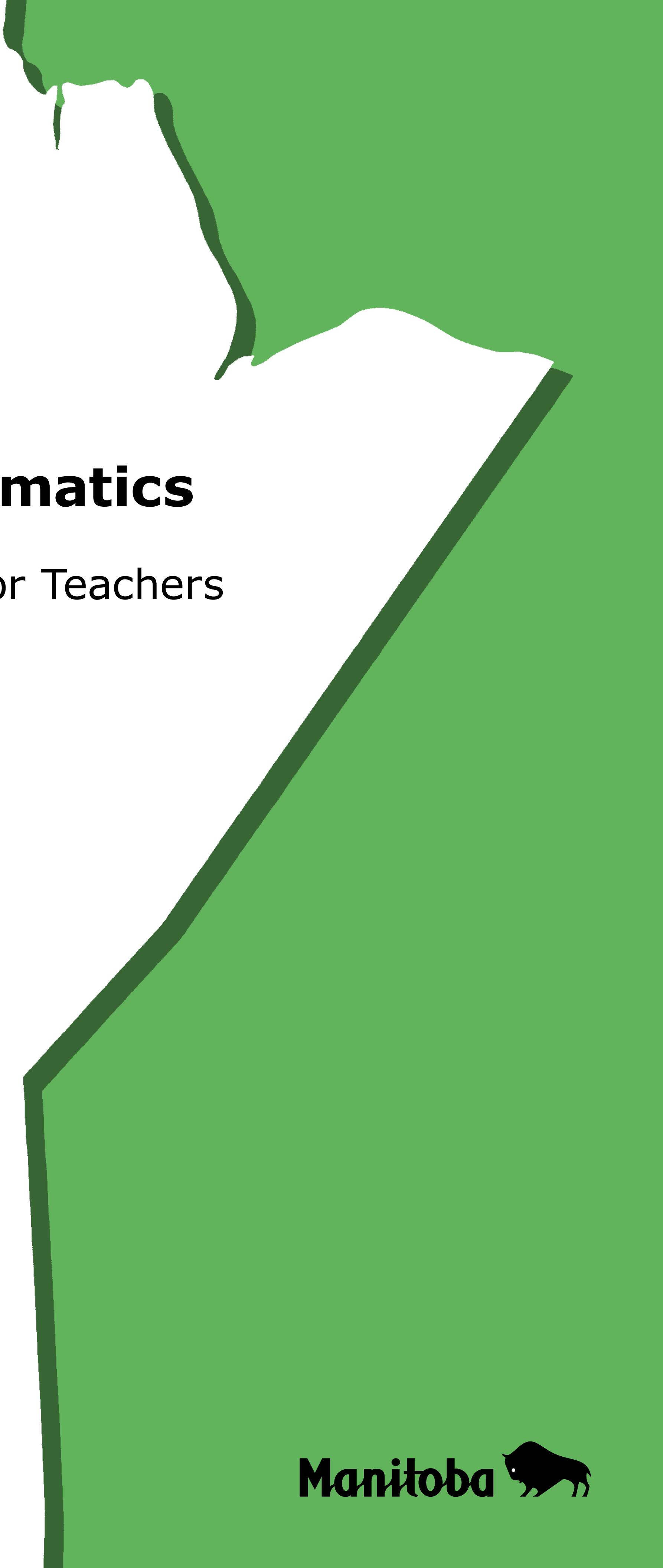
Grade 4 Mathematics

Support Document for Teachers



Manitoba

GRADE 4 MATHEMATICS

Support Document for Teachers

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List of Blackline Masters (BLMs)

Grade 4 Mathematics Blackline Masters

Number (N)

Patterns and Relations (PR)

Shape and Space (SS)

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INTRODUCTION

Purpose of This Document

Grade 4 Mathematics: Support Document for Teachers provides various suggestions for instruction, assessment strategies, and learning resources that promote the meaningful engagement of mathematics learners in Grade 4. The document is intended to be used by teachers as they work with students in achieving the learning outcomes and achievement indicators identified in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)* (Manitoba Education).

Background

Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes is based on *The Common Curriculum Framework for K–9 Mathematics*, which resulted from ongoing collaboration with the Western and Northern Canadian Protocol (WNCP). In its work, WNCP emphasizes

- common educational goals
- the ability to collaborate and achieve common goals
- high standards in education
- planning an array of educational activities
- removing obstacles to accessibility for individual learners
- optimum use of limited educational resources

The growing effects of technology and the need for technology-related skills have become more apparent in the last half century. Mathematics and problem-solving skills are becoming more valued as we move from an industrial to an informational society. As a result of this trend, mathematics literacy has become increasingly important. Making connections between mathematical study and daily life, business, industry, government, and environmental thinking is imperative. The Kindergarten to Grade 12 mathematics curriculum is designed to support and promote the understanding that mathematics is

- a way of learning about our world
- part of our daily lives
- both quantitative and geometric in nature

Beliefs about Students and Mathematics Learning

The Kindergarten to Grade 8 mathematics curriculum is designed with the understanding that students have unique interests, abilities, and needs. As a result, it is imperative to make connections to all students' prior knowledge, experiences, and backgrounds.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with unique knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem-solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Conceptual understanding: comprehending mathematical concepts, relations, and operations to build new knowledge. (Kilpatrick, Swafford, and Findell 5)

Procedural thinking: carrying out procedures flexibly, accurately, efficiently, and appropriately.

Problem solving: engaging in understanding and resolving problem situations where a method or solution is not immediately obvious. (OECD 12)

First Nations, Métis, and Inuit Perspectives

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural, and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning, and learn mathematics best when it is contextualized and not taught as discrete content.

Many First Nations, Métis, and Inuit students come from cultural environments where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks).

Affective Domain

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom learning activities, persist in challenging situations, and engage in reflective practices.

Teachers, students, and parents[*] need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward reaching these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessment of personal goals.

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[*] In this document, the term *parents* refers to both parents and guardians and is used with the recognition that in some cases only one parent may be involved in a child's education.

Early Childhood

Young children are naturally curious and develop a variety of mathematical ideas before they enter Kindergarten. Children make sense of their environment through observations and interactions at home, in daycares, in preschools, and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home.

Activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities.

Positive early experiences in mathematics are as critical to child development as early literacy experiences are.

Mathematics Education Goals for Students

The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between mathematical knowledge and skills and their applications
- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world

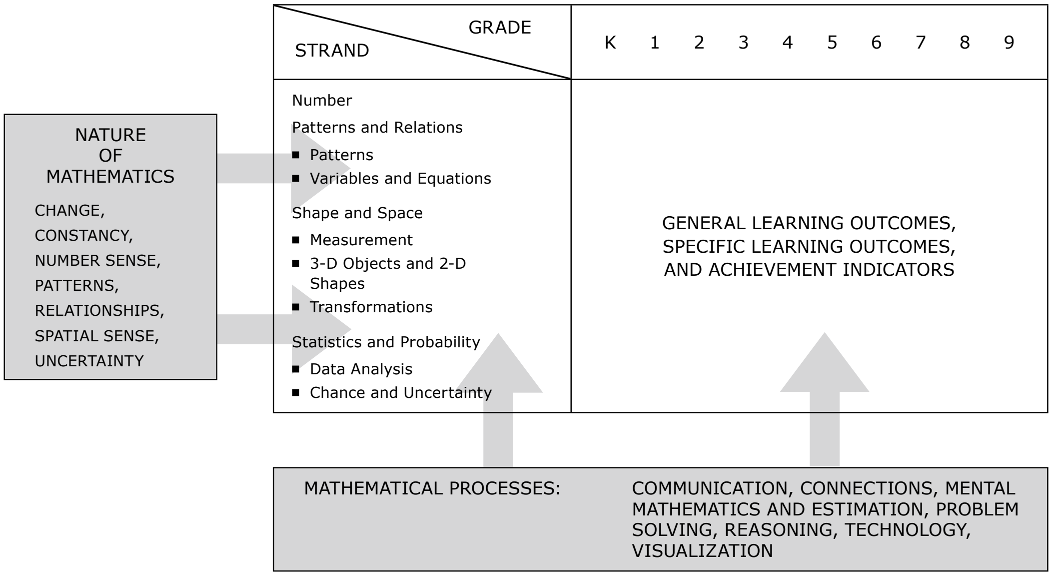
Mathematics education must prepare students to use mathematics to think critically about the world.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

CONCEPTUAL FRAMEWORK FOR KINDERGARTEN TO GRADE 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

There are critical components that students must encounter in mathematics to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The common curriculum framework incorporates these seven interrelated mathematical processes, which are intended to permeate teaching and learning:

- **Communication [C]:** Students communicate daily (orally, through diagrams and pictures, and by writing) about their mathematics learning. They need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. This enables them to reflect, to validate, and to clarify their thinking. Journals and learning logs can be used as a record of student interpretations of mathematical meanings and ideas.
- **Connections [CN]:** Mathematics should be viewed as an integrated whole, rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—concrete, pictorial, and symbolic (the symbolic mode consists of oral and written word symbols as well as mathematical symbols). The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas.
- **Mental Mathematics and Estimation [ME]:** The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills. Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense.
- **Problem Solving [PS]:** Students are exposed to a wide variety of problems in all areas of mathematics. They explore a variety of methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problems.
- **Reasoning [R]:** Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.
- **Technology [T]:** The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.
- **Visualization [V]:** Mental images help students to develop concepts and to understand procedures. Students clarify their understanding of mathematical ideas through images and explanations.

These processes are outlined in detail in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.

Strands

The learning outcomes in the Manitoba curriculum framework are organized into four strands across Kindergarten to Grade 9. Some strands are further subdivided into substrands. There is one general learning outcome per substrand across Kindergarten to Grade 9.

The strands and substrands, including the general learning outcome for each, follow.

Number

- Develop number sense.

Patterns and Relations

- Patterns
  - Use patterns to describe the world and solve problems.
- Variables and Equations
  - Represent algebraic expressions in multiple ways.

Shape and Space

- Measurement
  - Use direct and indirect measure to solve problems.
- 3-D Objects and 2-D Shapes
  - Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
- Transformations
  - Describe and analyze position and motion of objects and shapes.

Statistics and Probability

- Data Analysis
  - Collect, display, and analyze data to solve problems.
- Chance and Uncertainty
  - Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Learning Outcomes and Achievement Indicators

The Manitoba curriculum framework is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators:

- **General learning outcomes** are overarching statements about what students are expected to learn in each strand/substrand. The general learning outcome for each strand/substrand is the same throughout the grades from Kindergarten to Grade 9.
- **Specific learning outcomes** are statements that identify the specific skills, understanding, and knowledge students are required to attain by the end of a given grade.
- **Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success.

In this document, the word *including* indicates that any ensuing items **must be addressed** to meet the learning outcome fully. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are **not requirements that must be addressed** to meet the learning outcome fully.

Summary

The conceptual framework for Kindergarten to Grade 9 mathematics describes the nature of mathematics, the mathematical processes, and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Learning activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes, and lead students to an understanding of the nature of mathematics through specific knowledge, skills, and attitudes among and between strands. *Grade 4 Mathematics: Support Document for Teachers* is meant to support teachers to create meaningful learning activities that focus on formative assessment and student engagement.

ASSESSMENT

Authentic assessment and feedback are a driving force for the suggestions for assessment in this document. The purposes of the suggested assessment activities and strategies are to parallel those found in *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning* (Manitoba Education, Citizenship and Youth). These include the following:

- assessing *for*, *as*, and *of* learning
- enhancing student learning
- assessing students effectively, efficiently, and fairly
- providing educators with a starting point for reflection, deliberation, discussion, and learning

Assessment *for* learning is designed to give teachers information to modify and differentiate teaching and learning activities. It acknowledges that individual students learn in idiosyncratic ways, but it also recognizes that there are predictable patterns and pathways that many students follow. It requires careful design on the part of teachers so that they use the resulting information to determine not only what students know, but also to gain insights into how, when, and whether students apply what they know. Teachers can also use this information to streamline and target instruction and resources, and to provide feedback to students to help them advance their learning.

Assessment *as* learning is a process of developing and supporting metacognition for students. It focuses on the role of the student as the critical connector between assessment and learning. When students are active, engaged, and critical assessors, they make sense of information, relate it to prior knowledge, and use it for new learning. This is the regulatory process in metacognition. It occurs when students monitor their own learning and use the feedback from this monitoring to make adjustments, adaptations, and even major changes in what they understand. It requires that teachers help students develop, practise, and become comfortable with reflection, and with a critical analysis of their own learning.

Assessment *of* learning is summative in nature and is used to confirm what students know and can do, to demonstrate whether they have achieved the curriculum learning outcomes, and, occasionally, to show how they are placed in relation to others. Teachers concentrate on ensuring that they have used assessment to provide accurate and sound statements of students' proficiency so that the recipients of the information can use the information to make reasonable and defensible decisions.

Overview of Planning Assessment

	Assessment *for* Learning	**Assessment *as* Learning**	**Assessment *of* Learning**
Why Assess?	■ to enable teachers to determine next steps in advancing student learning	■ to guide and provide opportunities for each student to monitor and critically reflect on his or her learning and identify next steps	■ to certify or inform parents or others of student's proficiency in relation to curriculum learning outcomes
Assess What?	■ each student's progress and learning needs in relation to the curriculum outcomes	■ each student's thinking about his or her learning, what strategies he or she uses to support or challenge that learning, and the mechanisms he or she uses to adjust and advance his or her learning	■ the extent to which each student can apply the key concepts, knowledge, skills, and attitudes related to the curriculum outcomes
What Methods?	■ a range of methods in different modes that make a student's skills and understanding visible	■ a range of methods in different modes that elicit the student's learning and metacognitive processes	■ a range of methods in different modes that assess both product and process
Ensuring Quality	■ accuracy and consistency of observations and interpretations of student learning ■ clear, detailed learning expectations ■ accurate, detailed notes for descriptive feedback to each student	■ accuracy and consistency of a student's self-reflection, self-monitoring, and self-adjustment ■ engagement of the student in considering and challenging his or her thinking ■ the student records his or her own learning	■ accuracy, consistency, and fairness of judgments based on high-quality information ■ clear, detailed learning expectations ■ fair and accurate summative reporting
Using the Information	■ provide each student with accurate descriptive feedback to further his or her learning ■ differentiate instruction by continually checking where each student is in relation to the curriculum outcomes ■ provide parents or guardians with descriptive feedback about student learning and ideas for support	■ provide each student with accurate, descriptive feedback that will help him or her develop independent learning habits ■ have each student focus on the task and his or her learning (not on getting the right answer) ■ provide each student with ideas for adjusting, rethinking, and articulating his or her learning ■ provide the conditions for the teacher and student to discuss alternatives ■ the student reports his or her learning	■ indicate each student's level of learning ■ provide the foundation for discussions on placement or promotion ■ report fair, accurate, and detailed information that can be used to decide the next steps in a student's learning

Source: Manitoba Education, Citizenship and Youth. *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2006, 85.

INSTRUCTIONAL FOCUS

The Manitoba curriculum framework is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of learning outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Integration of the mathematical processes within each strand is expected.
- Problem solving, conceptual understanding, reasoning, making connections, and procedural thinking are vital to increasing mathematical fluency, and must be integrated throughout the program.
- Concepts should be introduced using manipulatives and gradually developed from the concrete to the pictorial to the symbolic.
- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Use educational resources by adapting to the context, experiences, and interests of students.
- Collaborate with teachers at other grade levels to ensure the continuity of learning of all students.
- Familiarize yourself with exemplary practices supported by pedagogical research in continuous professional learning.
- Provide students with several opportunities to communicate mathematical concepts and to discuss them in their own words.

"Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students" (Ontario 24).

DOCUMENT ORGANIZATION AND FORMAT

This document consists of the following sections:

- **Introduction:** The Introduction provides information on the purpose and development of this document, discusses characteristics of and goals for Early Years learners, and addresses First Nations, Métis, and Inuit perspectives. It also gives an overview of the following:
  - **Conceptual Framework for Kindergarten to Grade 9 Mathematics:** This framework provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.
  - **Assessment:** This section provides an overview of planning for assessment in mathematics, including assessment *for, as*, and *of* learning.
  - **Instructional Focus:** This discussion focuses on the need to integrate mathematics learning outcomes and processes across the four strands to make learning experiences meaningful for students.
  - **Document Organization and Format:** This overview outlines the main sections of the document and explains the various components that make up the various sections.
- **Number:** This section corresponds to and supports the Number strand for Grade 4 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013).*
- **Patterns and Relations:** This section corresponds to and supports the Patterns and Variables and Equations substrands of the Patterns and Relations strand for Grade 4 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013).*
- **Shape and Space:** This section corresponds to and supports the Measurement and 3-D Objects and 2-D Shapes substrands of the Shape and Space strand for Grade 4 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013).*
- **Statistics and Probability:** This section corresponds to and supports the Data Analysis substrand of the Statistics and Probability strand for Grade 4 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013).*
- **Blackline Masters (BLMs):** Blackline masters are provided to support student learning. They are available in *Microsoft Word* format so that teachers can alter them to meet students' needs, as well as in *Adobe* PDF format.
- **Bibliography:** The bibliography lists the sources consulted and cited in the development of this document.

Guide to Components and Icons

Each of the sections supporting the strands of the Grade 4 Mathematics curriculum includes the components and icons described below.

Enduring Understanding(s):

These statements summarize the core idea of the particular learning outcome(s). Each statement provides a conceptual foundation for the learning outcome. It can be used as a pivotal starting point in integrating other mathematics learning outcomes or other subject concepts. The integration of concepts, skills, and strands remains of utmost importance.

General Learning Outcome(s):

General learning outcomes (GLOs) are overarching statements about what students are expected to learn in each strand/substrand. The GLO for each strand/substrand is the same throughout Kindergarten to Grade 8.

SPECIFIC LEARNING OUTCOME(S):	**ACHIEVEMENT INDICATORS:**
Specific learning outcome (SLO) statements define what students are expected to achieve by the end of the grade. A code is used to identify each SLO by grade and strand, as shown in the following example: **4.N.1** The first number refers to the grade (Grade 4). The letter(s) refer to the strand (Number). The last number indicates the SLO number. [C, CN, ME, PS, R, T, V] Each SLO is followed by a list indicating the applicable mathematical processes.	Achievement indicators are examples of a representative list of the depth, breadth, and expectations for the learning outcome. The indicators may be used to determine whether students understand the particular learning outcome. These achievement indicators will be addressed through the learning activities that follow.

PRIOR KNOWLEDGE

Prior knowledge is identified to give teachers a reference to what students may have experienced previously. Teachers should assess students' prior knowledge before planning instruction.

BACKGROUND INFORMATION

Background information is identified to give teachers knowledge about specific concepts and skills related to the particular learning outcome(s).

MATHEMATICAL LANGUAGE

Lists of terms students will encounter while achieving particular learning outcomes are provided. These terms can be placed on mathematics word walls or used in a classroom mathematics dictionary. *Kindergarten to Grade 8 Mathematics Glossary: Support Document for Teachers* (Manitoba Education, Citizenship and Youth) provides teachers with an understanding of key terms found in Kindergarten to Grade 8 mathematics. The glossary is available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/cur/math/supports.html.

LEARNING EXPERIENCES

Suggested instructional strategies and assessment ideas are provided for the specific learning outcomes and achievement indicators. In general, learning activities and teaching strategies related to specific learning outcomes are developed individually, except in cases where it seems more logical to develop two or more learning outcomes together. Suggestions for assessment include information that can be used to assess students' progress in their understanding of a particular learning outcome or learning experience.

Assessing Prior Knowledge:

Observation Checklist:

Assessing Understanding:

Suggestions are provided for assessing prior to and after lessons, and checklists are provided for observing during lessons to direct instruction.

- **Achievement indicators appropriate to particular learning experiences are listed.**

Suggestions for Instruction

The instructional suggestions include the following:

- **Materials/Resources:** Outlines the resources required for a learning activity.
- **Organization:** Suggests groupings (individual, pairs, small group, and/or whole class).
- **Procedure:** Outlines detailed steps for implementing suggestions for instruction.

Some learning activities make use of BLMs, which are found in the Blackline Masters section in *Microsoft Word* and *Adobe* PDF formats.

PUTTING THE PIECES TOGETHER

Putting the Pieces Together tasks, found at the end of the learning outcomes, consist of a variety of assessment strategies. They may assess one or more learning outcomes across one or more strands and may make cross-curricular connections.

Notes

GRADE 4 MATHEMATICS

Number

Grade 4: Number (4.N.1, 4.N.2)

Enduring Understandings:

Numbers can be represented in a variety of ways (e.g., using objects, pictures, and numerals).

Place value patterns are repeated in large numbers, and these patterns can be used to compare and order numbers.

The position of a digit in a number determines the quantity it represents.

There is a constant multiplicative relationship between the places.

Essential Questions:

How many different ways can a number be represented?

How does changing the order of the digits in a number affect its placement on a number line?

How are place value patterns repeated in numbers?

How does the position of a digit in a number affect its value?

SPECIFIC LEARNING OUTCOME(S):	**ACHIEVEMENT INDICATORS:**
4.N.1 Represent and describe whole numbers to 10 000, pictorially and symbolically. [C, CN, V]	➛ Read a four-digit numeral without using the word "and" (e.g., 5321 is five thousand three hundred twenty-one, NOT five thousand three hundred AND twenty-one). ➛ Write a numeral using proper spacing without commas (e.g., 4567 or 4 567, 10 000). ➛ Write a numeral 0 to 10 000 in words. ➛ Represent a numeral using a place value chart or diagrams. ➛ Describe the meaning of each digit in a numeral. ➛ Express a numeral in expanded notation (e.g., 321 = 300 + 20 + 1). ➛ Write the numeral represented in expanded notation. ➛ Explain the meaning of each digit in a 4-digit numeral with all digits the same (e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens, and the fourth digit two ones).

SPECIFIC LEARNING OUTCOME(S):		ACHIEVEMENT INDICATORS:
4.N.2	Compare and order numbers to 10 000. [C, CN]	➛ Order a set of numbers in ascending or descending order, and explain the order by making references to place value. ➛ Create and order three 4-digit numerals. ➛ Identify the missing numbers in an ordered sequence or between two benchmarks on a number line (vertical or horizontal). ➛ Identify incorrectly placed numbers in an ordered sequence or between two benchmarks on a number line (vertical or horizontal).

PRIOR KNOWLEDGE

Students may have had experience

- representing and describing numbers to 1000, concretely, pictorially, and symbolically
- comparing and ordering numbers to 1000 (999)
- illustrating, concretely and pictorially, the meaning of place value for numerals to 1000 (hundreds, tens, and ones)

BACKGROUND INFORMATION

As a convention, the word *and* is reserved for the reading of decimal numbers. The reading of number words such 625 should be read as “six hundred twenty-five.” Many people, especially adults, use *and* inappropriately. Have students listen for and record examples of the misuse of the word *and*.
Note: In some other countries numbers are read using *and*.

Four-digit numbers can be written with or without a space between the hundreds and the thousands digits. Writing numbers that are five or more digits requires a space between the thousands and hundreds place (10 000).

Note: Students will see commas used in many resources and situations.

Meaningful real-life contexts (e.g., population data from a social studies unit) should be explored in order to help students develop an understanding of the relative size (magnitude) of numbers.

According to Kathy Richardson in her book, *How Children Learn Number Concepts: A Guide to the Critical Learning Phases* (145), in order for students to understand the structure of thousands, hundreds, tens, and ones they need to be able to

- count one thousand as a single unit
- know the total instantly when the number of thousands, hundreds, tens, and ones is known
- mentally add and subtract 10 and 100 to/from any four-digit number
- know the number of thousands that can be made from any group of hundreds, and the number of hundreds left over (e.g., 15 hundreds is 1 thousand and 5 hundreds)
- describe any number from 1000 to 10 000 in terms of its value in ones, or tens, or hundreds (e.g., 3400 is 34 hundreds, 3400 ones, and 3 thousand and 4 hundred)
- determine the total value of groups of thousands, hundreds, tens, and ones by reorganizing them into all possible thousands, hundreds, tens with leftover ones (e.g., 6 thousands, 27 hundreds, 45 ones can be reorganized to make 8745)

Preventing Misconceptions: The way we talk about concepts/ideas can create misconceptions for students. For example: Students are shown the number 168 and asked, “How many tens are in this number?” Generally, the expected response is “6” but in fact, there are 16 tens in 168. Rephrasing the question to ask, “How many tens are in the tens place in this number?” may help prevent misconceptions.

MATHEMATICAL LANGUAGE

place value
thousand
hundreds
tens
ones
expanded notation
numeral
digit

benchmark
vertical
horizontal
greatest
least
ascending order
descending order

Assessing Prior Knowledge

Interview:

Give students a 3-digit number such as 264. Have them explain the meaning of each digit using base-10 materials, Digi-Blocks, or teacher/student-made representations, to support their explanation.

The student is able to

☐ use materials to represent a 3-digit number
☐ explain that the first digit represents 2 hundreds (e.g., two hundred blocks)
☐ explain that the second digit represents 6 tens (e.g., six ten blocks)
☐ explain that the third digit represents 4 ones (e.g., four single blocks)

Paper-and-Pencil Task:

1. Roll a 0-to-9 die three times. Record the numbers. (If any of the numbers are the same, roll the die again.)

   Make as many 3-digit numbers as you can.

   Order the numbers from greatest to least.
2. Choose one of the numbers you made. Explain the value of each digit. Use pictures and words.
3. Choose another number. Represent it in at least 6 different ways using what you know about place value.

- **Read a four-digit numeral without using the word "and" (e.g., 5321 is five thousand three hundred twenty one, NOT five thousand three hundred AND twenty one).**
- **Write a numeral using proper spacing without commas (e.g., 4567 or 4 567, 10 000).**
- **Write a numeral 0 to 10 000 in words.**
- **Represent a numeral using a place value chart or diagrams.**
- **Describe the meaning of each digit in a numeral.**
- **Express a numeral in expanded notation (e.g., 321 = 300 + 20 + 1).**
- **Write the numeral represented in expanded notation.**
- **Explain the meaning of each digit in a 4-digit numeral with all digits the same (e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens, and the fourth digit two ones).**

Representing Numbers

Students should be able to represent numbers in standard form, expanded notation, words, and with models such as tent/arrow cards, base-10 materials, money, and place-value charts.

Standard form is the usual form of a number, where each digit is in its place value.
Example: twenty-nine thousand three hundred four is written as 29 304

Expanded notation is a way to write a number that shows the value of each digit.
Example: 4556 = 4000 + 500 + 50 + 6

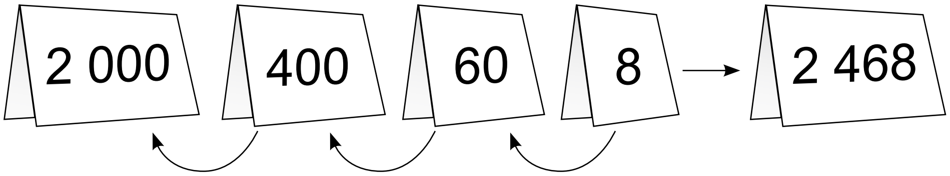
Suggestions for Instruction

BLM
4.N.1.1

- **Standard Form, Expanded Form, and Words:** This can be part of a Number of the Day routine. See BLM 4.N.1.1 for an example of a Number of the Day.
- **Tent Cards:** Place value tents/arrows help students to see the relationship between a digit and its value based on its position in the number.

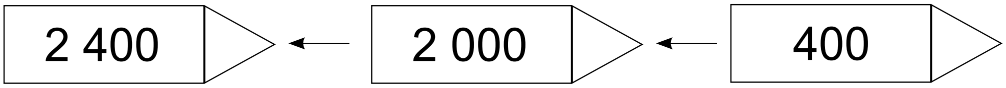
  Tent cards can be used to build numbers from their expanded form. They nest one on top of the other. They can also be used to move from the standard form to the expanded form (pulling apart the number). They can be downloaded from http://www.edu.gov.mb.ca/k12/cur/math/games/index.html.

  Example:



- **Arrow cards** are a set of place value cards with an arrow on the right side. They can be organized horizontally or vertically to represent numbers in expanded notation. Cards can be overlapped by lining up the arrows to form multi-digit numbers.

  Example:



- **Base-10 Materials:** These are proportional materials, which means that each block is 10 times larger than previous one (e.g., the flat is 10 times as large as the long).

Have students use the blocks to solve problems such as the following:

- Make the number that is one less than 1000.
  - If you have ten longs, what is the total value?
  - If you were able to break up the thousands block, how many flats would you have? How many longs? How many ones blocks?
  - Make the number 3468 with the blocks.
  - Make the number 2008.
  - Use five base-10 blocks. Make six different numbers. Each number must have at least one thousand block. Record your answers using pictures and numbers.
  - Problem: Samuel has seven base-10 blocks. The value of these blocks is more than 3000 and less than 3902. Which blocks might Samuel have chosen? Give four possible answers and explain your choices.

Extension: Find all the possible numbers.

- **Place-Value Chart:** Build numbers in the place-value chart. Be sure to include numbers with zeroes.

Example: Show the number 3 057.

Thousands			Ones		
hundreds	tens	ones	hundreds	tens	ones
		3	0	5	7

Transfer the information on the place value chart to standard form. (5 902)

Thousands			Ones		
hundreds	tens	ones	hundreds	tens	ones
		5	9	0	2

Note: Placing numbers on the place-value chart and transferring them from the chart can become a rote procedure that students can often accomplish without understanding. Using non-standard place value representations can challenge student thinking and allow them to demonstrate their understanding.

Examples:

Show 3 thousands, 46 tens, 8 ones on the chart.

Thousands			Ones		
hundreds	tens	ones	hundreds	tens	ones
		3	4	6	8

Write the number shown on the chart in standard form. (1523)

Thousands			Ones		
hundreds	tens	ones	hundreds	tens	ones
			14	12	3

- **Money:** Money can be used as a representation. Ask questions such as, “A large swimming pool costs $4 982.00. If you paid for it with hundred dollar bills, how many would you need? If you paid with ten dollar bills, how many would you need? If you paid with loonies, how many would you need?

  Pictures/charts can also be used.

$100	$10	$1
		

- **Make the Number?** Students write numbers following the directions given.

  Example:

  Write two different numbers that match the directions.

  1. 2 in the thousands place and 4 in the hundreds place (Answers may be a variety of numbers such as 2400, 2410, 2456, 2479, etc., but there must be a 2 in the thousands place and 4 in the hundreds place.)
  2. 8 in the tens place and 5 in the hundreds place
  3. 7 in the thousands place and 3 in the ones place
  4. 9 in the tens place and 6 in the thousands place

BLM
4.N.1.2

- **Renaming Numbers:** As a grouping or sorting activity, use a set of cards that have different ways of representing numbers. (If used for grouping, decide on the number of groups needed and then use one number for each group.)

  Example:

  In order to make 4 groups of 5, use a set such as the following:

4230	1305	2087	4387
4000 + 200 +30	1000 +300 + 5	2000 + 80 + 7	4000 + 300 +80 +7
423 tens	130 tens 5 ones	208 tens 7 ones	3 th 13 h 8 t 7 ones
3 th 12 h 3 t	1 th 2 h 10 t 5 ones	1 th 10 h 8 t 7 ones	438 tens 7 ones
4 th 1 h 13 tens	1305 ones	207 tens 17 ones	4387 ones

  Randomly pass out the cards and have students find their group members.

- **Calculator Wipe It Out!** The object of the activity/game is to wipe/zero out one or more digits from the display using subtraction. Initially the digits should all be different.

  Example:

  Students enter the number 3268 on their calculator. Ask them to “wipe out” only the numeral 6 or to make the display show 3208. Explain what they subtracted and why they chose that particular number. Students should also communicate what they will do before they press the buttons, and what number will be gained by removing the digit.

  Variation of the game: Use addition.

Example: How can you use addition to wipe out the 6? (Add 40)

Alternative ways to play the game:

Example: Enter 4537.

- Using addition, turn the 7 into a 2.
- Using subtraction, turn the 5 into a 3.
- Wipe out more than one place value position (e.g., Make the display show 4007).
- Make your display show 2000.
- Make your display show 0.

- **Place Value Game:**

BLM 4.N.1.3

Materials: a spinner with place value positions (BLM 4.N.1.3) and a 0-to-9 spinner (BLM 4.N.1.3) to be shared, and a white board or other erasable surface (page protector) with a place-value chart (BLM 4.N.1.3) for each player

Directions:

1. Each player draws a place-value chart on their board.

   Example:

TH	H	T	O

2. Player 1 spins the 0-to-9 spinner and the place-value spinner and enters the number in the correct position on their board. If the place is already filled, their turn is over.
3. The first person to complete their chart scores 10 points, the second 8 points, and so on.

   Extension: Bonus points can be given to the player with the largest/smallest number.
4. The game ends when a player reaches the point goal (set at the start of the game).

Assessing Understanding: Paper-and-Pencil Task

1. Dictate the numbers and have students record.

   4 651

   2 075

   1 902

   8 364

   5 008

2. Write the following numbers in words.

   7 268 ____

   4 080 ____

   5 921 ____

   6 004 ____

3. Write the following numbers in expanded form.

   1 634 ____

   9 999 ____

   2 100 ____

   7 305 ____

4. Explain the value of each digit in the number 4444.

   ____

   ____

   ____

   ____

5. Fill in the blanks to make these true:

   6070 = _____ hundreds + _____ tens

   3254 = _____ hundreds + _____ ones

   1280 = _____ tens

   2900 = _____ hundreds or _____ tens

- **Order a set of numbers in ascending or descending order, and explain the order by making references to place value.**
- **Create and order three 4-digit numerals.**
- **Identify the missing numbers in an ordered sequence or between two benchmarks on a number line (vertical or horizontal).**
- **Identify incorrectly placed numbers in an ordered sequence or between two benchmarks on a number line (vertical or horizontal).**

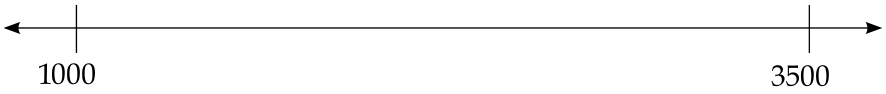
Suggestions for Instruction

Students should be exposed to both vertical (thermometers, measuring cups, etc.) and horizontal number lines. Discussions related to the importance of scale (the distance/difference between the reference points) will assist students in determining the placement of a number in relative position.

- **Number Line:** Provide a number line with end/reference points identified. Have students place given whole numbers on the number line.

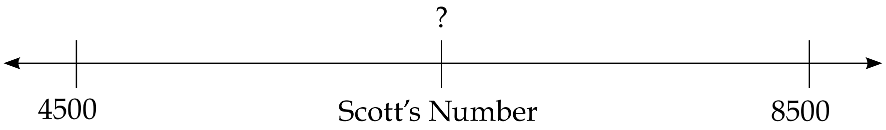
Examples:

1.



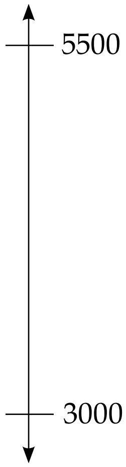
Where would 2450 be on the number line?

2.



Scott placed a number on the number line. What might his number be? Explain your answer.

3. Place 4750 on the number line.



Suggestion: Set up a clothesline (a string held up by a couple of magnets) in the classroom. Write numbers on tent cards (paper that is folded so that it loops over the string). Identify the end points (reference points). Have students place given numbers on the line and then justify their placement.

