (9.12 ÷ 2 will be between 4 and 5)
   b) Person #1: Uses traditional algorithm
   c) Person #2: Uses the base-10 blocks
   d) Person #3: Uses algorithm 1

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine products and quotients using the standard algorithms.
- ❒ Explain and model the relationship between algorithms and place value.

Suggestions for Instruction

- **Solve multiplication and division problems in context using personal strategies, and record the process.**
- **Refine personal strategies, such as mental math, to increase their efficiency when appropriate (e.g., 4.46 ÷ 2 think 446 ÷ 2 = 223, and then use front-end estimation to determine the placement of the decimal 2.23).**

Materials:

- BLM 6.N.8.8: Question Sheet

Organization: Small groups

Procedure:

1. Circulate BLM 6.N.8.8 to students in small groups and have them solve the questions.
2. Have students communicate their group answers and strategies by participating in a whole-class group share. Groups can take turns sharing an answer to a question and other students can compare their group's strategies, ask questions, and share solutions.

Observation Checklist

☑ Observe students' responses to determine if they can do the following:

- ❒ Solve problems using personal strategies.
- ❒ Record the problem-solving process.
- ❒ Refine personal strategies to increase efficiency.

Calculate the Distance

Purpose: The purpose of this activity is for students to connect decimals to real life. Students will need to apply their knowledge of decimals, measurements, and recording and analysis of data. Students will need some prior knowledge, such as multiplying and dividing by single-digit whole numbers and multiples of 10. The following processes are demonstrated by this task: communication, connections, mental mathematics and estimation, problem solving, and reasoning.

Curricular Links: This task can be linked to science.

Materials/Resources:

- paper and pencil
- metre stick
- poster-sized paper

Students will need to measure lengths in different areas of the school.

Organization:

- Four groups

Inquiry:

Scenario

Students will be working in four separate groups. Each group will measure the distance to one of these rooms: library, gym, music room, and lunch room.

Procedure

Teacher:

1. Divide the students into four groups.
2. Assign a room to each group (library, gym, music room, and lunch room).
3. Distribute the directions to each group.
4. Tell students to follow the directions.

Students:

1. Each group needs to measure the exact distance in metres and the decimal parts of a metre (extra centimetres) from the classroom door to the door of the designated room.
2. Record the exact distance in metres and the decimal parts of a metre from the classroom door to the door of the designated room.
3. Record how many times per week you go from the classroom to the designated room.

4. Make a chart like the one provided below:

Room	Number of Metres (Use Decimals to Record the Exact Number of Centimetres)	Number of Times per Week

5. Calculate how many metres you walk each week from the classroom door to the door of the designated room and back.
6. How many metres do you walk from the classroom door to the door of the designated room in four weeks?
7. Get together with another group.
8. Record the information from your chart and the other group's chart on the poster-sized paper.
9. Compare their distance in metres in a week to yours using estimation strategies and mental math.
10. Compare your distance in metres in four weeks to the distance of the other group, using estimation strategies and mental math.
11. Write up your observations.

Assessment:

Use the following observation checklist to assess students' learning.

The student can do the following:	Yes	No	Comment
Use the metre stick to to take measurements.			
Record the measurements accurately.			
Use decimals.			
Multiply decimals.			
Record data.			
Analyze data.			
Effectively communicate with peers.			
Cooperate with peers.			

Extension:

Taking it further

Based on the ideas in this task, create your own activity that involves multiplying or dividing decimals.

NOTES

Grade 8: Number (6.N.9)

Enduring Understanding(s):

When a mathematics question is composed of more than one operation, the solution depends on the order in which the operations are done.

It is important to follow a standardized order of operations so that everyone solving the identical problem will obtain the same answer.

