Grade 5 Mathematics

Support Document for Teachers

Manitoba

Grade 5 Mathematics

Support Document for Teachers

2014

Manitoba Education and Advanced Learning

Manitoba Education and Advanced Learning Cataloguing in Publication Data

Grade 5 mathematics [electronic resource] : support document
for teachers

Includes bibliographical references
ISBN: 978-0-7711-5901-5

1. Mathematics—Study and teaching (Elementary).
2. Mathematics—Study and teaching (Elementary)—Manitoba.
I. Manitoba. Manitoba Education and Advanced Learning.
372.7044

Manitoba Education and Advanced Learning
School Programs Division
Winnipeg, Manitoba, Canada

Print copies of this resource can be purchased from the Manitoba Text Book Bureau (stock number 80634). Order online at <www.mtbb.mb.ca>.

This resource is also available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/supports.html>.

Available in alternate formats upon request.

CONTENTS

List of Blackline Masters (BLMs) v

Grade 5 Mathematics Blackline Masters v

Grades 5 to 8 Mathematics Blackline Masters viii

Acknowledgements ix

Introduction 1

Overview 2

Conceptual Framework for Kindergarten to Grade 9 Mathematics 6

Assessment 10

Instructional Focus 12

Document Organization and Format 13

Number 1

Number (5.N.1) 3

Number (5.N.2) 19

Number (5.N.3) 31

Number (5.N.4) 51

Number (5.N.5) 67

Number (5.N.6) 85

Number (5.N.7) 99

Number (5.N.8, 5.N.9) 117

Number (5.N.10) 131

Number (5.N.11) 141

Patterns and Relations 1

Patterns and Relations (Patterns) (5.PR.1) 3

Patterns and Relations (Variables and Equations) (5.PR.2) 17

Shape and Space 1

Shape and Space (Measurement) (5.SS.1) 3

Shape and Space (Measurement) (5.SS.2) 15

Shape and Space (Measurement) (5.SS.3) 31

Shape and Space (Measurement) (5.SS.4) 43

Shape and Space (3-D Objects and 2-D Shapes) (5.SS.5) 55

Shape and Space (3-D Objects and 2-D Shapes) (5.SS.6) 69

Shape and Space (Transformations) (5.SS.7) 81

Statistics and Probability 1

Statistics and Probability (Data Analysis) (5.SP.1, 5.SP.2) 3

Statistics and Probability (Chance and Uncertainty) (5.SP.3, 5.SP.4) 19

Appendices 1

Bibliography 1

List of Blackline Masters (BLMs)

Grade 5 Mathematics Blackline Masters

Number (N)

BLM 5.N.1.1: Place Value

BLM 5.N.1.2: Place Value Headings

BLM 5.N.1.3: Which Do You Prefer—Chunky or Smooth?

BLM 5.N.2.1: Situation Cards

BLM 5.N.2.2: Estimation Situations

BLM 5.N.3.1: Tic-Tac-Toe Grids

BLM 5.N.3.2: Division Puzzle

BLM 5.N.3.3: Everyone Cards

BLM 5.N.3.4: Single Digit Multiplication Chart

BLM 5.N.3.5: Race around the Clock

BLM 5.N.4.1: Multiplication Problems

BLM 5.N.4.2: Game Sheet

BLM 5.N.4.3: Products

BLM 5.N.5.1: Multiplication Method

BLM 5.N.6.1: Division Problem Cards

BLM 5.N.6.2: Division Playing Board

BLM 5.N.7.1: Equivalent Fraction Cards

BLM 5.N.7.2: Fraction Cards

BLM 5.N.7.3: Recording Sheet

BLM 5.N.8&9.1: Decimal Equivalent Cards

BLM 5.N.10.1: Decimal Recording Sheet

BLM 5.N.10.2: Decimal Cards

BLM 5.N.11.1: Number Frames

BLM 5.N.11.2: Decimal Activity

Patterns and Relations (PR)

BLM 5.PR.1.1: Pattern Problem 1

BLM 5.PR.1.2: Pattern Problem 2

BLM 5.PR.1.3: Pattern Problem 3

BLM 5.PR.1.4: Pattern Problem 4

BLM 5.PR.1.5: Pattern Activity

BLM 5.PR.2.1: Equation Problem

BLM 5.PR.2.2: Equation Activity

BLM 5.PR.2.3: Word Problems

BLM 5.PR.2.4: Equation Activity

Shape and Space (SS)

BLM 5.SS.1.1: Perimeter and Area Recording Sheet

BLM 5.SS.2.1: Assessing Prior Knowledge

BLM 5.SS.2.2: Measurement Activity

BLM 5.SS.2.3: Line Cards

BLM 5.SS.2.4: Measurement Cards

BLM 5.SS.2.5: Measurement Table

BLM 5.SS.2.6: Line Segments

BLM 5.SS.2.7: I Have, Who Has . . .?

BLM 5.SS.3.1: Volume Activity

BLM 5.SS.4.1: Capacity Procedure

BLM 5.SS.6.1: Quadrilateral Name Cards

BLM 5.SS.6.2: Quadrilateral Cards

BLM 5.SS.6.3: Quadrilateral Activity Sheet

BLM 5.SS.7&8.1: Reflection Activity Sheet

BLM 5.SS.7&8.2: Transformation Cards

Statistics and Probability (SP)

BLM 5.SP.1&2.1: Prior Knowledge

BLM 5.SP.1&2.2: First- and Second-Hand Data

BLM 5.SP.1&2.3: Candy Graph

BLM 5.SP.3&4.1: Probability Label Cards

BLM 5.SP.3&4.2: Probability Activity

BLM 5.SP.3&4.3: Spinner Statements

BLM 5.SP.3&4.4: Experiment

BLM 5.SP.3&4.5: Mystery Spinner

Grades 5 to 8 Mathematics Blackline Masters

BLM 5–8.1: Observation Form

BLM 5–8.2: Concept Description Sheet #1

BLM 5–8.3: Concept Description Sheet #2

BLM 5–8.4: How I Worked in My Group

BLM 5–8.5: Number Cards

BLM 5–8.6: Blank Hundred Squares

BLM 5–8.7: Place-Value Chart—Whole Numbers

BLM 5–8.8: Mental Math Strategies

BLM 5–8.9: Centimetre Grid Paper

BLM 5–8.10: Base-Ten Grid Paper

BLM 5–8.11: Multiplication Table

BLM 5–8.12: Fraction Bars

BLM 5–8.13: Clock Face

BLM 5–8.14: Spinner

BLM 5–8.15: Thousand Grid

BLM 5–8.16: Place-Value Mat—Decimal Numbers

BLM 5–8.17: Number Fan

BLM 5–8.18: KWL Chart

BLM 5–8.19: Double Number Line

BLM 5–8.20: Algebra Tiles

BLM 5–8.21: Isometric Dot Paper

BLM 5–8.22: Dot Paper

BLM 5–8.23: Understanding Words Chart

BLM 5–8.24: Number Line

BLM 5–8.25: My Success with Mathematical Processes

BLM 5–8.26: Percent Circle

Acknowledgements

Manitoba Education and Advanced Learning wishes to thank the members of the Grades 5 to 8 Mathematics Support Document Development Team for their contribution to this document. Their dedication and hard work have made this document possible.

Writer	Betty Johns	University of Manitoba
Grade 5 to Grade 8 Mathematics Support Document Development Team (2006–2008)	Heidi Holst	Lord Selkirk School Division
	Linda Girling	Louis Riel School Division
	Darlene Willetts	Evergreen School Division
	Holly Forsyth	Fort La Bosse School Division
	Chris Harbeck	Winnipeg School Division
	Steven Hunt	St. Gerard School
	Jan Jebsen	Kelsey School Division
	Dianna Kicenko	Evergreen School Division
	Kelly Kuzyk	Mountain View School Division
	Judy Maryniuk	Lord Selkirk School Division
	Greg Sawatzky	Hanover School Division
Manitoba Education and Advanced Learning School Programs Division Staff	Heather Anderson Consultant (until June 2007)	Development Unit Instruction, Curriculum and Assessment Branch
	Carole Bilyk Project Manager	Development Unit Instruction, Curriculum and Assessment Branch
	Lee-Ila Bothe Coordinator	Document Production Services Unit Educational Resources Branch
	Linda Girling Project Leader (since January 2011)	Development Unit Instruction, Curriculum and Assessment Branch
	Lynn Harrison Desktop Publisher	Document Production Services Unit Educational Resources Branch
	Heather Knight Wells Project Leader (until December 2010)	Development Unit Instruction, Curriculum and Assessment Branch
	Grant Moore Publications Editor	Document Production Services Unit Educational Resources Branch

Introduction

Purpose of the Document

Grade 5 Mathematics: Support Document for Teachers provides various instructional activities, assessment strategies, and learning resources that promote the meaningful engagement of mathematics learners in Grade 5. The document is intended to be used as an aid to teachers as they work with students in achieving the prescribed outcomes and achievement indicators identified in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)* (Manitoba Education).

Background

Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes is based on *The Common Curriculum Framework for K–9 Mathematics*, which resulted from ongoing collaboration with the Western and Northern Canadian Protocol (WNCP). In its work, WNCP emphasizes

- common educational goals
- the ability to collaborate and achieve common goals
- high standards in education
- planning an array of educational activities
- removing obstacles to accessibility for individual learners
- optimum use of limited educational resources

The growing effects of technology and the need for technology-related skills have become more apparent in the last half century. Mathematics and problem-solving skills are becoming more valued as we move from an industrial to an informational society. As a result of this trend, mathematics literacy has become increasingly important. Making connections between mathematical study and daily life, business, industry, government, and environmental thinking is imperative. The Kindergarten to Grade 12 Mathematics curriculum is designed to support and promote the understanding that mathematics is

- a way of learning about our world
- part of our daily lives
- both quantitative and geometric in nature

Beliefs about Students and Mathematics Learning

The Kindergarten to Grade 8 Mathematics curriculum is designed with the understanding that students have unique interests, abilities, and needs. As a result, it is imperative to make connections to all students' prior knowledge, experiences, and backgrounds.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with unique knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem-solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Conceptual understanding: comprehending mathematical concepts, relations, and operations to build new knowledge. (Kilpatrick, Swafford, and Findell 5)

Procedural thinking: carrying out procedures flexibly, accurately, efficiently, and appropriately.

Problem solving: engaging in understanding and resolving problem situations where a method or solution is not immediately obvious. (OECD 12)

First Nations, Métis, and Inuit Perspectives

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural, and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning, and learn mathematics best when it is contextualized and not taught as discrete content.

Many First Nations, Métis, and Inuit students come from cultural environments where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

Affective Domain

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

Teachers, students, and parents* need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessment of personal goals.

* In this document, the term *parents* refers to both parents and guardians and is used with the recognition that in some cases only one parent may be involved in a child's education.

Middle Years Education

Middle Years education is defined as the education provided for young adolescents in Grades 5, 6, 7, and 8. Middle Years learners are in a period of rapid physical, emotional, social, moral, and cognitive development.

Socialization is very important to Middle Years students, and collaborative learning, positive role models, approval of significant adults in their lives, and a sense of community and belonging greatly enhance adolescents' engagement in learning and commitment to school. It is important to provide students with an engaging and social environment within which to explore mathematics and to construct meaning.

Adolescence is a time of rapid brain development when concrete thinking progresses to abstract thinking. Although higher-order thinking and problem-solving abilities develop during the Middle Years, concrete, exploratory, and experiential learning is most engaging to adolescents.

Middle Years students seek to establish their independence and are most engaged when their learning experiences provide them with a voice and choice. Personal goal setting, co-construction of assessment criteria, and participation in assessment, evaluation, and reporting help adolescents take ownership of their learning. Clear, descriptive, and timely feedback can provide important information to the mathematics student. Asking open-ended questions, accepting multiple solutions, and having students develop personal strategies will help students to develop their mathematical independence.

Adolescents who see the connections between themselves and their learning, and between the learning inside the classroom and life outside the classroom, are more motivated and engaged in their learning than those who do not observe these connections.

Adolescents thrive on challenges in their learning, but their sensitivity at this age makes them prone to discouragement if the challenges seem unattainable. Differentiated instruction allows teachers to tailor learning challenges to adolescents' individual needs, strengths, and interests. It is important to focus instruction on where students are and to see every contribution as valuable.

Mathematics Education Goals for Students

The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between mathematical knowledge and skills, and their application
- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world

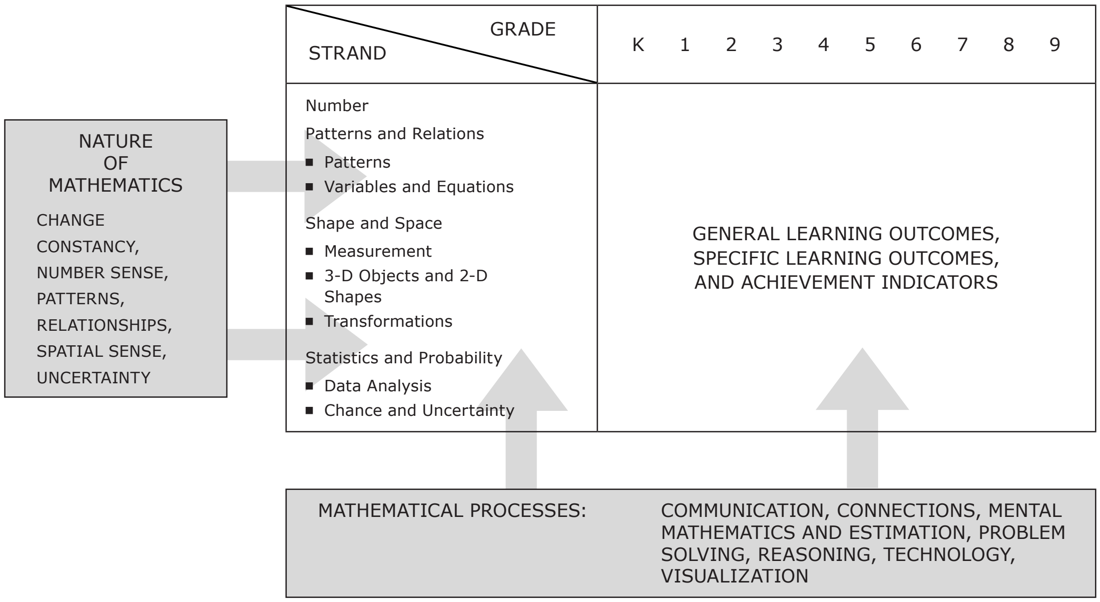
> Mathematics education must prepare students to use mathematics to think criticallly about the world.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

Conceptual Framework for Kindergarten to Grade 9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

There are critical components that students must encounter in mathematics in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The common curriculum framework incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

- **Communication [C]:** Students communicate daily (orally, through diagrams and pictures, and by writing) about their mathematics learning. They need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. This enables them to reflect, to validate, and to clarify their thinking. Journals and learning logs can be used as a record of student interpretations of mathematical meanings and ideas.
- **Connections [CN]:** Mathematics should be viewed as an integrated whole, rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—concrete, pictorial, and symbolic (the symbolic mode consists of oral and written word symbols as well as mathematical symbols). The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas.
- **Mental Mathematics and Estimation [ME]:** The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences and students should be provided with frequent opportunities to practise these skills. Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense.
- **Problem Solving [PS]:** Students are exposed to a wide variety of problems in all areas of mathematics. They explore a variety of methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problems.
- **Reasoning [R]:** Mathematics reasoning involves informal thinking, conjecturing, and validating—these help children understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.
- **Technology [T]:** The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.
- **Visualization [V]:** Mental images help students to develop concepts and to understand procedures. Students clarify their understanding of mathematical ideas through images and explanations.

These processes are outlined in detail in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.

Strands

The learning outcomes in the Manitoba curriculum framework are organized into four strands across Kindergarten to Grade 9. Some strands are further subdivided into substrands. There is one general learning outcome per substrand across Kindergarten to Grade 9.

The strands and substrands, including the general learning outcome for each, follow.

Number

- Develop number sense.

Patterns and Relations

- Patterns
  - Use patterns to describe the world and solve problems.
- Variables and Equations
  - Represent algebraic expressions in multiple ways.

Shape and Space

- Measurement
  - Use direct and indirect measure to solve problems.
- 3-D Objects and 2-D Shapes
  - Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
- Transformations
  - Describe and analyze position and motion of objects and shapes.

Statistics and Probability

- Data Analysis
  - Collect, display, and analyze data to solve problems.
- Chance and Uncertainty
  - Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Outcomes and Achievement Indicators

The Manitoba curriculum framework is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators.

- **General learning outcomes** are overarching statements about what students are expected to learn in each strand/substrand. The general learning outcome for each strand/substrand is the same throughout the grades from Kindergarten to Grade 9.
- **Specific learning outcomes** are statements that identify the specific skills, understanding, and knowledge students are required to attain by the end of a given grade.
- **Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success.

In this document, the word *including* indicates that any ensuing items **must be addressed** to meet the learning outcome fully. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are **not requirements that must be addressed** to meet the learning outcome fully.

Summary

The conceptual framework for Kindergarten to Grade 9 mathematics describes the nature of mathematics, mathematical processes, and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Learning activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes, and lead students to an understanding of the nature of mathematics through specific knowledge, skills, and attitudes among and between strands. The *Grade 5 Mathematics: Support Document for Teachers* is meant to support teachers to create meaningful learning activities that focus on formative assessment and student engagement.

Assessment

Authentic assessment and feedback are a driving force for the suggestions for assessment in this document. The purposes of the suggested assessment activities and strategies are to parallel those found in *Rethinking Classroom Assessment with Purpose in Mind: Assessment* for *Learning, Assessment* as *Learning, Assessment* of *Learning* (Manitoba Education, Citizenship and Youth). These include the following:

- assessing *for*, *as*, and *of* learning
- enhancing student learning
- assessing students effectively, efficiently, and fairly
- providing educators with a starting point for reflection, deliberation, discussion, and learning

Assessment *for* learning is designed to give teachers information to modify and differentiate teaching and learning activities. It acknowledges that individual students learn in idiosyncratic ways, but it also recognizes that there are predictable patterns and pathways that many students follow. It requires careful design on the part of teachers so that they use the resulting information to determine not only what students know, but also to gain insights into how, when, and whether students apply what they know. Teachers can also use this information to streamline and target instruction and resources, and to provide feedback to students to help them advance their learning.

Assessment *as* learning is a process of developing and supporting metacognition for students. Assessment *as* learning focuses on the role of the student as the critical connector between assessment and learning. When students are active, engaged, and critical assessors, they make sense of information, relate it to prior knowledge, and use it for new learning. This is the regulatory process in metacognition. It occurs when students monitor their own learning and use the feedback from this monitoring to make adjustments, adaptations, and even major changes in what they understand. It requires that teachers help students develop, practise, and become comfortable with reflection, and with a critical analysis of their own learning.

Assessment *of* learning is summative in nature and is used to confirm what students know and can do, to demonstrate whether they have achieved the curriculum outcomes, and, occasionally, to show how they are placed in relation to others. Teachers concentrate on ensuring that they have used assessment to provide accurate and sound statements of students' proficiency, so that the recipients of the information can use the information to make reasonable and defensible decisions.

Overview of Planning Assessment			
	Assessment *for* Learning	**Assessment *as* Learning**	**Assessment *of* Learning**
Why Assess?	■ to enable teachers to determine next steps in advancing student learning	■ to guide and provide opportunities for each student to monitor and critically reflect on his or her learning and identify next steps	■ to certify or inform parents or others of student's proficiency in relation to curriculum learning outcomes
Assess What?	■ each student's progress and learning needs in relation to the curriculum outcomes	■ each student's thinking about his or her learning, what strategies he or she uses to support or challenge that learning, and the mechanisms he or she uses to adjust and advance his or her learning	■ the extent to which each student can apply the key concepts, knowledge, skills, and attitudes related to the curriculum outcomes
What Methods?	■ a range of methods in different modes that make a student's skills and understanding visible	■ a range of methods in different modes that elicit the student's learning and metacognitive processes	■ a range of methods in different modes that assess both product and process
Ensuring Quality	■ accuracy and consistency of observations and interpretations of student learning ■ clear, detailed learning expectations ■ accurate, detailed notes for descriptive feedback to each student	■ accuracy and consistency of a student's self-reflection, self-monitoring, and self-adjustment ■ engagement of the student in considering and challenging his or her thinking ■ the student records his or her own learning	■ accuracy, consistency, and fairness of judgments based on high-quality information ■ clear, detailed learning expectations ■ fair and accurate summative reporting
Using the Information	■ provide each student with accurate descriptive feedback to further his or her learning ■ differentiate instruction by continually checking where each student is in relation to the curriculum outcomes ■ provide parents or guardians with descriptive feedback about student learning and ideas for support	■ provide each student with accurate, descriptive feedback that will help him or her develop independent learning habits ■ have each student focus on the task and his or her learning (not on getting the right answer) ■ provide each student with ideas for adjusting, rethinking, and articulating his or her learning ■ provide the conditions for the teacher and student to discuss alternatives ■ the student reports his or her learning	■ indicate each student's level of learning ■ provide the foundation for discussions on placement or promotion ■ report fair, accurate, and detailed information that can be used to decide the next steps in a student's learning

Source: Manitoba Education, Citizenship and Youth. *Rethinking Classroom Assessment with Purpose in Mind: Assessment* for *Learning, Assessment* as *Learning, Assessment* of *Learning*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2006, 85.

Instructional Focus

The Manitoba mathematics curriculum framework is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of learning outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Integration of the mathematical processes within each strand is expected.
- Problem solving, conceptual understanding, reasoning, making connections, and procedural thinking are vital to increasing mathematical fluency, and must be integrated throughout the program.
- Concepts should be introduced using manipulatives and gradually developed from the concrete to the pictorial to the symbolic.
- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Use educational resources by adapting to the context, experiences, and interests of students.
- Collaborate with teachers at other grade levels to ensure the continuity of learning of all students.
- Familiarize yourself with exemplary practices supported by pedagogical research in continuous professional learning.
- Provide students with several opportunities to communicate mathematical concepts and to discuss them in their own words.

"Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students" (Ontario 24).

Document Organization and Format

This document consists of the following sections:

- **Introduction:** The Introduction provides information on the purpose and development of this document, discusses characteristics of and goals for Middle Years learners, and addresses Aboriginal perspectives. It also gives an overview of the following:
  - **Conceptual Framework for Kindergarten to Grade 9 Mathematics:** This framework provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.
  - **Assessment:** This section provides an overview of planning for assessment in mathematics, including assessment *for*, *as*, and *of* learning.
  - **Instructional Focus:** This discussion focuses on the need to integrate mathematics learning outcomes and processes across the four strands to make learning experiences meaningful for students.
  - **Document Organization and Format:** This overview outlines the main sections of the document and explains the various components that comprise the various sections.
- **Number**—This section corresponds to and supports the Number strand for Grade 5 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Patterns and Relations:** This section corresponds to and supports the Patterns and Variables and Equations substrands of the Patterns and Relations strand for Grade 5 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Shape and Space:** This section corresponds to and supports the Measurement, 3-D Objects and 2-D Shapes, and Transformations substrands of the Shape and Space strand for Grade 5 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Statistics and Probability:** This section corresponds to and supports the Data Analysis and Chance and Uncertainty substrands of the Statistics strand for Grade 5 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Blackline Masters (BLMs):** Blackline Masters are provided to support student learning. They are available in *Microsoft Word* format so that teachers can alter them to meet students' needs, as well as in *Adobe* PDF format.
- **Bibliography:** The bibliography lists the sources consulted and cited in the development of this document.

Guide to Components and Icons

Each of the sections supporting the strands of the Grade 5 Mathematics curriculum includes the components and icons described below.

Enduring Understanding(s):

These summarize the core idea of the particular learning outcome(s). Each statement provides a conceptual foundation for the learning outcome. It can be used as a pivotal starting point in integrating other mathematical learning outcomes or other subject concepts. The integration of concepts, skills, and strands remains of utmost importance.

Essential Question(s):

These are used to guide students' learning experiences and may be useful when planning assessments. Inquiring into essential questions gives teaching and learning purposeful and meaningful focus for achieving the specific learning outcome(s).

Specific Learning Outcome(s):	**Achievement Indicators:**
Specific learning outcome (SLO) statements define what students are expected to achieve by the end of the grade. A code is used to identify each SLO by grade and strand, as shown in the following example: **5.N.1** The first number refers to the grade (Grade 5). The letter(s) refer to the strand (Number). The last number indicates the SLO number. [C, CN, ME, PS, R, T, V] Each SLO is followed by a list indicating the applicable mathematical processes.	Achievement indicators are examples of a representative list of the depth, breadth, and expectations for the learning outcome. The indicators may be used to determine whether students understand the particular learning outcome. These achievement indicators will be addressed through the learning activities that follow.

Prior Knowledge

Prior knowledge is identified to give teachers a reference to what students may have experienced previously. Teachers should assess students' prior knowledge before planning instruction.

Related Knowledge

Related knowledge is identified to indicate the connections among the Grade 5 Mathematics learning outcomes.

Background Information

Background information is identified to give teachers knowledge about specific concepts and skills related to the particular learning outcome.

Mathematical Language

Lists of terms students will encounter while achieving particular learning outcomes are provided. These terms can be placed on math word walls or used in a classroom math dictionary. *Kindergarten to Grade 8 Mathematics Glossary: Support Document for Teachers* (Manitoba Education, Citizenship and Youth) provides teachers with an understanding of key terms found in Kindergarten to Grade 8 mathematics. The glossary is available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/supports.html>.

Learning Experiences

Suggested teaching strategies and assessment ideas for the specific learning outcomes and achievement indicators are provided. In general, learning activities and teaching strategies related to specific learning outcomes are developed individually, except in cases where it seems more logical to develop two or more learning outcomes together. Suggestions for assessment include information that can be used to assess students' progress in their understanding of a particular learning outcome or learning experience.

Assessing Prior Knowledge:

Observation Checklist:

Assessing Understanding:

Suggestions are provided for assessing prior to and after lessons, and checklists are provided for observing during lessons to direct instruction.

Suggestions for Instruction

- **Achievement indicators appropriate to particular learning experiences are listed.**

The instructional suggestions include the following:

- **Materials/Resources:** Outlines the resources required for a learning activity.
- **Organization:** Suggests groupings (individual, pairs, small group, and/or whole class).
- **Procedure:** Outlines detailed steps for implementing suggestions for instruction.

Some learning activities make use of BLMs, which are found in the Blackline Masters section in *Microsoft Word* and *Adobe* PDF formats.

Putting the Pieces Together

Putting the Pieces Together tasks, found at the end of some learning outcomes, consist of a variety of assessment strategies. They may assess one or more learning outcomes across one or more strands and may make cross-curricular connections.

Grade 5 Mathematics

Number

Grade 5: Number (5.N.1)

Enduring Understandings:

The position of a digit in a number determines its value.

Each place value position is 10 times greater than the place value position to its right.

General Outcome:

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.N.1 Represent and describe whole numbers to 1 000 000. [C, CN, T, V]	➛ Write a numeral using proper spacing without commas (e.g., 934 567 and not 934,567). ➛ Describe the pattern of adjacent place positions moving from right to left. ➛ Describe the meaning of each digit in a numeral. ➛ Provide examples of large numbers used in print or electronic media. ➛ Express a given numeral in expanded notation (e.g., 45 321 = [4 x 10 000] + [5 x 1000] + [3 x 100] + [2 x 10] + [1 x 1] or 40 000 + 5000 + 300 + 20 + 1). ➛ Write the numeral represented in expanded notation.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Representing and describing whole numbers to 10 000 pictorially and symbolically
- Comparing and ordering whole numbers to 10 000
- Demonstrating an understanding of addition of numbers with answers to 10 000
- Demonstrating an understanding of subtraction of 3- and 4-digit numbers

Background Information

For students to work effectively with large numbers, they need to have a good understanding of the structure of our numeration system. The Hindu-Arabic, or base-10, numeration system that we use today originated in India around 500 CE, and was carried to other parts of the world by Arab people. The system gradually replaced the use of Roman numerals and the abacus in trade and commerce in Europe and, by the 16th century, was predominant. The features of the system that led to its acceptance and the computational procedures we use today include the following:

1. It consists of 10 **digits** (symbols), 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, that are used in combination to represent all possible numbers.
2. It has a base number. In this system, 10 ones are replaced by one group of 10, 10 tens are replaced by one hundred, 10 hundreds are replaced by one thousand, and so on. The number of objects grouped together is called the **base** of the system. Thus, the Hindu-Arabic system is a base-10 system.
3. It has **place value**. Each place in a numeral has its own value. For any place in the system, the next position to the left is 10 times greater and the position to the right is one-tenth as large.
4. It has a symbol for zero. The symbol has two functions. It is a placeholder in numerals like 5027, where it indicates there are no hundreds (or 50 hundreds). It is also the number that indicates the size of the set that has no objects in it.
5. It is additive and multiplicative. The value of a numeral is found by multiplying each place value by its corresponding digit and then adding all the resulting products. Expressing a numeral as the sum of its digits times their respective place values is called **expanded notation**. For example, the expanded notation for 8273 is (8 x 1000) + (2 x 100) + (7 x 10) + (3 x 1) or 8000 + 200 + 70 + 3.

Consequently, the focus of the learning experiences that follow is on helping students conceptualize the magnitude of large numbers and understanding the characteristics of our numeration system that allow us to read, write, and interpret the numerals for these numbers.

Mathematical Language

Base

Digit

Expanded notation

Hundred thousand

One million

Place value

Ten thousand

LEARNING EXPERIENCES

Assessing Prior Knowledge

Materials: BLM 5.N.1.1: Place Value

Organization: Individual

Procedure:

a) Tell students that in the next few lessons they will be learning about numbers greater than 10 000, but before they begin you need to find out what they already know about large numbers.

b) Ask students to complete the activity found on BLM 5.N.1.1.

Observation Checklist

Use students' responses to the questions to determine whether they can do the following:

- ❏ compare and order whole numbers in the thousands
- ❏ write numbers in words
- ❏ identify the place value position of the digits in a numeral
- ❏ identify the value of each digit in a numeral

- **Provide examples of large numbers used in print or electronic media.**

Materials: *A Million Dots* by Andrew Clements, calculators, stopwatch or timer with a second hand.

Organization: Whole class/Small groups

Procedure:

a) Ask students, "How many dots do you think you can draw in one minute? If we counted all the dots everyone in the class makes in one minute, how many dots do you think we would have altogether? Do you think we would have a million dots?"

b) Explain that a million is a big number and they are going to find out what a million dots looks like.

c) Read *A Million Dots*.

d) After reading the book, ask students whether they want to change their estimates of the number of dots that they can draw in one minute Have students draw dots for one minute. When they finish, have them suggest ways to count the dots. Encourage them to think about making groups of tens to facilitate the counting process.

e) Have students use the total number of dots that they make in one minute to determine how long it would take
   - one person to make a million dots
   - the class to make a million dots

f) Have each group decide what else they could do to show how big a million is. Help them devise and carry out a plan for showing the magnitude of the number. For example, students could determine
   - the length of 1 million loonies laid end to end
   - the number of pages a telephone book would need to have to list 1 million people
   - the number of boxes of toothpicks they would need to make a million

g) Have each group share their plans and what they found out about 1 million with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ make reasonable estimates
- ❑ solve computational problems with and without using technology
- ❑ develop and carry out a plan for solving a problem
- ❑ indicate that they have a sense of the magnitude of 1 million

- **Describe the pattern of adjacent place positions moving from right to left.**

Materials: Blank Hundred Square (BLM 5–8.6), scissors, tape, or stapler.

Organization: Small groups

Procedure:

a) Write the numbers 600 and 60 on the board or on an overhead. Ask students, "Would you rather have 600 pennies or 60 pennies?" Have students explain their reasoning.

b) Explain that the place of a digit within a number is important because it tells us the value of the digit, and today they will be learning more about place value.

c) Point to each digit in the numeral 600 and ask students, "What is the place value position of this digit?" Write students' responses on the board or overhead, and show students that the ones can be represented with a square, the tens with a strip of 10 squares, and hundreds with a grid of 100 squares.

d) Ask students, "What place value position comes next? How can we use the hundred squares to show 1000?" Let students explore different ways to arrange the hundred squares to make 1000. Each student should then make a 1000-strip by taping or stapling ten of the hundred squares together.

e) Ask students questions about the relationship between the different place value positions. For example:

- "How many hundreds in one thousand?"
- "How many times larger is one thousand than one hundred?"
- "How many tens are in one hundred?"
- "How many times larger is one hundred than ten?"
- "How many tens are in one thousand?"
- "How many times larger is one thousand than ten?"

f) Ask students, "What place value position comes next? How much larger than the thousands position should the new place value position be? Why do you think this? How can we use the 1000-strips to show the next place value position?" Have groups of 10 students staple or tape their 1000 strips together. When students finish making their 10 000 square strips, ask them questions about the relationship between the different place value positions similar to the ones in part (e).

g) Have students in each group work together to answer these questions: "What place value position comes next? What is the relationship of this position to the other place value positions? What would a model of this place value position look like?" Have each group share its answers with the other members of the class. Encourage students to explain their reasoning.

h) Repeat part (g) to introduce students to the millions position.

Observation Checklist

Observe students' responses to determine whether they can do the following:

❑ recognize that each place is 10 times greater than the place to its right

❑ describe the various relationships among the place value positions (e.g., 10 000 is 100 times greater than 100 and 1000 times greater than 10)

❑ describe place value positions to the millions

- **Write a numeral using proper spacing without commas.**

Materials: Paper and pencils, overhead copy of place value chart—whole numbers (BLM 5–8.7)

Organization: Whole class/Pairs

Procedure:

a) Write a number on the board or overhead (e.g., 62 893), and ask students, "How do you read the number? How do place value patterns help us read numbers?"

b) Show students a place value chart. Explain that when we read and write large numbers, we group the digits into threes. Each group of three forms a family. Each family has a different last name and is separated from the other families by a space. The family on the far right is the ones. The family to its immediate left is the thousands. The next family on the left is the millions. In each family, there is a place for ones, tens, and hundreds.

c) Tell students that, for the remaining time, they will be focusing on numbers in the thousands family. Record a number in the place value chart (e.g., 425 679), and explain how to read the number and what each digit in the number means. Do three or four more examples.

d) Have students work with their partner. Students need to sit so one person in a pair can see the board and the other one cannot. Write a number on the board (e.g., 286 164). Students facing the board read the number to their partner. Their partner writes the number down. Students then compare the number they wrote down with the number on the board. Repeat the activity several times, giving each student an opportunity to be both the "reader" and the "writer."

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ read a number correctly
- ❑ write a numeral correctly with proper spacing

- **Describe the meaning of each digit in a numeral.**

Materials: Number cards with the numbers 0 through 9 (BLM 5–8.5) with one number per card, and large strips of paper showing the place value headings (BLM 5.N.1.2), one for each group

Organization: Small groups (group size depends on the size of the numbers)

Procedure:

a) Put the place value column headings on the walls so they are just above the students' heads.

b) Say a number (e.g., 90 372). Students in each group must find the appropriate number cards, then arrange themselves into a line underneath the column headings showing the number you said. Encourage students to tell what each digit in the number means. Have students repeat the activity several more times.

c) Expand the place value column headings to include hundred thousands and have students form 6-digit numbers.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify the place value position of each digit in a number
- ❑ describe the meaning of each digit in a number

- **Describe the meaning of each digit in a numeral.**

Materials: None

Organization: Whole class

Procedure:

a) Tell students they will be doing some number calisthenics. Explain that they will be acting out a number you write on the board or overhead projector. They must act out the number by doing in sequence:

   - as many hops on their left foot as specified by the value of the digit in the hundred-thousands
   - as many jumping jacks as the value of the digit in the ten-thousands position
   - as many clap-your-hands as in the value in the thousands position
   - as many touch-your-toes as the value in the hundreds position
   - as many hops on their right foot as the value of the digits in the tens place
   - as many finger snaps as the value of the digits in the ones position

   For example, for the number 243 167, students would do

   - 2 hops on their left foot
   - 4 jumping jacks
   - 3 clap-your-hands
   - 1 touch-your-toes
   - 6 hops on their right foot
   - 7 finger snaps

b) Have students act out 527 483; 298 645; and 738 295. When students are familiar with the movements for each place value position, have them act out 3-digit, 4-digit, 5-digit, and 6-digit numbers.

c) Vary the activity by having students choose the numbers that they act out.

Observation Checklist

Observe students' responses to determine whether they can do the following:

❏ identify the correct place value position of each digit in the number

❏ identify the value of each digit in a number

■ **Write a numeral using proper spacing without commas.**

Materials: Paper and pencils

Organization: Whole class

Procedure:

a) Write four or five numbers on the board that use different arrangements of the same digits. For example:

10 053 10 503 10 530 13 005 13 530

b) Read one of the numbers (e.g., ten thousand five hundred thirty). Ask the students to tell you which one you chose. Have students explain how they knew which number you read. Encourage students to describe what the "zeros" in each numeral mean.

c) Continue reading the numbers and having students identifying them. When they finish identifying all the numbers, have them order the numbers from smallest to largest and then write the numbers in expanded notation.

d) Repeat the activity using different sets of numbers.

e) Vary the activity by using six-digit numbers instead of five-digit numbers.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ describe the meaning of each digit in a numeral
- ❑ describe the pattern of place value positions moving from right to left
- ❑ identify the place value position of each digit in a numeral
- ❑ express a numeral in expanded notation

- **Write a numeral using proper spacing without commas.**
- **Express a given numeral in expanded notation.**

Materials: Dice

Organization: Small groups

Procedure:

a) Distribute six dice to each group. Tell students that they will be tossing the dice five times. The first time they roll the dice they should create a six-digit number with the numbers that they roll. They should record the number and then write it in expanded notation. Next, they should remove one die and roll the remaining five dice to create a five-digit number. Again, they should write the number in both standard notation and expanded notation. Students should continue removing a die, creating a number with the numbers that are rolled, and recording the numbers in standard notation and expanded notation until they have one die left.

b) Have each group share its results with the other members of the class.

c) Vary the activity by having students write the number in standard notation and in words.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ write a numeral using proper spacing without commas
- ❏ express a given numeral in expanded notation
- ❏ write a numeral in expanded notation

- **Write a numeral using proper spacing without commas.**
- **Express a given numeral in expanded notation.**
- **Write the numeral represented in expanded notation.**

Materials: Calculators, overhead of the excerpt from *Which Do You Prefer – Chunky or Smooth?* (BLM 5.N.1.3)

Organization: Individual

Procedure:

a) Have students read the excerpt shown below. Ask them to rewrite the number words using numerals and rewrite the numerals using number words. In addition, have students write each of the numbers in expanded notation.

In her book called *Which Do You Prefer – Chunky or Smooth?*, Heather Brazier tells us the following:

> On an average day in Canada …we consume eighty thousand, eight hundred forty-nine kilograms of peanut better. Of the total, 20 212 kg are chunky… (46)

b) Have students figure out how much smooth peanut better must be eaten by Canadians on an average day. Have them write their answer as a numeral and in words.

c) Throughout the year, have students bring in examples of large numbers that they find in newspapers or magazines. Keep a class chart that shows the number in numerals, expanded form, and words.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ write numbers in word form
- ❏ write the numeral for a number written in words
- ❏ write a numeral in expanded notation
- ❏ provide examples of large numbers used in print or electronic media

- **Describe the meaning of each digit in a numeral.**

Materials: Calculators

Organization: Whole class

Procedures:

a) Ask students to show 83 247 on their calculators. Tell them that their goal is to change the 2 to 0 (zero it) by subtracting one number. When students finish, ask them the following:
   - "What number do you have on your calculator now?"
   - "What number did you subtract to wipe out the 2?"
   - "Why did you subtract that number?"

b) Continue asking students to show five-digit and six-digit numbers on their calculators. After you name a number for them to show on their calculators, ask them to zero a digit in one of the place value positions. Encourage students to describe what they did and why they did it to zero a digit.

c) Ask students to add a number to wipe out a digit (e.g., adding 4 can wipe out the 6 in 506).

d) Vary the activity by telling students that they can use either addition or subtraction to wipe out a digit.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify the place value position of each digit in a numeral
- ❏ identify the value of each digit in a numeral
- ❏ use technology to compute sums and differences

Materials: Number cards (BLM 5–8.5) (one set per student)

Organization: Groups of three or four

Procedure:

a) Tell students that they will be playing a place value game. Explain how to play the game.

   1. Hand each student a complete set of cards. Once students are in the group, all cards should be combined together.
   2. Shuffle the cards and lay them face down in the playing area.
   3. Players take turns drawing five cards from the deck.
   4. Players arrange the cards in their hands so that they have the largest possible number.
   5. One player says, "Let's see the numbers" and everyone lays their cards face up in front of them. A card cannot be moved after it has been placed face-up on the playing surface.
   6. Players take turns reading the number that they created. The player who has the largest number and reads the number correctly wins a point.
   7. The winner is the person with the most points after five rounds of the game.

b) Demonstrate how to play the game and answer any questions that students may have. Have students play the game.

c) Vary the game by having students

   - draw six cards instead of five
   - create the smallest possible number with their cards

Observation Checklist

Observe students' responses to determine whether they can do the following:

❑ read 5-digit and 6-digit numbers correctly

❑ use place value concepts to determine which of two or more numbers is the largest

❑ use place value concepts to determine which of two or more numbers is the smallest

- **Write a numeral using proper spacing without commas.**
- **Describe the meaning of each digit in a numeral.**

Materials: Calculators, paper and pencils

Organization: Pairs

Procedure:

a) Tell students that they will be playing a game called "Give and Take." Explain how to play the game.

   1. Players write down a six-digit number containing no zeros and no identical digits. Players keep their numbers hidden from each other throughout the game.
   2. Players take turns being the giver and the taker. Each player tries to increase his or her number by taking digits from the other player.
   3. A turn begins when the asker says: "Give me your x's, where 'x' can be any digit from 1 through 9." (e.g., "Give me your 7s.").
   4. If that digit is in the giver's number, the giver announces its place value (e.g., "You get 700." If the digit is not in the giver's number, the giver announces this by saying, "You get zero.").

      Note that the value of a digit that is asked for depends on its position in the giver's number. If 7 is asked for and the number is 325 714, then the giver says, "You get 700." If the giver's number is 372 514, then the giver says " you get 70 000."
   5. As soon as the giver responds with the number, the asker adds that amount to his or her number (e.g., + 700) and the giver subtracts that amount from his or her number (e.g., –700).
   6. Players' numbers change with each new addition or subtraction. Players always use the most recent form of their numbers when adding, subtracting, or announcing the place value of a digit. Players keep track of their changing number by adding and subtracting from their original number and its successors. For example:

      325 714 + 20 000 = 345 714

      345 714 – 5 000 = 340 714
   7. If the same digit appears two or more times in a giver's number during play, the giver can say either of its values (e.g., for 845, 218, the giver can say 8 and not mention the 800 000).
   8. The game ends after each player has had five turns as asker. Players check each other's addition and subtractions. The player with the largest number is the winner.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify the correct place value position of each digit
- ❏ identify the value of each digit in a number
- ❏ use calculators correctly to determine sums and differences
- ❏ write a numeral with the proper spacing with no commas

NOTES

Grade 5: Number (5.N.2)

Enduring Understandings:

Computational estimations produce approximate answers.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
5.N.2 Apply estimation strategies, including ■ front-end rounding ■ compensation ■ compatible numbers in problem-solving contexts. [C, CN, ME, PS, R, V]	➛ Provide a context for when estimation is used to ■ make predictions ■ check reasonableness of an answer ■ determine approximate answers ➛ Describe contexts in which overestimating is important. ➛ Determine the approximate solution to a problem not requiring an exact answer. ➛ Estimate a sum or product using compatible numbers. ➛ Estimate the solution to a problem using compensation, and explain the reason for compensation. ➛ Select and use an estimation strategy to solve a problem. ➛ Apply front-end rounding to estimate ■ sums (e.g., 253 + 615 is more than 200 + 600 = 800) ■ differences (e.g., 974 – 250 is close to 900 – 200 = 700) ■ products (e.g., the product of 23 x 24 is greater than 20 x 20 or 400 and less than 25 x 25 or 625) ■ quotients (e.g., the quotient of 831 ÷ 4 is greater than 800 ÷ 4 or 200)

Prior Knowledge

Students may have had experience with the following:

- Adding whole numbers with sums less than 10 000
- Subtracting whole numbers with differences less than 10 000
- Using different strategies to estimate sums and differences
- Multiplying a 1-digit whole number times a 2-digit or 3-digit whole number
- Using a personal strategy to estimate a product
- Dividing a 2-digit whole number dividend by a 1-digit whole number divisor
- Using a personal strategy to estimate a quotient

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of multiplication (1- and 2-digit multipliers and up to 4-digit multiplicands)
- Demonstrating an understanding of division (1- and 2-digit divisors and up to 4-digit dividends)

Background Information

Computational estimation is the process of determining approximate answers to computational problems. Students who are skillful estimators have a good grasp of basic facts, place value, and the operations of addition, subtraction, multiplication, and division. They are also adept at mental mathematics and flexible in their use of estimation strategies, such as the ones described below.

Front-End Estimation:

Front-end rounding involves identifying the most significant (left-most) digits in a question, performing the appropriate operation, and determining the place value of the digits. For example:

- 654 + 714 + 435 is more than 1700 since 6 + 7 + 4 = 17 (and annex the zeros)
  (or since 600 + 700 + 400 = 1700)
- 532 − 285 is approximately 300 since 5 − 2 = 3 (and annex the zeros)
  (or since 500 − 200 = 300)
- 4 x 728 is more than 2800 since 4 x 7 = 28 (and annex the zeros)
  (or since 4 x 700 = 2800)
- 926 ÷ 3 is more than 300 since 9 ÷ 3 = 3 (and annex the zeros)
  (or since 900 ÷ 3 = 300)

Note: It is important for teachers to emphasize estimation skills. Discourage students from calculating first, then estimating (e.g., "I know 2.5 + 4.7 is 7.2, so I will estimate it is close to 7.").

Although front-end rounding can be used with any operation, it is most powerful when adding and multiplying. With these two operations, the computation is always underestimated.

Compatible Numbers:

This strategy involves searching for pairs of numbers that are easy to compute. When using this strategy, students look at all the numbers in a problem, and change or round the numbers so they can be paired usefully with another number. It is particularly effective for division. For example, in the question 2270 ÷ 6, rounding the dividend to 2300 (the closest hundred) or 2000 (the closest 1000) does not facilitate the estimation process. However, rounding it to 2400 (a compatible number because it is divisible by 6) makes estimating the quotient easier.

This strategy is also useful for addition. For example, when adding several numbers, students look for numbers that can be paired or grouped together to make multiples of 10.

about 100

25 + 45 + 63 + 81 or about 100 < 27, 45, 63, + 81 > about 100

about 100

Therefore, the sum of 27 + 45 + 63 + 81 is about 200

Compensation:

Compensation involves refining, or adjusting, an original estimate that was obtained with another strategy. For example, the front-end estimation of 220 for the sum 86 + 23 + 72 + 55 can be adjusted to 240, since 6 + 3 and 2 + 5 (the numbers in the ones position) are both close to 10. Similarly, the front-end estimation of 2400 for 43 x 62 can be adjusted to 2600 since (3 x 60) + (2 x 40) would be greater than 200. Also, for 44 x 54, for example, you can round one number up and one number down and compute 50 x 50 for an estimate of 2500, rather than front-end rounding for an estimate of 2000.

In many instances, different strategies can be applied to the same problem. The choice of strategies depends on the students, the numbers, and the operations involved. Teachers need to help students become aware of the various strategies and help them develop confidence in their ability to estimate. To do this, they need to

- engage students in discussions about the strategies they used to estimate the solution to a computational problem (Sharing strategies can lead to the development and use of new strategies.)
- accept a range of estimates in order to help students understand that there is no one "right" estimate
- encourage students to identify real-world situations that involve estimations

- incorporate estimation throughout their instructional programs (Like problem solving, estimation should not be taught in isolated units.)

Mathematical Language

Annex

Approximate

Compatible numbers

Compensation

Estimate

Estimation

Front-end rounding

Learning Experiences

Assessing Prior Knowledge

Materials: None

Organization: Individual

Procedure:

a) Tell students you need to know what they know about computational estimation so you can help them become better estimators. To find out what they know, give them some problems to estimate. Tell students that you will show them several problems, one at a time, and they will have an appropriate amount of time (decide this based on individual students—approximately 30 seconds) to estimate the solution to each problem. They must record their estimate before their time is up.

b) Give students the following problems:

1. 84 + 27 + 35 + 62
2. 892 + 154
3. 4821 + 3179
4. 628 – 147
5. 5372 – 3124
6. 8 x 12
7. 4 x 356
8. 86 ÷ 4

c) Have students share their estimates and the strategies they used to determine them.

Observation Checklist

❑ Check students' responses to the problems to determine whether they can estimate the solutions to addition, subtraction, multiplication, and division problems. Use the class discussion to find out what strategies students use to make their estimates.

- **Provide a context for when estimation is used.**

Materials: Markers and newspapers/magazines

Organization: Whole class

Procedure:

a) Tell students that they are going to investigate the use of estimated and exact numbers.

b) Give students copies of different newspapers. Ask them to circle the numbers used in the headlines and articles. Next, have students review the context for the use of each circled number to determine whether the numbers in the headlines or articles refer to exact or estimated (approximate) values. For example, have students decide whether these statements taken from a newspaper refer to exact or estimated values:

   - One million people evacuated from New Orleans
   - The condo resold for $134 000
   - Last year, 13 142 tons of scrap metal were recycled

c) Engage students in a discussion about the numbers they found in the newspapers. Encourage them to explain why they think a given number is exact or estimated. Have students discuss why estimated numbers are often used in newspaper articles (e.g., estimated numbers are easier to interpret and use).

Observation Checklist

Observe students' responses to determine whether they can do the following:

❑ identify real-world examples of estimated numbers

❑ distinguish between exact and estimate (approximate) numbers

❑ give a reasonable explanation why a number is either exact or estimated

Materials: Situation Cards (BLM 5.N.2.1) and index cards

Organization: Small groups

Procedure:

a) Give each group a card with one of the situations on it.

b) Ask students to decide whether the situation on their card requires an estimated answer or an exact answer, and to list the reasons for their response.

c) Have each group read its situation to the other members of the class, and explain why they think the situation requires an estimated or exact answer.

d) Have each group create a situation card. Each group should record, on a separate piece of paper, the reasons why they think the situation they created requires an estimated or exact answer. Have the groups exchange cards and decide whether the new situation they were given requires an estimated or exact answer and why they think so. Each group should compare its response with the response of the group who created the situation.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ provide a context for when estimation is used to approximate an answer
- ❑ provide a context for when estimation is used to predict an answer
- ❑ distinguish between situations that require an exact answer and those that require an estimated answer
- ❑ give reasonable explanations of why a situation requires either an exact or estimated answer

- **Provide a context for when estimation is used.**
- **Determine the approximate solution to a problem not requiring an exact answer.**

Materials: None

Organization: Pairs/Whole class/Small groups

Procedure:

a) Present students with the following problem:

> The 28 students in Mr. Nelson's fifth-grade class are planning a Halloween party. The students decided to make a fruit punch for everyone to drink at the party. They know that a can of juice makes eight cups of punch. How many cans of juice should they buy?

b) Give students time to solve the problem, and then have them share their solutions with their partner.

c) Have students discuss their solutions and share their reasoning with the other members of the class. Help them recognize that there are times we need an estimate because there is not enough information to determine an exact answer (e.g., we do not know how thirsty students will be).

d) Ask each group to identify other situations that require an estimate because there is not enough information to compute an exact answer.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify situations that require an estimate because there is not enough information to compute an exact answer
- ❑ provide a context when estimating is used to make predictions
- ❑ provide reasonable estimates
- ❑ give reasonable explanations for their estimates

- **Describe contexts in which overestimating is important.**

Materials: Copies of estimation situations (BLM 5.N.2.2)

Organization: Pairs/Large group

Procedure:

a) Ask students to read each of the situations, and decide whether an overestimate or underestimate is needed.

b) Have students discuss their answers with their partners. Then have students share their answers and the reasons for them with the other members of the class.

c) Have students describe other estimation situations and decide whether an underestimate or an overestimate is needed.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify and describe contexts in which overestimating is important
- ❏ identify and describe contexts in which underestimating is important

- **Determine the approximate solution to a problem not requiring an exact answer.**

Materials: Number tiles or number cards (BLM 5–8.5), calculators

Organization: Individual/Pairs/Whole class

Procedure:

a) Ask students to complete the following activity:

   Explain that they must use the number tiles 4–9 to create 3-digit by 1-digit multiplication problems. The products of the problems must be as close to the target as possible. They get three tries for each target number. They should record each problem they create and its solution. Tell them they can use their calculators to find the solution to the problems they create.

   Target

   _____ _____ _____ x _____ = 5000

   _____ _____ _____ x _____ = 8000

   _____ _____ _____ x _____ = 7000

   _____ _____ _____ x _____ = 4000

b) When students finish, have them compare their estimates with a partner and explain the strategies they used to create the problems.

c) Have students share the strategies they used to create the problems with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ select and use an estimation strategy to estimate the product of two numbers
- ❑ make reasonable adjustments to their estimates of the product of two numbers
- ❑ explain the strategy they used to estimate the product of two numbers

Materials: Dice, calculators, paper and pencil

Organization: Pairs

Procedure:

a) Tell students that they will be playing an estimation game with their partner. Explain how the game is played.