- **Roll the Dice:** Students roll a 0-to-9 die four times and record the numbers shown as a 4-digit number on an erasable surface or on paper. Have students use their numbers to arrange themselves from greatest to least (descending order). This can be made more challenging by doing it without talking.

  Note: It is important that students are aware that when comparing two numbers with the same number of digits, the digit with the greatest value should be focused on first. For example, when asked to explain why one number is greater or less than another, they might say that 2541 is less than 3652 because 2541 is less than 3 thousands while 3652 is more than 3 thousands. When comparing 5367 and 5489, students will begin comparing the thousands and move to the right.

- **Find the Error:** Prepare sets of numbers that have been ordered from least to greatest (ascending order) or greatest to least but with one or two errors. Have students identify the error(s) and then write them in the correct order.

  Example:

4000	4004	4040	4404	4044	4400
			X	X	X

  Correct order: 4000 4004 4040 4044 4400 4404

  Have students make of sets for the class to solve.

- **What Number Fits?** Give two reference points and have students write a number that fits between them.

  Example:

  Write a number that lies between

  5100 and 5200

  3199 and 4019

  8490 and 9500

  1250 and 1285

- **More and Less:** Use a double set of 0-to-9 digit cards for each student. Dictate a 4-digit number and have them make it with their digit cards (e.g., 4251). Give directions such as the following:
  - Make the number that is 200 more than 4251.
  - Make the number that is 1000 less than 4251.
  - Make the number that is 7 more than 4251.
  - Make the number that is 40 more than 4251.
  - Make the number that is 900 more than 4251.

  Observe students as they work. Do they have to remake the number from scratch or do they change only the place value position(s) affected?

- **Greater or Less Than:** Have students compare numbers in different ways. The comparisons should reference the understanding of place value in explanations.

  Ask questions such as:

  A. Which number is greater? Why?

  1. 6005 or 6050
  2. 4209 or 4029
  3. 3124 or 3214
  4. 7642 or 6742

  B. Fill in the missing digits so that the first number is greater than the second number.

  1. 5☐21 > 5☐21
  2. ☐250 > 6368
  3. 20☐9 > 2049
  4. 7306 > 7☐☐6

  Note: The use of the greater than (>) and less than (<) symbols are not taught formally until Middle Years. However, the symbols can be introduced earlier. The symbols are conventions of mathematics and should be introduced once students have a solid understanding of the concepts of greater than and less than. (Try to have students determine and share their own ways to remember symbols. For example, "I put 2 dots [colon] beside the larger number and 1 dot beside the smaller number and then I join the dots to make the symbol.")

- **Mystery Number:** Have students write Mystery Number riddles for the class to solve.

  Examples:

  1. I am a 4-digit number between 4500 and 6000.
     I am odd.
     I am a multiple of 5.
     The digit in the thousands place is repeated in the ones place.
     The sum of my digits is 17.
     The digit in the tens place is 2 more than the digit in the ones place.
     What number am I? (5075)
  2. I am a 4-digit number.
     I am even.
     The digit in the ones place is 4 times larger than the digit in the thousands.
     The digit in the tens place is 7 less than the digit in the ones place.
     The digit in the hundreds place is 5 more than the digit in the tens place.
     The sum of my digits is 17.
     What number am I? (2618)

- **Twenty Questions:** Think of a 4-digit number. Place dashes on the board to indicate the number of digits. Students ask questions to determine the number. Keep a tally of the number of questions asked. If the number is guessed in less than 20 questions, the students win. If not, the teacher/leader wins. (After modelling by the teacher, students should assume the role of leader for this game.)

  Example:

  ☐☐☐☐

  Question examples:

  - "Is there a three in the tens place?"
  - "Is the number greater than 5000?"
  - "Is there a 5 anywhere in the number?" (A yes doesn't mean that the 5 is then placed on one of the blanks. Students would still have to determine its position in the number through additional questions.)
  - "Is the number odd?"
  - "Does the number have more than 20 tens?"

- **Higher or Lower:** Students play in groups of three (2 players and 1 leader). The leader secretly writes down a 4-digit number and then gives players the range (e.g., "The number is between 5 000 and 6 000"). Each player draws a number line, marking the reference points.

  The first player gives a possible number, and the leader tells them whether the number is higher or lower than the one chosen. The players record the response on their number lines. The game continues in this manner until one player gives the correct number.

Have students discuss the strategies they used to determine the secret number.

- **Guess My Number:** Prepare a card/piece of paper (a strip of masking tape will work) with a 4-digit number written on it for each student. Tape one card on each student's back. Students ask their classmates questions requiring a "yes" or "no" answer in order to determine their number. Limit the questions they can ask to one per classmate. (e.g., Am I greater than 5000? Am I less than 6000? Am I an even number? Am I a multiple of 10?)

  When all numbers have been identified, have students line up in order (ascending/descending).

Assessing Understanding: Performance Task/Observation/Interview

Materials: a deck of playing cards with the face cards and tens removed (aces count as 1 and the jokers count as 0) or use 4 sets of 0-to-9 numeral cards.

Organization: Work with a small group of students (4 or 5).

Directions:

Player A turns over 4 cards from the deck. Each player then arranges the cards to make a different 4-digit number. Player A records the numbers on individual pieces of paper/cards and keeps them in a pile. Players each take turns turning over four cards and recording the group's 4-digit numbers. Play continues until each student has had a turn.

Have each player order their numbers in ascending or descending order.

Ask students to

☐ read each number

☐ explain how they know their ordering is correct

☐ pick one of their numbers and identify the place value of each numeral

☐ pick one of the numbers and identify the number before and after

☐ pick one of the numbers and represent it in as many ways as they can (words, expanded form, base 10)*

☐ count forwards/backwards by tens/hundreds/thousands from one of the numbers

* Students can be doing this while the teacher interviews individual students.

Extend the activity by combining all of the number cards and have the group order them in ascending or descending order. Observe the process.

Grade 4: Number (4.N.3)

Enduring Understandings:

Quantities can be taken apart and put together.

Addition and subtraction are inverse operations.

There are a variety of appropriate ways to estimate sums and differences depending on the context and the numbers involved.

Essential Questions:

How can symbols be used to represent quantities, operations, or relationships?

How can strategies be used to compare and combine numbers?

What questions can be answered using subtraction and/or addition?

How can place value be used when adding or subtracting?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.N.3 Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals), concretely, pictorially, and symbolically, by • using personal strategies • using the standard algorithm • estimating sums and differences • solving problems [C, CN, ME, PS, R]	➛ Model addition and subtraction using concrete materials and visual representations, and record the process symbolically. ➛ Determine the sum of two numbers using a personal strategy (e.g., for 1326 + 548, record 1300 + 500 + 74). ➛ Determine the difference of two numbers using a personal strategy (e.g., for 4127 – 238, record 238 + 2 + 60+ 700 +3000 + 127 or 4127 – 27 – 100 – 100 – 11). ➛ Model and explain the relationship that exists between an algorithm, place value, and number properties. ➛ Determine the sum and difference using the standard algorithms of vertical addition and subtraction. (Numbers are arranged vertically with corresponding place value digits aligned.) ➛ Describe a situation in which an estimate rather than an exact answer is sufficient. ➛ Estimate sums and differences using different strategies (e.g., front-end estimation and compensation). ➛ Solve problems that involve addition and subtraction of more than 2 numbers. ➛ Refine personal strategies to increase efficiency when appropriate (e.g., 3000 – 2999 should not require the use of an algorithm).

PRIOR KNOWLEDGE

Students may have an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2-, and 3-digit numerals) by

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially, and symbolically.

They may be able to describe and apply mental math strategies for adding and subtracting two 2-digit numerals including

- adding from left to right
- taking one addend to the nearest multiple of 10 and then compensating
- using doubles
- taking the subtrahend to the nearest multiple of ten and then compensating
- thinking of addition

They may be able to apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context.

They may be able to recall addition and related subtraction facts to 18.

BACKGROUND INFORMATION

There are many different types of addition and subtraction problems. Students should have experience with all types.

<table>
<tr><th colspan="4">Addition</th><th>Both
+ and -</th></tr>
<tr><th>Result Unknown (a + b = ?)</th><th>Change Unknown (a + ? = c)</th><th>Start Unknown (? + b = c)</th><th>Combine (a + b = ?)</th><th>Compare</th></tr>
<tr><td>Pat has 8 marbles. Her brother gives her 4. How many does she have now?
(8 + 4 = ?)</td><td>Pat has 8 marbles but she would like to have 12. How many more does she need to get?
(8 + ? = 12)</td><td>Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start with?
(? + 4 = 12)</td><td>Pat has 8 blue marbles and 4 green marbles. How many does she have in all?
(8 + 4 = ?)</td><td rowspan="4">Pat has 8 blue marbles and 4 green marbles. How many more blue marbles does she have?
(8 – 4 = ? or 4 + ? = 8)
Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have?
(8 – 4 = ? or 4 + ? = 8)</td></tr>
<tr><th colspan="4">Subtraction</th></tr>
<tr><th>Result Unknown (a – b = ?)</th><th>Change Unknown (a – ? = c)</th><th>Start Unknown (? – b = c)</th><th>Combine</th></tr>
<tr><td>Pat has 12 marbles. She gives her brother 4 of them. How many does she have left?
(12 – 4 = ?)</td><td>Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother?
(12 – ? = 8)</td><td>Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start with?
(? – 4 = 8)</td><td>Pat has 12 marbles. 8 are blue and the rest are green. How many are green?
(12 – 8 = ?)</td></tr>
</table>

The **standard algorithm** is a procedural method for performing a mathematical computation. It should be introduced after students have demonstrated a conceptual understanding of the operations through the use of concrete materials, visual representations, and personal strategies. The term *traditional algorithm* is used to indicate the symbolic algorithm traditionally taught in North America.

Front-end estimation: A method for estimating an answer to a calculation problem by focusing on the front-end or left-most digits of a number (e.g., 2356 + 1224 is estimated to be 2000 + 1000 = 3000).

Compensation: This strategy involves rounding one quantity up and the other down. For example, 1 752 + 648 would be thought of as 1 700 + 700. The 1 700 is a low estimate for 1 752 so the 648 is estimated as 700 (a high estimate) in order to compensate.

MATHEMATICAL LANGUAGE

Operations:
addition
add
sum
total
more
subtraction
subtract
difference
less
take away
story problem
number sentence
estimate
addition fact
subtraction fact
strategy
standard algorithm
regroup
exchange
front-end estimation
compensation

Instructional Strategies: Consider the following guidelines for teaching addition and subtraction:

- Teach through problem solving.
- Select thought-provoking problems that are meaningful for students (relate to their own lives).
- Ensure that students understand the problem without inadvertently directing them to the way to solve the problem.
- Have students estimate the answer to the problem first.
- Ensure that students have access to manipulatives if they need them.
- Provide time for students to think individually before having them share/discuss with their partner, group, or whole class.
- Circulate, listen, observe, encourage, and/or question without telling or evaluating strategies. Carefully selected questions can help students move forward when they are “stuck.”
- Once a solution has been reached, have students compare the answer with their initial estimate.
- Orchestrate the sharing and critiquing of strategies. Which strategies worked? Which strategy was the most efficient? Have students justify their solutions.
- Have students create their own problems. An addition or subtraction number sentence can be provided or just the answer (e.g., The answer is 1250. What is the question?).

Assessing Prior Knowledge: Paper-and-Pencil Task

A. Solve the problems.

Be sure to show your work.

1. The students in Mrs. Johnson's class collected aluminum cans for recycling. Jana collected 214 cans. Mason collected 206 cans, and Marilyn collected 255 cans. How many cans did they collect altogether?
2. The elementary school has 457 students. If 232 of the students are boys, how many girls are in the school?
3. Simone has two jars of buttons. One jar has 326 buttons and the other jar has 387 buttons. How many buttons does Simone have altogether?
4. The answer is 236. What is the question?
   Write an addition problem that has an answer of 236.
5. The answer is 154. What is the question?
   Write a subtraction problem that has an answer of 154.

B. Show two different ways to solve each question.

218 + 407 = ______ 683 – 364 = ______

- **Model addition and subtraction using concrete materials and visual representations, and record the process symbolically.**
- **Determine the sum of two numbers using a personal strategy (e.g., for 1326 + 548, record 1300 + 500 + 74).**
- **Determine the difference of two numbers using a personal strategy (e.g., for 4127 – 238, record 238 + 2 + 60+ 700 +3000 + 127 or 4127 – 27 – 100 – 100 – 11).**
- **Model and explain the relationship that exists between an algorithm, place value, and number properties.**
- **Determine the sum and difference using the standard algorithms of vertical addition and subtraction. (Numbers are arranged vertically with corresponding place value digits aligned.)**
- **Solve problems that involve addition and subtraction of more than 2 numbers.**
- **Refine personal strategies to increase efficiency when appropriate (e.g., 3000 – 2999 should not require the use of an algorithm).**

Suggestions for Instruction

There are many different strategies that can be used for addition and subtraction.

Possible Strategies for Addition

Each of the following are strategies to calculate 1382 + 126.

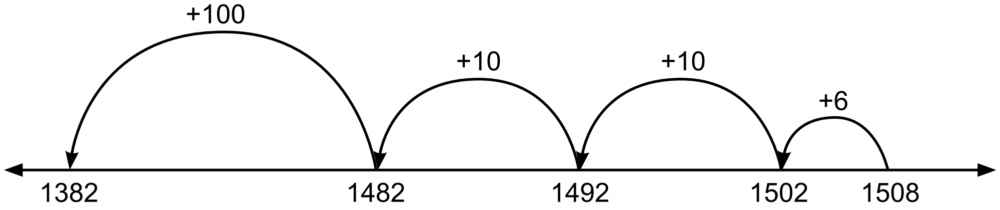
Breaking Up Numbers Using Place Value (Split Strategy)

This method requires place value understanding.

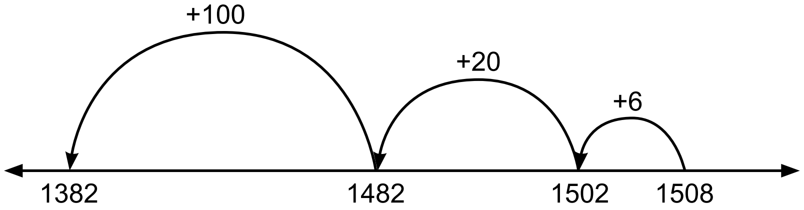
1382 + 126 = 1508
(1000 + 300 + 80 + 2) + (100 + 20 + 6)
1000 + 400 + 100 + 8 = 1000 + 500 + 8 = 1508

Note: As the size of the numbers increases, it is more difficult for students to use this method mentally. This strategy is easily demonstrated with base-10 blocks.

Empty Number Line (Jump Strategy)



or



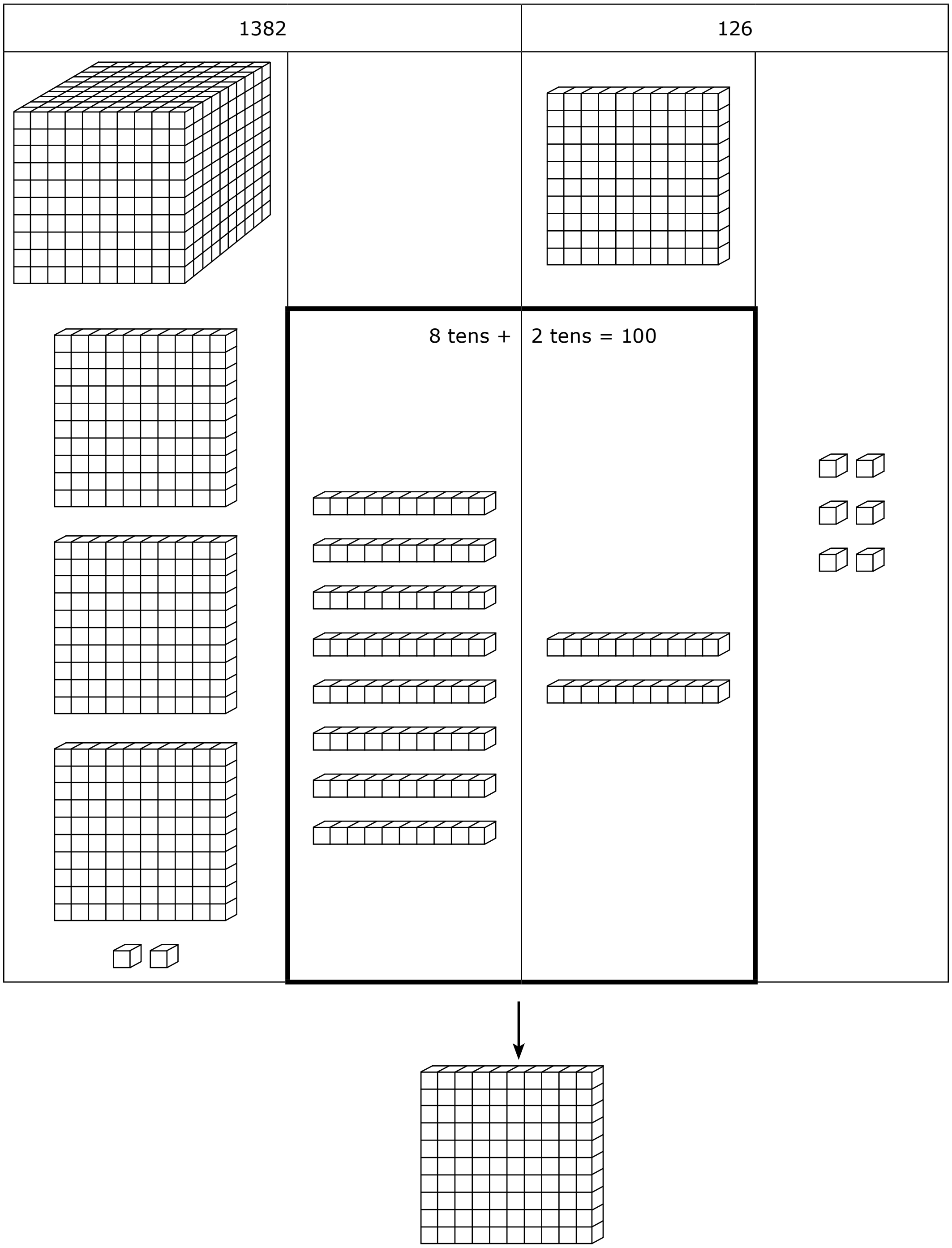
There are other possibilities.

Making "nice" or "friendly" numbers

1382 + 126 = 1382 + 18 + 108 = 1400 + 108 = 1508
because 1382 needs 18 more to get to 1400 and then only 108 are left to add on.

Note: Students need to use their knowledge of compatible number pairs for 10 and be able to extend this knowledge to pairs for 100 in order to be able to use this strategy.

Use representations of materials such as base-10 blocks.



(1 x 1000) + (5 x 100) + (8 x 1) = 1508

Therefore 1382 + 126 = 1508

Modified Standard Algorithm

$$\begin{array}{r l} 1382 & \\ \underline{+\ \ 126} & \\ 8 & = 2 + 6 \\ 100 & = 80 + 20 \\ 400 & = 300 + 100 \\ \underline{1000} & = 1000 + 0 \\ 1508 & \end{array}$$

Standard Algorithm

$$\begin{array}{r} 1\overset{1}{3}82 \\ \underline{+\ \ 126} \\ 1508 \end{array}$$

Possible Strategies for Subtraction

Each of the following are strategies to calculate 1382 – 126.

Breaking Up Numbers Using Place Value

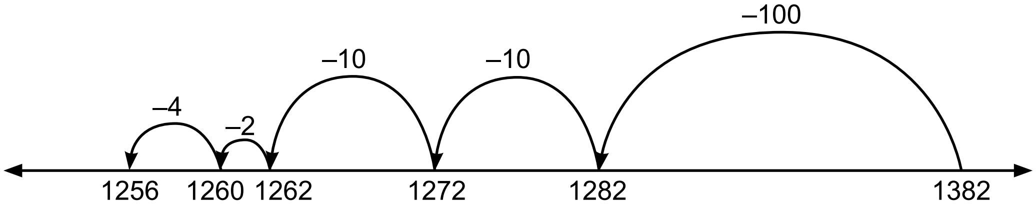
This method requires place value understanding.

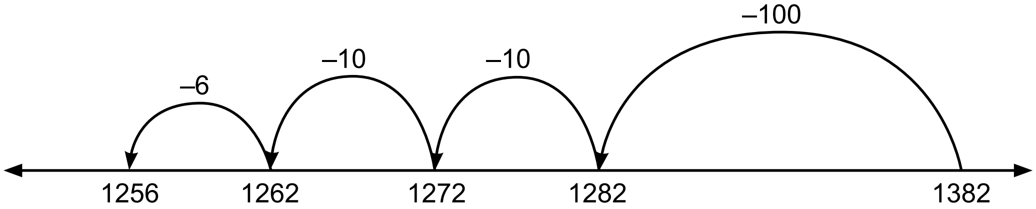
1382 – **126 =**

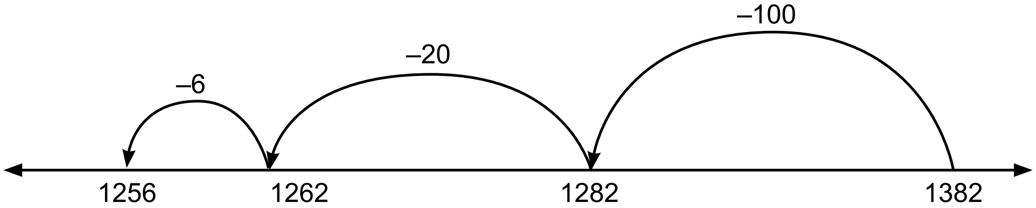
(1000 + 300 + 80 + 2) – 100 + 20 + 6 =

1000 + (300 – 100) + (70 – 20) + (12 – 6) = 1000 + 200 + 50 + 6 = 1256

Empty Number Line (Jump Strategy)







There are other possibilities.

Making "nice" or "friendly" numbers

Add 4 to both numbers.

(1382 + 4) – (126 + 4) $\longrightarrow$ 1386 – 130 = 1256

Renaming

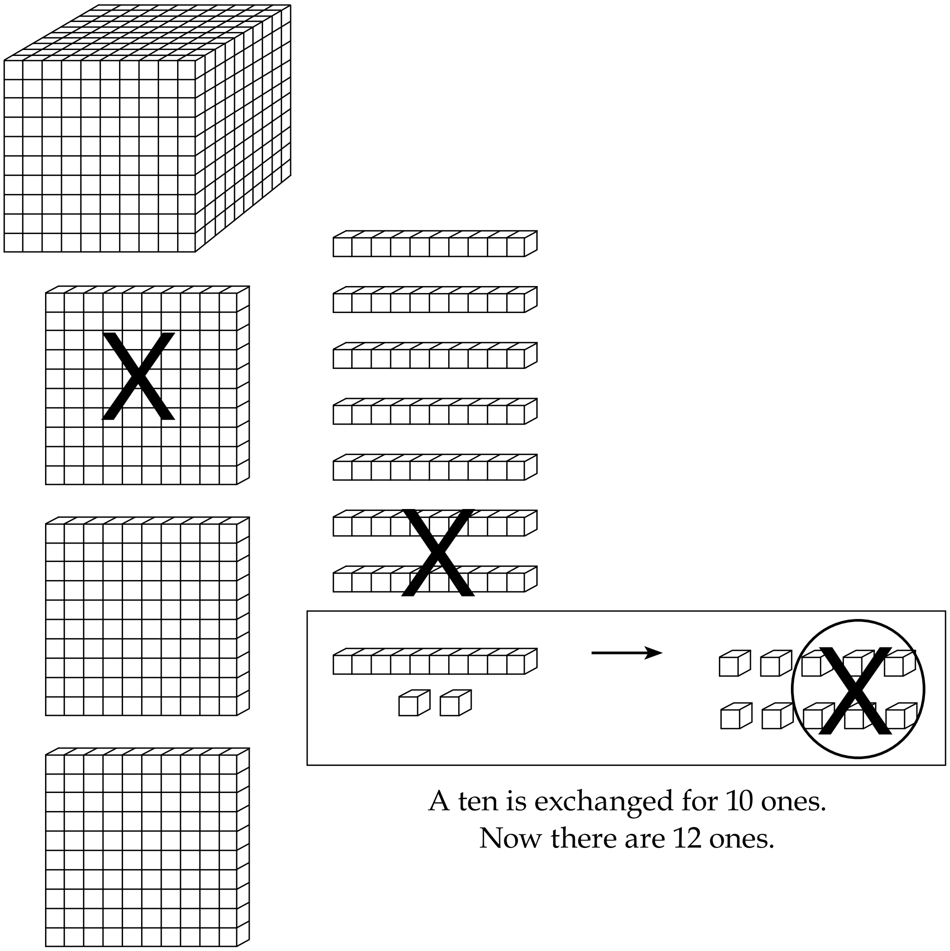
This strategy relies on the student's sense of number.

5 000 – 2 674 =
4 999 + 1 (renamed the 5000)
<u>– 2 674</u>
2 325 + 1 = 2 326

or

One can be subtracted from each number before subtracting (4 999 – 2 673).

Use representations of materials such as base-10 blocks.



A ten is exchanged for 10 ones.
Now there are 12 ones.

1000 + (300 – 100) + (70 – 20) + (12 – 6)

Therefore 1382 – 126 = 1256

Standard Algorithm

$$\begin{array}{r} 13{}^{7}\cancel{8}{}^{1}2 \\ \underline{-\ \ 126} \\ 1256 \end{array}$$

Note: Students will also develop their own strategies.

Example: Use negative numbers

$$\begin{array}{rl} 1382 & \\ \underline{-\ 126} & \\ -\ 4 & (2-6) \\ 60 & (80-20) \\ 200 & (300-100) \\ \underline{1000} & \\ 1256 & (1000+200+60-4) \end{array}$$

Assessing Understanding: Paper-and-Pencil Task

Jonah and Savana were given the question 1382 + 126 to solve.

Jonah solved it this way.	Savana solved it this way.
$\begin{array}{r} 1382 \\ \underline{+\ 126} \\ 8 \\ 100 \\ 400 \\ \underline{1000} \\ 1508 \end{array}$	$\begin{array}{r} 1{}^{1}382 \\ \underline{+\ 126} \\ 1508 \end{array}$

Both students got the correct answer.

How are their methods the same? How are they different?

Note: It is important to model correct vocabulary. In looking at the standard algorithm, the terms *carrying* and *borrowing* are no longer used because they have no real mathematical meaning with respect to the operations. Instead, the terms *regroup, exchange,* or *trade* should be used in their place.

- **Multi-step Problems:** Students should have opportunities to solve problems that involve the addition or subtraction of more than one number.

  Examples:

  1. On Monday, there were 4128 visitors to the Lower Fort Gary. 2709 of them were adults and the rest were children. How many children visited the fort? The first 2890 visitors received a small Canadian flag. How many visitors did not receive a flag?
  2. There were 3670 bags of cotton candy sold at the fair. 1565 of them were pink and 1005 were blue. The rest were green. How many green bags were sold?
  3. In July, 3889 people flew from Winnipeg to Vancouver. In August, the number of people who flew from Winnipeg to Vancouver was 1335 more than in July. How many people flew from Winnipeg to Vancouver in the July and August altogether?
  4. Mark was downloading apps to his phone. The first app he downloaded was 177 kb, the second was 446 kb, and the last was 207 kb. What was the total size (in kb) of all the apps he downloaded?

Using the Bar Model to Support Part-Whole Understanding for Addition and Subtraction

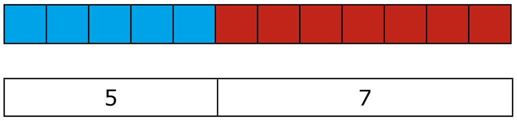
The Bar Model is a problem-solving strategy that originates from Singapore. It is used to help students convert the data from a word problem into concrete visual images. These images can then be converted into relevant mathematical expressions or number sentences.

This model also helps students see that different problems in a variety of contexts share the same mathematical structure therefore they can be visualized in the same way.

If students have not had prior experience with model drawing, it is recommended that they begin by using physical objects. The objects can be organized in a linear fashion and then eventually be replaced by a model drawing (bar).

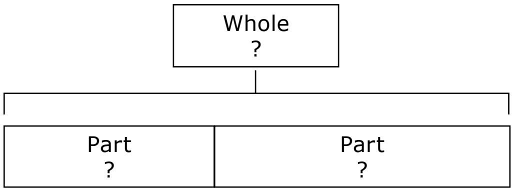
Example:

John had 5 blue candies and 7 red candies. How many candies did he have altogether?



5 + 7 = 12

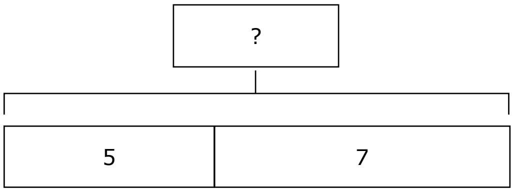
In problems involving addition and subtraction, there are three possible unknowns. When the value of two of them is known the third can be found.



Addition Types

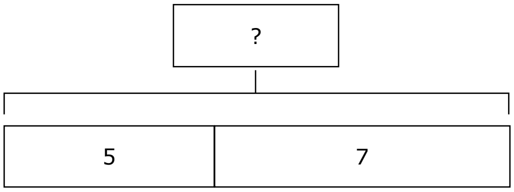
Two Quantities Combined

I have 5 blue candies and 7 red candies. How many candies do I have?



A Quantity Is Increased

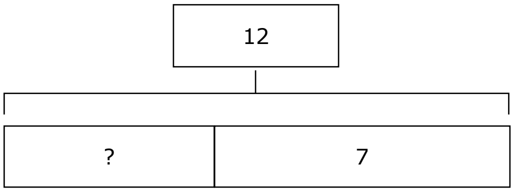
I have 5 blue candies and I buy 7 red candies. How many candies do I have?



Subtraction Types

Take Away

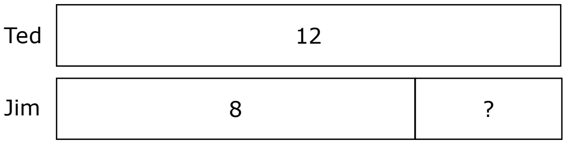
I had 12 candies. I gave 7 away. How many candies do I have now?



The whole is known along with one of the parts. The whole is partitioned and one of the parts removed to identify the missing part.

Comparison or Difference

Ted has 12 candies. Jim has 8 candies. How many more does Ted have?



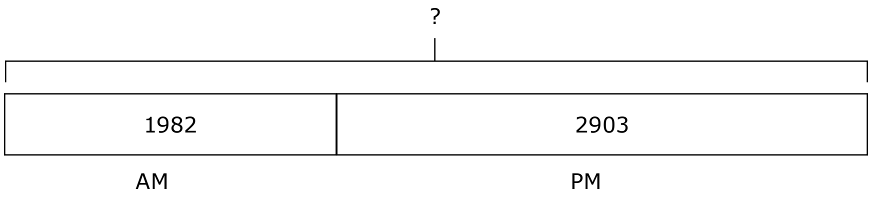
Example Using a Multi-step Problem:

At the fair, 1982 hotdogs were sold in the morning and 2903 were sold in the afternoon.

How many hotdogs were sold altogether?

How many more hotdogs were sold in the afternoon than in the morning?

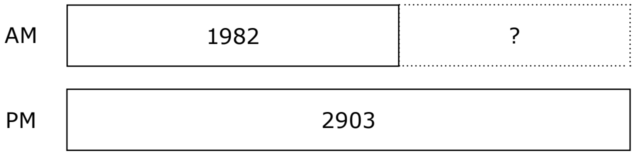
Part A



1982 + 2903 = 4885

There were 4885 hotdogs sold all together.

Part B



2903 – 1982 = 921

There were 921 more hotdogs sold in the afternoon than were sold in the morning.

- **Describe a situation in which an estimate rather than an exact answer is sufficient.**
- **Estimate sums and differences using different strategies (e.g., front-end estimation and compensation).**

Suggestions for Instruction

- **Estimation Strategies:** Students need to be introduced to a variety of estimation strategies such as the following:
  - **Front-end estimation:** In this strategy, only the digit with the largest place value is used even though the estimate may be low. For example, 127 + 238 is estimated to be 100 + 200 = 300.
  - **Compensation:** This strategy involves rounding one quantity up and the other down. In doing so, one number might be overestimated in order to compensate for the other number being underestimated. For example, 1 752 + 648 would be thought of as 1 700 + 700. The 1 700 is a low estimate for 1 752 so the 648 is estimated at 700 (a high estimate) in order to compensate.
  - **Rounding:** 1 439 + 352 is estimated to be 1440 + 350 = 1790 or 1400 + 400 = 1800. Note: This is not to be taught in a formal/structured manner.

  Students should have multiple opportunities to estimate not only in mathematics but in other subject areas as well.

- **Estimate or Exact Answer?** Students should be able to determine when an exact answer is required or when an estimate is sufficient based on the situation. Give students problems. Have them decide if an exact answer or an estimate is needed, and then justify their choice.

  Examples:

  1. Lee has 482 hockey cards, 173 baseball cards, and 198 football cards. Does Lee have more than 1000 cards in all? (estimate)
  2. Amy empties her piggy bank. She counts $104.50 in quarters, $75.10 in dimes and $27.75 in nickels. How much money was in the piggy bank? (exact)
  3. The 120 Grade 4 students are going on a field trip. The teachers have collected 87 permission slips so far. How many permission slips still need to be returned? (exact)
  4. The school concert was held on two nights. On Wednesday, there were 652, and on Thursday there were 571 people. About how many people attended the concert? (estimate due to the way the question is asked)

- **Modelling Estimation Strategies:** Present students with the following problem:

  You read 175 pages on the first day, 198 pages on the second day, and 150 pages on the third day. About how many pages did you read over the three days?

  - Have students represent the numbers using base-10 materials.
  - Use the base-10 materials to model front-end rounding e.g., 175 → 100, 198 → 100, and 150 → 100. Using this strategy the estimation would be 300 pages. Ask students if this is a good estimate for the answer.
  - Focus their attention on the remaining base-10 blocks. Point out that the remaining (75 + 98 + 50) blocks would together make at least 200. Therefore, adding on another 200 would compensate for the values that were dropped off when using the front-end strategy. An estimate of 500 (100 + 100 + 100 + 200) is a better estimate.
  - Make sure that students understand that the front-end strategy and compensation used together enable them to make a more reasonable estimate.

  Model the same process for subtraction where compensation is used to subtract more from the initial front-end estimate.

  Example:

  Estimate the answer to 410 – 395.

  - Represent the numbers using base-10 blocks. Using front-end estimation—510 → 500 and 395 → 300 therefore the estimate (500 – 300) is 200.
  - The students should see that there are still 95 blocks remaining after the hundreds are compared that were initially to be subtracted from the 510. Therefore, since 95 is close to 100, an additional 100 should be subtracted from the initial estimate. 500 – 300 – 100 = 100.

- **Writing Problems:** Have students write their own addition and subtraction problems. Some of the problems should require an estimate only and others should require both an estimate and a calculation (exact answer).

Assessing Understanding: Paper-and-Pencil Task

Present students with the following problems.