General Learning Outcome(s):

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
6.N.9 Explain and apply the order of operations, excluding exponents (limited to whole numbers). [CN, ME, PS, T]	➛ Demonstrate and explain with examples why there is a need to have a standardized order of operations. ➛ Apply the order of operations to solve multi-step problems with or without technology.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of multiplication (1- and 2-digit multipliers and up to 4-digit multiplicands) to solve problems
- Demonstrating an understanding of division (1- and 2-digit divisors and up to 4-digit dividends) with and without concrete materials, and interpreting remainders to solve problems
- Demonstrating an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals)

Related Knowledge

Students should be introduced to the following:

- Demonstrating and explaining the meaning of and preservation of equality

Background Information

In previous years, students have learned about the importance that numbers play in our daily lives. They also learned to perform the four basic mathematical operations: addition, subtraction, multiplication, and division.

In Grade 6, students will learn to recognize that the order in which the four mathematical operations are performed is very important. Mathematicians realized that a standardized order of operations is required if everyone is to produce the same answer to a multi-operation question.

Without a standardized rule, a multi-operation question can be solved in a different order, yielding a variety of replies.

Order of Operations

Order of operations is a specified sequence in which mathematical operations are expected to be performed. An arithmetic expression is evaluated by following these ordered steps:

1. Simplify within grouping symbols, such as parentheses or brackets, starting with the innermost.
2. Apply exponents—powers and roots.
3. Perform all multiplications and divisions in order from left to right.
4. Perform all additions and subtractions in order from left to right.

A common way to remember this is to use the acronym BEDMAS: Brackets, Exponents, Division, Multiplication, Addition, Subtraction.

Note: In Kindergarten to Grade 7, exponents are not used. Students are first exposed to squares and square roots in Grade 8, and in Grade 9 students will revisit the order of operations and work with exponents.

Mathematical Language

order of operations

LEARNING EXPERIENCES

Assessing Prior Knowledge

Organization: Whole class

Procedure:

1. Tell students you want to check what they remember about the four mathematical operations.
   a) Ask students to solve questions, such as the following:

      23 + 15=

      68 – 29=

      15 × 8=

      63 ÷ 7=
2. Have a class discussion. Ask questions such as the following:
   a) "How did you know which mathematical operation you had to use?" Discuss the importance of the symbols representing the four operations (i.e., addition, subtraction, multiplication, and division).
   b) "Was there an order, or a rule, you had to follow to solve the questions?"
   c) "Would you get the same answer if you reversed the order?"

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve simple questions using any one of the four operations.
- ❒ Understand the four operations.
- ❒ Understand the need for specific symbols.
- ❒ Understand the need for specific order.

Suggestions for Instruction

- **Demonstrate and explain with examples why there is a need to have a standardized order of operations.**

Materials:

- pencil and paper
- BLM 6.N.9.1: One Solution, Two Solutions?

Organization: Whole class

Procedure:

1. Place on the overhead a transparency of BLM 6.N.9.1. Make sure the two solutions are covered, showing the question part only, as shown below.

> **BLM 6.N.9.1: One Solution, Two Solutions?**
>
> Bonny and Jenny loved to compare their work. Yesterday, their math teacher assigned the following question for homework:
>
> $3 + 5 \times 7 - 2 + 9 \div 3 =$
>
> Both girls decided that the question needs to be split into many parts.

2. Provide students with the following instructions:
   a) Read the question carefully.
   b) Discuss with your group members how you would solve the problem.
   c) Do not solve the problem.
3. Have a class discussion on what each group decided as to how to solve the problem.
4. Then say "Let's see how Bonny and Jenny solved the problem."
5. Now show the solution part of BLM 6.N.9.1, as shown below.

Bonny solved the problem like this:	Jenny solved the problem like this:
$3 + 5 = 8$	$5 \times 7 = 35$
$8 \times 7 = 56$	$9 \div 3 = 3$
$56 - 2 = 54$	$3 + 35 = 38$
$54 + 9 = 63$	$38 - 2 = 36$
$63 \div 3 = 21$	$36 + 3 = 39$
Then, Bonny stated:	Then, Jenny stated:
$3 + 5 \times 7 - 2 + 9 \div 3 = 21$	$3 + 5 \times 7 - 2 + 9 \div 3 = 39$

