Grade 5 Mathematics

Appendices

APPENDICES

APPENDIX 1: MODELS FOR DIVIDING

This appendix focuses on dividing 2 numbers by building understanding through the use of models, non-traditional algorithms, and traditional algorithms.

Note: The term *traditional algorithm* is used to indicate the symbolic algorithm traditionally taught in North America. Throughout the world, many other algorithms are traditionally used.

Base-10 Blocks

Base-10 blocks can be used to represent the operations of addition, subtraction, multiplication, and division of whole numbers.

Students must have a fluent understanding of the numeric values for the model. If they lack this understanding, their attention will be focused on trying to make sense of the model instead of learning to compute with whole numbers. Have students physically separate the blocks, or cut paper grids, to help them attach meaningful values to the representations. Spend time naming various combinations of blocks and creating representations of various whole numbers to develop fluency.

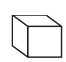
Base-10 grid paper serves as a two-dimensional representation of the base-10 blocks (see BLM 5-8.10: Base-Ten Grid Paper).

Note: It is important that students work flexibly with various representations to develop their understanding of operations with whole numbers, rather than memorizing the steps without understanding their meaning.

Representations:

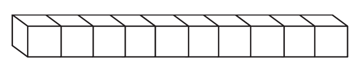
If the cube represents a whole,

- its value is 1



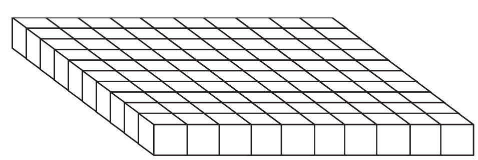
Then the rod represents ten, and

- its value is 10



Then the flat represents one hundred, and

- its value is 100



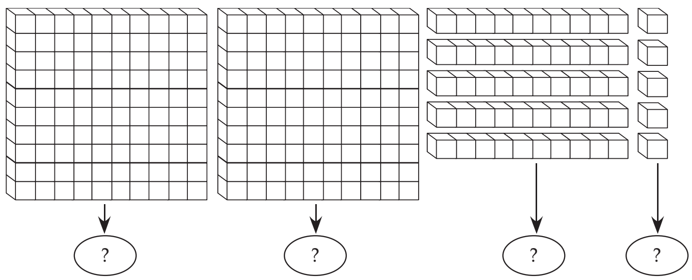
Example:

255 ÷ 4

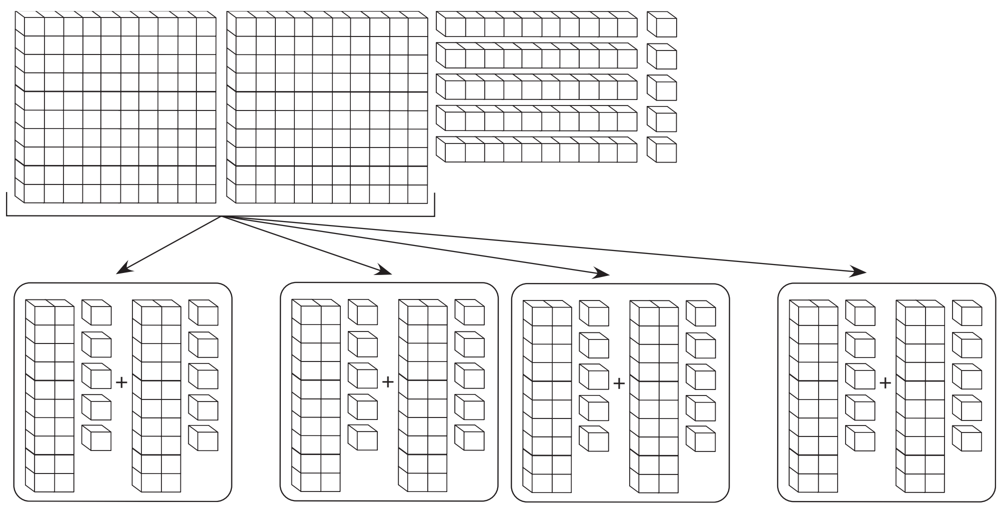
Estimate:

Think: 400 ÷ 4 is 100, so 200 ÷ 4 is 50.
My answer must be slightly more than 50.

Use models to help find an exact number. What is 255 divided into 4 groups?