1. Your family is going to visit friends in Calgary 1320 kilometres away. On the first day they travel 650 km and stop in Moose Jaw. Will you have to travel more or less than 700 km to reach your destination on the second day?
2. The book you are reading has 525 pages. If you read 220 pages the first day and 230 pages the second day, how many more pages do you have to read in order to finish the book?
3. A jogger jogs 1300 m the first day and 1800 m the second day. About how far did she jog in all?
4. On Saturday, 4012 people registered to run in the marathon. If 1278 of them were males, about how many were females?
5. In order to move to a new level in the video game, you need at least 2000 points. If you have 1254 points, how many more points do you need before you can move to the next level?
6. On Saturday, 1500 people visited the zoo, and on Sunday 2800 people visited. Approximately how many people visited the zoo over the weekend?
    a. Have students decide which problems can be answered with an estimate only and which problems require calculation as well as an estimate. (Note: Estimating is necessary for every problem because estimates help to determine the reasonableness of the calculated answer.)
    b. Have students answer the problems independently.
    c. Have students share their solutions and strategies with one another or in an interview.

The student

- ☐ explains the meaning of the problem and justifies why only an estimate is needed or why a calculated answer is necessary as well
- ☐ uses compensation as well as front-end rounding to estimate the sum or difference
- ☐ explains clearly the strategies used in estimating and how he or she knows that the resulting estimate is reasonable

Grade 4: Number (4.N.4, 4.N.5)

Enduring Understandings:

Multiplication and division are inverse operations.

Multiplication is repeated addition.

Division is repeated subtraction.

Essential Questions:

How can skip-counting and arrays be used to demonstrate multiplication and division?

How are addition and multiplication related?

How are subtraction and division related?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.N.4 Explain the properties of 0 and 1 for multiplication, and the property of 1 for division. [C, CN, R]	➛ Explain the property for determining the answer when multiplying numbers by one. ➛ Explain the property for determining the answer when multiplying numbers by zero. ➛ Explain the property for determining the answer when dividing numbers by one.
4.N.5 Describe and apply mental mathematics strategies, such as • skip-counting from a known fact • using halving/doubling • using doubling and adding one more group • using patterns in the 9s facts • using repeated doubling to develop an understanding of basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, PS, R] Recall of the multiplication and related division facts up to 5 × 5 is expected by the end of Grade 4.	➛ Provide examples for applying mental mathematics strategies: • skip-counting from a known fact (e.g., for 6 × 3, think 5 × 3 = 15, then 15 + 3 = 18) • halving/doubling (e.g., for 4 × 3, think 2 × 6 = 12) • using a known double and adding one more group (e.g., for 3 × 7, think 2 × 7 = 14, then 14 + 7 = 21) • repeated doubling (e.g., for 4 × 6, think 2 × 6 = 12 and 2 × 12 = 24) • use ten facts when multiplying by 9 (e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63) • halving (e.g., for 30 ÷ 6, think 15 ÷ 3 = 5) • relating division to multiplication (e.g., for 64 ÷ 8, think 8 × ☐ = 64).

PRIOR KNOWLEDGE

Students may have

- represented and explained multiplication (to 5 x 5) using equal groups and arrays
- modelled multiplication using concrete and visual representations, and recorded the process symbolically
- related multiplication to repeated addition
- related multiplication to division and division to multiplication
- created and solved problems in context that involve multiplication
- represented and explained division using equal sharing and equal grouping
- modelled equal sharing and equal grouping using concrete and visual representations, and recorded the process symbolically
- related division to repeated subtraction
- created and solved problems in context that involve equal sharing and equal grouping

BACKGROUND INFORMATION

Terminology

Multiplication: A mathematical operation of combining groups of equal amounts; repeated addition; the inverse of division.

Product: The number obtained when two or more factors are multiplied (e.g., in 6 × 3 = 18, 18 is the product).

Division: A mathematical operation involving two numbers that tells how many groups there are or how many are in each group.

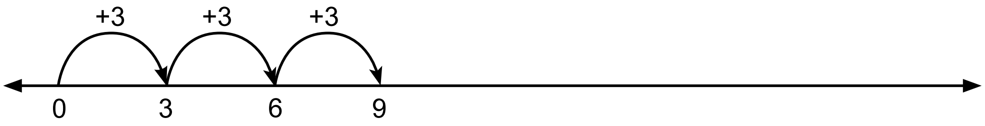
Quotient: The answer to the division of two numbers (in 12 ÷ 3 = 4, the quotient is 4).

Array: A set of objects or numbers arranged in an order, usually in rows and/or columns.

Meanings of Multiplication at the Grade 4 Level

1. Repeated addition

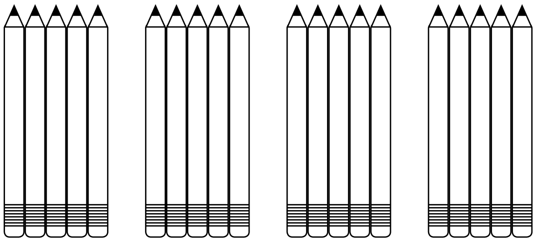
   For example: 3 + 3 + 3 = 9



2. Equal groups or sets

   For example:

   Pencils come in packages of 5.

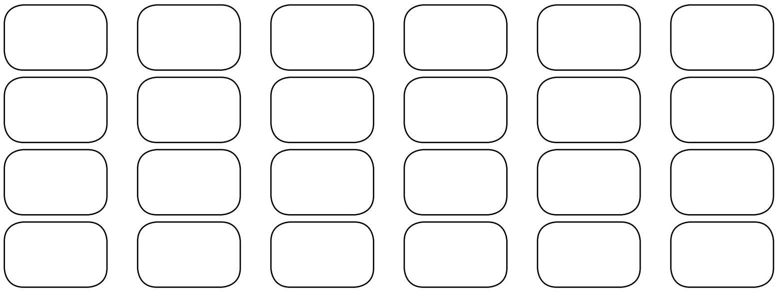


   How many pencils are in 4 packages?

3. An array

   For example:

   A classroom has 4 rows with 6 desks in each row.



   How many desks are in the classroom?

BLM 4.N.5.1

BLM 4.N.5.1 is a picture of a 10 x 10 dot array that can be used in a student's tool kit to help them with problem solving. Print off the page and place it in a plastic sheet protector. Students can then use a white board marker to show different problems.

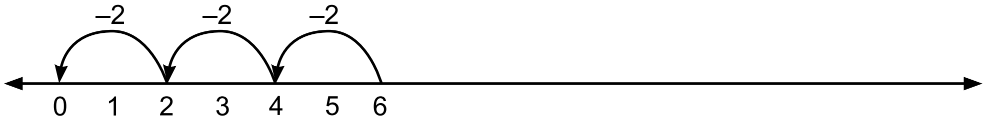
Multiplication Problems

In a multiplication problem both the number of objects in each group and the number of groups are given. The total number of objects is the unknown.

Meanings of Division at the Grade 4 Level:

1. Repeated Subtraction

   For example: $6 \div 2 = 3$ is the number of times you can subtract groups of 2 before you get to 0.

   $6 - 2 - 2 - 2 = 0$

   $6 \div 2 = 3$

2. Equal Sharing (Partitive)

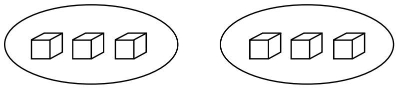
   For example: $6 \div 3 = 2$ is the amount each person gets if you share 6 things equally among 3 people.

   $6 \div 3 = 2$

3. Equal Grouping (Quotative)

   For example: $6 \div 3 = 2$ is the number of equal groups of 3 you can make with 6 things.

   $6 \div 3 = 2$

Note: Division should be taught together with multiplication so that students can see the inverse relationship between the two operations.

Preventing Misconceptions: Common misconceptions students may develop include multiplication makes bigger and division makes smaller. This is not true when working with fractions or decimals less than one.

Example: $\frac{1}{2} \times 8 = 4$, $8 \div \frac{1}{2} = 16$, $8 \times 0.25 = 2$ or $8 \div 0.25 = 32$

MATHEMATICAL LANGUAGE

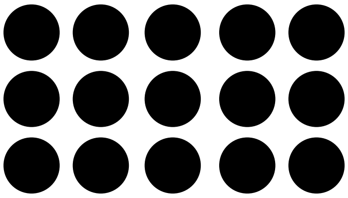
sets of
groups of
multiply
multiplication
product
quotient
divide
division
equal groups
sharing
array
times
skip-counting
halving
doubling
property
properties

LEARNING EXPERIENCES

Assessing Prior Knowledge: Paper-and-Pencil Task

1. What does this array show?

   Write two multiplication number sentences and two division number sentences.



2. The Grade 4 class is playing a game.

   The teacher wants them to be in equal groups with no remainders.

   If there are 20 students in the class, how many different sizes of equal groups can you make?

3. The answer to a multiplication question is 12.

   What might the question be?

4. My friend said that multiplication is repeated addition and division is repeated subtraction. Explain what s/he means. Use words, pictures, number lines, and/or numbers and symbols in your explanation.

Look for evidence that the student understands that

☐ an array can represent both operations
☐ multiplication is repeated addition
☐ division is repeated subtraction
☐ multiplication and division are inverse operations

- **Explain the property for determining the answer when multiplying numbers by one.**
- **Explain the property for determining the answer when multiplying numbers by zero.**
- **Explain the property for determining the answer when dividing numbers by one.**

Note: It is important that students have multiple opportunities to solve problems, create their own problems, and interact with concrete and visual models related to multiplication and division by 1 and multiplication by 0. Through these experiences, students will develop their understanding of the properties and then be able to come to their own generalizations.

Identity Property of Multiplication: Any number multiplied by one is equal to the original number.

Identity Property of Division: Any number divided by 1 is equal to the original number.

Zero Property of Multiplication: Any number multiplied by zero is equal to zero.

This approach is far more effective than just giving students arbitrary rules.

Suggestions for Instruction

- **Exploring Multiplication by 1:** Have students represent 5 x 1 using materials, an array, and a number line. Have students represent 1 x 5 using materials, an array, and a number line. Ask students what they notice about their representations (answers). (The answers are always the same as the number being multiplied by 1.)

  Extension: Will this be true for larger numbers? Have students justify their thinking.

- **Is there a difference?** Have students use materials to show 1 group of 6 objects and 6 groups of 1 object. What is the same about their models and what is different?

  Exploring Division by 1:

  Materials: 1-to-6 or 1-to-9 die.

  Procedure: Students roll the die and record the number shown. Have them model with materials and a number line the division of their number by 1. Repeat two more times. What do they notice about their answers?

  Extension: Do they think that this will be true for larger numbers? Have students explain/justify their thinking.

- **Similarities and Differences:** Have students represent the following problems using materials.
  - Paul has 8 cookies. He puts them in a bag. How many cookies are in the bag?
  - Paul has 8 cookies. If he puts 1 cookie in each bag, how many bags can he make?

  How are their representations the same? How are they different?
- **Problem Writing:** Have students create and share their own problems that involve multiplying or dividing by 1.
- **Multiplication by 0:** Have students represent 5 x 0 and 0 x 5 using materials such as paper plates to represent groups and/or a number line. What do they notice about their answers? Will the result be the same for any number multiplied by 0? Explain your thinking.

BLM
4.N.4.1

- **Equation Match:** Prepare a set of cards from BLM 4.N.4.1 for each pair of students. Have students find the matching representations and justify their thinking.

Assessing Understanding: Paper-and-Pencil Task

1. Write a note to your parent(s)/guardian(s)/caregiver(s) explaining what you have learned about multiplying and dividing any number by 1. Use pictures, number lines, and words.
2. Is she correct? Explain/show how you know.

The student is able to

☐ give a generalization for multiplying by 1

☐ give a generalization for dividing by 1

☐ support their generalizations using pictures and/or number lines

☐ explain why multiplication by zero always results in an answer of zero

- **Provide examples for applying mental mathematics strategies:**
    - **skip-counting from a known fact (e.g., for 6 x 3, think 5 x 3 = 15, then 15 + 3 = 18)**
    - **halving/doubling (e.g., for 4 x 3, think 2 x 6 = 12)**
    - **using a known double and adding one more group (e.g., for 3 × 7, think 2 × 7 = 14, then 14 + 7 = 21)**
    - **repeated doubling (e.g., for 4 x 6, think 2 x 6 = 12 and 2 x 12 = 24)**
    - **use ten facts when multiplying by 9 (e.g., for 9 × 6, think 10 × 6 = 60, and 60 – 6 = 54; for 7 × 9, think 7 × 10 = 70, and 70 – 7 = 63)**
    - **halving (e.g., for 30 ÷ 6, think 15 ÷ 3 = 5)**
    - **relating division to multiplication (e.g., for 64 ÷ 8, think 8 × ☐ = 64).**

Mental Math

Note: The development of mental math strategies is greatly enhanced by sharing and discussion. Students should be given the freedom to adapt, combine, and invent their own strategies.

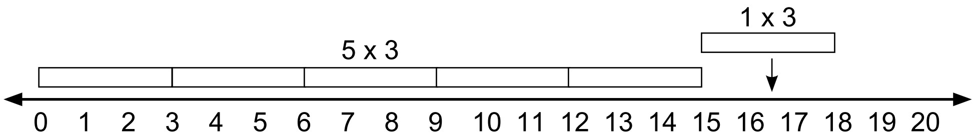
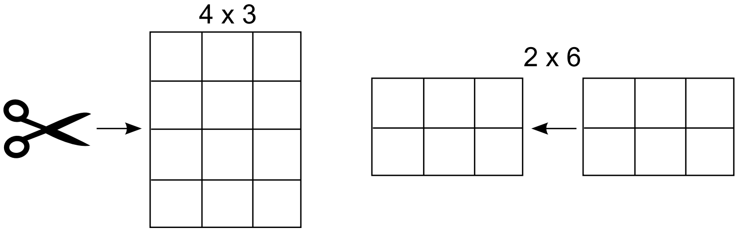
Students should be able to apply these strategies to larger numbers.

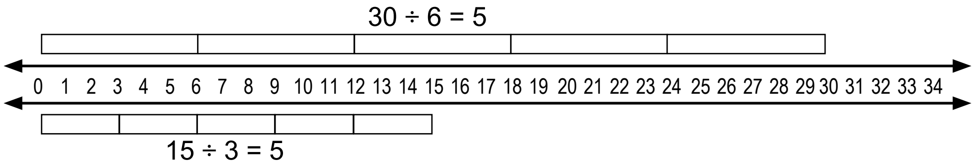
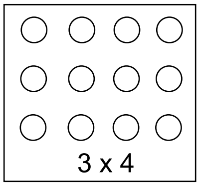
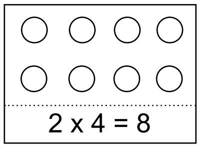
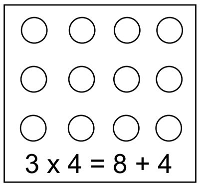
BACKGROUND INFORMATION

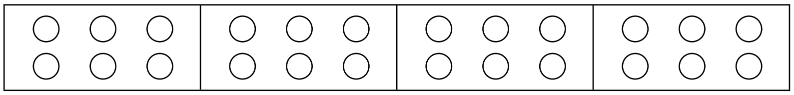
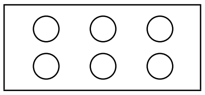
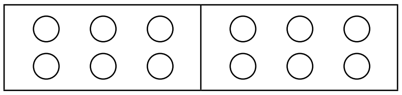
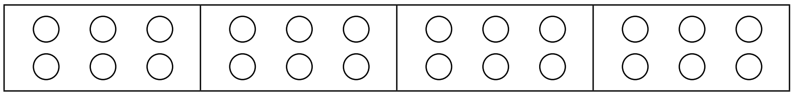
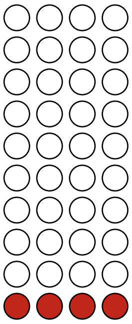
An understanding of the multiplication and division properties is needed in order for students to be able to develop and use mental math strategies. These properties include:

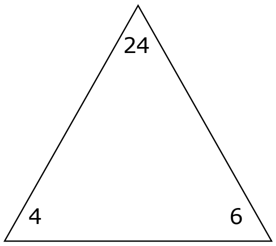
- **Commutative property of multiplication:** Numbers can be multiplied in any order. (e.g., 3 × 4 = 4 × 3). An array model can help to demonstrate this property.
- **Associative Property:** When three or more numbers are multiplied together, it doesn’t matter in which order they are grouped or associated. For example, 5 × 2 × 4 = (5 × 2) × 4 = 5 × (2 × 4).
- **Distributive Property:** The distributive property refers to the idea that one or both of the factors in a multiplication question can be decomposed into two or more parts and each part multiplied separately and then added [e.g., 9 x 7 is equivalent to (9 × 5) + (9 × 2)].

Suggestions for Instruction

Strategy	**Teaching Strategies**
Skip-Counting from a known fact	Prerequisite knowledge: Students should be able to skip-count forward and backward by 2s, 3s, 4s, 5s, and 10s. Cuisenaire rods can help to make this strategy visible for students. (**Note:** Use a metre stick as a number line. The rods are 1 to 10 cm in length, so they will match the centimetre markings on the metre stick.) Example: For 6 x 3: "I know that 5 x 3 is 15 so for 6 x 3 I just need to add one more rod/group."  Have students connect the strategy to larger numbers. Example: For 6 x 30: Think "5 x 30 is 150, so 6 x 30 is 150 + 30 = 180."
Halving/Doubling	Halving and doubling can make multiplication calculations easier. Using grid paper to make arrays and then cutting them apart and reassembling them can help make this strategy visible for students. Example: For 4 x 3: Use grid paper to make the 4 x 3 array and cut it out. Cut the array in half and reassemble it to show 2 x 6. Students will be able to see that the total number of squares did not change; therefore, the two representations are equal.  Have students apply this strategy to larger numbers. Example: 6 x 50 can be thought of as 3 x 100.

Strategy	**Teaching Strategies**
Halving	Using the strategy of halving both the dividend and the divisor can help make division calculations easier. Using a double number line and Cuisenaire rods can help students understand/see that dividing both the dividend and the divisor by two results in the same answer that they would get if no changes were made to the original question. Example:  Have students apply this strategy to larger numbers. Example: 160 ÷ 4 can be thought of as 80 ÷ 2.
Using a known double and adding one more group BLM 4.N.5.1	This strategy is similar to using the doubles plus 1 strategy in addition except that it is one more group that is being added. Using array cards (see BLM 4.N.5.1) can help to make this strategy visible for students.  Initial array card  Fold the bottom flap up to cover the last row of the array.  Unfold to show one more group.

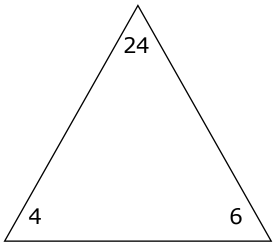
Strategy	Teaching Strategies
Repeated Doubling	This strategy is useful when multiplying by 4 or by 8. Using paper strips and arrays can help to make this strategy visible for students. Use a strip of paper. Fold it in half and then in half again. Unfold the paper and make dot arrays for the number being multiplied by 4 or 8. Example: For 4 x 6 the original strip looks like this.  Fold the strip so that one section is showing.  Open it up to show the double.  2 x 6 = 12 Finally, open it up to show all four sections.  2 x 12 = 24 Have students make their own representations of the repeated doubling strategy.
Use ten facts when multiplying by 9	Although not considered basic facts, most Grade 4 students know how to multiply by 10. This knowledge can be applied when multiplying by 9. Two-colour counters can be used to help make this strategy visible. Example: For 9 x 4: Have students use the counters to make 10 groups of 4, all of one colour. Then have them turn over last counter in each column to represent the extra group of four that needs to be subtracted. 10 x 4 = 40  40 – 4 = 36 Have students extend this strategy to larger numbers. For 40 x 9: Think "40 x 10 = 400, but I need to take away the extra group of 40 so 400 – 40 = 360. 40 x 9 = 360."

Strategy	Teaching Strategies
Relating division to multiplication BLM 4.N.5.2	Thinking multiplication is often an easier way of solving a division question. Students need to be able to understand the relationship between the two operations. Triangle flash cards can support this understanding. Example: For this flashcard students can see that  4 x 6 = 24 6 x 4 = 24 24 ÷ 4 = 6 24 ÷ 6 = 4 Using triangular flashcards: Display the card with one of the numbers covered. Students have to figure out the hidden number. Example: If the 24 (product) is covered students need to multiply 4 x 6 (factors) to find the answer. If the 4 or the 6 is covered students need to either divide or to "think multiplication" to find the answer. For example, if the 4 is covered, students can think "24 ÷ 6 = ?" or "6 x ? = 24" **Match Game:** See BLM 4.N.5.2.

Assessing Understanding: Interview

Ask the student to

- explain how knowing 4 × 6 helps find the product/answer for 8 × 6.
- explain how knowing 7 × 10 can help someone find the answer for 7 × 9.
- write 25 ÷ 5 = ? as a multiplication question/number sentence.
- show how they can solve 5 x 64 using the doubling and halving strategy.
- use counters to show why 7 × 6 is the same as (5 × 6) + (2 × 6).
  Note: Students are not expected to use parentheses.
- explain why 2 × 7 is equal to 7 × 2.
- write two multiplication number sentences and two division number sentences for the following flashcard



The student understands and can use the following mental math strategies and/or properties:

☐ repeated doubling
☐ using ten facts when multiplying by 9 (build down)
☐ relate multiplication to division
☐ doubling and halving
☐ distributive property
☐ commutative property

- **Recall of the multiplication and related division facts up to 5 x 5 is expected by the end of Grade 4.**

BACKGROUND INFORMATION

Stages of Basic Fact Acquisition

Learning math facts is a developmental process where the focus of instruction is on thinking and building number relationships. Facts become automatic for students through repeated exposure and practice.

Arthur Baroody identifies three stages through which students typically progress in acquiring basic facts:

- **Counting Strategies:** The student uses objects (e.g., blocks, counters, fingers) or verbal counting to determine the answer. For example, for 3 x 7, the student starts with 7 and skip counts saying, "7, 14, 21."
- **Reasoning:** The student uses known information (i.e., known facts and relationships) to logically determine the answer of an unknown fact. For example, for 3 x 7, the student says, "2 x 7 or double 7 is 14, and add one more set of seven is 21."
- **Automaticity or Mastery:** Student produces efficient (fast and accurate) answers. For example, for 3 x 7 the student quickly answers, "It is 21; I just know it."

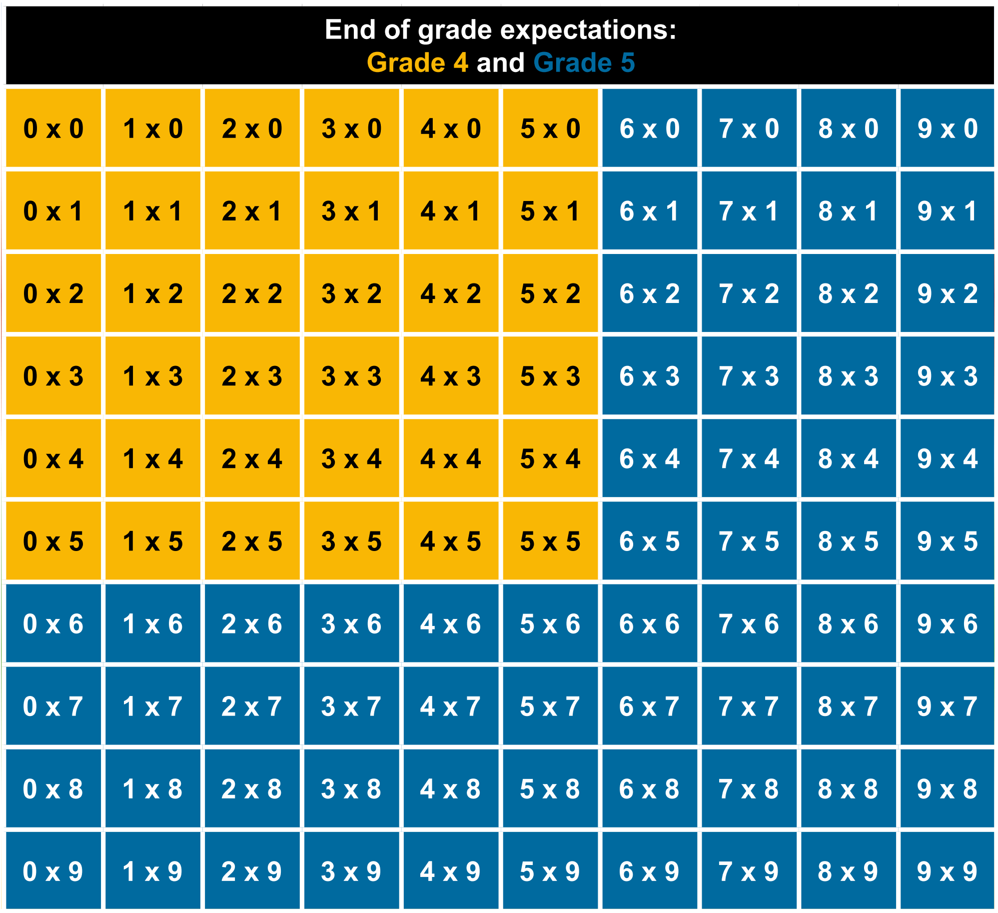
Assessing the Facts

Basic facts should be assessed through observation, interviews, games, self-assessment, and strategy-focused paper/pencil practice. Although the goal is to have students recall facts in a reasonable time frame, researchers strongly caution against the use of timed tests.

Timed tests may lead to math anxiety in some students. "Timed tests as well as other speed-related materials (such as flash cards) cause slow, strong mathematical thinkers to become discouraged in class, develop math anxiety, and turn away from the subject" (Boaler 471). Math anxiety, in turn, causes a negative impact on students who use higher-level strategies, ones that rely on working memory, because the anxiety interferes with the working memory (Ramirez, Gunderson, Levine, and Beilock). This indicates that some of the best mathematical thinkers are often those most negatively affected by timed testing.

Basic Facts for Grade 4

Multiplication facts to 81

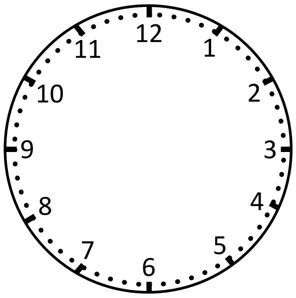


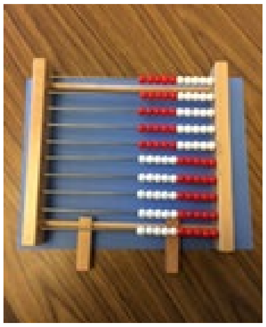
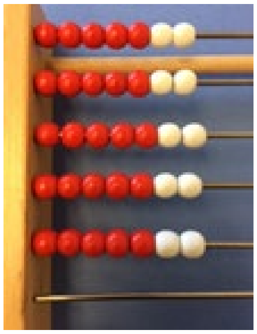
End of grade expectations:
Grade 4 and Grade 5

0 x 0	1 x 0	2 x 0	3 x 0	4 x 0	5 x 0	6 x 0	7 x 0	8 x 0	9 x 0
0 x 1	1 x 1	2 x 1	3 x 1	4 x 1	5 x 1	6 x 1	7 x 1	8 x 1	9 x 1
0 x 2	1 x 2	2 x 2	3 x 2	4 x 2	5 x 2	6 x 2	7 x 2	8 x 2	9 x 2
0 x 3	1 x 3	2 x 3	3 x 3	4 x 3	5 x 3	6 x 3	7 x 3	8 x 3	9 x 3
0 x 4	1 x 4	2 x 4	3 x 4	4 x 4	5 x 4	6 x 4	7 x 4	8 x 4	9 x 4
0 x 5	1 x 5	2 x 5	3 x 5	4 x 5	5 x 5	6 x 5	7 x 5	8 x 5	9 x 5
0 x 6	1 x 6	2 x 6	3 x 6	4 x 6	5 x 6	6 x 6	7 x 6	8 x 6	9 x 6
0 x 7	1 x 7	2 x 7	3 x 7	4 x 7	5 x 7	6 x 7	7 x 7	8 x 7	9 x 7
0 x 8	1 x 8	2 x 8	3 x 8	4 x 8	5 x 8	6 x 8	7 x 8	8 x 8	9 x 8
0 x 9	1 x 9	2 x 9	3 x 9	4 x 9	5 x 9	6 x 9	7 x 9	8 x 9	9 x 9

http://www.edu.gov.mb.ca/k12/cur/math/facts/chart.pdf

Specific Fact Strategies

Multiplication by	Strategies
2	Connect to addition—doubling
3	Double and add one more group
4	Double, double
5	■ Relate to an analog clock—skip-counting by 5s  ■ Multiply by 10 and then divide by 2.

Multiplication by	Strategies
6	■ Multiply by 5 and then add one more group. ■ Multiply by 3 and then double.
7	Split the 7 into 5 + 2. Multiply by 5 and then add the multiplication by 2. For example, 7 x 4 ➝ (5 x 4) + (2 x 4) The 100-bead abacus can help students see how this strategy works.   5 x 7 The red beads show 5 x 5. The white beads show 5 x 2. Students can clearly see the multiplication by 5 and by 2.
8	■ Double, double, and double ■ Multiply by 4 and then double
9	■ Multiply by ten and then subtract one group. ■ Students might use the patterns in the nine times table as a strategy. 1 x 9 = 9 2 x 9 = 18 3 x 9 = 27 4 x 9 = 36 5 x 9 = 40 6 x 9 = 54 7 x 9 = 63 8 x 9 = 72 9 x 9 = 81 10 x 9 = 90 ■ Patterns: ▪ Looking at the products in the column, the ones are decreasing by one and the tens are increasing by 1. ▪ The sum of the digits in the product always add up to 9. ▪ The tens digit is always one less than the number being multiplied by 9. For example, for 8 x 9, the product will have a 7 in the tens place and a two (7 + ☐ = 9) in the ones place.

- The book *The Best of Times* by Greg Tang can be used to help students develop the basic facts strategies. Each two-page spread deals with a specific times table. There is a poem to introduce the strategy, and then examples that include both the basic facts as well as the application of the strategy to larger numbers.

Grade 4: Number (4.N.6, 4.N.7)

Enduring Understandings:

Flexible methods of calculation in multiplication and division involve decomposing and composing numbers in a wide variety of ways.

Flexible methods of calculation in multiplication and division require a strong understanding of the operations and the properties of the operations.

There are a variety of appropriate ways to estimate products and quotients depending on the context and the numbers involved.

Essential Questions:

How can materials be used to model multiplication and division?

How can arrays be used to model multiplication and division?

How are multiplication and division related? How can this relationship help with calculations?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.N.6 Demonstrate an understanding of multiplication (2- or 3-digit numerals by 1-digit numerals) to solve problems by • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products [C, CN, ME, PS, R, V]	➛ Model a multiplication problem using the distributive property [e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5)]. ➛ Use concrete materials, such as base-10 blocks or their pictorial representations, to represent multiplication, and record the process symbolically. ➛ Create and solve a multiplication problem that is limited to 2 or 3 digits by 1 digit. ➛ Estimate a product using a personal strategy (e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250). ➛ Model and solve a multiplication problem using an array, and record the process. ➛ Solve a multiplication problem and record the process.

Specific Learning Outcome(s):	**Achievement Indicators:**
4.N.7 Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by • using personal strategies for dividing with and without concrete materials • estimating quotients • relating division to multiplication [C, CN, ME, PS, R, V]	(It is not intended that remainders be expressed as decimals or fractions.) ➢ Solve a division problem without a remainder using arrays or base-10 materials. ➢ Solve a division problem with a remainder using arrays or base-10 materials. ➢ Solve a division problem using a personal strategy, and record the process. ➢ Create and solve a word problem involving a 1- or 2-digit dividend. ➢ Estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5).

BACKGROUND INFORMATION

Students should be encouraged to use a variety of strategies for multiplication and division for the following reasons:

- Sometimes a particular strategy makes more sense to one student than to another.
- Sometimes a strategy works better for a particular set of numbers.
- Being familiar with a variety of strategies allows students to use one strategy to calculate, and then use another strategy to check the answer (justify their answer).

Vocabulary

Terms for Multiplication

$$\underbrace{3 \times 4}_{\text{factors}} = \underset{\text{product}}{\underset{\downarrow}{12}}$$

Terms for Division

$$\underset{\text{dividend}}{\underset{\swarrow}{25}} \div \underset{\text{divisor}}{\underset{\searrow}{6}} = \underset{\text{quotient}}{\underset{\searrow}{4}}{}^{r1} \searrow \text{remainder}$$

Distributive Property: The distributive property refers to the idea that one or both of the factors in a multiplication question can be decomposed into two or more parts and each part multiplied separately and then added [e.g., 9 x 7 is equivalent to (9 × 5) + (9 × 2)].

MATHEMATICAL LANGUAGE

multiply
multiplication
factor
product
divide
division
quotient
remainder
divisor
expanded form
array
base 10
estimate
estimation

LEARNING EXPERIENCES

Assessing Prior Knowledge

Materials: Math journals

Organization: Individual/Whole class

Procedure:

1. Ask students to solve each of the following problems in two different ways:
   a. Rosa is planning to arrange 48 books on six shelves. If she puts an equal number of books on each shelf, how many books will she put on each shelf?
   b. Mark has a six-page photo album. How many pictures does Mark have if each page holds eight pictures?
2. Have students share their solutions with the other members of the class. Encourage students to explain their reasoning by asking questions, such as the following:
   - Which strategy did you use to solve the problem?
   - What is another strategy you could use to solve the problem?
   - Will the strategy work for other problems involving division (multiplication)? Show me.
   - Which strategy do you prefer to use? Why?

Observation Checklist

Use students' responses to determine which strategies students know. Also, examine their responses to determine whether they can do the following:

- ☐ identify problem situations that call for the operation of multiplication
- ☐ identify problem situations that call for the operation of division
- ☐ describe and apply a thinking strategy to determine the product or quotient of two whole numbers
- ☐ describe and apply more than one thinking strategy to determine the product or quotient of two whole numbers

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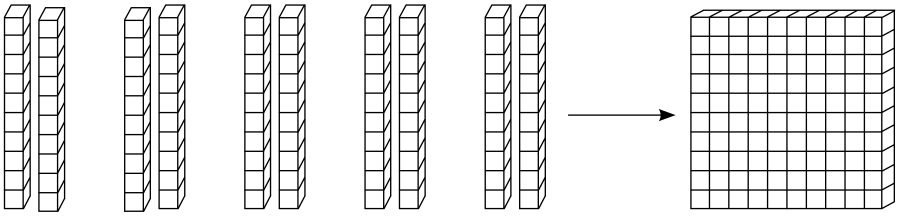
- **Use concrete materials, such as base-10 blocks or their pictorial representations, to represent multiplication, and record the process symbolically.**

Suggestions for Instruction

- Have students use base-10 blocks to model 5 × 20 (five groups of 20)

  Example:

  5 × 20 = 100

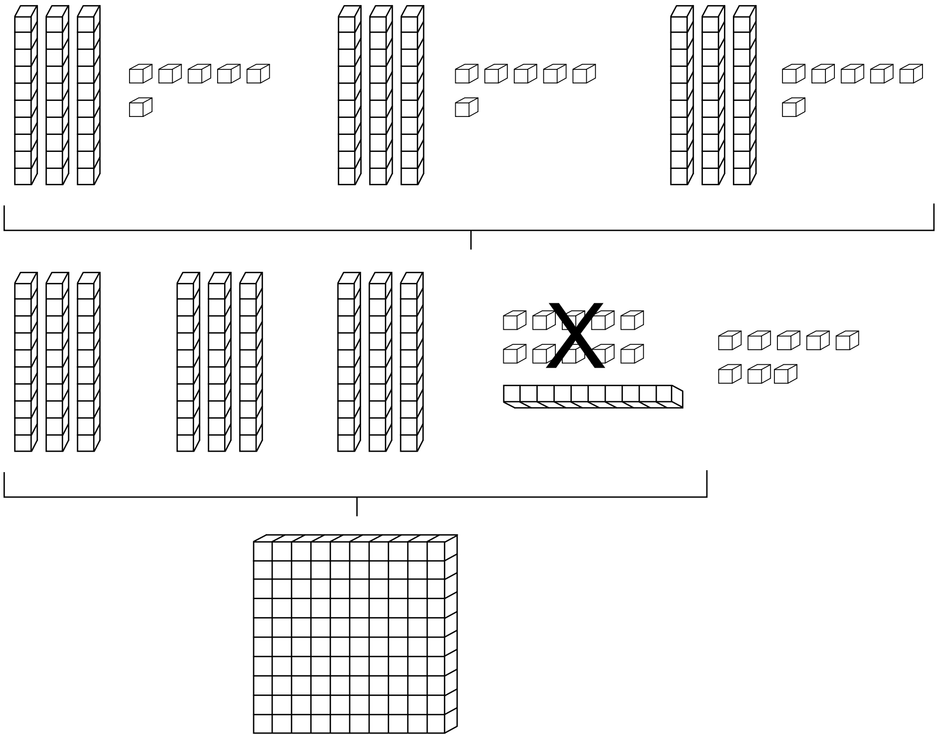
  Have students model 4 × 10, 3 × 10, 6 × 10.