   1. One player tosses three dice and creates a 3-digit number with the numbers that are rolled. This number becomes the dividend.
   2. The other player tosses one die, and the number that is rolled becomes the divisor. Both players should record the problem.
   3. Players record the problem, then quickly and silently write an estimate of the quotient of the two numbers. Players should not take more than 10 or 15 seconds to write their estimates. A 15-second timer could be used to time student estimates, or a third student could ensure a fair time period has elapsed.
   4. Players use a calculator to find the quotient of the two numbers. Each player's score is the difference between his or her estimate and the exact answer.
   5. The person with the lowest score after five rounds of the game is the winner.

b) Demonstrate how to play the game and answer any questions students may have. Have students play the game.

c) Have students share some of the problems they created and the strategies that they used to estimate the quotient.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ select and use an estimation strategy for division
- ❏ use technology to determine the solution to a division problem
- ❏ describe the strategy they used to determine an estimate

- **Apply front-end rounding to estimate sums, differences, products, and quotients.**
- **Estimate the solution to a problem using compensation, and explain the reason for compensation.**

Materials: Paper and pencil

Organization: Whole class

Procedure:

a) Present students with the following problem:

> Mrs. Martin's class was estimating the sums of addition problems. Matty said that the sum of 66 + 23 + 74 is about 150. How did Matty get her estimate?

b) Have students share their ideas about how the sum was estimated. Then ask, "Is the sum of 66 + 23 + 74 over or under 150? How do you know? How could you get a closer estimate of the sum?"

c) Explain that when we use the front-end rounding strategy to estimate the sum of two or more numbers, we can always adjust our estimate by looking at the other digits in the problem. For example, if we look at the digits in the ones position in the problem 66 + 23 + 74, 6 + 3 is close to 10. So the sum of the ones is greater than 10. Therefore, we can adjust our estimate to 160.

d) Do two or three more examples, and then ask students to use the front-end rounding strategy to estimate the sum of the following problems, and then adjust their estimates to get a closer approximation of the solution.

- 62 + 49 + 88 + 21
- 14 + 23 + 85 + 91
- 19 + 30 + 83 + 54 + 57

d) Have students share their estimates. Encourage them to describe how they adjusted their estimates to get a closer approximation.

e) Have students use front-end rounding to estimate the sums of problems with 3- and 4-digit numbers, and adjust their estimates to get a closer approximation.

f) Use a similar approach to help students learn how to adjust problems involving subtraction, multiplication, and division.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ use front-end rounding to estimate the sum (difference, product, quotient) of two or more numbers
- ❏ estimate the solution to a problem using compensation, and explain the reasons for compensating

N OTES

Grade 5: Number (5.N.3)

Enduring Understandings:

Proficiency with the basic facts facilitates estimation and computation with larger and smaller numbers.

Multiplication and division are inverse operations.

General Outcome:

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.N.3 Apply mental math strategies to determine multiplication facts and related division facts to 81 (9 x 9). [C, CN, ME, R, V] Recall of multiplication facts to 81 and related division facts is expected by the end of Grade 5.	➛ Describe the mental mathematics strategy used to determine a basic fact, such as ■ skip-count up by one or two groups from a known fact (e.g., if 5 x 7 = 35, then 6 x 7 is equal to 35 + 7 and 7 x 7 is equal to 35 + 7 + 7) ■ skip-count down by one or two groups from a known fact (e.g., if 8 x 8 = 64, then 7 x 8 is equal to 64 – 8 and 6 x 8 is equal to 64 – 8 – 8) ■ halving/doubling (e.g., for 8 x 3 think 4 x 6 = 24) ■ use patterns when multiplying by 9 (e.g., for 9 x 6, think 10 x 6 = 60, then 60 – 6 = 54; for 7 x 9, think 7 x 10 = 70, and 70 – 7 = 63) ■ repeated doubling (e.g., if 2 x 6 is equal to 12, then 4 x 6 is equal to 24, and 8 x 6 is equal to 48) ■ repeated halving (e.g., for 60 ÷ 4, think 60 ÷ 2 = 30 and 30 ÷ 2 = 15) ■ relating multiplication to division facts (e.g., for 7 x 8, think 56 ÷7 =) ■ use multiplication facts that are squares (1 x 1, 2 x 2, up to 9 x 9) ➛ Refine personal strategies to increase efficiency (e.g., for 7 x 6, use known square 6 x 6 + 6 instead of repeated addition 6 + 6 + 6 + 6 + 6 + 6 + 6).

Prior Knowledge

Students may have had experience with the following:

- Explaining the properties of 0 and 1 for multiplication and the property of 1 for division
- Describing and applying mental mathematics strategies, such as
  - skip-counting from a known fact
  - using halving, doubling
  - using doubling and adding one more group
  - using patterns in the 9s facts
  - using repeated doubling

  to develop an understanding of basic multiplication facts to 9 x 9 and related division facts

Recall of the multiplication and related division facts up to 5 x 5 is expected by the end of Grade 4.

- Using arrays to represent multiplication facts

Background Information

Calculations people do on a daily basis involve knowing basic math facts. For this reason, basic facts continue to be an integral part of the mathematics curriculum. In the Early Years, students are expected to recall some facts and use strategies to determine others in order to help them learn more sophisticated mathematics. If students entering the Middle Years do not have the strategies to determine or recall the basic facts, then teachers need to teach the strategies and help students work towards these skills.

Thinking strategies provide students with different approaches for arriving at an answer. Students also strengthen their number sense and learn to adapt these strategies when working with larger numbers. Once students have had time to practice these strategies in game or activity settings, then different methods can be implemented to help students develop and maintain the ability to recall and be able to determine the facts that are appropriate for the grade level.

"Learning math facts is a developmental process where the focus of instruction is on thinking and building number relationships. Facts become automatic for students through repeated exposure and practice. When a student recalls facts, the answer should be produced without resorting to inefficient means, such as counting. When facts are automatic, students are no longer using strategies to retrieve them from memory." (Manitoba Education. *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*, p. 12)

As part of a balanced mathematics program, it is useful to be able to add, subtract, multiply, and divide quickly. It is also important to know facts without having to rely on inefficient methods of working them out. “While computational recall is important, it is only a part of a comprehensive mathematical background that includes more complex computation, an understanding of mathematical concepts, and the ability to think and reason to solve problems.” (Seeley 1).

In *About Teaching Mathematics*, Marilyn Burns writes the following:

> What about using timed tests to help children learn their basic facts? This makes no instructional sense. Children who perform well under time pressure display their skills. Children who have difficulty with skills, or who work more slowly, run the risk of reinforcing wrong learning under pressure. In addition, children can become fearful and negative toward their math learning. Also, timed tests do not measure children’s understanding. . . . It doesn’t ensure that students will be able to use the facts in problem-solving situations. Furthermore, it conveys to children that memorizing is the way to mathematical power, rather than learning to think and reason to figure out answers (2000, p. 157).

According to John A. Van de Walle (2006), speed (using timed tests) “is effective only for students who are goal oriented and who can perform in pressure situations. The pressure of speed can be debilitating and provides no positive benefits. The value of timed tests as a learning tool can be summed up as follows:

Timed Tests

- Cannot promote reasoned approaches to fact mastery
- Will produce few long-lasting results
- Reward few
- Punish many
- Should generally be avoided

“If there is any defensible purpose for a timed test of basic facts it may be for diagnosis—to determine which combinations are mastered and which remain to be learned. Even for diagnostic purposes there is little reason for a timed test more than once every couple of months” (pp. 95–96).

Research shows that timed tests can be more harmful than beneficial and are associated with lowering the level of fact mastery (Isaacs & Carroll, 1999). Prematurely demanding speed of fact retrieval can cause anxiety and can undermine understanding (Isaacs & Carroll). Timed tests cause many students to become quicker at immature approaches (Isaacs & Carroll; Ezbicki, 2008). For this reason, it is recommended that students master strategies for efficient fact retrieval prior to practicing those facts for fluency (accuracy and speed) (Woodward, 2006). Timed tests discourage the use of thinking strategies, as students are less likely to explore the more sophisticated strategies necessary to make progress if they are being timed (Isaacs & Carroll).

Daily timed tests, and even weekly or monthly timed tests, are unnecessary (Isaacs & Carroll, 1999). However, timed tests can be used every few months or so to assess fact proficiency (Isaacs & Carroll).

Principles and Standards for School Mathematics defines computational fluency as "... having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate *flexibility* in the computational methods they choose, *understand* and can explain these methods, and produce accurate answers *efficiently*. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships" (p. 152).

Many of the following activities should be used multiple times throughout the school year to encourage repeated exposure and practice for the basic facts.

Mathematical Language

Divide

Dividend

Division

Divisor

Factor

Multiplication

Multiply

Product

Quotient

Learning Experiences

Assessing Prior Knowledge

Materials: Math journals, Mental Math Strategies checklist (BLM 5–8.8)

Organization: Individual/Whole class

Procedure:

a) Ask students to solve each of the following problems in two different ways:

- Rosa is planning to arrange 48 books on six shelves. If she puts an equal number of books on each shelf, how many books will she put on each shelf?
- Mark has a six-page photo album. How many pictures does Mark have if each page holds eight pictures?

b) Have students share their solutions with the other members of the class. Encourage students to explain their reasoning by asking questions, such as:

- "Which strategy did you use to solve the problem?"
- "What is another strategy you could use to solve the problem?"
- "Will the strategy work for other problems involving division (multiplication)? Show me."
- "Which strategy do you prefer to use? Why?"

Observation Checklist

Use students' responses and BLM 5–8.8 to determine which strategies students know. Also, examine their responses to determine whether they can do the following:

- ❏ identify problem situations that call for the operation of multiplication
- ❏ identify problem situations that call for the operation of division
- ❏ describe and apply a thinking strategy to determine the product or quotient of two whole numbers
- ❏ describe and apply more than one thinking strategy to determine the product or quotient of two whole numbers

- **Describe the mental mathematics strategy used to describe a basic fact.**
- **Refine personal strategies to increase efficiency.**

Materials: Activity sheets that show the same array repeated about eight times. For example:

1. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	2. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	3. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	4. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx
5. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	6. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	7. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx	8. xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx xxxxxxx

Organization: Individual/Whole class

Procedure:

a) Show students an 8 x 7 array. Ask them to split the array to show a strategy for finding the product of 8 x 7 and have them describe the corresponding number sentences. For example:

xxxxxxx
xxxxxxx
xxxxxxx
xxxxxx
xxxxxxx
xxxxxxx
xxxxxxx

xxxxxxx

7 x 7 = 49
1 x 7 = 7

Therefore, 8 x 7 = 49 + 7 = 56

Ask students to split the remaining arrays in different ways to show the various strategies that can be used to find the product of 8 x 7. Have them record the corresponding number sentences for each strategy that they find.

b) Have students share the strategies they found with the other members of the class. Ask, "What strategy is the easiest? Why do you think this?"

c) Repeat the activity for other multiplication facts and have students compare the strategies that they find for each fact.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ describe the mental math strategy used to determine a multiplication fact such as skip counting by 1 or 2 groups from a known fact
- ❑ identify and apply different mental math strategies to determine a multiplication fact such as doubling, repeated doubling (e.g., 8 x 7 = [2 x 7] + [2 x 7] + [2 x 7] + [2 x 7] = 56), or doubling plus one or two groups (e.g., 8 x 7 = [3 x 7] + [3 x 7] + [2 x 7] = 56)

Materials: Paper and pencils

Organization: Whole class/Pairs

Procedure:

a) Have students study the following array and then ask them how knowing the facts of 5 can help them with other facts.

x x x x x x
x x x x x x
x x x x x x
x x x x x x
x x x x x x
x x x x x x
x x x x x x

5 x 6 = 30
2 x 6 = 12

Therefore, 7 x 6 = 42.

b) Have students describe how using a "think 5 facts" strategy can help them determine these fact problems. If necessary, have students draw the corresponding arrays.

9 x 6 =

8 x 3 =

6 x 4 =

7 x 7 =

c) Have students make a list of other facts that they could determine easily using a "think 5 facts" strategy. Have them share their lists with their partner and describe how they would use the strategy to solve each fact that they listed.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ describe how the "facts of 5" strategy can be used to determine a basic multiplication fact
- ❑ identify multiplication facts that can be determined using a "facts of 5" strategy

Materials: Paper and pencil

Organization: Pairs

Procedure:

a) Give students a fact problem such as 6 x 8. Have the first student in each pair do one part of the problem (e.g., 4 eights is 32). The second student must finish the problem—in this case, 2 eights is 16. The first student then adds the two parts together to determine the product. Have the students record the strategy that they used. Encourage students to split the problem into parts that are easy to find.

b) Repeat the activity several times, but have students switch roles.

c) Have students share the strategies they used with the other members of the class and discuss which strategies are the easiest and most efficient to use.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify and apply different mental math strategies to determine a multiplication fact
- ❑ describe the mental math strategies used to determine a multiplication fact

Materials: Paper and pencil

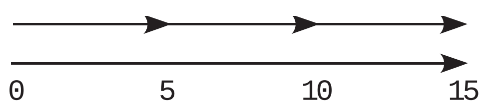
Organization: Whole class

Procedure:

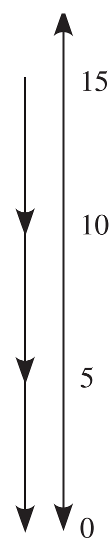
a) Ask students to write four number sentences using only the numbers 3, 5, and 15. When they finish, ask them to show the facts (3 x 5 = 15, 5 x 3 = 15, 15 ÷ 3 = 5, and 15 ÷ 5 = 3) on number lines.

For example:

3 x 5 = 15



15 ÷ 5 = 3



c) Ask students what they notice about the facts and how you showed them on the number lines. (Students should notice the inverse relationship between multiplication and division, although they may use the terms "opposite" or "backwards.")

d) Have students write four math sentences for each of the following triplets of numbers:

6 7 42

8 3 24

4 5 20

9 8 72

e) Ask students how a division fact can be determined by thinking a multiplication fact. Then have them use the relationship between multiplication and division to describe the thinking strategy for solving the following division problems. For example, for 36 ÷ 9 = , think 'some number' x 9 = 36 since 4 x 9 = 36, 36 ÷ 9 must equal 4.

- 48 ÷ 6 =	16 ÷ 4 =
- 63 ÷ 7 =	27 ÷ 3 =
- 18 ÷ 3 =	40 ÷ 8 =
- 32 ÷ 4 =	49 ÷ 7 =
- 56 ÷ 8 =	72 ÷ 9 =

Observation Checklist

Observe students' responses to determine whether they can do the following:

❑ identify the multiplication and division facts that can be associated with a given triplet of numbers

❑ identify the inverse of a multiplication fact

❑ identify the inverse of a division fact

❑ describe how a "think multiplication" strategy can be used to recall a division fact

❑ recognize the inverse relationship between multiplication and division

Materials: A deck of cards with the face cards removed.

Organization: Small groups

Caution: In some communities, playing cards are seen as a form of gambling and discouraged. Please be aware of local sensitivities before introducing this activity.

Procedure:

a) Tell students that they are going to play a variation of the game " I spy" with the members of their group. Explain how the game is played.

   1. Lay the cards on the playing surface face up in five rows of 8.
   2. Players take turns challenging each other to find two cards that have a specific product. The two cards must be next to each other horizontally, vertically, or diagonally. For example, if a 5 and a 3 are next to each other, a player could say, "I spy two cards whose product is 15."
   3. The other players look for the cards. The player who finds the right combination takes the two cards. If the combination cannot be found then the player who posed the "I spy" question takes the two cards.
   4. If a player makes an error or there is no such combination of cards, nobody collects any cards and the next player takes his or her turn.
   5. As cards are removed, the remaining cards should be rearranged to fill in the spaces.
   6. The game is over when all the cards have been picked up. The winner is the player with the most cards.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

Variation: Instead of using playing cards, copy four sets of number cards for each student (BLM 5–8.5). Omit the "0" card.

Observation Checklist

Observe students' responses to determine which

- ❏ facts that they can recall easily
- ❏ facts that they have difficulty recalling

Materials: Tic-tac-toe grids (BLM 5.N.3.1)

Organization: Pairs

Procedure:

a) Tell students that they will be playing "Multiplication Tic-tac-toe." Explain how the game is played.

1. Players decide who will be "X" and who will be "O."
2. Each player lists the numbers from 1 to 9 in a column beside the multiplication grid.
3. The first player crosses out any number in his or her column of numbers.
4. Beginning with the second player, the game proceeds in this manner. During a turn, a player crosses off any number in his or her column of 9 numbers that has not been crossed off. The player then multiplies that number by the last number crossed off by his or her opponent. If the product is on the tic-tac-toe board and not yet crossed off the player places an x or O over the product. For example:
   - player 1 crosses off the number 9 in his or her column of numbers
   - player 2 crosses off the number 7 in his or her column of numbers (Since 7 x 9 = 63, player 2 places an X [or O] over the 63 on the grid.)
   - player 1 crosses off 5 in his or her column of numbers (Since 5 x 7 = 35, player 1 places an O [or X] over the 35 on the grid.)
5. The game ends when any of the following occur:
   - a player gets three marks in a row (as in tic-tac-toe)
   - all of the numbers on the grid are marked off with either an X or an O
   - all nine numbers in a player's column of numbers are marked off

b) Demonstrate the game and answer any questions that students might have. Have the students play the game.

c) Have students play the game using these grids

d) Have students make their own multiplication tic-tac-toe grids and use them to play the game with their partner.

e) Have students discuss the strategies that they used to win the game.

Observation Checklist

Observe students' responses to determine which

- ❑ facts that they can recall easily
- ❑ facts that they have difficulty recalling
- ❑ strategies they are using to win the game

Materials: Copies of the division puzzle (BLM 5.N.3.2)

Organization: Individual/Pairs

Procedure:

a) Tell students that their task is to find the ten division facts that are hidden horizontally and vertically in the puzzle. Explain that two adjacent squares can be used to form a 2-digit number and show them that the numbers 8, 1, 9, and 9 in the first row form the fact 81 ÷ 9 = 9

b) Have students find the remaining facts. They should circle each fact that they find and then compare their answers with their partner.

c) Vary the activity by creating multiplication puzzles or by creating combined multiplication and division puzzles.

d) Have students create their own fact puzzles and exchange them with the other members of the class.

Observation Checklist

Check students' responses for the following facts:

- ❏ 81 ÷ 9 = 9   49 ÷ 7 = 7   56 ÷ 7 = 8   24 ÷ 3 = 8
- ❏ 63 ÷ 7 = 9   40 ÷ 5 = 8   9 ÷ 3 = 3   42 ÷ 6 = 7
- ❏ 16 ÷ 4 = 4   25 ÷ 5 = 5

Materials: A set of 27 cards for each group, two cards for each of the numbers 1 through 9 (BLM 5–8.5), and nine cards with the word “everyone” on it and a number from 1 through 9 under it (BLM 5.N.3.3); one-minute timers, paper and pencils

Organization: Small groups

Procedure:

a) Tell students that they will be playing a game involving the basic facts for division. Explain how the game is played.

   1. Shuffle the cards and place them face down on the playing area.
   2. The player whose birthday comes first starts the game.
   3. The first player turns over the top card. The player has two minutes to write as many division facts as he or she can that have the number on the card as a quotient. The second player acts as the timer and says “Stop!” when two minutes are up.
   4. The first player receives one point for each correct division fact that has the number on the card as the quotient.
   5. The second player turns over the next card and writes as many division facts as he or she can that have the number on the card as a quotient. The first player acts as the timer, and says “Stop!” when two minutes are up. The second player receives one point for each correct fact.
   6. If a player turns over an “everyone” card, all players write down as many division facts as they can that have the number on the card as a quotient. One player volunteers to be the timekeeper, and says “Stop!” when two minutes are up. Each player receives one point for each fact that has the number on the card as a quotient.
   7. The first player to get 50 points is the winner.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

Observation Checklist

Observe students’ responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling

Materials: Number cards (BLM 5–8.5), observation form (BLM 5–8.1)

Organization: Groups of 3

Procedure:

a) Tell students that they will be playing a game involving the basic facts for multiplication and division. Explain how the game is played.

   1. Shuffle the cards and place them face down in a pile in the centre of the playing area.
   2. Two students sit facing each other while the third student sits so they can see the other two. The two players who are facing each other are the guessers. The third student is the caller.
   3. Each guesser chooses one card from the deck without looking at the card and holds it up to his or her forehead.
   4. The caller states the product of the numbers on the cards.
   5. The guesser who is the first to figure out what number is on his or her card wins both cards.
   6. The player who has the most cards after 10 rounds is the winner.
   7. The winner becomes the caller for the next game.

b) Demonstrate how to play the game and answer any questions students might have. Have students play the game.

Observation Checklist

Observe students' responses to determine which facts they

- ❑ recall easily
- ❑ have difficulty recalling
- ❑ use a strategy for
- ❑ have difficulty using a strategy for

Use the observation form (BLM 5–8.1) to assess how well students work together.

Materials: Single digit multiplication chart activity (BLM 5.N.3.4)

Organization: Whole class and individual

Procedure:

Demonstrate the patterns that exist in the multiplication chart. Have students colour in the facts according to each pattern.

a) The doubles:

x	0	1	2	3	4	5	6	7	8	9
0										
1			2							
2		2	4	6	7	10	12	14	16	18
3			6							
4			8							
5			10							
6			12							
7			14							
8			16							
9			18							

b) The five facts (clock facts): 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60

c) The "nifty nines": Each decade is one less than the number of nines and each answer's digits add up to nine.

9 x 9 = **8**1

8 x 9 = **7**2

7 x 9 = **6**3

6 x 9 = **5**4

5 x 9 = **4**5

4 x 9 = **3**6

3 x 9 = **2**7

d) The zeros and the facts multiplied by one.

e) Facts with 3: Doubles and one more group.

f) Facts with 4: Double Double

g) The last nine facts: These are often the most difficult for students: 6 x 6, 6 x 8, 7 x 6, 8 x 6, 7 x 8, 8 x 7, 7 x 7, and 8 x 8. Have students suggest strategies for learning these facts.

By the end, students should have a fully coloured multiplication chart by fact pattern.

More practice with multiplication facts and charts can be found at the following websites:

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=155
- http://eworkshop.on.ca/edu/resources/guides/NSN_vol_3_Multiplication.pdf
- http://dvl.ednet.ns.ca/mental-math
- http://www.aplusmath.com/cgi-bin/Homework_Helper/mtable
- http://www.mathsisfun.com/tables.html
- http://nzmaths.co.nz/taxonomy/term/195
- https://www.det.nsw.edu.au/eppcontent/glossary/app/.../factsheet/4024.pdf

Observation Checklist

Observe students' responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling
- ❏ use patterns for recall
- ❏ have other strategies to recall

Use the observation form (BLM 5–8.1) to assess students.

Materials: Blank multiplication chart—numbers from 0 to 9 across the top and 0 to 9 along the side, three pencil crayons—green, yellow, and red

Organization: Individual

Procedure:

On a blank multiplication chart, have students shade in the answers to the multiplication facts that they know for sure with the green pencil crayon. If they think they know, have them shade the square in yellow. For those facts they do not know, have them shade the square in red. Walk around while students are doing this and point to a green square to ensure students know these facts periodically. For the red facts, have students write these down for study, practise, and review. Put into a math portfolio to revisit throughout the year.

Usually students know the zeros, the ones, the fives, the nines, doubles, etc., but have trouble with the 6 x 7, 8 x 7, etc. Students should practise the ones they don't know.

Observation Checklist

Observe students' responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Materials: Race Around the Clock activity (BLM 5.N.3.5) or hand-draw the clock shape on individual white boards, 9-sided number cube

Organization: Individual

Procedure:

Have students write the numbers from 1 to 9 in the boxes on the inside edge of the clock in random order and placement. **Note:** Some numbers will be repeated. Roll the number cube and students write the number rolled in the centre of the clock. Say "go," and students write the multiplication fact for each number around the clock face in the outside circle as quickly as they can.

Note: This is not a timed testing activity. Each student can assess her or his own progress and practise the facts they don't know.

Observation Checklist

Observe students' responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Materials: Individual white boards or sheets of paper/student notebook or journal

Organization: Individual

Procedure:

Students work on multiplication fact practice exercises. Say a math fact (4 x 7) and students write their answer on the left side of the T-chart. After 5 seconds, say the answer (28) and the students write that answer on the righthand side opposite their answer. If the student is correct, the answers will match; if not, they will know they must practise that fact. Practice 10 to 15 facts each day. The important part of this exercise is for students to hear the correct answer to the multiplication fact and then to reinforce or correct that fact.

Student Answer	Teacher Answer

Variation: Instead of using playing cards, copy four sets of number cards for each student (BLM 5–8.5). Omit the "0" card.

Note: This activity can also be used for addition, subtraction, or division facts. The 10 to 15 facts of the day may be all one operation or may be a combination of operations.

Observation Checklist

Observe students' responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Materials: Playing cards with face cards removed

Caution: In some communities, playing cards are seen as a form of gambling and should be discouraged. Please be aware of local sensitivities before introducing this activity.

Organization: Pairs

Procedure:

To practise multiplication facts, have students divide a deck of cards with the face cards removed into two piles. Each student takes a pile and they will play against each other. Each student turns his or her first card over and the first person to say the multiplication fact wins the hand. Play continues. Matching players of like abilities makes a more enjoyable game.

Variation: Instead of using playing cards, copy four sets of number cards for each student (BLM 5–8.5). Omit the "0" card.

Observation Checklist

Observe students' responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Materials: Playing cards with face cards removed and 6-sided number cube

Caution: In some communities, playing cards are seen as a form of gambling and should be discouraged. Please be aware of local sensitivities before introducing this activity.

Organization: Pairs

Procedure:

Roll the number cube to arrive at target number (e.g., 4). Have students turn over five playing cards or do this using an overhead projector, document camera, or smart board (e.g., 4 5 1 8 7). The challenge is for students to arrive at the target number using these 5 numbers and all four operations. A possible solution is $\underline{8} \times \underline{5} = 40$, $40 \div \underline{4} = 10$, $10 - \underline{7} = 3$, $3 + \underline{1} = 4$.

Observation Checklist

Observe students' responses to determine which facts they

- ❏ recall easily
- ❏ have difficulty recalling

Use the observation form (BLM 5–8.1) to assess students.

Grade 5: Number (5.N.4)

Enduring Understandings:

There are many strategies that can be used to compute the answers to computational problems.

Strategies for computing the answers to computational problems involve taking apart and combining numbers in a variety of ways.

General Outcome:

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.N.4 Apply mental mathematics strategies for multiplication, such as ■ annexing then adding zeros ■ halving and doubling ■ using the distributive property [C, ME, R]	➛ Determine the products when one factor is a multiple of 10, 100, or 1000 by annexing zero or adding zeros (e.g., for 3 x 200, think 3 x 2 and then add two zeros). ➛ Apply halving and doubling when determining a product (e.g., 32 x 5 is the same as 16 x 10). ➛ Apply the distributive property to determine a product involving multiplying factors that are close to multiples of 10 [e.g., 98 x 7 = (100 x 7) – (2 x 7)].

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Modelling a multiplication problem using the distributive property
- Multiplying a 1-digit number times a 2-digit whole number or 3-digit whole number
- Using arrays to represent multiplication problems
- Adding and subtracting whole numbers less than 10 000
- Connecting concrete representations of multiplication problems with symbolic representations

Related Knowledge

Students should be introduced to the following:

- Determining multiplication facts to 81 and the related division facts

Background Information

The term **mental math** is most commonly used to describe computation that is done without paper and pencil or any calculation device such as a computer or calculator. A focus on mental math can help students become more adept at reasoning with numbers and enable them to gain new insights into operations and number relationships. It can also help them become adept at estimating, a skill that has become more important because of its practicality and the widespread use of computers and calculators.

Mental math usually involves the use of nonstandard algorithms such as repeated doubling or doubling and halving. Perhaps the most commonly used mental math strategy is the dropping and reattaching of common zeros. For example, to find the product of 3 x 70, think 3 x 7 = 21 and then "tack on" the zero that was dropped to get 210. The terminology " add zero" should be avoided, since it is misleading. 21 + 0 is 21 not 210.

Teachers can help students become adept at mental computation by making mental math an integral part of their instructional programs. In particular, they need to

- help students develop mental math strategies that make sense to them
- provide frequent practice sessions that are about 10 minutes in duration
- help students develop confidence by gradually increasing the complexity of the mental computations
- encourage students to use mental math whenever possible
- encourage students to develop their own mental math strategies
- make sure that students know the difference between mental math and estimation
- model the use of mental math and estimation

Mathematical Language

Doubling

Factor

Halving

Product

Multiple

Multiplication

Assessing Prior Knowledge

Note: This activity is also used to assess students' readiness for outcome 5.N.5.

Materials: Paper and pencils

Organization: Individual/Whole class

Procedure:

a) Ask students to solve the following problem in two different ways.
   - There are eight rows of chairs in the school auditorium. If each row has 45 chairs, how many chairs are there altogether?

b) Have students share their solutions and strategies with the other members of the class. Record the strategies students use on the board or overhead, and encourage discussion by asking questions, such as:
   - "Is there another strategy you could use to solve the problem? What is it?"
   - "Which strategy is easier to use? Why do you think it is easier?"
   - "Will the strategy work for other problems involving multiplication? Show me."
   - "Which strategy do you prefer to use? Why?"

Observation Checklist

Use students' responses to determine which strategies they know. Also, examine their responses to determine whether they can do the following:

- ❑ identify problem situations that require the operation of multiplication
- ❑ determine the correct product of a 1-digit whole number times a 2-digit whole number
- ❑ use more than one strategy to solve a multiplication problem involving a 1-digit whole number times a 2-digit whole number.
- ❑ determine the product of a 1-digit number times a 2-digit number using the distributive property.

- **Determine the products when one factor is a multiple of 10, 100, or 1000 by annexing zero or adding zeros.**

Materials: Base-10 blocks, strings of 100 beads, or other place-value materials, centimetre grid paper (BLM 5–8.9), copies of multiplication problems (BLM 5.N.4.1)

Organization: Pairs/Whole class

Procedure:

a) Have students explore multiplying by powers of ten by asking them to solve the problems. Let students know that they can use materials or draw diagrams to help them solve the problems.

b) Have students explain the strategies that they used to solve the problems. Encourage students to discuss the similarities among the problems and any patterns that they see by asking them questions such as:
   - "How are the problems alike?"
   - "How are the solutions alike?"
   - "Why do you think this is so?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recognize that the problems describe multiplication situations
- ❑ use models (materials or diagrams) appropriately to solve the problems
- ❑ represent the problems symbolically (e.g., 4 x 20 = 80)
- ❑ recognize that the problems involve calculating the products of multiples of 10
- ❑ recognize that the solutions involve a power or multiple of 10

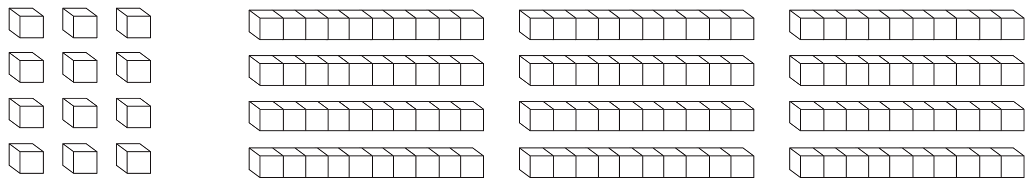
Materials: Base-10 blocks and calculators

Organization: Whole class

Procedure:

a) Show students these arrays.

4 x 3 ones 4 x 3 tens



4 x 3 = 12 4 x 3 tens = 12 tens = 120

Have students compare the two arrays and find the products. Encourage discussion by asking:

- "How are the two arrays alike?" (Both have 4 rows of three.)
- "How are the two arrays different?" (The first has ones in each row, the second has tens.)
- "How are the products different?" (In the first we have ones; in the second we have tens.)

b) Repeat part (a) several times. For example, have students compare these arrays and record the corresponding multiplication sentences.

- 8 x 2 ones and 8 x 2 tens ⟶ 8 x 2 = 16 and 8 x 2 tens = 16 tens = 160
- 3 x 8 ones and 3 x 8 tens ⟶ 3 x 8 = 24 and 3 x 8 tens = 24 tens = 240
- 7 x 5 ones and 7 x 5 tens ⟶ 7 x 5 = 35 and 7 x 5 tens = 35 tens = 350
- 2 x 9 ones and 2 x 9 tens ⟶ 2 x 9 = 18 and 2 x 9 tens = 18 tens = 180
- 4 x 6 ones and 4 x 6 tens ⟶ 4 x 6 = 24 and 4 x 6 tens = 24 tens = 240
- 9 x 3 ones and 9 x 3 tens ⟶ 3 x 9 = 27 and 9 x 3 tens = 27 tens = 270

Help students generalize their findings by asking them questions such as:

- "What patterns do you see?"
- "What conclusions can you draw?"
- "What do you know about the product of a number times a multiple of 10?"
- "What rule can you use when multiplying a number of ones by a multiple of 10?"

c) Provide students with a variety of mental math exercises. For example, ask students to solve these problems mentally:

- 8 x 60
- 3 x 70
- 20 x 4
- 60 x 5
- 9 x 40
- 30 x 5
- 10 x 7

d) Extend students' knowledge of multiplying by a multiple of ten by having them use their calculators to solve problems like the following:

■ 15 x 10	125 x 10
■ 15 x 20	125 x 20
■ 15 x 30	125 x 30
■ 15 x 40	125 x 40
■ 15 x 50	125 x 50
■ 15 x 60	125 x 60
■ 15 x 70	125 x 70
■ 15 x 80	125 x 80
■ 15 x 90	125 x 90

Have students record any patterns that they see and determine a rule for multiplying a number by a multiple of 10.

e) Use a similar procedure for multiplying ones times hundreds and ones times thousands.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recognize that ones times tens is tens
- ❑ recognize that when multiplying by ten, the product has a zero in the ones position
- ❑ find the product of ten times a number
- ❑ recognize the rule that to find the product of a number times a multiple of ten, multiply the number times the digit in the tens position, then "tack on" a zero to show tens

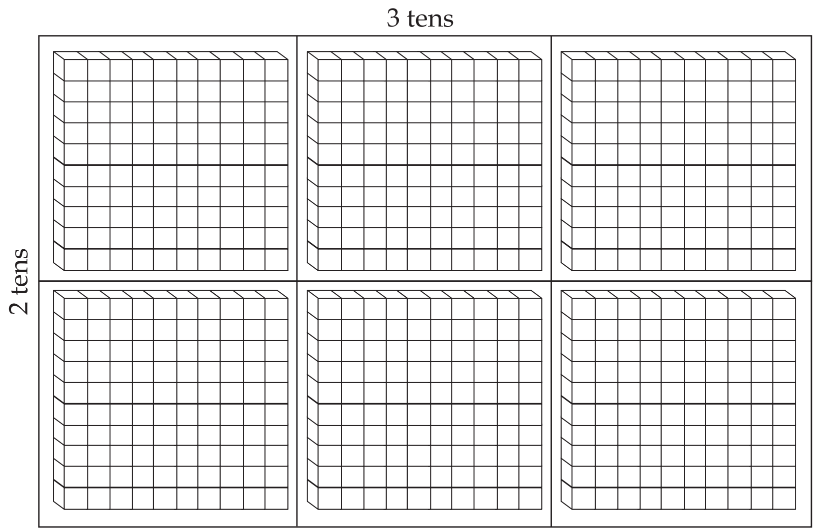
Use similar criteria for multiplying by 100 and 1000.

Materials: Base-10 blocks, base-10 grid paper (BLM 5–8.10), or graph paper

Organization: Whole class

Procedure:

a) Ask students to use the base-10 blocks or graph paper to make a 2-tens by 3-tens array.



When students finish, ask the following questions:

- "How many rows are there?" (2 tens)
- "How many columns are there?" (3 tens)
- "How many hundreds are there?" (6 hundreds)
- "What multiplication sentence does the array illustrate?"
(2 tens x 3 tens = 6 hundreds; 20 x 30 = 600)

b) Repeat part (a) several times. For example, have students make these arrays and record the corresponding multiplication sentences.

- 1 ten x 6 tens → 1 tens x 6 tens = 6 hundreds → 10 x 60 = 600
- 5 tens x 3 tens → 5 tens x 3 tens = 15 hundreds → 50 x 30 = 1500
- 8 tens x 4 tens → 8 tens x 4 tens = 32 hundreds → 80 x 40 = 3200
- 3 tens x 1 ten → 3 tens x 1 ten = 3 hundreds → 10 x 30 = 300
- 9 tens x 5 tens → 9 tens x 5 tens = 45 hundreds → 90 x 50 = 4500
- 6 tens x 6 tens → 6 tens x 6 tens = 36 hundreds → 60 x 60 = 3600

Help students generalize their findings by asking them questions such as:

- "What patterns do you see?"
- "What conclusions can you make?"
- "What do you know about the product of a multiple of ten times a multiple of ten?"
- "What can you use to find the product of a multiple of ten times a multiple of ten?"

c) Provide students with a variety of mental math activities. For example, ask students to solve these problems mentally:

- 20 x 40
- 30 x 60
- 70 x 20
- 90 x 80
- 10 x 60
- 40 x 50
- 20 x 50
- 70 x 60

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recognize that tens x tens is hundreds
- ❑ recognize that when multiplying a multiple of ten times a multiple of ten, there are zeros in the ones and the tens position of the product
- ❑ find the product of a multiple of ten times a multiple of ten
- ❑ recognize the rule that to find the product of a multiple of ten times a multiple of ten, find the product of the digits in the tens position, then "tack on" two zeros to show hundreds

Materials: Calculators and copies of the game sheet cut into strips (BLM 5.N.4.2)

Organization: Pairs

Procedure:

a) Tell students that they will be playing a game with their partner. Explain how to play the game.

   1. Players take turns selecting tasks from the pile.
   2. The other player uses his or her calculator to carry out the task. If the player can perform the task, he or she scores a point. For example, suppose a player is given the task to change 2 into 120 using multiplication, in one input. The player enters 2 into his or her calculator, and then inputs “x 60 =” to get 120.
   3. The player with the most points after 10 rounds of the game is the winner.

b) Demonstrate how to play the game and answer any questions students might have. Have students play the game.

Observation Checklist

Observe students’ responses to determine whether they can do the following:

- ❏ recognize multiples of 10
- ❏ recognize multiples of 100
- ❏ determine the missing factor in multiplication problems involving multiples of 10 and 100

- **Apply halving and doubling when determining a product.**

Materials: Counters

Organization: Whole class

Procedure:

a) Have students make a rectangular array with eight rows of three counters. Ask them how many counters they have altogether. “What strategy did you use to find the total?” “What is another way you could find the total?” Focus on partitioning strategies, such as double four threes or five threes and three threes. Record the appropriate number sentence(s) on the board. Record the number sentence 8 x 3 = 24 on the board or overhead.

x x x
x x x
x x x
x x x
x x x
x x x
x x x
x x x

b) Ask students to work out how they can change their array to show 4 x 6. If necessary, show students how the 8 x 3 array can be partitioned to illustrate 4 x 6. Ask, “How many counters do you have altogether?” Record the number sentence 4 x 6 = 24 on the board and note that 8 x 3 and 4 x 6 have the same product. Record 8 x 3 = 4 x 6. Ask students why they think this is so. Help students make the connection between their actions on the materials and the generalization that one set of factors can be changed into the other by doubling and halving.

x x x	x x x x x x
x x x	x x x x x x
x x x	x x x x x x
x x x	x x x x x x
x x x	
x x x	
x x x	
x x x	

c) Have students use their counters to test if the following statements are true:

$4 \times 5 = 2 \times 10$

$2 \times 8 = 4 \times 4$

$4 \times 3 = 2 \times 6$

$4 \times 10 = 8 \times 5$

d) Ask students to match each of the following multiplication problems with an equivalent problem. Have students use their counters to check their answers.

1.	3 x 16	6 x 4
2.	20 x 3	3 x 28
3.	7 x 4	6 x 8
4.	9 x 4	14 x 2
5.	6 x 14	10 x 6
6.	12 x 2	18 x 2

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ double a number
- ❑ find half of a number
- ❑ recognize that the product is the same if one factor is doubled and the other factor is halved
- ❑ determine an equivalent multiplication problem by doubling and halving its factors

Materials: Paper and pencil

Organization: Whole class/Small groups

Procedure:

a) Tell students that they will be doing a mental math activity. Explain that you will be giving them some multiplication problems. You will only say a problem once and they will only have enough time to record the answer.

b) Give students the following problems:

1. What is 6 doubled?
2. What is half of 14?
3. What is 10 doubled?
4. What is 8 doubled?
5. What is half of 24?
6. What is half of 18?
7. What is 3 doubled plus half of 10?
8. What is half of 4 plus 11 doubled?
9. What is 12 doubled plus 5 doubled?
10. What is half of 30 plus half of 20?

c) Have students share their answers with the other members of the class.

d) Have each group discuss the following question: "What mental math strategies can you use to find the product of 15 x 18?" Have the groups share their strategies. Encourage students to discuss the advantages and disadvantages of using each of the suggested strategies.

e) If no one suggests doubling and halving, show students the strategy. Explain that one way to determine the product of 15 x 18 is to double 15 and multiply it by half of 18. Since 15 doubled is 30 and half of 18 is 9, the product of 15 times 18 is the same as 30 times 9, which is equal to 270. Do two or three more examples such as

1. 32 x 21
2. 16 x 42
3. 8 x 84
4. 4 x 168
5. 2 x 336
6. 1 x 672

f) Ask students to use the doubling and halving strategy to determine the following products:

1. 12 x 45
2. 16 x 15
3. 14 x 25
4. 8 x 35
5. 18 x 25

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ double a number
- ❑ find half a number
- ❑ determine the product of two numbers by applying a doubling and halving strategy

- **Apply the distributive property to determine a product involving multiplying factors that are close to multiples of 10.**

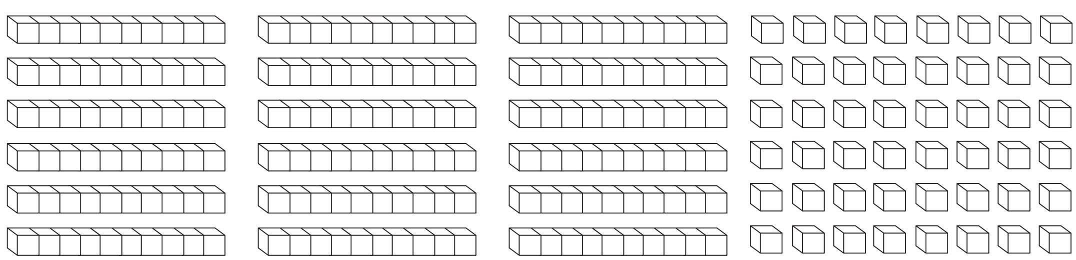
Materials: Base-10 blocks or counters

Organization: Whole class

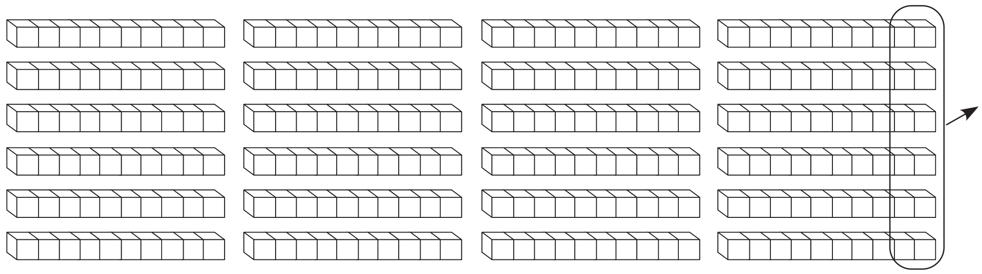
Procedure:

a) Present students with the following problem:
   - Ellen keeps the stamps she collects in a book. There are 38 stamps on each page of her book. If there are six pages in her book, how many stamps does she have altogether?

b) Ask students to identify a strategy that they would use to solve the problem. As students explain their strategy, model it with the materials. If no one suggests using the distributive property of multiplication over subtraction, explain the strategy while modelling it with materials.



There are 6 pages with 38 stamps on each page, so we need to make a 6 x 38 array. If we add 2 to each 38, we have six 40s or 240. Now take the 12 that we added away, which leaves 228.



c) Do two or three more examples and then ask students to solve the following problems using the strategy. Let students use materials to help them solve the problems.

1. 3 x 29 = ______
2. 7 x 18 = ______
3. 4 x 49 = _______
4. 2 x 58 = _______
5. 8 x 28 = _______

d) Have students share their answers and explain how they used the strategy to solve each of the problems.

e) Work with students to represent their solutions concretely (i.e., with base-10 blocks), pictorially (as above), and symbolically (i.e., 6 x 38 = 6 x 40 — 6 x 2 or "6 groups of 38 is the same as 6 groups of 40 subtract 6 groups of 2).

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ apply the distributive property to determine a product when one of the factors is close to a multiple of ten
- ❑ calculate the correct product of a 1-digit number times a multiple of ten
- ❑ calculate the correct difference between two whole numbers
- ❑ explain how to use the strategy to find the product when one of the factors is close to a multiple of ten

Materials: Copies of the Products activity (BLM 5.N.4.3) and base-10 materials

Organization: Individual/Partner

Procedure:

a) Have students complete the activity.

b) Have students check their answers with their partner. If discrepancies arise, have students use materials to determine the correct answer.

c) Have students discuss the strategies that they used to find the products in part B. Encourage students to describe the strategies that they used and explain why they choose them. Encourage students to use concrete, pictorial, and symbolic representations.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ apply the distributive property to determine a product when one of the factors is close to a multiple of 10
- ❏ calculate the correct product of a 1-digit number times a multiple of 10
- ❏ calculate the correct difference between two whole numbers
- ❏ recognize when to use the distributive strategy and the halving and doubling strategy

Notes

Grade 5: Number (5.N.5)

Enduring Understandings:

There are a variety of strategies that can be used to compute the answers to computational problems.

Strategies for computing the answers to computational problems involve taking apart and combining numbers in a variety of ways.

General Outcome:

Develop number sense.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.N.5 Demonstrate an understanding of multiplication (1- and 2-digit multipliers and up to 4-digit multiplicands), concretely, pictorially, and symbolically, by ■ using personal strategies ■ using the standard algorithm ■ estimating products to solve problems. [C, CN, ME, PS, V]	➛ Illustrate partial products in expanded notation for both factors [e.g., for 36 x 42, determine the partial products for (30 + 6) x (40 + 2)]. ➛ Represent both 2-digit factors in expanded notation to illustrate the distributive property [e.g., to determine the partial product of 36 x 42, (30 + 6) x (40 + 2) = 30 x 40 + 30 x 2 + 6 x 40 + 6 x 2 = 1200 + 60 + 240 + 12 = 1512]. ➛ Model the steps for multiplying 2-digit factors using an array and base-10 blocks, and record the process symbolically. ➛ Describe a solution procedure for determining the product of two 2-digit factors using a pictorial representation, such as an area model. ➛ Model and explain the relationship that exists between an algorithm, place value, and number properties. ➛ Determine products using the standard algorithm of vertical multiplication. (Numbers arranged vertically and multiplied using single digits, which are added to form a final product.) ➛ Solve a multiplication problem in context using personal strategies, and record the process. ➛ Refine personal strategies such as mental math strategies to increase efficiency when appropriate [e.g., 16 x 25 think 4 x (4 x 25) = 400].

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Using personal strategies to solve multiplication problems involving a 1-digit number times a 2-digit or 3-digit number
- Using arrays to represent multiplication problems
- Connecting concrete representations of multiplication problems with symbolic representations
- Modelling a multiplication product using the distributive property
- Estimating the product of a 1-digit number and a 2-digit or 3-digit number
- Adding and subtracting whole numbers less than 10 000
- Demonstrating an understanding of area of regular 2-D shapes

RELATED KNOWLEDGE

Students should be introduced to the following:

- Determining multiplication facts to 81
- Determining the product of two numbers when one of the factors is a multiple of 10, 100, or 1000
- Determining the product of a multiple of 10 times a multiple of 10

BACKGROUND INFORMATION

An **algorithm** is a system of finite procedures for solving a particular class of problems. The best known algorithms are the traditional paper-and-pencil procedures for adding, subtracting, multiplying, and dividing. Along with these standard algorithms, mathematics instruction should include an emphasis on understanding through mental mathematics, estimation, the use of technology, the development of invented procedures, and the use of alternative algorithms, such as area model multiplication, and adding up to solve subtraction problems.

By encouraging students to develop their own computation strategies and allowing them to use alternative algorithms, the emphasis in mathematics instruction is shifted to reasoning, problem solving, and conceptual understanding. Providing students with opportunities to invent their own strategies and use alternative algorithms enhances their number and operation sense. Students become more flexible in their thinking, more aware of the different ways to solve a problem, and more adept at selecting the most appropriate procedure for solving a problem. Discussion of the algorithms or strategies and their relationship to place value and number properties can also help students develop better reasoning and communication skills.

The standard algorithm for multiplication is where numbers are arranged vertically and multiplied using single digits, which are added to form a final product. Students are expected to use the standard algorithm as one of the tools for computation.

When teaching the traditional algorithm for multiplication, it is important to follow the concrete, pictorial, and symbolic sequence of teaching. The important idea is to allow students to construct meaning, not memorize procedures without understanding. Students' misconceptions (or their fuzzy understandings) can be reinforced by a poorly understood algorithm. Students should be able to explain the relationship that exists among an algorithm, place value, and number properties.

Teachers can facilitate students' understanding and use of a variety of computational strategies by

- providing a supporting and accepting environment
- allowing time for exploration and experimentation
- embedding computational tasks in real-life situations
- allowing students to discuss, analyze, and compare their solution strategies
- encouraging discussion that focuses on place value and number properties when defending the choice of a particular algorithm or strategy
- understanding that a child needs to be efficient at computation and that this looks different for each student

Note: For further information regarding using models and methods of division to build understanding, see Appendix 2: Models for Multiplying 2-Digit Numbers by 2-Digit Numbers. Also see resources such as the following:

- "Big Ideas for Teaching Mathematics Grades 4–8" by Marian Small (found in Chapter 2 of *Varied Approaches for Multiplication and Division*)
- "Making Math Meaningful to Canadian Students K–8" by Marian Small (found in Chapter 10 of *Computational Strategies: Operations with Whole Numbers*)
- *Teaching Student Centered Mathematics* by John A. Van de Walle

Mathematical Language

Array

Expanded notation

Factor

Multiplication

Multiply

Partial product

Product

Assessing Prior Knowledge

Note: This activity is also used to assess students' readiness for outcome 5.N.4.

Materials: Paper and pencils

Organization: Individual/Whole class

Procedure:

a) Ask students to solve the following problem in two different ways:
   - There are eight rows of chairs in the schools' auditorium. If each row has 45 chairs, how many chairs are there altogether?

b) Have students share their solutions and strategies with the other members of the class. Record their strategies on the board or overhead, and encourage discussion by asking questions, such as the following.
   - "Is there another strategy you could use to solve the problem? What is it?"
   - "Which strategy is easier to use? Why do you think it is easier?"
   - "Will the strategy work for other problems involving multiplication? Show me."
   - "Which strategy do you prefer to use? Why?"

Observation Checklist

Use students' responses to determine which strategies they know. Also, examine their responses to determine whether they can do the following:

- ❑ identify problem situations that require the operation of multiplication
- ❑ determine the correct product of a 1-digit number times a 2-digit number
- ❑ use more than one strategy to solve a multiplication problem involving a 1-digit number times a 2-digit number
- ❑ determine the product of a 1-digit number times a 2-digit number using the distributive property

- **Model the steps for multiplying 2-digit factors using an array and base-10 blocks, and record the process symbolically.**
- **Describe the solution procedure for determining the product of two 2-digit factors using a pictorial representation, such as an area model.**

Materials: Dot paper

Organization: Whole class

Procedure:

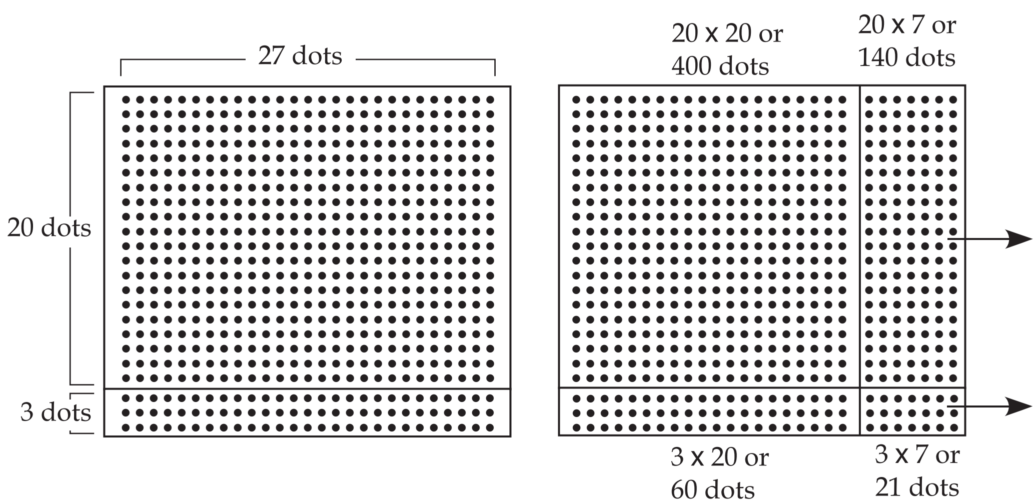
Note: Before beginning this task, you may need to determine the students' understanding of area as the amount of space taken up by a 2-D shape.

a) Present students with the following situation and ask them what they need to do to solve the problem:
   - There are 23 rows of cars in the shopping mall's parking lot. Each row has 27 cars in it. How many cars are parked in the lot?

b) Have students model the scenarios using base-10 blocks. Each unit cube represents one parking stall.

c) Have students draw a border around an array (23 x 27) that represents the cars in the parking lot. Ask them to partition the array in a way that will make it easier to find the total number of dots.

d) Have students share how they partitioned the array. Introduce the following ways of partitioning the array if students do not suggest them, and discuss how they are related. Students should not yet be required to record the process symbolically.