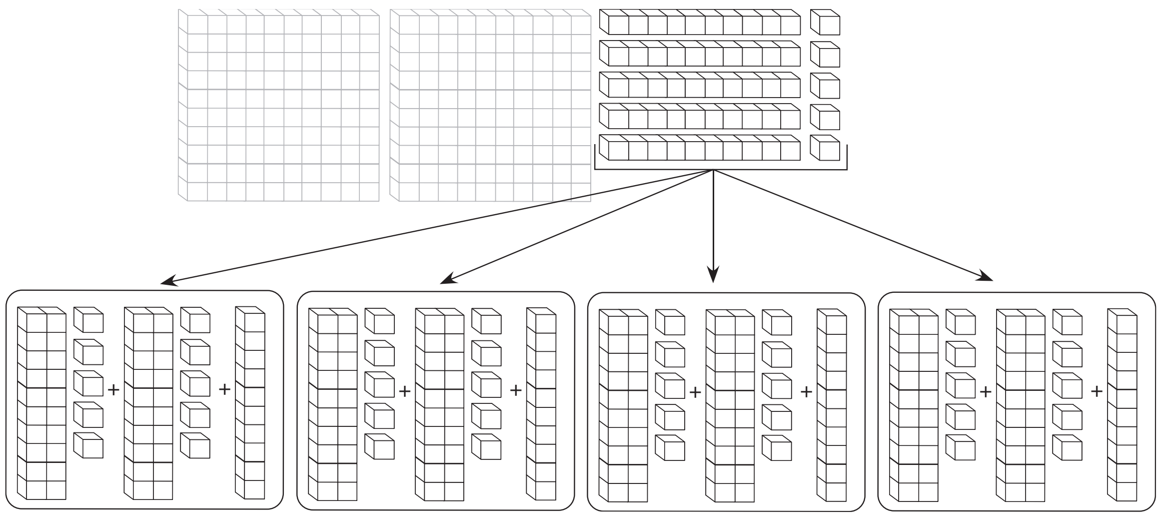


Think: "I can break each hundred into 4 groups. There is 25 in each group for every hundred I divide."

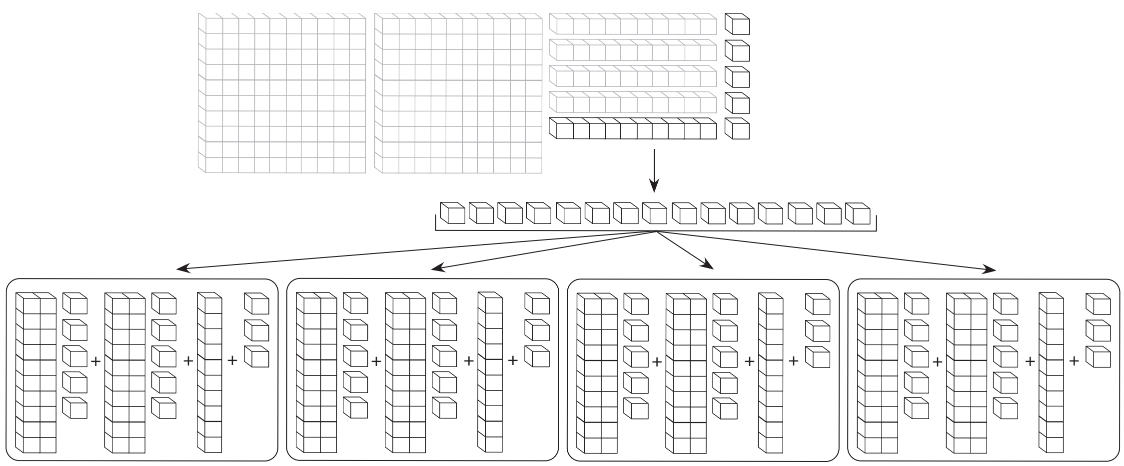


Think: “After I divide the 200 into 4 groups, there is nothing left over.”

Think: “I can move one whole group of 10 to each group.”



Think: “That leaves one ten (or an additional 10 ones). There are 15 ones left, so I can move 3 more to each group.”



Think: “There are 3 left, so 255 ÷ 4 is 63 R3 or $63\frac{3}{4}$ or 63.75.”

Strategic Division

Using strategic division means using one's number sense and confidence with place value in order to select numbers that are easier to divide. It can also help you to guide students toward an understanding of long division, as seen in Example 3.

Example 1:

$255 \div 4$

$$
\begin{array}{rl}
4\overline{)255} & \text{Take away:} \\
\underline{-100} & \text{25 groups of 4} \\
155 & \\
\underline{100} & \text{25 groups of 4} \\
55 & \\
\underline{-40} & \text{10 groups of 4} \\
15 & \\
\underline{-12} & \underline{\text{3 groups of 4}} \\
3 & \text{63 groups of 4}
\end{array}
$$

"So $255 \div 4$ is 63 R3 or $63\frac{3}{4}$ or 63.75."