  What do they observe? *All of the answers end in a zero. If the zero is covered the number remaining is the product of the multiplier and the numeral in the tens place (basic facts).*

  Note: It is important that teachers avoid telling students that when multiplying by multiples of ten you just add a zero. They need to recognize the patterns to understand that when multiplying by 10 there is always a zero in the one's place.

  Repeat the activity using 100. Students will see that when multiplying by 100, there is always a zero in both the one's place and the ten's place.

- Have students use base-10 blocks to model 3 x 36.



3 groups of 36 → 9 groups of ten and 18 ones → (9 tens + 1 ten) + 8 ones = 108

1 ten and 8 ones

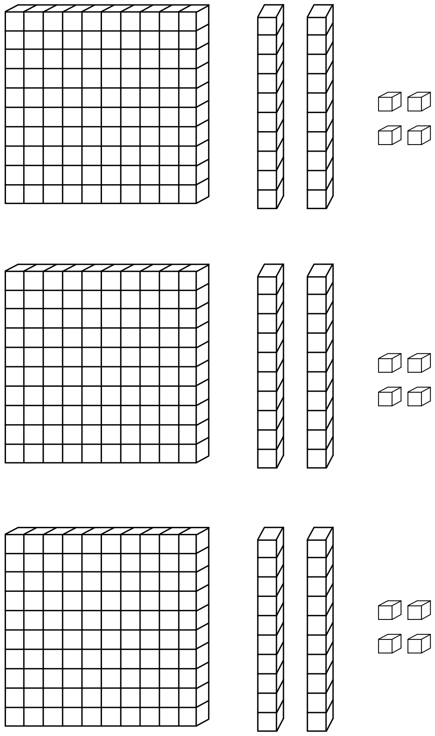
- Have students solve problems using the base-10 blocks. Provide opportunities for students to explain the process orally as well as symbolically.

  Examples:

  - Mark swims 280 laps in the pool each week. How many laps does he swim in 4 weeks?
  - Julie has a stamp collection. She puts 25 stamps on each page of her binder. If she has filled 7 pages of her binder, how many stamps has she collected?

Assessing Understanding

- Use base-10 blocks to show how to multiply 3 × 207. Use pictures and numbers to record your work.
- What multiplication problem does this picture show?



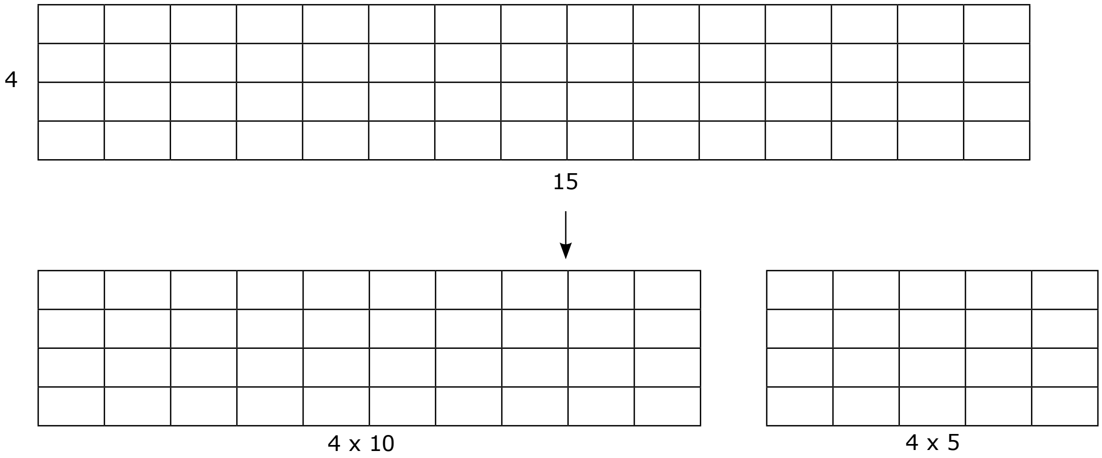
Explain how to use the picture to solve the problem.

- **Model and solve a multiplication problem using an array, and record the process.**

Suggestions for Instruction

Arrays can be made using grid paper or colour (square) tiles.

Example: 4 × 15



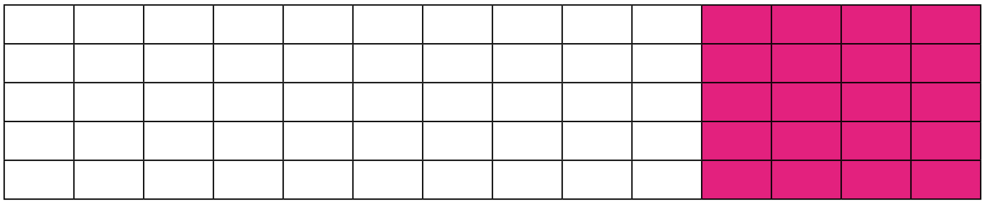
It is easier to multiply if the 15 is decomposed into 10 and 5.

$(4 \times 10) + (4 \times 5) \longrightarrow 40 + 20 = 60$

Decomposing numbers using arrays serves as an introduction to using the distributive property.

Assessing Understanding

- What multiplication problem does this array show? (5 × 14)



- Explain how you can use the array to solve the problem.

  Use the problem 6 x 18. Explain why you might chose to split (decompose) the 18 into 10 and 8.

  Is there another way that you might chose to use this strategy for this problem? (Students might break the problem down into (6 x 9) + (6 x 9).)

- **Model a multiplication problem using the distributive property [e.g., 8 × 365 = (8 × 300) + (8 × 60) + (8 × 5)].**
- **Create and solve a multiplication problem that is limited to 2 or 3 digits by 1 digit.**
- **Solve a multiplication problem and record the process.**

PRIOR KNOWLEDGE

In order to be successful using the distributive property students need to be able to represent 2-digit and 3-digit numbers in expanded form.

Note: Students should have a good understanding of 2-digit × 1-digit multiplication before introducing 3-digit × 1-digit multiplication.

The distributive property can be recorded in several ways.

Examples:

4 × 52
4 × 52 = (4 × 50) + (4 × 2) = 208
4 × 52 = (2 × 52) + (2 × 52) = 208

Suggestions for Instruction

- **Solving Multiplication Problems:** Ask students to solve problems using expanded form.

  Examples:

  - Troy has $34 in his bank. Sarah has 3 times as much in her bank. How much money does Sarah have in her bank?
  - There are 28 students in each of the four Grade 4 classes in the school. How many Grade 4 students are in the school?
  - Health Canada says that children should have 60 minutes of exercise each day. How many minutes should a child have in one week?
  - Eva’s pedometer shows that it takes 428 steps to go around the perimeter of the gym. If Eva goes around the gym five times, how many steps will her pedometer show?
  - The school is having a pancake breakfast. Three hundred seventy-six tickets have been sold. If each person will be served 3 pancakes, how many pancakes do they need to prepare?

- **What is the problem?** Have students create and solve problems (using the distributive property) for given factors.

  Examples:

  - Create and solve a problem that includes the factors 5 and 231.
  - Create and solve a problem that includes the factors 6 and 345.

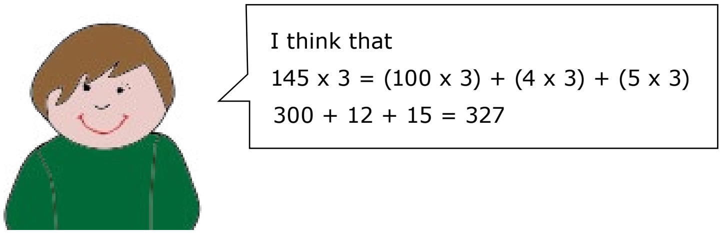
  Extension: Use dice (1 to 6 or 1 to 9) to determine the factors.

  Example:

  Students roll one die to determine the multiplier and then two or three dice to determine the multiplicand.

  Note: Place value dice could also be used.

Assessing Understanding: Paper-and-Pencil Task or Interview



Do you agree with his solution? Explain your thinking.

- **Estimate a product using a personal strategy (e.g., 2 × 243 is close to or a little more than 2 × 200, or close to or a little less than 2 × 250).**

Suggested Estimation Strategies:

- Make "friendly" numbers by rounding (one or both factors) to the nearest multiple of 10 or 100 (e.g., 4 × 62 is about 4 × 60 = 240).
- Round one factor up and one down (e.g., 33 × 8 is about 30 × 10 = 300).
- Using front-end estimation (e.g., 219 × 9 is about 200 × 9 = 1800).

Note: Students should estimate before solving a problem/question.

LEARNING EXPERIENCES

- **Is it reasonable?** Present students with a scenario along with estimations that have been done using different strategies. Have students determine which estimate is the most reasonable based on the scenario.

  Example:

  - Zoe has 4 pieces of ribbon. Each piece is 38 cm long. About how many centimetres of ribbon does she have?

    Possible estimates:

    Using front-end estimation: 4 x 30 = 120

    Using "friendly" numbers: 4 x 40 = 160

    Which estimate is more reasonable? Justify your thinking.

Assessing Understanding: Paper-and-Pencil Task or Interview

- Give an estimate for each of the following and explain the strategy you used.
  - 3 x 87
  - 211 x 9
  - 59 x 4
- Jacob drives 425 km each day. About how far will he have travelled after four days? Explain your thinking.
- Explain the strategy used for each of these estimations.
  - 431 x 4 = 400 x 4 or 1600
  - 68 x 8 = 70 x 8 or 560
  - 43 x 8 = 40 x 10 or 400

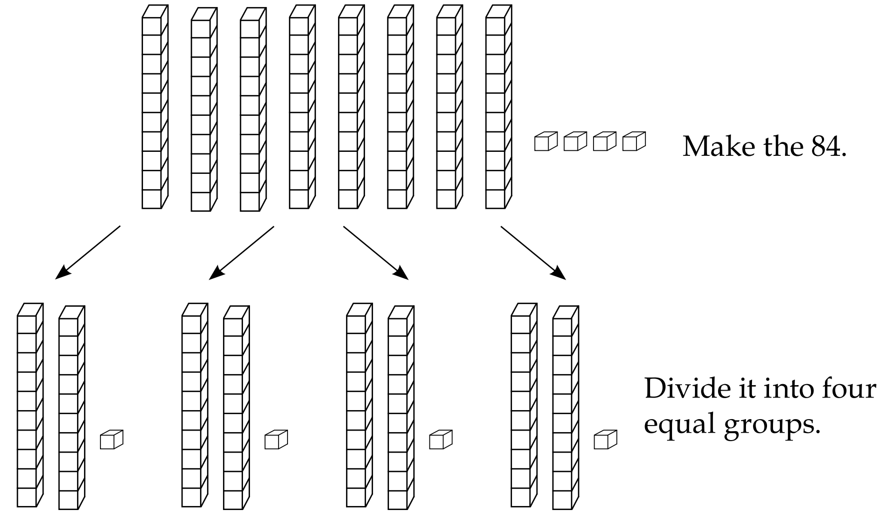
- **Solve a division problem without a remainder using arrays or base-10 materials.**
- **Solve a division problem with a remainder using arrays or base-10 materials.**

Suggestions for Instruction

- **Using Base-10 Materials**

  Example:

  84 ÷ 4

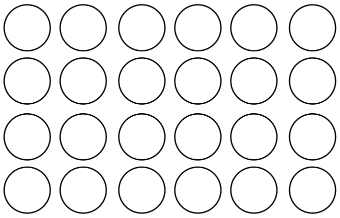


84 = 80 + 4

(80 ÷ 4) + (4 ÷ 4) = 20 + 1

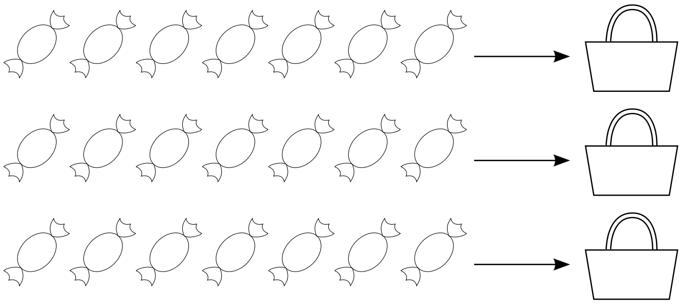
20 + 1 = 21

- **Using an Array:** It is important that students understand the relationship between multiplication and division. Have students write the number sentences represented by this array.



4 × 6 = 24 6 × 4 = 24 24 ÷ 4 = 6 24 ÷ 6 = 4

- **Using an Array to Solve a Problem:** You have 21 candies to share equally into 3 treat bags. How many candies does each bag get? Use an array to solve the problem.

  Show the candies in a 3 x 7 array.

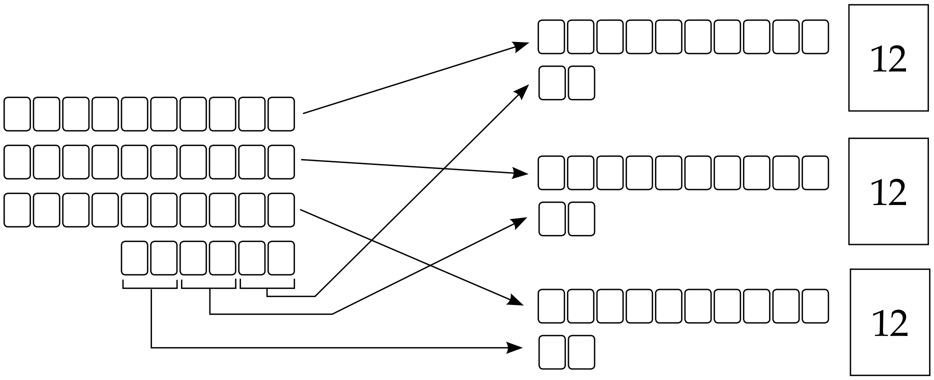
  Demonstrate the division using sharing—one for the first bag, one for the second, and one for the third, et cetera.

  Represent the action with a number sentence.

  21 ÷ 3 = 7 where 21 represents the number of candies, 3 represents the number of bags and 7 represents the number of candies in each bag.

- **Arrays and 2-Digit Division:** Jonas has 36 hockey cards on 3 pages of his binder. How many cards are on each page?

  Demonstrate the solution.

  Represent the 36 cards using counters.

  Demonstrate that the cards can be arranged in an array using the equal sharing process.

  Record the process symbolically.

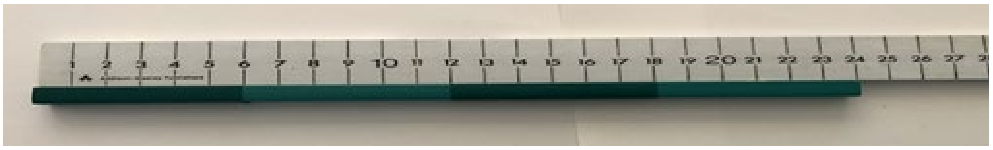
  30 ÷ 3 = 10 and 6 ÷ 3 = 2

  10 + 2 = 12 cards per page

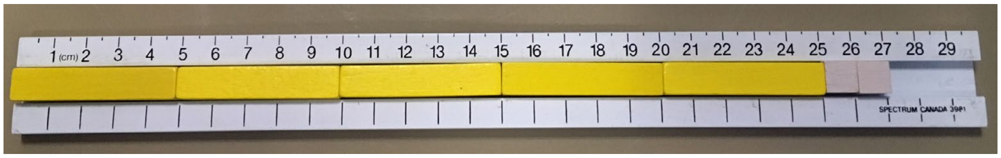
  Extension: Ask student to represent this problem using a multiplication number sentence (3 × ? = 36)

- **Using Cuisenaire Rods:** Cuisenaire rods can help make the division process visible.

  Example: 24 ÷ 6 = 4

  Showing remainders: 27 ÷ 5 = 5 R2

Assessing Understanding: Performance

1. Give students the following problems. Have them use manipulatives to solve them. Ask them to explain their strategies.
   - There are 48 chocolates in a box. If Martha eats 4 chocolates each day, how many days will the box last?
   - The baker puts 6 cookies in each package. If there are 45 cookies, how many packages can the baker make? Will there be cookies left over?
2. Ask students to use a model to explain to their partner how to share 65 gumballs among 4 friends.
3. Roll two dice (1 to 9) to create a 2-digit dividend. Arrange the order of the digits so that when divided by 7, it will give you the lowest remainder. Record the remainder after each roll. Total the score (reminders) after 5 rolls. The player with the lowest score wins.

   Example:

   If you roll a 4 and a 5, decide if the dividend will be 45 or 54.

   45 ÷ 7 = 6 R3 and 54 ÷ 7 = 7 R5 so it is better to chose 45 as the dividend. 3 is recorded as the score for that roll.

   Extension: Try the game using different divisors.
4. Present the following problem.

   Mrs. Wong bought a package of 52 pencils to be shared equally among her 4 children. Mr. Rodriquez bought a package of 40 pencils to be shared equally among his 3 children. Which children received the most pencils—the Wong family or the Rodriquez family?

   Use an array to solve the problem.

   Are there any pencils left over?

5. Ask students to model three different division questions of their choice using base-10 blocks. Have them write the division sentence for each.

- **Solve a division problem using a personal strategy, and record the process.**

Suggestions for Instruction

- **Personal Strategies:** Encourage students to come up with their own strategies. Have them explain how their strategies work.

  Ask students if they used place value in their strategies. If so, have them explain how it was used.

  Note: Students are not expected to use the standard algorithm. Some students, however, may have seen it used outside of the classroom. If it is suggested acknowledge it as a strategy but encourage students to find other ways to solve the problems.

  Strategy examples: 36 ÷ 6 = ____

  a. $36 \div 6 = \overset{5}{(30 \div 6)} + \overset{1}{(6 \div 6)}$

  $$5 + 1 = 6$$

  b. $$\begin{array}{r} 5 + 1 = 6 \\ 6\overline{)\ \ 30 + 6} \end{array}$$

  c. $$\begin{array}{l} \quad 5 + 1 = 6 \\ 6\overline{)\,36 \qquad\quad} \\ \ \ \underline{-\,30} \\ \qquad 6 \\ \ \ \underline{\ \ -6} \end{array}$$

  d. Repeated subtraction

  36 – 6 = 30

  30 – 6 = 24

  24 – 6 = 18

  18 – 6 = 12

  12 – 6 = 6

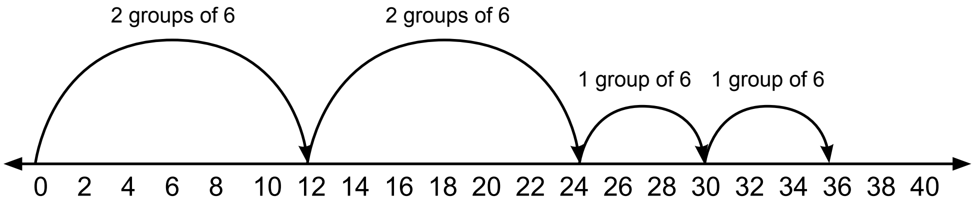
  6 – 6 = 1

  There are 6 sixes in 36.

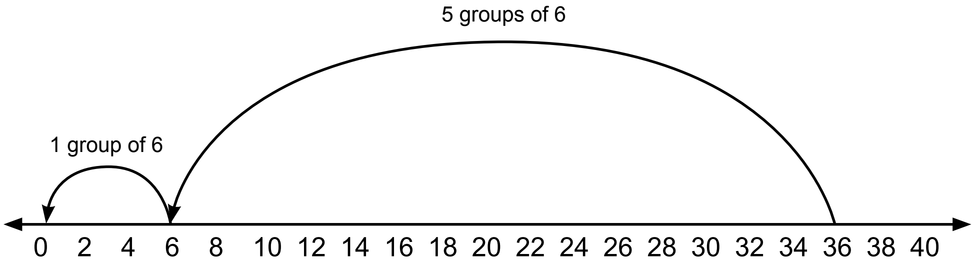
e. I know that two sixes are 12 and that 12 and 12 are 24. That is 4 sixes. 36 – 24 = 12 and that is another two sixes so there are 4 + 2 or 6 sixes in 36.

f. Number Line

Building Up—Adding



Building Back—Subtracting



g.

$$\begin{array}{r|l} 6\overline{)36} & \\ \underline{-30} & \left.\begin{array}{l} 5 \\ 1 \end{array}\right] 5 + 1 = 6 \\ 6 & \\ \underline{-6} & \\ 0 & \end{array}$$

- ***A Remainder of One*** by Elinor J. Pinczes, illustrated by Bonnie MacKain: This is the story of the Queen's 25th marching corps. Joe, one of the bugs, wants to march in the parade but every time they group the 25 troupe members he seems to be the odd one out—the remainder of one. The troupe keeps regrouping until there is no remainder.

  Use this book to have students model the division for each regrouping.
- Extension: Students could create their own story involving remainders.

Assessing Understanding

Paper-and-Pencil Task:

Have students use two different strategies to calculate 54 ÷ 6.

Interview:

Ask students to explain how to use 92 = 30 + 30 + 30 + 2 to calculate 92 ÷ 5.

- **Create and solve a word problem involving a 1- or 2-digit dividend.**

Suggestions for Instruction

- **Number Draw:** Prepare number tickets or a spinner with the digits 1 to 9. Prepare number tickets or a spinner (could also use tens and ones place value dice) with 2-digit numbers. Have students select one 1-digit number and one 2-digit number and then create and solve a division problem using personal strategies. Repeat.
- **Multiplication, Division, or Both?** Present students with problems. Have students determine whether the problem can be solved using multiplication, division, or both operations.

  Examples:

  - Joey places five rows of 24 chairs in the gym. Tracy places twice as many chairs as Joey. How many chairs are placed in the gym?
  - Ninety-six apples are shared equally into 4 baskets. If one of the baskets is shared equally among 8 people, how many apples does each person get?
  - Sarah and Jim buy three pieces of chocolate fudge that each cost 28¢. If they share the cost of the fudge equally, how much does each person pay?

  Have students work in pairs to classify the problems as multiplication, division, or both multiplication and division. Have the students represent each problem with appropriate number sentences and then solve the problem.

- **Make It Multiplication:** Give students a set of problems. Have them write both a division number sentence and a multiplication number sentence that could be used to solve the problem.

  Examples:

  1. Kate has 32 beads to share equally with her two friends. How many beads will each friend get? (e.g., 32 ÷ 2 = ?, 2 x ? = 32 or ? x 2 = 32)
  2. There are 91 stickers to be shared equally with 7 people. How many stickers will each person get?
  3. You travel 84 km in 3 days. If you travel the same distance each day, how far did you travel each day?
  4. Sixty students are going on a bus to the park. If 3 students can fit on each seat, how many seats are needed for the whole group?

Assessing Understanding

- **Paper and Pencil:** The answer is 6. What is the question? Have students create a division problem with a quotient of 6.
- **Interview:** Ask the student to explain how to use multiplication to solve the following: Ms. Hardy's 4 children have 52 stickers to share equally. How many stickers will each child get?

- **Estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5).**

Note: Using facts and fact strategies can help students make more reasonable estimates for division problems.

BACKGROUND INFORMATION

When estimating students might need to change one or both numbers so that familiar multiplication and division facts can be used.

Examples:

- 43 ÷ 5 is about 45 ÷ 5 = 9, or 43 ÷ 5 is about 40 ÷ 5 = 8
- 33 ÷ 8 is about 36 ÷ 9 = 4, or 33 ÷ 8 is about 28 ÷ 7 = 4

Possible Estimation Strategies for Division:

- Nearest Multiple of 10: Present the following problem:

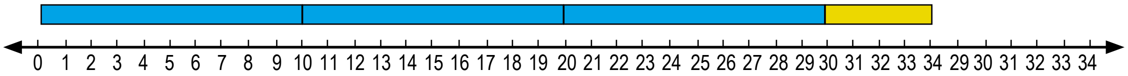
  Mandy has 34 cm of string. About how many pieces of string, each 5 cm long, can be cut from this string?

  Draw attention to the word *about.* This indicates that an estimation rather than an exact answer is required.

  Base-10 materials can be used to focus on the place values of the numbers. Cuisenaire rods and a centimetre ruler could also be used.

  Model your thinking (think aloud) as you decide how to estimate.

  Example, using Cuisenaire rods:



  "I cannot make 34 multiplying by 5. I see that 34 is close to 30 and I can make 30 multiplying by 5. 34 is also close to 40 and I can make 40 multiplying by 5. I am going to use 30 because it is closer to 34 (only 4 away) than 40 is (6 away).

  30 ÷ 5 = 6 so about 6 pieces of string can be cut from the string."

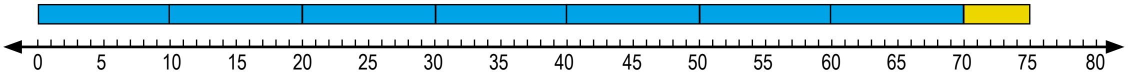
  When does this strategy work? Present other examples and ask students to decide if the nearest multiple of 10 strategy will work (e.g., 41 ÷ 7, 68 ÷ 9, 23 ÷ 5).

  Students should understand that this is a good strategy if the divisor divides evenly into the multiple of 10.

- Compatible Numbers with Compensation: This strategy that can be used when the divisor in the problem does not divide evenly into multiples of ten.

  Present the following problem:

  You have 75 cm of lace to share equally among 9 craft projects. About how much lace would each project receive?



  Think aloud:

  "Nine does not divide evenly into 70 or 80. I can see that 75 is halfway between 70 and 80 but 9 does not divide evenly into 75. What number close to 75 is divisible by 9? I know that 72 is divisible by 9 so I will use 72 ÷ 9 to estimate the quotient. 72 ÷ 9 = 8 so each project will get about 8 cm of lace."

Suggestions for Instruction

- **Estimate or Calculate?** Present students with problems and have them determine if the problem requires an estimate or an exact answer (calculation).

  Examples:

  - Mason rode his bike every day for 7 days. He cycled 47 km altogether. About how far did he cycle each day?
  - Eva put $8 in her bank each week. How long did it take her to save $64?
  - Ian has $80. About how many books could he buy if each book costs $9?
  - Pencils come in packages of 24. If they are shared equally among 4 children, how many pencils will each child get?
  - The tennis ball factory puts 3 balls in each package. If there are 42, how many packages can they make?

Assessing Understanding: Performance and Interview

- Give pairs of students the following questions:

  43 ÷ 6 = ____

  67 ÷ 5 = ____

  Have them estimate a quotient and then explain their strategy to their partner. Explain why their estimate is high or low.

- Give the student the following division problems:
  - The football team's score was 38. If touchdowns are worth 7 (including the convert), about how many touchdowns might they have scored?
  - Over the past 6 days Mark has eaten a total of 27 cookies. About how many cookies did he eat each day?

  Have the student estimate and explain their reasoning.

- Ask the student to describe a situation in which
  - the quotient is determined using an estimate
  - the quotient is determined using an exact calculation

PUTTING THE PIECES TOGETHER: MATH INFORMATION NIGHT

Organization:

Individual or small group activity

Materials:

Large poster paper, markers, et cetera

Directions:

Your class is planning a math information night for parents. In order to help the parents understand what you have been learning about multiplication and division, you are going to create a poster showing the strategies that you have learned.

Criteria:

- Each strategy is labelled.
- An example of the strategy and an explanation of how it works is included.
- Examples include both 2-digit and 3-digit numbers.
- Examples of estimation strategies are included.

Look for the following:

☐ The student/group has included
  - an array
  - base-10 representations
  - distributive property

☐ Examples used are appropriate for the strategy

☐ Explanations are organized and easy to follow

☐ Estimation strategies are included

Grade 4: Number (4.N.8)

Enduring Understandings:

Fractions are numbers with magnitudes.

A fraction represents a part of a whole or a set.

Fractions can be compared using a variety of models.

The size of the fractional part depends on the size of the whole.

Equal parts do not have to look the same, but they must be the same size or have the same amount of the whole.

Essential Questions:

What is a fraction?

Where do you use fractions in everyday life?

What is the numerator?

What is the denominator?

How are the numerator and denominator related?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.N.8 Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to • name and record fractions for the parts of a whole or a set • compare and order fractions • model and explain that for different wholes, two identical fractions may not represent the same quantity • provide examples of where fractions are used [C, CN, PS, R, V]	➛ Represent a fraction using concrete materials. ➛ Identify a fraction from its concrete representation. ➛ Name and record the shaded and non-shaded parts of a set. ➛ Name and record the shaded and non-shaded parts of a whole. ➛ Represent a fraction pictorially by shading parts of a set. ➛ Represent a fraction pictorially by shading parts of a whole. ➛ Explain how denominators can be used to compare two unit fractions. ➛ Order a set of fractions that have the same numerator, and explain the ordering. ➛ Order a set of fractions that have the same denominator, and explain the ordering. ➛ Identify which of the benchmarks 0, $\frac{1}{2}$, or 1 is closest to a fraction. ➛ Name fractions between two benchmarks on a number line (vertical or horizontal). ➛ Order a set of fractions by placing them on a number line (vertical or horizontal) with benchmarks. ➛ Provide examples where two identical fractions may not represent the same quantity (e.g., half of a large apple is not equivalent to half of a small apple; half of ten berries is not equivalent to half of sixteen berries). ➛ Provide an example of a fraction that represents part of a set, and a fraction that represents part of a whole, from everyday contexts.

PRIOR KNOWLEDGE

Students may have had experience in Grade 3 exploring parts of a whole that has been divided into “fair shares” or equal-sized pieces. They have also described situations in which fractions were used and compared fractions of the same whole with like denominators.

BACKGROUND INFORMATION

In Grade 4, the focus continues to be on developing a solid understanding of fractions less than or equal to one. Students will continue to build their understanding of fractional parts of a whole. They will also work with fractions that represent parts of a set.

Terminology

- **Fraction:** A number that represents part of a whole, part of a set, or a quotient in the form $\frac{a}{b}$, which can be read as *a* divided by *b*.
- **Numerator:** The number above the line in a fraction that can state one of the following:
  - the number of equal parts in a set to be considered
  - the number of equal parts of a whole to be considered
- **Denominator:** The number below the line in a fraction that can state one of the following:
  - the number of elements in a set
  - the number of equal parts into which the whole is divided
- **Unit Fraction:** A fraction with a numerator of 1.
- **Set:** Any collection of things, without regard to their order. The members (or elements) of a set could be numbers, names, shapes, and so on.

Suggested Manipulatives:

Fraction circles, fraction pieces, square (colour) tiles, Cuisenaire Rods, number lines, geoboards, paper (for folding), and egg cartons.

MATHEMATICAL LANGUAGE

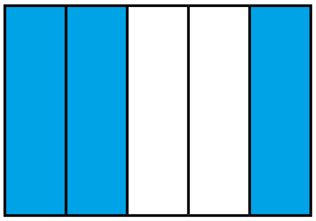
fraction
numerator
denominator
fair share
whole
one whole
half
third
quarter
fifth
sixths
eighths
tenths
one of ____ equal parts, set

Assessing Prior Knowledge

- Draw a picture for these fractions:

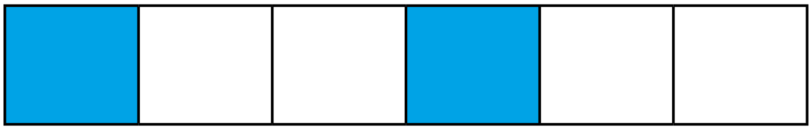
$\frac{3}{4}$ $\frac{2}{6}$

- What fraction of each shape is shaded? What fraction is not shaded?



shaded ______

unshaded ______



shaded ______

unshaded ______

- Sam says that his fraction has a denominator of 8 and a numerator of 3. Draw a picture to match Sam's fraction.
- Order these fractions from the smallest to the largest:

$\frac{7}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{4}{8}$ $\frac{6}{8}$

The student understands that

☐ the denominator represents the total number of parts the whole has been divided into

☐ the numerator represents the number of shaded parts or the parts being focused on

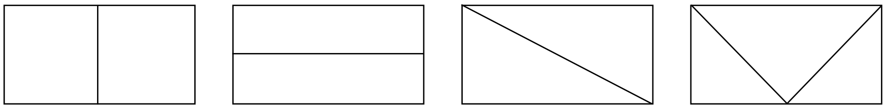
☐ when ordering fractions with the same denominator the larger the numerator the larger the fraction

- **Represent a fraction using concrete materials.**
- **Identify a fraction from its concrete representation.**
- **Name and record the shaded and non-shaded parts of a set.**
- **Name and record the shaded and non-shaded parts of a whole.**
- **Represent a fraction pictorially by shading parts of a set.**
- **Represent a fraction pictorially by shading parts of a whole.**

Suggestions for Instruction

- **Paper Folding:** Give students a rectangular piece of paper and have them fold the paper to show $\frac{1}{2}$. Compare the representations.

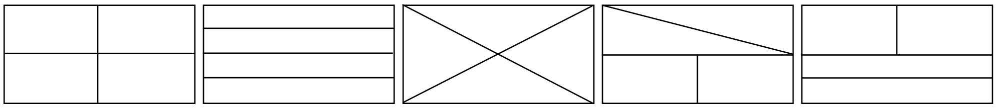
  Possible representations:

  Ask students to prove that their representation shows $\frac{1}{2}$ of the whole. (Students can cut out the shapes and overlap them to prove that they are congruent.)

  Repeat the activity having students show $\frac{1}{4}$.

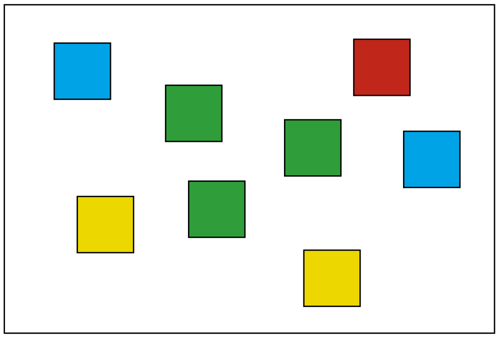
  Possible representations:

  Have students explain how they know that the non-congruent shapes represent $\frac{1}{4}$ of the whole.

- **Square (Colour) Tile Sets:** Have students choose 8 tiles. Have them identify the fraction of the set each colour represents.
  Note: Begin the activity by limiting students to use two colours only.