6. Tell students to look at each solution carefully, and then discuss the following with their group members:
   a) What was different in the two solutions?
   b) Did the solution the group decided on match either girl's solution?
   c) What was the same? What was different?
   d) What would you suggest to Bonny and Jenny to do so they could both get the same solution?
7. Discuss the following with the whole class:
   a) the students' observations about this question and the solutions
   b) what is needed in order to assure that everyone solving the same question will arrive at the same answer
   c) why there is a need to have a standardized order of operations
8. Have students give examples that show there is a need to have a standardized order of operations.
9. Explain why a standardized order of operations would ensure sameness of replies.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand a multi-step problem.
- ❒ Take apart a multi-step problem.
- ❒ Realize that a rule is needed for solving a multi-step problem.
- ❒ Demonstrate and explain with examples why there is a need to have a standardized order of operations.

Suggestions for Instruction

- **Demonstrate and explain with examples why there is a need to have a standardized order of operations.**

Materials:

- pencil and paper

Organization: Whole class/pairs

Procedure:

1. Write on the board the following question:

   $8 + 3 \times 4 - 12 \div 2 =$

2. Provide students with the following instructions:
   a) Copy the question.
   b) Discuss with your partner the different ways you could solve this problem if a standardized order of operations did not exist.
   c) Show the different ways you could solve this problem if a standardized order of operations did not exist.
3. Have a class discussion about why it is important to have a standardized order of operations.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize that without a standardized order of operations, a multi-step problem could be solved in a variety of ways.
- ❒ Realize that a rule is needed for solving a multi-step problem.
- ❒ Demonstrate and explain with examples why there is a need to have a standardized order of operations.

Suggestions for Instruction

- **Demonstrate and explain with examples why there is a need to have a standardized order of operations.**

Materials:

- pencil and paper

Organization: Individual

Procedure:

1. Provide students with the following instructions:
   a) Work individually on this task.
   b) Construct an example of a math question that includes the four operations: addition, subtraction, multiplication, and division.
   c) Show the variety of ways you could solve the problem if there was no standardized order of operations.
2. Have students write in their journals why it is important to have a standardized order of operations.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Provide an example of a multi-step problem that shows a need for a standardized order of operations.
- ❒ Demonstrate and explain with examples why there is a need to have a standardized order of operations.

Suggestions for Instruction

- **Apply the order of operations to solve multi-step problems with or without technology.**

Materials:

- pencil
- BLM 6.N.9.2: Use Your Pencil: Set A

Organization: Individual/whole class

Procedure:

1. Distribute to each student a copy of BLM 6.N.9.2.
2. Provide students with the following instructions:
   a) Work individually using a pencil only.
   b) Apply the order of operations in solving each question.
3. When the set is completed, discuss with the class how the order of operations helped solve the multi-step problems.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Work individually on a set of problems.
- ❒ Solve problems using a pencil only.
- ❒ Apply the order of operations to solve multi-step problems without technology.

Suggestions for Instruction

- **Apply the order of operations to solve multi-step problems with or without technology.**

Materials:

- pencil
- BLM 6.N.9.3: Use Your Pencil: Set B

Organization: Individual/pairs

Procedure:

1. Distribute to each student a copy of BLM 6.N.9.3.
2. Provide students with the following instructions:
   a) Work individually using a pencil only.
   b) Apply the order of operations in solving each question.
   c) When the set is completed, compare your work with your partner.
3. When the set is completed, discuss with your partner whether the order of operations helped you both get the same solution to the multi-step problems.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Work individually on a set of problems.
- ❒ Cooperate with a partner.
- ❒ Apply the order of operations to solve multi-step problems without technology.

Suggestions for Instruction

- **Apply the order of operations to solve multi-step problems with or without technology.**

Materials:

- calculator
- BLM 6.N.9.4: Use Your Calculator: Set A
- BLM 6.N.9.5: Use Your Calculator: Set B

Organization: Individual/small groups

Procedure:

1. Use two overhead projectors.
   a) Place a transparency of BLM 6.N.9.4 on one overhead projector
   b) Place a transparency of BLM 6.N.9.5 on the second overhead projector
2. Provide students with the following instructions:
   a) Choose either set A or set B to solve.
   b) Work individually using a calculator.
   c) Apply the order of operations in solving each question.
   d) When the set is completed, compare your work with the other group members who chose the same set as you.
   e) Discuss how the order of operations helped you all get the same solution to the multi-step problems.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Work individually on a set of problems.
- ❒ Cooperate with group members.
- ❒ Use a calculator to solve a problem.
- ❒ Apply the order of operations to solve multi-step problems with technology.