Example 2:

$$
\begin{array}{r}
25 \ + \ 25 \ + \ 10 + \ 3 \quad \text{R3} \\
4\overline{)100 \ + 100 + 40 + 15}
\end{array}
$$

"So $255 \div 4$ is 63 R3 or $63\frac{3}{4}$ or 63.75."

Example 3:

"How many whole groups of 4 are in 25?"
"6"
"So there are 60 groups of 4 in 250."

"How many whole groups of 4 are in 2?"
"None."
"Let's ignore the 200 for now."

$$\begin{array}{r} 063 \\ 4\overline{)255} \\ \underline{-240} \\ 15 \\ \underline{12} \\ 3 \end{array}$$

"How many whole groups of 4 are in 15?"
"3"

"So 255 ÷ 4 is 63 R3 or $63\frac{3}{4}$ or 63.75."

Long Division

Long division is a more compact algorithm used to show the division of multi-digit numbers. Once students have an understanding of the above models and methods, and they are confident with their understanding of division and place value, the "traditional algorithm" for division can be a quick and precise method of dividing. In your modelling of this method, be sure to use appropriate mathematical language so as not to reinforce misconceptions about place value.

Example:

$$\begin{array}{r} 63 \text{ R3} \\ 4\overline{)255} \\ \underline{-24} \\ 15 \\ \underline{-12} \\ 3 \end{array}$$

Note: There are certainly circumstances where mental mathematics is the most efficient way to divide numbers.

Example:

319 ÷ 3 can be thought about as the following:

- "319 can be broken into 300 + 19"
- "What is 300 ÷ 3"
    - "100"
- "And 19 ÷ 3?"
    - "6 with one remaining"
- "So, 319 ÷ 3 must be 100 plus 6 R 1 or $106.\overline{3}$."

Remainders

Students should become familiar with interpreting remainders based on the context of the question being asked.

- A *whole number* remainder is used in situations where you are treating division as equal sharing or equal grouping with objects that cannot be shared or grouped in their whole state (i.e., people, rocks, pens).

  For example, the school parliament is planning an intramural floor hockey tournament. In order to run the event, they need four teams. If 57 students have signed up to play, how many students are on each team? (57 ÷ 4 = 14 R 1, so there will be 14 players on each team with one extra player, so one team would have 15 players.)

- A *decimal* remainder is used in situations where you are treating division as equal sharing or equal grouping with objects that can be shared or grouped, and that can be described using decimals (i.e., score on a test, money, average height of a basketball team).

  For example, the total height of all players on the girls' basketball team is 1592 cm. If there are 10 girls on the basketball team, what is the average height of each athlete? (1592 ÷ 10 = 159 R 2, and since 2 ÷ 10 is 0.2, the average height of the athletes on the girls' basketball team is 159.2 cm.)

- A *fractional* remainder is used in situations where you are treating division as equal sharing or equal grouping with objects that can be shared or grouped, and that can be described using fractions (i.e., imperial measurements, time).

  Note: Sometimes it makes sense to express the fraction in simplest form, and sometimes it does not.

  For example, Drayson is making a dog house and doesn't want to waste too much lumber. The scrap 2×4 that his mom is letting him use is 63 inches long. If he wants to cut four lengths of wood from each 2×4, how long will each piece be?

  ($63 \div 4 = 15$ R 3 or $15\frac{3}{4}$, so he will have four pieces of wood, each with a length of $15\frac{3}{4}$ inches)

NOTES

APPENDIX 2: MODELS FOR MULTIPLYING

This appendix focuses on multiplying numbers by building understanding through the use of models, non-traditional algorithms, and traditional algorithms.

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Base-10 Blocks

Base-10 blocks can be used to represent the operations of addition, subtraction, multiplication, and division of whole numbers.

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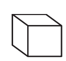

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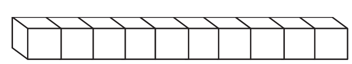
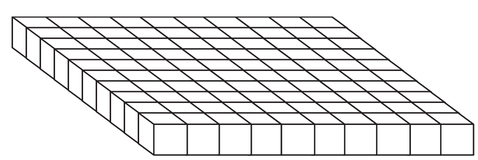