  Example:

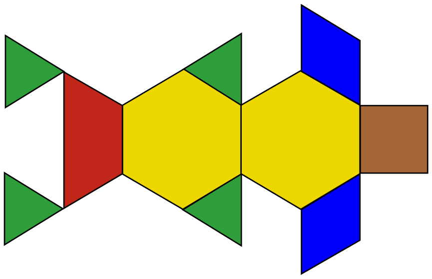
  🟦 $\frac{2}{8}$ 🟨 $\frac{2}{8}$ 🟩 $\frac{3}{8}$ 🟥 $\frac{1}{8}$

  Repeat using a different number of tiles.

- **Pattern Block Sets:** Have students make a simple design with the pattern blocks. Have them identify the fraction of the design/set that each shape represents.

  * Initially have students use only 2 shapes for their designs.

  Example:

  🟫 $\frac{1}{10}$ ◆ $\frac{2}{10}$

  ▲ $\frac{4}{10}$ ⏢ $\frac{1}{10}$

  ⬢ $\frac{2}{10}$

Extension: Ask students to create designs that are

- $\frac{1}{4}$ yellow and $\frac{3}{4}$ green
- $\frac{2}{6}$ blue and $\frac{4}{6}$ red
- $\frac{1}{6}$ green, $\frac{2}{6}$ yellow, and $\frac{3}{6}$ blue

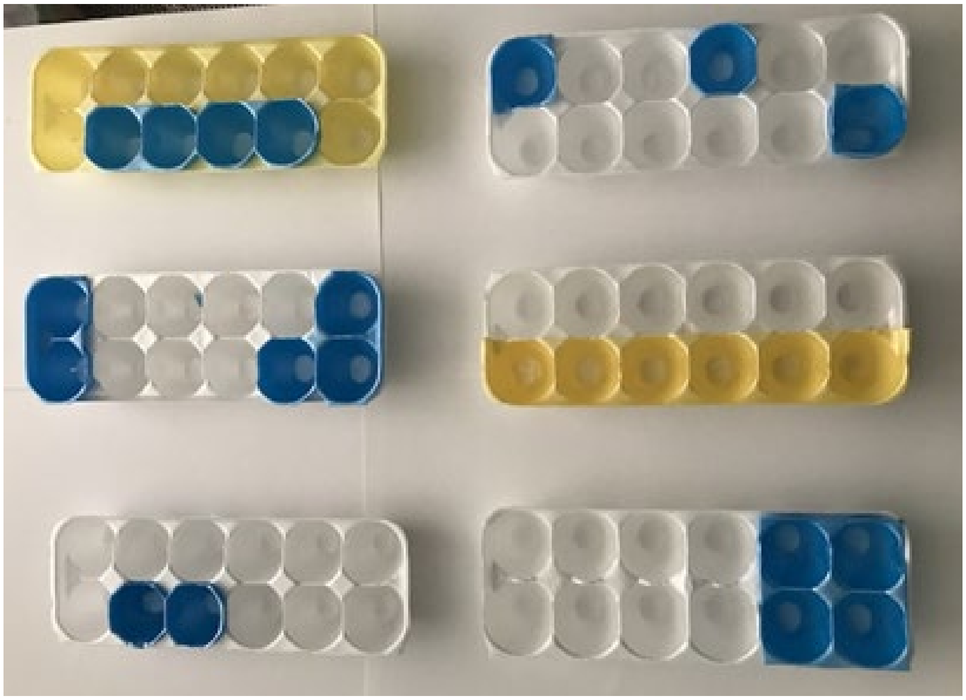
Have students make a design of their choice and then share their designs with a partner. Have the partner identify the fraction of the design/set that each shape represents.

- **Egg Carton Fractions:** Egg cartons are easy to obtain and easy to cut apart.

  Have students write a fraction for both the coloured/shaded part of the egg carton and the uncoloured/unshaded part of the carton.

  Note: Students may describe the coloured section in more than one way (e.g., for the carton with 6 coloured parts, students might record the fraction as $\frac{6}{12}$).

  Examples:



Extension: Cut out multiples of 2, 3, 4, and 6 cups so that students manipulate them to see that, for example, 1 group of three cups is $\frac{3}{12}$ but is also $\frac{1}{4}$ because 4 of the groups of 3 fit into the 12 cups in the carton.

BLM
4.N.8.1

- **Show the Fraction:** Give students a fraction. Have them represent it pictorially as part of a whole and as part of a set.

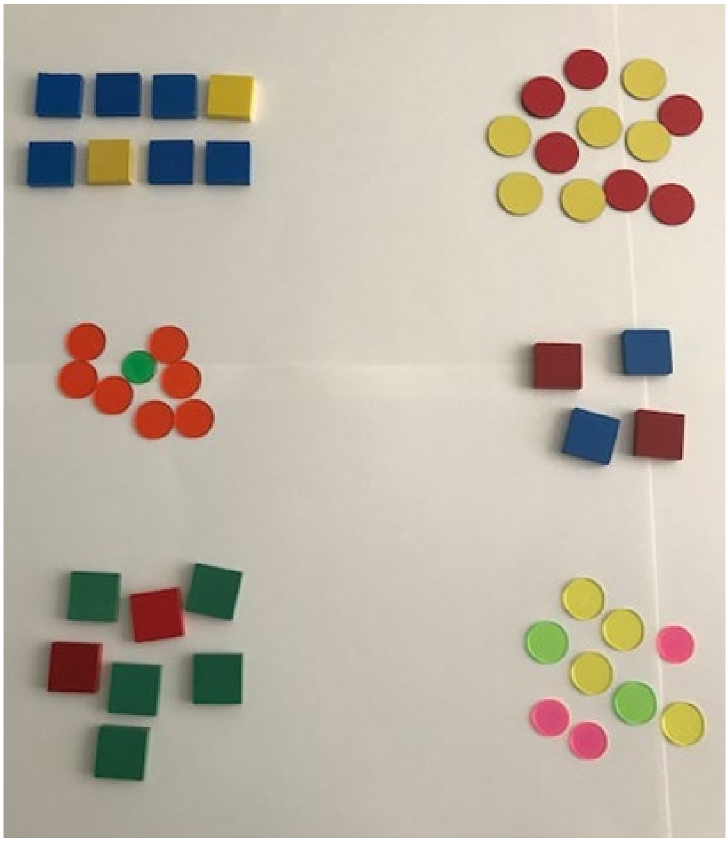
  Example:

  Directions: Draw a picture of each fraction as part of a whole and as part of a set.

Fraction	of a whole	of a set
$\frac{3}{4}$		
$\frac{1}{3}$		
$\frac{2}{5}$		
$\frac{4}{8}$		

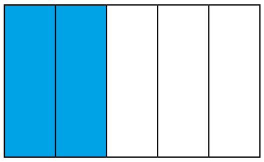
Assessing Understanding: Paper and Pencil

- What fraction does each colour represent in these sets?

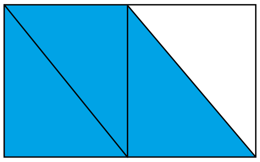


- Write the fraction representing
    - the shaded part of the diagram
    - the unshaded part of the diagram

a.



b.



- Draw a diagram to show $\frac{3}{4}$ as
    - part of a whole
    - part of a set
- Sam has a dozen (12) eggs. He used 3 for his cookie recipe. What fraction of the eggs did he use?

Assessing Understanding: Performance Task

- In this task, students will be able to demonstrate their understanding of fractions of a set. Have paper squares available to represent cheese slices. Students can cut the squares to help them solve the following problems:
    - Show how 4 people can share 3 cheese slices.
    - Show how 3 people can share 2 cheese slices.
    - Show how 12 people can share 6 cheese slices.
    - Show how 6 people can share 4 cheese slices.

Recording Sheet Example:

Draw a picture of the divided cheese.

___ cheese slices shared by ___ people.

Each person's share is ___ (fraction).

- **Explain how denominators can be used to compare two unit fractions.**
- **Order a set of fractions that have the same numerator, and explain the ordering.**
- **Order a set of fractions that have the same denominator, and explain the ordering.**

Suggestions for Instruction

- **Paper Folding:** Give students a strip of paper. Give the following directions:

  1. Fold the paper in half. Open it up. How many sections do you see? (2) What fraction is represented by each section/part? ($\frac{1}{2}$) Record the fraction. Refold the paper.
  2. Fold the paper in half again. Open it up. How many sections do you see? (4) What fraction is represented by each section/part? ($\frac{1}{4}$) Record the fraction. Refold the paper.
  3. Repeat two more times.

  Ask students what they noticed about the sections as they were doing the folding.

  Each time the paper was folded the number of sections/parts increased but the size of the sections/parts got smaller.

  Extension: Fold paper strips separately and label the fraction.

$\frac{1}{1}$

$\frac{1}{2}$	$\frac{1}{2}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

  The visual representation can help students conclude that the larger the denominator the smaller the fraction.

  Exit Slip: Would you rather have $\frac{1}{5}$ of a chocolate bar or $\frac{1}{10}$ of a chocolate bar? Explain your thinking.

- **Ordering Fractions:** Same Numerator

  The students in Mr. Grove's class were colouring fractions of paper strips. Josh coloured $\frac{2}{7}$ of his strip, Anna coloured $\frac{2}{5}$ of her strip, Levi coloured $\frac{2}{3}$ of his strip, and Sara coloured $\frac{2}{8}$ of her strip. Order the fractions from least to greatest. Explain your thinking using pictures and words.

  (Paper strips should be made available.)

- **Ordering Fractions:** Same Denominator

  Ms. Ang is baking four different items for the school bake sale. Each recipe calls for eggs. Ms. Ang has one dozen (12) eggs.

  - Cookies use $\frac{4}{12}$ of the eggs.
  - Cupcakes use $\frac{1}{12}$ of the eggs.
  - Brownies use $\frac{2}{12}$ of the eggs.
  - Lemon tarts use $\frac{6}{12}$ of the eggs.

  Order the fractions from the greatest number of eggs used to the least number of eggs used. Explain your thinking using pictures and words.

  (Egg cartons should be available, if needed, to help support student thinking.)

Assessing Understanding: Interview

BLM 4.N.8.2

1. Prepare a set of cards with fractions that have the same numerator and different denominators or use BLM 4.N.8.2.

$\frac{3}{8}$	$\frac{3}{4}$	$\frac{3}{5}$
$\frac{3}{10}$	$\frac{3}{9}$	$\frac{3}{12}$
$\frac{3}{7}$	$\frac{3}{3}$	$\frac{3}{6}$

Place the cards face down on the table. Have the student select 4 of the cards and order them from least to greatest. Ask them to explain their thinking.

BLM 4.N.8.3

2. Prepare a set of cards with fractions that have the same denominator and different numerators or use BLM 4.N.8.3.

$\frac{6}{10}$	$\frac{4}{10}$	$\frac{1}{10}$
$\frac{8}{10}$	$\frac{2}{10}$	$\frac{7}{10}$
$\frac{3}{10}$	$\frac{5}{10}$	$\frac{10}{10}$

Place the cards face down on the table. Have the student select 5 of the cards and order them from least to greatest. Ask them to explain their thinking.

The student understands that

- ☐ the greater the denominator the smaller the fraction
- ☐ when ordering fractions with a common denominator, the larger the numerator the larger the fraction

- **Identify which of the benchmarks 0, $\frac{1}{2}$, or 1 is closest to a fraction.**
- **Name fractions between two benchmarks on a number line (vertical or horizontal).**
- **Order a set of fractions by placing them on a number line (vertical or horizontal) with benchmarks.**

Suggestions for Instruction

Note: The most important benchmarks or referents for fractions are 0, $\frac{1}{2}$, and 1. These benchmarks help students understand the relative size of fractions. Noticing the relationships between the numerators and the denominators can help students determine the placement of a fraction related to the benchmarks.

- **What Am I Thinking?** Tell students that you are thinking of a fraction between 0 and 1. What might my fraction be? Record student responses.

  Repeat having students suggest possible fractions:

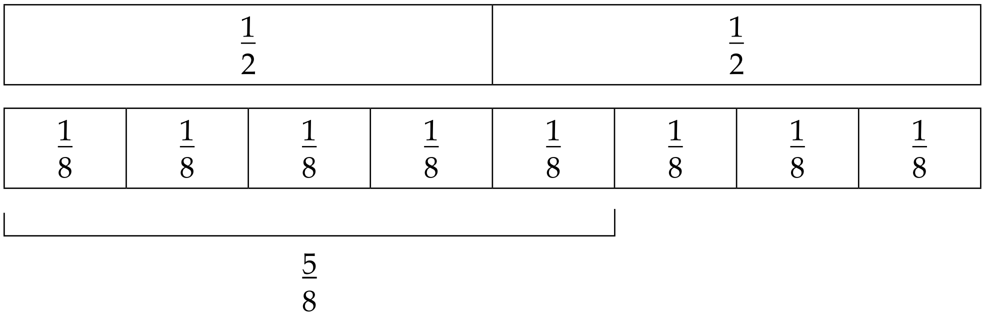
  - between 0 and $\frac{1}{2}$
  - between $\frac{1}{2}$ and 1

  Have students explain how they know that their fraction belongs between the given benchmarks.

BLM
4.N.8.4

- **Less than $\frac{1}{2}$ or Greater than $\frac{1}{2}$:** Have students use fraction bars to determine whether a given fraction is less than or greater than $\frac{1}{2}$.
  Example:

  Is $\frac{5}{8}$ less than or greater than $\frac{1}{2}$? Students can compare the $\frac{5}{8}$ to $\frac{1}{2}$ to see that $\frac{5}{8}$ is greater than $\frac{1}{2}$.



BLM
4.N.8.5

- **Less than $\frac{1}{2}$ or Greater than $\frac{1}{2}$ Sort:** Prepare a set of fraction cards or use BLM 4.N.8.5. Have students work with a partner or in small groups to sort them into two groups—Less than $\frac{1}{2}$ and Greater than $\frac{1}{2}$. Have students explain their placement.

$\frac{5}{8}$	$\frac{3}{4}$	$\frac{4}{10}$
$\frac{6}{9}$	$\frac{3}{7}$	$\frac{2}{3}$
$\frac{4}{12}$	$\frac{2}{5}$	$\frac{2}{4}$

BLM
4.N.8.6

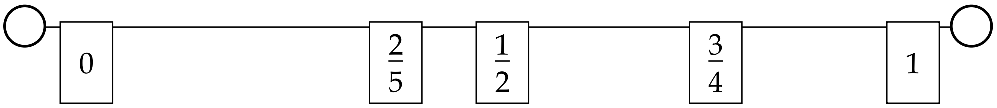
- **Close to 0 or Close to 1?** Prepare a set of fraction cards or use BLM 4.N.8.6. Have students work with a partner or small group to sort them into two groups—Close to 0 and Close to 1. Have them explain their placement.

  Example:

$\frac{1}{8}$	$\frac{9}{10}$	$\frac{3}{4}$
$\frac{2}{7}$	$\frac{2}{10}$	$\frac{7}{9}$
$\frac{5}{6}$	$\frac{4}{12}$	$\frac{1}{2}$

- **"Clothesline" Fractions:** Set up a "clothesline" number line using 2 strong magnets and a long piece of string. Make tent cards with 0, $\frac{1}{2}$, 1 along with other fractions less than 1. Place the benchmarks (0, $\frac{1}{2}$, and 1) on the line. Have students place a few of the remaining fractions on the number line and justify their placement.

  Example:



Note: Having every student in the class place a fraction is very time consuming and can result in a lack of focus on the part of the students. Instead, select two or three students and have them place a fraction.

This activity could become a class routine with different students selected each day.

This also provides formative assessment data.

Assessing Understanding

Performance:

Have students place fractions on the clothesline number line or paper number line (benchmarks included) and explain their placement.

Interview:

Use cards from the learning experiences above. Have the student sort the fractions into three groups—Close to 0, Close to $\frac{1}{2}$, Close to 1—and justify their placement.

Paper-and-Pencil Task:

Matt sorted fractions into two groups.

Here is his sort.

$\frac{1}{5}$ $\frac{1}{3}$ $\frac{2}{8}$

$\frac{9}{10}$ $\frac{6}{7}$ $\frac{10}{12}$

What might his sorting rule be? Explain your thinking.

- **Provide examples where two identical fractions may not represent the same quantity (e.g., half of a large apple is not equivalent to half of a small apple; half of ten berries is not equivalent to half of sixteen berries).**

Suggestions for Instruction

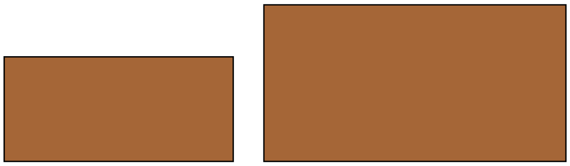
Note: It is important that students understand that a fraction represents the relationship between the part and the whole but that it does not provide information about the size of the whole or the size of the parts.

- **When Does Size Matter?** Show students two fruits or vegetables that have an obvious size difference (e.g., an apple and a grape, an orange and a watermelon, a pea and a potato). Cut each one in half. Point out that although the fraction we write for each piece is $\frac{1}{2}$, the halves are not the same because of the difference in the size of the fruits/vegetables.
  Have students suggest other examples.

- **Pizza Dilemma:** Kim and her friend Erin went out to eat, and each one ordered a pizza. After eating $\frac{1}{4}$ of each of their pizzas, Kim noticed Erin had a lot more pizza left. How could this happen? Explain using pictures and words. (Erin's pizza is larger than Kim's.)

- **Chocolate Bar Challenge:** James has 2 chocolate bars to share with his 6 friends.

  Here are the chocolate bars.

  James decides to divide each chocolate bar into thirds and then give $\frac{1}{3}$ to each friend.

  Do you agree with his plan? Why or why not?

Assessing Understanding: Journal Entry

- Ian saw a poster in the classroom that said:

  When
  comparing
  fractions,
  the whole
  matters.

  Explain what the poster means.

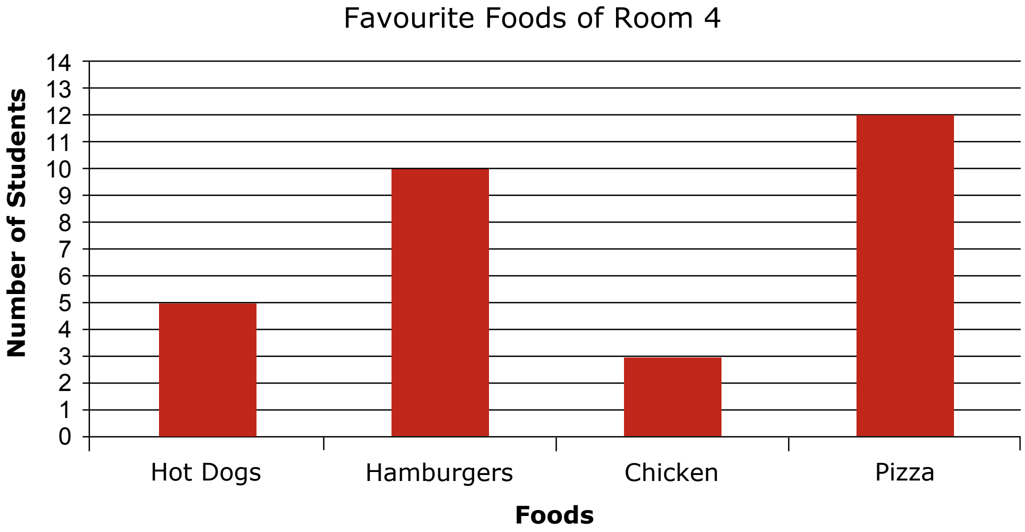
  The student understands that, when comparing fractions, the whole must be the same size for each fraction.

- **Provide an example of a fraction that represents part of a set, and a fraction that represents part of a whole, from everyday contexts.**

Suggestions for Instruction

- **Fraction Hunt:** Have students find examples of both fractions of a set and fractions of a whole from real-world contexts. Have them present their findings using pictures and symbols.

  Possible examples might include
  - Food—measuring for recipes
  - Time—fractions of an hour, day, month, year, etc.
  - Measurement tools
  - Classroom contexts—using student attributes (e.g., fraction of students with dark hair, wearing runners, etc.)
- **Classroom Routine(s):**
  - Have the daily attendance recorded as a fraction (e.g., fraction of students absent, fraction of students present).
  - Although Grade 4 students might not celebrate 100 Day, counting the number of days in school could be recorded as a fraction, either counting to 100 day (e.g., $\frac{40}{100}$) or using the total number of school days for the school year (e.g., $\frac{40}{186}$).
- **Connecting with Statistics:** Show students this graph. Have them summarize the findings using fractions of the set of 30 students (e.g., $\frac{5}{30}$ of the students like hot dogs).



Assessing Understanding: Performance

- Have the students
    - decide on a survey questions to ask the students in the classroom
    - gather the data
    - represent the data in graph form
    - summarize their findings using fractions of the set (number of students surveyed)

Criteria (connect to Statistics and Probability):

- survey question is clear
- categories/choices are appropriate
- graph is complete (labels, title, scale, bars)
- data is accurately represented on the graph
- summary of the data is accurately represented in fraction form

Putting the Pieces Together: The Birthday Party

Present the following problems related to a birthday party.

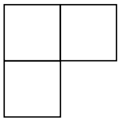
1. Alex's birthday is in September. Instead of telling his friends the date, he gives these clues:

   My birthday is $\frac{1}{2}$ the way through the month.

   What is the date of Alex's birthday? (15th)

2. Alex helped his mother bake the birthday cake. The recipe called for 3 eggs. If there were 7 eggs left in the egg carton, what fraction of the remaining eggs did they use? ($\frac{3}{7}$)

3. This picture represents $\frac{1}{4}$ of the cake.

   Draw the whole cake.

4. Alex is turning 9. He uses blue and yellow candles for the cake. What combinations of candles are possible? Record your answers in fraction form.

   Example:

Possible Combinations	
Yellow	**Blue**
$\frac{1}{9}$	$\frac{8}{9}$
	Etc.

5. Alex has 10 friends coming to the party. $\frac{2}{5}$ of his guests are girls, the rest are boys. How many girls are coming to the party? (4) How many boys are coming? (6)
6. The party starts at 12:00 p.m. and ends at 4:00 p.m. What fraction of the whole day will the party take up? ($\frac{4}{24}$)
7. Ten different games were played at the party.

   May won $\frac{4}{10}$. Sal won $\frac{2}{10}$, Mark won $\frac{1}{10}$, and Jonas won $\frac{3}{10}$.
   Order the children from the greatest number of wins to the least number of wins. (May $\frac{4}{10}$, Jonas $\frac{3}{10}$, Sal $\frac{2}{10}$, Mark $\frac{1}{10}$)
   What fractions of the guests did not win any of the games? ($\frac{6}{10}$)
8. Alex puts 12 treats in each goodie bag.

   In each bag,

   - $\frac{1}{4}$ of the treats are chocolates (3 × 10 = 30)
   - $\frac{7}{12}$ of the treats are candies (7 × 10 = 70)
   - $\frac{2}{12}$ of the treats are small toys (2 × 10 = 20)

   How many of each treat does Alex need to make 1 goodie bag?

   How many of each treat does he need to make 10 goodie bags?

   Recording sheet:

Treats	**Number needed for 1 bag**	**Number needed for 10 bags**
Chocolates		
Candies		
Toys		

The student

- is able to apply their understanding of fractions of a whole in problem-solving contexts
- is able to apply their understanding of fractions of a set in problem-solving contexts
- is able to make connections between fractions and real-world contexts

Grade 4: Number (4.N.9, 4.N.10, 4.N.11)

Enduring Understandings:

Decimals are an extension of our place value system.

Fractions and decimals are related.

Essential Questions:

How are decimals connected to place value?

How are fractions and decimals related?

Specific Learning Outcome(s):	Achievement Indicators:
4.N.9 Describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically. [C, CN, R, V]	➛ Write the decimal for a concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure. ➛ Represent a decimal using concrete materials or a pictorial representation. ➛ Explain the meaning of each digit in a decimal with all digits the same. ➛ Represent a decimal using money values (pennies and dimes). ➛ Record a money value using decimals. ➛ Provide examples of everyday contexts in which tenths and hundredths are used. ➛ Model, using manipulatives or pictures, that a tenth can be expressed as hundredths (e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies).
4.N.10 Relate decimals to fractions (to hundredths). [CN, R, V]	➛ Read decimals as fractions (e.g., 0.5 is zero and five-tenths). ➛ Express orally and in written form a decimal in fractional form. ➛ Express orally and in written form a fraction with a denominator of 10 or 100 as a decimal. ➛ Express a pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or $\frac{15}{100}$). ➛ Express orally and in written form the decimal equivalent for a fraction (e.g., $\frac{50}{100}$ can be expressed as 0.50).

SPECIFIC LEARNING OUTCOME(S):	**ACHIEVEMENT INDICATORS:**
4.N.11 Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by • using compatible numbers • estimating sums and differences • using mental math strategies to solve problems. [C, ME, PS, R, V]	➢ Predict sums and differences of decimals using estimation strategies. ➢ Solve problems, including money problems, which involve addition and subtraction of decimals, limited to hundredths. ➢ Determine the approximate solution of a problem not requiring an exact answer. ➢ Estimate a sum or difference using compatible numbers. ➢ Count back change for a purchase.

PRIOR KNOWLEDGE

Students have had no formal instruction related to decimals prior to Grade 4.

BACKGROUND INFORMATION

Possible misconceptions:

- Students may mistakenly believe that the place value places to the right of the decimal point have the same names as those on the left (e.g., the first place to the right is the tens not the tenths).
- Students think that the larger the decimal the larger the number because they are applying what they understand about whole numbers to decimals.
- Students may ignore the decimal point altogether when reading decimal numbers.

Reading Decimal Numbers: It is important that students are able to read decimal numbers correctly. Avoid using the term *point* because it has no mathematical meaning to students. Instead, use *and* to represent the decimal (e.g., 2.5 should be read as two and five tenths, not two point five).

When writing numbers less than one use a zero in the one's place. This helps emphasis that the decimal is less than 1.

Suggested Manipulatives: money, metre stick, ruler, egg cartons, hundred chart, hundred bead frame (Rekenrek), ten frames, base-10 blocks, Cuisenaire rods, number lines, measuring tape, and grid paper.

MATHEMATICAL LANGUAGE

- fraction(s)
- decimals
- tenths
- hundredths
- decimal point

LEARNING EXPERIENCES

Assessing Prior Knowledge

- Ask students what they know about decimals. Where have they seen them before?
- Show students this number $2.35. Ask them to read it. What does the 2 represent? What does the 35 represent?

- **Write the decimal for a concrete or pictorial representation of part of a set, part of a region, or part of a unit of measure.**
- **Represent a decimal using concrete materials or a pictorial representation.**
- **Explain the meaning of each digit in a decimal with all digits the same.**
- **Represent a decimal using money values (pennies and dimes).**
- **Record a money value using decimals.**
- **Provide examples of everyday contexts in which tenths and hundredths are used.**
- **Model, using manipulatives or pictures, that a tenth can be expressed as hundredths (e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies).**
- **Read decimals as fractions (e.g., 0.5 is zero and five-tenths).**
- **Express orally and in written form a decimal in fractional form.**
- **Express orally and in written form a fraction with a denominator of 10 or 100 as a decimal.**
- **Express a pictorial or concrete representation as a fraction or decimal (e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or $\frac{15}{100}$).**
- **Express orally and in written form the decimal equivalent for a fraction (e.g., $\frac{50}{100}$ can be expressed as 0.50).**

Suggestions for Instruction

Note: Begin decimals by working with tenths. Students come to understand that the unit (1) can be divided into 10 equal parts (tenths) and, to be able to record these parts, another place value has been added to the right of the one's place separated by a decimal point to show that it is a fractional part.

- **Ten Frames:** Ten frames are a familiar manipulative material and, as a result, students can easily apply their prior experience to their work with decimals.

  Use a set of ten frames. Select a ten frame. Have students identify the number of filled/shaded spaces as a fraction. Explain how the fraction can be written as a decimal.

  Have students record the number shown as a fraction and as a decimal.

  Have students record both the fraction and decimal for both the shaded and unshaded portions of each ten frame.

  Example:

●	●	●	●	●
●	●			

Shaded/filled in	Unshaded/not filled in
Fraction: $\frac{7}{10}$	Fraction: $\frac{3}{10}$

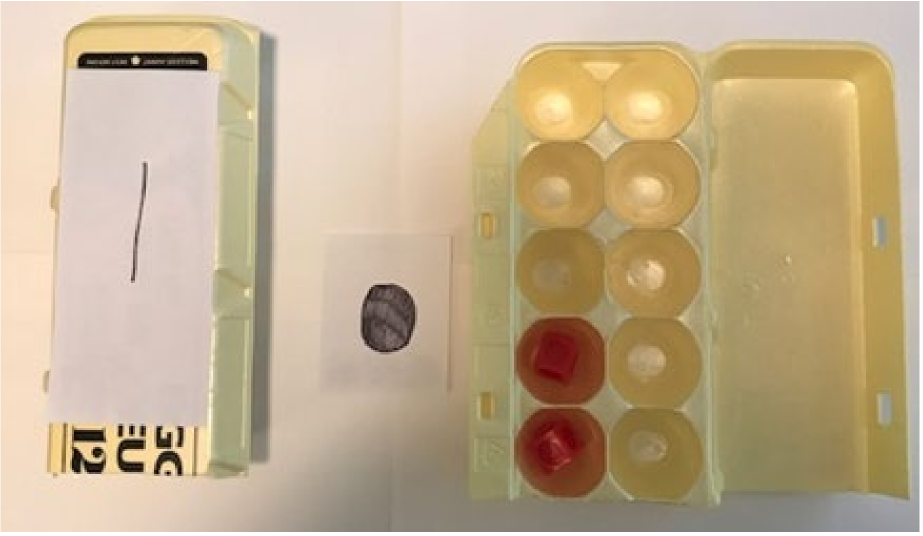
- **Egg Carton Ten Frames:** Keep the egg carton intact (lid included). Cut off two of the cups of the egg carton. Cut off the portion of the lid covering the two cups. Label the remaining portion of the lid with the number 1. The cup portion serves as a ten frame. When the ten frame is full the lid can be closed and students will see it as one whole.

  Example:

  The ten frame shows 0.7.

The student rolls a die and adds 5 more cubes.

3 cubes fill the ten frame, and the lid is closed and moved to the left of the decimal. The remaining 2 cubes are placed in a new ten frame.



Students can see that they now have one whole and 2 tenths. The visual representation supports the recording as 1.2.

Extension: Have students play a Race to Two or Three game.

Organization:

Students play with a partner.

Materials:

Each person needs egg carton ten frames and cubes, die (1 to 6 or 1 to 9), and a recording sheet.

Directions:

Students take turns rolling the die and adding cubes to match the number rolled to the ten frame. When a ten frame is filled, the lid is closed, and it is moved to the left of the decimal. Students record after each roll.

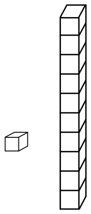
The first person to reach two or three (or more) is the winner.

Example:

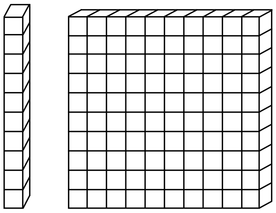
Roll #	Amount rolled	Total
1	0.7	0.7
2	0.5	0.7 +0.5 = 1.2
3	0.2	1.2 + 0.2 = 1.4

- **Tenths as Parts of Different Wholes:** Tenths can be represented using different materials or wholes. Students need experiences with a variety of these materials.

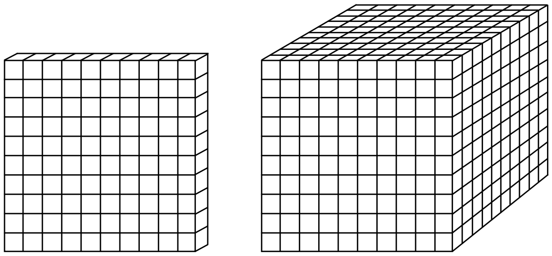
  Base-10: Note: Using base-10 materials for decimals can be confusing for some students because they are used to working with these materials for whole numbers (e.g., If the flat (100) is the whole, it is important that students understand that the flat is now 1 (or the whole) and not 100 as it is with whole numbers).



If the ten is the whole, the ones cube is one tenth of the ten.



If the hundred is the whole, the ten is one tenth of the hundred.



If the thousand is the whole, the hundred is one tenth of the thousand.

  Metre Stick: When exploring a metre stick students can see the following:

  - If the centimetre is the whole, 1 millimetre is one tenth of the centimetre.
  - If the decimetre is the whole, 1 centimetre is one tenth of the decimetre.
  - If the metre is the whole, 1 decimetre is one tenth of the metre.

  Money: Money is a non-proportional model for decimal numbers. Some students may find this model more difficult.

  Note: Although the penny is no longer in circulation in Canada, it is still used in electronic transactions. Most sets of "play" money include pennies. Pennies can also be made using labelled counters or paper/foam discs.

  As students work with money, they will see the following:

  - A penny is one tenth of a dime or one hundredth of a loonie/dollar (0.01).
  - A dime is one tenth of a loonie/dollar or 0.1.
  - A loonie/dollar is 1 whole.

- **Clothesline Number Line:** Place a zero at one end of the line and 1 at the other end. Have students place tent cards with numbers such as 0.7, 4 tenths, 0.5, et cetera, on the line and justify their placement.

BLM
4.N.9.1

BLM
4.N.9.2

- **Hundredth Squares/Grids:** Hundredth squares/grids are excellent tools to use to help develop an understanding of hundredths.

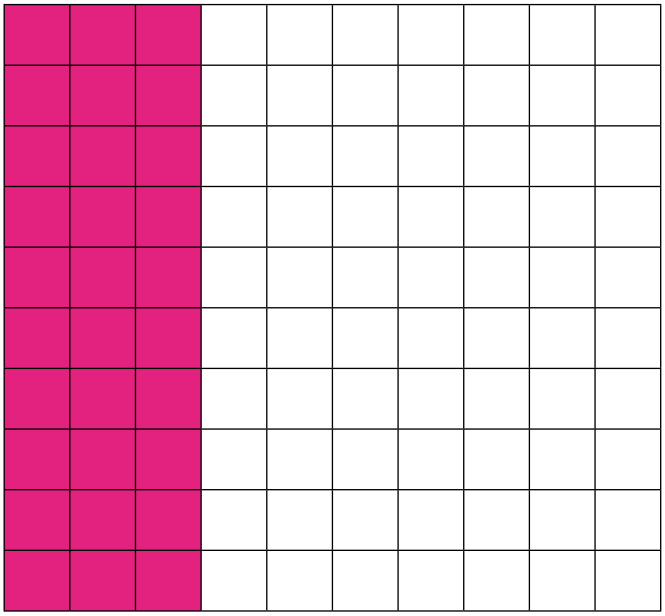
  Begin by having students shade in only tenths on grids. Have them identify the fraction and the decimal for both the shaded and unshaded parts of the grid.

  Example:

  Show 0.3 on the hundredth grid.

  Write the fraction and the decimal for the shaded part of the grid.

  Write the fraction and the decimal for the unshaded part of the grid.