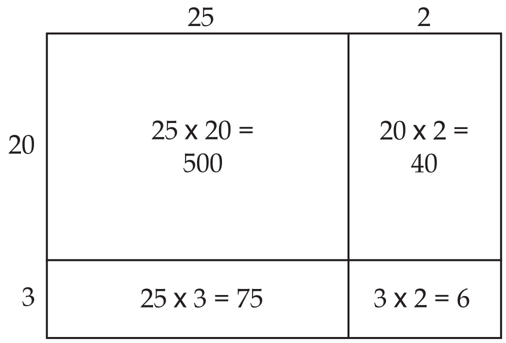


Since $20 \times 27 = 20 \times (20 + 7) = (20 \times 20) + (20 \times 7)$

and $3 \times 27 = 3 \times (20 + 7) = (3 \times 20) + (3 \times 7)$

then $(20 + 3) \times 27 = (20 \times 20) + (20 \times 7) + (20 \times 3) + (3 \times 7)$

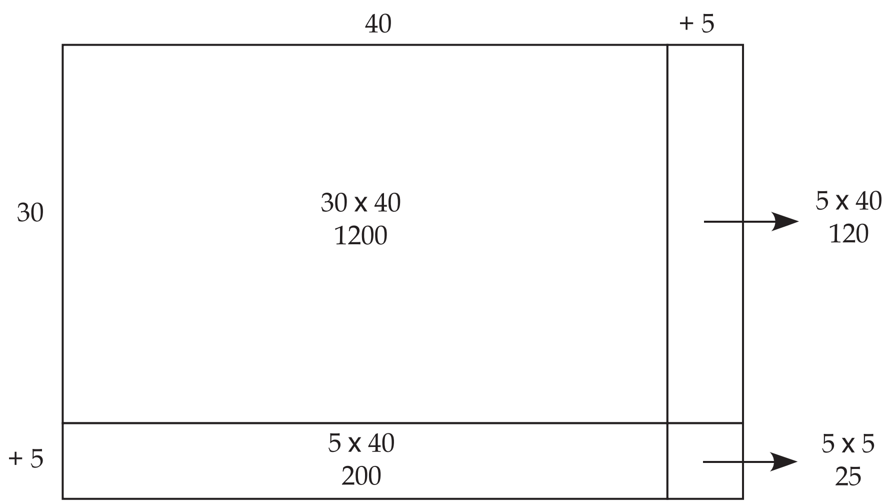
Or



Total Area = 500 + 40 + 75 + 6
Total Area = 621

Emphasize that the whole numbers can be "broken apart" into more convenient pieces in order to make the computation easier. Make sure that students understand that this will not affect their answer.

e) Do another example of using place value to find the product of two numbers—that is, show students how the product of 35 x 45 can be found by partitioning an array into four parts that can be associated with the expanded forms of the factors and finding the sum of the parts.



Once students become comfortable, they can draw the representation without needing the appropriate number of dots.

Use the sum of the individual (partial) products to find the total value of the original product:

35 x 45

= (30 + 5) x (40 + 5)

= (30 x 40) + (30 x 5) + (5 x 40) + (5 x 5)

= 1200 + 150 + 200 + 25

= 1575

f) Ask students to solve each of the following problems by partitioning an array and finding the sum of the parts, as illustrated in part (e). Have them represent these sentences concretely and pictorially in as many ways as possible.

- 42 x 15
- 22 x 37
- 21 x 24
- 18 x 23

Observation Checklist

Examine students' responses to determine whether they can do the following:

- ❑ illustrate partial products in expanded notation for both factors
- ❑ represent both 2-digit factors in expanded notation to illustrate the distributive property
- ❑ describe a solution procedure for determining the product of two 2-digit numbers using pictorial representations
- ❑ calculate correct sums and products
- ❑ model a 2 x 2 digit multiplication concretely and pictorially

Examine students' responses to determine whether any errors are due to

- ❑ carelessness
- ❑ not knowing a basic multiplication or addition fact
- ❑ a procedural error (e.g., renaming and regrouping incorrectly when finding the sum of two numbers or not finding all the partial products in a multiplication problem)

- **Illustrate partial products in expanded notation for both factors.**
- **Represent both 2-digit factors in expanded notation to illustrate the distributive property.**
- **Model the steps for multiplying 2-digit factors using an array and base-10 blocks, and record the process symbolically.**
- **Describe a solution procedure for determining the product of two 2-digit factors using a pictorial representation, such as an area model.**

Materials: Base-10 blocks

Organization: Whole class/Pairs

Procedure:

a) Ask students to solve the following problem:

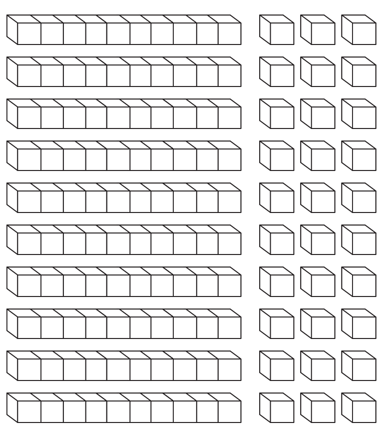
- The students enrolled in the community centre's swimming program are lined up in rows to get their picture taken. There are 12 rows with 13 students in each row. How many students are enrolled in the swimming program?

b) Have students share their solutions to the problem. Encourage them to discuss the strategies they used by asking the following questions:

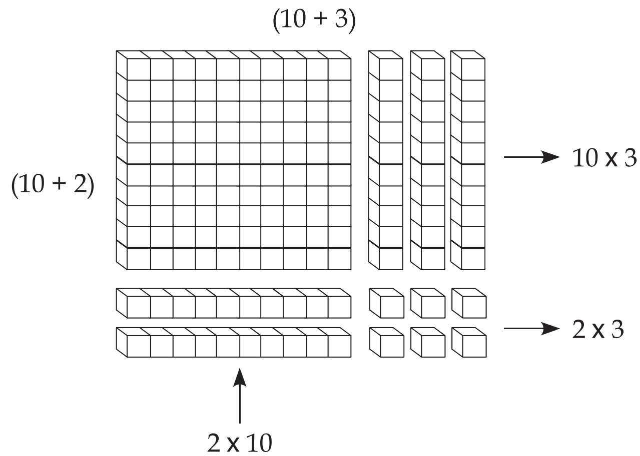
- "How did you find your answer?"
- "Will your strategy work for other problems?"
- "Is there another strategy you could use?"
- "Which strategy is more efficient?"

c) If students do not suggest using the distributive property, illustrate the strategy by making an array with the base-10 blocks. The array should consist of 12 rows of 13 blocks (or 13 rows of 12 blocks).

12 x 13 = 12 (10 + 3)

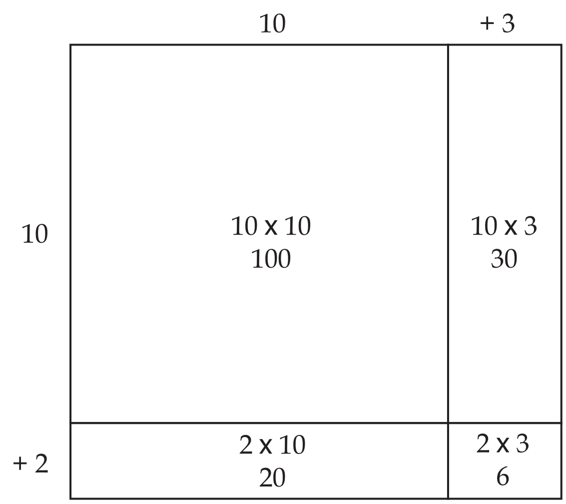


Since 10 longs equal 1 flat and 10 units equal 1 long, the array can be simplified by exchanging these blocks.



d) Model several more 2 x 2 digit multiplication scenarios using base-10 blocks.

e) Emphasize that the array has been partitioned into four parts. Each part represents a partial product. These four parts can be shown in a diagram called an area model:



The product of the original question is simply the sum of each for the four sections.

f) The partial products are the result of expressing each factor in expanded notation, and multiplying each addend in the first factor by each addend in the second factor. The product is the sum of the partial products and can be expressed symbolically:

$$12 \times 13 = (10 + 2) \times (10 + 3) = (10 \times 10) + (10 \times 3) + (2 \times 10) + (2 \times 3) = 156.$$

This can also be represented symbolically as:

$$\begin{array}{rl} 12 & \\ \times\ 13 & \\ \hline 6 & (3 \times 2) \\ 30 & (3 \times 10) \\ 20 & (10 \times 2) \\ 100 & (10 \times 10) \\ \hline 156 & \end{array}$$

g) Do one or two more examples, then have students use the base-10 blocks to solve these problems. Encourage students to record the procedure they used for each problem, concretely, pictorially, and symbolically.

- 15 x 25
- 18 x 27
- 21 x 26
- 22 x 28
- 32 x 31

h) Students should choose one of the questions and explain how each representation relates to place value and number properties.

Note: As students get more practice with multiplication, another symbolic representation can be presented:

$$\begin{array}{rl} 12 & \\ \times\ 13 & \\ \hline 36 & (3 \times 12) \\ 120 & (10 \times 12) \\ \hline 156 & \end{array}$$

Observation Checklist

Check students' responses to determine whether they can do the following:

- ❑ illustrate partial products in expanded notation for both factors
- ❑ represent both 2-digit factors in expanded notation to illustrate the distributive property
- ❑ model the procedure for finding the product of two 2-digit factors using an array and base-10 blocks
- ❑ record the procedure symbolically
- ❑ solve a multiplication problem in context using personal strategies, and record the procedure concretely, pictorially, and symbolically

Examine students' responses to determine if a mistake is due to

- ❑ carelessness
- ❑ not knowing a basic multiplication or addition fact
- ❑ a procedural error (e.g., renaming and regrouping incorrectly when finding the sum of two numbers or not finding all the partial products in a multiplication problem)

- **Illustrate partial products in expanded notation for both factors.**
- **Represent both 2-digit factors in expanded notation to illustrate the distributive property.**

Materials: Number cards (BLM 5–8.5), paper and pencils.

Organization: Groups of 2–4/Whole class

Procedure:

a) Tell students that they will be playing a game involving multiplication of 2-digit numbers. Explain how the game is played.

1. Shuffle the cards and place them face down in a pile in the centre of the playing area.
2. Each player draws four cards and arranges them to form two 2-digit whole numbers that will give him or her the largest possible product.
3. Players place their numbers face up in front of them. Each player writes his or her numbers as the sum of tens and ones, then multiplies each member of the first number times each member of the second number.

   For example, if 1, 3, 5, and 7 are drawn, a player can form the numbers 71 and 53, then find the product of the two numbers by multiplying (70 + 1)(50 + 3)—that is, (70 +1)(50 + 3) = (70 x 50) + (70 x 3) + (1 x 50) + (1 x 3) = 3500 + 210 + 50 + 3 = 3763.

4. Players check each other's calculation. The player with the largest product gets one point.
5. The winner is the player with the most points after five rounds.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Have students discuss the strategies that they used to win the game. Begin the discussion by asking questions, such as "What four cards did you draw? How did you decide which numbers to form with the cards you drew? What other numbers could you have made? How did you decide which numbers would give you the largest product?"

d) Vary the activity by having students

- use the four cards to form a 1-digit by 3-digit multiplication to give the largest possible product
- use four cards and ask students to form either a 2-digit by 2-digit or a 1-digit by 3-digit multiplication
- use five cards to form a 2-digit by 3-digit or 1-digit by 4-digit multiplication
- use six cards to form a 3-digit by 3-digit or 2-digit by 4-digit multiplication
- have students arrange the cards to form the smallest product

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ illustrate partial products in expanded notation for both factors
- ❏ present both 2-digit factors in expanded notation to illustrate the distributive property
- ❏ determine the correct product of two 2-digit whole numbers
- ❏ give reasonable estimates of the product of two 2-digit numbers
- ❏ recognize that the larger the factors, the greater the product

- **Multiply 2-digit numbers.**

Materials: Copies of the basic facts multiplication table (BLM 5–8.11)

Organization: Whole class

Procedure:

a) Show students a copy of the multiplication table and ask them to describe any patterns they see.

b) Show students the square labelled "A"

2	3
4	6

and have them find the product of the opposite pairs of vertices (2 x 6 and 3 x 4). Ask them what they notice about the products.

c) Now show students the square labelled "B"

18	21	24	27
24	28	32	36
30	35	40	45
36	42	48	54

and have them find the product of the opposite pairs of vertices (18 x 54 and 36 x 27). Again, ask students what they notice about the products. Ask students if they think this would be true for other squares.

d) Have students work with their partners to find the product of the opposite pairs of vertices of 10 different squares on the multiplication table. Make sure students vary the size of the squares.

e) Have students discuss their findings with the other members of the class.

f) Extend the activity by asking students to determine whether the pattern they found for squares also holds for rectangles.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recall the basic facts for multiplication
- ❑ determine the correct product of a 1-digit number times a 2-digit number
- ❑ determine the correct product of a 2-digit number times a 2-digit number
- ❑ use a strategy for calculating the product of two numbers that is mathematically correct and efficient and can be generalized (applied to other multiplication problems)
- ❑ recognize that the products of the opposite pairs of vertices of squares (and other rectangles) on the multiplication table are equal

Materials: Number cards (BLM 5–8.5), dice, paper and pencils

Organization: Pairs

Procedure:

a) Tell students that they will be playing the game "target." Explain how the game is played.

1. Shuffle the cards and place them face down in a pile in the centre of the playing area (the cards will need to be reshuffled after each round of play).
2. Players take turns rolling a die and drawing four cards.
3. On a turn, a player rolls the die and uses the chart shown below to determine the target range of the product.

Number Rolled	Target Range
1	500 or less
2	501 — 1000
3	1001 — 3000
4	3001 — 5000
5	5001 — 7000
6	more than 7000

4. The player then draws four cards and uses them to form two numbers whose product the player thinks falls within the target range. The player does not have to use all four cards. A number cannot begin with a zero.

5. The player multiplies the two numbers. The other player checks his or her partner's calculations. If the product is in the target range, he or she gets one point. If the product is outside the range, no points are awarded.
6. The first player to get five points is the winner.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Vary the activity by having students draw five or six cards.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ give reasonable estimates of the product of a 2-digit number times a 1-digit or 2-digit number
- ❏ determine the product of 2-digit numbers using the distributive property
- ❏ use concrete or pictorial representations to determine the product of two numbers
- ❏ use an efficient strategy for finding the product of two numbers

- **Illustrate partial products in expanded notation for both factors.**
- **Represent both 2-digit factors in expanded notation to illustrate the distributive property.**
- **Model and explain the relationship that exists between an algorithm, place value, and number properties.**
- **Determine products using the standard algorithm of vertical multiplication.**

Materials: Copies of Multiplication Methods activity (BLM 5.N.5.1)

Organization: Whole class/Small Groups (3 or 4)

Procedure:

a) Explain to students that they will be practising three different strategies or algorithms for finding the products of two numbers. Although the strategies could be called different names, for this activity, we will call them

   i) Area Model

   ii) Vertical Method

   iii) Compressed Vertical Method

b) As a whole class, go through the following example. Each of these three methods should have been discussed with students prior to the activity and the first example is to activate their knowledge.

Multiply 26 x 57. Use each of the three methods and be prepared to explain the method using appropriate mathematical language.

Method 1: Area Model

Work:

	20 +	6
50 +	1000	300
7	140	42

1000 + 300 + 140 + 42 = 1482

Or

$$\begin{array}{r} 1000 \\ 300 \\ 140 \\ +\quad 42 \\ \hline 1482 \end{array}$$

Explanation:

Write each factor in expanded notation.

Find each of the four partial products:

50 x 20 = 1000
50 x 6 = 300
7 x 20 = 140
7 x 6 = 42

Add the four partial products.

Method 2: Vertical Method

Work:

$$\begin{array}{r} 26 \\ \times\quad 57 \\ \hline 42 \\ 140 \\ 300 \\ 1000 \\ \hline 1482 \end{array}$$

Explanation:

Find the partial products:

7 x 6 = 42
7 x 20 = 140
50 x 6 = 300
50 x 20 = 1000

Add the partial products.

Note: When finding the partial products, remember that the 2 in 26 is not a 2 but 20. Similarly, the 5 in 56 is not a 5 but 50.

Method 3: Compressed Vertical Method

Work:	Explanation:
$\begin{array}{r} {}^{3}4\;\;26 \\ \times\;\;57 \\ \hline 182 \\ 1300 \\ \hline 1482 \end{array}$	Find the partial products: 7 x 26 = 182* 50 x 26 = 1300** Add the partial products.

Notes:

* To find the partial product 7 x 26, multiply 7 x 6 to get 42. Place the 2 below the line in the ones place and put the 4 beside the 2 to show that you need to add 4 tens to the product of 7 and 20 or 4 to the product of 7 and 2 tens. Since 7 x 2 = 14, and 14 + 4 = 18, you have 18 tens so you can write 18 to the left of the 2 to show 18 tens. You really did 7 x 26 = (7 x 20) + (7 x 6) = 140 + 42 = 182.

** To find the partial product 50 x 26, you can think of the partial product 5 x 26 if you place a zero in the ones place under the 2. Now proceed as in Note 1: multiply 5 x 6 to get 30. Put the 0 in the tens place under the 8 and put the 3 beside the 2 to show that you need to add 3 to the product of 5 and 2. Since 5 x 2 + 3 = 13, put 13 to the left of the digits you just put on the second row. Note that you were really multiplying 50 x 20 and adding 300 to get 1000 + 300 (or 50 x 26 = (50 x 20) + 50 x 6) = 1000 + 300 = 1300).

c) Arrange the students in groups of 3 (or 4). Have the students number off from 1 to 3 (or 4). The students will each solve multiplication problems using a specified method. They will then discuss their solutions and talk about the efficiency of each method.

d) There will be 3 rounds for this activity. For each round, assign a particular method to each group member

- Round 1: Area Model: Person 1 (and Person 4)
  Vertical Method: Person 2
  Compressed Vertical Method: Person 3
- Round 2: Area Model: Person 2
  Vertical Method: Person 3 (and Person 4)
  Compressed Vertical Method: Person 1
- Round 3: Area Model: Person 3
  Vertical Method: Person 1
  Compressed Vertical Method: Person 2 (and Person 4)

e) For each round, write the multiplication question and the answer on the board. Each person in the group is to do her or his own work. After everyone in the group is finished, have the students discuss the following questions:

Did we all get the right answer? If no, have students check the incorrect answers to try to correct the mistake.

- Which method seemed to be the most efficient? Explain.

f) After the discussion, the group should work together to explain each step in each method.

g) Questions such as the following could be used (be sure to vary the size of the factors each time so students get practice with different types of questions):

- 9 x 483 = 4347
- 52 x 149 = 7748
- 6743 x 28 = 188 804

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ determine the product of two numbers using the distributive property
- ❑ explain the steps in each of the three procedures
- ❑ record each procedure symbolically
- ❑ use place value knowledge when performing multi-digit multiplication

Examine students' responses to determine if a mistake is due to:

- ❑ carelessness
- ❑ not knowing a basic multiplication or addition fact
- ❑ procedural error (e.g., renaming and regrouping incorrectly when finding the sum of two numbers or not finding all the partial products in a multiplication problem)

Grade 5: Number (5.N.6)

Enduring Understandings:

There are many strategies that can be used to determine the answers to computational problems.

Strategies for computing the answers to computational problems involve taking apart and combining numbers in a variety of ways.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	**Achievement Indicators:**
5.N.6 Demonstrate an understanding of division (1- and 2-digit divisors and up to 4-digit dividends), concretely, pictorially, and symbolically, and interpret remainders by ■ using personal strategies ■ using the standard algorithm ■ estimating quotients to solve problems. [C, CN, ME, PS]	➛ Model the division process as equal sharing using base-10 blocks, and record it symbolically. ➛ Explain that the interpretation of a remainder depends on the context: ■ ignore the remainder (e.g., making teams of 4 from 22 people) ■ round up the quotient (e.g., the number of five passenger cars required to transport 13 people) ■ express remainders as fractions (e.g., five apples shared by two people) ■ express remainders as decimals (e.g., measurement or money) ➛ Model and explain the relationship that exists between algorithm, place value, and number properties. ➛ Determine quotients using the standard algorithm of long division. (The multiples of the divisor are subtracted from the dividend). ➛ Solve a division problem in context using personal strategies, and record the process. ➛ Refine personal strategies, such as mental math strategies to increase efficiency when appropriate (e.g., 860 ÷ 2 think 86 ÷ 2 = 43 then 860 ÷ 2 is 430).

Prior Knowledge

Students may have had experience with the following:

- Solving division problems involving 2-digit whole number dividends by 1-digit whole number divisors
- Using personal strategies to solve division problems involving 1-digit whole number divisors and 2-digit whole number dividends
- Estimating the quotients of division problems involving 1-digit whole number divisors and 2-digit whole number dividends
- Relating division to multiplication
- Adding and subtracting whole numbers to 10 000

Related Knowledge

Students should be introduced to the following:

- Determining basic multiplication and division facts to 81
- Using strategies such as front-end rounding, compensation, and compatible numbers to estimate the answers to computational problems
- Dividing multiples of 10 and 100 by whole numbers less than 10

Background Information

An algorithm is a system of finite procedures for solving a particular class of problems. The best known algorithms are the traditional paper-and-pencil procedures for adding, subtracting, multiplying, and dividing. Along with these standard algorithms, mathematics instruction should include an emphasis on understanding through mental mathematics, estimation, the use of technology, the development of invented procedures, and the use of alternative algorithms, such as equal sharing, equal grouping, and strategic dividing.

By encouraging students to develop their own computation strategies and allowing them to use alternative algorithms, the emphasis in mathematics instruction has shifted to reasoning, problem solving, and conceptual understanding. Providing students with opportunities to invent their own strategies and use alternative algorithms enhances their number and operation sense. Students become more flexible in their thinking, more aware of the different ways to solve a problem, and more adept at selecting the most appropriate procedure for solving a problem. Discussion of the algorithms or strategies and their relationship to place value and number properties can also help students develop better reasoning and communication skills.

The standard algorithm for division is subtracting multiples of the divisor from the dividend. Students are expected to use the standard algorithm as one of the tools for computation.

When teaching the traditional algorithm for division, it is important to follow the concrete, pictorial, and symbolic sequence of teaching. The important idea is to allow students to construct meaning, not memorize procedures without understanding. Students' misconceptions (or their fuzzy understandings) can be reinforced by a poorly understood algorithm. Therefore, be sure that a student has a thorough understanding of the operations and of taking apart and combining numbers (i.e., using place value) before beginning work on developing the traditional algorithm. Students should be able to explain the relationship that exists among an algorithm, place value, and number properties.

Teachers can facilitate students' understanding and use of a variety of computational strategies by

- providing a supporting and accepting environment
- allowing time for exploration and experimentation
- embedding computational tasks in real-life situations
- allowing students to discuss, analyze, and compare their solution strategies
- encouraging discussion that focuses on place value and number properties when defending the choice of a particular algorithm or strategy
- understanding that a child needs to be efficient at computation and that this looks different for each student

Note: For further information regarding using models and methods of division to build understanding, see Appendix 1: Models for Dividing 3-Digit Numbers by 1-Digit Numbers.

Mathematical Language

Division

Divisible

Divisor

Dividend

Quotient

Remainder

Assessing Prior Knowledge

Materials: Math journals

Organization: Individual/Whole class

Procedure:

a) Present students with the following situation:
   - Manuel’s calculator is broken. He needs to find the quotient of 57 ÷ 6, but he has forgotten how to divide. Help him out by explaining how he can find the quotient.

b) Have students share their answers. Record their strategies on the board or overhead and encourage discussion of them by asking questions, such as the following:
   - “What strategy could Manuel use to find the quotient of 57 ÷ 6?”
   - “Will your strategy work for other division problems? Show me.”
   - “What is another strategy he could use?”
   - “How are the strategies alike? How do they differ?”
   - “Which strategy do you prefer? Why?”

Observation Checklist

Observe students’ responses to determine whether they can do the following:

- ❏ correctly use the terms *dividend*, *divisor*, *quotient*, *remainder*, and *divide*
- ❏ recognize that division involves partitioning into equal parts
- ❏ recognize that division involves forming equal-sized groups
- ❏ describe a strategy that is mathematically correct
- ❏ use an efficient strategy that can be used for all division problems
- ❏ recognize that the division is uneven (i.e., there is a remainder of 3)

- **Model the division process as equal sharing using base-10 blocks, and record it symbolically.**

Materials: Base-10 materials such as base-10 blocks or digi-blocks

Organization: Whole class

Procedure:

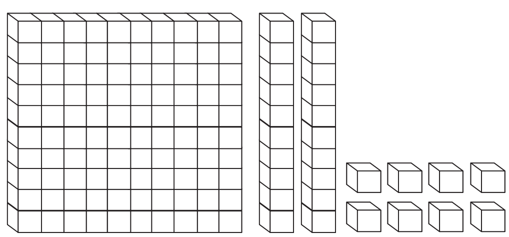
a) Ask students to solve the following problem:

- Marcy works in the library. She needs to put the same number of books on four shelves. There are 128 books. How many books should she put on each shelf?

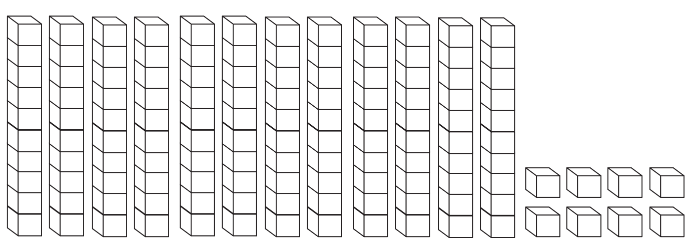
b) Have the students share their solutions with the other members of the class. Encourage discussion by asking questions, such as the following:

- "What strategy did you use to determine the solution to the problem?"
- "Is there another strategy you could use to solve the problem?"
- "How are the strategies alike?"
- "Which strategy is more efficient? Why do you think this?"
- "How do you know your solution is correct?"

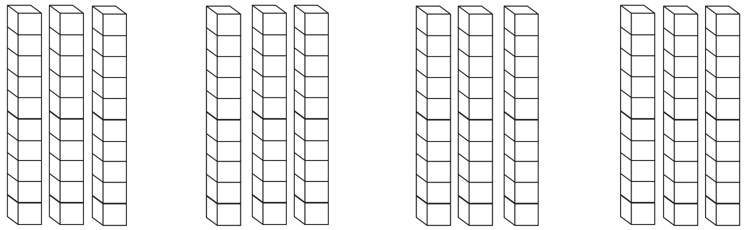
c) Model the division process as equal sharing if students do not suggest this strategy. For example, show students 128 with the blocks.



Explain one strategy by telling students that 128 needs to be partitioned into four equal parts. Since there are not enough flats (100s) to put one in each part, the flat can be exchanged for 10 longs so 128 becomes 12 tens and 8 ones.

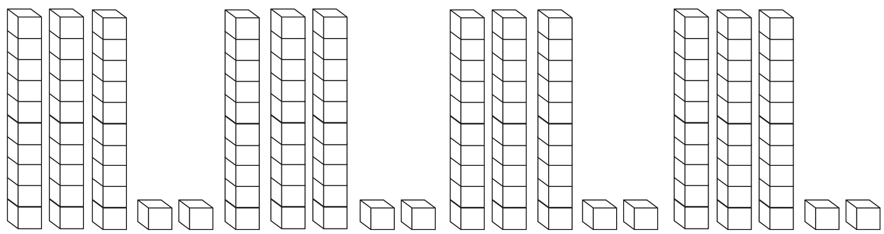


Next, partition the tens into four equivalent parts.



Then partition the ones into four equivalent parts.

Now, put the pieces together to make four parts with 32 in each part.



Record the process as 128 ÷ 4 = 12 tens and 8 ones ÷ 4 = 3 tens and 2 ones, or 32, or as

$$\begin{array}{r}\text{3 tens, 2 ones}\\ 4\overline{)\text{12 tens, 8 ones}}\end{array} \text{ or } \begin{array}{r}30+2\\ 4\overline{)120+8}\end{array}$$

d) Do two or three more examples, and then have students use the strategy to solve the following problems. Have students record their answers symbolically.

- 156 ÷ 3 =
- 216 ÷ 4 =
- 317 ÷ 2 =
- 438 ÷ 5 =
- 624 ÷ 4 =

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ model the division process as equal sharing using base-10 blocks
- ❑ record the division process symbolically

- **Model the division process as equal sharing using base-10 blocks, and record it symbolically.**
- **Explain that the interpretation of a remainder depends on the context.**
- **Solve a division problem in context using personal strategies, and record the process.**

Materials: Base-10 materials such as base-10 blocks or digi-blocks.

Organization: Whole class/Small groups

Procedure:

a) Have students solve the following problem:
   - Max and Joshua are responsible for setting up tables for the school's banquet. Eight people can sit at each table. How many tables do Max and Joshua need to set up if there are 150 people going to the banquet?

b) Have students share their answers and discuss how the remainder should be interpreted. Encourage students to explain why the quotient needs to be increased by 1.

c) Explain that remainders are a common occurrence in division, and how they are interpreted depends on the problem statement.

d) Have each group of students solve the following problems and decide how the remainder should be interpreted.
   - Some students will be wearing sashes for the spring school dance festival. Four metres of ribbon are needed for each sash. Mr. Sanchez has 135 metres of ribbon. How many sashes can he make?
   - Sally has 26 cookies to share evenly with three friends. How many cookies does each get?
   - Marshall saved $157.00. He spent half of the money he saved to buy three video games. How much did he spend on the video games?
   - Seventy-four people are going on a camping trip. Six people can go in each car. How many cars are needed?

e) Have the groups share their answers and discuss how they interpreted the remainder. Encourage students to explain their reasons for how they handled the remainder. Summarize the discussion by stressing that there are four ways the remainder can be handled. Depending on the problem statement, the remainder can be expressed as a fraction or a decimal; it can be dropped or it can require the quotient to be increased by 1.

f) Assign each group one of the ways that a remainder can be interpreted, and ask them to write a problem that involves the interpretation that you gave them. Have the groups give their problem to the other members of the class to solve and decide how to interpret the remainder.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ model the division process as equal sharing using base-10 materials
- ❑ solve a division problem using personal strategies, and record the process
- ❑ interpret the remainder with respect to the context
- ❑ write a division problem that involves dropping the remainder, increasing the quotient by 1, expressing the remainder as a fraction, or expressing the remainder as a decimal

- **Model the division process as equal sharing using base-10 blocks, and record it symbolically.**
- **Explain that the interpretation of a remainder depends on the context.**

Materials: Dice, base-10 blocks or digi-blocks

Organization: Pairs

Procedure:

a) Tell students that they will be doing an activity involving division. Explain that they will be rolling a die four times and using the four numbers that they roll to create as many division problems as they can that have no remainder. Each problem that they create must involve a 3-digit whole number dividend and a 1-digit whole number divisor. Let students know that they can use the materials to help them determine whether there is a remainder, and that they should record both the numbers that they rolled and the problems that they created that have no remainders.

   If they are convinced that there are no problems that can be created that have a remainder of zero, they should record the numbers that they rolled, and explain in writing why they think this is so.

b) Do an example of the activity with the students. For instance, suppose 2, 5, 6, and 4 are rolled. One problem that has no remainder that can be made with the numbers is $564 \div 2$; another is $256 \div 4$.

c) Have students repeat the activity, but this time have them create problems with the largest possible remainder.

Observation Checklist

Analyze students' responses to determine whether they can do the following:

- ❑ devise a strategy determining all the possible problems that can be considered
- ❑ solve division problems in context using personal strategies, and record the process
- ❑ use a strategy that is efficient, mathematically correct, and can be generalized (work for all problems)
- ❑ recognize division problems that have no remainders
- ❑ recognize division problems that have remainders
- ❑ identify the largest possible remainder for a given divisor

■ Solve a division problem in context using personal strategies, and record the process.

Materials: Division problem cards (BLM 5.N.6.1), timer or clock with second hand, calculator for each group.

Organization: Groups of 3/Whole class

Procedure:

a) Tell students that they will be playing a game that involves estimating quotients. Explain how the game is played.
   1. Shuffle the cards and place them face down in the middle of the playing area.
   2. One student in each group acts as the timer. The other two students play against each other.
   3. The timer turns over a card and says, "Go!" The other two students now have 10 seconds to write down their estimate of the quotient.
   4. When the 10 seconds are up, the timer says, "Stop!" and the players must put their pencils down. The timer then uses the calculator to find the quotient.
   5. The player whose estimate is closest to the actual answer wins the card. If there is a tie, no player receives a card.
   6. The game is over when there are no cards left. The player with the most cards wins the game.

b) Demonstrate how the game is played and answer any questions that students might have. Have students play the game.

c) Have students discuss the strategies they used to estimate the quotients. Begin the discussion by asking questions, such as the following:
   - "How did you estimate the quotient of 748 ÷ 2?"
   - "How can you get a closer estimate?"
   - "Is there another strategy you could use to estimate the quotient?"

d) Have the winner of the game become the timer and play the game again.

e) Have students make up their own division problem cards and use them to play the game.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ give reasonable estimates of the quotients
- ❏ use technology to solve division problems
- ❏ use strategies such as "front-end rounding" and "compensation" to determine reasonable estimates of the quotient
- ❏ determine basic facts for division

- **Model the division process as equal sharing using base-10 blocks, and record the process symbolically.**
- **Explain that the interpretation of a remainder depends on the context.**
- **Solve a division problem in context using personal strategies, and record the process.**

Materials: Base-10 blocks or digi blocks for students who need them; paper and pencils

Organization: Whole class/Small groups

Procedure:

a) Ask students to solve the following problem: 24 ÷ 3 = . Explain that there are other problems related to 24 ÷ 3 that also have a quotient of 8, but these problems have a remainder. For example: 25 ÷ 3 is 8 with a remainder of 1. Ask students to find another problem related to 24 ÷3 that has a quotient of 8 and a remainder of 2 (26 ÷ 3). Have students find all the division problems with a remainder related to 36 ÷ 4 = (37 ÷ 4 = , 38 ÷ 4 = , and 39 ÷ 4 =).

b) Assign each group one of the division problems shown below, and ask them to find all the related division problems as well as the remainder for each problem.

- 150 ÷ 5 = (151 ÷ 5 = , 152 ÷ 5 = , 153 ÷ 5 = , 154 ÷ 5 =)
- 432 ÷ 6 =
- 217 ÷ 7 =
- 344 ÷ 8 =
- 918 ÷ 9 =

c) Have each group share its findings with the other members of the class and record their answers on the board. Encourage discussion by asking questions, such as the following:

- "What patterns do you see?"
- "If you divide a 634 by 12, how many related division problems could you write? Why?"
- "What do you know about the remainders when you divide by 12?"
- "When you divide a number, what is the smallest remainder you can have?"
- "What is the largest remainder you can have?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ model the division process as equal sharing using base-10 blocks
- ❑ solve division problems in context using a personal strategy and record the process symbolically
- ❑ recognize that the larger the divisor, the more related division problems
- ❑ recognize that the smallest possible remainder is 0 and the largest possible remainder is always 1 less than the divisor
- ❑ recognize that the number of possible remainders is the same as the divisor (e.g., when dividing by 3, there are 3 possible remainders: 0, 1, and 2)

- **Explain that the interpretation of a remainder depends on the context.**
- **Solve a division problem in context using personal strategies, and record the process.**

Materials: Two dice and a playing board for each group (BLM 5.N.6.2). One die should have the numbers 1, 3, 5, 7, 7, and 9 written on it and the other die should have the numbers 1, 2, 4, 6, 6, and 8 written on it.

Organization: Groups with 3 or 4 students

Procedure:

a) Tell students that they will be playing a game involving remainders in division. Explain how the game is played.

   1. Players take turns, rotating clockwise.
   2. During a turn, a player crosses off any unused number on the playing board. He or she then chooses one of the dice and rolls it. Next, the player divides the number rolled into the number that was crossed off and finds the remainder.
   3. The remainder for the division problem is the player's score for that round.

      For example, a player crosses off 113 and chooses to roll the die with the numbers 1, 3, 5, 7, 7, and 9 on it. If the number rolled is a 5, the player divides 5 into 113 and gets a quotient of 22 with a remainder of 3. The player's score for that round is 3.
   4. If a player notices a mistake that another player makes, he or she gets that player's score for the turn.
   5. The game ends when all the numbers have been crossed off. The player with the largest cumulative score is the winner.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Have students play the game again, but this time the player with the smallest cumulative score is the winner.

d) Have students create their own division game board and use it to play the game with their partner.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve division problems in context using a personal strategy, and record the process symbolically
- ❑ use a strategy that is efficient, is mathematically correct, and can be generalized
- ❑ develop a strategy for playing the game
- ❑ recognize problems that have been solved incorrectly

For further information, see

- Ontario Education. *Number Sense and Numeration, Grades 4 to 6*. Volume 4. "Learning about Division in the Junior Grades."
- *Big Ideas from Dr. Small* by Marian Small. "Grades 4 to 8: Whole Number Operations. Varied Approaches for Multiplication and Division." p. 34.

NOTES

Grade 5: Number (5.N.7)

Enduring Understandings:

Equivalent fractions are fractions that represent the same value.

Different strategies can be used to compare fractions with unlike denominators.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
5.N.7 Demonstrate an understanding of fractions by using concrete and pictorial representations to ■ create sets of equivalent fractions ■ compare fractions with like and unlike denominators [C, CN, PS, R, V]	➣ Create a set of equivalent fractions and explain why there are many equivalent fractions for any fraction using concrete materials. ➣ Model and explain that equivalent fractions represent the same quantity. ➣ Determine if two fractions are equivalent using concrete materials or pictorial representations. ➣ Formulate and verify a rule for developing a set of equivalent fractions. ➣ Identify equivalent fractions for a fraction. ➣ Compare two fractions with unlike denominators by creating equivalent fractions. ➣ Position a set of fractions with like and unlike denominators on a number line (vertical or horizontal), and explain strategies used to determine the order.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Demonstrating an understanding of fractions less than or equal to one using concrete and pictorial representations
- Naming and recording fractions for the parts of a whole or a set
- Comparing and ordering fractions with like numerators or like denominators
- Providing examples of where fractions are used
- Relating fractions to decimals

BACKGROUND INFORMATION

Equivalent fractions are fractions that represent the same value. For example, $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$ are different names for the same number. Students need an understanding of equivalence in order to compare, order, add, and subtract fractions.

Students often find fractions confusing and difficult to comprehend. Difficulties with learning fractions can arise from instruction that emphasizes procedural knowledge rather than conceptual knowledge. They also arise because the whole number concepts that students learned do not always apply to fractions. For example, when the numerators are the same and the denominators are different, the larger of two fractions is determined by comparing denominators using order ideas that are the inverse of those for whole numbers. For example, 4 is less than 5, but $\frac{1}{5}$ is less than $\frac{1}{4}$. However, when the denominators are the same, the larger of two fractions is determined by comparing the numerators using whole number concepts. For example, 3 is less than 5 so $\frac{3}{8}$ is less than $\frac{5}{8}$.

To help students overcome their difficulties with learning fractions, instruction should focus on developing concepts rather than on abstract rules. Learning experiences that emphasize exploration and the manipulation of a variety of concrete materials and pictorial representations are key to helping students develop meaning for fraction concepts. Rules for manipulating fractions should only be introduced after students develop an understanding of the concepts. If the rules are introduced too soon, students end up memorizing them. Rules that are memorized without understanding are often forgotten or applied inappropriately.

Mathematical Language

Denominator

Equivalent

Fraction

Greater than

Less than

Numerator

Learning Experiences

Assessing Prior Knowledge

Materials: Copies of concept description sheet (BLM 5–8.2)

Organization: Individual

Procedure:

a) Tell students that in the next few lessons they will be learning about fractions, but before they begin you need to find out what they already know about fractions.

b) Ask students to complete the concept development sheet. Let them know that it is all right if they cannot think of anything to put in a section. They will have another opportunity to complete the sheet when they learn more about fractions.

c) When students finish, begin a discussion of fractions by asking, "What is a fraction? What is an example of a fraction?" As the discussion progresses, clear up any misconceptions students may have and make sure that they see a variety of examples and non-examples.

d) Have students complete the concept development sheet again.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recognize that a whole can be a region or a collection of objects
- ❑ recognize that the parts of a whole must be equivalent
- ❑ give appropriate examples and non-examples of fractions
- ❑ write a symbol for a fraction
- ❑ know the terms "denominator" and "numerator"

- **Determine if two fractions are equivalent using concrete materials or pictorial representations.**
- **Create a set of equivalent fractions and explain why there are many equivalent fractions for any fraction using concrete materials.**
- **Identify equivalent fractions for a fraction.**

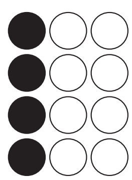
Materials: Two-coloured chips

Organizations: 3 groups of students

Procedure:

a) Give each student 12 chips and divide the students into three groups: A, B, and C. Give each group different instructions. For example, tell Team A to show $\frac{1}{3}$, Team B to show $\frac{2}{6}$, and Team C to show $\frac{4}{12}$.

b) When students finish, have them compare their results. Students should note that everyone has turned over four chips and the instructions yield the same results. Explain that $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{4}{12}$ are equivalent fractions because they are different names for the same amount.



$$\frac{1}{3}=\frac{2}{6}=\frac{4}{12}$$

c) Give different instructions that yield equivalent results. For example, ask group A to show $\frac{2}{3}$, group B to show $\frac{4}{6}$, and group C to show $\frac{8}{12}$. Ask questions such as, "Why is $\frac{2}{3}$ equivalent to $\frac{4}{6}$?" "Is $\frac{2}{3}$ the same as $\frac{4}{6}$ even when you start with 18 chips?" "Why?" Have students verify their answer by manipulating the chips.

d) Continue the activity, but change the number of chips and the instructions that result in equivalent fractions.

e) Vary the activity by having one team give the instruction (e.g., show $\frac{1}{2}$ and the other two teams give instructions that they think will yield the same results when carried out). Have students carry out the instructions to see if they create equivalent fractions.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ determine if two fractions are equivalent using concrete materials
- ❑ create a set of equivalent fractions and explain why they are equivalent
- ❑ identify equivalent fractions for a fraction

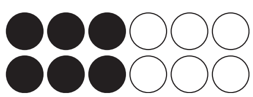
Materials: Egg cartons and counters

Organization: Pairs

Procedure:

a) Have students use their egg cartons and counters to show you $\frac{1}{12}$ of a dozen. Then ask students to show you $\frac{2}{12}$ of a dozen. Now have students show you $\frac{1}{6}$ of a dozen. When students show two counters in their egg cartons to illustrate $\frac{1}{6}$ of a dozen, say, "But you just said that is $\frac{2}{12}$ of a dozen!" Ask, "Which is it: $\frac{1}{6}$ or $\frac{2}{12}$? Why are they equivalent?"

b) Ask students to use their egg cartons to find as many equivalent fractions as they can (students should find fractions equivalent to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{5}{6}$). Have students record their findings pictorially and symbolically.



$$\frac{1}{2} = \frac{3}{6} = \frac{6}{12}$$

c) Ask students to find two fractions that are not equivalent and explain why they are not equivalent. Again, have students record their findings pictorially and symbolically.

d) Have students share their findings with the other members of the class. Encourage students to explain why two fractions are equivalent or not equivalent.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ model and explain that equivalent fractions represent the same quantity
- ❑ distinguish between equivalent and non-equivalent fractions
- ❑ identify equivalent fractions for a fraction
- ❑ determine if two fractions are equivalent using concrete materials or pictorial representations

- **Model and explain that equivalent fractions represent the same quantity.**
- **Determine if two fractions are equivalent using concrete materials or pictorial representations.**
- **Identify equivalent fractions for a fraction.**

Materials: Fraction blocks or fraction bars

Organization: Small groups

Procedure:

Note: Colours may vary based on materials used. Be sure to determine values before you begin.

a) Ask students to show you the yellow block. Tell them that this block represents one-half of the whole pink fraction block. Have them name each of the other blocks as a fraction of the pink block.

b) Ask students to use their blocks to find out
   - how many reds fit on the yellow block, and to write a number sentence that shows the relationship between the red and yellow blocks $\left(\frac{2}{4} = \frac{1}{2}\right)$
   - how many blues fit on the yellow block, and to write a number sentence that shows the relationship
   - how many greens fit on the yellow block, and to write a number sentence that shows the relationship
   - what one-eighth would look like, and to find out how many eights would fit on the yellow block (Have them write a number sentence that shows this relationship.)

c) Ask students to list all the fractions they found that are equivalent to one-half $\left(\text{Answer: } \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{6}{12} = \frac{4}{8}\right)$, and to describe any patterns or relationships that they see.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ name the fractional part of each block in relation to the larger pink block
- ❑ create a set of equivalent fractions and explain why there are many equivalent fractions using concrete materials
- ❑ model and explain that equivalent fractions represent the same quantity
- ❑ identify patterns or relationships (e.g., the denominator is twice the numerator)

- **Determine if two fractions are equivalent using concrete materials or pictorial representations.**
- **Identify equivalent fractions for a fraction.**

Materials: Two-coloured chips, fraction blocks or fraction bars

Organization: Pairs

Procedure:

a) Tell students that you will be giving them some fractions and it is their job to determine if they are equivalent. Explain that they can use chips or fraction blocks to help them decide.

b) Have students determine whether the following fractions are equivalent:

- $\frac{3}{5}$ and $\frac{9}{15}$
- $\frac{2}{8}$ and $\frac{4}{12}$
- $\frac{3}{6}$ and $\frac{9}{18}$
- $\frac{7}{10}$ and $\frac{14}{20}$
- $\frac{5}{9}$ and $\frac{18}{27}$

c) Have students share their answers with the other members of the class. Encourage discussion by asking questions, such as the following:
   - "How do you know that $\frac{3}{5}$ is equivalent to $\frac{9}{15}$?"
   - "Why is $\frac{2}{8}$ not equivalent to $\frac{4}{12}$?"
   - "What fraction is equivalent to $\frac{4}{12}$? How do you know?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify equivalent fractions
- ❑ distinguish between equivalent and non-equivalent fractions
- ❑ model and explain that two equivalent fractions represent the same amount

- **Determine if two fractions are equivalent using concrete materials or pictorial representations.**

Materials: A set of 40 equivalent fraction cards for each group of students (BLM 5.N.7.1).

Organization: Groups of 2 to 4 students

Procedure:

a) Tell students that they will be playing a game involving equivalent fractions. Explain how the game is played.
   1. Deal five cards to each player. The remaining cards are placed in a pile face down in the middle of the playing area.
   2. Players take turns asking another player for a card. For example, "Please give me the card that shows $\frac{1}{2} = \frac{2}{4}$." If the player has the card, he or she must give it to the asker. When a player receives a requested card, the player lays the two matched cards aside.
   3. If the player does not have the requested card, the asker draws a card from the pile in the middle of the playing area. If it matches a card in his or her hand, the cards are set aside. If it does not match any of the cards in his or her hand, the player keeps the card.
   4. The game is over when one player runs out of cards. The player with the most pairs is the winner.

b) Demonstrate how to play the game and answer any questions students may have. Have students play the game.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify pictorial representations of equivalent fractions
- ❑ match pictorial representations of equivalent fractions with symbolic statements
- ❑ determine if two fractions are equivalent using concrete materials or pictorial representations

Materials: Math journals

Organization: Individual

Procedure:

a) Ask students to answer the following question in their math journals:

- Are these fractions equivalent? Explain how you reached your answer.

$$\frac{3}{4} \qquad \frac{6}{12}$$

b) Have students share their answers with the other members of the class. Encourage them to explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ determine if two fractions are equivalent
- ❑ explain how they know $\frac{3}{4} \neq \frac{6}{12}$ or demonstrate using pictorial representations that $\frac{3}{4} \neq \frac{6}{12}$ (If pictorial representations are used, make sure their diagrams match their explanations.)

- **Identify equivalent fractions for a fraction.**
- **Create a set of equivalent fractions and explain why there are many equivalent fractions for any fraction using concrete materials.**
- **Determine if two fractions are equivalent using concrete materials or pictorial representations.**
- **Formulate and verify a rule for developing a set of equivalent fractions.**

Materials: Two-coloured chips and a set of fraction cards for each group of students (BLM 5.N.7.2)

Organization: Small groups

Procedure:

a) Ask students to sort the cards into piles according to how much of each diagram is shaded. Have students turn over each pile and record the fractions on the back of the cards. For example, $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}$, and $\frac{18}{24}$. Have them describe any patterns they see, and then ask them to name other fractions that belong to each set and explain how they know they are equivalent.

b) Have students complete each of the following patterns. Tell them that they can draw pictures or use materials to help them complete each pattern.

$$\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \_\_\_\_, \_\_\_\_, \_\_\_\_$$

$$\_\_\_\_, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \_\_\_\_, \_\_\_\_, \frac{7}{28}$$

$$\_\_\_\_, \_\_\_\_, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \_\_\_\_, \_\_\_\_, \_\_\_\_$$

c) Ask students to share their answers and explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify equivalent fractions for a fraction
- ❑ create a set of equivalent fractions and explain why there are many equivalent fractions
- ❑ determine if two fractions are equivalent using pictorial representations
- ❑ formulate a rule for determining equivalent fractions

- **Compare two fractions with unlike denominators by creating equivalent fractions.**

Materials: Egg cartons, counters, fraction bars (BLM 5–8.12), Cuisenaire rods, or fraction blocks, clock face (BLM 5–8.13)

Organization: Individual or pairs

Procedure:

a) Ask students to use their egg cartons and counters to answer questions, such as identifying which is larger among the following:
   - $\frac{1}{4}$ dozen or $\frac{1}{3}$ dozen
   - $\frac{7}{12}$ dozen or $\frac{1}{2}$ dozen
   - $\frac{3}{4}$ dozen or $\frac{4}{6}$ dozen
   - $\frac{1}{2}$ dozen or $\frac{5}{12}$ dozen
   - $\frac{2}{3}$ dozen or $\frac{5}{6}$ dozen

b) Have students share their answers and explain the strategies they used to determine which fraction is the larger.

c) If an analog clock is not in the classroom, draw a picture of a clock face on the board and then ask questions such as identifying which is greater among the following:
   - $\frac{1}{2}$ hour or $\frac{5}{6}$ hour
   - $\frac{2}{4}$ hour or $\frac{1}{3}$ hour
   - $\frac{5}{12}$ hour or $\frac{2}{3}$ hour
   - $\frac{3}{4}$ hour or $\frac{1}{2}$ hour
   - $\frac{10}{12}$ hour or $\frac{2}{4}$ hour
   - $\frac{2}{3}$ hour or $\frac{7}{12}$ hour

d) Repeat the activity, but have students use a different material. For example, have students use fraction bars to answer questions such as identifying which is larger among the following:

- $\frac{5}{8}$ or $\frac{2}{3}$
- $\frac{1}{6}$ or $\frac{1}{4}$
- $\frac{3}{10}$ or $\frac{5}{6}$
- $\frac{3}{4}$ or $\frac{7}{8}$
- $\frac{3}{4}$ or $\frac{11}{12}$
- $\frac{1}{3}$ or $\frac{2}{4}$
- $\frac{5}{12}$ or $\frac{3}{8}$
- $\frac{1}{2}$ or $\frac{2}{6}$

e) Have students share their answers and explain the strategies they used to determine which fraction is the larger.

Observation Checklist

Observe students to determine whether they can do the following:

- ❏ represent a given fraction with concrete materials
- ❏ compare two fractions with unlike denominators using concrete materials

Materials: Graph paper

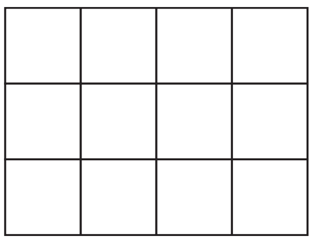
Organization: Small groups/Whole class

Procedure:

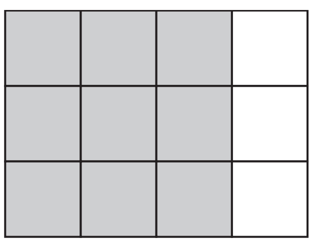
a) Present students with the following problem:

- Sammy had a piece of graph paper on his desk. His teacher asked, “Which is larger, $\frac{2}{3}$ or $\frac{3}{4}$?” After a few minutes, Sammy said, “I think I see a new way to find out.” How do you think Sammy did it?