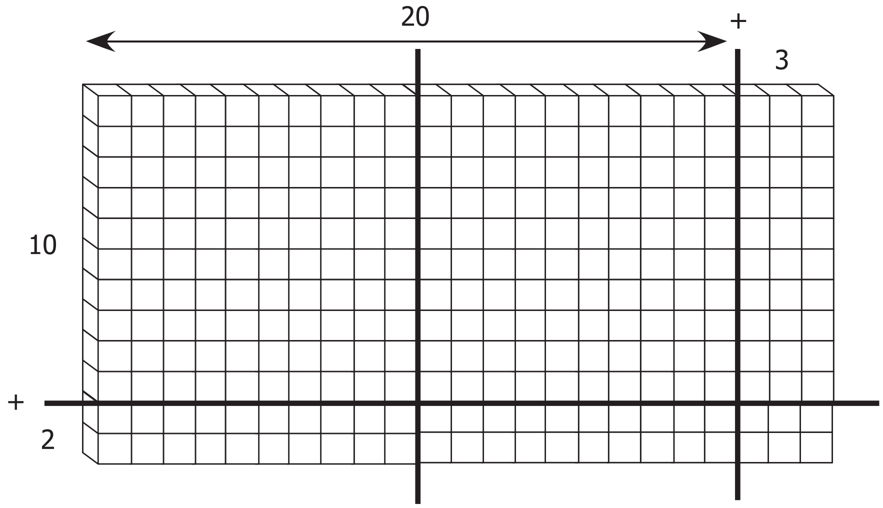
If the flat represents 100,

- its value is 100



Example:

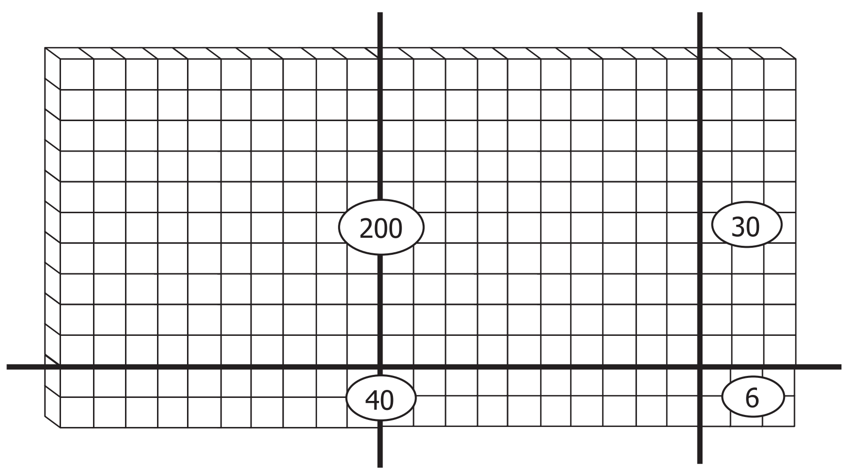
23 × 12



To determine the partial products, think of numbers that are easier to multiply. For example, represent numbers according to place value and think:

- "What is 10 × 20?" "200"
- "What is 10 × 3?" "30"
- "What is 20 × 2?" "40"
- "What is 3 × 2?" "6"

"So then, 23 × 12 must be 200 + 30 + 40 + 6, or 476."



It is important to establish a convention of keeping the blocks organized, as it will help with developing future representations.

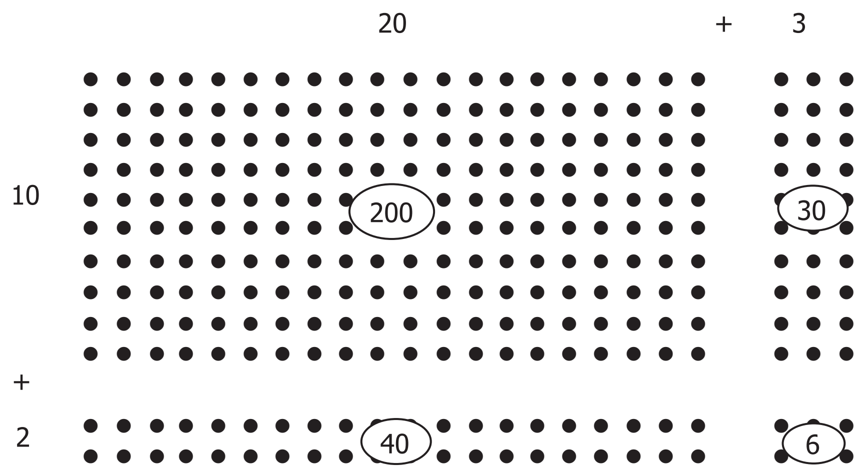
Base-10 blocks can be used to represent the active understanding of multiplication as a specific number of groups of a specific size, or the non-active array representation of a quantity.

When the blocks are arranged as a rectangle, the rectangle may be rotated and the quantity does not change. This is a verification of the commutative property of multiplication. The orientation of the array has no effect on the result, but in some places a convention has been established of representing the first number horizontally and the second vertically.