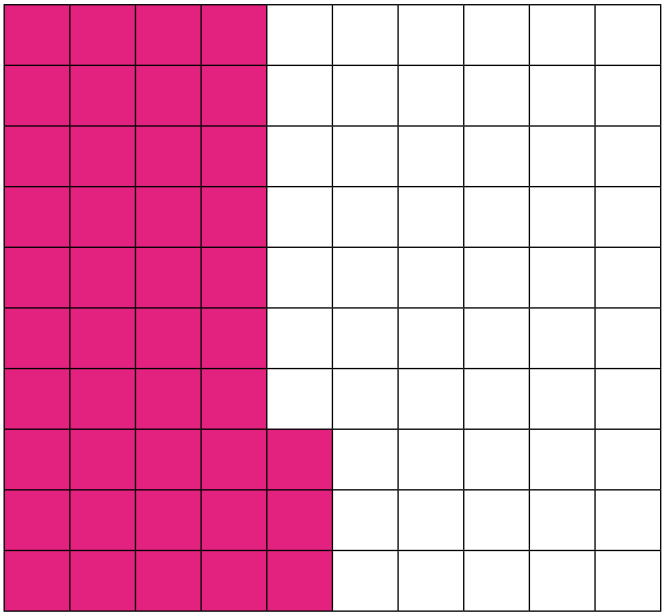
$\frac{3}{10}$ or 0.3 of the grid is shaded.

$\frac{7}{10}$ or 0.7 of the grid is unshaded.

Once students are confident with the tenths, introduce the hundredths place value.

Prepare cards with tenths and hundredths and have students select a card, and shade in the number shown. Have them label the fraction and the decimal for both the shaded and unshaded portions of the grid.

Example: Show 0.43 on the hundredth grid.



$\frac{43}{100}$ or 0.43 of the grid is shaded.

$\frac{57}{100}$ or 0.57 of the grid is unshaded.

Note: It is important that students understand that tenths can be written as hundredths (e.g., 0.8 can also be written as 0.80; 80 cents is written as $0.80 in decimal form).

- **Representing Hundredths with a Variety of Materials**

  Money: Have students represent decimal values with money and money values as decimals.

  Examples:

  a. How much money is shown in each picture? Record using decimals.

($1.35)

($2.73)

  b. Show each money value using coins or pictures of coins.

  $1.90 $3.46 $2.07

  Extension: Have students bring in grocery store flyers. Have students choose two items from the flyer and show and record at least two different ways (using cash) that they could pay for each item.

  Example:

Item and Cost	First Way	Second Way
a litre of milk for $1.89	1 loonie, 3 quarters, 1 dime, 4 pennies	4 quarters, 8 dimes, 1 nickel and 4 pennies

- **What's My Value?** Give students decimals in which all digits are the same. Have students identify the value of each digit.

  Example: In the number 4.44 what is the value of each of the digits? Explain using base-10 blocks, money, or hundredth grids, numbers, and words.

- **Decimals Scavenger Hunt:** Have students find examples of decimal use in their everyday life (home, school, TV, newspapers, other). Have them shared with the class. Prepare a class poster or book with the examples.

  Possible examples: items packaged in tens, fingers and toes, statistics for sports, gas prices, prices in flyers, et cetera

Assessing Understanding

Performance:

Prepare a set of tent cards with decimals representing tenths and hundredths. Have the student select 4 cards and place them on the clothesline number line (or paper number line) and justify the placement.

Example:

0.60	0.54	0.3
0.98	0.26	0.05
0.17	0.83	0.77

Performance:

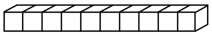
Provide a collection of play money. Ask the student to take a small handful of coins. Have the student count the money and then record the amount as a decimal.

Have the student show $2.56 using the coins in two different ways.

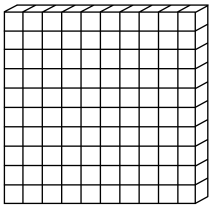
Performance:

Give students the following problem.

represent hundredths



represent tenths



represent wholes

If you use ten base-10 blocks, what number can you make? Give at least 8 possible answers.

Paper-and-Pencil Task/Journal Entry:

How are fractions and decimals related? Explain using pictures and words.

Interview:

Have the student explain where decimals are used in everyday life.

Paper-and-Pencil Task

Show the student a ten frame. Have them write a fraction and decimal for both the shaded and unshaded parts.

Show the student a hundredth grid with 0.64 shaded in. Have the student write a fraction and a decimal for both the shaded and unshaded portions.

- **Predict sums and differences of decimals using estimation strategies.**
- **Solve problems, including money problems, which involve addition and subtraction of decimals, limited to hundredths.**
- **Determine the approximate solution of a problem not requiring an exact answer.**
- **Estimate a sum or difference using compatible numbers.**
- **Count back change for a purchase.**

BACKGROUND INFORMATION

Mental math strategies used for the addition and subtraction of whole numbers can be used with decimal numbers.

Examples:

- Compatible (Friendly) Numbers: For example, for 0.73 + 0.24, think 0.73 is close to 0.75 and 0.24 is close to 0.25, so my estimate is 0.75 + 0.25 = 1.
- Front-End Addition: For example, for 42.5 + 31.8, think the nearest multiple of ten to 42.5 is 40 and the nearest multiple of ten to 31.8 is 30, so my estimate is 40 + 30 = 70.
- Front-End Subtraction: For example, for 5.38 – 2.41, think 5.38 is close to 5 and 2.41 is close to 2, so my estimate is 5 – 2 = 3.
- Rounding: For example, for 5.38 + 2.87, think 5.38 is close to 5 and 2.87 is close to 3, so my estimate is 5 + 3 = 8.

Suggestions for Instruction

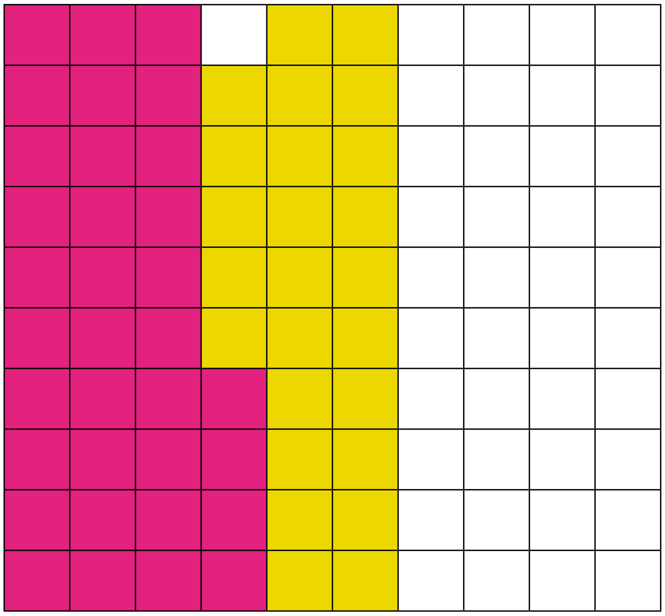
- **Adding and Subtracting from $1.00:** Provide students with hundredth squares. Have them model addition and subtraction from 1 using the squares.

  Note: The hundredth square can also be used to model addition and subtraction from $1.00. (Students should estimate before solving each problem.)

Examples:

Addition

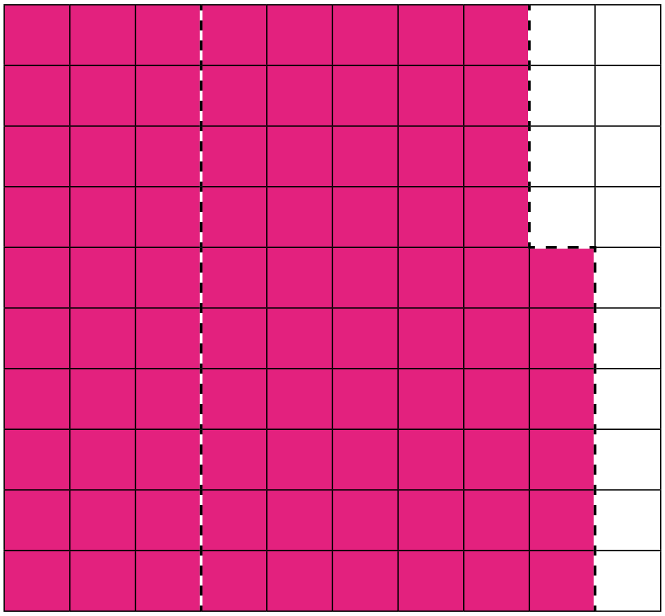
0.34 + 0.25



Shade in the 0.34. Shade in the 0.25. It is easy for students to see that the total is 0.59.

Subtraction

0.86 – 0.56



Shade in the 0.86. Cut off the unshaded portion. Using the shaded portion, cut out the 0.56. The remaining portion is the solution.

- **Base-10 Modelling:** Give students addition and subtraction questions and have them use base-10 blocks to model the process. (Have students estimate first.)

  Examples:

  - Marg has 1.35 m of red material and 2.67 m of blue material. How much material does she have altogether?
  - John has $25 to spend. If he buys a game for $19.56, how much money will he have left?
  - Ken ran 13.4 km on Monday, 12.82 km on Tuesday, and 14.07 km on Wednesday. How many kilometres did he run over the three days?

    The following week Ken ran a total of 35.2 km. In which week did he run the farthest? How much farther?
  - Ms. Allan's grocery bill came to $70.06. If she paid with a $100 bill, how much change did she receive?

- **Spending Spree:** Have students bring in flyers. Ask students to do the following:

  - Pick a flyer.
  - Choose 3 or 4 items that they might like to buy from the flyer.
  - Find the total cost of the items.

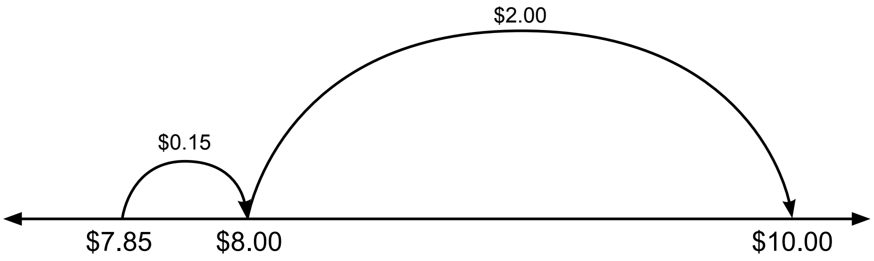
  Have students record their work. (Pictures of the items can be cut out and put in their notebooks/journals.)

- **Classroom Store:** Have students bring in food boxes/containers, small toys, et cetera. Have students price the items. Students can take turns being the customers and the store clerks. (Be sure that students and clerks estimate before calculating an exact answer.) Play money should be used for payment.

  Have students practise counting back change.

- **Counting Back Change:** Use a number line to model counting on as you give back change.

  Example: Making change from $10 for a $7.85 purchase.



Assessing Understanding

Interview:

Ask the student to estimate the answers to these problems and explain the strategy they used.

- 3.32 – 2.15
- 42.55 + 23.07
- $20 – $16.78

Performance:

Prepare cards with money values less than $20.

Example:

$12.75	$9.98	$15.35
$13.65	$11.40	$8.05

Place the cards face down on the table. Tell the student that they have a $20 bill. Have them select a card and then show how they would count back the change. Repeat one more time.

Performance:

Jenna bought a new pair of jeans for $28.97. She paid with a $50 bill.

- What was her change? Explain your thinking using models, words, and pictures.
- What other bills could Jenna have used to pay for the jeans? What change would she receive?

Paper-and-Pencil Task:

Create an addition problem and a subtraction problem that use decimal numbers. Show the solution.

GRADE 4 MATHEMATICS

Patterns and Relations

Grade 4: Patterns and Relations (4.PR.1, 4.PR.2, 4.PR.3, 4.PR.4)

Enduring Understandings:

Patterns show order in the world.

Patterns can be found in many different forms.

Graphic organizers can be used to solve problems.

Essential Questions:

What is the increasing or decreasing unit in the pattern?

What strategies can be used to continue an increasing or decreasing pattern?

What strategies can be used to continue a numerical sequence?

How is the pattern increasing or decreasing?

How can graphic organizers help solve problems?

How is the pattern increasing or decreasing?

Specific Learning Outcome(s):	**Achievement Indicators:**
4.PR.1 Identify and describe patterns found in tables and charts, including a multiplication chart. [C, CN, PS, V]	➛ Identify and describe a variety of patterns in a multiplication chart. ➛ Determine the missing element(s) in a table or chart. ➛ Identify error(s) in a table or chart. ➛ Describe the pattern found in a table or chart.
4.PR.2 Reproduce a pattern shown in a table or chart using concrete materials. [C, CN, V]	➛ Create a concrete representation of a pattern displayed in a table or chart. ➛ Explain why the same relationship exists between the pattern in a table and its concrete representation.
4.PR.3 Represent and describe patterns and relationships using charts and tables to solve problems. [C, CN, PS, R, V]	➛ Extend patterns found in a table or chart to solve a problem. ➛ Translate the information provided in a problem into a table or chart. ➛ Identify and extend the patterns in a table or chart to solve a problem.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.PR.4 Identify and explain mathematical relationships using charts and diagrams to solve problems. [CN, PS, R, V]	➣ Complete a Carroll diagram by entering data into correct squares to solve a given problem. ➣ Determine where new elements belong in a Carroll diagram. ➣ Solve a problem using a Carroll diagram. ➣ Identify a sorting rule for a Venn diagram. ➣ Describe the relationship shown in a Venn diagram when the circles intersect, when one circle is contained in the other, and when the circles are separate. ➣ Determine where new elements belong in a Venn diagram. ➣ Solve a problem by using a chart or diagram to identify mathematical relationships.

PRIOR KNOWLEDGE

Students may have

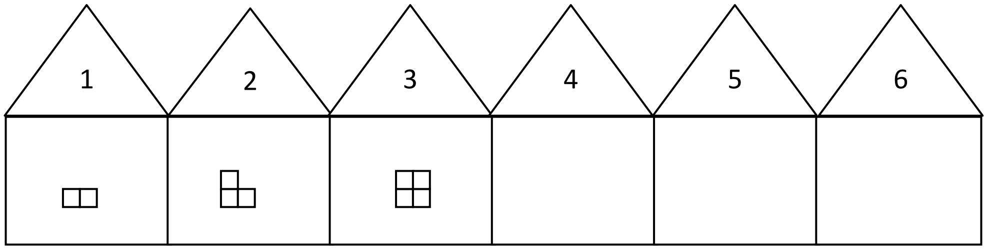
- worked with repeating, increasing, and decreasing patterns
- identified patterns on a hundred chart, addition table, and calendar
- worked with numerical patterns with numbers to 1000

BACKGROUND INFORMATION

Repeating and growing/increasing/decreasing patterns consist of a series of related elements—each new element is related to the previous in some manner. Students must be able to identify the relationship in order to understand the pattern.

Encourage students to make connections with numbers by presenting the pattern with numerical term positions.

Example:

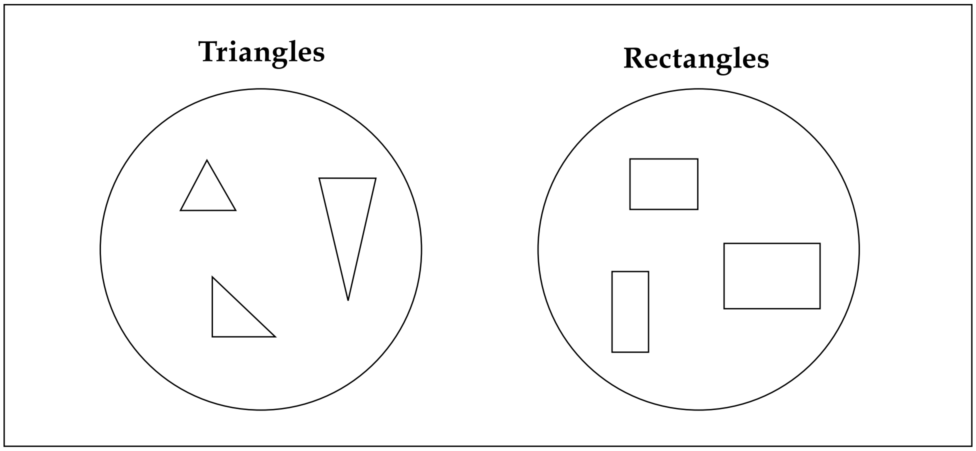


Increasing and decreasing patterns are patterns in which the basic core pattern grows/shrinks or changes in a predictable way.

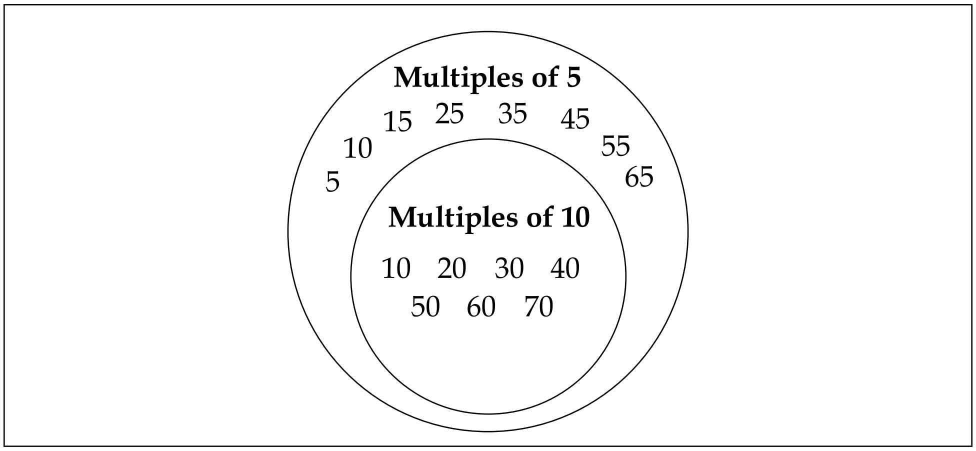
Venn Diagrams: There are three types of Venn diagrams: discrete sets, set and subset, and intersecting sets.

Examples of comparing two sets:

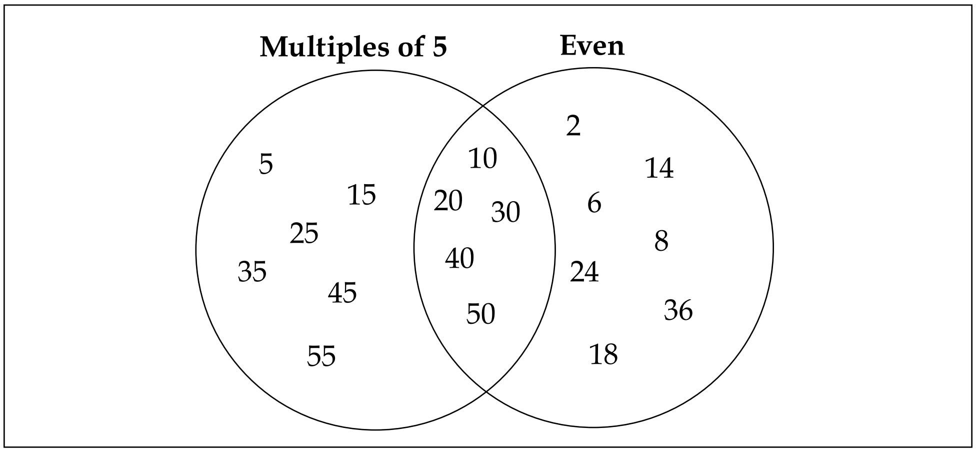
- **Discrete Sets:** The attributes being compared have nothing in common.



- **Set and Subset:** One set is a subset of the other. One of the attributes is contained in the set of the other attribute.



- **Intersecting Sets:** The attributes are shared by some members of both sets. The intersection shows the set of numbers that are both multiples of 5 and even.



Note: Numbers, objects, shapes, et cetera, that do not fit either attribute are placed outside the circles but inside the rectangle because they are still part of the whole set.

Carroll Diagrams: A Carroll diagram is a chart used to sort and display data by attributes. The diagram is done in a yes/no way. The diagrams are named after the mathematician and author (*Alice in Wonderland*), Lewis Carroll.

Example:

Sort the following numbers on the Carroll diagram:

2, 5, 6, 8, 10, 11, 14, 15, 18, 20, 24, 25, 27

	Even	**Not Even**
Multiple of 5	10, 20	5, 15, 25
Not a multiple of 5	2, 6, 8, 14, 18, 24	11, 27

MATHEMATICAL LANGUAGE

pattern
decreasing pattern
increasing pattern
element
extend
reproduce
rule
Venn diagram
Carroll diagram
table
attribute
set
column
row
diagonal

LEARNING EXPERIENCES

Assessing Prior Knowledge

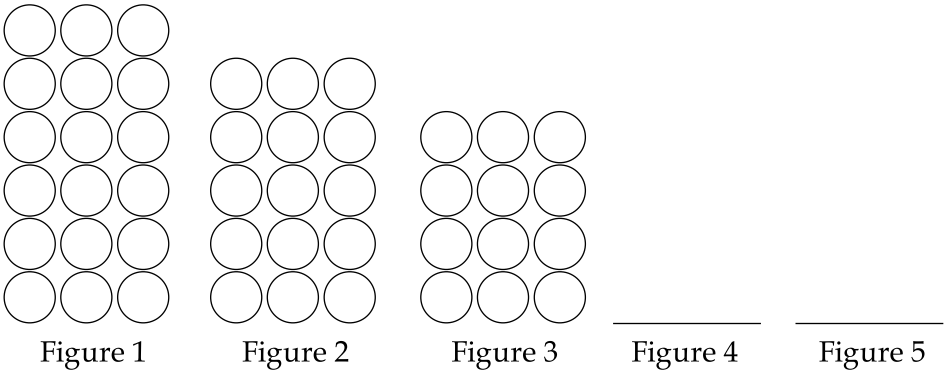
BLM
4.PR.1.1

1. Ask students to do the following:
   a. Find an increasing and a decreasing pattern on the hundred chart. Identify the pattern rules.

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

   b. Extend the pattern.



What is the pattern rule?

c. Create a decreasing pattern. Explain your pattern rule.
d. Give an example of an increasing or decreasing pattern in the environment.

The student is able to

☐ identify an increasing pattern on the hundred chart
☐ identify a decreasing pattern on the hundred chart
☐ identify a pattern rule for an increasing pattern
☐ identify a pattern rule for a decreasing pattern
☐ extend an increasing pattern
☐ create a decreasing pattern and describe the pattern rule
☐ give an example of an increasing or decreasing pattern in the environment

- **Identify and describe a variety of patterns in a multiplication chart.**

Suggestions for Instruction

BLM
4.PR.1.2

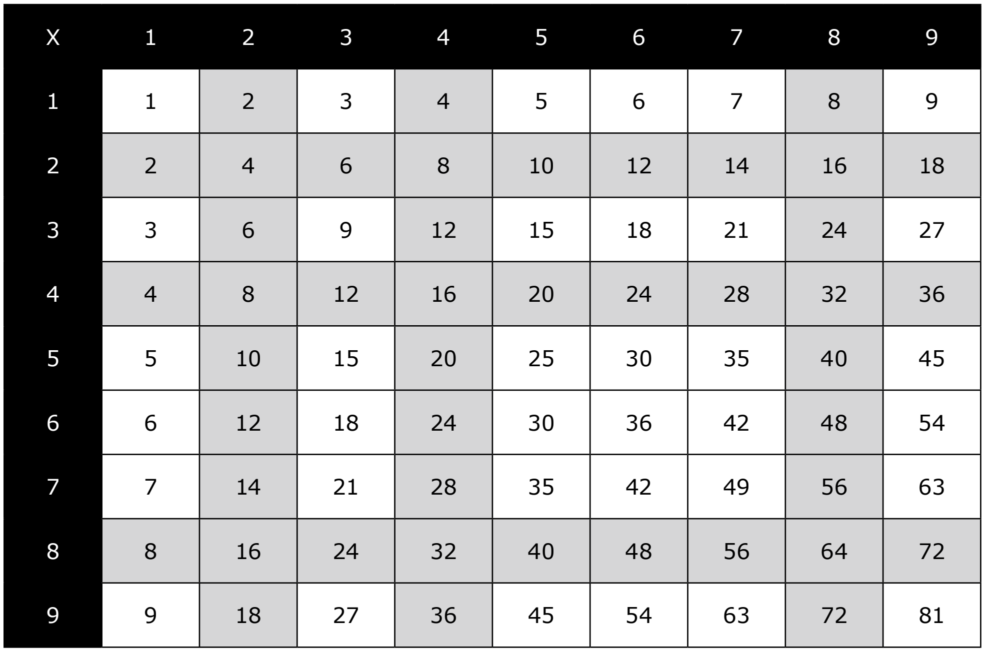
- **Exploring Patterns:** Work with a partner or small group. Select one pattern on the multiplication table. Describe the pattern. Explain why the pattern occurs. Share your pattern with the class.

  Possible patterns might include the following:

  - skip counting patterns in each row and column
  - the products in the 1, 3, 5, and 7 rows and columns alternate between even and odd numbers
  - the products on the diagonal are all square numbers (A square number is the product of a number multiplied by itself.)
  - the products on either side of the diagonal are mirror images
  - in the 9 column and row the sum of the digits in each product is 9
  - the products in the 4 column and row are double those in the 2 column and row
  - the products in the 8 row and column are double those in the 4 row and column and four times those in the 2 column and row
  - the products in the 6 column and row are twice those in the 3 column and row

- Create a class chart showing the patterns identified in the multiplication chart. Use small charts and have students shade/highlight the pattern they are describing.

  Example:



X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

- **Connect to Number Operations/Basic Facts:** Have students explain how the identified pattern might be a useful strategy to use when they are multiplying.

  Example:

  "The products in the 8 row and column are double the products in the 4 row and column and four times the products in the 2 row and column.

  "I can use this pattern when I am multiplying by 4 and 8. For example, if I multiply 32 x 4, I can multiply 32 x 2 and then double the product.

  "If I multiply 32 x 8, I can multiply 32 x 2 and then double the product twice to get the answer (32 x 2 = 64, 64 x 2 = 128, 128 x 2 = 256)."

- **Determine the missing element(s) in a table or chart.**
- **Identify error(s) in a table or chart.**
- **Describe the pattern found in a table or chart.**
- **Create a concrete representation of a pattern displayed in a table or chart.**
- **Explain why the same relationship exists between the pattern in a table and its concrete representation.**

Suggestions for Instruction

- Use a book such as *Anno's Magic Seeds* by Mitsumasa Anno. Have students use materials/pictures to represent the pattern in the story. The author provides support in the illustrations.

  Demonstrate how to transfer this information to a table or chart. Discuss the relationship between the concrete/pictorial representation and the data on the chart.

  Note: Students have not done any formal work with tables or charts in previous grades.

- **What's the Pattern?** Describe the pattern(s) on the following chart.

Number of triangles	**Number of sides**
1	3
2	6
3	9
4	12
5	15
6	18
…	…
10	30

  Note: Students will quickly see that the triangle column increases by 1 and that the sides column skip counts by 3s. Help them to see the relationship between the number of triangles and the corresponding number of sides (the number of sides is 3 times the number of triangles). Seeing this relationship will assist students in predicting the number of sides for the "nth" number of triangles.

Provide students with multiple opportunities to work with the pattern relationships on tables and charts.

- Look at the numbers provided in the table:

spiders	1	2	3		5	6
legs			24	32		48

  - What number do you think is missing from the top row? Why?
  - What numbers are missing from the bottom row? Why?
  - Complete the chart.
  - Describe the pattern.
  - Write number sentences which show how to calculate the number of legs on 1 spider, 2 spiders . . . 6 spiders."

- Marc made a chart for this pattern.

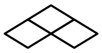


Figure 1

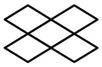


Figure 2

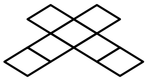


Figure 3

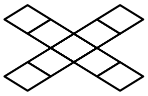


Figure 4

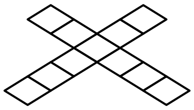


Figure 5

This is the chart he made:

Figure	1	2	3	4	5
Number of boxes	3	5	8	9	11

Compare the pattern with the chart and give feedback to Marc on how he did. Use the pattern rule to help you.

- Make a colour tile/unifix cube pattern to match the information on the chart.

Figure	1	2	3	4	5
Number of cubes	1	4	9	16	25

Assessing Understanding: Performance Task

Student Directions:

1. Use materials such as colour tiles, counters, cubes, or base-10 blocks to make an increasing or decreasing pattern.
2. Record your pattern in your math journal/notebook.
3. Use your pattern to make a table or chart. Explain how you know that your table or chart matches your pattern.
4. Explain your pattern rule.
5. Share your chart with a partner. Have your partner try to make the pattern using the same materials. Compare the patterns. Do both patterns support your pattern rule?

The student is able to

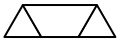
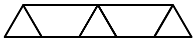
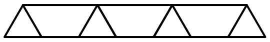
☐ transfer a concrete pattern to a chart or table
☐ identify the pattern rule
☐ explain how both representations support/reflect the pattern rule
☐ transfer a pattern from a chart to a concrete representation

- **Translate the information provided in a problem into a table or chart.**
- **Identify and extend the patterns in a table or chart to solve a problem.**

Suggestions for Instruction

Students should have many opportunities to work with pattern problems.

Samples:

- 
  Figure 1
  
  Figure 2
  
  Figure 3
  - If this pattern continues, how many triangles and how many trapezoids will there be in Figure 8?
  - Create a table or chart to show the pattern.
  - Identify the pattern rule.
  - Use the rule to solve the problem.

- Sara had a lemonade sale for a week in the summer. She started to record the number of glasses sold each day but forgot after three days. Sara noticed that there was a pattern in the number of sales.

  Complete the table to extend the pattern. Describe the pattern rule.

Day of the Week	Number of Glasses Sold
Monday	8
Tuesday	10
Wednesday	12
Thursday	
Friday	
Saturday	
Sunday	

- A robin comes to the birdfeeder every 5 days and a blue jay comes by every 3 days. Today, the robin and blue jay both came to the birdfeeder. How many days will it be before the robin and the blue jay both come on the same day again? Use a chart to help you solve the problem.

- Jean has been exercising.

  On the first day he did 1 pushup.

  On the second day he did 2 pushups.

  On the third day he did 4 pushups.

  On the fourth day he did 7 and on the fifth day he did 11.

  If this pattern continues, how many pushups will he do on the tenth day?

- The bus made 16 stops before arriving at Summerville. It picked up one passenger at the first stop, three at the second stop, five at the third stop, seven at the fourth stop, and nine at the fifth stop. This pattern continued. How many people were on the bus when it arrived in Summerville?

- When Holly checked a book out of the library, she read this notice: If a book is 1 day overdue the fine is 1¢, 2 days overdue the fine is 2¢, 3 days—4¢, 4 day—8¢, and so on. If Holly's book is 7 days overdue, how much is her fine?

- The baseball tickets at the stadium were going on sale at 4:00 p.m. At 1:00 p.m. 15 people were in line waiting to buy tickets. Every 15 minutes 10 more people got in line. At what time were 85 people waiting in line? How many people were in line at 2:15 p.m.?

- On Sunday the genie granted Matt three wishes. On Monday he used each wish to wish for three more wishes. On Tuesday he used each wish to wish for 3 more wishes. How many wishes will he have by Saturday?

Assessing Understanding

- Observe students as they work with problems.

  Ask questions such as the following:

  - What information does the problem give you?
  - What is the question?
  - What labels will you use for your chart/table?
  - What pattern do you see?
  - What is the pattern rule?
  - How can you use the rule to solve the problem?

- Have students write their own pattern problems. Use these problems at a centre or station for others to solve.

- **Challenge Problem:** Give students a partially completed chart or table. Have them write a pattern problem based on this information.

  Example:

Number of guests	1	2	3	4	5	6	7
Number of ice cream scoops	2	4	6				

  Student response:

  Lucas wants to buy an ice cream cone for each of the 7 guests at his birthday party. If each guest gets 2 scoops of ice cream, how many scoops will Lucas have to buy altogether?

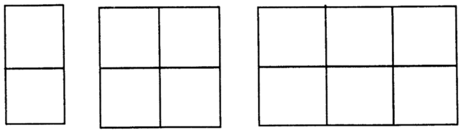
Assessing Understanding: Paper-and-Pencil Task

- Juan was learning to type. He decided to practise each day and to test himself at the end of each day. His score on the first day was 10 words per minute. On the second day, his score was 21 words per minute. On the third day, his score was 32 words per minute. If he continued at this rate, how many words would Juan write on the seventh day? On the tenth day?

  Show your thinking.

- Gary used toothpicks to make this pattern. How many toothpicks does he need for the next shape in his pattern?

  Gary said that he needed 51 toothpicks to make the tenth shape in the pattern. Was Gary right? Use a chart to prove your conclusion.



Assessing Understanding: Performance Task

- When 2 square tables are pushed together, 6 people can be seated. Eighteen people are coming to dinner. How many square tables are needed to make 1 long table to seat everyone?

  Sample Rubric:

  Needs ongoing help:

  - attempts to draw tables pushed together
  - calculations/drawings are incomplete to arrive at the number of tables needed

  Approaching expectations:

  - draws 8 tables to explain how many tables are needed

  Meeting expectations:

  - may or may not draw a diagram, but uses a mathematical operation or numerical relationship to explain how many tables are needed

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- **Complete a Carroll diagram by entering data into correct squares to solve a given problem.**
- **Determine where new elements belong in a Carroll diagram.**

Suggestions for Instruction

Note: Carroll diagrams and Venn diagrams are interchangeable. Students should be able to transfer data from one to the other.

- Have students use a Carroll diagram to represent data (limited to 2 choices) collected from the class.

  Example:

  Do you have a dog or a cat?

	Dog	No dog
Cat		
No cat		

Do you like white milk or chocolate milk?

	White	Not white
Chocolate		
Not chocolate		

BLM
4.PR.4.1

- Solve problems using a Carroll diagram.

Examples:

a. The 24 students in Mrs. Lee's class completed a survey about pets. Thirteen students said they have a dog. Five students said they have a cat. One student said they do not have a pet. How many students have both a dog and a cat?

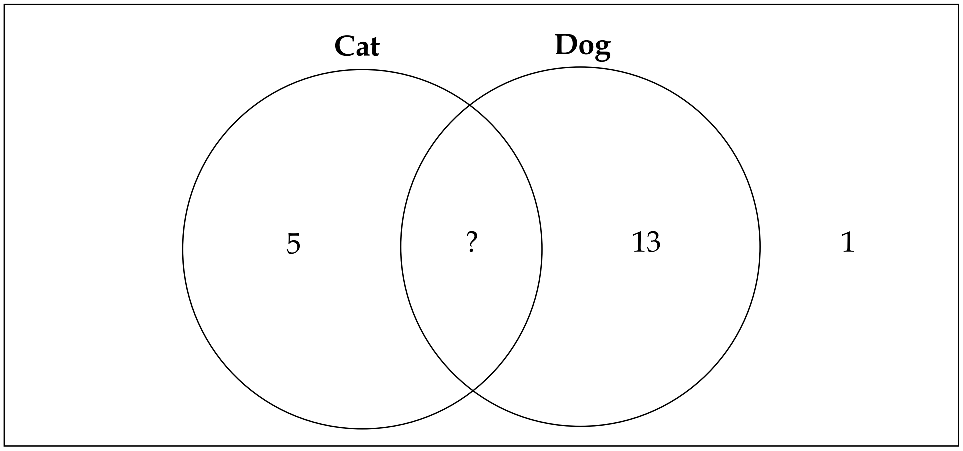
	Dog	No dog
Cat	?	5
No cat	13	1

The total number of students represented on the chart so far is 19. This leaves 5 students who have both a cat and a dog.

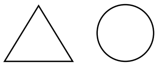
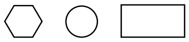
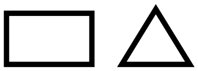
Have students transfer this information to a Venn diagram.