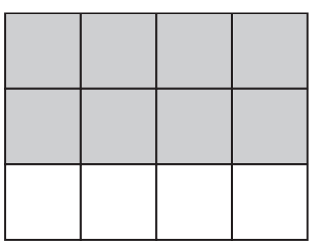
b) Have each group explain how they think Sammy used the graph paper to help him compare the two fractions. If none of the groups use the graph paper to find common denominators, explain that Sammy could have used the graph paper to mark off a rectangle that was four squares one way and three squares the other way.



Each column is $\frac{1}{4}$ of the rectangle, so $\frac{3}{4}$ would look like this:



Each row is $\frac{1}{3}$, so $\frac{2}{3}$ would look like this:



$\frac{3}{4} = \frac{9}{12}$ and $\frac{2}{3} = \frac{8}{12}$, $\frac{9}{12}$ is greater than $\frac{8}{12}$, so $\frac{3}{4}$ is greater than $\frac{2}{3}$.

c) Do two or three more examples, then ask students to use the method to decide which fraction is larger:

- $\frac{3}{4}$ and $\frac{4}{6}$
- $\frac{3}{5}$ and $\frac{2}{3}$
- $\frac{4}{9}$ and $\frac{5}{6}$
- $\frac{2}{7}$ and $\frac{4}{5}$
- $\frac{3}{4}$ and $\frac{2}{5}$

d) Have students share their answers. Encourage them to describe and show how they used the graph paper to help them find fractions with common denominators.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ represent fractions pictorially
- ❏ compare two fractions with unlike denominators by creating equivalent fractions
- ❏ describe and show how they found equivalent fractions

Materials: Spinner with the numbers 1-12 on them (BLM 5–8.14), copies of the recording sheet (BLM 5.N.7.3), fraction strips, pattern blocks or other fraction manipulatives, an overhead transparency of the recording sheet

Organization: Pairs/Whole class

Procedure:

a) Tell students that they will be playing a game with their partner that involves comparing fractions. Explain how to play the game.

1. Players take turns spinning the spinner four times.
2. After each spin, the player writes the number in one of the boxes or on one of the lines beside the bottom box. Once a number has been written, it cannot be changed. After four spins, there will be a fraction (formed by the numbers in the boxes) and two "rejected" numbers that can be put in the trash can.

3. After both players have created their fractions, they can use the materials or draw pictures to model their fractions and decide which is the larger. The players then write a sentence to show the comparison $\left(\text{e.g., } \frac{3}{6} > \frac{1}{4} \text{ or } \frac{3}{4} = \frac{9}{12}\right)$.

4. The player who made the larger fraction scores one point. If the fractions are equivalent, both players score a point.

5. The winner is the player with the most points after four rounds.

c) Demonstrate how to play the game and answer any questions students might have. Have students play the game.

d) Vary the game by having one player spin the spinner four times and both players use the same numbers to complete the game board.

e) Have students discuss their experience playing the game. Encourage them to describe any strategy they used that worked well for them.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ model fractions appropriately using concrete and pictorial representations
- ❏ compare fractions with unlike denominators by
  - creating equivalent fractions
  - using concrete materials or pictorial representations
  - using a personal strategy
- ❏ recognize equivalent fractions
- ❏ show symbolically which of two fractions is larger $\left(\text{e.g., } \frac{2}{3} > \frac{1}{2}\right)$
- ❏ recognize that if the denominators are the same, the fraction with the larger numerator is the larger
- ❏ recognize that if the numerators are the same, the fraction with the smaller denominator is the larger

Materials: Math journals and pencils

Organization: Individual/Whole class

Procedure:

a) Write these questions on the board or overhead.
   - "Which is larger: $\frac{6}{8}$ or $\frac{4}{5}$? How do you know?"

b) Ask students to think about the questions and then record their answers in their math journals.

c) When students finish, have them share their answers. Encourage them to discuss the strategies they used by asking them questions, such as:
   - "How do you know that $\frac{4}{5}$ is larger than $\frac{6}{8}$?"
   - "Is there another strategy you could use to show that $\frac{4}{5}$ is larger than $\frac{6}{8}$?"
   - "Which strategy do you prefer? Why?"

Observation Checklist

Observe students' responses to determine the strategy they used to find the larger fraction. For example, some students may find the larger fraction by

- ❏ creating equivalent fractions with like denominators $\left(\text{e.g., } \frac{32}{40} \text{ and } \frac{30}{40}\right)$
- ❏ creating equivalent fractions with like numerators $\left(\text{e.g., } \frac{12}{15} \text{ and } \frac{12}{16}\right)$
- ❏ comparing the fractions to a benchmark (e.g., deciding whether $\frac{6}{8}$ or $\frac{4}{5}$ is closer to 1)

Also, observe students' responses to determine whether they can do the following:

- ❏ communicate their ideas effectively
- ❏ use appropriate diagrams or pictures (when used in their explanation)
- ❏ recognize that the more pieces into which a whole is divided, the smaller the pieces.
- ❏ recognize equivalent fractions
- ❏ use appropriate procedures to find equivalent fractions

- **Position a set of fractions with like and unlike denominators on a number line (vertical or horizontal), and explain strategies used to determine the order.**

Materials: Fraction cards (BLM 5.N.7.2), vertical and horizontal number lines marked with 0, $\frac{1}{2}$, and 1.

Organization: Whole class/Pairs

Procedure:

a) Show students the cards with fractions on them, and explain that their task is to place the fractions in order from smallest to largest. Select one card and prop it on the chalkboard tray. Show the other cards one at a time and have a student place them on the tray. Have students explain their reasoning.

b) Draw a number line on the board and mark the points 0, $\frac{1}{2}$, and 1. Have students place each fraction on the number line and explain their reasoning.

c) Have students work with their partner to position each of the following sets of fractions on the number line:

- $\frac{1}{12}, \frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}, \frac{11}{12}$
- $\frac{1}{16}, \frac{2}{8}, \frac{3}{8}, \frac{9}{16}, \frac{7}{8}$
- $\frac{1}{9}, \frac{1}{4}, \frac{1}{3}, \frac{5}{9}, \frac{2}{3}, \frac{6}{8}$

d) Have students create their own set of fractions to order and place on the number line. Have them justify, in writing, their placement of the fractions.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ use an appropriate (mathematically correct) strategy for comparing fractions with unlike denominators
- ❑ position a set of fractions with like and unlike denominators on a number line
- ❑ explain the strategies that they used to position the fractions

NOTES

Grade 5: Number (5.N.8, 5.N.9)

Enduring Understandings:

Decimals are symbols for common fractions whose denominators are powers of ten.

Decimals are an extension of the base-10 numeration system.

Fractions and decimals can be used interchangeably.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
5.N.8 Describe and represent decimals (tenths, hundredths, thousandths), concretely, pictorially, and symbolically. [C, CN, R, V]	➛ Write the decimal for a concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure. ➛ Represent a decimal using concrete materials or a pictorial representation. ➛ Represent an equivalent tenth, hundredth, or thousandth for a decimal, using a grid. ➛ Express a tenth as an equivalent hundredth and thousandth. ➛ Express a hundredth as an equivalent thousandth ➛ Describe the value of each digit in a decimal.
5.N.9 Relate decimals to fractions (tenths, hundredths, thousandths). [CN, R, V]	➛ Write a decimal in fractional form. ➛ Write a fraction with a denominator of 10, 100, or 1000 as a decimal. ➛ Express a pictorial or concrete representation as a fraction or decimal (e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $\frac{250}{1000}$).

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of fractions less than or equal to one using concrete and pictorial representations
- Naming and recording fractions for the parts of a whole or a set
- Comparing and ordering fractions with like numerators or like denominators
- Providing examples of where fractions are used
- Describing and representing decimals (tenths and hundredths) concretely, pictorially, and symbolically
- Relating fractions to decimals (to hundredths)

Related Knowledge

Students should be introduced to the following:

- Using concrete or pictorial representations to create equivalent fractions
- Measuring the length of an object in millimetres, centimetres, or metres
- Stating the relationship between millimetres and centimetres, centimetres and metres, millimetres and metres

Background Information

Knowledge of decimals is necessary to deal effectively with everyday situations involving money, measurement, probability, and statistics. However, many students lack the understanding of decimals needed to deal with these situations in meaningful ways. Many of their misconceptions about decimals stem from their efforts to apply whole number concepts to decimals. For example, some students believe that 0.143 is greater than 0.43 because 143 is larger than 43, while others believe that 0.150 is ten times larger than 0.15 since 150 is ten times larger than 15.

Instruction that focuses on the meaning of decimals can help students overcome or avoid these misconceptions. In particular, learning experiences need to emphasize two interpretations of decimals: first, decimals are just another symbol for common fractions whose denominators are powers of ten; and second, decimals are an extension of the base-10 numeration system. Allowing students to manipulate concrete and pictorial representations of decimals and helping them make connections between their actions on these representations and the symbols for decimals can facilitate their understanding of these two interpretations.

Mathematical Language

Decimal

Decimal point

Denominator

Equivalent

Fraction

Hundredths

Numerator

Tenths

Thousandths

Learning Experiences

Assessing Prior Knowledge

Materials: Paper and pencil

Organization: Individual/Whole class

Procedure:

a) Tell students that in the next few lessons they will be learning about decimals, but before they begin you need to find out what they already know about decimals. Have students write a letter telling you what they know about decimals.

b) When students finish, have them share what they know about decimals with the other members of the class. Use the discussion to clear up any misconceptions they might have about decimals.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ relate fractions to decimals (to hundredths)
- ❑ describe and represent decimals pictorially and symbolically
- ❑ identify examples and non-examples of decimals
- ❑ identify examples of where decimals are used
- ❑ recognize that decimals are an extension of the place value system

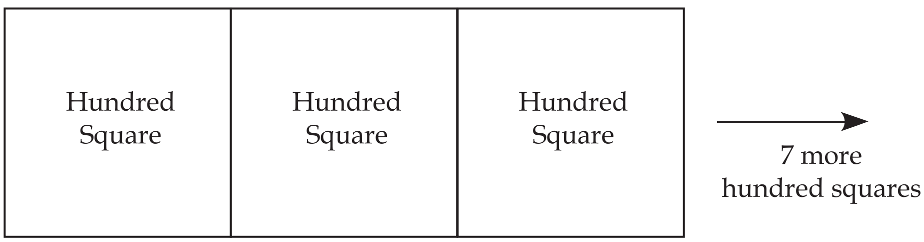
- **Write the decimal for a concrete or pictorial representation of a part of a set, part of a region, or part of a unit of measure.**
- **Represent a decimal using concrete materials or a pictorial representation.**
- **Represent an equivalent tenth, hundredth, or thousandth for a decimal, using a grid.**

Materials: Hundred squares (BLM 5–8.6) and tape

Organization: Whole class

Procedure:

a) Give each student 10 hundred squares. Tell them that the squares are pieces of a whole and that they will be using them to learn about decimals. Ask students to arrange the pieces in a strip to show a whole. Have them tape the pieces together, as shown below.



b) Name the piece "one whole strip." Ask students, "How many hundred squares make one whole strip? If there are 10 hundred squares in a whole, what is a hundred square called (1 tenth)? How many rows of ten small squares are there? How do you know? If there are 100 columns of ten small squares, what is each column of ten small squares called (1 hundredth)? How many small squares are in the whole? How do you know? If there are 1000 small squares in the whole, what is each small square called (1 thousandth)?"

c) Ask students to count parts of the strips by tenths. Have them point to each tenth as they count. Next, have them count by hundredths. Tell students that since it would take too long to count by thousandths, you want them to count by 10 thousandths. Ask, "What part of the whole is 10 thousandths (1 hundredth)?" Have students point to each 10 thousandth as they count 10 thousandths, 20 thousandths, …, 100 thousandths, …, 1000 thousandths.

d) Write the decimals 0.1 and 0.01 on the board or overhead. Ask students to use their strips to show what each decimal means. Then ask, "How do you think we should write the decimal for one-thousandth? Why?" Discuss that it makes sense to use the next place to the right for thousandths.

e) Write the following decimals on the board. Have students read each decimal and illustrate it with their strip.

- 0. 007
- 0. 015
- 0. 065
- 0. 105
- 0. 504
- 0. 253
- 0. 183
- 0. 724
- 0. 965
- 0. 008

Note: Students will need this "thousands strip" for a later activity.

Observation Checklist

Observe students' responses to determine the strategy they used to find the larger fraction. For example, some students may find the larger fraction by

- ❑ identifying tenths, hundredths, and thousandths
- ❑ counting by tenths, hundredths, and 10 thousandths
- ❑ reading decimals correctly
- ❑ associating symbols for decimals with a concrete representation
- ❑ representing decimals with concrete materials

- **Represent a decimal using concrete materials or a pictorial representation.**

Materials: Metre sticks (one for each pair of students) and an overhead transparency of a metre stick

Organization: Pairs

Procedure:

a) Ask students to find 0.1 of their metre stick. Record the decimal on the board or overhead, and then ask students to explain their choice.

b) Repeat the activity—that is, have students use their metre sticks to show the following:

- 0.01
- 0.008
- 0.001
- 0.680
- 0.040
- 0.308
- 0.125
- 0.882
- 0.268
- 0.913

Write the decimals on the board or overhead and have students explain their choices.

c) Vary the activity by asking students questions such as:
- “Is 0.001 closer to 0 or to 1?”
- “Is 0.275 closer to 0.200 or 0.300?”
- ”What decimals come between 0.775 and 0.780?”
- ”What decimal comes immediately before 0.431?”
- ”What decimal comes immediately after 0.599?”

Observation Checklist

Observe students’ responses to determine whether they can do the following:

- ❑ represent decimals with concrete materials
- ❑ associate symbols for decimals with concrete materials
- ❑ identify the decimal that comes immediately before or after a given decimal
- ❑ identify decimals that come between two given decimals

- **Represent a decimal using concrete materials or a pictorial representation.**
- **Express a tenth as an equivalent hundredth and thousandth.**
- **Express a hundredth as an equivalent thousandth.**
- **Describe the value of each digit in a decimal.**

Materials: The thousandths strips that students made and used in a prior activity for these outcomes

Organization: Whole class/Pairs

Procedure:

a) Have students decide what part of their strip represents 0.600. Next, ask students to use their strip to help them name a decimal equivalent to 0.600 (0.6 and 0.60). If students experience difficulty completing this task, have them count by tenths (and then hundredths) until 0.600 is reached.

b) Repeat the activity, but this time ask students to decide what part of their strip represents 0.350. Then have students use their strip to find a decimal equivalent to 0.350 (0.3 + 0.05 and 0.35). Continue to give students other thousandths and ask them to find decimal equivalents.

c) Have students use their strips to find decimals equivalent to 0.45 (0.450, 0.40 + 0.05, 0.4 + 0.05) and 0.3 (0.30 and 0.300).

d) Have students work with their partner to find decimals that are equivalent to

- 0.125 (0.1 + 0.20 + 0.005, 0.12 + 0.005, 0.1 + 0.025)
- 0.328
- 0.8
- 0.630
- 0.72
- 0.034
- 0.295
- 0.044
- 0.638

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ represent a decimal using concrete materials
- ❑ describe the value of each digit in a decimal
- ❑ express tenths as equivalent hundredths and thousandths
- ❑ describe thousandths as equivalent tenths and hundredths
- ❑ describe hundredths as equivalent tenths and thousandths

Materials: Thousandth grid (BLM 5–8.15) (Students should use one grid for each decimal number.), crayons or markers

Organization: Whole class/Small groups

Procedure:

a) Ask students to use the grid paper to represent these decimals.

0.2	0.4	0.7	0.9
0.20	0.40	0.70	0.90
0.200	0.400	0.700	0.900

b) When students finish, ask questions about the relationships among the decimals, such as:

- "What do you notice about two-tenths, twenty-hundredths, and two-hundred-thousandths?"
- "How are these decimals alike?"
- "How do they differ?"

c) Have students use the grid paper to find decimals that are equivalent to 0.10, 0.300, 0.5, 0.60, and 0.8. Encourage students to discuss their findings by asking questions similar to the following:

- "What decimals are equivalent to ten hundredths? How do you know?"
- "How are the decimals alike?"
- "How do they differ?"
- "What rule do your observations suggest?"

d) Have groups consider these questions: "What is the difference between the values of the 5 in (1) and the values in (2)? What does this tell you? How do you know?"

1. 5, 50, 500
2. 0.5, 0.50, 0.500

e) Have the groups share their answers with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ demonstrate the meaning of a decimal by representing it pictorially
- ❏ represent an equivalent tenth, hundredth, or thousandth using a grid
- ❏ recognize and explain the similarities and differences among equivalent decimals
- ❏ represent a tenth as an equivalent hundredth or thousandth
- ❏ recognize that annexing a zero to the right of a whole number changes the place and value of each digit
- ❏ recognize that annexing a zero to the right of a decimal changes the name but not the place or value of each digit
- ❏ describe the value of each digit in a whole number or decimal

Materials: Base-10 blocks or digi-blocks and a place value mat (BLM 5–8.16)

Organization: Whole class

Procedure:

a) Show students the large block and tell them that, for this activity, the block will represent one, a flat will represent one-tenth, the long will represent one-hundredth, and the small cube will represent one-thousandth. To help students get used to the new number names for the pieces, ask them to use their blocks to show you:

- two-tenths five-hundredths six-thousandths
- nine-tenths two-hundredths three-thousandths
- five-tenths four-hundredths nine-thousandths

b) Continue asking students to show you different numbers with their base-10 blocks until they can do it quickly and easily.

c) Represent the following decimals with the blocks and ask students to write the corresponding symbols for the decimals.

- 2.156
- 0.189
- 1.057
- 3.420
- 1.903
- 5.341
- 3.204
- 2.651

Encourage students to discuss their answers by asking them questions such as:

- "What decimal did you write?"
- "What is the value of the 1 in 2.156?"
- "What is the place value of the 8 in 0.189?"
- "What is the value of the 2 in 3.204? In 3.420?"
- "What decimal is equivalent to 3.420?"

d) Vary the activity by naming a decimal (e.g., 1.254) and having students represent it with their blocks. After they represent the number, ask them to record the corresponding symbol (decimal). Again, encourage students to discuss their answers by asking questions similar to the ones in part (c).

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ write the decimal that corresponds to its concrete representation
- ❑ represent a decimal using concrete or pictorial representations
- ❑ state the value of each digit in a decimal
- ❑ name the place value position of each digit in a decimal
- ❑ recognize equivalent decimals

- **Write a decimal in fractional form.**
- **Write a fraction with a denominator of 10, 100, or 1000 as a decimal.**
- **Express a pictorial or concrete representation as a fraction or decimal.**

Materials: Thousandths grid (BLM 5–8.15) and coloured pencils

Organization: Pairs

Procedure:

a) Give students several copies of the thousandths grid. Have students observe that each grid is a 20-by-50 rectangle containing 1000 small squares.

b) Ask students to shade in 350 squares, and then write under the grid the decimal and fraction names for the shaded-in squares.

c) Next, have students shade in 250 squares. Ask students to write the basic fraction and decimal name for the shaded squares $\left(\frac{250}{1000} \text{ and } 0.250\right)$ Encourage students to use their grids to help them find equivalent numbers. Ask:
   - "What decimal is equivalent to $\frac{250}{1000}$? How do you know?"
   - "What fraction with a smaller numerator and denominator is equivalent to $\frac{250}{1000}$? $\left(\frac{25}{100} \text{ or } \frac{1}{4}\right)$ How do you know?"
   - "What other fraction is equivalent to $\frac{250}{1000}$? How do you know?"

d) Repeat the activity several times. For example, have students shade
   - 125 squares
   - 184 squares
   - 375 squares
   - 500 squares
   - 600 squares
   - 750 squares
   - 100 squares
   - 267 squares

   Encourage students to write as many decimal and fraction names as they can for each shaded grid.

e) Have students share their answers and explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ express a pictorial representation as a fraction
- ❑ express a pictorial representation as a decimal
- ❑ identify equivalent fractions for a pictorial representation
- ❑ identify equivalent decimal for a pictorial representation
- ❑ explain their reasoning

- Write a decimal in fractional form.
- Write a fraction with a denominator of 10, 100, or 1000 as a decimal.

Materials: Fraction and decimal equivalent cards (BLM 5.N.8&9.1)

Organization: Pairs

Procedure:

a) Tell students that they will be playing a game that involves matching decimals with their equivalent fractions.

b) Tell students that they will need to shuffle their cards and spread them face up on their playing area. When you say go, they should begin matching the fraction/decimal cards without saying anything to their partner. The first pair to match the decimal/fraction cards correctly wins.

c) Demonstrate how to play the game and answer any questions students might have. Have them play the game.

d) Repeat the activity, but this time put a time limit on it. For example, give students a minute to complete the activity. The pair that matches the most number of cards correctly wins.

e) Have students make their own equivalent fraction/decimal cards and use them to play the game.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ associate a fraction with a denominator of 10 with an equivalent decimal
- ❑ associate a fraction with a denominator of 100 with an equivalent decimal
- ❑ associate a fraction with a denominator of 1000 with an equivalent decimal
- ❑ identify fractions and decimals that are not equivalent

Materials: Fraction and decimal equivalent cards (BLM 5.N.8&9.1)

Organization: Small groups

Procedure:

a) Tell students that they will be playing concentration with the fraction/decimal cards.

b) To play the game, have students spread the cards face down on the playing area. Have students take turns turning over two cards. If the cards match, the player keeps the cards and takes another turn. If the cards do not match, the player turns them back over and the next player takes a turn. Play continues until all the cards have been matched. The player with the most cards is the winner.

c) Demonstrate how to play the game and answer any questions students might have. Have students play the game.

d) Have students play the game again with the fractions/decimal equivalent cards that they made (see previous activity).

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ associate a fraction with a denominator of 10 with an equivalent decimal
- ❏ associate a fraction with a denominator of 100 with an equivalent decimal
- ❏ associate a fraction with a denominator of 1000 with an equivalent decimal
- ❏ identify fractions and decimals that are not equivalent

Materials: A coin and paper and pencils

Organization: Whole class

Procedure:

a) Tell students that their job is to record the numbers that you will be reading to them. They can record a number as a decimal or as a fraction (they must choose one representation, but can change which they choose for each new number). They will score a point if they record the number correctly. After they record the number, you will toss a coin. If the coin lands heads up, everyone who wrote the number correctly as a decimal scores another point. If the coin lands tails up, everyone who wrote the number correctly as a fraction scores another point. Students keep their own score.

b) Read these numbers to students:

- three thousandths
- one-hundred-fifteen thousandths
- sixty-two thousandths
- eighty thousandths
- forty hundredths
- six tenths
- one-hundred-five thousandths
- nine tenths
- five-hundred-three thousandths
- three hundredths

c) Select 10 more numbers to read to the class. Repeat the activity, but this time let the student who scored the most points read the numbers.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ associate an oral representation of a number with the correct decimal symbol for the number
- ❑ associate an oral representation of a number with the correct fraction symbol for the number
- ❑ read a decimal numeral correctly
- ❑ read a fraction correctly

Notes

Grade 5: Number (5.N.10)

Enduring Understandings:

Decimals are an extension of the base-10 numeration system.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
5.N.10 Compare and order decimals (tenths, hundredths, thousandths) by using ■ benchmarks ■ place value ■ equivalent decimals [CN, R, V]	➛ Order a set of decimals by placing them on a number line (vertical or horizontal) that contains the benchmarks 0.0, 0.5, 1.0. ➛ Order a set of decimals including only tenths using place value. ➛ Order a set of decimals including only hundredths using place value. ➛ Order a set of decimals including only thousandths using place value. ➛ Explain what is the same and what is different about 0.2, 0.20, and 0.200. ➛ Order a set of decimals including tenths, hundredths, and thousandths using equivalent decimals.

Prior Knowledge

Students may have had experience with the following:

- Describing and representing decimals (tenths and hundredths)

Related Knowledge

Students should be introduced to the following:

- Describing and representing decimals to thousandths concretely, pictorially, and symbolically

Mathematical Language

Decimal

Equivalent

Benchmark

Tenths

Hundredths

Thousandths

Learning Experiences

- **Order a set of decimals including only tenths using place value.**
- **Order a set of decimals including only hundredths using place value.**

Materials: Math journals, base-10 blocks, place value mats (BLM 5–8.16)

Organization: Pairs

Procedure:

a) Show students a flat and tell them that, for this activity, a flat will represent one, a long will represent one-tenth, and the small cube will represent one-hundredth. To help students get used to the new number names for the pieces, ask them to use their blocks to show you the following:

- nine-tenths
- two-tenths
- three-tenths
- one-hundredth
- six-hundredths
- fourteen-hundredths

b) Continue asking students to show you different numbers with their base-10 blocks until they can do it quickly and easily.

c) Have students use their blocks to represent each pair of numbers and decide which number is larger. Ask students to record their answers.

- 0.5 and 0.8
- 0.2 and 0.7
- 1.3 and 1.2
- 7.8 and 7.9
- 0.34 and 0.54
- 0.21 and 0.12
- 0.53 and 0.08
- 0.07 and 0.42

d) Vary the activity by asking students to

- make a number larger than a given number (e.g., ask students to make a number greater than 2.6)
- make a number smaller than a given number (e.g., ask students to make a number less than 0.36)
- make a number between two given numbers (e.g., ask students to make a number between 0.1 and 0.4)
- look at each number in a pair of numbers, decide which number is greater, and then check their answers with the blocks (e.g., ask students to circle the larger number [0.51 or 0.43], and then use their blocks to check their answers)

e) Ask students to answer the following questions in their math journals:

- "Which number is smaller: 2.8 or 2.3? How do you know?"
- "Which number is larger: 0.53 or 0.06? How do you know?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ order a set of decimals including only tenths using place value
- ❑ rename hundredths as tenths and hundredths (e.g., Do students recognize that fifty-three–hundredths is the same as five-tenths and three-hundredths)
- ❑ order a set of decimals including only hundredths using place value
- ❑ identify numbers greater than (or less than) a given number
- ❑ identify numbers that come between two given numbers

- **Order a set of decimals including only tenths using place value.**
- **Order a set of decimals including only hundredths using place value.**
- **Order a set of decimals including only thousandths using place value.**

Materials: One die per group, recording sheet (BLM 5.N.10.1)

Organization: Small groups

Procedure:

a) Tell students that they will be playing a game called "Less Than." Explain how the game is played.

   1. Designate one player to roll the die. When the die is rolled, players write the resulting number in one of their boxes before the die is rolled again. Once a number is written on a line, it cannot be changed.
   2. A round of the game consists of four rolls of the die. If the two numbers that are generated from the four rolls of the die are in the correct order, the player scores one point. If the numbers are not in the correct order, the player scores a zero.
   3. Play four rounds. The winner is the player with the most points.

b) Demonstrate how to play the game and answer any questions students may have. Have students play the game.

c) Vary the game by having students

   - create and order three decimal numbers (___ · ___ < ___ · ___ < ___ · ___)
   - create and order decimals in the hundredths or thousandths (e.g., 0 · ____ ____ < 0 · ____ ___)

Observation Checklist

Depending on the version of the game that is played, check students' responses to determine whether they can order a set of decimals

- ❑ including only tenths using place value
- ❑ including only hundredths using place value
- ❑ including only thousandths using place value

Materials: Paper and pencils

Organization: Individual

Procedure:

a) Have students solve problems like the following:

- The split times for Dimitri and Euclid were 10.321 and 10.209 seconds respectively. Who had the fastest time?
- The masses of five eggs were as follows: 0.042 kg, 0.053 kg, 0.051 kg, 0.049 kg, and 0.056 kg. Place the masses in order from smallest to largest.
- To make a miniature toy car, you need tires with a width between 0.465 cm and 0.472 cm. Will a tire with a width of 0.469 cm work? Explain your answer.
- At a swimming competition, June scored 9.80, Nora scored 9.75, Debbie scored 9.79, and Alexia scored 9.81. What must Tina score to win the competition? Explain your answer.

b) Have students share their solutions to the problems and explain their reasoning.

Observation Checklist

Check students' responses to determine whether they can do the following:

- ❑ solve problems involving the comparing and ordering of decimals to thousandths
- ❑ compare and order hundredths
- ❑ compare and order thousandths

Materials: Paper and pencil

Organization: Individual

Procedure:

a) Ask students to name a decimal that is

- greater than 5.9 and less than 6
- greater than 9 and less than 9.1
- greater than 0.63 and less than 0.64
- greater than 8.9 and less than 9.15
- greater than 7.8 and less than 7.62

b) Have students use the digits 0 through 9 to complete the answer to each statement. A digit cannot be used more than once.

- greater than 0.52 but less than 0.53 — 0.5__ __
- greater than 0.614 but less than 0.62 — 0.6 __ __
- greater than 83.07 but less than 83.079 — 83.0 __ __
- greater than 367.821 but less than 367.831 — 376.8 __ __

Observation Checklist

Monitor students' responses to determine whether they can do the following:

❑ identify a number between two given numbers

❑ compare and order decimals including only tenths

❑ compare and order decimals including only hundredths

❑ compare and order decimals including only thousandths

- **Order a set of decimals by placing them on a number line (vertical or horizontal) that contains the benchmarks, 0.0, 0.5, 1.0.**

Materials: Decimal cards (BLM 5.N.10.2), as well as grid paper, metre sticks, or base-10 blocks available for students who would like to use them

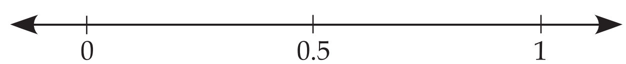
Organization: Pairs or small groups/Whole class

Procedure:

a) Give each pair of students a set of cards and ask them to sort the cards into three groups: those that are close to zero, those that are close to five-tenths, and those that are close to 1.

b) When students finish sorting their cards, have them share their answers with the other members of the class. Encourage students to explain their reasoning by asking them questions such as:
   - "How do you know when a decimal is close to five-tenths?"
   - "How do you know when a number is close to zero?"
   - "How do you know when a number is close to one?"
   - "Are the number of decimal places important in determining the size of a decimal? Why?"
   - "Which decimals are hardest to order? Why?"

c) Draw either a horizontal or vertical number line on the board and label the points 0, 0.5, and 1.



Ask students to estimate where one-tenth would be on the number line. Have a student tape a card showing one-tenth on the number line. Continue having students estimate and show where the numbers on their decimal cards would be on the number line.

d) Ask each pair of students to make their own decimal cards that show hundredths and thousandths (e.g., 0.20 and 0.132). Have them exchange their cards with another pair and then sort the cards into three groups: decimals that are close to one, decimals that are close to five-tenths, and decimals close to zero.

e) Have students share the numbers and their explanations of how they grouped them with the other members of the class. Encourage them to explain their reasoning by asking them questions similar to those in part (b).

f) Select several of the number cards that the students created and have students estimate and show where the numbers are on the number line.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify decimals that are close to the benchmarks of 0, 0.5, and 1
- ❏ explain how they know when a decimal is close to 0, 0.5, and 1
- ❏ order a set of decimals consisting of tenths and hundredths using place value
- ❏ order a set of decimals consisting of hundredths and thousandths using place value
- ❏ order a set of decimals by placing them on a number line that contains the benchmarks 0, 0.5, and 1
- ❏ order a set of tenths, hundredths, and thousandths by using equivalent decimals

■ Order a set of decimals including tenths, hundredths, and thousandths using equivalent decimals.

Materials: Blank dice (Write the decimals 0.4, 0.3, 0.9, 0.74, 0.04, and 0.60 on each die.)

Organization: Pairs

Procedure:

a) Tell students that they are going to play a place value game caller "Larger." Explain how the game is played.

   1. Both players roll a die at the same time. After a roll, each player records the resulting decimal numbers in a table like the one shown below.

My Number																
My Partner's Number																

   2. Players circle the larger decimal number. The player with the larger number scores one point. If a tie occurs, circle both decimal numbers and give each player a point.
   3. The winner of the game is the person with the most points after 20 rounds of the game.

b) Demonstrate how the game is played and answer any questions students may have. Have students play the game.

c) Vary the game by writing different decimals on the dice.

Observation Checklist

Observe students' responses to the game to determine whether they can do the following:

- ❑ order sets of decimals using place value
- ❑ order tenths and hundredths using equivalent decimals

- **Order a set of decimals including only thousandths using place value.**

Materials: Players' averages and batting criteria (see below), nine pieces of paper for each student or pair of students

Organization: Individual or pairs/Large group

Procedure:

a) Present students with the following scenario:

- You have just been hired to coach the school's softball team. You have to make up the batting order for the game today but there is no one around who can help you. You know from your coaching experience that a player's batting average, on-base average, and slugging average are good indications of where a player should bat in the lineup. These are the averages of your players:

Player	Batting Average	On-Base Average	Slugging Average
Arnez	.310	.335	.414
Britton	.415	.477	.318
Brock	.297	.331	.355
Charles	.338	.355	.614
Jackson	.429	.461	.586
Lamar	.323	.360	.576
Rolen	.273	.341	.358
Santos	.248	.305	.315
White	.423	.416	.399

b) Tell students that they must make up cards that list each player's statistics, and then arrange them in a batting order that is most closely aligned to the following criteria. When they finish, they should make a lineup card to share with the rest of the class.

Player	Batting Average	On-Base Average	Slugging Average
1	High	Highest	Low–Medium
2	High	Medium–High	Medium
3	Highest	High	High
4	Medium	Medium	Highest
5	Medium	Medium	High
6	Medium	Medium	Medium
7	Low–Medium	Low–Medium	Low–Medium
8	Low–Medium	Low–Medium	Low–Medium
9	Lowest	Lowest	Lowest

c) Before students begin working on the activity, make sure they understand that the higher the batting average, the more often the player gets a hit; the higher the on-base average, the more often the player gets on base; and the higher the slugging average, the more often a player gets an extra base hit (doubles, triples, and homers).

d) Have students share their lineups with the other members of the class. Encourage them to explain why their lineups satisfy the given criteria.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ order a set of decimals in the thousandths using place value
- ❏ create a lineup card that fits the criteria
- ❏ explain the reasons for the placement of each batter in the lineup

Grade 5: Number (5.N.11)

Enduring Understandings:

Adding and subtracting decimals is similar to adding and subtracting whole numbers.

General Outcome:

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
5.N.11 Demonstrate an understanding of addition and subtraction of decimals (to thousandths), concretely, pictorially, and symbolically, by ■ using personal strategies ■ using the standard algorithms ■ using estimation ■ solving problems [C, CN, ME, PS, R, V]	➣ Estimate a sum or difference using front-end estimation (e.g., for 6.3 + 0.25 + 306.158, think 6 + 306, so the sum is greater than 312) and place the decimal in the appropriate place. ➣ Correct errors of decimal point placements in sums and differences without using paper and pencil. ➣ Explain why keeping track of place value positions is important when adding and subtracting decimals. ➣ Predict sums and differences of decimals using estimation strategies. ➣ Solve a problem that involves addition and subtraction of decimals, to thousandths. ➣ Model and explain the relationship that exists between an algorithm, place value, and number properties. ➣ Determine the sum and difference using the standard algorithms of vertical addition and subtraction. (Numbers are arranged vertically with corresponding place value digits aligned.) ➣ Refine personal strategies, such as mental math, to increase efficiency when appropriate (e.g., 3.36 + 9.65 think, 0.35 + 0.65 = 1.00, therefore, 0.36 + 0.65 = 1.01 and 3 + 9 = 12 for a total of 13.01).

Prior Knowledge

Students may have had experience with the following:

- Using compatible numbers when adding and subtracting decimals (to hundredths)
- Estimating the sums and differences of problems involving addition and subtraction of decimals (to hundredths)
- Using mental math strategies to solve problems involving addition and subtraction of decimals (to hundredths)

Related Knowledge

Students should be introduced to the following:

- Describing and representing decimals to thousandths concretely, pictorially, and symbolically
- Comparing and ordering decimals to thousandths
- Modelling and explaining the relationship between mm and cm units and mm and m units

Mathematical Language

Addition
Difference
Estimate
Subtraction
Sum

Assessing Prior Knowledge

Materials: Paper and pencil

Organization: Individual

Procedure:

a) Ask students to solve the following problems:

   1. Roberta was getting ready for the first day of school. She bought a set of five pens for $2.57, a binder for $4.35, and a box of three-hole paper for $5.15. What was the total cost of her school supplies?
   2. Joe is saving money to buy a new video game. He has $19.50, and tomorrow Mr. Mitchell is giving him $4.75 for mowing his lawn. If the video game cost $39.95, how much more money does Joe need to save?

b) Have students share their solutions to the questions. Encourage them to explain the strategies they used to solve the problems.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify the operation(s) needed to solve a computational problem
- ❏ calculate sums and differences of decimals (limited to hundredths)
- ❏ solve one- and two-step problems
- ❏ use appropriate strategies to solve addition and subtraction problems involving decimals

- **Solve a problem that involves addition and subtraction of decimals, to thousandths.**

Materials: Base-10 blocks, two different-coloured dies for each group, and a place value mat (BLM 5–8.16)

Organization: Small groups

Procedure:

a) Show students the large block and explain that they will be playing a game that involves letting the block represent one. Ask, “If the block represents one, what does the flat represent? Why? What does the long represent? Why? What does the small cube represent? Why?”

b) To help students become familiar with the new names for the base-10 block, ask them to use their blocks to show
   - 5 tenths
   - 8 tenths
   - 6 hundredths
   - 3 hundredths
   - 12 hundredths
   - 2 thousandths
   - 7 thousandths
   - 15 thousandths

   Continue to ask students to represent different decimals with the blocks until they can do it quickly and easily.

c) Tell students that the game they will be playing is called “Race to One.” Explain how the game is played.
   1. Let one coloured die represent hundredths and the other coloured die represent thousandths.
   2. Players take turns rolling the die and using their blocks to represent the number on their place value mats.
   3. When players get 10 cubes, they trade them for a long. When they have 10 longs, they trade them for a flat, and when they have 10 flats, they trade them for a block.
   4. The first player to get a block wins the game.

d) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

e) After students have played the game several times, have them use a recording sheet like the one shown below.

Turn	Number Rolled	Total
1	0.006	0.006
2	0.012	0.018

f) Repeat the activity for subtraction—that is, have students play "Race to Zero." For this version of the game, students start with the large block and, on each turn, remove the number of blocks that represents the number rolled on the die. The first player to remove all of his or her blocks is the winner. After students have played the game several times, have them use a recording sheet like the one shown above.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ use concrete materials to solve problems involving addition and subtraction of decimals to thousandths
- ❑ make groups of ten to trade, when necessary
- ❑ exchange materials for smaller or larger units when necessary

Materials: Decimal grid paper that shows thousandths (BLM 5–8.15), metre sticks, and base-10 blocks

Organization: Individual/Large group

Procedure:

a) Tell students that they will be solving some problems involving decimals. Explain that they can use a strategy of their own choosing and that grid paper, metre sticks, and base-10 blocks are available if they want to use materials to help them solve the problems.

b) Present students with these problems:

- Marco bought two bananas. The mass of the first banana was 0.057 kg, and the mass of the second was 0.45 kg. What was the combined mass of the bananas?
- At the swim meet, Marsha scored 9.234 points for her high dive. Maxine scored 6.192 points for her high dive. How much higher was Marsha's score than Maxine's?
- Nadia has a piece of string that is 1.12 m long. If she uses 0.509 m to tie a package, how much string does she have now?
- Dimitri ran the first leg of a race in 10.123 seconds, the second leg of the race in 9.72 seconds, and the last leg of the race in 11.658 seconds. How long did it take him to run the entire race?

c) When students finish solving a problem, have them share their solutions with the other members of the class. Encourage students to explain the strategies they use to solve the problems.

d) Continue to give students problems like those above.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify the operation needed to solve the problem
- ❑ solve problems involving addition and subtraction of decimals to thousandths
- ❑ use a variety of strategies to solve a problem
- ❑ calculate correctly

- **Place the decimal point in a sum or difference using front-end estimation.**
- **Explain why keeping track of place value positions is important when adding and subtracting decimals.**
- **Predict sums and differences of decimals using estimation strategies.**

Materials: Paper and pencils

Organization: Pairs/Whole class

Procedure:

a) Tell students that you will be giving them some problems. Explain that you do not want them to find the answers to the problems. Instead, you want them to list everything that they know about them. For example, there are two things we know about this problem:

   0.3 + 0.7 + 0.5 = _____

   The answer is in tenths and the answer is greater than one.

b) Ask students to make a list of everything they know about the following problems:

   1. 8.245 + 0.28 + 1.35
   2. 15.921 – 9.468
   3. 11.03 + 4.8 + 12.143
   4. 10.186 – 4.795
   5. 19.823 + 0.45 + 6.782
   6. 4 – 0.357

c) Have students share their answers with the other members of the class. Encourage students to explain their reasoning by asking them questions such as:

   - "How do you know the answer is about 9?"
   - "How do you know the answer is in thousandths?"
   - "Why is it important to keep track of place value positions when adding? Subtracting?"
   - "How do you know that the answer is less than 3?"

d) Instead of having students list everything they know about a problem, have them list what the answer cannot be. For example, the answer to the problem 5.123 + 4.382 cannot be less than 9. The total number in the thousandths place cannot be greater than 10.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ predict sums and differences of decimals using estimation strategies (e.g., front-end estimation or front-end estimation with compensation)
- ❑ explain why it's important to keep track of place value positions when adding and subtracting decimals
- ❑ predict the place of the decimal point in sums and differences using front-end estimation

- **Explain why keeping track of place value positions is important when adding and subtracting decimals.**
- **Solve a problem that involves the addition and subtraction of decimals, limited to thousandths.**

Materials: Number cards (BLM 5–8.5) (one set for each student), number frames (BLM 5.N.11.1)

Organization: Pairs

Procedure:

a) Ask students to arrange cards 1–6 on their number mats to create a problem with the largest possible sum. Have them record their solution. Have students share their solutions with the other members of the class and explain their reasoning.

b) Repeat the activity. Have students use cards 1–6 to create problems with the
   - smallest possible sum
   - largest possible difference
   - smallest possible difference

c) Have students use the digit cards 0, 1, 4, 6, 7, and 9 to create problems with the
   - largest possible sum
   - smallest possible sum
   - largest possible difference
   - smallest possible difference

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve problems involving addition and subtraction of decimals (limited to thousandths)
- ❑ compare and order decimals to thousandths
- ❑ identify the place value position of each digit in a decimal (limited to thousandths)
- ❑ identify the value of each digit in a decimal (limited to thousandths)
- ❑ calculate sums and differences involving decimals using appropriate strategies

- **Predict sums and differences of decimals using estimation strategies.**
- **Solve a problem that involves addition and subtraction of decimals, to thousandths.**

Materials: Copies of the activity (BLM 5.N.11.2)

Organization: Individual

Procedure:

a) Have students complete the activity.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve problems involving addition and subtraction of decimals (limited to thousandths)
- ❑ predict sums and differences involving decimals using estimation strategies
- ❑ calculate sums and differences involving decimals using appropriate computational strategies

Materials: Paper and pencils, number fan (BLM 5–8.17)

Organization: Pairs/Whole class

Procedure:

a) Ask students to make a decimal greater than 0.8 using their number fans. Make a list of their suggestions to show them a variety of answers.

b) Tell students that they will be going on a hunt for decimals. Explain that you will be giving them different clues and their job is to work with their partner to find an answer that fits the clue and to show their response on a number fan.

   1. A decimal between 3.25 and 3.26
   2. Two decimals whose sum is 9.346
   3. Two decimals with a difference of 0.821
   4. Three decimals whose sum is 4.734

c) Have students share their answers. Make a list of their answers to each statement so students can see a variety of solutions. Encourage students to share the strategies that they used to determine their answers.

d) Have each pair of students make up clues and exchange them with another pair of students.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ solve problems involving addition and subtraction of decimals (limited to thousandths)
- ❏ order decimals including tenths, hundredths, and thousandths
- ❏ recognize the relationship between operations
- ❏ calculate sums and differences involving decimals using appropriate computational strategies
- ❏ explain the strategies they used to determine the answers to the statements

Planning a Party

Purpose:

The intent of this investigation is to have students apply their knowledge of number and measurement concepts to a real-world situation. In particular, the investigation is designed to reinforce students' ability to

- apply estimation strategies
- compute whole numbers
- demonstrate an understanding of fractions
- represent decimals to thousandths
- compare and order decimals
- add and subtract decimals
- demonstrate an understanding of units of measure and their relationship to each other

In addition, the investigation enhances students' ability to

- solve problems
- reason mathematically
- make connections to other subjects (science and ELA)
- make connections to the real world
- communicate mathematically

Materials/Resources: Price list and purchase order (see below)

Organization: Small groups

Procedure:

a) Present the following situation:
   - You have been asked to plan a party for the students in your class. You need to order food, beverages, and entertainment for the party. You must plan two hours of entertainment. You have $25.00 plus $2.00 per person to spend on the party.

b) Tell students that they should use the following price list to decide which items they would order for the party. Explain that they can specify the flavours of the drink and food that they want (e.g., apple juice and chocolate cake). They can also name a specific movie or game (sports or video) that they would like to have at the party. Anything else they need for the party, such as plates and utensils, will be provided.

Price List:

Beverages:

No name pop	$0.98 for 2 L
Brand name pop	$1.98 for 2 L
No name juice	$1.12 for 1 L
Brand name juice	$2.04 for 1 L
Milk	$5.00 for 4 L

Food:

Apples	4 for $1.00
Watermelon	1/2 for $3.50
No name chips	$4.99 for a 500 g bag
Brand name chips	$1.25 for 100 g bag
Hot dogs	$2.79 per dozen
Hot dog buns	$2.50 for 10
Microwave popcorn	$2.99 for 3 bags
20 cm by 30 cm cake	$12.00
40 cm by 60 cm cake	$20.00
Cupcakes/muffins	$0.50 each
Large carrots	$1.44 for 12 carrots
Baby carrots	$2.99 for 30 carrots
Party sub	$15.00 for 10 people
Individual subs	$1.75 per person

Entertainment:

Movie rental	$5.49
Game system rental	$21.75
Video game rental	$4.26
Karaoke machine	$28.15
Community centre rental	$11.00 per hour
Sporting equipment rental	$1.50 per person
Sports centre rentals	$2.25 per person

c) Explain that when they have decided on what items they want for the party, they should complete the following purchase order:

Purchase Order

Item	Reasons	Amount You Will Buy?	How Much Each Person Will Get	Cost
			Total Cost	

d) Have students complete the investigation and then share their plans for the party. Encourage students to explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ give reasonable estimates of the amount of food and drink needed for the party
- ❏ calculate correctly the cost of an item
- ❏ calculate correctly the amount of an item needed
- ❏ provide appropriate reasons for choosing an item
- ❏ calculate correctly the total cost of the party
- ❏ calculate the cost of the party to be less than or equal to the amount of money available
- ❏ plan entertainment that is within the two-hour limit.
- ❏ plan entertainment that is fun and inclusive for all
- ❏ select food and drinks that meet everyone's needs
- ❏ select food and drinks that reflect the school's policy on nutrition

GRADE 5 MATHEMATICS

Patterns and Relations

Grade 5: Patterns and Relations (Patterns) (5.PR.1)

Enduring Understandings:

Number patterns and relationships can be represented using variables.

General Outcome:

Use patterns to describe the world and solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
5.PR.1 Determine the pattern rule to make predictions about subsequent elements. [C, CN, PS, R, V]	➛ Extend a pattern with or without concrete materials, and explain how each element differs from the preceding one. ➛ Describe, orally or in writing, a pattern using mathematical language, such as one more, one less, five more. ➛ Write a mathematical expression to represent a pattern, such as $r + 1$, $r - 1$, $r + 5$. ➛ Describe the relationship in a table or chart using a mathematical expression. ➛ Determine and explain why a number is or is not the next element in a pattern. ➛ Predict subsequent elements in a pattern. ➛ Solve a problem by using a pattern rule to determine subsequent elements. ➛ Represent a pattern visually to verify predictions.

Prior Knowledge

Students may have had experience with the following:

- Describing, extending, comparing, and creating increasing patterns using manipulatives, diagrams, and numbers
- Describing, extending, comparing, and creating decreasing patterns using manipulatives, diagrams, and numbers
- Identifying and describing patterns found in tables including a multiplication chart
- Adding and subtracting whole numbers less than 10 000

- Multiplying whole numbers less than 100 by whole numbers less than 10
- Dividing whole numbers less than 100 by whole numbers less than 10

Background Information

Although there are different types of patterns, the learning experiences that follow focus on **increasing/decreasing patterns**. The elements that make up these patterns are called terms. Each term builds on the previous term. Consequently, these patterns are often referred to as **growing patterns**. For example, 2, 4, 6, 8, 10. . . and 1, 2, 4, 8, 16. . . are two common increasing patterns.

Using a table to model an increasing/decreasing pattern can help students organize their thinking. It can also help them generalize the patterns symbolically. There are two types of generalizations (rules) that can be made: recursive and explicit. A **recursive generalization** tells how to find the value of a term given the value of the preceding term. An **explicit generalization** expresses the relationship between the value of the term and the term number. For example, consider this pattern:

♥ ♥♥♥ ♥♥♥♥♥ ♥♥♥♥♥♥♥ ♥♥♥♥♥♥♥♥♥

The pattern can be organized into a table like this.

Term	1	2	3	4	5
Term Value	1	3	5	7	9

The recursive generalization that describes this pattern is $n + 2$, since the value of each term is two more than the preceding term. If the pattern were continued, the value of the sixth term would be 11 since 9 + 2 = 11. However, to find the value of the 100th term, you would need to find the value of each of the 99 preceding terms.

It is easier to predict the value of subsequent terms with an explicit generalization. Notice that in the above pattern, if you double a term number and subtract 1, you get the value of the term. For example:

> The value of the third term is $2 \times 3 - 1 = 5$ and the value of the fifth term is $2 \times 5 - 1 = 9$. Thus, the explicit generalization that describes the pattern is $2n - 1$. If the pattern were continued, the value of the 100th term would be 199, since $2 \times 100 - 1 = 199$.

When helping students recognize patterns, it is important to remember that they may not see the pattern in the same way as you. Therefore, it is important that you ask students to explain their thinking. Having students describe their reasoning can also help them realize that often there is more than one way to look at a pattern.

Mathematical Language

Decreasing pattern

Increasing pattern

Pattern

Term

Term number

Term value

Learning Experiences

Assessing Prior Knowledge

Materials: K-W-L charts (BLM 5–8.18) and large chart paper sheets

Organization: Pairs/Whole class

Procedure:

a) Tell students that in the next few lessons they will be learning about patterns, but before they begin you need to find out what they already know about them.

b) Pose the question: "What is a pattern?" Have students think about the question and then share their thoughts with their partners. Ask students to work with their partners to complete the first two columns of their K-W-L chart.

c) Post the large sheets of chart paper on the board. Make a class K-W-L chart by asking each pair of students to share their ideas with the rest of the class and to write their responses on the chart paper. Encourage students to explain what they know about patterns by asking them questions such as

- "What is a pattern?"
- "How do you know the example you gave is a pattern?"
- "What comes next in your pattern? How do you know?"
- "How can you describe your pattern?"
- "Is there another way you could describe your pattern?"

d) Ask students to record what they learned from the discussion in the third column of their chart.

e) Have students maintain and revisit their chart throughout the unit on patterns.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ define what a pattern is
- ❏ identify examples and non-examples of patterns
- ❏ identify different types of patterns (e.g., Do they know what a repeating pattern is? Do they know what an increasing pattern is?)
- ❏ extend a pattern.
- ❏ describe a pattern
- ❏ use the vocabulary associated with patterns correctly (e.g., Do they use words such as "core," "repeating," "increasing/decreasing," and "term" correctly?)

Use their responses to clear up any misconceptions they might have about patterns and to enhance their ability to create, extend, analyze, and describe patterns.

- **Extend a pattern with or without concrete materials, and explain how each element differs from the preceding one.**
- **Describe, orally or in writing, a pattern using mathematical language.**
- **Write a mathematical expression to represent a pattern.**
- **Describe the relationship in a table or chart using a mathematical expression.**
- **Predict subsequent elements in a pattern.**
- **Represent a pattern visually to verify predictions.**

Materials: Coloured tiles, overhead of the problem, and copies of the worksheet (BLM 5.PR.1.1).

Organization: Whole class/Pairs

Procedure:

a) Present Problem A to students.

   Encourage students to discuss the pattern by asking them questions, such as the following:

   - "What does Angela's pattern look like?"
   - "How many tiles will Angela need to make the fifth term of her pattern?"
   - "Draw the next two terms in Angela's pattern. Was your prediction right?"
   - "How many tiles do you think Angela will need to make the tenth term? Why?"

b) Begin to fill in a table. Explain that a table can help them think about the pattern. Fill in the values for the first three terms. Relate each number you write to the pattern.

Term	1	2	3							
Term Value	4	5	6							

c) Tell students that they should work with their partner to find out how many tiles Angela needs to make the tenth term, and record their findings on the following worksheet.

d) Have students share their answers with the other members of the class and explain their reasoning. Draw a completed table showing the number of tiles needed for the first 10 terms, and ask students how they could find the number of tiles needed for any term in the pattern.

Encourage students to look at the relationship between a term and the number of tiles in the term. (Students should note that the number of tiles is always three more than the term number, so the rule for finding the number of tiles in any term of the pattern is $n + 3$). Show students how to record the rule using a mathematical expression.