The array also serves as a model for area. It represents the area covered by a rectangle with a length of the multiplicand and a width of the multiplier, or vice versa.

Example:

23×12



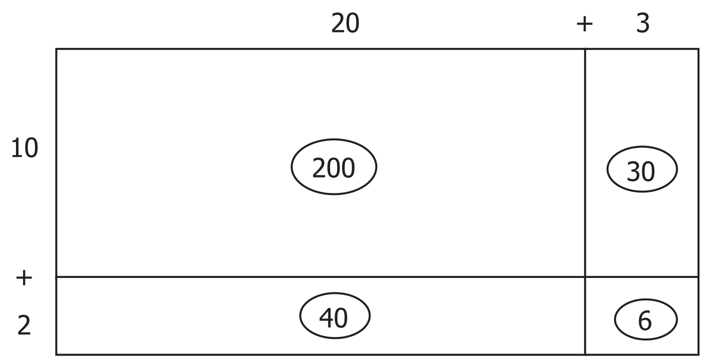
The partial products are determined in the same way as they are in the previous example.

The Area Model

Once students have an understanding of multiplication using base-10 blocks and/or base-10 paper, and they have a thorough understanding of the numerical value they represent, they can simply move to an area model. In this model, the length represents the multiplicand and the width represents the multiplier, or vice versa; however, the lengths and widths do not have to represent an exact measurement.

Example:

23×12

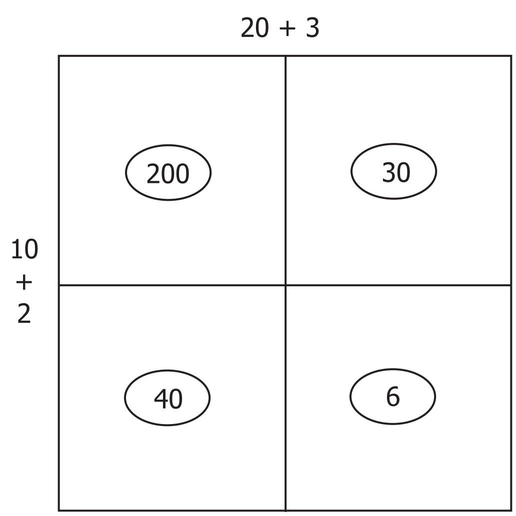


The partial products are determined in the same way as they are in the previous example.

This model is sometimes referred to as the "window-box" method, and is simplified further by completely ignoring the comparative sizes of the length and the width.

Example:

23×12



The partial products are determined in the same way as they are in the previous example.

Distributive Property

Once students have an understanding of multiplication using place value and are confident "rearranging" the numbers, representing multiplication using the distributive property will allow them to multiply symbolically.

Example:

23×12

Think $(20 + 3) \times (10 + 2)$

$= (20 \times 10) + (20 \times 2) + (3 \times 10) + (3 \times 2)$

$= 200 + 40 + 30 + 6$

$= 276$

Long Multiplication

Once students have an understanding of multiplication using place value, representing multiplication using this "long method" can lead them toward a better understanding of the traditional algorithm. In your modelling of this method, be sure to use appropriate mathematical language so as not to reinforce misconceptions about place value.

Example:

23×12

23		
× 12	Think:	
6	"What is 3 × 2?"	"6"
40	"What is 2 × 20?"	"40"
30	"What is 10 × 3?"	"30"
+ 200	"What is 10 × 20?"	"200"
276	"So 23 × 12 is 6 + 40 + 30 + 200 = 276."	

Compact Multiplication (The "Traditional Algorithm")

Once students have an understanding of the above models and methods, and they are confident with their understanding of multiplication and place value, the "traditional algorithm" for multiplication can be a quick and precise method of multiplying. In your modelling of this method, be sure to use appropriate mathematical language so as not to reinforce misconceptions about place value.

Example:

23×12

$$\begin{array}{r} 23 \\ \times\ 12 \\ \hline 46 \\ +\ 230 \\ \hline 276 \end{array}$$

Think:

"What is 3×2?"	"6"
"What is 2×20?"	"40"
"What is 10×3?"	"30"
"What is 10×20?"	"200"

"So 23×12 is $6 + 40 + 30 + 200 = 276$."

Note: There are certainly circumstances where mental mathematics is the most efficient way to multiply numbers.

Example:

24×15 can be thought about as:

- "What is 10×24?"
  - "240"
- "And half of that is?"
  - "120"
- "And so 24×15 must be?"
  - "240 + 120, so 360"