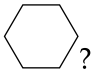
BLM
4.PR.4.2

Example:

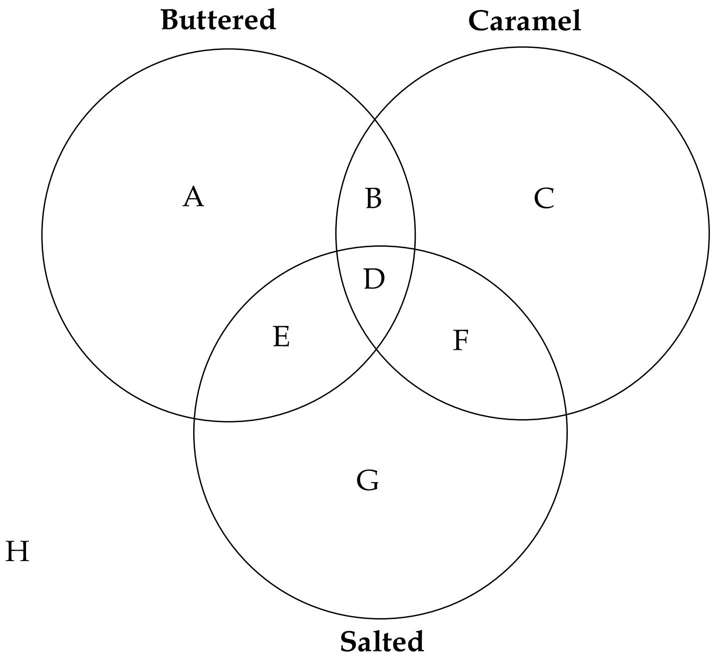


b. Meg sorted attribute blocks using a Carroll diagram.

	Large	Not large
Thin	A 	B 
Not thin	C 	D 

Where should she place this block ? Explain your choice.

- **Three-Circle Venn to Carroll Diagram:**
  How Do You Like Your Popcorn?



How do you like your popcorn?	Buttered		Not Buttered	
	Salted	Not Salted	Salted	Not Salted
Caramel	D	B	F	C
Not Caramel	E	A	G	H

Assessing Understanding: Performance Task

Student Directions:

1. Design a survey question with two possible responses (not yes or no).
2. Collect the data from your class.
3. Represent your data using a Carroll diagram.
4. Survey an adult in the room but do not include their data on the diagram.
5. Instead provide the data separately along with the question, “Where does this belong on the diagram?”
6. Share your findings with the class. Have someone explain where the additional data belongs on your diagram.

Note: Before beginning work on the task, have the class develop the assessment criteria.

Criteria might include the following:

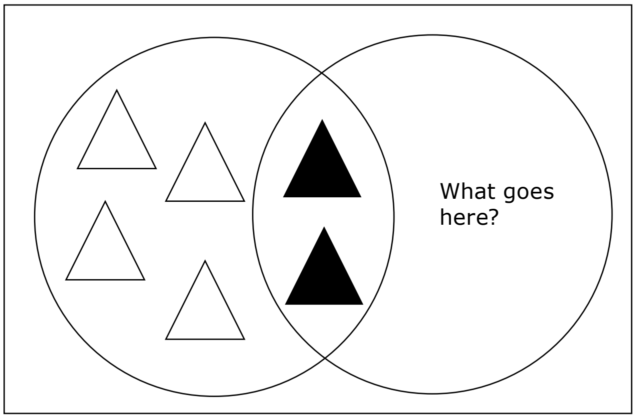
- Survey question is developed.
- Data is collected.
- Carroll diagram is created with correct labels based on the question.
- Data is correctly represented on the diagram.
- Additional data is placed correctly.

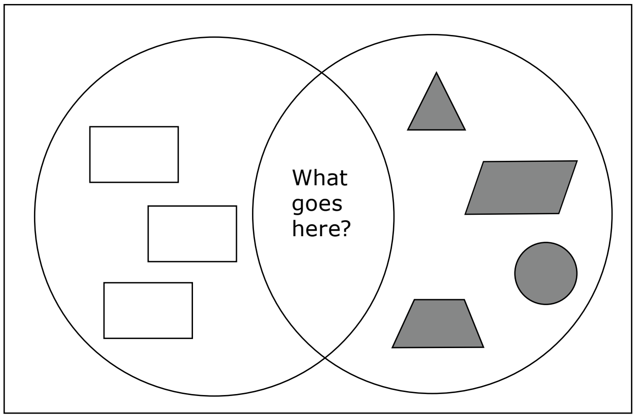
- **Identify a sorting rule for a Venn diagram.**
- **Describe the relationship shown in a Venn diagram when the circles intersect, when one circle is contained in the other, and when the circles are separate.**
- **Determine where new elements belong in a Venn diagram.**
- **Solve a problem by using a chart or diagram to identify mathematical relationships.**

Suggestions for Instruction

- Provide students with concrete Venn diagrams with objects in only 2 of the 3 sections. Ask: "What objects would go in the empty section?"

  Examples:





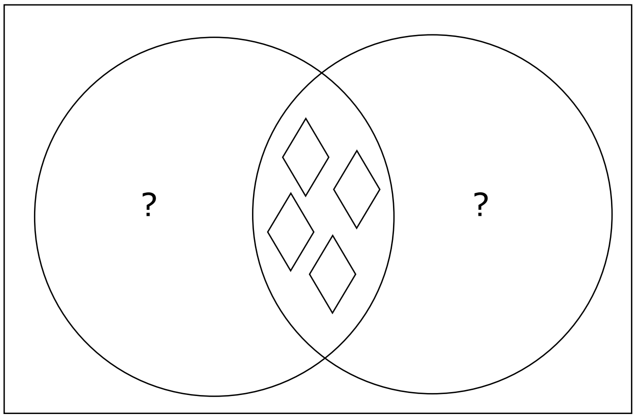
Note: Initially, students should have a collection of objects from which to choose.

Do students

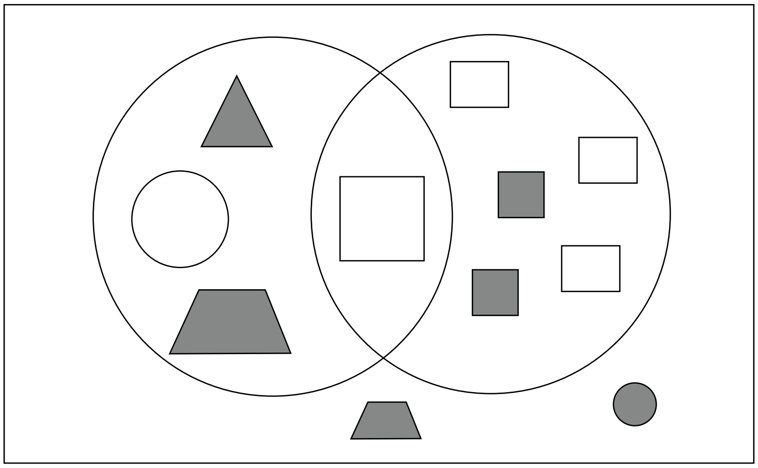
- correctly identify the attributes?
- recognize the intersection?
- use appropriate language?
- need assistance? (If they do, they need more practice.)

Repeat a similar activity using Carroll diagrams.

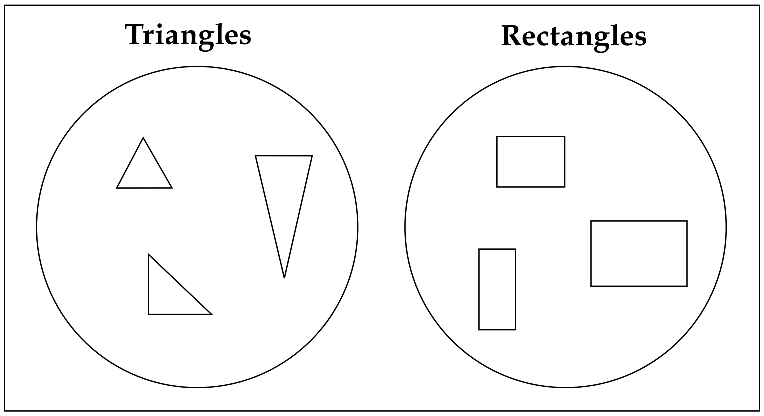
- **Extension:** A more difficult task for students is to ask them to determine the characteristics of the two sets when only the intersection of the sets is shown.



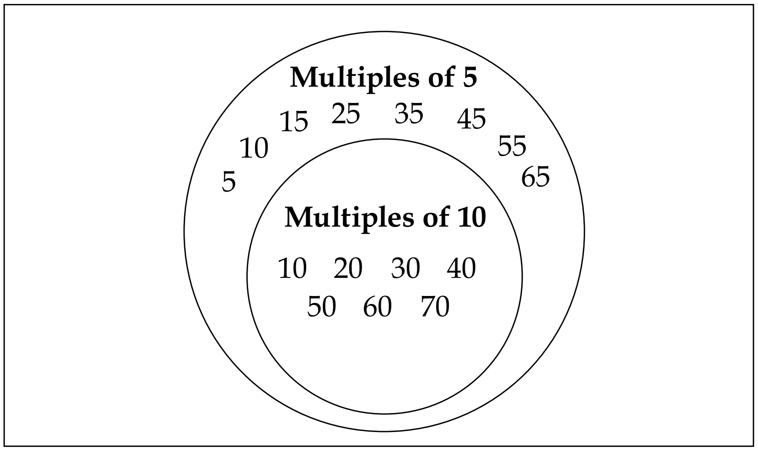
- Give students a set of presorted materials in a Venn diagram (see below). Ask: "What are the attributes of these sets?"



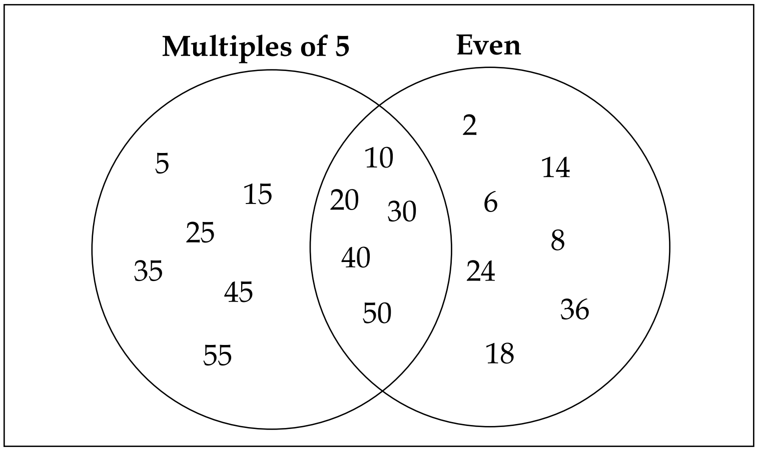
- Have students explain why these objects are sorted on the Venn diagram below in this way.



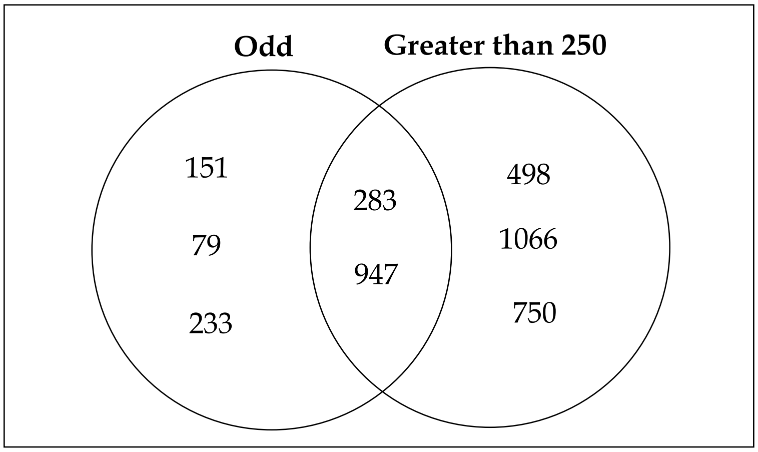
- Have students explain why the numbers are sorted in this way on the Venn diagram below.



- Have students explain why the numbers are sorted in this way on the Venn diagram below.



- Tyler sorted a set of numbers on the Venn diagram below.



Where do the numbers below belong on Tyler's Venn diagram?

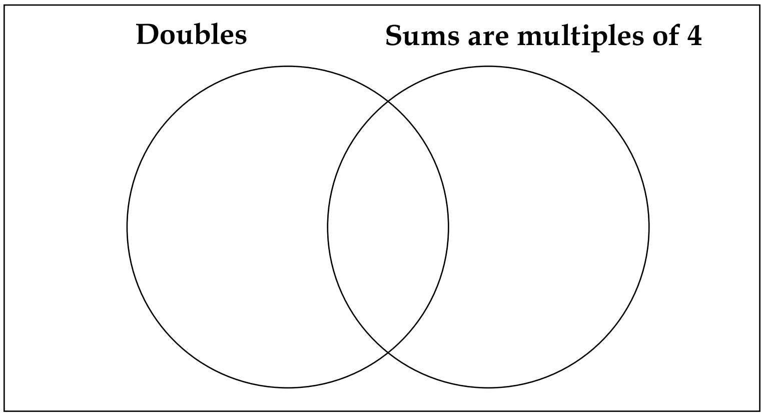
608   311

Explain your thinking.

Assessing Understanding: Paper-and-Pencil Task

1. Sort these addition facts onto the Venn diagram:

4 + 4 =	8 + 4 =	7 + 3 =	3 + 3 =
6 + 6 =	9 + 7 =	12 + 0 =	

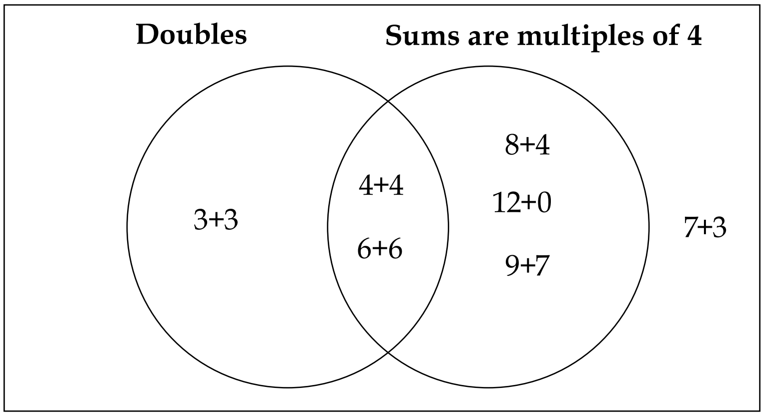


a. Explain why you placed 12 + 0 where you did.

b. Explain why you placed 7 + 3 where you did.

c. Transfer this information to a Carroll diagram.

Scoring Rubric:



Needs support:

- has 1 error in sorting
- places 7 + 3 incorrectly
- has limited reason for 7 + 3 placement

Approaching:

- has 1 error in sorting
- places 7 + 3 correctly
- states 1 or 2 reasons for 7 + 3 placement

Meets expectations:

- has no errors
- lists the 2 reasons why 7 + 3 is on the outside

Student Self-Assessment: Have students add to the chart several times during the year (perhaps close to reporting periods). Student samples could accompany the self-assessment to provide evidence of the learning.

Name:				
		Patterns		
	September	November	March	June
I can				
I can				
I can				

Example:

Name:				
		Patterns		
	September	November	March	June
I can	identify and describe patterns in a multiplication table	use Venn diagrams to sort		
I can	describe patterns in a chart or table	use Carroll diagrams to sort		
I can	identify the pattern rule on a chart or table	solve problems using a table or chart		

Notes

Grade 4: Patterns and Relations (4.PR.5, 4.PR.6)

Enduring Understandings:

“Equals” indicates equivalent sets.

Unknown quantities can be found by using the balance strategy.

Number patterns and relationships can be represented using variables.

Essential Questions:

How is a number sentence like a balance scale?

What does the equal sign mean?

What is the purpose of a symbol in an equation?

What strategies can be used to solve the unknown in an equation?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.PR.5 Express a problem as an equation in which a symbol is used to represent an unknown number. [CN, PS, R]	➛ Explain the purpose of the symbol, such as a triangle or circle, in an addition, subtraction, multiplication, or division equation with one unknown (e.g., 6 = 36 ÷ □). ➛ Express a pictorial or concrete representation of an equation in symbolic form. ➛ Identify the unknown in a story problem, represent the problem with an equation, and solve the problem concretely, pictorially, or symbolically. ➛ Create a problem in context for an equation with one unknown.
4.PR.6 Solve one-step equations involving a symbol to represent an unknown number. [C, CN, PS, R, V]	➛ Solve a one-step equation using manipulatives. ➛ Solve a one-step equation using guess and test. ➛ Describe, orally, the meaning of a one-step equation with one unknown. ➛ Solve an equation when the unknown is on the left or right side of the equation. ➛ Represent and solve an addition or subtraction problem involving a “part-part-whole” or comparison context using a symbol to represent the unknown. ➛ Represent and solve a multiplication or division problem involving equal grouping or partitioning (equal sharing), using symbols to represent the unknown.

PRIOR KNOWLEDGE

Students may be able to

- demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100)
- record equalities and inequalities symbolically using the equal symbol or the not-equal symbol
- solve one-step addition and subtraction equations involving symbols representing an unknown number

BACKGROUND INFORMATION

The equal symbol represents a relation between two equal quantities. In other words, the expression on the left-hand side of the equal symbol represents the same quantity as the expression on the right-hand side of the equal symbol.

Many students have misconceptions about the equal symbol. Many think that the equal symbol means “give answer.” As a result they have difficulty with questions such as the following:

4 + ___ = 7	Students will add across the equal sign and fill the blank with 11.
___ = 2 + 5	Students will say that the question itself is incorrect because the blank is on the wrong side.
3 + 4 = 5 + ___	Students will add all the numbers and put 12 in the blank.

Equation: A mathematical sentence stating that two expressions are equal. An equation contains an equal sign (=).

Equal sign: A symbol that means two things have the same amount, size, number, or value.

Equality: A mathematical statement indicating that two quantities (or expressions) are in balance; two expressions that are equivalent (e.g., 2 + 5 + 1 = 4 + 4).

MATHEMATICAL LANGUAGE

same
more
less
equal
not equal
balance
match
equal sign
equal symbol
inequality
equality
symbol
unknown
equation

LEARNING EXPERIENCES

Assessing Prior Knowledge

Present the following equations. Have students fill in the missing numbers.

a. 16 + □ = 17 + 5
b. △ – 6 = 17 – 5
c. 32 + 19 = ○ + 20
d. 100 = 64 + ⬡

As students work, ask questions such as the following:

- What is the question asking?
- What strategy did you use to solve the problem?

Students are able to

☐ identify what the question is asking
☐ solve the equation maintaining the balance on either side of the equal sign
☐ explain the strategy used

- **Explain the purpose of the symbol, such as a triangle or circle, in an addition, subtraction, multiplication, or division equation with one unknown (e.g., 6 = 36 ÷ □).**
- **Express a pictorial or concrete representation of an equation in symbolic form.**
- **Describe, orally, the meaning of a one-step equation with one unknown.**
- **Solve an equation when the unknown is on the left or right side of the equation.**

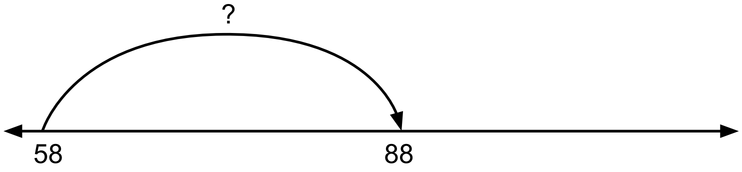
Suggestions for Instruction

- Provide a variety of opportunities for students to "read" equations with one unknown. For example, 6 = 36 ÷ □ can be read as "6 is the same as 36 divided by what number?" or as "36 divided by what number equals 6?"

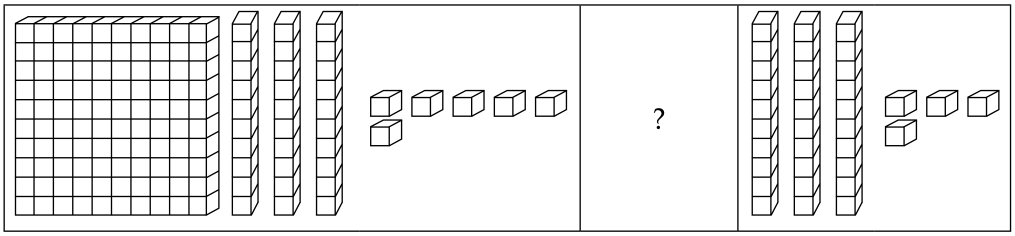
  By "reading" the equation the student is able to demonstrate their ability to interpret the meaning/purpose of the symbol.
- Have students write equations to match concrete or pictorial representations.

  Examples:

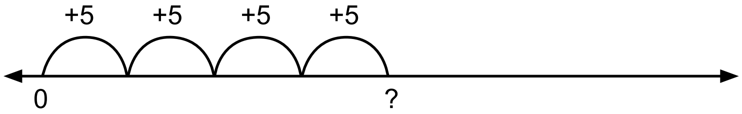
1.



2.



3.



- **Identify the unknown in a story problem, represent the problem with an equation, and solve the problem concretely, pictorially, or symbolically.**
- **Create a problem in context for an equation with one unknown.**
- **Solve a one-step equation using manipulatives/guess and test.**

Suggestions for Instruction

- Give students addition and subtraction word problems. Have them write an equation using a symbol for the unknown.

  Example:

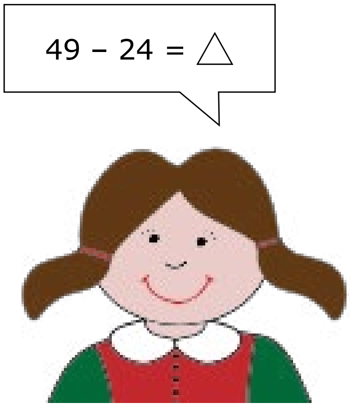
  Directions: Write an equation/number sentence for each problem. Use a symbol for the missing part.

  1. Chiara has 34 candies. She eats some and now she has 25 left. How many candies did she eat?
  2. Jim is saving money to buy a new video game. The game costs $28. He has saved $16 so far. How much more does he need to save?
  3. Len added 56 and 49. What answer did he get?

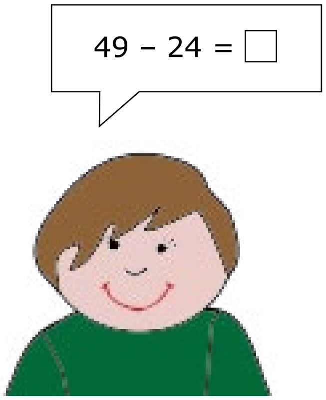
- The Grade 4 class was given the following problem:

  There are 49 students in the two Grade 4 classrooms. If there are 24 students in the first room, how many are in the second room?

  Jill wrote:



  John wrote:



  Who is correct? Explain your thinking.

  Explanation should include the following:

  - Both students are correct.
  - It doesn't matter what symbol you use to represent the unknown.

- **Represent and solve an addition or subtraction problem involving a "part-part-whole" or comparison context using a symbol to represent the unknown.**

Suggestions for Instruction

There are 28 students in Mr. Martin's class and 32 students in Mrs. Powell's class.

How many more students are there in Mrs. Powell's class than in Mr. Martin's class?

Note: Students can use a bar model to represent the problem. The bar model is a problem-solving (visualization) strategy taught in the Singapore Math program. Students represent problems by drawing bars to show how the known elements of the problem relate to one another and to the question. The bar model is a powerful way to represent part-part-whole relationships.

Mrs. Martin's class	28	?
Mrs. Powell's class	32	

$28 + \triangle = 32$

- **Represent and solve a multiplication or division problem involving equal grouping or partitioning (equal sharing), using symbols to represent the unknown.**

Suggestions for Instruction

- Sandra has 54 candies. She wants to make 6 goodie bags for her party. How many candies should she put in each bag?

Sandra's candies	54					
Goodie bags	?					

$54 \div 6 = \square$

- Marc has 28 marbles. He gives 4 marbles to each of his friends. How many friends got marbles from Marc?

Marc's marbles	28	
Friends	4	...?...

$28 \div 4 = \square$

Assessing Understanding: Paper-and-Pencil Task

Write an equation/number sentence for each problem using a symbol to represent the unknown. Solve the problem.

1. Sarah has 26 bubble gum pieces. She buys 7 more. How many pieces of gum does she have now?
2. Alex is inviting 5 friends to his party. He has 20 party favours. How many party favours will each friend get?
3. At the beginning of the school year each student in Mrs. Leckie's class has 6 pencils. If Mrs. Leckie has 20 students, how many pencils does the class have altogether at the beginning of the year?
4. Harold has 234 stickers in his collection. He decides to give 58 stickers to his sister. How many stickers does he have left in his collection?
5. Mrs. Ames's cookie recipe calls for 2 eggs. If Mrs. Ames has one dozen eggs, how many cookie recipes can she make?

Notes

GRADE 4 MATHEMATICS

Shape and Space

Grade 4: Shape and Space (Measurement) (4.SS.1, 4.SS.2)

Enduring Understandings:

The attributes of a clock help to read time.

Reading time on a clock can help communicate the measurement of time.

Elapsed time is the measure of the duration of an event.

A given time of day can be represented in more than one way.

The attributes of a calendar help to read and record dates and help to organize events in life.

Essential Questions:

How do we use time throughout the day?

How can the time on a clock be read and recorded?

How can the date be recorded in several ways?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.SS.1 Read and record time using digital and analog clocks, including 24-hour clocks. [C, CN, V]	➛ State the number of hours in a day. ➛ Express the time orally and numerically from a 12-hour analog clock. ➛ Express the time orally and numerically from a 24-hour analog clock. ➛ Express the time orally and numerically from a 12-hour digital clock. ➛ Describe time orally and numerically from a 24-hour digital clock. ➛ Describe time orally as “minutes to” or “minutes after” the hour. ➛ Explain the meaning of AM and PM, and provide an example of an activity that occurs during the AM and another that occurs during the PM.
4.SS.2 Read and record calendar dates in a variety of formats. [C, V]	➛ Write dates in a variety of formats (e.g., yyyy/mm/dd, dd/mm/yyyy, March 21, 2006, dd/mm/yy). ➛ Relate dates written in the format yyyy/mm/dd to dates on a calendar. ➛ Identify possible interpretations of a given date (e.g., 06/03/04).

PRIOR KNOWLEDGE

Students have had experience

- relating the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years)
- determining the number of days in any month using a calendar
- solving a problem involving the number of minutes in an hour or the number of days in a given month
- creating a calendar that includes days of the week, dates, and personal events

BACKGROUND INFORMATION

Clocks and calendars are used to organize the daily activities in our lives. Clocks and calendars should be used as tools to measure time like rulers measure length.

Time is abstract and it cannot be seen, so it is a challenge to fully understand. Students need many experiences relating the passage of time to events in their lives. They need to understand that the duration of an event/activity is determined by its start and end times and that there are standard units that we can use to describe the duration. Students should develop personal referents (length of a class or television show) to understand duration.

An **analog clock** is a clock with a minute hand and an hour hand.

A **digital clock** is a clock on which the time is displayed numerically (e.g., the time is displayed as 12:22).

The abbreviation, A.M., is the short form for *ante meridiem* meaning "being before midday" and P.M. is the abbreviation for *post meridiem* meaning "being after midday."

Note: It is also acceptable to write AM and PM in the following ways:

- 11:30 am
- 11:30 AM
- 11:30 A.M.
- 11:30 a.m.
- 11:30am

It is also important to have students benefit from having experiences with time throughout the day rather than just engaging in activities that narrowly focus on telling time. The reading and recording of time is best learned through authentic learning experiences that happen throughout the year and can be integrated into other subject areas and events to make it meaningful to students.

Through the learning outcome 4.SS.2, students will become aware of the variety of ways dates can be recorded. There are many different ways that dates are written in numeric format. There are several acceptable formats that can be used. The International Organization for Standardization has indentified a standard notation that many countries, including Canada, have adopted. It starts with the year, then the month, followed by the day (yyyy-mm-dd). June 12, 2016, would be recorded 2016-06-12.

MATHEMATICAL LANGUAGE

minutes
hours
days
weeks
months
years
seconds
time
o'clock
analog clock
digital clock
12-hour clock
24-hour clock
AM
PM
calendar
hours
days
weeks
months
years

Assessing Prior Knowledge: Interview

Ask students the following:

- Would you use minutes or would you use hours to measure
  - the length of the school day?
  - the length of recess?

  Explain your choices.
- Milo's video is 85 minutes long. Is that more or less than an hour? Explain how you know.
- Sally's mother said that she could play at her friend's for either two hours or for 150 minutes. Which one should she choose if she wants to play for as long as she can? Explain your choice.

The student

☐ understands when to use minutes and hours to measure the passage of time

☐ understands that there are 60 minutes in an hour

☐ applies this information in a problem-solving situation

☐ understands that there are 60 seconds in a minute

Interview: Ask students the following:

- Can you show what the date is on the calendar? Have them write it out.
- How many months are there in one year?
- How many days are in one week?
- How many days are in two weeks?

The student understands

☐ what a calendar is, and how to read it

☐ the relationship between days, weeks, and months

Suggestions for Instruction

- **Exploration of Time:** Telling time will likely be a concept that is familiar to students. Whether the concept was formally or informally introduced to students, they will know something about it. When formally teaching the concept, it is good to focus on the essential questions:
    - Why do I need standardized units of measurement?
    - Why do I need to tell the time?
    - What is important about telling time?
    - How does telling time help us in our lives?

  These questions will lead into a discussion that will help engage students and also help you assess what students know and feel about the concept. Student learning and engagement increase when students become aware of their learning and make connections to other concepts. By making connections, students draw on or add to their understanding.

BLM 4.SS.1.1

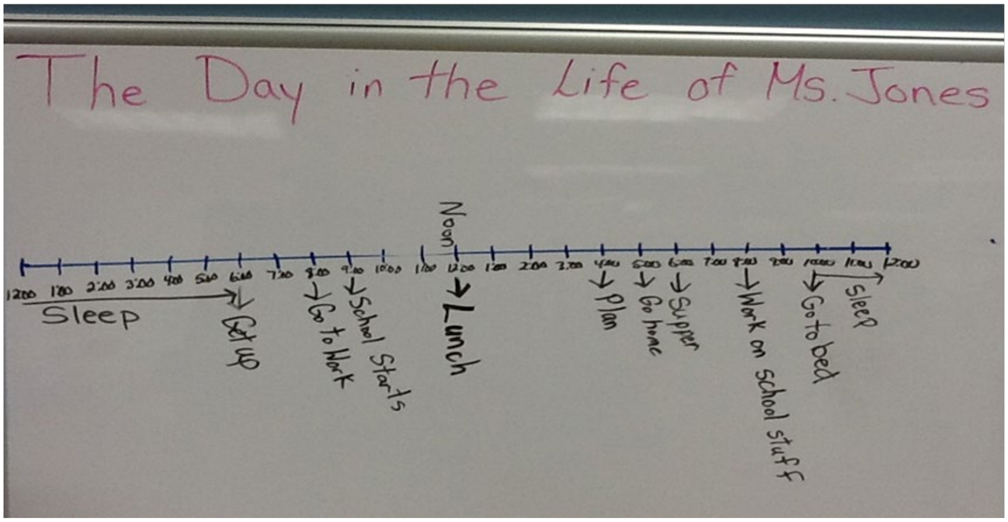
- **Telling Time Makes Me Think Of:** Activate the formal unit on time by using the BLM 4.SS.1.1, *Telling Time Makes Me Think Of . . .*, as way to get a good idea of what students know about time and the related vocabulary. Each student will record words, phrases, numbers, and pictures of what they know about time. As a whole class or in small groups, have students share what they have written and explain their choices. Throughout the discussion, help students make connections to other math concepts, connect to prior knowledge, and recognize how interconnected math concepts relate to everyday life. Throughout the year, have students go back to the page to expand on their thoughts.

- Explore familiar occurrences/events associated with particular times (time of day, week, year) within the students' local and extended community. This would be an ideal time to make links to the learning outcomes of the social studies curriculum.

- **State the number of hours in a day.**

Suggestions for Instruction

- **The Day in the Life of ________:** While time is often described using the 12-hour clock, students should realize that there are 24 hours in a day. Tell the students that the timeline shows all the hours in a day. Discuss and record events of your day over a 24-hour period using a timeline. Discuss the relationship between the hours in a day and the timeline. Ask students why the times are recorded twice. This can lead to discussion about how to differentiate between AM and PM. Start the discussion off by asking students why it is important to say the time of day when describing time.



- **Explain the meaning of AM and PM, and provide an example of an activity that occurs during the AM and another that occurs during the PM.**

Suggestions for Instruction

BLM 4.SS.1.2

- **Student Timeline:** Have students use BLM 4.SS.1.2, *Timeline: The Day in the Life of* _______, or have them create their own timeline. If possible, make the timeline on cardstock. The students will be able to use this timeline for the next learning experience. Explain to students that 12 noon is the bridge between AM and PM times, and that times between 12 midnight and 12 midday are AM times, and that times between 12 midday and 12 midnight are PM times.

- **AM and PM Sort:** Have students write different activities from their timeline on index cards. Have students sort the cards into AM or PM hours.

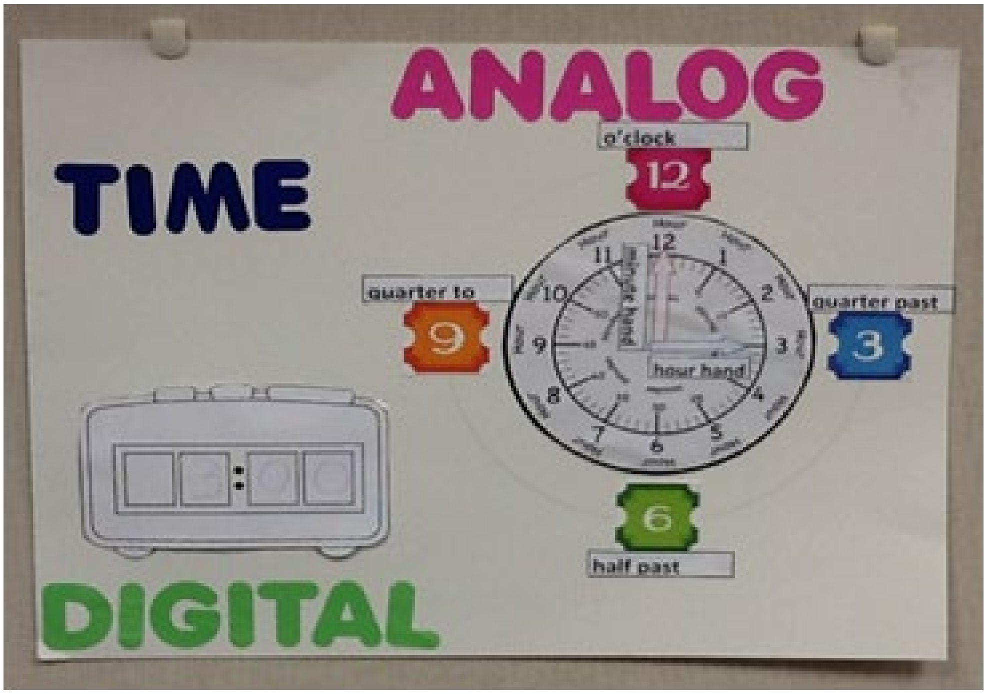
BLM 4.SS.1.3

- **AM and PM Activities:** Have students list the activities they do on the hour throughout the day and record them using BLM 4.SS.1.3, *AM and PM Activities.*
- Find out about arrival and departure times of boats, ferries, planes, buses, or trains in your community. Have students bring in tickets from the different modes of transportations and compare times on them.