Term	1	2	3	4	5	6	7	8	9	10
Term Value	4	5	6	7	8	9	10	11	12	13

Observation Checklist

Observe students' responses to determine whether they can do the following:

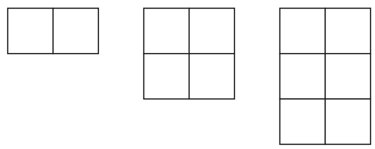
- ❑ extend a pattern with or without concrete materials, and explain how each element differs from the preceding one
- ❑ describe, orally and in writing, a pattern using mathematical language, such as "one more," etc.
- ❑ write a mathematical expression to represent a pattern, such as $n + 3$
- ❑ describe the relationship in a table or chart using a mathematical expression
- ❑ predict subsequent elements in a pattern
- ❑ solve a problem by using a pattern rule to predict subsequent elements
- ❑ represent a pattern visually to verify predictions

Materials: Coloured tiles and pattern blocks

Organization:

Procedure:

a) Ask students to use the coloured tiles to construct and then draw the next two terms in this pattern.



b) Encourage students to discuss the patterns by asking them questions, such as the following:

- "What patterns do you notice?"
- "How does the second term differ from the first term? The third term?"
- "Are there other patterns that describe how the pattern grows? What are they?"

c) Ask students to make a table of values for the pattern, and then predict the number of tiles needed to make the tenth term.

Term	1	2	3	4	5	6	7	8	9	10
No. of Tiles	2	4								20

d) Have students explain the pattern that they used to predict the tenth term. Ask them to write a rule for finding the value of any term in the pattern ($2n$), and then use their rule to predict the number of tiles in the 25th term.

e) Have students draw or use pattern blocks to create a pattern that uses the same numbers as the tile pattern they just discussed. Have them share their patterns with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ extend a pattern with or without the use of concrete materials, and explain how each element differs from the preceding one
- ❑ describe, orally or in writing, a pattern using mathematical language, such as "one more" or "one less"
- ❑ write a mathematical expression to represent a pattern, such as n +2
- ❑ describe the relationship in a table or chart using a mathematical expression
- ❑ predict subsequent elements in a pattern
- ❑ recognize that different patterns can have the same rule
- ❑ create a rule that fits a given pattern

Materials: Paper, pencil, and an overhead of the problem (BLM 5.PR.1.2)

Organization: Whole class

Procedure:

a) Present the following problem:

- A fence is constructed using posts and boards. Between adjacent posts is one board, as shown on the transparency. How many boards will you need if you build a fence with 90 posts?

b) Have students discuss the problem and suggest strategies for solving it. Try their strategies. If students do not think of creating a pattern, ask them to draw a picture to show what the fence would look like if it were made with three posts. Four posts? Then ask, "How many boards are needed if the fence is made with three posts? Four posts? How many boards do you think are needed if the fence is made with five posts? Draw a picture to check your prediction. Were you right? What could you do to find out the number of boards needed for a fence with 90 posts?"

c) Ask students to work with their partner to solve the problem.

d) Have students share their answers and explain their reasoning. Write their strategies on the board and encourage students to describe a rule that they could use to find the number of boards needed to make any number of posts (n – 1, where n represents the number of posts).

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve a problem by using a pattern rule to determine subsequent elements
- ❑ describe orally a pattern using mathematical language
- ❑ represent a pattern visually to verify predictions
- ❑ construct a table of values
- ❑ describe the relationship in a table or chart using mathematical expressions
- ❑ explain why a number is or is not the next element in a pattern

- ■ **Extend a pattern with or without concrete materials, and explain how each element differs from the preceding one.**
- ■ **Describe, orally or in writing, a pattern using mathematical language.**
- ■ **Write a mathematical expression to represent a pattern.**
- ■ **Predict subsequent elements in a pattern.**

Materials: Pattern blocks and overhead or copies of the problem (BLM 5.PR.1.3)

Organization: Whole class

Procedure:

a) Give students the following activity:

- ■ Miguel started to build this pattern with the triangle pattern blocks.

1. Construct and draw the next two terms in Miguel's pattern.
2. Complete the table of values:

Term	1				
No. of Triangles	3				

3. Predict the tenth term of the pattern.
4. Describe using mathematical symbols any pattern (rule) you used to determine the tenth term.

b) Have students share their solutions with the other members of the class and explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ extend a pattern using concrete materials, and explain how each element differs from the preceding one
- ❑ describe a pattern using mathematical language, such as "one more"
- ❑ write a mathematical expression to represent a pattern, such as $3r$
- ❑ predict subsequent elements in a pattern

Materials: Paper and pencils

Organization: Whole class

Procedure:

a) Present the following problem.

- Mr. Olsen's class was studying addition. Corry, who likes to add, decided to create a pattern involving the sums of two numbers. These are the first three terms in his pattern:

0 + 1 = 1	2 + 0 = 2	3 + 0 = 3
1 + 0 = 1	0 + 2 = 2	0 + 0 = 3
	1 + 1 = 2	1 + 2 = 3
		2 + 1 = 3

If Corry continues his pattern, how many different ways will he write 75 as the sum of two whole numbers?

b) Have students discuss the problem and suggest strategies they could use to solve the problem. Have students try their strategies. If no one suggests trying to find a rule to describe the pattern, ask students to write the next two term terms in Corry's pattern and then create a table of values.

No.	1	2	3						
No. of Ways	2	3	4						

c) Ask, "How many different ways can Corry write 10 as the sum of two whole numbers? 15? 75? What rule can you use to describe the number of ways any whole number can be written as the sum of two whole numbers?" (n + 1)

d) Extend the activity by asking: "How many different ways can you write the number 75 as the sum of two whole numbers if combinations that use the same addends but in a different order only count as one way? For example, 0 + 1 and 1 + 0 would only count as one way."

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ extend a pattern and explain how each element differs from the preceding one
- ❑ describe orally a pattern using mathematical language, such as "one more"
- ❑ describe the relationship in a table or chart using mathematical language
- ❑ solve a problem by using a pattern rule to determine subsequent elements
- ❑ predict subsequent elements in a pattern
- ❑ determine and explain why a number is or is not the next number in a pattern

■ Represent a pattern visually to verify predictions.

Materials: Overhead of the pattern (BLM 5.PR.1.4), paper and crayons or markers

Organization: Individual/Whole class

Procedure:

a) Ask students to complete the following activity:
   1. Here are two rules: $n + 5$ and $4n$.

      Ask students, "Which rule fits this pattern? How do you know the rule fits the pattern?"
   2. Design a pattern that fits the other rule.
   3. Ask them to explain how they know their pattern fits the rule.

b) Have students share their answers with the other members of the class. Encourage them to explain their reasoning.

c) Have students revisit their K-W-L chart. Have them share their ideas and add them to the class K-W-L chart.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify that the rule represents a given pattern
- ❏ create a pattern that fits a given rule
- ❏ recognize that different patterns can have the same rule

- ■ **Extend a pattern with or without concrete materials, and explain how each element differs from the preceding one.**
- ■ **Describe, orally or in writing, a pattern using mathematical language.**
- ■ **Write a mathematical expression to represent a pattern.**

Materials: Copies of the activity sheet (BLM 5.PR.1.5)

Organization: Individual

Procedure:

a) Ask students to complete the activity sheet.

b) Have students share their answers with the other members of the class. Encourage students to explain their reasoning.

c) Have students revisit their K-W-L chart. Have students share their ideas and add them to the class K-W-L chart.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ extend a pattern without concrete materials
- ❏ explain how each element differs from the preceding one
- ❏ describe orally a pattern using mathematical language, such as "one more than"
- ❏ write a mathematical expression to represent a pattern, such as $n + 2$
- ❏ determine and explain why a number is or is not the next element in a pattern

How Tall?

Purpose:

The purpose of this investigation is to have students apply their knowledge of number concepts, patterns, and measurement. In particular, the investigation was designed to reinforce students' ability to

- apply estimation strategies
- recall multiplication and division facts
- demonstrate an understanding of fractions
- represent decimals to thousandths
- relate fractions to decimals
- measure the length of objects
- determine the rule for a pattern to make predictions

The specific concepts and skills that students demonstrate will depend on the strategy that they use to solve the problem.

In addition, the investigation is designed to enhance students' ability to

- solve problems
- reason mathematically
- communicate mathematically
- make connections

Materials: Metre sticks and several copies of a large hand (the size of bristol board)

Organization: Small groups

Procedure:

a) Hang copies of the large hand around the room before students come into the room.
b) Have students look around the room and ask them what they notice. Tell students that they just had a visitor in the room and he left his handprints all over the room. Ask, "Who could have left these prints? How big do you think this person is?"
c) Tell students that their job is to figure out how tall the visitor was. Have the groups discuss the problem and devise a strategy for determining the height of the visitor.
d) Have the groups carry out their strategy for solving the problem.
e) Have the groups share their solutions with the other members of the class and explain their reasoning.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ use mathematical language correctly
- ❏ carry out mathematical procedures correctly, such as measuring and computing
- ❏ use mathematics confidently to solve problems
- ❏ use a variety of strategies to ensure the correctness of their solution
- ❏ communicate and reason mathematically
- ❏ contribute to the mathematical discussion

Extension:

Examine proportions in the book *Jim and the Beanstalk* by Raymond Briggs.

- The giant claims that he is going to eat three fried boys on a piece of toast. How big would the toast have to be to fit three fifth-graders?
- How big would the giant be based on the size of his/her toast?

NOTES

Grade 5: Patterns and Relations (Variables and Equations) (5.PR.2)

Enduring Understandings:

Number patterns and relationships can be represented by variables.

General Outcome:

Represent algebraic expressions in multiple ways.

Specific Learning Outcome(s):	Achievement Indicators:
5.PR.2 Solve problems involving single-variable (expressed as symbols or letters), one-step equations with whole-number coefficients, and whole-number solutions. [C, CN, PS, R]	➛ Express a problem in context as an equation where the unknown is represented by a letter variable. ➛ Solve a single-variable equation with the unknown in any of the terms (e.g., $n + 2 = 5$, $4 + a = 7$, $6 = r - 2$, $10 = 2c$). ➛ Create a problem in context for an equation.

Prior Knowledge

Students may have had experience with the following:

- Expressing a problem as an equation in which a symbol is used to represent an unknown number
- Solving one-step equations involving a symbol to represent an unknown number

Background Information

The learning experiences in this section focus, in part, on translating word problems into equations and then solving them. This is not new to students. In the Early Years, students were introduced to whole-number operations through routine problems. In the beginning, students solved these problems using concrete and pictorial representations. Later, they translated these problems into equations, often using an empty square to represent the unknown value.

Consequently, the learning experiences provide students with additional experience with solving routine problems and, at the same time, beginning the transition to using letters to represent unknown quantities. They also serve as an informal introduction to the terms "equation," "mathematical expression," and "variable." These terms are defined as follows:

An **equation** is a mathematical sentence stating that one or more quantities are equal. Equations that contain variables, such as $3 + x = 21$ and $2y + 3 = 15$, are sometimes referred to as open sentences, while equations that have no variables, such as $3 + 5 = 8$ and $24 \div 3 = 8$, are referred to as closed sentences.

A **mathematical expression** comprises numbers, variables, and operation signs, but does not contain a relational symbol such as $=$, $\neq$, $<$, $>$, $\leq$, and $\geq$. For example, $6x + 3$ and $\frac{x}{4} - 8$ are mathematical expressions.

A **variable** is a symbol for a number or group of numbers in a mathematical expression or equation.

Mathematical Language

Equation
Solution
Unknown

Assessing Prior Knowledge

Materials: None

Organization: Pairs/Whole class

Procedure:

a) Tell students they will be solving some problems involving equations, but before you can give them these problems you need to know what they already know about equations.

b) Pose this question: "What is an equation?" Have students think about the question for a few moments, then share their answer with their partner.

c) Have students share their answers with the other members of the class. Encourage discussion by asking students questions, such as the following:

- "What is an example of an equation?"
- "What does the equation tell you?"
- "Is 6 x 7 an equation? Why or why not?"
- "Is $5 = 14 - 9$ an equation? Why or why not?"
- "Is $n \div 4 = 16$ an equation?"
- "When do you use equations?"

d) Write students' responses on the board or overhead. Place their responses under the headings "Things we know about equations" and "Things we need to think about." Use the list to help you plan subsequent lessons.

Observation Checklist

Observe students' responses to determine whether they can do the following:

❏ describe what an equation is

❏ recognize examples and non-examples of equations

❏ provide examples of equations that contain variables (unknown quantities)

❏ provide examples of equations that do not contain variables

❏ recognize that an expression can be on either side of the equal sign (e.g., $5 = 14 - 9$ is the same as $14 - 9 = 5$)

- **Express a problem in context as an equation where the unknown is represented by a letter variable.**

Materials: Overhead of the problems (BLM 5.PR.2.1), copies of the activity (5.PR.2.2)

Organization: Pairs/individual

Procedure:

a) Ask students to work with their partner to solve the following problem:
   - Make sure students understand what the phrase “best fits” means (i.e., the equation should reflect the structure of the problem. For example, if the action in the problem indicates items are being combined, the equation should express the addition of quantities in the order they are given). Also, stress that they do not have to solve the equation.

b) Give students Problem A. Have students share their answer and explain their reasoning.

c) Give students Problem B. Have students share their answer and explain their reasoning.

d) Have students complete the activity sheet.

Observation Checklist

Examine students’ responses to determine whether they can do the following:

- ❏ identify the equation that reflects the structure of the problem
- ❏ express a problem in context as an equation where the unknown is represented by a letter variable

Materials: Math journal

Organization: Individual/Whole class

Procedure:

a) Present this problem:

- Nancy and Jessica were asked to write an equation for this story. I want to buy 35 pencils. Pencils come in packages of 7. How many packages do I need to buy? Nancy wrote $7 \times n = 35$ and Jessica wrote $35 \div 7 = n$. Who is right? Why?

b) Ask students to think about the problem and then record their answer in their math journals.

c) Have students discuss their answer with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify an equation that fits a given story problem
- ❑ recognize that both equations fit the problem

- **Express a problem in context as an equation where the unknown is represented by a letter variable.**
- **Solve a single-variable equation with the unknown in any of the terms.**

Materials: Copies of the word problems (BLM 5.PR.2.3)

Organization: Whole class

Procedure:

a) Ask students to write an equation to fit a problem and then solve it. Make sure students know that they should use a letter to represent the unknown value.

b) Have students share their answer and explain their reasoning. Encourage discussion of the problem by asking them questions, such as the following:

- "What equation did you use to solve the problem?"
- "Did anyone use a different equation? What is it?"
- "Do the equations have the same solution?"
- "How do you know your answer is correct?"
- "What strategy did you use to solve the problem?"
- "Did anyone use a different strategy? What is it?"

c) Repeat the activity with similar problems.

d) Ask students to write their own problems. Have them give their problems to the other members of the class and ask them to solve them. Make a class booklet of the problems that they write, and share the book with another class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ express a problem in context as an equation where the unknown is represented by a letter variable
- ❑ solve a single variable equation where the unknown is represented by a letter variable
- ❑ create a problem in context for an equation
- ❑ use appropriate strategies to solve an equation
- ❑ justify the solution to an equation

- **Express a problem in context as an equation where the unknown is represented by a letter variable.**
- **Solve a single-variable equation with the unknown in any of the terms.**
- **Create a problem in context for an equation.**

Materials: Paper and pencils

Organization: Small groups

Procedure:

a) Ask the groups to select one equation from each of the following lists:

$x + 9 = 25$	$25 - y = 14$	$26 = 2c$
$7 + r = 35$	$y - 8 = 16$	$5x = 60$
$z = 25 + 83$	$x = 62 - 23$	$4k = 32$

b) Ask the groups to write one word problem for each equation that they chose.

c) Have each group exchange its problems with another group. Have the groups solve each other's problems.

d) Have several groups share their problems and solutions with the other members of the class. Encourage students to discuss their solutions by asking them questions, such as the following:

- "What equation did you use to solve the problem?"
- "Why did you choose that equation?"
- "Is there another equation that could be used to solve the problem?"
- "How do you know your solution is correct?"
- "What strategy did you use to solve the equation?"
- "Is there another strategy that could be used to solve the problem? What is it?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ create a problem in context for an equation
- ❏ solve a single variable equation with the unknown in any of the terms
- ❏ express a problem in context as an equation where the unknown is represented by a letter variable
- ❏ use an appropriate strategy to solve an equation
- ❏ justify their solution to an equation

- **Solve a single-variable equation with the unknown in any of the terms.**

Materials: Copies of the activity (BLM 5.PR.2.4)

Organization: Individual/Whole class

Procedure:

a) Have students complete the activity. Do an example with the class before letting students work on their own.

b) Have students share their answers with the other members of the class. Encourage students to justify their solutions and explain the strategy that they used to solve the equations.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ solve a single variable equation with the unknown represented by a letter variable
- ❑ identify the variable in an equation
- ❑ use an appropriate strategy to solve the equation
- ❑ justify their solution to an equation

Grade 5 Mathematics

Shape and Space

Grade 5: Shape and Space (Measurement) (5.SS.1)

Enduring Understandings:

There is no direct relationship between perimeter and area.

General Outcome:

Use direct or indirect measurement to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
5.SS.1 Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and draw conclusions. [C, CN, PS, R, V]	➢ Construct or draw two or more rectangles for a given perimeter in a problem-solving context. ➢ Construct or draw two or more rectangles for a given area in a problem-solving context. ➢ Illustrate that for any perimeter, the square or shape closest to a square will result in the greatest area. ➢ Illustrate that for any perimeter, the rectangle with the smallest possible width will result in the least area. ➢ Provide a real-life context for when it is important to consider the relationship between area and perimeter.

Prior Knowledge

Students may have had experience with the following:

- Estimating, measuring, and recording the length, width, and height of objects to the nearest metre and centimetre
- Estimating, measuring, and recording the perimeter of regular and irregular shapes
- Constructing shapes with a given perimeter
- Estimating, measuring, and recording the area of regular and irregular shapes with a given perimeter
- Estimating, measuring, and recording the area of regular and irregular shapes
- Constructing shapes with a given area
- Distinguishing between perimeter and area
- Identifying and describing patterns found in tables and charts
- Identifying polygons

Students may also have had experience with the following:

- Perimeter is the distance around a shape
- Area is the amount of surface within a region

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of measuring length in millimetres, and distinguish rectangles from other quadrilaterals
- Recognizing that all squares are rectangles

Background Information

Perimeter is the distance around a shape. Students often confuse this concept with **area**, the amount of surface a shape covers. Involving students in actual measuring experiences can help them distinguish between these two concepts. For example, activities that have students completely covering shapes with square units can help them understand the meaning of area.

Moreover, students often have misconceptions about the relationship between perimeter and area. Two of the most common are the following:

1. the longer the perimeter, the larger the area
2. perimeter and area increase at the same rate

For example, some students have the mistaken belief that if the perimeter is doubled, the area will double. Therefore, the intent of the learning experiences in this section is to help students overcome their misconceptions by having them explore the perimeter and area of rectangles. The learning experiences are also designed to help students recognize at least five generalizations about the relationship between these two measurements:

- If only the perimeter (area) of a rectangle is given, its area (perimeter) cannot be determined.
- Increasing the perimeter (area) of a rectangle does not necessarily increase the area (perimeter) of the rectangle.
- If the length (width) of a rectangle is fixed, then increasing its perimeter will increase its area.
- The square has the largest area among rectangles that have the same perimeter.
- The square has the smallest perimeter among rectangles that have the same area.

Mathematical Language

Area

Length

Perimeter

Polygon

Rectangle

Square

Width

Assessing Prior Knowledge

Materials: Centimetre grid paper (BLM 5–8.9)

Organization: Individual

Procedure:

a) Ask students to use the centimetre grid paper to draw the following:
- A polygon with a perimeter of 10 cm
- A polygon with a perimeter greater than 15 cm
- A polygon with an area of 12 cm^2
- A polygon whose area is greater than 15 cm^2 and less than 25 cm^2

Inside of each shape that they draw, have students write the name of the polygon and its perimeter or area measurement.

b) Present the students with the following problems:
- Kelly wants to make a wooden frame for the picture his aunt drew for him. Does Kelly need to measure the perimeter or the area of the picture to find out how much wood he needs? What unit of measurement do you think he should use? Explain your answers.
- Mr. Lien wants to cover the bulletin board in his classroom with a piece of paper. Does he need to measure the perimeter or the area of the bulletin board to find out how much paper he needs? What unit of measurement do you think he should use? Explain your answers.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ construct a polygon with a given perimeter
- ❑ construct a polygon with a given area
- ❑ distinguish between perimeter and area
- ❑ understand that perimeter is the distance around a shape
- ❑ understand that area is the amount of surface inside a region
- ❑ name polygons according to the number of sides that they have
- ❑ know that perimeter is measured in linear units and that area is measured in square units
- ❑ justify their selection of a unit of measurement

- **Construct or draw two or more rectangles for a given perimeter in a problem-solving context.**
- **Illustrate that for any perimeter, the square or shape closest to a square will result in the greatest area.**
- **Illustrate that for any perimeter, the rectangle with the smallest possible width will result in the least area.**

Materials: Square tiles, centimetre grid paper (BLM 5–8.9), and recording sheet (BLM 5.SS.1.1)

Organization: Small group/Whole class

Procedure:

a) Present students with the following problem:
   - Mrs. Zahn and Mr. Stewart have gardens that are rectangular in shape. The perimeter of Mrs. Zahn's garden is 16 metres and the perimeter of Mr. Stewart's garden is 20 metres. Is Mr. Stewart's garden larger than Mrs. Zahn's?

   Make sure students understand the problem by asking:
   - "What do you know about Mrs. Zahn's garden?"
   - "What do you know about Mr. Stewart's garden?"
   - "What question do you need to answer?"

b) Have students write down what they think the answer is, and then share it with the other members of their group.

c) Next, have students draw rectangles on the centimetre grid paper to show why they think their answer is correct, and then share their drawings with the other members of their group.

d) Challenge students by asking: "Are there other rectangles that have perimeters of 16 metres? Are there other rectangles that have perimeters of 20 metres? What are their areas?" Have students in each group use the square tiles or centimetre grid paper to find other rectangles that have perimeters of 16 metres and 20 metres.

e) Encourage students to organize their work by having them record their findings in the table provided (see BLM 5.SS.1.1).

f) Have students in each group analyze their tables and record any patterns or relationships that they find.

g) Ask each group to present its findings to the other members of the class, as well as its conclusion regarding whose garden—Mrs. Zahn's or Mr. Stewart's—is larger.

Observation Checklist

Check students' work to determine whether they can do the following:

- ❏ construct or draw two or more rectangles for a given perimeter in a problem-solving context
- ❏ recognize that they cannot tell for sure whether Mr. Stewart's garden is larger than Mrs. Zahn's—that is, if only the perimeter of a rectangle is given, its area cannot be determined
- ❏ recognize patterns and relationships, such as
  - the square has the largest area among rectangles with the same perimeter
  - the rectangle with the smallest width has the least area
  - increasing the perimeter does not necessarily increase the area
  - if the length of a rectangle is fixed, increasing its perimeter increases its area

Materials: Square tiles, centimetre grid paper (BLM 5–8.9), and recording sheet (BLM 5.SS.1.1)

Organization: Small group/Whole class

Procedure:

a) Present students with the following problem:
   - A farmer has 36 metres of fencing material. He is planning to use all of the fencing material to make a rectangular pen for his sheep. What is the largest pen he can make for his sheep?

   Make sure students understand the problem by asking:
   - "What does the farmer want to do?"
   - "How much fencing material does he have?"
   - "What do you need to find out?"

b) Have students write down what they think the answer to the problem is and share their answer with the other members of their group.

c) Next, ask, "How many different rectangular pens can the farmer make with 36 metres of fencing material?"

d) Have students in each group use the square tiles or centimetre grid paper to find all the rectangles that have a perimeter of 36 units. Have students record their findings in the table provided (see BLM 5.SS.1.1).

e) Have students analyze their findings and record any patterns and relationships that they find.

f) Have each group share with the other members of the class its solution to the problem and any other patterns that it finds.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ construct or draw two or more rectangles with a given perimeter in a problem-solving context
- ❑ recognize that the pen with the largest area is a square with sides six metres in length—that is, the square has the largest area among rectangles with the same perimeter
- ❑ recognize patterns and relationships, such as
  - the rectangle with the smallest width has the least area
  - the closer the rectangle is to a square, the closer the area is to the maximum area

- **Construct or draw two or more rectangles for a given area in a problem-solving context.**
- **Illustrate that for any perimeter, the square or shape closest to a square will result in the greatest area.**
- **Illustrate that for any perimeter, the rectangle with the smallest possible width will result in the least area.**
- **Provide a real-life context for when it is important to consider the relationship between area and perimeter.**

Materials: *Spaghetti and Meatballs for All* by Marilyn Burns, square tiles, and recording table (BLM 5.SS.1.1)

Organization: Whole class/Small group

Procedure:

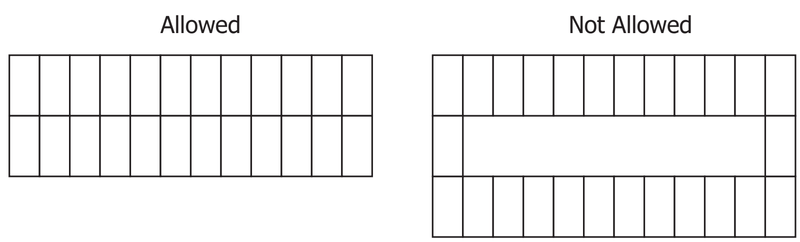
a) Read *Spaghetti and Meatballs for All*. As you read the story, have the students use the square tiles to model what is happening with the tables.

b) Have students discuss the problem with the table arrangements. Begin the discussion by asking, "Why does Mrs. Comfort keep saying the table arrangements won't work?" Have students work with their partner to find different ways of arranging eight tables. Have them decide which arrangement is the best.

c) Pose the problem: "Mrs. Comfort has 24 square tables. If she pushes the tables together to form a rectangle, what is the highest number of people she can sit around the rectangle?"

d) Make sure the students understand the problem by asking them the following questions:

- "How many square tables does Mrs. Comfort have?"
- "What does Mrs. Comfort do with the tables?"
- "What do you need to find out?"
- "What is one way that Mrs. Comfort can push the tables together to form a rectangle?" (Make sure the students recognize that the rectangular arrangements cannot have any spaces in the middle.)



- "How many people can Mrs. Comfort sit around the table?"

e) Explain that the number of tables pushed together represents the area and the number of people who can sit around the table represents the perimeter. Then, ask, "Are there other rectangles that Mrs. Comfort can make that have an area of 24 square units? Can she seat the same number of people around each rectangle?"

f) Have the students work with their partners to determine all the rectangles that can be made with 24 tiles. Encourage students to record their findings in the table provided (see BLM 5.SS.1.1).

g) Have students analyze their findings and record any patterns and relationships that they find.

h) Ask students to share their findings and their conclusion as to which rectangle Mrs. Comfort should make if she wants to seat the most people with the rest of the class.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ construct or draw two or more rectangles for a given area in a problem-solving context
- ❑ recognize that Mrs. Comfort can seat the most people around a 1 x 24 rectangle—that is, among rectangles with the same area, the one with the smallest width has the greatest perimeter
- ❑ recognize patterns and relationships, such as the following:
  - The closer the rectangle is to a square, the smaller its perimeter.
  - If two rectangles have the same area, they do not necessarily have the same perimeter.
  - If only the area of a rectangle is given, its perimeter cannot be determined.

- **Construct or draw two or more rectangles for a given area in a problem-solving context.**
- **Illustrate that for any perimeter, the square or shape closest to a square will result in the greatest area.**

Materials: Math journals, square tiles, and square centimetre paper (BLM 5–8.9)

Organization: Individual/Large group

Procedure:

a) Pose the following problem:
   - Mr. Santos is making a rectangular flower garden in his backyard. If the area of the garden is 36 m^2, what is the least amount of fencing that he needs to enclose the garden?

b) Make sure the students understand the problem by asking them the following questions:
   - "What is Mr. Santos making?"
   - "How big is the garden?"
   - "What do you need to find out?"

c) Tell students that they can use the square tiles or the centimetre grid paper to help them solve the problem. Have the students record their solutions in their math journals.

d) Have students share their answers and the strategies they used to solve the problem.

Observation Checklist

Check students' work to determine whether they can do the following:

- ❑ recognize that different rectangles can have the same area
- ❑ recognize that the least amount of fencing that is needed is 24 metres

Putting the Pieces Together

Design a Clubhouse

Purpose:

The purpose of this investigation is to have students apply the concepts of perimeter and area to a problem-solving situation. In particular, it is designed to enhance students' ability to

- differentiate between perimeter and area
- construct rectangles with a given perimeter or area
- maximize or minimize the area of a rectangle with a fixed perimeter
- maximize or minimize the perimeter of a rectangle with a fixed area

In addition, the investigation is designed to enhance students' ability to

- communicate mathematically
- connect mathematics to real-world situations
- solve problems
- reason mathematically

Materials/Resources: Centimetre grid paper (BLM 5–8.9), coloured centimetre grid paper, square tiles, scissors, and glue

Organization: Whole class/Small groups

Procedure:

a) Present students with the following situation:

You and your friends have decided to build a rectangular clubhouse. You plan to build your clubhouse in a section of the schoolyard with an area of 200 m^2. You have decided that your clubhouse must have the following:

- The largest possible floor space
- Two rugs (One rug must have a perimeter of 24 m and cover the largest possible area, and the other rug must have a perimeter of 16 m that covers the least possible area.)
- At least two doors with a width of one metre

- A play area that takes up at least ¼ of the floor space. No furniture can be placed in the play area
- A rectangular seating area with a perimeter of 12 m
- A rectangular table with an area of 4 m^2

You also decide that the clubhouse can have other items as long as they are not placed in the play area.

b) Explain that each group must draw up a plan for the clubhouse that includes the dimensions of each item in the list of specifications. Tell students that they can draw their plan for the clubhouse on the white centimetre grid paper and use the colour centimetre paper to indicate the furniture and the rugs. They should let each square centimetre on the grid paper represent one square metre.

c) Help students develop the criteria for assessing their plans for a clubhouse.

d) Have students work on their plans for a clubhouse.

e) Have each group present its design for a clubhouse to the other members of the class. Encourage students to describe the dimensions of each item in their clubhouse and how they determined its size.

Observation Checklist

❑ Use the rubric provided and the student-developed criteria to assess students' attainment of outcome 5.SS.1 during the completion of the project.

	3	2	1
Distinguishes between perimeter and area	Determines the perimeter of a rectangle by finding the distance around it. Determines the area of a rectangle by finding the number of square units it covers.	Determines the perimeter of a rectangle ***or*** the area of a rectangle.	Is not able to determine the perimeter of a rectangle.
Constructs rectangles with a given perimeter	Constructs a rectangle with a given perimeter.	Constructs a rectangle with a given perimeter with support.	Is not able to construct a rectangle with a given perimeter.
Recognizes relationships	Recognizes that a square has the largest area among rectangles that have the same perimeter. Recognizes that, among rectangles that have the same perimeter, the one with the smallest width has the least area. Recognizes that a square has the smallest perimeter among rectangles with the same area.	Recognizes that a square has the largest area among rectangles with the same perimeter with support. Recognizes that, among rectangles that have the same perimeter, the one with the smallest width has the least area with support. Recognizes that a square has the smallest perimeter among rectangles with the same area with support.	Does not recognize that a square has the largest area among rectangles with the same perimeter. Does not recognize that, among rectangles that have the same perimeter, the one with the smallest width has the least area. Does not recognize that a square has the smallest perimeter among rectangles with the same area.

Grade 5: Shape and Space (Measurement) (5.SS.2)

Enduring Understandings:

All measurements are comparisons.

Length, area, volume, capacity, and mass are measurable properties of objects.

The unit of measure must be of the same nature as the property being measured.

General Outcome:

Use direct or indirect measurement to solve problems.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.SS.2 Demonstrate an understanding of measuring length (mm) by ■ selecting and justifying referents for the unit mm ■ modelling and describing the relationship between mm and cm units, and between mm and m units [C, CN, ME, PS, R, V]	➛ Provide a referent for one millimetre and explain the choice. ➛ Provide a referent for one centimetre and explain the choice. ➛ Provide a referent for one metre and explain the choice. ➛ Show that 10 millimetres is equivalent to 1 centimetre using concrete materials (e.g., ruler). ➛ Show that 1000 millimetres is equivalent to 1 metre using concrete materials (e.g., metre stick). ➛ Provide examples of when millimetres are used as the unit of measure.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Estimating, measuring, and recording the length, width, and height of objects to the nearest metre or centimetre
- Describing the relationship between a metre and a centimetre
- Identifying referents for a cm and a m
- Demonstrating an understanding of fractions less than one

Students may also have had experience with the terms *length*, *width*, *height*, and *perimeter*.

Related Knowledge

Students should be introduced to the following:

- Multiplying and dividing whole numbers by 10s, 100s, and 1000s
- Reading, writing, interpreting, and using decimal notation for 10ths, 100ths, and 1000ths
- Relating fractions to decimals
- Describing orally and in writing the rule for a pattern

Background Information

Measurement is the process of comparing a unit of measure with a measurable property of an object or phenomenon. The process consists of the following:

1. Identifying the property to be measured
2. Selecting an appropriate unit of measure
3. Repeatedly matching the unit with the property or phenomena being measured
4. Counting the number of units

By the end of the 18th century, units of measure varied greatly within and between countries. The lack of standard units made trading with other cultures difficult to carry out. To remedy this situation, the French National Assembly in 1790 asked the Academy of Science to develop a common system of measurement. The system it developed is known as the metric system. The units of measure developed by the academy have evolved into the Système International d'Unités (abbreviated SI), which was established in 1960. The SI is governed by the General Conference on Weights and Measures, which makes changes in the system to reflect the latest advances in science and technology. Even though there are differences between the two systems, SI is still referred to as the metric system.

Because of its simplicity, all but a few countries have adopted the metric system. Its simplicity arises from its use of the following:

1. A small number of base units
2. The decimal system
3. A uniform set of prefixes that apply to each area of measurement

These prefixes—the most common of which are shown below—indicate multiples or subdivisions of the base units.

Prefix	Meaning
kilo (k)	1000 units
hecto (h)	100 units
deka (da)	10 units
deci (d)	0.1 unit
centi (c)	0.01 unit
milli (m)	0.001 unit

In the Early and Middle Years, students are introduced to length, area, volume, capacity, and mass. Their measurement of these properties involves the units listed in the chart below, and can be either direct or indirect. **Direct measurements** involve selecting a unit and comparing it directly with the object (e.g., using a metre stick to measure the height of a table). **Indirect measurements** are made when a unit cannot be placed directly on the object (e.g., finding the height of a flagpole or the area of a country). Often, objects can be measured indirectly by comparing them with things that can be measured (e.g., finding the height of a tree by measuring its shadow).

Quantity	Units	Symbol
Length	kilometre metre centimetre millimetre	km m cm mm
Area	square metre square centimetre	m^2 cm^2
Volume	cubic metre cubic centimetre	m^3 cm^3
Capacity	litre millilitre	L mL
Mass	kilogram gram	kg g

Learning experiences that require students to use measuring instruments in realistic situations are key ingredients in helping them understand the concepts and skills involved in measurement systems. In particular, these experiences can help students understand the following:

- The measure of a unit is always 1
- The unit must be of the same nature as the property that is being measured
- The unit must be repeatedly matched with the property being measured without any gaps or overlaps (This process is known as **unit iteration**.)
- One unit may be more appropriate than another to measure the property of an object
- There is an inverse relationship between the number of units and the size of the unit
- A smaller unit gives a more exact measurement
- A measurement must include both a number and a unit
- When the same units are used, measurements can be easily compared

Estimating—that is, making a reasonable judgment about the approximate amount of a quantity—also plays an important role in the development of students' understanding of measurement systems. A focus on estimating enables students to create a mental frame of reference for the size of units and their relationships to each other. It also helps them judge the reasonableness of their measurements.

Mathematical Language

Centimetre

Estimate

Height

Length

Measurement

Metre

Millimetre

Referent

Width

Learning Experiences

Assessing Prior Knowledge

Materials: Assessment activity sheet (BLM 5.SS.2.1) and cm rulers

Organization: Individual

Procedure:

Have students complete the assessment activity sheet (BLM 5.SS.2.1).

- **Provide a referent for one millimetre and explain the choice.**
- **Show that 10 millimetres is equivalent to 1 centimetre using concrete materials (e.g., ruler).**
- **Provide examples of when millimetres are used as the unit of measure.**

Materials: cm rulers with mm marked on them, a cm ruler that can be projected on the overhead or an overhead of a cm ruler, toothpicks, safety pins, index cards, crayons, math scribblers, soda straws, and the activity sheet (BLM 5.SS.2.2)

Organization: Whole class/Individual

Procedure:

a) Ask students to draw a metre stick and make sure that they include all the markings. When students finish their drawings, have them share their pictures and explain what the markings on their metre sticks mean. Use the discussion to determine what the students already know about mm so you can clear up any misconceptions that they may have.

b) Tell students that they will be learning about a new unit of linear measure called a millimetre. Place a cm ruler on the overhead and point out that there are 10 spaces between consecutive centimetres. Tell students that each space represents 1 millimetre. Write the word *millimetre* on the board or overhead, and show students the symbol for the unit.

c) Have students take out their cm rulers. Ask them to find the number of millimetres between

   - the 1 cm mark and the 2 cm mark
   - the 10 cm mark and the 11 cm mark
   - the 15 cm mark and the 17 cm mark
   - the 20 cm mark and the 23 cm mark

d) Have students show these points on their rulers:

- 20 mm
- 45 mm
- 85 mm
- 120 mm

e) Tell students that millimetres are used to measure the lengths of small objects. Have them identify objects that they would measure with this unit.

f) Have students complete the activity sheet (BLM 5.SS.2.2). Remind students of the symbol for millimetre.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ make reasonable estimates
- ❑ read and use their cm rulers correctly to determine the lengths of objects
- ❑ record measurements correctly (e.g., recorded measurements include both a number and the unit)
- ❑ recognize when it's appropriate to use mm as a unit of measure (Note: Many of the objects that students name could also be measured in cm or m.)

- **Provide a referent for one millimetre and explain the choice.**
- **Provide a referent for one centimetre and explain the choice.**
- **Provide a referent for one metre and explain the choice.**

Materials: Metre sticks and cm rulers

Organization: Whole class/Partners

Procedure

a) Discuss the importance of estimation in measurement. For example, talk about how good estimating skills can help an individual recognize when an error in measurement is made and the consequences of using an incorrect measurement.

b) Have students identify examples of situations when estimating the lengths of objects would be beneficial. For example:
   - "We need to wrap a gift. Do we have enough ribbon to wrap the package?"
   - "We want to put a new shelving unit in the room. Is the room high enough for the shelving unit?"
   - "We want to store some books. Is the box we have wide enough?"

c) Ask students to estimate the length of the room to the nearest metre. Record their responses on the board. Have students share their strategies for estimating the length of the room and discuss why their estimates varied.

d) Discuss the importance of personal referents in the estimating process. Explain that a personal referent is a familiar object, one that they see or use regularly whose measure is known. They can think of this object when they are estimating the length of an unknown object (e.g., the length of a baseball bat is approximately 1 metre). When estimating the length of an unknown object, they can think of a bat and visualize how may "bats long" the object is.

e) Have the students work with a partner to find five common objects that are approximately
   - 1 mm in length, width, or height
   - 1 cm in length, width, or height
   - 1 m in length, width, or height

f) Have students share their referents for each unit with the rest of the class.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ provide reasons why estimating is an important skill
- ❑ give examples of situations in which estimating would be beneficial
- ❑ identify appropriate referents for 1 mm, 1 cm, and 1 m

Materials: Decks of 20 cards (one side of each card should have a letter on it; the other side should have a line segment drawn on it [BLM 5.SS.2.3]), an answer sheet listing the length of the line segment on each card

Organization: Pairs

Procedure:

a) Tell students that they will be playing an estimating game called "Metric 210" with their partner. Explain how the game is played.

   1. Shuffle the cards and lay them face down on the playing surface.
   2. Players take turns taking a card from the top of the pile until one of them estimates that he or she has a total length of 210 mm and stops the game by saying "I have the line." This player may get rid of any one card that pushes the total over 210 mm. The player then states an estimate for the total length of the remaining cards.
   3. The player uses the answer sheet to determine his or her score. The player's score is determined by adding the difference between the estimate and the actual length to the difference between the actual length and 210.
   4. The winner is the player with the lowest total score after five rounds of the game.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Vary the game by having students estimate the lengths of the line segments in cm. A round of the game is over when a student thinks he or she has reached a length of 21 cm.

Observation Checklist

Observe students to determine whether they can do the following:

- ❏ understand the rules for playing the game
- ❏ give reasonable estimates of the lengths of the line segments
- ❏ calculate the total length of the lines correctly
- ❏ calculate their scores correctly

Materials: A deck of measurement cards (BLM 5.SS.2.4), straight edges, a centimetre ruler, and a piece of paper for each player

Organization: Pairs

Procedure:

a) Tell students that they will be playing a variation of the game Metric 210. Explain how the new version of the game is played.

   1. Shuffle the cards and place them face down on the playing surface.
   2. Each card in the deck represents a mm length.
   3. The first player turns over a card and uses a straight edge to draw a line segment he or she estimates to be the same length as the number of mm on the card (e.g., if the player turns over a 30, he or she draws, without measuring, a line segment that he or she thinks is 30 mm in length and records the length above the line segment).
   4. The second player turns over a card and uses a straight edge to draw a line segment he or she estimates to be the same length as the number of mm on the card. The second player then records the length above the line segment.
   5. The first player turns over a line card, and extends, without measuring, his or her line segment the number of mm shown on the card. The first player then records the length above the line segment. For example, if the first player draws a 30 and then a 50, his or her paper would look like this:

   6. Play continues in this fashion until one player has a line segment he or she estimates is 210 mm in length and stops the game by saying, "I have the line." If the player thinks the last extension of his or her line segment pushes the total length beyond 210 mm, he or she can state an estimate greater than 210 mm.
   7. The player measures the line segment to determine his or her score. The player's score is determined by adding the difference between the actual length and the estimated length to the difference between the actual length and 210 mm.
   8. The winner is the player who has the lowest score after five rounds of the game.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Vary the game by changing mm to cm (BLM 5.SS.2.4). A round of the game is over when a student draws a line segment he or she estimates is 21 cm in length.

Observation Checklist

Observe students to determine whether they can do the following:

- ❑ understand the rules of the game
- ❑ make reasonable estimates
- ❑ determine the length of the line segments by using and reading their rulers correctly
- ❑ record measurements correctly (e.g., recorded measurements include both the number and the unit)
- ❑ calculate their scores correctly

Materials: Metre sticks, cm rulers, table to record their findings (BLM 5.SS.2.5)

Organization: Small groups

Procedure

a) Tell students that they will be playing an estimating and measuring game with the members of their group. Explain how to play the game.
   1. Players take turns naming a unit of measure (m, cm, mm) and an object that everyone can see.
   2. Everyone records an estimate of the length of the object in the stated unit.
   3. The player who named the object measures it. The player whose estimate is the closest to the actual measurement gets one point. If there is a tie, all players with the best estimate get one point.
   4. The winner of the game is the first person to get five points.

b) Have students record in the table provided their choice of objects, their estimates, and the actual measurements (5.SS.2.5).

c) Demonstrate how the game is played and answer any question students might have. Have students play the game.

Observation Checklist

Observe students to determine whether they can do the following:

- ❑ understand the rules of the game
- ❑ determine the lengths of objects by using and reading a metre stick or cm ruler correctly
- ❑ record measurements correctly (e.g., recorded measurements include both a number and the unit of measure)
- ❑ make reasonable estimates
- ❑ select an appropriate unit of measure

- **Show that 10 millimetres is equivalent to 1 centimetre using concrete materials (e.g., ruler).**
- **Show that 1000 millimetres is equivalent to 1 metre using concrete materials (e.g., metre stick).**

Materials: cm rulers and line segments (BLM 5.SS.2.6)

Organization: Individual

Procedure:

a) Give students a copy of BLM 5.SS.2.6, and tell them that they should measure each line segment twice: The first time, they should measure the line segment to the nearest cm; the second time, they should measure the line segment to the nearest mm.

b) Encourage students to record their findings in the table provided in BLM 5.SS.2.6.

c) Ask students to study their tables and record any patterns they see.

d) Have students share their findings with the other members of the class. Encourage students to state a rule that describes the relationship between cm and mm.

e) Check students' understanding of the relationship between cm and mm by asking:
   - "How many mm are in 1 cm? 2 cm? 3 cm? 4 cm? 8 cm? 15 cm? 50 cm? n?"
   - "If the length of an object is given in cm, how can you find how long it is in mm without measuring?"
   - "If 1 cm = 10 mm, what part of a cm is 1 mm? 2 mm? 4 mm? 8 mm? 10 mm?"
   - "If an object is 7 mm long, how long is it in cm?"
   - "If an object is 35 mm long, how long is it in cm?"
   - "If an object is 83 mm long, how long is it in cm?"
   - "If the length of an object is given in mm, how can you find how long it is in cm without measuring?"

f) Show students how to record the relationship between mm and cm.
   - 1 cm = 10 mm
   - 1 mm = 0.1 cm

g) Help students understand the relationship between m and mm by asking:
   - "How many cm are in 1 metre?"
   - "How many mm are in 1 cm?"
   - "If there are 10 mm in 1 cm and 100 cm in 1 metre, how many mm are in 1 metre?"

   Have students use a metre stick to show why their answers are correct.

h) Check students' understanding of the relationship between mm and m by asking:

- "How many mm are in 1 m? 2 m? 3 m? 5 m? 8 m? 10 m?"
- "If the length of an object is given in m, how can you find how long it is in mm without measuring?"
- "If there are 1000 mm in one m, what part of a m is 1 mm? 2 mm? 10 mm? 25 mm? 100 mm?"
- "If an object is 1000 mm long, how long is it in m?"
- "If an object is 3000 mm long, how long is it in m?"
- "If an object is 6000 mm long, how long is it in m?"
- "If the length of an object is given in mm, how can you find how long it is in m without measuring?"

i) Show students how to record the relationship between m and cm.

- 1 m = 1000 mm
- 1 mm = 0.001 m

Emphasize that "milli" means thousandths so 1 mm means 1 thousandth of a metre.

Observation Checklist

Observe students to determine whether they can do the following:

- ❑ find the lengths of line segments by using and reading their cm rulers correctly
- ❑ record measurements correctly (e.g., recorded measurements include both a number and the unit)
- ❑ recognize the relationship between mm and cm
- ❑ recognize the relationship between mm and m
- ❑ convert cm to mm and vice versa
- ❑ convert mm to m and vice versa

Materials: *I have, who has…?* cards (BLM 5.SS.7)

Organization: Whole class

Procedure:

a) Tell students that they will be playing a metric version of the game "I have, who has…?". Explain that each student will get one card (some students may get two cards if there are fewer than 30 students in the class). One student will start the game by reading his or her card, and the person who has the answer to the question posed by this student reads his or her card. Play continues in this fashion until it gets back to the person who started the game.

b) After the students have played the game several times, have them make their own metric conversion *I have, who has…?* game and play it with the other members of the class.

Variation: Have students work in groups of 2 or 3, giving them several of the cards. Play the game as a class as you would in (a) and (b) above. This gives students the opportunity to engage with more than one card.

Observation Checklist

Monitor students' responses to determine whether they

- ❏ multiply or divide by tens, hundreds, or thousands
- ❏ add, subtract, multiply, or divide numbers other than powers of 10
- ❏ know the relationship between m, cm, and mm

Materials: cm rulers, stir sticks, books, erasers, soda cans, and pencil cases

Organization: Pairs/Whole class

Procedure:

a) Present the following problem to students:

- Martin drew a line that was 64 mm long. His friend Zack measured the line segment and said that it was 6.4 cm long. Is Zack right? How do you know?

b) Make sure students understand the problem by asking:

- "How long is the line that Martin drew?"
- "What else do you know?"
- "What do you need to find out?"

c) Have students work with their partner to solve the problem. When they finish, have them share their solutions with the other members of the class, and discuss why the two measurements are the same.

Note: Some students will be able to solve the problem by reasoning while others will need to use their cm rulers and draw the line.

d) Check students' understanding of how to express measurements to the nearest tenth of a cm by asking:

- "What is 37 mm expressed in cm?"
- "What is 93 mm expressed in cm?"
- "What is 58 mm expressed in cm?"
- "What is 116 mm expressed in cm?"

Have students use their cm rulers to justify their answers.

e) Have students make and complete the following table:

Object	Estimated Length	Length to Nearest Tenth of a cm	Length to Nearest mm
A stir stick			
Thickness of a book			
An eraser			
Distance around a can of soda			
Width of a pencil case			

f) Have students express each of the following measurements in mm:

- 25.1 cm
- 85.6 cm
- 37.9 cm
- 12.2 cm

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ record a measurement to the nearest tenth of a cm
- ❑ convert mm to the nearest tenth of a cm and vice versa
- ❑ understand that 10 mm is the same as 1 cm
- ❑ read and interpret measurements expressed in decimals
- ❑ make reasonable estimates

Caution: In some communities, playing cards are seen as a form of gambling and discouraged. Please be aware of local sensitivities before introducing this activity.

Materials: Deck of cards for each group

Organization: Small groups of 2 to 4 students

Procedure:

a) Tell students they will be playing the *Metric Convert* game, and explain how it is played.

    1. Shuffle the cards and place them face down on the playing area.
    2. The numbers on the cards represent mm. Let aces = 1 mm, jacks = 11 mm, queens = 12 mm, and kings = 13 mm.
    3. One player turns over a card and places it in the centre of the playing area so everyone can see it.
    4. The first player to convert mm to cm correctly takes the card (e.g., if an 8 is turned over, the first player to say 0.8 [8 tenths] cm wins the card).
    5. If there is a tie or an error is made, the card is put back into the deck and the cards are reshuffled.
    6. The person who wins the cards turns over the next card.
    7. The game proceeds in this fashion until there are no cards.
    8. The person with the most cards is the winner.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Vary the game by having the students convert from

    - cm to mm
    - m to cm
    - m to mm
    - mm to m

Observation Checklist

Observe students to determine whether they

- ❏ know the relationships between mm, cm, and m
- ❏ calculate correctly

NOTES

Grade 5: Shape and Space (Measurement) (5.SS.3)

Enduring Understandings:

All measurements are comparisons.

Length, area, volume, capacity, and mass are measurable properties of objects.

The unit of measure must be of the same nature as the property of the object being measured.

General Outcome:

Use direct or indirect measurement to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
5.SS.3 Demonstrate an understanding of volume by ■ selecting and justifying referents for cm^3 or m^3 units ■ estimating volume by using referents for cm^3 and m^3 ■ measuring and recording volume (cm^3 or m^3) ■ constructing rectangular prisms for a given volume [C, CN, ME, PS, R, V]	➣ Identify the cube as the most efficient unit for measuring volume and explain why. ➣ Provide a referent for a cubic centimetre and explain the choice. ➣ Provide a referent for a cubic metre and explain the choice. ➣ Determine which standard cubic unit is represented by a given referent. ➣ Estimate the volume of a 3-D object using personal referents. ➣ Determine the volume of a 3-D object using manipulatives and explain the strategy. ➣ Construct a rectangular prism for a given volume. ➣ Explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same volume.

Prior Knowledge

Students may have had experience with the following:

- Using direct comparison to compare the volume of two objects
- Identifying attributes of objects that can be compared
- Demonstrating an understanding of measurement as a process of comparing by filling
- Describing and constructing rectangular prisms
- Measuring the lengths of objects in m or cm

Background Information

The terms *volume* and *capacity* are often used interchangeably. For the purposes of the learning experiences in this section and the section that follows, a distinction will be made. Volume is the amount of space an object occupies or, if the object is hollow, the amount of space inside the object. Volume is measured in cubic centimetres (cm^3) or cubic metres (m^3).

Capacity is the maximum amount of liquid a container can hold. Capacity is measured in litres (L) and millilitres (mL).

Mathematical Language

Cubic unit (centimetre and metre)

Dimension

Rectangular prism

Length (width, height)

Less (least) volume

More (greatest) volume

Same volume

Volume

LEARNING EXPERIENCES

- **Identify the cube as the most efficient unit for measuring volume and explain why.**
- **Determine the volume of a 3-D object using manipulatives and explain the strategy.**

Materials: A variety of small boxes, cubes, marbles, other 3-D shapes such as a triangular prism or pyramid, and sand

Organization: Whole class/Small group

Procedure:

a) Show students the insides of two empty boxes. Ask, “Which box has more space inside? How can we tell for sure?”

b) Explain that volume is the amount of space inside a container or the number of units needed to fill the container. Ask, “What unit do you think we should use to measure volume?”

c) Give each group a box and three possible units: marbles, cubes, and triangular prisms (or any other shape). Tell students that their task is to determine which unit is best for measuring volume. Explain that they will be measuring the volume of their box three times. Each time they will completely fill the box with one of the units. Explain that when filling the box they should lay the units carefully on the bottom of the box, record the number used, and then fill the box layer by layer. Ask students to record the total number of units used as well as their observations on the appropriateness of the unit.

d) Have students share their observations about the different units. Help them recognize that the cube is the best unit to use because it is easy to stack and there are no gaps or overlaps when filling the containers (e.g., have students pour sand into a box filled with marbles to show them that there are gaps between the marbles).