Suggestions for Instruction

- **Apply the order of operations to solve multi-step problems with or without technology.**

Materials:

- BLM 5–8.6: Hundred Chart
- three 10-sided dice
- paper

Organization: Whole group

Procedure:

1. Roll three numbers and record them on the board.
2. Allow students one minute to use the three numbers and the order of operations to create as many different answers as they can. They will record their number sentences on paper. For example, if the numbers rolled were 4, 9, and 2, students could write the following:

   $4 + 9 + 2 = 15$

   $9 - 4 \times 2 = 1$

   $(9 - 4) \times 2 = 10$

   Etc.
3. Have students mark the numbers they find onto BLM 5-8.6.
4. The first student to create a 3-by-3 grid (or whatever you choose) wins.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Work individually on a set of problems.
- ❒ Cooperate with group members.
- ❒ Use a calculator to solve a problem.
- ❒ Apply the order of operations to solve multi-step problems with technology.

Tina's Ten Turkeys

Purpose: The purpose of this activity is for students to connect the order of operations to real life. Students will need to apply their knowledge of the four basic mathematical operations (addition, subtraction, multiplication, and division), as well as the use of brackets. They will also need to be able to draw simple geometric shapes, such as a triangle, square, and rectangle. The processes that are demonstrated by this task are problem solving, reasoning, communication, and connection.

Curricular Links: This task can be linked to reading and science.

Materials/Resources:

- BLM 6.N.9.6: Tina's Ten Turkeys
- Scrap paper
- Recipe cards
- Poster paper
- Markers

Organization:

- Small groups

Procedure:

Teacher's instructions:

Arrange the classroom so that it is conducive to small-group work.

Hand out the following to each group:

- Several pieces of scrap paper
- Three recipe cards
- A poster-sized paper
- A marker
- A copy of BLM 6.N.9.6
- A copy of the instruction sheet

Students' instructions:

1. Work together.
2. Have one group member carefully read "Tina's Ten Turkeys" to the group.
3. Decide amongst yourselves who will do each task, and use your geometry set to do the following:
   a) Draw a large triangle on one recipe card.
   b) Draw a large square on the second recipe card.
   c) Draw a large rectangle on the third recipe card.

4. Above the triangle, write "omelet."
5. Above the square, write "waffle."
6. Above the rectangle, write "cake."
7. Inside the triangle, write the numeral representing the number of eggs it takes to make an omelet.
8. Inside the square, write the numeral representing the number of eggs it takes to make a waffle.
9. Inside the rectangle, write the numeral representing the number of eggs it takes to make a cake.
10. Discuss each of the following before you do them on your scrap paper:
    a) Show how using the order of operations helps you find out how many turkey eggs Tina would have used up during week 1.
    b) Show how using the order of operations helps you find out how many turkey eggs Tina would have leftover at the end of week 1.
    c) Show how using the order of operations helps you find out how many turkey eggs Tina would have used up during week 2.
    d) Show how using the order of operations helps you find out how many turkey eggs Tina would have used up during the two-week period.
    e) Show how using the order of operations helps you find out how many turkey eggs Tina would have leftover at the end of week 2.
11. Discuss and check your work.
12. Make corrections if needed.
13. Use the marker to write the corrected version of your work on the poster-sized paper.
14. Place the poster on the board.

Assessment:

Use the following observation checklist to assess students' learning.

The student can do the following:	Yes	No	Comment
Use numerals and mathematical operations to solve a word problem.			
Use the order of operations correctly.			
Explain the order of operations.			
Analyze their work.			
Effectively communicate with peers.			
Cooperate with peers.			

Extension:

Taking it further

Based on the ideas in this task, create your own activity using the order of operations.