Observation Checklist

Observe students to determine whether they can do the following:

- ❑ measure the volume of the box correctly (e.g., completely fill the box with a unit and count the number of units used)
- ❑ record both the number and the unit of measure
- ❑ recognize that the cube is the most efficient unit for measuring volume and explain why

- **Determine the volume of a 3-D object using manipulatives and explain the strategy.**

Materials: Small boxes and cubes

Organization: Pairs or small groups

Procedure:

a) Give each group four or five boxes. Have students label the boxes A, B, C, D.…

b) Have students look at the labelled boxes, and decide which one they think has the smallest volume and which one has the largest volume. Ask them to put the boxes in order from the smallest volume to the largest volume and to record the order they have decided on.

c) Have students measure the volume of each box to the nearest whole unit and record their measurements in a table like the one shown below. Have students record the actual volume of the containers and compare it with their estimated volume.

Box	Estimated Volume	Actual Volume
A		
B		
C		
D		

d) Have each group share its findings with the rest of the class. Encourage students to discuss the strategies they used to determine the volume of the boxes, particularly when the number of cubes was not an exact fit (e.g., when there was some space around the layers).

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ compare and order containers according to their volume
- ❑ make reasonable estimates of the volume of containers
- ❑ determine the volume of an object using manipulatives and explain the strategy
- ❑ record measurements that include both a number and the unit
- ❑ use the terms more (greatest) volume, less (least) volume, and the same volume correctly

- **Identify the cube as the most efficient unit for measuring volume and explain why.**
- **Provide a referent for a cubic centimetre and explain the choice.**

Materials: Different sizes of boxes, different sizes of cubes, a white rod from a set of Cuisenaire rods, and centicubes

Organization: Small groups

Procedure:

a) Present students with the following situation:
   - A box Nicky measured has a volume of 18 cubic units. She gave the box to Cathy. When Cathy measured the volume of the box, she found it had a volume of 26 cubic units. Could both girls' measurements be correct? Why or why not?

b) Encourage students to devise and carry out a plan to prove their assertions about the situation.

c) Have students share their results and reasoning with the other members of the class. Encourage students to discuss the need for a standard unit of measure and the reasons why it is important to use common units (e.g., to facilitate communications, business, and trade).

d) Show students a white rod from the set of Cuisenaire rods or a centicube, and explain that in the metric system a cubic centimetre is one of the units used to determine the volume of an object. Show students the word and the symbol for the unit.

e) Tell students that they will be using the white rods (or centicubes) to complete the following activity:
   1. Find a container that has a volume that is
      - greater than 80 cm^3
      - less than 40 cm^3
      - between 50 and 60 cm^3
   2. Find as many objects as you can that have a volume of 1 cm^3.

f) Have students share their findings with the rest of the class. Encourage students to discuss the strategies they used to find the containers and the objects they found that are approximately 1 cm^3. Start a class list of objects that have a volume of 1 cm^3. Encourage students to look outside the classroom for objects that have a volume of approximately 1 cm^3 and add them to list.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recognize the need for a standard unit
- ❑ determine the volume of an object using manipulatives and explain the strategy
- ❑ identify objects that have a volume of approximately 1 cm^3

- **Estimate the volume of a 3-D object using personal referents.**
- **Determine the volume of a 3-D object using manipulatives and explain the strategy.**
- **Construct a rectangular prism for a given volume.**
- **Explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same volume.**

Materials: Centimetre grid paper (BLM 5–8.9), centicubes, scissors; tape, copies of the instructions for the activity (BLM 5.SS.3.1), and observation form (BLM 5–8.1)

Organization: Small groups

Procedures:

a) Tell students that they will be using BLM 5.SS.3.1 to complete an investigation involving volume.

b) Help students determine what should be included in their reports and the criteria for evaluating them. Encourage students to consider such things as the accuracy of their measurements, the strategies they used to determine the volumes of the open boxes, and the clarity of their explanation of what happens to the volume as the dimensions of the open boxes change.

Observation Checklist

- ❑ Use the observation form (BLM 5–8.1) to observe how well students work together.

- **Estimate the volume of a 3-D object using personal referents.**
- **Determine the volume of a 3-D object using manipulatives and explain the strategy.**

Material: Centicubes, small boxes such as a shoebox or a cereal box, a list of the volumes of the boxes, and math journals

Organization: Small groups/Whole class

Procedure:

a) Give each group four or five small boxes and only enough centicubes to cover the bottom of each box separately, plus enough to make one stack the height of each box.

b) Ask students to estimate the volume of each box, and to record their estimates in their math journals. Explain that there are not enough centicubes to completely fill any box; however, they can use the centicubes that they have been given to help them make their estimates. When they finish estimating the volumes of the boxes, they should compare their estimates with the list of the volumes of the boxes that you have prepared.

c) Have students share the strategies they used to estimate the volumes of each box.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ explain the strategy they used to determine the volume of the boxes
- ❑ use a referent to make reasonable estimates of the volume of the boxes

- **Construct a rectangular prism for a given volume.**
- **Explain that many rectangular prisms are possible for a given volume by constructing more than one rectangular prism for the same volume.**

Materials: Centicubes, or the white rods from a set of Cuisenaire rods, or multilink cubes

Organization: Pairs

Procedure:

a) Tell students that they will be making rectangular prisms with their centicubes and determining their volumes. Show students a rectangular prism made with 10 cubes. Ask students what the volume of the prism is, and how they know.

b) Have students complete the following activity:

1. Construct a rectangular prism with a volume of
   - 12 cm^3
   - 16 cm^3

   Record the dimensions of the prisms you made.
2. Build two rectangular prisms, side by side, so that one prism has a volume of 6 cm^3 more than another. Record the dimensions of each prism.
3. Make two rectangular prisms with the same length, with one wider and shorter than the other, but with different volumes. Record the dimensions of each prism.
4. Make two rectangular prisms with the same length, with one wider and shorter than the other, but with the same volume. Record the dimensions of each prism.
5. Make as many rectangular prisms as you can that have a volume of 24 cm^3. Record the dimensions of each prism that you make.
6. Make three rectangular prisms with the following dimensions:
   - 6 cm x 6 cm x 6 cm
   - 3 cm x 12 cm x 6 cm
   - 3 cm x 9 cm x 8 cm

   Find the volume of each prism. Record the dimensions of each prism and its volume.
7. Think about the rectangular prisms that you made. What can you conclude about the volume of prisms? Record your observations.

When students finish each part of the activity, have them share their results with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ construct a rectangular prism with a given volume
- ❑ recognize that different rectangular prisms are possible for a given volume
- ❑ recognize that rectangular prisms with different dimensions can have the same volume
- ❑ recognize that if one dimension of two rectangular prisms is the same, the volume of the prisms is not necessarily the same
- ❑ recognize that the volume of a rectangular prism is dependent on its dimensions

- **Provide a referent for a cubic metre and explain the choice.**
- **Determine which standard cubic unit is represented for a given referent.**

Materials: Metre sticks, plasticine, cardboard, scissors, and tape

Organization: Small groups/Whole class

Procedure:

a) Ask students to think of containers they see inside and outside of school whose volume should be measured in cm^3. Keep a list of their suggestions. Finish the discussion by asking, “Are there any containers or objects that are too large to be measured with a cm^3?”

b) Have each group develop a list of containers or objects that would require a larger unit of measure. When students finish, have them share their list with the other members of the class.

c) Explain that in the metric system the volumes of very large items or containers are measured in cubic metres. Ask students to show with their hands how large they think a cubic metre is.

d) Have each group make a model of a cubic metre. Some groups can make their cubic metre using 12 metre sticks (or wooden dowels 1 metre in length) joined with plasticine or masking tape, while other groups can draw and cut out six 1 metre squares from heavy cardboard and join the squares with masking tape.

e) Have students estimate how many students they think will fit into a cubic metre. Have them try it out and then discuss how their estimates compared with the actual number and the reasons why they may vary.

Observation Checklist

Observe students’ responses to determine whether they can do the following:

- ❑ identify containers or objects whose volumes should be measured in cubic centimetres
- ❑ identify containers or objects whose volumes should be measured with larger units
- ❑ make a cubic metre

- **Provide a referent for a cubic metre and explain the choice.**
- **Determine which standard cubic unit is represented for a given referent.**
- **Estimate the volume of a 3-D object using manipulatives and explain the strategy.**

Materials: Models of cubic metres

Organization: Whole class/Small groups

Procedure:

a) Have students refer to their models of a cubic metre to estimate whether the following objects have a volume greater than, less than, or about the same as a cubic metre:
   - their desk
   - a pop machine
   - a filing cabinet
   - a garbage can
   - a dump truck
   - a stove

   Encourage students to explain their reasons for their estimates.

b) Ask each group to make a list of items inside and outside of the classroom whose volume could be measured in cubic metres. Have the groups share their lists and explain the reasons for their choices.

c) Have each group refer to its model of a cubic metre to estimate the volume of
   - their classroom
   - the school gym
   - the principal's office

   Have the groups share their estimates and the strategies they used to determine the volumes of the rooms.

d) Have students discuss the question: "Does a cubic metre have to be a cube?"

   Note: Students should recognize from the previous activity that a cubic metre does not need to be a cube since prisms with different dimensions can have the same volume.

e) Have students collect and fill one of the cardboard cubic metres they made with an item they would like to give to charity (e.g., students could give a cubic metre of clothes that they have outgrown). Have students write a letter to the community and other classes in the school explaining what they are doing and inviting them to help collect the item they have chosen.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❏ identify objects or containers whose volume could be measured in cubic metres
- ❏ give reasonable estimates of containers or items whose volume could be measured in cubic centimetres
- ❏ explain the strategies they used to estimate the volumes of large containers and objects

NOTES

Grade 5: Shape and Space (Measurement) (5.SS.4)

Enduring Understandings:

All measurements are comparisons.

Length, area, volume, capacity, and mass are measurable properties of objects.

The unit of measure must be of the same nature as the property of the object being measured.

General Outcome:

Use direct or indirect measurement to solve problems.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.SS.4 Demonstrate an understanding of capacity by ■ describing the relationship between mL and L ■ selecting and justifying referents for mL or L units ■ estimating capacity by using referents for mL or L ■ measuring and recording capacity (mL or L) [C, CN, ME, PS, R, V]	➢ Demonstrate that 1000 millilitres is equivalent to 1 litre by filling a 1-litre container using a combination of smaller containers. ➢ Provide a referent for a litre and explain the choice. ➢ Provide a referent for a millilitre and explain the choice. ➢ Determine which capacity unit (mL or L) is represented by a given referent. ➢ Estimate the capacity of a container using personal referents. ➢ Determine the capacity of a container using materials that take the shape of the inside of the container (e.g., a liquid, rice, sand, beads), and explain the strategy.

Prior Knowledge

Students may have had experience with the following:

- Identifying attributes of objects that can be measured
- Using direct comparison to compare the capacity of two objects
- Demonstrating an understanding of measurement as a process of comparing by filling
- Demonstrating an understanding of whole numbers less than 10 000
- Demonstrating an understanding of addition and subtraction of whole numbers with answers less than 10 000

Related Knowledge

Students should be introduced to the following:

- Providing a referent for one millimetre, one centimetre, and one metre

Background Information

The terms *volume* and *capacity* are often used interchangeably. For the purposes of the learning experiences in this section and the previous section, a distinction will be made. Volume is the amount of space an object occupies or, if the object is hollow, the amount of space inside the object. Volume is measured in cubic centimetres (cm^3) or cubic metres (m^3).

Capacity is the maximum amount of liquid a container can hold. Capacity is measured in litres (L) and millilitres (mL).

Mathematical Language

Capacity

More capacity

Less capacity

Same capacity

Estimate

Litre

Referent

Millilitre

Learning Experiences

- **Determine the capacity of a container using materials that take the shape of the inside of the container (e.g., a liquid, rice, sand, beads), and explain the strategy.**

Materials: A variety of containers (some of which should be transparent), funnels, water, sand (or any other material that will take the shape of containers), paper towels, sponges, and markers

Organization: Whole class/Small groups

Procedure:

a) Explain that we often hear expressions, such as the following:
   - "The room was filled to capacity."
   - "They played to a capacity crowd."

   Ask, "What does the word 'capacity' mean? How can we find the capacity of an object?"

b) Explain that in math we use the term *capacity* to describe how much liquid a container can hold, and to determine the capacity of a container we need a unit of measure.

c) Show students a transparent container. Show students how to measure the capacity of the container by using another smaller transparent container as the unit of measure. Repeat this activity two or three times to make sure students understand how to measure the capacity of a container.

d) Give each group four or five containers. Have students select one of their containers to be the unit of measure and label the other containers A, B, C, D….

e) Have students look at the labelled containers and decide which one they think has the smallest capacity and which one has the largest capacity. Ask them to put the containers in order from the smallest capacity to the largest capacity, and to record the order they have decided on.

f) Have students give their unit a name. Have them measure each container and record their measurements in a table like the one shown below. Have students record the actual order of the containers, and compare it with their estimated order.

Container	Estimated Capacity	Actual Capacity
A		
B		
C		
D		

g) Have each group share its findings with the rest of the class. Encourage them to describe how the real order of the containers compared with their estimated order.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ use the terms "more capacity," "less capacity," and "the same capacity" correctly
- ❑ measure correctly (students completely fill the unit over and over again until the container being measured is full)
- ❑ record their measurements correctly (includes both a number and the unit)
- ❑ give reasonable estimates of capacity
- ❑ compare and order containers according to their capacities

Materials: A wide variety of containers, masking tape, spoons, large plastic glasses or jars, water, markers, paper towels, sponges, and procedure steps (BLM 5.SS.4.1)

Organization: Pairs

Procedure:

a) Have students use the following procedure to make their own measuring device:
   1. Put a piece of masking tape down the side of a glass (or jar).
   2. Fill a small container with spoonfuls of water, keeping track of the number of spoonfuls needed to fill it.
   3. Empty the water in the small container into the glass.
   4. Mark the level of the water and the number of spoonfuls on the tape.
   5. Fill the small container again. Empty the water into the glass. Mark the level of the water and the total number of spoonfuls.
   6. Continue filling and marking the glass until the top is reached.

b) Make sure the students know how to read and use their measuring device. Have them use their measuring device to find the capacity of five different containers in two different ways. Have the students record their findings in the chart provided in BLM 5.SS.4.1.

c) Have students write a paragraph describing the two different ways they found the capacity of their containers. Have students share and discuss their methods with the other members of the class.

d) Have students exchange their containers with another group. Have them use their measuring device to determine the capacity of these containers and record their findings in a table.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ use the terms "more capacity," "less capacity," and "the same capacity" correctly
- ❑ determine the capacity of a container by filling it with water using their measuring device
- ❑ determine the capacity of a container by filling it, and then pouring its contents into the measuring device to see how much it holds
- ❑ measure correctly (e.g., completely fill the container)
- ❑ read their measuring devices correctly
- ❑ record their measurements correctly (include both the number and the unit)

- **Estimate the capacity of a container using personal referents.**
- **Determine the capacity of a container using materials that take the shape of the inside of the container (e.g., a liquid, rice, sand, beads), and explain the strategy.**

Materials: Containers (some containers should be greater than a litre, less than a litre, and equal to a litre), water, sand (and other material that takes the shape of a container), student-made measuring devices, litre measuring devices, masking tape, and markers

Organization: Small groups

Procedure:

a) Give each group the same two containers. Have some of the groups use their measuring devices to determine the capacity of the containers. Have other groups select another container to be their unit. Have these groups name their unit and find the capacity of their containers.

b) Have students share their measurements. List their measurements on the board and ask why they differ. Ask students what they could do so everyone would get the same measurement. Help students recognize the need for a standard unit of measure and the reasons why it's important to use standard units (e.g., the use of standard units facilitates business and trade).

c) Tell students that in the metric system the litre is the standard unit of measure for capacity. Show students an unmarked litre container and tell them that a litre is the amount of the liquid it can hold. Also, show students how to write the word and the symbol for the unit.

d) Give each group five or six containers. Have the students label the containers from A to F and then make a list in their math journal of the containers they think are less than a litre, the same as a litre, and larger than a litre.

e) Have students use the unmarked litre containers to measure the capacity of each container. Explain that they should not fill any container higher than the bottom part of the neck of the container. Ask students to write the letter of each container in their math journal, and record whether its capacity is greater than a litre, less than a litre, or the same as a litre.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ determine the capacity of a container using materials that take the shape of the container
- ❑ measure correctly (e.g., fill the litre-measuring container and the containers they are measuring to the right levels)
- ❑ record measurements properly (e.g., use the correct symbol for a litre)
- ❑ make reasonable estimates of capacity

Materials: Litre-measuring containers, a variety of containers, water, sand, or any other material that takes the shape of a container, a pitcher, a water pail, and a wastepaper basket

Organization: Whole class/Small groups

Procedure:

a) Have students provide examples of when they would need to estimate the capacity of a container, and discuss how they can ensure that their estimates are reasonable.

b) Ask each group to find two common containers they can use as a referent for a litre.

c) Have the groups share their referents with each other and keep a class list of referents for a litre.

d) Show students a large pitcher, a wastepaper basket, and an empty water pail. Ask them to think of their referent and then estimate the capacity of each container. Have the students check their estimates by measuring each item.

e) Ask students to estimate the capacity of a bathtub. Help them devise and carry out a plan to check their estimates.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❏ provide a referent for a litre and explain their choice
- ❏ make reasonable estimates of the capacities

- **Provide a referent for a millilitre and explain the choice.**
- **Estimate the capacity of a container using personal referents.**
- **Determine the capacity of a container using materials that take the shape of the inside of the container (e.g., a liquid, rice, sand, beads), and explain the strategy.**

Materials: Beakers calibrated in mL, graduated cylinders calibrated in mL, an eyedropper, baby food jars, tin cans, small milk cartons, small soda cans, pickle jars, ketchup bottles, water, paper towels, funnels, and sponges, capacity table (BLM 5.SS.4.1)

Organization: Whole class/Small group

Procedure:

a) Show students a small container, such as an empty tuna can or empty baby food jar, and ask them how they could find the capacity of the container.

b) Explain that to find the capacity of smaller containers, we need a new unit of measure. The unit that is commonly used is the millilitre. Tell students that the millilitre is a very small unit about the size of a drop from an eyedropper. Fill an eyedropper with water and show students several drops so they can begin to conceptualize how large the unit is.

c) Explain that because the unit is so small, we often use measuring devices that are marked off in millilitres. Show students different measuring devices that are calibrated in mL, and explain how they should use them to find the capacity of a container.

d) Have students measure the capacity of each object listed below in two different ways and record their results in the table from 5.SS.4.1.

- **Demonstrate that 1000 millilitres is equivalent to 1 litre by filling a 1-litre container using a combination of smaller containers.**

Materials: A 500 mL beaker, a 250 mL beaker, a 100 mL beaker, and a 50 mL beaker; unmarked litre containers, water, funnels, paper towels, and math journals

Organization: Small groups/Whole class

Procedure:

a) Show students the litre container and tell them that their job is to determine the number of mL in a litre.

b) Have students estimate the number of 50 mL beakers of water it will take to fill the litre container. Have them record their estimates in a table like the one shown below.

Beaker	Estimated Number of Beakers	Actual Number of Beakers	Total Number of mL
50 mL			
100 mL			
250 mL			
500 mL			

c) Have students check their estimates by filling the litre container with 50 mL beakers of water and record their results in the table.

d) Repeat the activity using the 100 mL beaker, the 250 mL beaker, and the 500 mL beaker.

e) Have students compare their results with another group. Ask them what they can conclude about the relationship between a mL and a litre.

f) Give students the following problem and have them record their solution in their math journals:

  - Jessi has a container that holds 1425 mL of liquid. Is Jessi's container smaller than or larger than a litre? How do you know? How much larger or smaller than a litre is Jessi's container?

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ demonstrate that there are 1000 mL in a litre using a variety of smaller containers
- ❑ measure correctly (fill the beakers properly)
- ❑ record the measurements correctly
- ❑ solve a problem involving the relationship between millilitres and litres

Materials: Cards numbered from 0 to 9 (BLM 5–8.5), paper and pencil

Organization: Small groups/Whole class

Procedure:

a) Tell students that they will be playing a game involving the relationship between a litre and a millilitre. Explain how the game is played

   1. Players should make the following grid on their papers:

   2. Shuffle the cards and place them face down on the playing area.
   3. Turn over one card. Players decide where they want to write that number on their grid. Once a number has been placed on the grid, it cannot be changed.
   4. The next card is turned over and the players now place this number on their grids. Play continues until six numbers have been turned over and each player has placed the numbers on his or her grid.
   5. The players add the millilitre quantities and the player or players closest to 1 litre receive one point.
   6. Reshuffle the cards, make new grids, and play the game again.
   7. Continue playing the game. The first player to reach 10 points is the winner.

b) Demonstrate how the game is played and answer any questions students might have. Have students play the game.

c) Vary the game so that the player with the sum closest to 500 mL wins a point.

Observation Checklist

Observe students to determine whether they

- ❑ know the relationship between mL and litres
- ❑ calculate correctly

Putting the Pieces Together

Planning a Healthy Meal

Purpose:

The purpose of this investigation is to have students apply their knowledge of capacity to a real-world situation. In particular, it is designed to reinforce students' abilities to

- measure the capacity of containers
- estimate the capacity of containers
- record the capacity of containers

The investigation is also designed to enhance students' abilities to

- communicate mathematically
- solve problems
- reason mathematically
- connect mathematics to real-world situations and other subject areas (PE/HE)

Materials/Resources

- Centimetre measuring cubes
- Assorted containers
- Food groups guide (can be found on the Internet)
- Water, sand, or other material that takes the shape of a container
- Cylinders or beakers calibrated in mL
- Paper towels
- Markers

Organization: Whole class/Small groups

Procedure:

a) Tell students that each group will be responsible for planning a healthy breakfast or lunch. Since the capacity of the human stomach is approximately 1 litre, the meal they planned should not contain more than 800 mL of food. In planning their meal, they should

- use the food guide to help them select foods from each food group
- include foods that are available locally
- indicate the quantity of each food in mL

b) Have students design their meals. When they finish planning their meal, have them find a container with the same capacity as each item on their menu. Have students label each container by indicating the item of food it represents and its capacity.

c) Have each group display its menu and corresponding containers. Have students explain why their meals are nutritious and how the capacities of the different items add up to an 800 mL meal.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ plan a meal that meets the criteria specified in part (a)
- ❏ make reasonable estimates of the capacities of containers
- ❏ measure the capacity of containers correctly
- ❏ record the capacity of containers correctly

NOTES

Grade 5: Shape and Space (3-D Objects and 2-D Shapes) (5.SS.5)

Enduring Understandings:

Shapes are distinguished by their properties.

General Outcome:

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationship between them.

Specific Learning Outcome(s):	Achievement Indicators:
5.SS.5 Describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes, that are ■ parallel ■ intersecting ■ perpendicular ■ vertical ■ horizontal [C, CN, R, T, V]	➛ Identify parallel, intersecting, perpendicular, vertical, and horizontal edges and faces on 3-D objects. ➛ Identify parallel, intersecting, perpendicular, vertical, and horizontal sides on 2-D shapes. ➛ Provide examples from the environment that show parallel, intersecting, perpendicular, vertical, and horizontal line segments. ➛ Find examples of edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, and horizontal in print and electronic media, such as newspapers, magazines, and the Internet. ➛ Draw 2-D shapes or 3-D objects that have edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, or horizontal. ➛ Describe the faces and edges of a given 3-D object using terms such as parallel, intersecting, perpendicular, vertical, or horizontal. ➛ Describe the sides of a 2-D shape using terms such as parallel, intersecting, perpendicular, vertical, or horizontal.

Prior Knowledge

Students may have had experience with the following:

- Identifying cubes, spheres, cones, cylinders, pyramids, triangular prisms, and rectangular prisms
- Identifying triangles, squares, rectangles, and circles
- Identifying the faces, edges, and vertices of 3-D objects
- Sorting regular and irregular polygons including triangles, quadrilaterals, pentagons, hexagons, and octagons according to the number of sides

Related Knowledge

Students should be introduced to the following:

- Identifying and sorting quadrilaterals

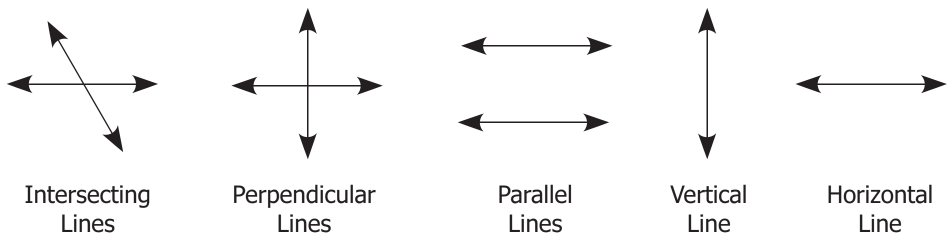
Background Information

Points, lines, and planes are the building blocks of geometry. These concepts are undefined and, like number, they are abstractions that cannot be seen or touched. Students' understanding of these concepts evolves from their experiences with physical objects (e.g., the tip of a pencil, the corner of a table or block, and the dot drawn on a piece of paper suggest the idea of a point to students).

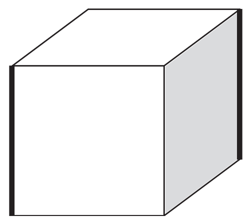
Lines are sometimes described as a set of points extending endlessly in two directions. They have length but no other dimension. Physical models, such as a rope stretched out, a wire held taut, and the centre line on a highway, can help students develop an understanding of this concept. A **line segment** is part of a line. It consists of two endpoints and all the points between them. Examples of line segments include the rungs of a ladder, the edges of a box, and the bars in a grill.

A **plane** is two-dimensional. Any smooth, flat surface, such as a tabletop, a floor, or a ceiling, can be thought of as a plane. However, each of these models is only a part of a plane because a plane extends infinitely in two directions.

Two lines in a plane can intersect or be parallel to each other. **Intersecting lines** have one point in common; **parallel lines** have no points in common. The distance between them is the same everywhere. Sometimes lines intersect at right angles. These lines are **perpendicular**. Because students are not introduced to angles until Grade 6, perpendicular lines are described as two lines that form "square" corners. In addition, lines can be either horizontal or vertical. A **horizontal line** is a line that is parallel to the horizon. A **vertical line** is a line that is at right angles to the horizon. Students usually describe horizontal lines as going across, and vertical lines as going up and down. However, their perception of whether a line is horizontal or vertical might differ according to their perspective.



When describing the edges of prisms, some students may think that any two line segments that do not intersect are parallel. For example, consider the cube shown below:



Some students may think the two dark edges are parallel since they do not intersect. However, these edges lie in different planes and therefore are not parallel.

Mathematical Language

Cone	Parallel lines
Cube	Perpendicular lines
Cylinder	Pyramid
Edge	Rectangular prism
Face	Sphere
Horizontal line	Triangular prism
Intersecting line	Vertex (Vertices)
Line	Vertical line
Line segment	

Assessing Prior Knowledge

Materials: A set of 3-D objects that includes a cone, sphere, cylinder, a pyramid, a cube, a rectangular prism, and a triangular prism

Organization: Whole class/Individual

Procedure:

a) Put the 3-D objects in a place where all students can see them. Tell the students that you will be asking them some questions about the shapes to find out what they already know about them.

b) Tell students they can look at the shapes to help them identify the 3-D objects or parts of objects that fit the following clues:

1. I have six faces all the same size and shape. (cube)
2. I am formed by the intersection of two faces. (edge)
3. Two of my faces are circular. (cylinder)
4. I am the point where three or more edges meet. (vertex)
5. I have six rectangular faces. (rectangular prism)
6. I have no flat faces. (sphere)
7. My shape is found on every pyramid. (triangle)
8. We are the faces found on a triangular prism. (triangle and rectangle)
9. I am one face of a cone. (circle)

Observation Checklist

- ❏ Use students' responses to the questions to determine whether further review on the identification and characteristics of 3-D objects and 2-D shapes is needed.

- **Provide examples from the environment that show parallel, intersecting, perpendicular, vertical, and horizontal line segments.**

Materials: Copies of the concept description sheet (BLM 5–8.2).

Organization: Individual/Whole class

Procedure:

a) Tell students that in the next few lessons they will be learning about lines and today they will be discussing what a line is. Before beginning the discussion, you want them to write down what they already know about lines.

b) Have students complete the concept description sheet. Let students know that it is alright if they cannot think of anything to put in a section. They will have another opportunity to complete the sheet when they learn more about lines.

c) When students finish, begin a discussion by asking, "What is a line? What are some examples of lines?" As the discussion progresses, clear up any misconceptions students may have about lines and make sure they see a variety of examples and non-examples.

d) Have students add to their concept description.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ describe the characteristics of a line (e.g., it continues indefinitely in two directions)
- ❑ identify examples of a line
- ❑ identify non-examples of a line

Material: A long rope or a skein of yarn

Organization: Whole class

Procedure:

Note: This activity could be done in the gym or outside.

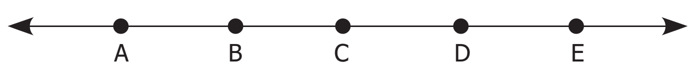
a) Take students outside to the playground. Stretch the rope across the playground. Have students hold onto the rope and hold it taut. Tell students that the rope represents a line that keeps going forever (e.g., if they were to tie another piece on the end and pull it taut, the line would continue). Have students discuss what things the line would go through as it goes beyond each end of the rope. Discuss how they think they could show that the rope/yarn would continue on.

b) Tell students that everyone holding onto the line is a point and the space between each pair of them is a line segment or part of a line. Name the line segments using the students' names (e.g., line segment Jack and Josie). Call out the names of several line segments. Each time you call out a line segment, have the named students hold on to the rope and the students between them let go to show how long the line segment is. Have students take turns naming line segments.

c) Ask students to remember who is standing on either side of them. Return to the classroom and draw an arrow on the chalkboard. Put an arrow on either end to indicate that the line goes on forever. Indicate points on the line and write the names of the students under them.

d) Have students discuss the differences between their experiences outdoors and the ideas represented by the line on the board. For example, students should note that the arrows on the line indicate that the line goes on forever, and the points where students' names appear show that a line segment has definite ends.

e) Explain that in math we use a double arrow to indicate a line and capital letters instead of students' names to indicate points on the line. Draw another line on the board. Name several line segments and have students identify where they are on the line.



Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ recognize that a line extends infinitely in two directions
- ❑ recognize that a line segment is part of a line
- ❑ name line segments correctly

- **Identify parallel, intersecting, perpendicular, vertical, and horizontal sides on 2-D shapes.**
- **Provide examples from the environment that show parallel, intersecting, perpendicular, vertical, and horizontal line segments.**
- **Draw 2-D shapes or 3-D objects that have edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, or horizontal.**
- **Describe the faces and edges of a 3-D object using terms such as parallel, intersecting, perpendicular, vertical, or horizontal.**

Materials: Stir sticks or toothpicks, a mat or a rug

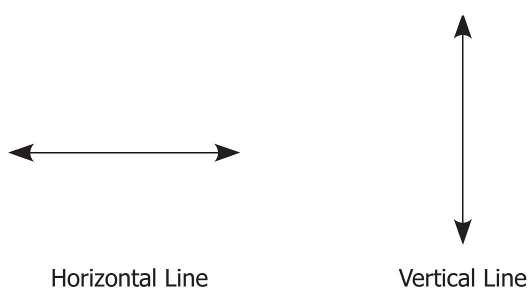
Organization: Whole class/Small group

Procedure:

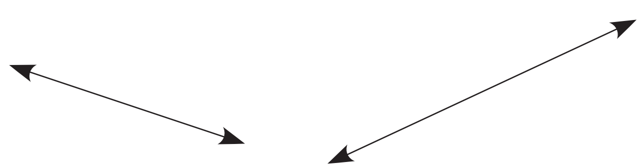
a) Have two or three students come to the front of the class. Ask them to lie down on the floor. Explain that when the students are lying on the floor they are horizontal. Now ask the students to stand up. Explain that when the students are standing up they are vertical.

b) Ask different students to either lie down or stand up. Have the other students indicate whether the students are horizontal or vertical.

c) Draw a horizontal and a vertical line on the board. Explain that, in math, lines can be horizontal or vertical. Lines that are lying down are horizontal and those that are standing up straight (go up and down on their paper) are vertical.



d) Draw several lines on the board like the ones shown below:



Ask, "Are these lines horizontal? Why or why not? Are the line segments vertical? Why or why not?"

e) Ask students to use the stir sticks to show the following:
- A vertical line segment
- A horizontal line segment
- A line segment that is neither horizontal or vertical
- A vertical line segment that crosses a horizontal line segment
- A horizontal line segment that crosses a line segment that is not vertical
- A horizontal line segment that crosses three vertical line segments
- A vertical line segment that crosses two line segments that are neither horizontal nor vertical
- Four horizontal line segments
- Two vertical line segments and three horizontal line segments
- A vertical line segment and two horizontal line segments and a line segment that is neither vertical nor horizontal

f) Have students identify whether the edge of their desk is a horizontal or a vertical line segment. Have them identify two or three other horizontal or vertical line segments in the room.

g) Ask each group to make a list of horizontal and vertical line segments that they see in the school. Have them share their lists with the other members of the class.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ use the terms "horizontal" and "vertical" correctly
- ❑ make vertical and horizontal line segments
- ❑ identify examples of horizontal and vertical line segments in the environment.
- ❑ identify line segments that are neither horizontal nor vertical

- **Identify parallel, intersecting, perpendicular, vertical, and horizontal edges and faces on 3-D objects.**
- **Identify parallel, intersecting, perpendicular, vertical, and horizontal sides on 2-D shapes.**
- **Draw 2-D shapes or 3-D objects that have edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, or horizontal.**
- **Describe faces and edges of a 3-D object using terms such as parallel, intersecting, perpendicular, vertical, or horizontal.**
- **Describe the sides of 2-D shapes using terms such as parallel, intersecting, perpendicular, vertical, or horizontal.**

Materials: 20 stir sticks or toothpicks for each pair of students, cubes, rectangular prisms, pyramids, and triangular prisms

Organization: Pairs/Whole class

Procedure:

a) Show students a square made out of stir sticks. Have them identify the shape, as well as the horizontal and vertical line segments that form it.

b) Have students make as many shapes as they can with their stir sticks. Explain that a shape can be made with any number of stir sticks, but it must have only horizontal and vertical lines. Ask students to draw a picture of each shape they make. Tell students that they should write the name of each shape under it, and label the horizontal and vertical line segments.

c) Give a 3-D object to each pair of students. Ask students to discuss their shape with their partner, and then write a description of their shape in their math journal. Explain that they should write the name of the shape, and then use words and pictures to explain which edges and faces of their shape are horizontal and which are vertical.

Observation Checklist

Check students' work to determine whether they can do the following:

- ❑ construct 2-D shapes that have vertical and horizontal lines
- ❑ draw 2-D shapes that have vertical and horizontal lines
- ❑ identify the horizontal and vertical line segments on a 2-D shape
- ❑ identify the names of 2-D shapes
- ❑ identify the horizontal and vertical edges and faces of a 3-D object
- ❑ draw 3-D objects that have vertical and horizontal edges and faces

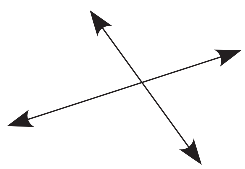
- **Identify parallel, intersecting, perpendicular, vertical, and horizontal edges and faces on 3-D objects.**
- **Identify parallel, intersecting, perpendicular, vertical, and horizontal sides on 2-D shapes.**
- **Provide examples from the environment that show parallel, intersecting, perpendicular, vertical, and horizontal line segments.**
- **Draw 2-D shapes or 3-D objects that have edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, or horizontal.**

Materials: Stir sticks and orange pattern block squares

Organization: Small groups

Procedure:

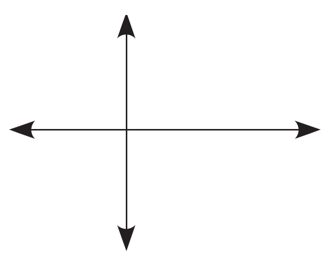
a) Draw the following line segments on the board or overhead. Explain that these lines are called intersecting lines because they cross each other.



Ask students to use their stir sticks to show

- three different pairs of intersecting line segments
- two line segments that do not intersect
- A line segment intersected by more than one line segment

b) Explain that sometimes lines intersect in a special way. Ask students what is special about how these two line segments intersect.



Explain that these lines are special because they form "square corners." Demonstrate this by placing the orange squares at the intersection of the lines.

Tell students that these lines are perpendicular.

Have students make three different pairs of perpendicular line segments with their stir sticks. Have them use the orange squares to show that each pair of lines forms a "square corner."

Ask students to make a pair of line segments that are not perpendicular and explain why they are not perpendicular.

c) Ask students what is special about this pair of lines (figure 1). Explain that lines that never meet are parallel. Demonstrate that the lines never meet by placing orange squares between the two lines (figure 2) and having students note that the distance between the two lines is the same everywhere.

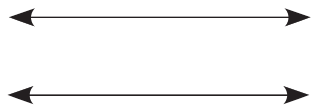


Figure 1

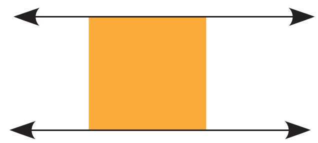


Figure 2

Ask students to make three different pairs of parallel line segments with their stir sticks. Have them demonstrate that the line segments are the same distance apart by using the orange squares or their rulers.

Have students make a pair of lines that are not parallel and explain why they are not parallel.

d) Ask each group to identify examples of parallel, intersecting, and perpendicular line segments inside and outside the classroom. Have them draw and label a diagram of each line segment pair, and list under each diagram real-world examples of the line segment pair.

e) Have each group share its examples of each type of line segment pair with the other members of the class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

❏ construct pairs of line segments that are parallel, perpendicular, and intersecting

❏ identify line segments that are not parallel

❏ identify line segments that are not intersecting

❏ identify line segments that are not perpendicular

❏ identify real-world examples of parallel, perpendicular, and intersecting line segments

- **Identify parallel, intersecting, perpendicular, vertical, and horizontal sides on 2-D shapes.**
- **Draw 2-D shapes or 3-D objects that have edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, or horizontal.**

Materials: Paint or watercolours, black marker, and samples of Piet Mondrian artwork (can be found on the Internet)

Organization: Large group/Individual

Procedure:

a) Show students a picture of Piet Mondrian's artwork. Explain that Piet Mondrian was a Dutch painter who was famous for paintings that he called compositions.

b) Ask students to describe the picture. Encourage them to discuss the types of lines and shapes he used to create the picture.

c) Have students use black markers and watercolours to create a picture in the style of Piet Mondrian.

d) Display students' artwork on walls and conduct a gallery walk so students can look at each other's work.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ identify parallel, intersecting, perpendicular, vertical, and horizontal lines
- ❏ create a piece of artwork that is comprised of parallel, intersecting, perpendicular, vertical, and horizontal lines

Materials: Pattern blocks

Organization: Pairs

Procedure:

a) Ask students to use their pattern blocks to complete the following activity. Have them draw a sketch of each shape that they make.

1. Use two different blocks to make a shape with
   - exactly two pairs of parallel sides
   - exactly one pair of parallel sides
   - no parallel sides
2. Use three different blocks to make a shape with
   - exactly three pairs of parallel sides
   - exactly two pairs of parallel sides
   - exactly one pair of parallel sides
   - no parallel sides
3. What is the largest number of pairs of parallel sides of a shape you can make with
   - two pieces?
   - three pieces?
   - four pieces?
4. Can you use six different pattern blocks to make a shape with no parallel sides?

b) Have students share their shapes with the other members of the class. Encourage students to identify lines that are parallel, perpendicular, intersecting, vertical, and horizontal.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ identify parallel, intersecting, perpendicular, horizontal, and vertical line segments on 2-D shapes
- ❑ draw 2-D shapes with parallel, intersecting, perpendicular, horizontal, and vertical line segments

- **Identify parallel, intersecting, perpendicular, vertical, and horizontal edges and faces on 3-D objects.**
- **Describe the faces and edges of a 3-D object using terms such as parallel, intersecting, perpendicular, vertical, or horizontal.**

Materials: Cubes, rectangular prisms, square-based and triangular-based pyramids, and triangular prisms, cards with the names of the 3-D objects on them, one name per card (e.g., cube, rectangular prism, etc.), stir sticks or toothpicks, and plasticine

Organization: Small groups

Procedure:

a) Show students a triangular prism and ask them to describe it. Encourage students to identify the faces and edges that are parallel, intersecting, perpendicular, vertical, and horizontal.

b) Give each group a set of the 3-D objects. Have students take turns describing one of the shapes to the other members of their group. Encourage students to point out the faces and edges that are parallel, intersecting, perpendicular, vertical, and horizontal.

c) Give each student a card. Tell students you will be describing the characteristics of 3-D objects. If the shape on their card has that characteristic, they should stand up and show their card to the other members of the class. For example, if I say, "I am a 3-D object that has three pairs of parallel faces," then students who have cards with "cube" and "rectangular prism" written on them should stand up. The other members of the class have to check the cards to make sure that the right shapes have been identified.

d) Show students how to join toothpicks with plasticine to build a 3-D object. Have students use the materials to build 3-D objects that fit each set of characteristics.
   - Each edge is perpendicular to eight other edges. The edges are not all the same length. (rectangular prism)
   - There are six edges. No edges are perpendicular. (triangular pyramid)
   - The vertical edges are perpendicular to the horizontal edges. There are three vertical edges. (triangular prism)

e) Have students discuss questions like the following:
   - "Why are the roofs of most houses not parallel to the ground?"
   - "Why are the shelves of a bookcase parallel to the floor?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ identify faces of 3-D objects that are parallel, intersecting, perpendicular, horizontal, and vertical
- ❑ identify the edges of 3-D objects that are parallel, intersecting, perpendicular, horizontal, and vertical

Grade 5: Shape and Space (3-D Objects and 2-D Shapes) (5.SS.6)

Enduring Understandings:

Shapes are distinguished by their properties.

General Outcome:

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationship between them.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
5.SS.6 Identify and sort quadrilaterals, including ■ rectangles ■ squares ■ trapezoids ■ parallelograms ■ rhombuses according to their attributes. [C, R, V]	➛ Identify and describe the characteristics of a pre-sorted set of quadrilaterals. ➛ Sort a set of quadrilaterals and explain the sorting rule. ➛ Sort a set of quadrilaterals according to the lengths of the sides. ➛ Sort a set of quadrilaterals according to whether or not opposite sides are parallel.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Identifying quadrilaterals
- Identifying and explaining mathematical relationships using a Venn diagram

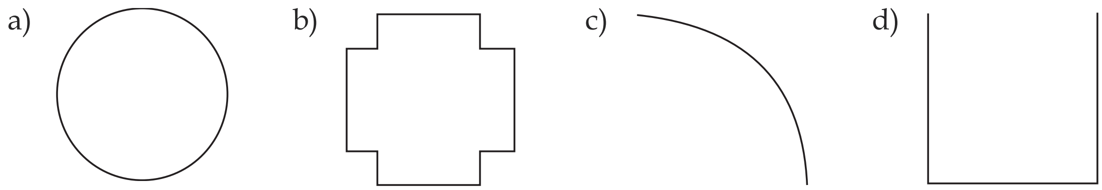
RELATED KNOWLEDGE

Students should be introduced to the following:

- Identifying parallel, perpendicular, horizontal, and intersecting line segments

BACKGROUND INFORMATION

A simple closed curve is a curve that does not cross itself and can be drawn by starting and stopping at the same point (e.g., in the diagram below, figures (a) and (b) are simple closed curves while (c) and (d) are curves that are not closed).



Polygons are simple closed curves formed by the union of line segments. In the diagram above, (b) is the only polygon since it is both a simple closed curve and made up of line segments. The line segments that form the polygon are the **sides** of the polygon. A point where two sides meet is a vertex of the polygon.

Polygons can be classified according to the number of sides they have. The most common classifications are: **triangle** (three sides), **quadrilateral** (four sides), **pentagon** (five sides), **hexagon** (six sides), **heptagon** (7 sides), **octagon** (8 sides), **nonagon** (9 sides), **decagon** (10 sides), and **dodecagon** (12 sides). Other polygons are commonly referred to as *n*-gons, where *n* is the number of sides. For example, an eleven-sided polygon can be referred to as an 11-gon and a 14-sided polygon can be referred to as a 14-gon.

Quadrilaterals can be classified according to the number of parallel sides that they have. The definition of each type of quadrilateral is given below.

Trapezium—A quadrilateral with **no pairs** of parallel sides.

Trapezoid—A quadrilateral with **at least one pair** of parallel sides.

Some texts define a trapezoid as a quadrilateral with **exactly one pair** of parallel sides. If the support material you are using defines a quadrilateral in this way, students should be shown both definitions. This can help them understand that mathematics is not a rigid subject and that mathematicians do not always agree on the definition of a concept.

Parallelogram—A quadrilateral in which **each pair** of opposite sides is parallel. The opposite sides of parallelograms are also congruent (same length).

Rectangle—A parallelogram that has four right angles.

Rhombus—A parallelogram that has four congruent sides.

Square—A parallelogram that has four congruent sides and four right angles.

Students can be asked to examine how the different definitions affect their solutions to problems involving trapezoids. Even though the learning experiences focus on quadrilaterals that have parallel sides, some of the activities include quadrilaterals that have no parallel sides. This has been done to avoid giving students a flawed concept of a quadrilateral.

Mathematical Language

Congruent
Polygon
Parallel
Parallelogram
Perpendicular
Quadrilateral
Rectangle
Rhombus (Rhombuses or Rhombi)
Set
Side
Square
Square corner
Trapezoid
Vertex (Vertices)

Learning Experiences

Assessing Prior Knowledge

Materials: Concept description sheet (BLM 5–8.2).

Organization: Individual/Whole class

Procedure:

a) Tell students that they will be learning about a family of shapes called quadrilaterals, but before they begin you need to find out what they already know about this shape.

b) Have students complete the concept description sheet. Let students know that it is all right if they cannot think of anything to put in a section. They will have another opportunity to complete the sheet after they have learned more about the shape.

c) When students complete the sheet, begin a discussion of their responses by asking, "What is a quadrilateral? What does it look like?" As the discussion progresses, make sure students see a variety of examples and non-examples. In particular, students should see examples of quadrilaterals that have no parallel sides

Observation Checklist

When the discussion ends, have students add to the concept description sheet to determine whether they can do the following:

- ❑ recognize that a quadrilateral is a four-sided polygon
- ❑ give appropriate examples and non-examples of quadrilaterals

- **Identify and describe the characteristics of a pre-sorted set of quadrilaterals.**

Materials: Five envelopes (one labelled trapezoid, one labelled square, one labelled rectangle, one labelled parallelogram, and one labelled rhombus), three different cut-outs of each quadrilateral, one large sheet of paper, and one marker for each group

Organization: Small group/Large group

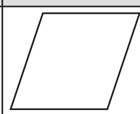
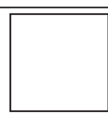
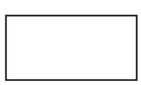
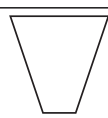
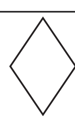
Procedure:

a) Place the cut-outs into the appropriate envelopes and then divide the class into five groups. Give each group a large sheet of paper, an envelope, and a marker.

b) Tell students that each group has a different type of quadrilateral and that their task is to teach the other groups about their quadrilateral. To do this, they need to look at the examples of the quadrilateral in their envelopes and determine its characteristics. Let students know that they should pay particular attention to the sides and vertices of their quadrilateral. Tell students that they should record the name of their quadrilateral and its characteristics on the large sheet of paper.

c) Have each group post their sheet of paper in the front of the room and tell the other members of the class about their quadrilateral. Help students identify any characteristics they might have missed.

d) Have students make a graphic organizer to help them learn the characteristics of the different quadrilaterals and their relationship to each other (e.g., students can make a chart like the one shown below).

Quadrilateral	Diagram	At least one pair of parallel sides	Two pairs of parallel sides	All sides congruent	Opposite sides congruent	All square corners
Parallelogram						
Square						
Rectangle						
Trapezoid						
Rhombus						

Observation Checklist

❑ Observe students' responses to make sure that for each quadrilateral they have correctly identified the characteristics shown in the chart.

Materials: Stir sticks or straws, and scissors

Organization: Whole class

Procedure:

a) Have the students use the stir sticks to make quadrilaterals
   - whose opposite sides are congruent
   - that is not a rhombus
   - that has at least one pair of parallel sides
   - that has no parallel sides
   - that has four square corners
   - that is neither a square nor a rectangle and has two pairs of parallel sides
   - that has one square corner

b) When students finish making each shape, ask them:
   - "What shape did you make?"
   - "How do you know that it is a quadrilateral?"
   - "Is there another shape that you could have made? What is it? How does it differ from the shape you made?"
   - "What other characteristics does your shape have?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

❑ construct and identify a quadrilateral with the given characteristic(s)

❑ describe the characteristics of each quadrilateral that they make

❑ identify other quadrilaterals that have the same characteristics as the one that was given

❑ describe how squares, rectangles, parallelograms, trapezoids, and rhombuses differ from each other

❑ recognize that there are other quadrilaterals besides squares, rectangles, trapezoids, parallelograms, and rhombuses

Materials: Tangrams

Organization: Individual

Procedure:

a) Have the students use the tangram pieces to make two different

- rectangles
- parallelograms
- trapezoids
- squares
- rhombuses

Let students know that they can use two or more of the tangram pieces to make each shape.

b) Have students place each shape they make on a piece of paper and trace around it. Ask them to write the name of the shape underneath their drawing and write a sentence stating which tangram pieces they used to make the shape.

Observation Checklist

Check students' work to see whether they can do the following:

❏ make two different rectangles, parallelograms, trapezoids, and squares

❏ correctly identify each quadrilateral that they made and the tangram pieces that they used to make it

❏ spell the names of the quadrilaterals correctly

Materials: Quadrilateral cards (BLM 5.SS.6.1)

Organization: Whole group

Procedure:

a) Give each student a card with a name of a quadrilateral on it.

b) Tell students that they are going to play a game called "Name that Quadrilateral." Explain that you will be describing a characteristic of a quadrilateral. Students who have a card with the name of a quadrilateral with that characteristic on it should stand up and show their card to the rest of the class (e.g., if I say, "I am a quadrilateral whose sides are all congruent, then students who have "square" or "rhombus" written on their card should stand up. The rest of the class checks to see whether students have identified the right quadrilaterals.).

c) Vary the game by selecting five students to be panel members. Give each panel member a card with a name of a quadrilateral on it and tell him or her to keep it hidden from the rest of the class. Tell students that everyone will have a chance to ask a panel member a question about his or her quadrilateral and the only question students can't ask is: "What is your quadrilateral?" The game is over when every student has had an opportunity to ask a question. The person who correctly identifies the quadrilateral on each panel member's card is the winner.

Observation Checklist

❑ Observe students' responses to determine whether they identify and describe the characteristics of the different quadrilaterals as shown in the chart.

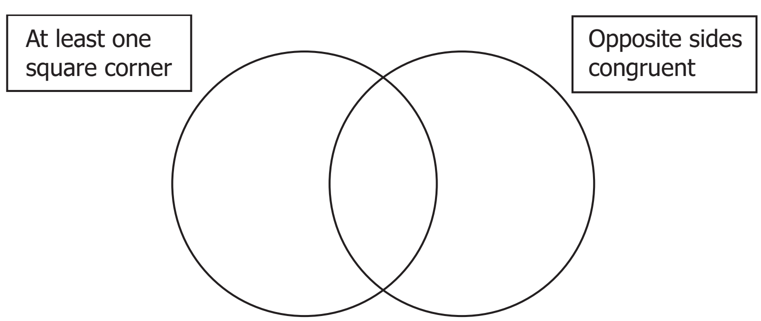
- **Sort a set of quadrilaterals and explain the sorting rule.**
- **Sort a set of quadrilaterals according to the lengths of the sides.**
- **Sort a set of quadrilaterals according to whether or not opposite sides are parallel.**

Materials: A set of quadrilateral cards and a set of rule cards for each pair of students (BLM 5.SS.6.2); two loops of string or yarn for each pair of students

Organization: Pairs

Procedure:

a) Have the students lay the string loops and the label cards "at least one square corner" and "opposite sides congruent" on their workspace, as shown below.



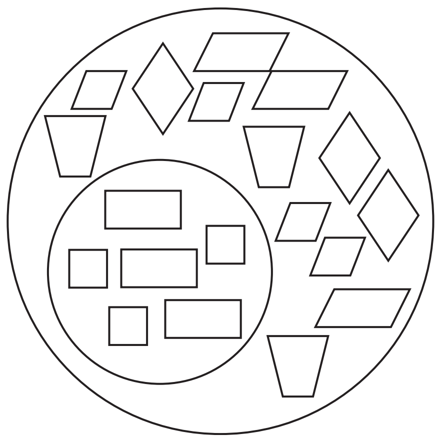
b) Have students sort their quadrilaterals into the appropriate sets. When students finish sorting the shapes, have them describe their solutions and explain how they knew where to place each quadrilateral.

c) Repeat the activity. Have the students sort the quadrilaterals into sets with

- at least one pair of parallel sides/ all sides congruent
- all square corners/two pairs of parallel sides
- no parallel sides/at least one pair of parallel sides

Have students make up their own rules for sorting the quadrilaterals.

d) Vary the activity by showing students pre-sorted sets and asking them to describe the rules that were used to sort the quadrilaterals. For example, show students the following set and ask them to identify the rule that was used to sort the quadrilaterals.



Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ sort the set of quadrilaterals according to the stated rule
- ❑ explain how they knew where each quadrilateral belonged
- ❑ recognize the relationships among the quadrilaterals, such as
  - all squares are rectangles
  - all parallelograms are trapezoids, using the definition of trapezoids as quadrilaterals with **at least one pair** of parallel sides
  - all rectangles, squares, and rhombuses are parallelograms
  - all squares are rhombuses

- **Sort a set of quadrilaterals and explain the sorting rule.**

Materials: Quadrilateral activity sheet (BLM 5.SS.6.3)

Organization: Whole class/Individual

Procedure:

a) Ask students to complete the activity.

b) Have students share their responses with the other members of the class.

Observation Checklist

Check students' responses to the questions to determine whether they can do the following:

- ❑ recognize the characteristics of rectangles, squares, trapezoids, rhombuses, and parallelograms
- ❑ recognize the relationships among quadrilaterals, such as the following:
  - All squares are rectangles
  - All squares are rhombuses
  - All rectangles are parallelograms
  - All squares are parallelograms
  - All parallelograms, rectangles, squares, and rhombuses are trapezoids, using the definition of trapezoids as quadrilaterals with **at least one pair** of parallel sides
  - All rhombuses are parallelograms

A Parallel World

Purpose:

The purpose of this activity is to have students recognize real-world examples of lines and quadrilaterals. In particular, the investigation is designed to enable students to identify real-world examples of

- faces and edges of 3-D objects and sides of 2-D shapes that are examples of parallel, intersecting, perpendicular, vertical, and horizontal lines
- rectangles, squares, trapezoids, parallelograms, and rhombuses

In addition, the investigation is designed to enhance students' ability to

- communicate mathematically
- use technology
- connect mathematical concepts to each other and the real world

Materials/Resources:

- Digital camera or video camera*
- Computer

Organization: Large group/Small groups

Procedure:

a) Tell students that they will be creating a digital scrapbook (or a video recording). Explain that each group is responsible for taking pictures of examples of lines and quadrilaterals that they find either inside or outside of school. When they finish taking their pictures, they will create a digital scrapbook. Each picture in their scrapbook must include a description of the types of lines and quadrilaterals found in the picture.

b) Help students determine the guidelines they should follow when taking their pictures (e.g., students need to consider the amount of time needed to find and take the pictures, their conduct as they move within and outside the school, and the responsibilities of each group member).

c) Have students take their pictures and create their scrapbooks.

d) Have students choose a picture to present to the class. Ask them to explain why they chose the picture and where in the picture they see lines and quadrilaterals. Encourage them to identify the types of lines and quadrilaterals found in their picture.

* If digital cameras or computers are not available, have students find pictures of lines and quadrilaterals in magazines and newspapers.

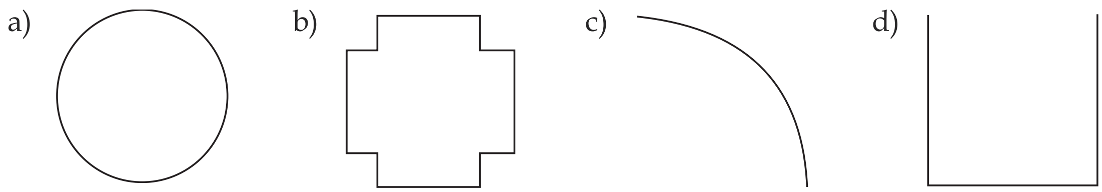
Observation Checklist

Use the following rubric to assess student mastery of learning outcomes *for* and *of* learning (during and at the completion of the activity).

3	2	1
Scrapbook includes: ▪ an example of each type of line ▪ an example of at least 3 different quadrilaterals	Scrapbook includes: ▪ an example of 3 or 4 types of lines ▪ an example of at least 2 different quadrilaterals	Scrapbook includes: ▪ examples of 1 or 2 types of lines ▪ an example of 1 type of quadrilateral
All lines are correctly identified.	Some lines are correctly identified.	A line is correctly identified.
All quadrilaterals are correctly identified.	Some quadrilaterals are correctly identified.	A quadrilateral is correctly identified.
Written description is clear. Mathematical terms are used correctly.	Written description is clear. Some mathematical terms are used correctly.	Written description is not clear. Some mathematical terms are used correctly.

BACKGROUND INFORMATION

A simple closed curve is a curve that does not cross itself and can be drawn by starting and stopping at the same point (e.g., in the diagram below, figures (a) and (b) are simple closed curves while (c) and (d) are curves that are not closed).



Polygons are simple closed curves formed by the union of line segments. In the diagram above, (b) is the only polygon since it is both a simple closed curve and made up of line segments. The line segments that form the polygon are the **sides** of the polygon. A point where two sides meet is a vertex of the polygon.

Polygons are classified according to the number of sides they have. The most common classifications are: **triangle** (three sides), **quadrilateral** (four sides), **pentagon** (five sides), **hexagon** (six sides), **heptagon** (7 sides), **octagon** (8 sides), **nonagon** (9 sides), **decagon** (10 sides), and **dodecagon** (12 sides). Other polygons are commonly referred to as *n*-gons, where *n* is the number of sides. For example, an eleven-sided polygon can be referred to as an 11-gon and a 14-sided polygon can be referred to as a 14-gon.

Quadrilaterals can be classified according to the number of parallel sides that they have. The definition of each type of quadrilateral is given below.

Trapezium—A quadrilateral with **no pairs** of parallel sides.

Trapezoid—A quadrilateral with **at least one pair** of parallel sides.

Parallelogram—A quadrilateral in which **each pair** of opposite sides is parallel. The opposite sides of parallelograms are also congruent (same length).

Rectangle—A parallelogram that has four right angles.

Rhombus—A parallelogram that has four congruent sides.

Square—A parallelogram that has four congruent sides and four right angles.

Some texts define a trapezoid as a quadrilateral with exactly one pair of parallel sides. If the support material you are using defines a quadrilateral in this way, students should be shown both definitions. This can help them understand that mathematics is not a rigid subject and that mathematicians do not always agree on the definition of a concept. Students can also be asked to examine how the different definitions affect their solutions to problems involving trapezoids. Moreover, even though the learning experiences focus on quadrilaterals that have parallel sides, some of the activities include quadrilaterals that have no parallel sides. This has been done to avoid giving students a flawed concept of a quadrilateral.

Mathematical Language

Congruent	Rhombus (Rhombuses or Rhombi)
Polygon	Set
Parallel	Side
Parallelogram	Square
Perpendicular	Square corner
Quadrilateral	Trapezoid
Rectangle	Vertex (Vertices)

Learning Experiences

Assessing Prior Knowledge

Materials: Concept description sheet (BLM 5–8.2).

Organization: Individual/Whole class

Procedure:

a) Tell students that they will be learning about a family of shapes called quadrilaterals, but before they begin you need to find out what they already know about this shape.

b) Have students complete the concept description sheet. Let students know that it is all right if they cannot think of anything to put in a section. They will have another opportunity to complete the sheet after they have learned more about the shape.

c) When students complete the sheet, begin a discussion of their responses by asking, "What is a quadrilateral? What does it look like?" As the discussion progresses, make sure students see a variety of examples and non-examples. In particular, students should see examples of quadrilaterals that have no parallel sides

Observation Checklist

When the discussion ends, have students add to the concept description sheet to determine whether they can do the following:

- ❏ recognize that a quadrilateral is a four-sided polygon
- ❏ give appropriate examples and non-examples of quadrilaterals

■ **Identify and describe the characteristics of a pre-sorted set of quadrilaterals.**

Materials: Five envelopes (one labelled trapezoid, one labelled square, one labelled rectangle, one labelled parallelogram, and one labelled rhombus), three different cut-outs of each quadrilateral, one large sheet of paper, and one marker for each group

Organization: Small group/Large group

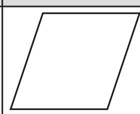
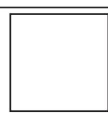
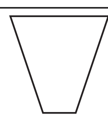
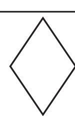
Procedure:

a) Place the cut-outs into the appropriate envelopes and then divide the class into five groups. Give each group a large sheet of paper, an envelope, and a marker.

b) Tell students that each group has a different type of quadrilateral and that their task is to teach the other groups about their quadrilateral. To do this, they need to look at the examples of the quadrilateral in their envelopes and determine its characteristics. Let students know that they should pay particular attention to the sides and "corners" of their quadrilateral. Tell students that they should record the name of their quadrilateral and its characteristics on the large sheet of paper.

c) Have each group post their sheet of paper in the front of the room and tell the other members of the class about their quadrilateral. Help students identify any characteristics they might have missed.

d) Have students make a graphic organizer to help them learn the characteristics of the different quadrilaterals and their relationship to each other (e.g., students can make a chart like the one shown below).

Quadrilateral	Diagram	At least one pair of parallel sides	Two pairs of parallel sides	All sides congruent	Opposite sides congruent	All square corners
Parallelogram						
Square						
Rectangle						
Trapezoid						
Rhombus						

Observation Checklist

❏ Observe students' responses to make sure that for each quadrilateral they have correctly identified the characteristics shown in the chart.

Materials: Stir sticks or straws

Organization: Whole class

Procedure:

a) Have the students use the stir sticks to make a quadrilateral

- whose opposite sides are congruent
- that is not a rhombus
- that has at least one pair of parallel sides
- that has no parallel sides
- that has four square corners
- that is neither a square nor a rectangle and has two pairs of parallel sides
- that has one square corner

b) When students finish making each shape, ask them:

- "What shape did you make?"
- "How do you know that it is a quadrilateral?"
- "Is there another shape that you could have made? What is it? How does it differ from the shape you made?"
- "What other characteristics does your shape have?"

Observation Checklist

Observe students' responses to determine whether they can do the following:

❏ construct and identify a quadrilateral with the given characteristic(s)

❏ describe the characteristics of each quadrilateral that they make

❏ identify other quadrilaterals that have the same characteristics as the one that was given

❏ describe how squares, rectangles, parallelograms, trapezoids, and rhombuses differ from each other

❏ recognize that there are other quadrilaterals besides squares, rectangles, trapezoids, parallelograms, and rhombuses

Materials: Tangrams

Organization: Individual

Procedure:

a) Have the students use the tangram pieces to make two different

- rectangles
- parallelograms
- trapezoids
- squares
- rhombuses

Let students know that they can use two or more of the tangram pieces to make each shape.

b) Have students place each shape they make on a piece of paper and trace around it. Ask them to write the name of the shape underneath their drawing and write a sentence stating which tangram pieces they used to make the shape.

Observation Checklist

Check students' work to see whether they can do the following:

- ❏ make two different rectangles, parallelograms, trapezoids, and squares
- ❏ correctly identify each quadrilateral that they made and the tangram pieces that they used to make it
- ❏ spell the names of the quadrilaterals correctly

Materials: Quadrilateral cards (BLM 5.SS.6.1)

Organization: Whole group

Procedure:

a) Give each student a card with a name of a quadrilateral on it.

b) Tell students that they are going to play a game called "Name that Quadrilateral." Explain that you will be describing a characteristic of a quadrilateral. Students who have a card with the name of a quadrilateral with that characteristic on it should stand up and show their card to the rest of the class (e.g., if I say, "I am a quadrilateral whose sides are all congruent, then students who have "square" or "rhombus" written on their card should stand up. The rest of the class checks to see whether students have identified the right quadrilaterals.).

c) Vary the game by selecting five students to be panel members. Give each panel member a card with a name of a quadrilateral on it and tell him or her to keep it hidden from the rest of the class. Tell students that everyone will have a chance to ask a panel member a question about his or her quadrilateral and the only question students can't ask is: "What is your quadrilateral?" The game is over when every student has had an opportunity to ask a question. The person who correctly identifies the quadrilateral on each panel member's card is the winner.

Observation Checklist

❑ Observe students' responses to determine whether they identify and describe the characteristics of the different quadrilaterals as shown in the chart.

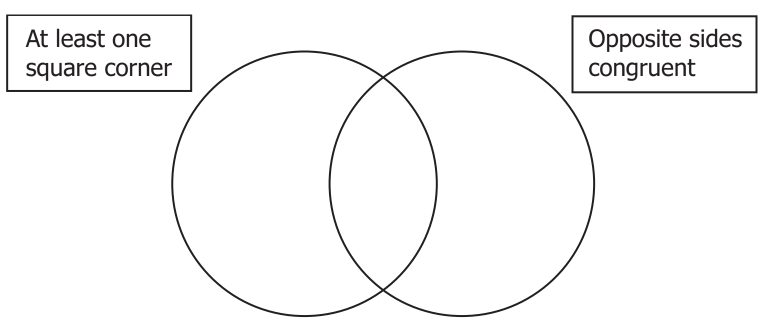
- **Sort a set of quadrilaterals and explain the sorting rule.**
- **Sort a set of quadrilaterals according to the lengths of the sides.**
- **Sort a set of quadrilaterals according to whether or not opposite sides are parallel.**

Materials: A set of quadrilateral cards and a set of rule cards for each pair of students (BLM 5.SS.6.2); two loops of string or yarn for each pair of students

Organization: Pairs

Procedure:

a) Have the students lay the string loops and the label cards "at least one square corner" and "opposite sides congruent" on their workspace, as shown below.



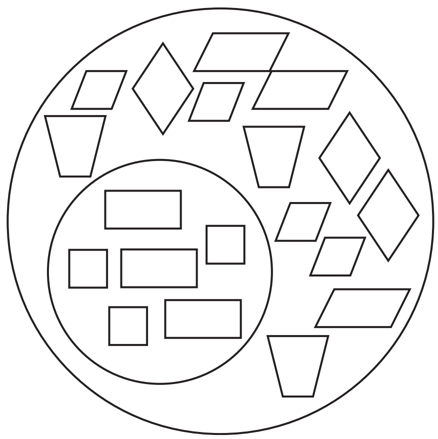
b) Have students sort their quadrilaterals into the appropriate sets. When students finish sorting the shapes, have them describe their solutions and explain how they knew where to place each quadrilateral.

c) Repeat the activity. Have the students sort the quadrilaterals into sets with

- at least one pair of parallel sides/ all sides congruent
- all square corners/two pairs of parallel sides
- no parallel sides/at least one pair of parallel sides

Have students make up their own rules for sorting the quadrilaterals.

d) Vary the activity by showing students pre-sorted sets and asking them to describe the rules that were used to sort the quadrilaterals. For example, show students the following set and ask them to identify the rule that was used to sort the quadrilaterals.



Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ sort the set of quadrilaterals according to the stated rule
- ❑ explain how they knew where each quadrilateral belonged
- ❑ recognize the relationships among the quadrilaterals, such as
  - all rectangles are squares
  - all parallelograms are trapezoids
  - all rectangles, squares, and rhombuses are parallelograms
  - all squares are rhombuses

- Sort a set of quadrilaterals and explain the sorting rule.

Materials: Quadrilateral activity sheet (BLM 5.SS.6.3)

Organization: Whole class/Individual

Procedure:

a) Ask students to complete the activity.

b) Have students share their responses with the other members of the class.

Observation Checklist

Check students' responses to the questions to determine whether they can do the following:

- ❏ recognize the characteristics of rectangles, squares, trapezoids, rhombuses, and parallelograms
- ❏ recognize the relationships among quadrilaterals, such as the following:
  - All squares are rectangles
  - All squares are rhombuses
  - All rectangles are parallelograms
  - All squares are parallelograms
  - All parallelograms, rectangles, squares, and rhombuses are trapezoids
  - All rhombuses are parallelograms

A Parallel World

Purpose:

The purpose of this activity is to have students recognize real-world examples of lines and quadrilaterals. In particular, the investigation is designed to enable students to identify real-world examples of

- faces and edges of 3-D objects and sides of 2-D shapes that are examples of parallel, intersecting, perpendicular, vertical, and horizontal lines
- rectangles, squares, trapezoids, parallelograms, and rhombuses

In addition, the investigation is designed to enhance students' ability to

- communicate mathematically
- use technology
- connect mathematical concepts to each other and the real world

Materials/Resources:

- Digital camera or video camera*
- Computer

Organization: Large group/Small groups

Procedure:

a) Tell students that they will be creating a digital scrapbook (or a video recording). Explain that each group is responsible for taking pictures of examples of lines and quadrilaterals that they find either inside or outside of school. When they finish taking their pictures, they will create a digital scrapbook. Each picture in their scrapbook must include a description of the types of lines and quadrilaterals found in the picture.

b) Help students determine the guidelines they should follow when taking their pictures (e.g., students need to consider the amount of time needed to find and take the pictures, their conduct as they move within and outside the school, and the responsibilities of each group member).

c) Have students take their pictures and create their scrapbooks.

d) Have students choose a picture to present to the class. Ask them to explain why they chose the picture and where in the picture they see lines and quadrilaterals. Encourage them to identify the types of lines and quadrilaterals found in their picture.

* If digital cameras or computers are not available, have students find pictures of lines and quadrilaterals in magazines and newspapers.

Observation Checklist

Use the following rubric to assess student mastery of learning outcomes *for* and *of* learning (during and at the completion of the activity).

3	2	1
Scrapbook includes: an example of each type of line; an example of at least 3 different quadrilaterals	Scrapbook includes: an example of 3 or 4 types of lines; an example of at least 2 different quadrilaterals	Scrapbook includes: examples of 1 or 2 types of lines; an example of 1 type of quadrilateral
All lines are correctly identified.	Some lines are correctly identified.	Not all lines are correctly identified.
All quadrilaterals are correctly identified.	Some quadrilaterals are correctly identified.	Quadrilateral is incorrectly identified.
Written description is clear. Mathematical terms are used correctly.	Written description is clear. Some mathematical terms are used correctly.	Written description is not clear. Some mathematical terms are used correctly.

NOTES

Grade 5: Shape and Space (Transformations) (5.SS.7)

Enduring Understandings:

The position of shapes can be changed by translating, rotating, or reflecting them.

General Outcome:

Describe and analyze position and motion of objects and shapes.

Specific Learning Outcome(s):	Achievement Indicators:
5.SS.7 Perform a single transformation (translation, rotation, or reflection) of a 2-D shape and draw and describe the image. [C, CN, T, V]	➛ Translate a 2-D shape horizontally, vertically, or diagonally, and describe the position and orientation of the image. ➛ Rotate a 2-D shape about a point, and describe the position and orientation of the image. ➛ Reflect a 2-D shape in a line of reflection, and describe the position and orientation of the image. ➛ Perform a transformation of a 2-D shape by following instructions. ➛ Draw a 2-D shape, translate the shape, and record the translation by describing the direction and magnitude of the movement (e.g., the circle moved 3 cm to the left). ➛ Draw a 2-D shape, rotate the shape, and describe the direction of the turn (clockwise or counter-clockwise), the fraction of the turn, and point of rotation. ➛ Draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection. ➛ Predict the result of a single transformation of a 2-D shape and verify the prediction.
5.SS.8 Identify a single transformation (translation, rotation, or reflection) of 2-D shapes. [C, T, V]	➛ Provide an example of a translation, a rotation, and a reflection. ➛ Identify a single transformation as a translation, rotation, or reflection. ➛ Describe a rotation by the direction of the turn (clockwise or counter-clockwise).

Prior Knowledge

Students may have had experience with the following:

- Identifying triangles, quadrilaterals, pentagons, hexagons, octagons, and circles

Related Knowledge

Students should be introduced to the following:

- Identifying vertical and horizontal lines
- Identifying rectangles, squares, trapezoids, rhombuses, and parallelograms
- Measuring the lengths of lines to the nearest cm or mm
- Identifying equivalent fractions

Background Information

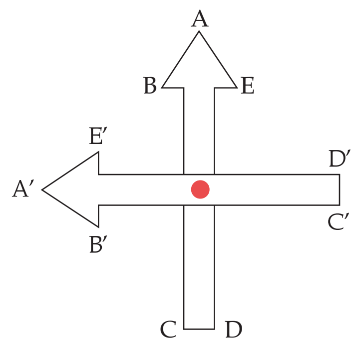
Transformations play an important role in the mathematics curriculum. In the Middle Years, the study of transformation can support students' work in patterning, algebra, problem solving, geometry, and statistics. In high school and beyond, the study of transformations helps students recognize the connections between algebra and geometry and enhances their understanding of other topics such as matrices, scaling, and complex numbers.

A **transformation** can be thought of as a change in the position, size, or shape of a figure. In the learning activities that follow, students are introduced to three transformations that change the position of a figure. Informally, these transformations are referred to as slides, flips, and turns. Formally, they are known as translations, reflections, and rotations. Students should know both the formal and informal terminology.

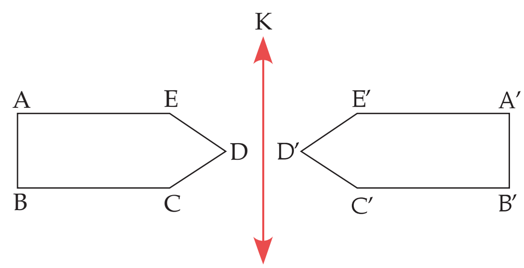
A **translation** "slides" a figure a fixed distance in a given direction. The figure and its translation are congruent (same size and shape) and face in the same direction. In the diagram shown below, square ABCD has been translated to a new position represented by square A′B′C′D′.

Note that Square A′B′C′D′, which is called the image of Square ABCD, is congruent to Square ABCD and faces in the same direction. The arrow indicates the distance and the direction of the translation.

A **rotation** “turns” a figure any number of degrees around a fixed point called the **centre of rotation**. The centre of rotation may be any point within or outside the figure. The figure and its **image** (the result of the transformation) are congruent but they may face in opposite directions (e.g., in the diagram below, the arrow ABCDE has been rotated 90° counter-clockwise about its midpoint). The image arrow A’B’C’D’E’ is congruent to Arrow ABCDE but faces in a different direction.



A **reflection** “flips” the figure over a line, creating a mirror image. The figure and its image are congruent but have different orientations. The line the figure is flipped over is called the **line of reflection** and it is the same distance from the figure as its image (e.g., in the diagram below, pentagon ABCDE has been flipped over line *k*).



Note that Pentagon A’B’C’D’E’ is congruent to Pentagon ABCDE but faces in the opposite direction. Line *k*, the line of reflection, is equidistant from the two pentagons.

Mathematical Language

Clockwise	Polygon
Counter-clockwise	Reflection (flip)
Congruent	Rotation (turn)
Diagonally	Transformation
Horizontal	Translation (slide)
Image	Vertical line
Line of reflection	

- **Translate a 2-D shape horizontally, vertically, or diagonally, and describe the position and orientation of the image.**
- **Rotate a 2-D shape about a point, and describe the position and orientation of the image.**
- **Reflect a 2-D shape in a line of reflection, and describe the position and orientation of the image.**
- **Perform a transformation of a 2-D shape by following instructions.**

Materials: Carpeted area or floor mats

Organization: Whole class

Procedure:

a) Have students lie down on a carpet or mat. Ask them to slide a short distance in one direction. Have them repeat the movement several times by asking them to slide up, down, and sideways. After each slide, ask, "What changed? What remained the same?" Emphasize that when a slide is made, the direction in which an object is pointing does not change.

b) Have students demonstrate flips. At the end of a flip, students should have changed from stomach to back or back to stomach. Discuss the different ways flips can be completed. For example, students may roll to the left or to the right, or over the feet or head. Have students try these different ways. After each flip, ask, "What changed? What remained the same?" Emphasize that when an object is flipped, its orientation changes. Ask students how this is different from looking at their reflection in the mirror. Emphasize that a true reflection of oneself would have exactly the same image, just in a different orientation.

c) Have students demonstrate turns. To perform a turn, students must keep either their feet or their heads (or belly button) at the same location for the duration of the turn. If the feet are the point (centre) of rotation, then the arms and head are used to move the body. If the head is the point (centre) of rotation, the feet are used to make the move. Have students turn all the way around or partway around. Have them turn in either a clockwise or counter-clockwise direction. After each turn, ask students, "What changed? What remained the same?" Discuss the fact that after a turn, the direction in which the head points is different, except when a complete turn is made.

d) Inform students that in the next few lessons they will be learning more about slides, flips, and turns.

Observation Checklist

- ❏ Observe students' responses to determine whether they are able to perform a slide, flip, and a turn.

- **Translate a 2-D shape horizontally, vertically, or diagonally, and describe the position and orientation of the image.**
- **Perform a transformation of a 2-D shape by following instructions.**

Materials: Grid paper and an overhead transparency of a grid

Organization: Whole class/Partners

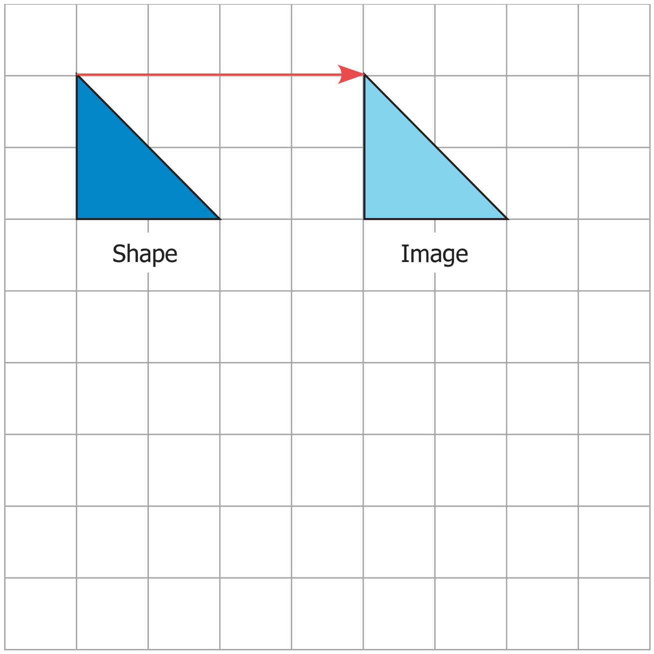
Procedure:

a) Remind students of the first activity by asking them to describe a slide. Discuss other objects in the environment that slide, such as drawers, sliding doors, and swings. Explain that to perform a slide, we need to know the distance and direction of the move.

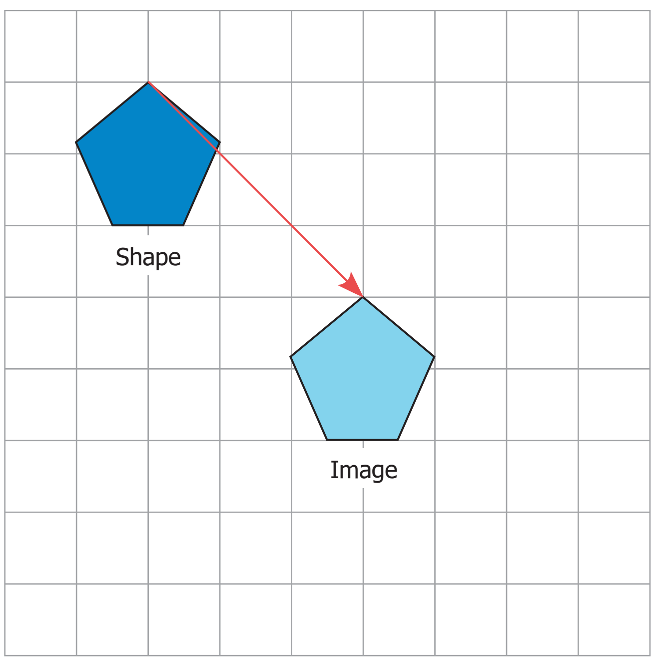
b) Tell students that they are going to perform a number of slides. Have them stand in a large open area or take them to the gym. Ask students to slide: one step to the right; one step up and two steps to the left; and three steps back and two steps to the right. After each slide, ask students, "What changed? What remained the same?"

c) Turn the activity into a game by playing "Simon Says." Tell students that if they move the wrong way or slide when Simon doesn't tell them to, they must take their seats. The last person standing is the winner.

d) Explain that another name for a slide is a translation. Draw a triangle on the overhead grid and show it to students. Tell students that to translate or slide the triangle, we need to know the distance and the direction of the move. Draw an arrow and explain that the arrow indicates the direction of the translation and its length describes the distance. Draw the image of the shape and explain that the original triangle has been translated horizontally four units to the right. Explain that the translated shape is called the image of the original shape. Ask, "How are the shape and its image alike? How do they differ?"



e) Do two or three more examples. Make sure you include an example where the shape is translated diagonally (e.g., in the figure shown below, the pentagon has been translated diagonally [on a slant] three units down and three units across).



f) Ask students to draw a shape on their grid paper and a slide arrow. Have them exchange papers with their partner. The partner must translate the shape according to the direction and length of the arrow. Have students repeat the activity several times.

g) Vary the activity by having students draw a shape and its image on grid paper, but not the slide arrow. Have them exchange papers with their partner. The partner must draw the slide arrow that corresponds to the translation. Have students repeat the activity several times.

Observation Checklist

Observe students' work to determine whether they can do the following:

❑ provide real-world examples of slides

❑ perform a transformation by following instructions

❑ translate a 2-D shape horizontally, vertically, and diagonally, and describe its position and orientation

❑ determine the direction and distance of a translation

❑ recognize that a shape and its image are congruent and face in the same direction

❑ draw a 2-D shape and translate the shape, and describe the distance and magnitude of the translation

- **Translate a 2-D shape horizontally, vertically, or diagonally, and describe the position and orientation of the image.**
- **Reflect a 2-D shape in a line of reflection, and describe the position and orientation of the image.**

Materials: Paint, paper, and black markers or crayons

Organization: Whole class/Individual

Procedure:

a) Remind students of the first activity by asking them to describe a flip. Explain that a flip is a mirror image. Tell students that they will be doing an activity that involves creating mirror images with their partners.

b) Have students stand up and face their partner. Ask students to: raise their right arm above their heads; bend their knees as if they were sitting; turn around and put their backs together; hold both arms straight out; and hop once on their left foot. Let one of the students be the leader and, without talking, make motions for the other students to follow. Then let another student be the leader.

c) Engage students in a discussion about their movements by asking them to describe how their movements were similar to and different from their partners. Explain that a mirror image or a flip is also called a reflection. Have students describe real-world occurrences of reflections, such as seeing one's reflection in a pool of water.

d) Give students a piece of paper and ask them to fold it in half vertically. Have students put dabs of different coloured paint on one side of the paper. Before the paint dries, have students fold the other half of the paper over the painted side and smooth it out. Have students open the picture to see the design.

e) Tell students that the fold in their paper is a line of reflection because it acts like a mirror. Have students draw a line segment along the fold line with a black marker or crayon and label the segment "reflection line."

f) Have students show their designs and their reflection to the entire class. Encourage students to describe why their pictures illustrate reflections. Display the designs in the classroom.

Observation Checklist

Observe students to determine whether they can do the following:

- ❑ provide real-world examples of reflections
- ❑ perform a transformation by following instructions
- ❑ reflect a 2-D shape over a line of reflection and describe its position and orientation

- **Reflect a 2-D shape in a line of reflection, and describe the position and orientation of the image.**
- **Perform a transformation of a 2-D shape by following instructions.**
- **Draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection.**

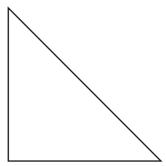
Materials: Miras or reflective plastic, copies of the activity sheet (BLM 5.SS.7&8.1), and rulers

Organization: Individual/Pairs

Procedure:

a) Show students how to use a Mira. Explain that the top and bottom of the Mira are different. The bottom of the Mira has a beveled drawing edge. When using the Mira to draw a figure, the beveled edge should always be facing the drawer.

b) Have students place a Mira on a piece of paper and draw a line along the drawing edge. Next, have students draw a triangle like the one shown below on the beveled side of the Mira. Have them look through the Mira and ask, “Do you see a reflection of the triangle you just drew?” Now have students reach around the Mira and draw the image of the triangle. Have students remove the Mira. Ask, “Does your drawing show a reflection? How do you know?”



c) Help students recognize that an object and its image are the same distance from the line of reflection by asking students to measure the distances from the vertices of the triangle (and its image) to the line of the reflection.

d) Have students repeat parts (b) and (c) several times using different shapes.

e) Give students a copy of the activity sheet (BLM 5.SS.7&8.1) and tell them to use their Miras to draw the line of reflection for each shape. After they draw the line of reflection, they should look through the Mira and draw the image of the shape.

f) Have students discuss the reflections. Encourage students to describe the distance the shape and its image are from the line of reflection and how the orientation of the image differs from the shape’s orientation.

Observation Checklist

Monitor students’ responses to determine whether they can do the following:

- ❑ draw a 2-D shape, reflect the shape, and identify the line of reflection and its distance from the shape and its image
- ❑ describe the orientation of the image
- ❑ recognize that the shape and its image are congruent
- ❑ perform a translation by following instructions

- **Reflect a 2-D shape in a line of reflection, and describe the position and orientation of the image.**
- **Perform a transformation of a 2-D shape by following instructions.**

Materials: Centimetre grid paper (BLM 5–8.9), Cuisenaire rods, mirrors, and black markers

Organization: Pairs or groups of four (if groups of four are used, divide each group into two teams of two)

Procedure:

a) Ask students to draw a black line horizontally across the middle of the grid paper. Tell them that the black line is the line of reflection.

b) Have one student in each pair arrange the Cuisenaire rods on the grid paper in some way on one side of the black line.

c) Have the other student in each pair build the reflection of the arrangement on the other side of the black line, and then use a Mira to check whether his or her arrangement is correct.

d) Have students continuing creating reflections, changing who "builds" and who "reflects."

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ recognize that an object and its reflection are the same distance from the line of reflection
- ❏ recognize that the orientation of the image is different from the orientation of the original shape
- ❏ recognize that the shape and its image are congruent
- ❏ translate the shape over a line of reflection, and describe its position and orientation

- **Perform a transformation of a 2-D shape by following instructions.**

Materials: None

Organization: Whole class

Procedure:

a) Remind students of the first activity by asking them to describe a turn. Encourage students to identify objects in their environment that turn, such as doorknobs, tires, and the hands on analog clocks. Explain that to turn a shape, we need to know how far and in what direction to turn it.

b) Have students stand up and face the front of the room. Have them turn around slowly until they see the front of the room again. Explain that they just made a full turn.

c) Ask students what they think they will see if they make a half-turn. Ask, “Will you be able to see me? What part of you will I be able to see?” Have students make a half-turn.

d) Ask students what they will see if they make a quarter-turn. Explain that what they see depends on the direction of their turn (e.g., if they turn counter-clockwise, they might see windows, and if they turn clockwise, they might see a bulletin board). Have students make: a quarter-turn clockwise; a three-quarter-turn counter-clockwise; a quarter-turn counter-clockwise, and a three-quarter-turn clockwise. Ask questions such as, “What are two ¼-turns clockwise equivalent to? What are three quarter-turns counter-clockwise equivalent to?” Have students perform the different turns to check their responses.

e) Have students practice making full-, quarter-, three-quarter-, and half-turns. Turn the activity into a game by playing “Simon Says.” Tell students that if they turn the wrong way or turn when Simon doesn’t tell them to, they must take their seats. The last person standing is the winner.

Observation Checklist

Observe students to determine whether they can do the following:

- ❑ understand the meaning of the terms clockwise and counter-clockwise
- ❑ make quarter-, half-, three-quarter-, and full turns in a clockwise direction
- ❑ make quarter-, half-, three-quarter-, and full turns in a counter-clockwise direction
- ❑ perform a transformation by following directions

- **Rotate a 2-D shape about a point, and describe the position and orientation of the image.**

Materials: Scissors, paper and pencil

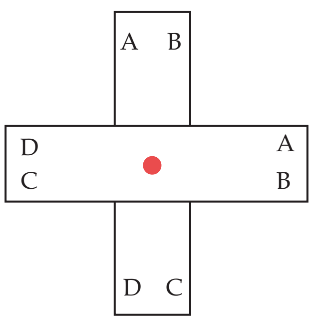
Organization: Individual

Procedure:

a) Ask students to draw a rectangle on a piece of paper. Have students label the vertices of their rectangle A, B, C, and D. Have them cut out another rectangle that is the same size as the rectangle that they drew on their paper, and label the vertices A, B, C, and D. Have students place the cut-out on their drawing so that the vertices of the two rectangles match.

b) Tell students another name for a turn is a rotation. Explain that to rotate the shape, we not only need to know the direction and how far to turn the shape, we also need to know the point (centre) of rotation. Any point on or off the shape can be used as the point (centre) of rotation.

c) Have students place the tip of a pencil on the centre of their rectangles and then have them rotate their rectangles a quarter-turn clockwise. Have students describe the position and orientation of the image.



Ask students to rotate their rectangles: ½-turn counter-clockwise; a ¾-turn clockwise, ¼-turn counter-clockwise; a ¾-turn counter-clockwise; and a ½-turn clockwise. After each rotation, have students describe the position and orientation of the rectangle. Have students discuss relationships, such as two ¼-turns is the same as a ½-turn; three ¼-turns is the same as a ¾-turn or a ½-turn plus a ¼-turn; and four ¼-turns is the same as a full turn.

d) Vary the activity by changing the point of rotation (e.g., let one of the vertices be the point of rotation, or select a point that is not on the rectangle as the point of rotation).

e) Repeat the activity using different shapes.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❏ understand the terms *clockwise* and *counter-clockwise*
- ❏ rotate a 2-D shape ¼-turn, ½-turn, ¾-turn, and a full turn, both clockwise and counter-clockwise, around any point in the interior or on the shape
- ❏ rotate a 2-D shape ¼-turn, ½- turn, ¾-turn, or a full turn from a point in the exterior of the shape
- ❏ describe the position and orientation of a 2-D shape after it has been rotated
- ❏ recognize that the shape and its image are congruent

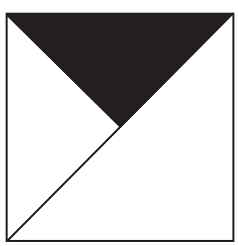
- **Rotate a 2-D shape about a point, and describe the position and orientation of the image.**
- **Perform a transformation of a 2-D shape by following instructions.**

Materials: Math journals

Organization: Individual

Procedure:

a) Have students draw in their math journals a square like the one shown below.



b) Ask students to draw a picture of how the square would look after it has been rotated around its centre point
   - ¼-turn clockwise
   - ½-turn clockwise
   - ¾-turn clockwise

c) Ask students to write a paragraph describing the position and orientation of the image after each rotation.

Observation Checklist

Check to determine whether students:

- ❏ know the term *clockwise*
- ❏ can perform a transformation by following instructions
- ❏ can rotate a 2-D shape ¼-turn, ½-turn, and ¾-turn
- ❏ can describe the direction and the orientation of the image of a rotation

- **Translate a 2-D shape horizontally, vertically, or diagonally, and describe the position and orientation of the image.**
- **Rotate a 2-D shape about a point, and describe the position and orientation of the image.**
- **Reflect a 2-D shape in a line of reflection, and describe the position and orientation of the image.**

Materials: Pattern blocks, math journals

Organization: Pairs/Individual

Procedure:

a) Present students with the following situation:

- Michaela wants to use the same pattern block to show her cousin how to rotate, translate, and reflect a shape. She is not sure which pattern block she should use to show her cousin all three moves.

b) Tell students it is their job to help Michaela decide which block she should use. Explain that if a block looks the same after it has been transformed, it is not a good model. Help them understand what you mean by this by demonstrating why a circle is not a good model for illustrating a reflection.

c) Have students translate, rotate, and reflect each pattern block. Encourage them to record their findings in a table like the one shown below:

Shape	Translation	Rotation	Reflection
Orange Square			
Blue Rhombus			
Red Trapezoid			
Yellow Hexagon			
Green Triangle			
Tan Rhombus			

d) Have students write a paragraph in the math journal explaining which shape Michaela should use and the reasons for their choice.

Observation Checklist

Check students' responses to determine whether they can do the following:

- ❑ translate a given 2-D shape and describe the position and orientation of the transformed shape
- ❑ rotate a given 2-D shape and describe the position and orientation of the transformed shape
- ❑ reflect a given 2-D shape and describe the position and orientation of the transformed shape

Materials: Geoboards, elastics, dot paper, a set of cards (BLM 5.SS.7&8.2) for each group

Organization: Groups of four

Procedure:

a) Tell students that they will be doing an activity involving translations, reflections, and rotations. Explain that each group will get four cards. The cards should be placed face down on the work area and each student should draw one. The student with the card that says "Original Position" makes a shape on his or her geoboard. The student who has the card stating "Rotation" creates a rotation of the original shape on his or her geoboard. The student who has the card stating "Reflection" creates a reflection of the original shape on his or her geoboard, and the one who has the card stating "Translation" creates a translation of the original shape on his or her geoboard. Students should record the original shape on their dot paper and the transformation that they performed. Underneath the transformation, they should describe how the shape was transformed (e.g., vertical translation two units up) and how the position and orientation of the shape changed.

b) Tell students that they have to find a way to verify that they have successfully translated, rotated, and reflected the original shape. When they have verified their transformations, they should exchange cards and do the activity again. Each student should have the opportunity to create, translate, rotate, and reflect a shape.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ translate a given shape horizontally, vertically, or diagonally, and describe the position and orientation of the image
- ❑ rotate a shape around a point and describe the position and orientation of the image
- ❑ reflect a shape across a line of reflection and describe the position and orientation of the image

- **Translate a 2-D shape horizontally, vertically, or diagonally, and describe the position and orientation of the image.**
- **Rotate a 2-D shape about a point, and describe the position and orientation of the image.**
- **Reflect a 2-D shape in a line of reflection, and describe the position and orientation of the image.**
- **Predict the result of a single transformation of a 2-D shape and verify the prediction.**
- **Identify a single transformation as a translation, rotation, or reflection.**
- **Describe a rotation by the direction of the turn (clockwise or counter-clockwise).**

Materials: A red trapezoid from the set of pattern blocks, a die, and math journals

Organization: Whole class

Procedure:

a) Tell students that they will be playing a game. Explain that you will be placing a trapezoid on the overhead and students will be taking turns rolling a die. After the die has been rolled, the overhead will be turned off and the student who rolled the die will transform the trapezoid according to the following rules:

   - If a 1 or 2 is rolled, the student will translate the trapezoid.
   - If a 3 or 4 is rolled, the student will reflect the trapezoid.
   - If a 5 or 6 is rolled, the student will rotate the trapezoid either clockwise or counter-clockwise.

   After the student has changed the position of the trapezoid, the overhead will be turned back on and the other members of the class must record in their math journals the transformation they think the student performed on the trapezoid.

b) Demonstrate the procedure for playing the game and answer any questions students may have.

c) Have students share their response to each round of the game. Encourage students to explain how the position and orientation of the trapezoid changed and why the transformation was a translation, rotation, or reflection.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ translate, rotate, and reflect a 2-D shape
- ❑ identify a transformation as a rotation, reflection, or translation
- ❑ describe a rotation by the direction of the turn (either clockwise or counter-clockwise)
- ❑ describe the position and orientation of a translated shape, a rotated shape, and a reflected shape

NOTES

Grade 5 Mathematics

Statistics and Probability

Grade 5: Statistics and Probability (Data Analysis) (5.SP.1, 5.SP.2)

Enduring Understandings:

Graphs are a way of organizing, representing, and communicating information.

General Outcome:

Collect, display, and analyze data to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
5.SP.1 Differentiate between first-hand and second-hand data. [C, R, T, V]	➢ Explain the difference between first-hand and second-hand data. ➢ Formulate a question that can best be answered using first-hand data and explain why. ➢ Formulate a question that can best be answered using second-hand data and explain why. ➢ Find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the Internet.
5.SP.2 Construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V]	➢ Determine the attributes (title, axes, intervals, and legend) of double bar graphs by comparing a set of double bar graphs. ➢ Represent a set of data by creating a double bar graph, label the title and axes, and create a legend with or without the use of technology. ➢ Draw conclusions from a given double bar graph to answer questions. ➢ Provide examples of double bar graphs used in a variety of print and electronic media, such as newspapers, magazines, and the Internet. ➢ Solve a problem by constructing and interpreting a double bar graph.

Prior Knowledge

Students may have had experience with the following:

- Collecting first-hand data and organizing it using tally marks, line plots, charts, and lists
- Constructing and interpreting bar graphs and pictographs involving many-to-one correspondence to draw conclusions
- Classifying objects or items into groups
- Making reasonable estimates of quantities
- Demonstrating an understanding of kilograms and grams
- Describing and representing decimals to hundredths

Related Knowledge

Students should be introduced to the following:

- Representing and describing whole numbers to 1 000 000

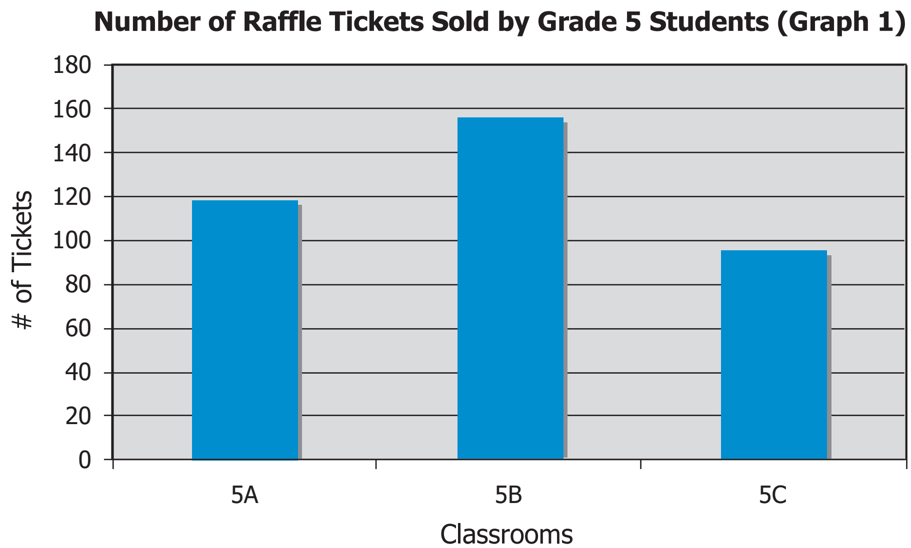
Background Information

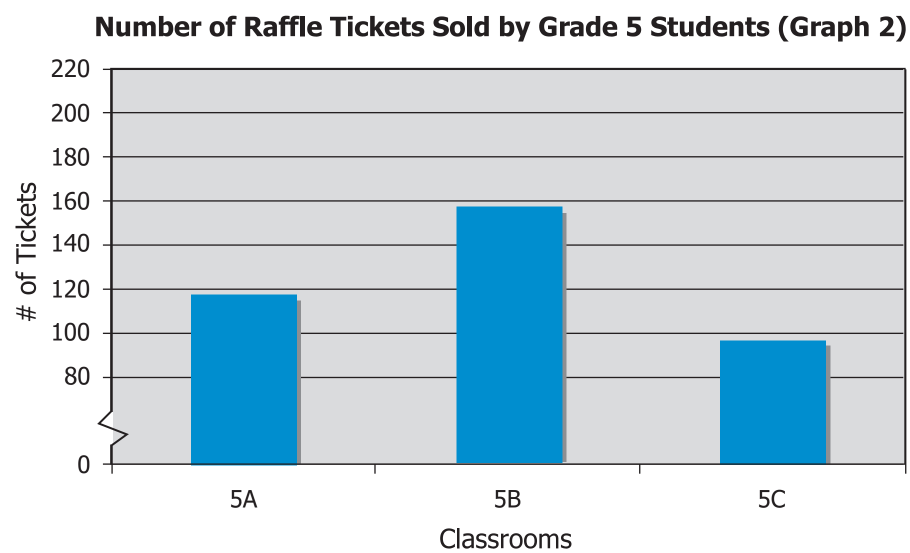
Graphs are visual displays of data that provide an overall picture of the information that has been collected. There are many types of graphs, the most common of which are bar graphs, line graphs, circle graphs, and pictographs. Before students can decide on which graph to use, they need to know what type of data have been collected. More specifically, the data that they collect can be either discrete or continuous. **Discrete data** involve observations that are separate and distinct and that can be counted. Shoe sizes, the number of students assigned to each classroom in a school, and the animals on a farm are all examples of discrete data. **Continuous data** involve observations that can take on many values within a finite or infinite interval. Height, temperature, and age are examples of continuous data. Bar graphs, circle graphs, and pictographs are used to illustrate discrete data. Line graphs are used to illustrate continuous data. In addition, students need to be aware of the advantages and disadvantages of using each type of graph. This will enable them to select an appropriate graph for their data and to defend their choices.

The data that students collect and display can be either first-hand or second-hand. **First-hand data** provide information that an individual obtains directly by asking questions, measuring, observing, or experimenting. Asking Grade 6 students what their favourite CD is, or observing the number of times the copy machine is used in a day, are examples of first-hand data. **Second-hand data** provide information that is readily available and only needs to be extracted from sources such as newspapers, electronic media, magazines, almanacs, and journals. Using a company's records to obtain information about the number of sales over a five-year period, and using census data to find information about Canadian households, are examples of second-hand data.

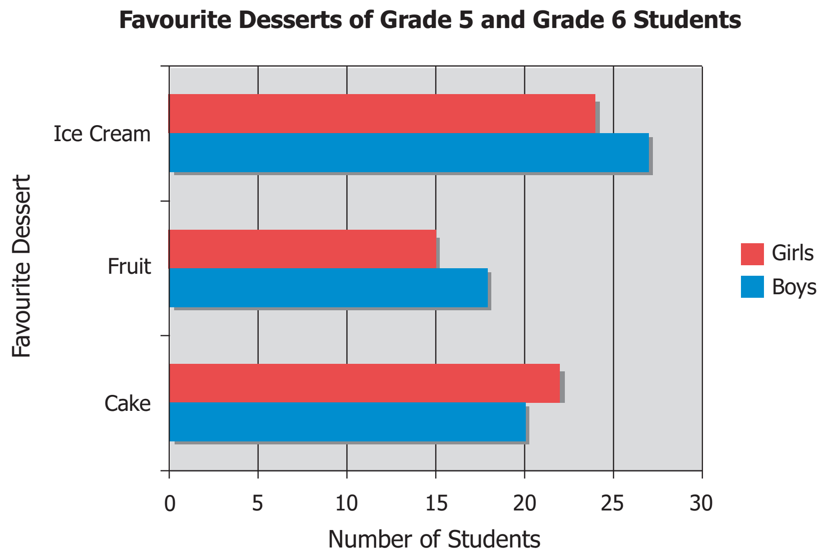
All graphs should have a title, labels, and neat and concise entry of data. When graphs show increments, those increments should start at 0, be of equal size, and be numbered. **The axes**—the horizontal and vertical line segments that divide the coordinate plane into quadrants—must also be labelled.

Bar graphs compare the frequency of discrete data. Data are displayed using a number of rectangles (bars) that are the same width. Each bar represents one of the categories that the data have been sorted into. The bars are displayed either horizontally or vertically with a space between them. The height (or length) of a bar represents the number of observations in that category. The numbers on the *y*-axis of a vertical bar graph or the *x*-axis of horizontal bar graph are called the scale. Sometimes a bar graph will have a squiggle in its scale. This means that part of the scale has been omitted. However, the use of a squiggle tends to give a misleading visual picture (see Graph 2 below).





Double bar graphs are used to make comparisons between and among sets of data (e.g., the double bar graph shown below compares the desserts favoured by boys with the desserts favoured by girls).



In addition to knowing the different types of graphs and how to construct them, students need to know how to read and interpret them. Curcio (2001) identifies three levels of graph comprehension: reading the data, reading between the data, and reading beyond the data.

Note: The Statistics Canada Website at <www.statcan.gc.ca> is an excellent source of second-hand data.

- **Reading the Data**

  This level of comprehension requires a literal reading of the graph. The reader simply "lifts" data explicitly stated in the graph, or the information found in the graph title and axes labels, from the graph. There is no interpretation at this level. Reading that requires this type of comprehension is a very low-level cognitive task.

- **Reading between the Data**

  This level of comprehension includes the interpretation and integration of the data in the graph. It requires the ability to compare quantities (e.g., greater than, tallest, smallest) and the use of other mathematical concepts and skills (e.g., addition, subtraction, multiplication, division) that allow the reader to combine and integrate data and identify the mathematical relationships expressed in the graph.

- **Reading beyond the Data**

  This level of comprehension requires the reader to predict or infer from the data by tapping existing schemata (i.e., background knowledge, knowledge in memory) for information that is neither explicitly nor implicitly stated in the graph. Whereas reading between the data might require the reader to make an inference that is based on the data presented in the graph, reading beyond the data requires that the inference be made on the basis of information in the reader's head, not in the graph.

Mathematical Language

Bar graph	Interval
Data	Legend
Double bar graph	Scale
Estimate	Second-hand data
First-hand data	Vertical axis
Horizontal axis	

Learning Experiences

Assessing Prior Knowledge

Materials: Graph paper, BLM 5.SP.1&2.1, markers or crayons, straight edge

Organization: Individual/Whole class

Procedure:

a) Have students complete the activity shown below (BLM 5.SP.1&2). Let them know that the purpose of the activity is to find out what they know about bar graphs.

   1. Louis is doing a project on animals. He found that animals have different heart rates (e.g., he found that a cow has a heart rate of 60 to 80 beats per minute). Here are some of the heart rates he found.

Cow	60 to 80	Cat	110 to 140
Horse	30 to 40	Chicken	300 to 350
Rabbit	140 to 160	Sheep	70 to 140

   Make a bar graph of the highest heart rate of each animal.

   2. Study your graph. List three conclusions you can draw from it.

   3. If you added the heart rate of a mouse to your graph, which animals do you think would have a slower heart rate than the mouse? Explain your answer.

b) Have students share their graph and conclusions with the rest of the class.

Observation Checklist

Use the following checklist to assess students' knowledge of bar graphs.

Makes a Bar Graph that Includes	Yes	No	Comments
A title			
Category labels			
A label for each axis			
Bars whose lengths correctly represent the number of observations in each category			
Bars the same width			
Spaces between bars			
An appropriate scale with equal increments that starts at zero			
An interpretation of bar graphs			
Reading between the data (Question 2)			
Reading beyond the data (Question 3)			

- **Explain the difference between first-hand and second-hand data.**
- **Formulate a question that can best be answered using first-hand data and explain why.**
- **Formulate a question that can best be answered using second-hand data and explain why.**
- **Represent a set of data by creating a double bar graph, label the title and axes, and create a legend with or without the use of technology.**

Materials: BLM 5.SP.1&2.2, two bar graphs, one illustrating first-hand data and one illustrating second-hand data, math journal

Organization: Pairs/Whole class

Procedure:

a) Tell students that in the next few lessons they will be doing some graphing. Explain that graphs are a quick and easy way to illustrate information. Ask students what a bar graph is and where they have seen one.

b) Show students the two bar graphs and have them discuss how they are alike and how they differ. Encourage students to think about how the information in the graphs was obtained.

c) Tell students that each of the following statements represents information that can be graphed. Have them sort the statements into two groups in as many ways as they can and to record their findings (BLM 5.SP.1&2.2).

- Names of the highest mountains in North America
- Students in the class who have pets
- The won/lost record of the Winnipeg Goldeyes
- The makes of cars parked in a shopping centre parking lot
- The sum of the numbers rolled when two dice are thrown
- The population of Canadian provinces
- The number of people who have immigrated to Manitoba
- How far students in Grade 7 can throw a softball
- The number of cm a plant grows over a five-week period
- The number of houses sold in Winnipeg last year

c) Have students share their classifications with the other members of the class.

d) Explain that information we collect ourselves is called first-hand data, and information that we get from other sources, such as newspapers, magazines or the Internet, is called second-hand data. Have students decide which statements are examples of first-hand data and which are second-hand data.

e) Introduce the never-ending graphing project. Tell students that they will be making a book of graphs. Each week, they will be making a graph that organizes information they have collected about themselves, their community, or any other topic of interest. Ask students what they would like to know about their classmates and their community. Make a list of their questions and have students determine which questions are best answered with first-hand data and which are best answered with second-hand data.

Note: Questions can be added to the list throughout the year. Each time a new question is added, students should discuss whether the question is best answered with first- or second-hand data. The graphing activities can be integrated into other subject areas (e.g., in language arts, students could make a graph illustrating the number of books they have read, and in science they can make a graph comparing distances different-sized balloons travel in a jet expulsion experiment. Also, the activities should be structured so students have opportunities to practice creating and interpreting different types of graphs [pictographs, bar graphs, and double bar graphs].).

f) Have students answer the following questions in their math journals. Encourage students to give examples that have not been discussed in class.

- "What are two examples of first-hand data"
- "What are two examples of second-hand data?"
- "What is the difference between first- and second-hand data?"

Observation Checklist

Examine students' responses to determine whether they can do the following:

- ❑ formulate a question that is best answered with first-hand data and explain why
- ❑ formulate a question that is best answered with second-hand data and explain why
- ❑ provide examples of first-hand data
- ❑ provide examples of second-hand data
- ❑ explain the difference between first- and second-hand data

- **Explain the difference between first-hand and second-hand data.**
- **Represent a set of data by creating a double bar graph, label the title and axes, and create a legend with or without the use of technology.**
- **Solve a problem by constructing and interpreting a double bar graph.**

Materials: Graph paper, markers, pink and yellow sticky notes

Organization: Whole class

Procedure:

a) Select one question that students have about their classmates (e.g., "What is your favourite pizza topping?"). Have students identify three different toppings (e.g., pepperoni, mushrooms, and sausage).

b) Have the girls record their favourite topping on the yellow sticky notes and the boys on the pink sticky notes. Write pepperoni, mushrooms, and sausage on the board and have students place their sticky notes under the appropriate heading and find the total number of students in each category.

c) Have students make a bar graph illustrating their favourite pizza topping and write a paragraph explaining what the graph means to them. When students finish writing their paragraphs, have them share their paragraph with a partner. The graph and paragraph can be the students' first entry into their graphing book (introduced in Part E of the previous learning experience).

d) Ask students how they can show how many girls like each topping and how many boys like each topping. Have students sort the sticky notes under each heading into two groups and find the total number in each category.

e) Show students how to make a double bar graph that compares girls' favourite pizza toppings with boys' favourite pizza toppings. Have them make a double bar graph along with you. Help students interpret their graph by asking questions such as the following:
   - "What topping do girls like the most? The least?"
   - "What topping do the boys like the most?"
   - "How many more girls like _____ than boys?"
   - "If you order pizza for the class and can only pick two toppings, which two would you pick? Why?"
   - "Does the graph illustrate first-hand or second-hand data? Explain."

f) Have students compare the bar graph of favourite pizza toppings with the double bar graph of favourite pizza toppings. Ask, "How are they alike? How do they differ?"

Observation Checklist

For Part C, use the following checklist to determine whether students can create and interpret an appropriate bar graph.

Makes a Bar Graph that Includes	**Yes**	**No**	**Comments**
A title			
Category labels			
A label for each axis			
Bars whose lengths correctly represent the number of observations in each category			
Bars the same width			
Spaces between bars			
An appropriate scale with equal increments that starts at zero			
An interpretation of bar graphs			
Reading between the data (Question 2)			
Reading beyond the data (Question 3)			

Observation Checklist

Observe students' responses to Parts E and F to determine whether they can do the following:

- ❏ make the double bar graph correctly
- ❏ interpret the double bar graph correctly
- ❏ recognize that both bar graphs and double bar graphs include a title, labelled axes, and a scale with equal increments that begin at 0
- ❏ recognize that double bar graphs need a legend
- ❏ recognize that bar graphs illustrate one set of data and double bar graphs compare two sets of data

- **Determine the attributes (title, axes, intervals, and legend) of double bar graphs by comparing a set of double bar graphs.**
- **Represent a set of data by creating a double bar graph, label the title and axes, and create a legend with or without the use of technology.**
- **Draw conclusions from a double bar graph to answer questions.**
- **Solve problems by constructing and interpreting a double bar graph.**

Materials: Two packages of assorted bags of candy for each group (e.g., two bags of jelly beans, two bags of gum drops, two bags of skittles), large graphing paper, markers, BLM 5.SP.1&2.3, BLM 5–8.4, paper plates for sorting the candy, self-assessment sheet

Organization: Groups of three or four

Procedure:

a) Tell students that they will be investigating what is inside a package of assorted candy. Explain that each group will get two packages of the same type of candy. There are different colours of candy in each package, and it is their job to determine whether the number of pieces of each colour in the package is the same or whether one colour appears more often than another. When they finish their investigation, they should be able to answer the question, "How assorted is a package of candies?"

b) Have each student complete the recording sheet (BLM 5.SP.1&2.3). Explain that the members of each group will need to work together to make a double bar graph that they will present to the rest of the class.

c) Have each group present its graph and its findings to the rest of the class.

d) Have the students compare all the double graphs. Ask them how the graphs are alike and how they differ and if there are any common patterns and relationships among them.

Observation Checklist

Monitor students' responses to determine whether they can do the following:

- ❑ make reasonable estimates of quantities
- ❑ identify the attributes (title, axes, intervals, and legend) of double bar graphs
- ❑ represent sets of data by creating a double bar graph that includes a title, labelled axes, appropriate intervals, and a legend without the use of technology
- ❑ draw conclusions from a double bar graph that illustrate that they can read between and beyond the data
- ❑ solve a problem using double bar graphs

Self-Assessment

Have students do a self-assessment (BLM 5–8.4) of how they work in a group.

- **Explain the difference between first-hand and second-hand data.**
- **Draw conclusions from a double bar graph to answer questions.**

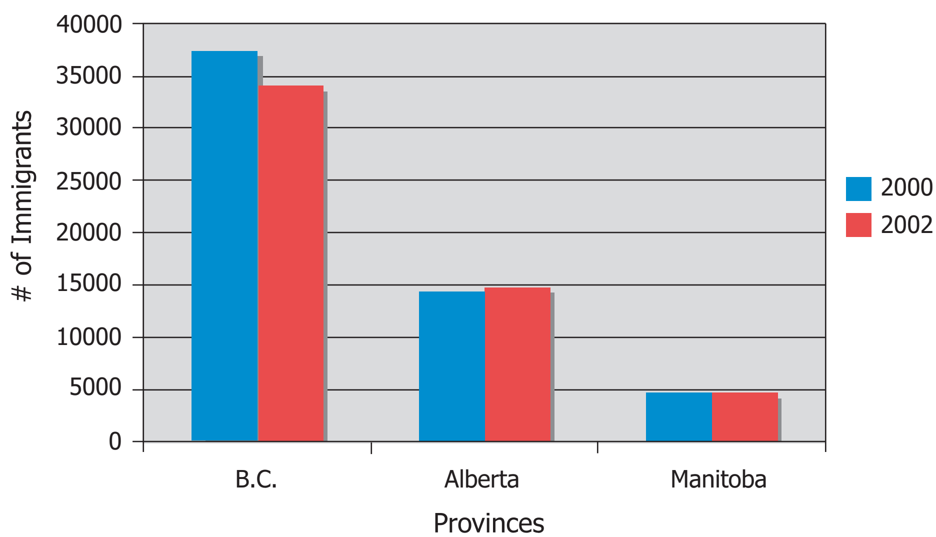
Materials: Copies of immigration map, math journals, calculators

Organization: Individual/Large group

Procedure:

a) Provide students with the following graph to analyze.

Number of Immigrants to Three Canadian Provinces



b) Ask students to record their answers to these questions in their math journals. Tell students that they can use a calculator to help them interpret the graph.

1. "What is the graph about?"
2. "Does the graph illustrate first-hand or second-hand data? Explain."
3. "What conclusions can you draw from the data?"
4. "What reasons can you give for the results?"
5. "If you add Ontario to the graph, do you think more people would have immigrated to Manitoba or to Ontario? Explain."

c) Have the students share their responses with the other members of the class. Encourage students to provide reasons for why more people immigrate to one province than another.

d) Ask students what questions they have as a result of analyzing the graph. Make a list of their questions and have students devise a plan for answering them.

Extension: Visit the Statistics Canada Website at <www.statcan.gc.ca> to see if the data have changed since 2008.

Observation Checklist

Check students' answers to determine whether they can do the following:

- ❑ differentiate between first-hand and second-data
- ❑ draw conclusions from a double bar graph that illustrate they can
  - ■ read between the data (e.g., A total of approximately 10 000 people immigrated to Manitoba in the two years)
  - ■ read beyond the data (e.g., More people would immigrate to Ontario than Manitoba)
- ❑ provide appropriate reasons for their conclusions (e.g., More people might have immigrated to B.C. because there were more jobs available)

- ■ **Explain the difference between first-hand and second-hand data.**
- ■ **Represent a set of data by creating a double bar graph, labelling the title and axes, and creating a legend with or without the use of technology.**
- ■ **Draw conclusions from a double bar graph to answer questions.**
- ■ **Solve a problem by constructing and interpreting a double bar graph.**

Materials: Empty cereal boxes, large graph paper, and markers

Organization: Individual or pairs

Procedure:

a) Show students an empty cereal box. Point out that there are many different types of cereal. Ask students why some cereals are considered to be better for them than others.

b) Explain that each box of cereal has a list of nutrients. Have students find the list of nutrients on one of their boxes. Ask them to find the number of grams of fibre in one serving of their cereal. Explain that cereals that are high in fibre and low in sugar content are considered healthier than those that are low in fibre and high in sugar.

c) Tell students that their task is to determine which cereal is better for them. Explain that they need to find the grams of fibre and sugar in three different cereals and represent this information on a double bar graph. Have students discuss whether they will be graphing first- or second-hand data, and how they know.

d) Have students write a paragraph describing their graph and all the conclusions that they can draw from it. Explain that they need to include a statement in their paragraph indicating which cereal is better for them and the reasons for their conclusion.

e) Have students share their graphs with the other members of the class, and explain which cereal they think is better for them. Encourage students to provide reasons for their conclusions.

Observation Checklist

Checks students' graphs and their conclusions about them to determine whether they can do the following:

- ❑ represent data on a double bar graph that includes a title, labelled axes, appropriate intervals, and a legend with or without the use of technology
- ❑ draw valid conclusions from the data that illustrate they can read between data
- ❑ solve a problem using a double bar graph

- **Represent a set of data by creating a double bar graph, labelling the title and axes, and creating a legend with or without the use of technology.**
- **Draw conclusions from a double bar graph to answer questions.**
- **Solve a problem by constructing and interpreting a double bar graph.**

Materials: A computer with a spreadsheet program, paper, and pencil

Organization: Individual or pairs

Procedure:

a) Ask, "Which city do you think is colder: Flin Flon or Montreal? Why?" Explain that one way to determine which city is colder is to compare the number of days the temperature in each city is less than or equal to zero.

b) Tell students they are going to find out which cities in Canada are the coldest. Have students go to <www.theweathernetwork.com> and select the statistics link. Tell them to pick any two cities in Canada and find the number of days in each month of the year that each city's temperature is less than or equal to zero.

c) Have students make a double bar graph using a computer spreadsheet program that shows the number of days in each month that each city's temperature is less than or equal to zero.

d) Have students make a list of questions about their graph. Ask them to prepare a poster consisting of the original data, the graph, and the questions that they developed.

e) Display the posters around the room and conduct a gallery walk. Have students answer the questions on each poster. Have students compare all the graphs to determine which city is the coldest.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ collect second-hand data from an electronic database
- ❑ create a double bar graph using technology
- ❑ draw valid conclusions from the data that illustrate they can read between and beyond the data
- ❑ ask appropriate questions about the data represented on a graph

NOTES

Grade 5: Statistics and Probability (Chance and Uncertainty) (5.SP.3, 5.SP.4)

Enduring Understandings:

Chance is an element of many aspects of our lives. The chance that an event will occur varies from impossible to certain.

General Outcome:

Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Learning Outcome(s):	Achievement Indicators:
5.SP.3 Describe the likelihood of a single outcome occurring, using words such as ■ impossible ■ possible ■ certain [C, CN, PS, R]	➛ Provide examples of events that are impossible, possible, or certain from personal contexts. ➛ Classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible, or certain. ➛ Design and conduct a probability experiment in which the likelihood of a single outcome occurring is impossible, possible, or certain. ➛ Conduct a probability experiment a number of times, record the outcomes, and explain the results.
5.SP.4 Compare the likelihood of two possible outcomes occurring, using words such as ■ less likely ■ equally likely ■ more likely [C, CN, PS, R]	➛ Identify outcomes from a probability experiment that are less likely, equally likely, or more likely to occur than other outcomes. ➛ Design and conduct a probability experiment in which one outcome is less likely to occur than the other outcome. ➛ Design and conduct a probability experiment in which one outcome is equally likely to occur as the other outcome. ➛ Design and conduct a probability experiment in which one outcome is more likely to occur than the other outcome.

Prior Knowledge

Although this is the first time that students should formally encounter the study of chance and uncertainty, students may have had experience with the following:

- Having an understanding of fractions

Background Information

Probability is a measure of the chance that an event will occur. The formal study of probability begins in Grade 5 with the development of the language of probability. Knowledge of the terms associated with probability facilitates students' understanding of the role chance plays in their lives and their awareness that some events are more probable than others.

The learning experiences in this substrand are divided into two parts. The first part focuses on the development of the terms *impossible*, *possible*, *certain*, *more likely*, *less likely*, and *equally likely* and their application to real-life events. The learning experiences in the second part have students applying the language of probability to predict and explain the outcomes (the results) of probability experiments. The research on probability indicates that students often have misconceptions about the outcomes of events. For example, if a coin is tossed three times and lands heads up each time, many students believe the next time the coin is tossed it is bound to land tails up because things will even out. Consequently, the learning activities in this section engage students in generating and analyzing data that will help them overcome their misconceptions and pave the way for a deeper understanding of theoretical probability.

Mathematical Language

Impossible

Possible

Certain

Less likely

More likely

Equally likely

LEARNING EXPERIENCES

- **Provide examples of events that are impossible, possible, or certain from personal contexts.**
- **Identify outcomes from a probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.**

Materials: *It's Probably Penny* by Loreen Leedy (ISBN-13:978-0-8050-7389-8/ ISBN-10:0-8050-7389-2)

Organization: Whole class

Procedure:

a) Read the book *It's Probably Penny* to the class.

b) Discuss the story. Begin the discussion by asking, "What was Penny's homework assignment? What event did Penny think was possible? What are some other events that are possible? What does possible mean?"

c) Create a class list of events that are
   - impossible
   - possible
   - likely
   - certain

   Have students explain why they think each event they name is impossible, possible, likely, or certain to occur.

Observation Checklist

Observe the students to determine whether they can do the following:

- ❏ identify events that are impossible, possible, certain, or unlikely
- ❏ provide valid reasons for why an event is impossible, possible, unlikely, or certain

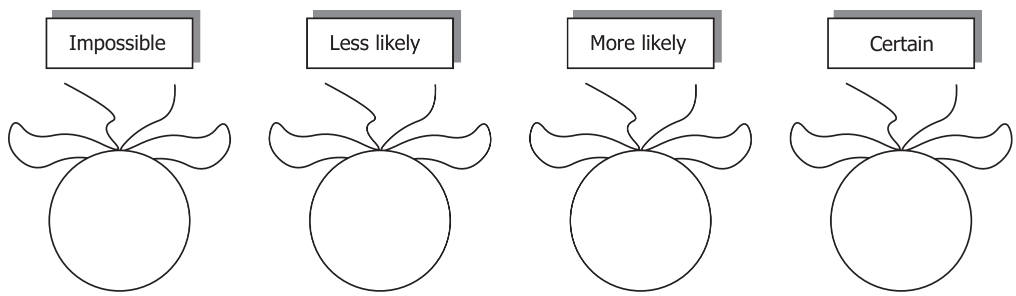
- **Classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible, or certain.**
- **Identify outcomes from a probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.**

Materials: Event cards and label cards BLM 5.SP.3&4.1, loops made of string or yarn

Organization: Pairs

Procedure:

a) Have the students place the string loops on their workspace and place a label card beside each set.



b) Ask students to place each of the given statements into one of the sets.

c) When students are finished sorting the statements, have them share their answers and explain their reasons for placing an event in a given set.

Observation Checklist

Monitor students to determine whether they have valid reasons for categorizing a given event as impossible, unlikely, possible, or certain, and whether they understand that

- ❑ if an event is impossible, it will not occur
- ❑ if an event is certain, it will occur
- ❑ if an event is possible, it may or may not occur
- ❑ some events are more possible than others

Material: Copies of the learning experience (BLM 5.SP.3&4.2)

Organization: Individual

Procedure:

a) Have students complete the following activity sheet (BLM 5.SP.3&4.2).

b) When students have finished answering the questions, have them share their answers and their reasons for them.

Observation Checklist

Observe students' responses to determine whether they understand the following:

- ❏ the more possibilities there are, the more likely an event will occur
- ❏ if an event is impossible, it will not occur
- ❏ if an outcome is certain, it is the only one
- ❏ if events are equally likely, they have the same chance of occurring

- **Provide examples from events that are impossible, possible, or certain from personal contexts.**

Materials: Journal/Learning log

Organization: Individual

Procedure:

Have the students complete the following statements in their math journals:

1. I am less likely to …..
2. I am more likely to …...
3. I am certain to …….
4. It is impossible for me to…..
5. It is equally likely that …..
6. It is possible that I will ……

Observation Checklist

Examine students' responses to determine whether they can do the following:

❑ identify events other than ones discussed in class that are certain, impossible, possible, more likely, less likely, and equally likely to occur

❑ understand the meaning of the terms *certain*, *impossible*, *possible*, *more likely*, *less likely*, and *equally likely*

- **Conduct a probability experiment a number of times, record the outcomes, and explain the results.**
- **Identify outcomes from a probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.**

Materials: A set of cards (BLM 5–8.5)

Organization: Whole class

Procedure:

Show students the cards and then spread them out face down on a table. Have one student select a card and show it to the rest of the class. Ask:

- "Is it possible to pick a number that is greater than the one just selected? Why or why not?"
- "Is it possible to pick a number that is less than the one just selected? Why or why not?"
- "Is the number more likely or less likely to be greater (less) than the one just selected? Why do you think so?"

Have a student pick another card so the class can check their prediction and discuss why their prediction may or may not have been correct.

Repeat the activity several times.

Observation Checklist

Observe students' responses to determine whether they are doing the following:

- ❑ basing their predictions on the relationships between the numbers
- ❑ using the terms *more likely* and *less likely* correctly
- ❑ providing valid reasons for their predictions

- **Conduct a probability experiment a number of times, record the outcomes, and explain the results.**
- **Identify outcomes from a probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.**

Material: Coloured tiles or blocks, paper bag

Organization: Whole class

Procedure:

a) Ask students, "If you put four blue tiles in a bag and then take one out without looking, can you be sure what colour tile you will get? Why or why not?"

b) Next, ask, "If you put a red tile in the bag with the blue tiles and take one out without looking, can you be sure what coloured tile you will get? Why or why not? Are you more likely or less likely to get a red tile? Why? Are you more likely or less likely to get a blue tile? Why?"

   Have students test their answer by placing four blue tiles and one red tile in a bag, and then asking several students to draw a tile from the bag and show it to the rest of the class. Have them put the tile back in the bag before asking another student to draw a tile. Have students keep a record of the number of times a blue tile is drawn and the number of times a red tile is drawn. Discuss the results.

c) Finally, ask, "If you put three red tiles in a bag with seven blue tiles and take one out without looking, is one coloured tile more likely to be drawn from the bag than the other? Why or why not? Are you more likely or less likely to get a red tile?"

   Have students test their answer by placing three red and seven blue tiles in the bag. Ask several students to draw a tile from the bag and show it to the rest of the class. Have them put the tile back into the bag before asking another student to draw a tile. Have students keep a record of the number of times a red tile is drawn and the number of times a blue tile is drawn. Discuss the results.

Observation Checklist

Monitor students' responses to the questions to determine whether they are doing the following:

- ❏ basing their predictions on the number of red and blue tiles
- ❏ using the terms *more likely* and *less likely* correctly
- ❏ providing valid reasons for their predictions
- ❏ explaining the results of the experiments

- **Identify outcomes from a probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.**

Materials: BLM 5.SP.3&4.3

Organization: Pairs

Procedure:

a) Have students examine a spinner (BLM 5.SP.3&4.3) and check their predictions by spinning the spinner 20 tines and recording their results. Complete the BLM.

b) When all pairs have checked the validity of these statements, discuss their results as a class.

Observation Checklist

Observe students' responses to determine whether they can do the following:

- ❑ understand that the spinner is more likely to stop on the section that is the greatest fraction of the whole
- ❑ understand the spinner is less likely to stop on the section that is the least fraction of the whole
- ❑ understand that sections of the spinner that are equivalent fractions of the whole are equally likely to occur
- ❑ provide valid reasons for their predictions
- ❑ explain the results of their experiment and why their results may differ from others

- **Identify outcomes from a probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.**

Materials: Two-coloured chips, paper cups, copies of the instruction for the experiment (BLM 5.SP.3&4.4)

Organization: Pairs

Procedure:

Have students complete activity sheet (BLM 5.SP.3&4.4).

Observation Checklist

Examine students' responses to determine whether they are doing the following:

- ❑ using the terms *more likely* and *less likely* correctly
- ❑ explaining the results of their experiment

- **Design and conduct a probability experiment in which the likelihood of a single outcome occurring is impossible, possible, or certain.**
- **Conduct a probability experiment a number of times, record the outcomes, and explain the results.**
- **Design and conduct a probability experiment in which one outcome is less likely to occur than the other outcome.**
- **Design and conduct a probability experiment in which one outcome is equally likely to occur as the other outcome.**
- **Design and conduct a probability experiment in which one outcome is more likely to occur than the other outcome.**

Materials: Copies of the Mystery Spinner activity sheet (BLM 5.SP.3&4.5)

Organization: Individual

Procedure:

Have students complete the activity sheet (BLM 5.SP.3&4.5).

Observation Checklist

Monitor students' responses to the spinning activities to determine whether:

- ❑ their spinners meet the conditions that were specified
- ❑ they use the terms *more likely*, *less likely*, *impossible*, *certain*, and *equally likely* correctly
- ❑ they refer to the fraction of the whole that each part of their spinner occupies to explain why the conditions that were specified are met

Putting the Pieces Together

Picturing Probability

Purpose:

The intent of this investigation is to have students demonstrate their understanding of the language of probability by depicting situations that illustrate different degrees of chance. In particular, the investigation is designed to reinforce the meaning and use of the terms

- impossible, possible, and certain
- less likely, equally likely, and more likely

The investigation is also designed to extend students ability to

- communicate mathematically
- use technology
- make connections to other subject areas (LwICT and ELA)
- make connections to real-world situations

Materials/Resources:

- Digital cameras
- Card readers (optional)
- Microphones
- Computer projectors
- Computers
- *Microsoft Photo Story*

Organization: Groups of three or four

Procedure:

a) Tell students that they will be using a digital camera to take six pictures. Each picture should illustrate an event whose likelihood of occurring can be described as impossible, possible, certain, equally likely, more likely, or less likely, using one picture for each word or phrase. The events can be ones they create or ones they actually see occurring.

   The pictures will be used to create a narrated *Microsoft Photo Story**. The narrated story should include the reasons why the event illustrated in each picture is included. The photo stories will be saved (a USB stick or other format can be used for this) and presented to the other members of the class.

   * *Microsoft Photo Story* is an application from Microsoft Office. It is available as a free download from the Microsoft website at <www.microsoft.com/windowsxp/using/digitalphotography/photostory/default.mspx>.

   Instead of using *Photo Story*, students could

   - make a *PowerPoint* presentation
   - dramatize the events they choose
   - make a "likelihood line" poster (order the events from least to most certain to occur)

b) Help students develop assessment criteria for their photo stories. The criteria should consider the following:

   - The correct use of mathematical vocabulary
   - A clear description of the event
   - Proper use of technology (with respect to the LwICT continuum)
   - Presentation skills

c) Have each group discuss how the terms can be illustrated. After they decide how each term can be represented, have them stage or find each event and take its picture.

d) Help students create their photo stories. Discuss with students how they can effectively, accurately, and concisely narrate their stores to entertain and engage their audience.

e) Have each group present its photo stories to the other members of the class.

f) Have students use the assessment criteria they developed to assess

   - themselves
   - their peers

Extension:

a) Have students use the following terms to describe events in the books *Cloudy with a Chance of Meatballs* by Judi Barret and *Dear Mr. Blueberry* by Simon James: *impossible*, *possible*, *certain*, *less likely*, *equally likely*, and *more likely*.

b) Have students look at different weather conditions and use the terms *possible*, *impossible*, *certain*, *less likely*, *equally likely*, and *more likely* to predict the weather.

c) Students may benefit from a connection between this book and Cluster 4: Weather Outcomes, found in the Manitoba Grade 5 Science curriculum.

NOTES

Grade 5 Mathematics

Appendices

Appendix 1: Models for Dividing

Appendix 2: Models for Multiplying

Appendices

Appendix 1: Models for Dividing

This appendix focuses on dividing 2 numbers by building understanding through the use of models, non-traditional algorithms, and traditional algorithms.

Note: The term *traditional algorithm* is used to indicate the symbolic algorithm traditionally taught in North America. Throughout the world, many other algorithms are traditionally used.

Base-10 Blocks

Base-10 blocks can be used to represent the operations of addition, subtraction, multiplication, and division of whole numbers.

Students must have a fluent understanding of the numeric values for the model. If they lack this understanding, their attention will be focused on trying to make sense of the model instead of learning to compute with whole numbers. Have students physically separate the blocks, or cut paper grids, to help them attach meaningful values to the representations. Spend time naming various combinations of blocks and creating representations of various whole numbers to develop fluency.

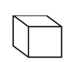
Base-10 grid paper serves as a two-dimensional representation of the base-10 blocks (see BLM 5-8.10: Base-Ten Grid Paper).

Note: It is important that students work flexibly with various representations to develop their understanding of operations with whole numbers, rather than memorizing the steps without understanding their meaning.

Representations:

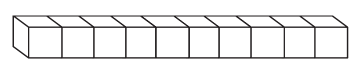
If the cube represents a whole,

- its value is 1



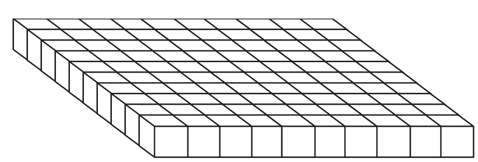
Then the rod represents ten, and

- its value is 10



Then the flat represents one hundred, and

- its value is 100



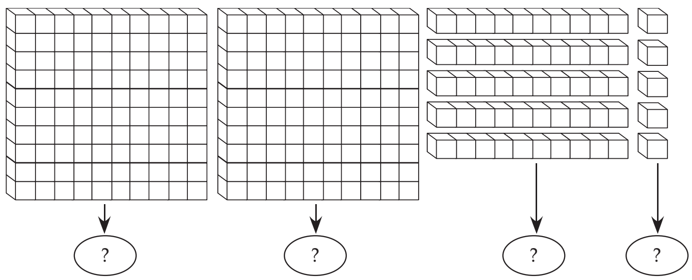
Example:

255 ÷ 4

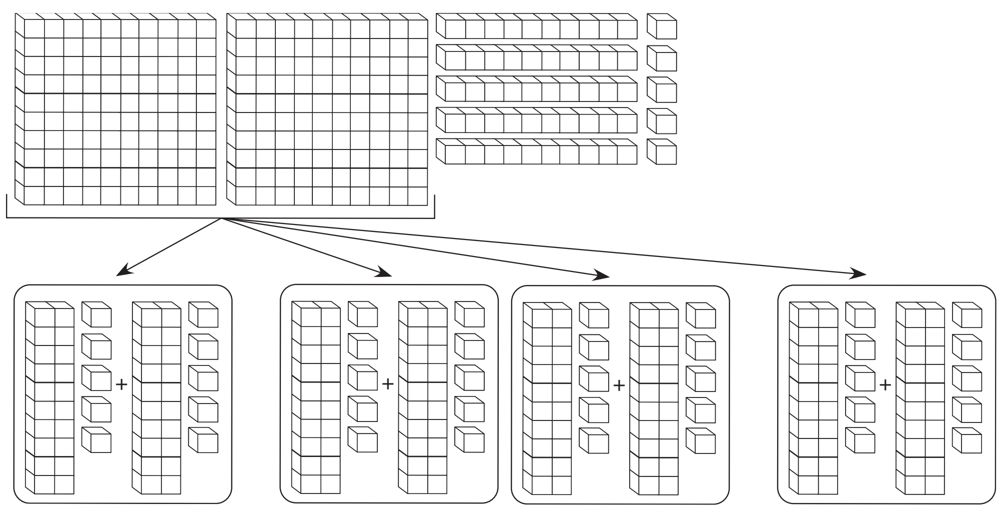
Estimate:

Think: 400 ÷ 4 is 100, so 200 ÷ 4 is 50.
My answer must be slightly more than 50.

Use models to help find an exact number. What is 255 divided into 4 groups?

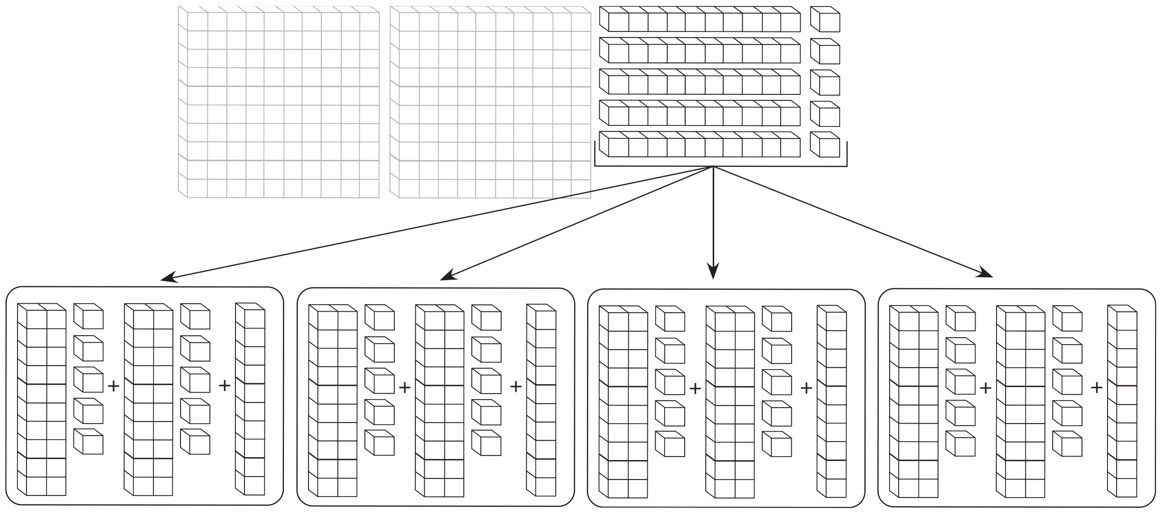


Think: "I can break each hundred into 4 groups. There is 25 in each group for every hundred I divide."

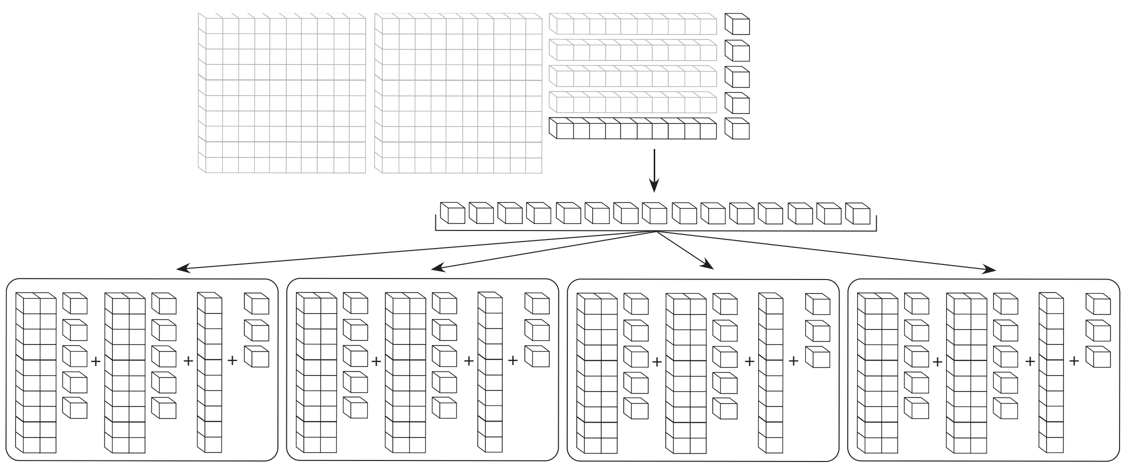


Think: “After I divide the 200 into 4 groups, there is nothing left over.”

Think: “I can move one whole group of 10 to each group.”



Think: “That leaves one ten (or an additional 10 ones). There are 15 ones left, so I can move 3 more to each group.”



Think: “There are 3 left, so 255 ÷ 4 is 63 R3 or $63\frac{3}{4}$ or 63.75.”

Strategic Division

Using strategic division means using one's number sense and confidence with place value in order to select numbers that are easier to divide. It can also help you to guide students toward an understanding of long division, as seen in Example 3.

Example 1:

255 ÷ 4

$4\overline{)255}$	Take away:
$\underline{-100}$	25 groups of 4
155	
$\underline{100}$	25 groups of 4
55	
$\underline{-40}$	10 groups of 4
15	
$\underline{-12}$	$\underline{\text{3 groups of 4}}$
3	63 groups of 4

"So 255 ÷ 4 is 63 R3 or $63\frac{3}{4}$ or 63.75."

Example 2:

$$\begin{array}{l} \quad 25 \; + \; 25 \; + \; 10 \; + \; 3 \quad \text{R3} \\ 4\overline{)100 \; + 100 + 40 + 15} \end{array}$$

"So 255 ÷ 4 is 63 R3 or $63\frac{3}{4}$ or 63.75."

Example 3:

"How many whole groups of 4 are in 25?"
"6"
"So there are 60 groups of 4 in 250."

"How many whole groups of 4 are in 2?"
"None."
"Let's ignore the 200 for now."

$$\begin{array}{r} 063 \\ 4\overline{)255} \\ \underline{-240} \\ 15 \\ \underline{12} \\ 3 \end{array}$$

"How many whole groups of 4 are in 15?"
"3"

"So 255 ÷ 4 is 63 R3 or $63\frac{3}{4}$ or 63.75."

Long Division

Long division is a more compact algorithm used to show the division of multi-digit numbers. Once students have an understanding of the above models and methods, and they are confident with their understanding of division and place value, the "traditional algorithm" for division can be a quick and precise method of dividing. In your modelling of this method, be sure to use appropriate mathematical language so as not to reinforce misconceptions about place value.

Example:

$$\begin{array}{r} 63\ \text{R3} \\ 4\overline{)255} \\ \underline{-24} \\ 15 \\ \underline{-12} \\ 3 \end{array}$$

Note: There are certainly circumstances where mental mathematics is the most efficient way to divide numbers.

Example:

319 ÷ 3 can be thought about as the following:

- "319 can be broken into 300 + 19"
- "What is 300 ÷ 3"
  - "100"
- "And 19 ÷ 3?"
  - "6 with one remaining"
- "So, 319 ÷ 3 must be 100 plus 6 R 1 or $106.\overline{3}$."

Remainders

Students should become familiar with interpreting remainders based on the context of the question being asked.

- A *whole number* remainder is used in situations where you are treating division as equal sharing or equal grouping with objects that cannot be shared or grouped in their whole state (i.e., people, rocks, pens).

  For example, the school parliament is planning an intramural floor hockey tournament. In order to run the event, they need four teams. If 57 students have signed up to play, how many students are on each team? (57 ÷ 4 = 14 R 1, so there will be 14 players on each team with one extra player, so one team would have 15 players.)

- A *decimal* remainder is used in situations where you are treating division as equal sharing or equal grouping with objects that can be shared or grouped, and that can be described using decimals (i.e., score on a test, money, average height of a basketball team).

  For example, the total height of all players on the girls' basketball team is 1592 cm. If there are 10 girls on the basketball team, what is the average height of each athlete? (1592 ÷ 10 = 159 R 2, and since 2 ÷ 10 is 0.2, the average height of the athletes on the girls' basketball team is 159.2 cm.)

- A *fractional* remainder is used in situations where you are treating division as equal sharing or equal grouping with objects that can be shared or grouped, and that can be described using fractions (i.e., imperial measurements, time).

  Note: Sometimes it makes sense to express the fraction in simplest form, and sometimes it does not.

  For example, Drayson is making a dog house and doesn't want to waste too much lumber. The scrap 2×4 that his mom is letting him use is 63 inches long. If he wants to cut four lengths of wood from each 2×4, how long will each piece be?

  ($63 \div 4 = 15$ R 3 or $15\frac{3}{4}$, so he will have four pieces of wood, each with a length of $15\frac{3}{4}$ inches)

NOTES

APPENDIX 2: MODELS FOR MULTIPLYING

This appendix focuses on multiplying numbers by building understanding through the use of models, non-traditional algorithms, and traditional algorithms.

Note: The term *traditional algorithm* is used to indicate the symbolic algorithm traditionally taught in North America. Throughout the world, many other algorithms are traditionally used.

Base-10 Blocks

Base-10 blocks can be used to represent the operations of addition, subtraction, multiplication, and division of whole numbers.

Students must have a fluent understanding of the numeric values for the model. If they lack this understanding, their attention will be focused on trying to make sense of the model instead of learning to compute with whole numbers. Have students physically separate the blocks, or cut paper grids, to help them attach meaningful values to the representations. Spend time naming various combinations of blocks and creating representations of various whole numbers to develop fluency.

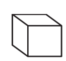
Base-10 grid paper serves as a two-dimensional representation of the base-10 blocks (see BLM 5-8.10: Base-Ten Grid Paper).

Note: It is important that students work flexibly with various representations to develop their understanding of operations with whole numbers, rather than memorizing the steps without understanding their meaning.

Representations:

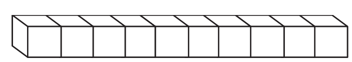
If the cube represents a whole,

- its value is 1



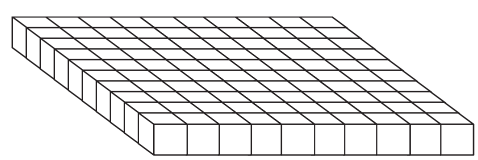
Then the rod represents 10, and

- its value is 10



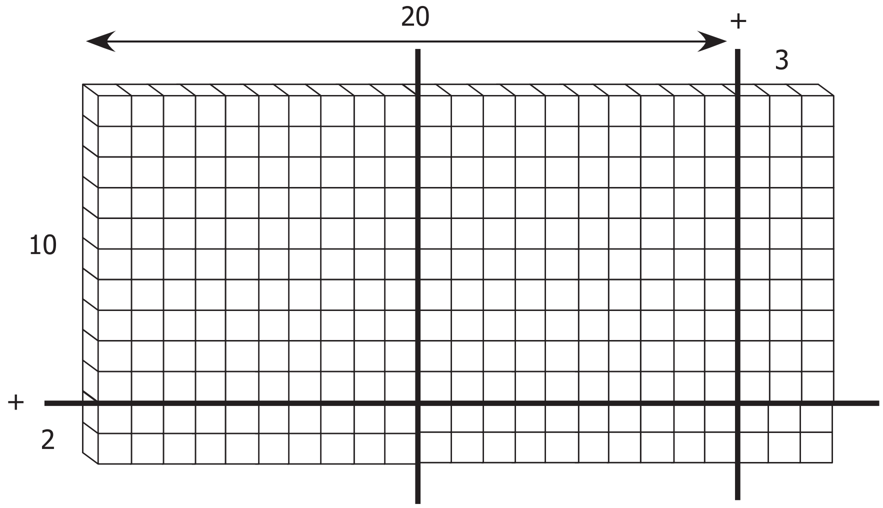
If the flat represents 100,

- its value is 100



Example:

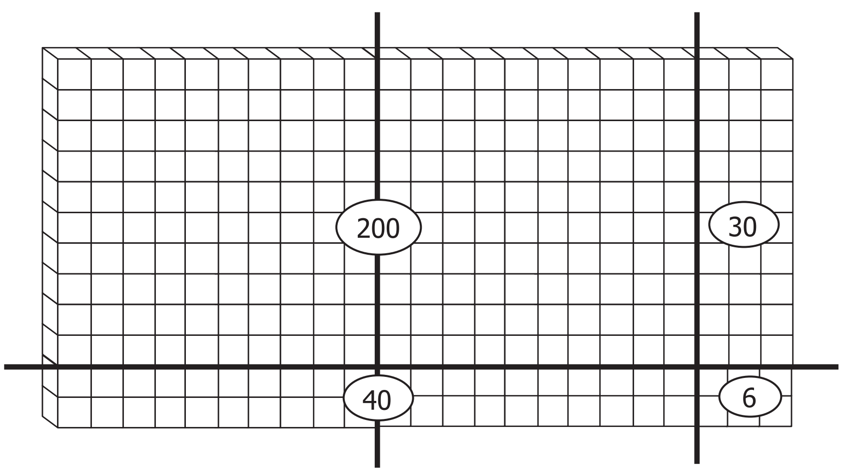
23 × 12



To determine the partial products, think of numbers that are easier to multiply. For example, represent numbers according to place value and think:

- "What is 10 × 20?" "200"
- "What is 10 × 3?" "30"
- "What is 20 × 2?" "40"
- "What is 3 × 2?" "6"

"So then, 23 × 12 must be 200 + 30 + 40 + 6, or 476."



It is important to establish a convention of keeping the blocks organized, as it will help with developing future representations.

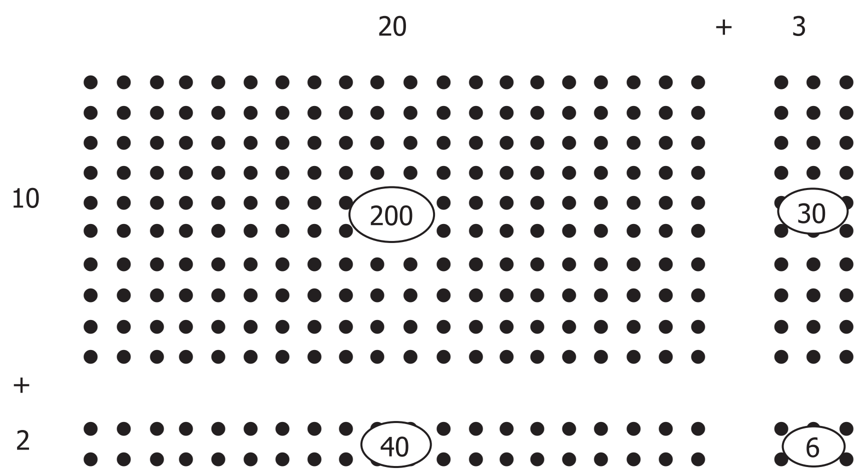
Base-10 blocks can be used to represent the active understanding of multiplication as a specific number of groups of a specific size, or the non-active array representation of a quantity.

When the blocks are arranged as a rectangle, the rectangle may be rotated and the quantity does not change. This is a verification of the commutative property of multiplication. The orientation of the array has no effect on the result, but in some places a convention has been established of representing the first number horizontally and the second vertically.

The array also serves as a model for area. It represents the area covered by a rectangle with a length of the multiplicand and a width of the multiplier, or vice versa.

Example:

23×12



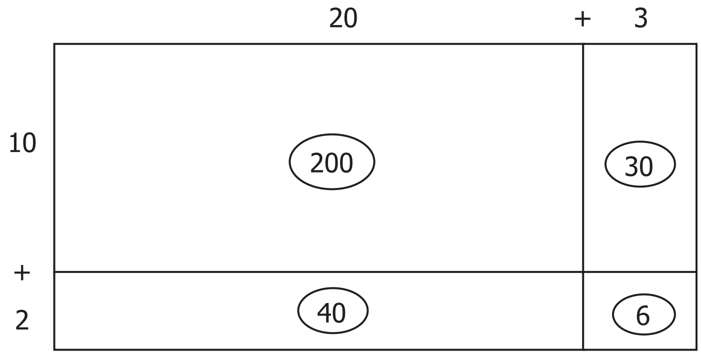
The partial products are determined in the same way as they are in the previous example.

The Area Model

Once students have an understanding of multiplication using base-10 blocks and/or base-10 paper, and they have a thorough understanding of the numerical value they represent, they can simply move to an area model. In this model, the length represents the multiplicand and the width represents the multiplier, or vice versa; however, the lengths and widths do not have to represent an exact measurement.

Example:

23 × 12

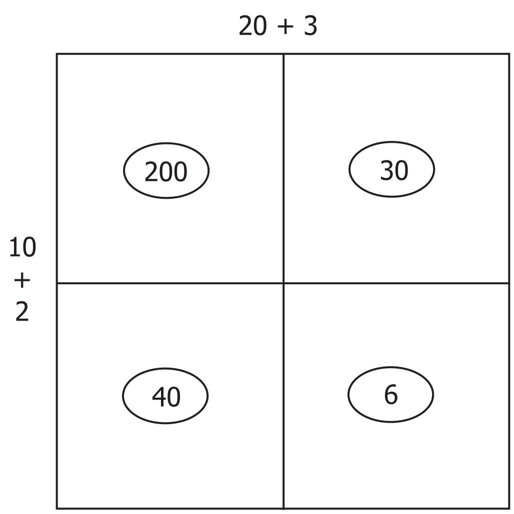


The partial products are determined in the same way as they are in the previous example.

This model is sometimes referred to as the "window-box" method, and is simplified further by completely ignoring the comparative sizes of the length and the width.

Example:

23 × 12



The partial products are determined in the same way as they are in the previous example.

Distributive Property

Once students have an understanding of multiplication using place value and are confident "rearranging" the numbers, representing multiplication using the distributive property will allow them to multiply symbolically.

Example:

23×12

Think $(20 + 3) \times (10 + 2)$

$= (20 \times 10) + (20 \times 2) + (3 \times 10) + (3 \times 2)$

$= 200 + 40 + 30 + 6$

$= 276$

Long Multiplication

Once students have an understanding of multiplication using place value, representing multiplication using this "long method" can lead them toward a better understanding of the traditional algorithm. In your modelling of this method, be sure to use appropriate mathematical language so as not to reinforce misconceptions about place value.

Example:

23×12

23		
× 12	Think:	
6	"What is 3 × 2?"	"6"
40	"What is 2 × 20?"	"40"
30	"What is 10 × 3?"	"30"
+ 200	"What is 10 × 20?"	"200"
276	"So 23 × 12 is 6 + 40 + 30 + 200 = 276."	

Compact Multiplication (The "Traditional Algorithm")

Once students have an understanding of the above models and methods, and they are confident with their understanding of multiplication and place value, the "traditional algorithm" for multiplication can be a quick and precise method of multiplying. In your modelling of this method, be sure to use appropriate mathematical language so as not to reinforce misconceptions about place value.

Example:

23×12

23	
$\underline{\times\ 12}$	Think:
46	"What is 3 × 2?" "6"
$\underline{+\ 230}$	"What is 2 × 20?" "40"
276	"What is 10 × 3?" "30"
	"What is 10 × 20?" "200"
	"So 23 × 12 is 6 + 40 + 30 + 200 = 276."

Note: There are certainly circumstances where mental mathematics is the most efficient way to multiply numbers.

Example:

24 × 15 can be thought about as:

- "What is 10 × 24?"
  - "240"
- "And half of that is?"
  - "120"
- "And so 24 × 15 must be?"
  - "240 + 120, so 360"

GRADE 5 MATHEMATICS

Bibliography

BIBLIOGRAPHY

Banks, James A., and Cherry Banks. *Multicultural Education: Issues and Perspectives*. 2nd Ed. Boston, MA: Allyn and Bacon, 1993.

Barret, Judi. *Cloudy with a Chance of Meatballs*. New York, NY: Aladdin Paperbacks, 1978.

Brazier, Heather. *Which Do You Prefer – Chunky or Smooth?* Toronto, ON: HarperCollins, 1992.

Briggs, Raymond. J*im and the Beanstalk.* New York, NY: Putnam & Grosset, 1970.

Burns, Marilyn. *Spaghetti and Meatballs for All*. New York, NY: Scholastic Inc., 2008.

Clements, Andrew. *A Million Dots.* New York, NY: Simon and Schuster, 2006.

Ezbicki, K. E. The Effects of a Math-Fact Fluency Intervention on the Complex Calculation and Application Performance of Fourth Grade Students (Doctoral dissertation, University of Massachusetts Amherst, 2008). Retrieved from Dissertations & Theses: Full Text (Publication No. AA13325323), 2008.

Gojak, Linda M. "Fluency: Simply Fast and Accurate? I Think Not!" *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2000.

Isaacs, A. C., and W. M. Carroll. Strategies for Basic-Fact Instruction. *Teaching Children Mathematics*, 6(8), 508–515, 1999.

James, Simon. *Dear Mr. Blueberry.* New York, NY: Aladdin Paperbacks, 1991.

Kilpatrick, J., J. Swafford, and B. Findell, eds. *Adding it Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press, 2001.

Leedy, Loreen. *It's Probably Penny*. New York, NY: Henry Holt and Co., 2007.

Manitoba Education. *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*. Winnipeg, MB: Manitoba Education, 2013. Available online at <www.edu.gov.mb.ca/k12/cur/math/framework_k-8/index.html>.

Manitoba Education, Citizenship and Youth. *Kindergarten to Grade 8 Mathematics: Manitoba Glossary.* Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2009.

______. *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2006.

OECD (Organisation for Economic Co-operation and Development). *PISA 2012 Field Trial Problem Solving Framework*. September 30, 2012. <www.oecd.org/pisa/pisaproducts/46962005.pdf>.

Ontario. Ministry of Education. *The Ontario Curriculum Grades 1–8: Mathematics.* Ontario: Ministry of Education, 2005. Available online at <www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf>.

Ontario Education. "Learning about Division in the Junior Grades." *Number Sense and Numeration, Grades 4 to 6*. Volume 4. Ontario Education, 2006.

Seeley, Cathy L. *Faster Isn't Smarter: Messages about Math, Teaching, and Learning in the 21st Century*. Sausalito, CA: Math Solutions, 2009.

Small, Marian. "Whole Number Operations: Varied Approaches for Multiplication and Division." *Big Ideas from Dr. Small: Grades 4–8*. Nelson Education, 2009.

Western and Northern Canadian Protocol (WNCP). *The Common Curriculum Framework for K-9 Mathematics.* Edmonton, AB: Governments of Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut Territory, Saskatchewan, and Yukon Territory, May 2006.

Printed in Canada
Imprimé au Canada