- **Express the time orally and numerically from a 12-hour analog clock.**
- **Express the time orally and numerically from a 12-hour digital clock.**
- **Describe time orally as "minutes to" or "minutes after" the hour.**

Suggestions for Instruction

- **Clock Collection:** Have students collect pictures of different clocks and watches from magazines, newspapers, and catalogues. Display the pictures and talk about the different ways the clocks display times. Introduce the students to analog and digital vocabulary. Review the different parts of clocks and make a class anchor chart. Try to find different types of analog and digital clocks to display in the room.



- **Make a Clock:** Have students make their own clocks by gluing a clock face to a paper plate. Have students attach two arrows made from heavy paper with a brass fastener to make hands. Have them move the hour hand on their clocks as you talk about the time. To get a BLM of clock faces go to http://lrt.ednet.ns.ca/PD/BLM/table_of_contents.htm.
- **Analog Clock:** Have students look at an analog clock. What do they notice? What do they wonder? Record their observations.

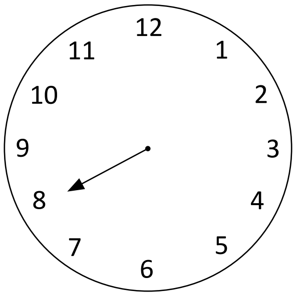
Example:

What we notice	What we wonder
The clock has the numbers 1 to 12.	Do all clocks have the numbers 1 to 12?
There are little lines in between the numbers.	Why are the lines there?
The clock has two hands—one short and one long.	Why are the hands different lengths?
The longer hand moves quickly around the clock face. The shorter hand moves very slowly.	Why do the hands move at different speeds?

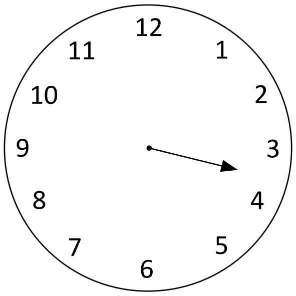
Guide students in finding answers to their wonderings.

- **Telling Time:** It is easier to tell the time on the hour, half past the hour, a quarter past the hour, and a quarter to the hour; however, many daily activities do not begin at these times. Where possible, connect telling time on the analog clock with meaningful events in the students' lives such as the daily class schedule or agenda.
- Review with students to show what "a little after the hour," "a little to the hour," and "half past" look like with the hour hand. For example, when it is "half past" the hour, the hour hand is pointed directly between the two numbers. Use a one-handed clock to help students understand and read analog clocks. Remove the minute hand from an old clock and set the short hand in varying places. Use language such as the following:

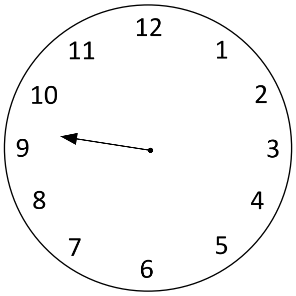
"It's about 8 o'clock."



"It's halfway between
3 o'clock and 4 o'clock."



"It's a little bit past
9 o'clock."



- **Making Connections to Digital Time:** Showing the time with a digital clock may be introduced concurrently with the time on an analog clock. Representing both ways will help students make connections.
- **Slit Clocks:** Have students make slit clocks if they need more examples of telling time to the hour. See below for steps and pictures. The slit clock helps students to tell the hour when it is not pointing directly at a number. When the number is beyond an hour, have them practise saying a little past the hour ("a little after 6:00 o'clock"). If the hand is a little before an hour, have them practise saying a little before the hour ("a little before 6:00 o'clock"). Have students practise with a partner. They can both show times and read the times to each other.

Step 1

On a paper plate, draw the numbers of a clock face. Cut a slit from the number 12 to the midway point of the plate.

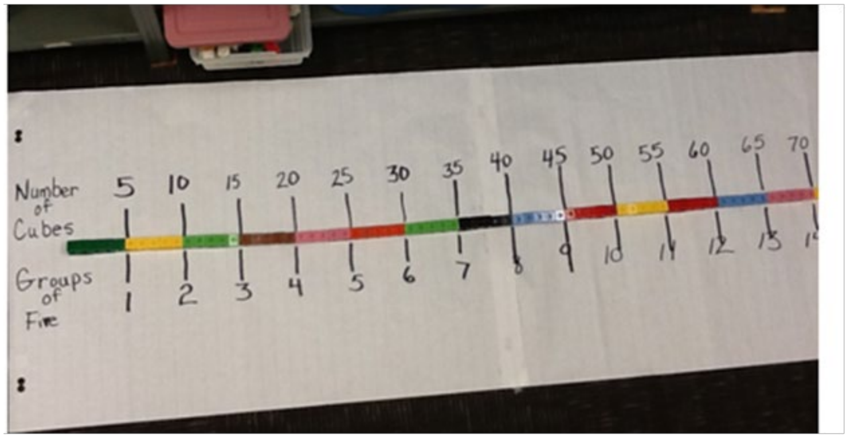
Step 2

Cut out the middle circle of another plate. Draw the hour hand onto the cut-out middle part, and cut along this hour hand.

Step 3

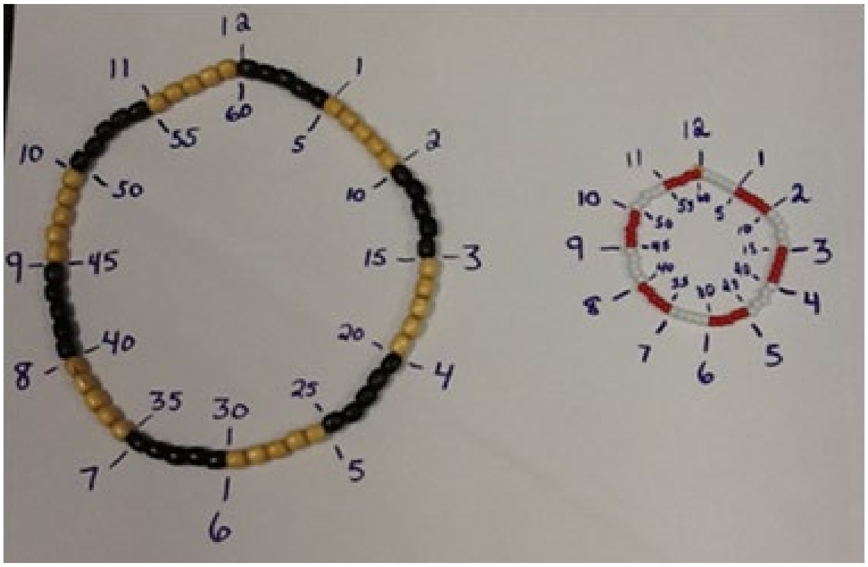
To complete the clock, slide the small circle through the slit on the clock. Move the inner circle from the back and hold the clock with the other hand.

- **Developing an Understanding of Minutes on the Clock (Part 1):** This activity may help students understand the minutes of a clock. Give each student five linking cubes of the same colour. On a few pieces of chart paper glued end to end, have students place their five linked cubes onto the chart paper. The goal is to create a number line with twelve groups of five linking cubes. Record each group of five by drawing a vertical line segment at the right end of the cubes. Discuss the relationships between the number of groups of five cubes and the total number of cubes.



- **Making Relationships to the Clock (Part 2):** Have an analog clock ready for display. Ask students if they see any relations between a clock and the number line that was created. You may want to remove the section of the cubed number line beyond twelve groups of five. Ask students if they could use the cubed number line to tell the time.

  Have students create a number line out of twelve groups of five linking beads and connect them on a string. Beads work well because they can be brought together end to end to make a clock. Make a model of a clock by having students use two colours, and alternate the groups of five with the two colours to a total of 60 beads. Have students label their clocks. Label the hour and interval of each group of five beads. This transitional clock is a visual image that shows both the number of groups of five beads (hours) and the total number of beads (minutes). Have a discussion about the minute hand by making a minute hand and using it to demonstrate how to tell minutes on a clock.



Larger beads can be used to make a class model.

Smaller beads can be used for students to make their own clocks.

BLM
4.SS.1.4

- **Analog and Digital Clocks:** Use BLM 4.SS.1.4, *Analog and Digital Clock Faces,* or have students draw two analog clock faces and two digital clocks. Ask them to answer the following questions, and have them record both times on both types of clock.
  - What time did you eat breakfast?
  - What time did you leave for school?

  Have the students choose one type of clock to work out the length of time between eating breakfast and going to school. Have a conversation about their choice and the differences between the clocks.

BLM
4.SS.1.5

- **Practise Telling Time:** Use BLM 4.SS.1.5, *Digital Time,* or purchase or make time dice. Partner students up. Have one partner roll the dice or pick a card and have the other partner show the time on a student clock model. Clock models can be made or purchased. You can also purchase inexpensive clocks at dollar stores.

- **Representing Time:** Explore all the different ways that time can be shown. Set an alarm clock to go off several times in a day. Have students record the time in their journals using analog, digital, and written forms.

Representing Time

Time	Numerically represented	Orally or in written form
3:35	■ 3:35	■ Thirty-five minutes after three o'clock ■ Twenty-five minutes to four o'clock ■ Thirty-five minutes past three o'clock ■ Twenty-five minutes before four o'clock ■ Three thirty-five
8:30	■ 8:30	■ Eight thirty ■ Thirty minutes after eight o'clock ■ Thirty minutes to nine o'clock ■ Half past eight
2:15	■ 2:15	■ Fifteen minutes after two o'clock ■ Forty-five minutes to three o'clock ■ Two fifteen ■ Quarter past two ■ Quarter past two o'clock
6:00	■ 6:00 ■ 6 o'clock	■ Six o'clock

- **Make Posters:** Have students make posters illustrating how to write and say the time. Have students give an example of what they would be doing during that time.

This is an analog clock.

The clock shows 10 minutes after 10 o'clock. It is the morning so I write the time 10:10 AM.

At school I would be reading at this time.

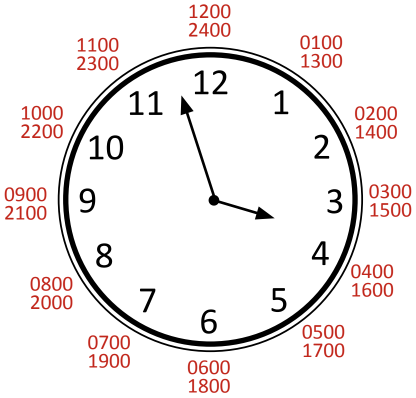
BLM 4.SS.1.6/ 4.SS.1.7

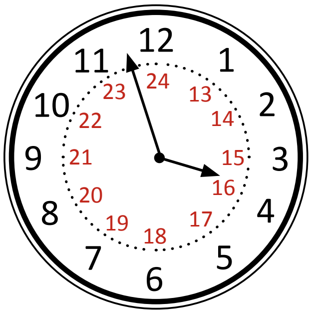
- **Match Game:** Have students make a class matching game. Ask each student to make two cards showing the same time, one analog and one digital. Place everyone's cards together in a learning centre so students can practise matching the two different times. Use BLM 4.SS.1.6, *Analog Clock Faces,* and BLM 4.SS.1.7, *Digital Clock Faces,* as possible templates.

- **Express the time orally and numerically from a 24-hour analog clock.**
- **Describe time orally and numerically from a 24-hour digital clock.**

Suggestions for Instruction

The 24-hour clock is a system used for telling time in which the day runs from midnight to midnight. The time on a 24-hour clock is shown as how many hours and minutes have passed since midnight. The 24-hour clock removes the confusion of whether the time is AM or PM. Students may have come across situations where the 24-hour clock is used. The time of day is written in the format of hh:mm in the 24-hour notation. The hour is always written as a 2-digit number; therefore, any hour below 10 has a zero before it.





BLM
4.SS.1.8

- **Find 24-Hour Clock Notations:** Show a few samples using the 24-hour time notations. BLM 4.SS.1.8, *24-Hour Clock Notations,* has some examples. Ask students what they notice about the time. Explain to them that the time is written using 24-hour notation. Show a 24-clock to the students and discuss what they see. Have students find other 24-hour notations and share them with the class.

- **Advantages and Disadvantages:** Discuss with the students the advantages of using a 24-hour clock. (You do not have to use AM or PM, and there is no confusion about what time of day it is.)

  Discuss with the students the disadvantages of using a 24-hour clock. (It can be hard to know what times like 15:00 mean because most people do not use a 24-hour clock to tell time.)

- **Reading and Recording:** Show different times on a 24- and 12-hour digital and analog clock. (Most digital alarm clocks will have a setting for 24-hour time.) Have students practise recording and saying the time.

Ask students if there is a good way to convert between 24-hour and 12-hour notation for times after noon. (You can add 12 to go from 12-hour clock notation to the 24-hour clock notation. You can subtract 12 to go from the 24-hour clock notation to a 12-hour clock notation.)

Note: Converting between the different formats takes practice. Post schedules that use all the different formats and encourage students to use and say the different formats throughout the year.

- The Sparklebox Teacher Resources website contains free printable clock visuals of different analog and digital formats. They can be reproduced for practice and matching activities. "Digital Times Teaching Resources" can be found at http://www.sparklebox.co.uk/maths/shape-space-measures/time/digital-times.html
- Once students have a good understanding of telling time, websites such as the following can help them practise telling the time:
  http://www.maths-games.org/time-games.html
  http://classroom.jc-schools.net/basic/math-time.html

Assessing Understanding: Paper-and-Pencil Task or Interview

"Fill in AM or PM next to each time":

James had finished breakfast. It was about 8:30 __(AM)__. James heard the phone ring. A friend called to see if he wanted to watch a movie after supper. His mother said yes but he had to be home at 9:30 __(PM)__. He asked whether he could stay overnight at a friend's. She agreed, but she wanted him home in the morning before 11:30 __(AM)__ because he had a doctor's appointment at 1:30 __(PM)__ the next day.

Have students choose an activity they do in a day and have them show the time on an analog and digital clock with the correct AM or PM notation.

Ask students: How do you know what the hour is when you tell time from an analog clock?

Could a digital clock read 3:62? Explain why.

Have students explain why the hour hand cannot be closer to the seven than to the eight if the time is 7:47?

Have students explain why the minute hand cannot point at the six when the hour hand points directly at the three?

Why would you use a 24-hour clock?

Show a time on a digital clock and ask students to

- tell you the time
- show the time on a 12-hour and 24 hour analog clock
- tell you two ways to read the time

Ask them what they could be doing at this time.

- **Write dates in a variety of formats (e.g., yyyy/mm/dd, dd/mm/yyyy, March 21, 2006, dd/mm/yy).**
- **Relate dates written in the format yyyy/mm/dd to dates on a calendar.**
- **Identify possible interpretations of a given date (e.g., 06/03/04).**

Suggestions for Instruction

- **Interpreting Different Date Formats:** Show students a date that can be read in only one way on the board (e.g., 15/04/2015) and ask the students to write it in words (April 15, 2015). After a discussion, write a date that can be misinterpreted (e.g., 08/11/2015) and ask the students to write the date in words (August 11, 2015, or November 8, 2015). Have students discuss what they wrote. (When the date is written with the year at the end, the day is listed first and then the month—smallest unit of time to the largest.)

  Show students a date with the year listed first (e.g., 2017/06/20). What is different in this format? (The month is listed after the year and the date is last—largest unit of time to the smallest.)

- **My Special Dates:** Have the students brainstorm important dates in their lives and have them write them down. Get the students to record each date using the different formats.
  - yyyy/mm/dd
  - dd/mm/yyyy
  - dd/mm/yy
  - June 12, 2015

- **Scavenger Hunt:** Have students engage in a scavenger hunt and have them bring in different formats of dates found in newspapers, calendars, tickets, posters, and magazines to discuss.
- **Changing Dates:** Have partners explore a special day in which the date fluctuates such as Labour Day. Have students record the dates over the past six years in different formats and present them in a poster format to the class.

Assessing Understanding: Paper-and-Pencil Task or Interview

Ask the students to point out the day's date on the calendar. Have them record and explain the date using the two formats.

Have the students write their birth date using four different formats.

Grade 4: Shape and Space (Measurement) (4.SS.3)

Enduring Understandings:

Objects have distinct attributes that can be measured with appropriate tools.

Standard units provide a common language for communicating measurement.

A measurement must contain a number and a unit.

Area tells how much material is required to cover a shape on the surface of an object.

Different rectangles have different areas.

Essential Questions:

How exact does a measurement have to be?

How does the length relate to measuring area?

What referent can you use to estimate area in centimetres?

What referent can you use to estimate area in metres?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.SS.3 Demonstrate an understanding of area of regular and irregular 2-D shapes by ■ recognizing that area is measured in square units ■ selecting and justifying referents for the units cm^2 or m^2 ■ estimating area by using referents for cm^2 or m^2 ■ determining and recording area (cm^2 or m^2) ■ constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area [C, CN, ME, PS, R, V]	➛ Describe area as the measure of surface recorded in square units. ➛ Identify and explain why the square is the most efficient unit for measuring area. ➛ Provide a referent for a square centimetre and explain the choice. ➛ Provide a referent for a square metre and explain the choice. ➛ Determine which standard square unit is represented by a referent. ➛ Estimate the area of a 2-D shape using personal referents. ➛ Determine the area of a regular 2-D shape and explain the strategy. ➛ Determine the area of an irregular 2-D shape and explain the strategy. ➛ Construct a rectangle for a given area. ➛ Demonstrate that many rectangles are possible for an area by drawing at least two different rectangles for the same area.

PRIOR KNOWLEDGE

Students may have had experience measuring length with both standard and non-standard units. They may have also had experience measuring mass and perimeter with non-standard units.

They may have explored the effect that changing the orientation of an object has on the measurements of its attributes (no change).

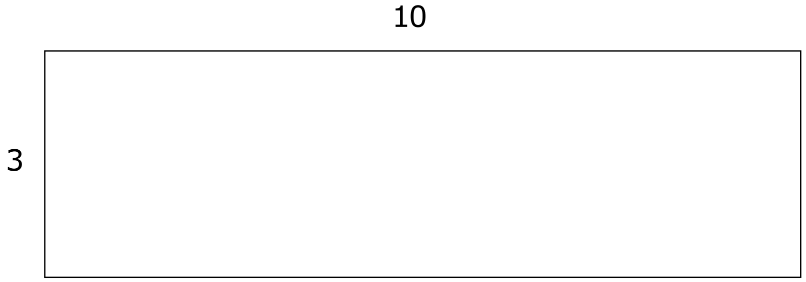
Students may have had no formal experiences with the concept of area.

BACKGROUND INFORMATION

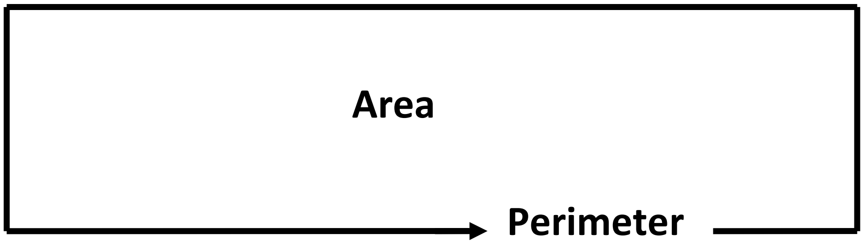
Area is the measure of the interior surface of a closed region or figure; area is measured in square units.

Example:

The area of the following rectangle is 30 square units.



A common misunderstanding of students is confusing perimeter and area. Using a visual reminder as seen below to distinguish between the two measurements may help students.



Centimetre and metre will be two standard units of measurement that students will use. It is important for students to have the time to discover personal referents for these standard units of measurement. Personal referents allow students to visualize measurement and make estimates more accurate. Area is measured in square units. If an object is measured in centimetres, the area would be recorded as cm^2.

Iteration means the act of repeating. In measurement, using a unit smaller than the object being measured and repeating it end-to-end is an example of iteration.

Referent is a point of reference used to compare in estimation (e.g., using the width of the baby finger as a referent for a centimetre).

Length is the distance from one end of an object to the other end, commonly measured in units of metres, centimetres, millimetres, and kilometres.

Width is one dimension of a 2-D or 3-D figure.

Tiling is to measure the area of shapes with units of measure that must fit together with no gaps or overlaps.

MATHEMATICAL LANGUAGE

area
regular
irregular
square units
referents
centimetre
metre
rectangle
shape
estimate

LEARNING EXPERIENCES

Assessing Prior Knowledge: Interview

Show students two pieces of paper that have the same area, but look different. Have students compare the two pieces of paper.

The student

- ☐ makes correct comparisons
- ☐ places one object on top of the other to measure, or dissects one piece of paper to see if it fits completely on top of the other
- ☐ uses standard or non-standard units of measure to compare

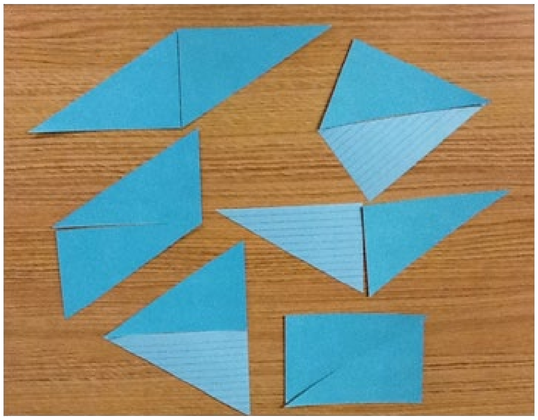
Show students a rectangle and ask: “If you change the position of the rectangle, do its measurements change? Explain your thinking.”

The student

- ☐ explains that the rectangle’s measurements have not changed, but the position has changed

- **Describe area as the measure of surface recorded in square units.**
- **Identify and explain why the square is the most efficient unit for measuring area.**
- **Provide a referent for a square centimetre and explain the choice.**
- **Provide a referent for a square metre and explain the choice.**
- **Determine which standard square unit is represented by a referent.**
- **Estimate the area of a 2-D shape using personal referents.**
- **Determine the area of a regular 2-D shape and explain the strategy.**
- **Determine the area of an irregular 2-D shape and explain the strategy.**
- **Construct a rectangle for a given area.**
- **Demonstrate that many rectangles are possible for an area by drawing at least two different rectangles for the same area.**

Suggestions for Instruction



- **Two-Piece Shapes:** Van de Walle, Karp, Lovin, and Bay-Williams suggest doing this activity as a precursor to developing a good understanding of area.

  Give each pair or triad of students six 3 x 5 inch index cards. Have them cut the rectangles on the diagonal, making two identical triangles for each card. Next, have them rearrange the triangles into different shapes. The rule is to have only sides of the same length matched up exactly, and to use only two triangles for each shape.

  Have each group of students find all the shapes that can be made. Discuss the area and shape of the different representations.

  Questions to ask students:

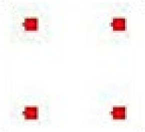
  - Does one shape have a greater area than the others?
  - Did one take more paper to make?

  The goal of the exploration is to have students understand that although each figure is differently shaped, all the representations have the same area (Van de Walle, Karp, Lovin, and Bay-Williams 324).

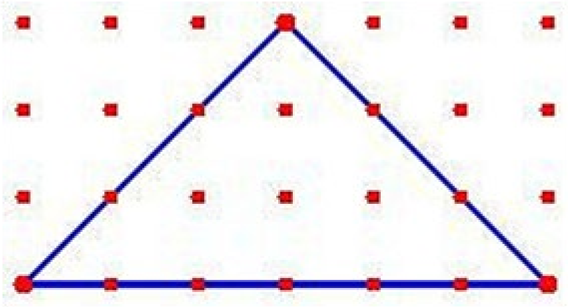
  Tangram puzzles can be used for the same purpose.

BLM 4.SS.3.1

- **What Is the Area?** Initial learning experiences should focus on developing the idea that area is measured by covering or tiling. It may be appropriate to start with non-standard units with students before measuring with standard metric units. Pass out BLM 4.SS.3.1 or draw three shapes that have similar areas (two rectangles and an irregular shape). Have students estimate which is the smallest and the largest of the three shapes. Have a set of manipulatives ready so students can measure the areas. The manipulatives can include tiles, Cuisenaire rods, base-10 blocks, unit cubes, pattern blocks, or cubes. It is important for students to have the opportunity to make decisions about which unit to use to measure area. This experience helps students build understanding of area. Discuss with the students their choices and the differences in the measurements. Ask why the numbers of units differ.
- **Area with a Geoboard:** Hand out a geoboard to each student or pair of students. Explain to the students that each square on the geoboard can represent one square unit. Display a shape to the students and have them find the area of the shape, stating the number of square units. After finding the area of a few shapes, state the area, and have students create shapes on their geoboard that have the given area. Have students justify their answers. Have the students compare their shapes. Facilitate the discussion so students see that many different shapes can be created for a given area.



One square unit



The area of this triangle
is 9 square units.

BLM 4.SS.3.2

- **Cover the Shape:** If more practice is needed in understanding the attributes of area, have students cover a pattern block mat. Make an outline using four to eight hexagons from a pattern block set. BLM 4.SS.3.2 can also be used. Ask students to cover the shape with each type of pattern block to determine the area. The discussion afterward should be on understanding the area does not change. The smaller the unit, the more units are needed to cover the same area. The discussion should also lead students to conclude that non-standard units are most efficient when the units tile.

The shape on BLM 4.SS.10 can be covered by the following:

- 7 hexagons
- 14 trapezoids
- 21 rhombi
- 42 triangles

- **Using Centimetre Cubes:** Once students understand the concept of area and realize the issues of using non-standard units, they can use standard units to measure. Marian Small notes that some students may be confused when area is described in terms of square units. Students need to understand that these squares can be dissected and rearranged to form many different shapes, all representing the same area (Small, *Grades K–3*).

- **Recording Area Measures in Standard Units:** To transition to standard measures, introduce square centimetres by using base-10 blocks or Cuisenaire rods (smallest unit) to measure because students can cover a shape with cubes and count to measure the area. These blocks measure 1 cm on each side. Have students measure and record different objects around the room with the centimetre cubes. After students have measured and recorded items, ask them how they recorded the measurements. Show how to record area measurements in square units. Spell out the measurement ("5 square centimetres") at first and then indicate the measurement with the exponent two ("5 cm^2"), which indicates that the units have two dimensions.

BLM
4.SS.3.3

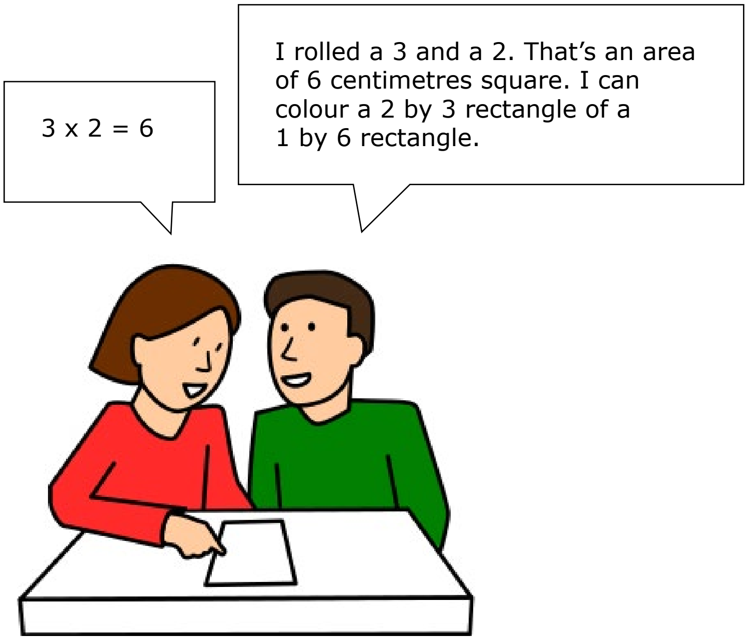
- **Using Grid Paper:** Introduce students to centimetre grid paper, such as BLM 4.SS.3.3. Have them explore how it can be used to measure area. Students may use centimetre cubes to compare it with the grid paper. Have the students use a centimetre ruler to measure the side of the centimetre grid paper to verify that each square is 1 cm^2. Have students practise measuring different classroom objects using the ruler. The grid paper can be copied onto transparency film to measure irregular objects more easily.

- **Estimating Area:** Ask students what they would do if they did not have a measuring tool to measure the area of an object. Facilitate the discussion toward how using referents could be a solution. Ask students what they could use as a referent for 1 cm^2 and have them explain their thinking. Using the width of a little finger can be used as a referent for about 1 cm. Gather a few different books and have the students estimate the area of the covers using the referent and then have them check by finding the area of the book covers.

- **Square Metres:** Introduce the concept that square metres can be used for measuring area. Ask students what items could be used to measure square metres. Discuss possible referents for 1 m^2 after marking off 1 m^2 on the floor using masking tape. This visual allows students to see what 1 m^2 looks like. Have students use their referents to estimate the area of a large section of a wall or floor. Use a square piece of paper that measures 1 m^2 to check the estimates. Have students practise measuring and recording using a metre stick.

BLM
4.SS.3.4

- **Cover the Area Game:** Students' understanding of area will deepen when they become aware that squares can be dissected and rearranged to form different shapes, all representing the same area. BLM 4.SS.3.4, *Cover the Area Game,* provides students with an opportunity to discover that shapes with the same area do not have to be congruent. The game also helps students make connections between arrays and multiplication. The game has students creating rectangles arranged in an array. Initially, students may count all the squares in the array, before they begin to understand that the total number of squares can be found by multiplying the number of squares in a row by the number of squares in a column. After the students play the game, a number discussion should be initiated. Ensure that students see how different rectangles were created but the area was the same.



- **Square Centimetres or Metres?** Provide students with centimetre-grid paper for measuring square centimetre units. Ask pairs of student to list four items that should be measured in square centimetres and four items that should be measured in square metres. Have students exchange lists and then estimate, measure, record, and order the items on each list.

Assessing Understanding: Paper-and-Pencil Task or Interview

Have students construct all the rectangles that have an area of 36 cm^2. Use grid paper to record each rectangle's dimensions.

Real life problem to solve:

Zeta wants a rectangular garden that is 24 m^2 for her backyard. What are the different gardens that she could have if the lengths and widths are whole numbers? How do you know that you have included all the possible garden plots?

The student

☐ names all the areas

☐ applies an understanding of patterns to solve the problem

Designing a Playground

Organization:

- Groups of two or three

Materials:

- 1-cm grid paper
- rulers, pencil crayons

Context:

The school has decided to build a new playground.

Students have been invited to submit proposals to the principal. Proposals must include a design of the playground area. The playground must consider the following guidelines:

- The playground must be in the shape of a rectangle.
- There must be three to five sections for equipment with space in between so that students are safe.
- Each structure should be different from the others.

Have students follow the design process (from science) to create their poster (discuss possible solutions, develop criteria, create a plan/diagram, make the prototype).

Each proposal should include the following:

- the plan for the playground, drawn on grid paper with each drawing on the plan labelled
- an explanation that the area of one grid square represents 1 m^2
- the equipment areas shown with area in m^2
- a written explanation of the plan
- an attached explanation of how each area was calculated

Look for an understanding of

☐ measurement vocabulary

☐ area as the amount of space a shape covers

☐ correct area calculations

☐ correct recording of measurements

☐ metric measurements

Notes

Grade 4: Shape and Space (Geometry) (4.SS.4, 4.SS.5)

Enduring Understandings:

Geometric shapes and objects can be classified by attributes.

Objects can be described and compared using geometric attributes.

Any 2-D shape or 3-D object can be created by combining and dissecting other shapes.

Essential Questions:

What are the attributes of a shape or object?

What are ways shapes or objects can be sorted?

How can a shape can be dissected and rearranged into other shapes to help describe the properties of the shape?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.SS.4 Solve problems involving 2-D shapes and 3-D objects. [C, CN, PS, R, V]	➛ Fill an outline with 2-D shapes (e.g., tangram pieces, pentominoes, or polygons). ➛ Reproduce 2-D shapes from drawings, real objects (tables, houses, letters of the alphabet), or attributes on geoboards. ➛ Reproduce a structure using 3-D objects (e.g., cubes, 3-D pentominoes).
4.SS.5 Describe and construct rectangular and triangular prisms. [C, CN, R, V]	➛ Identify and name common attributes of rectangular prisms from sets of rectangular prisms. ➛ Identify and name common attributes of triangular prisms from sets of triangular prisms. ➛ Sort a set of rectangular and triangular prisms using the shape of the base. ➛ Construct and describe a model of rectangular and triangular prisms using materials such as pattern blocks or modelling clay. ➛ Construct rectangular prisms from their nets. ➛ Construct triangular prisms from their nets. ➛ Identify examples of rectangular and triangular prisms found in the environment.

PRIOR KNOWLEDGE

Students may have had experience

- describing 3-D objects according to the shape of faces, and the number of edges and vertices
- sorting regular and irregular polygons according to the number of sides, including triangles, quadrilaterals, pentagons, hexagons, and octagons

BACKGROUND INFORMATION

As students progress with their understanding of geometric thinking, the goal is to have them be aware of geometric properties of two-dimensional shapes and three-dimensional objects. For example, students can see a rectangle and recognize it as a rectangle, but they should know the specific attributes that make it a rectangle. Students should know that rectangles have four sides in which opposite sides are of equal length and that rectangles have four square corners.

Students will learn about geometric properties as they handle and manipulate objects. Specific Learning Outcome 4.SS.4 encourages students to describe and reproduce two-dimensional shapes and three-dimensional objects. It also allows students to investigate the results of composing and decomposing shapes to develop spatial sense. Van de Walle, Karp, Lovin, and Bay-Williams describe spatial sense as "an intuition about shapes and the relationships among shapes. Spatial sense includes the ability to mentally visualize objects and spatial relationships, including being able to turn objects around in one's mind. It also includes a familiarity with geometric descriptions of objects and position" (Van de Walle, Karp, Lovin, and Bay-Williams 345).

Providing students the opportunity to solve puzzles using shapes will build an understanding of the shapes and foster their spatial and problem-solving skills.

Pierre van Hiele and Dina van Hiele-Geldof (cited in Van de Wall, Karp, Lovin, and Bay-Williams 346–351), mathematics teachers from the Netherlands in the 1950s, researched the development of geometrical thinking. Through their research they identified five sequential levels of geometric thought. Two of the levels are listed on the following page. Most Grade 4 students will be at level 0 or 1.

There are four characteristics of these levels of thought:

- The levels of geometric reasoning/understanding are sequential. Students must pass through all prior levels to arrive at any specific level.
- These levels are not age-dependent.
- Geometric instructional experiences have the greatest influence on advancement through the levels.
- Instruction or language at a higher level than the level of the student may inhibit learning.

Level 0 (sometimes labelled as Level 1): **Visual**

At this level students can name and recognize shapes by their appearance, but cannot specifically identify properties of shapes. Students may think that a rotated square is a diamond and not a square because it looks different from their visual image of square. Most students in Kindergarten to 3 will be at Level 0 (visualization).

Suggestions for instruction at this level include

- sorting, identifying, and describing shapes
- working with physical models
- seeing different sizes and orientations of the same shape in order to distinguish the characteristics of the shape and to identify features that are not relevant
- building, drawing, making, putting together, and taking apart 2-D shapes and 3-D objects

Level 1: Analysis (Some students may be at this stage.)

At this level students begin to be able to identify the properties of shapes. They use appropriate geometric vocabulary related to properties. They are able to move beyond less important features such as size or orientation in order to sort and classify shapes. They start to describe the relationship between shapes and their properties.

Suggestions for instruction at this level include

- focusing on properties (defining, measuring, observing, or changing) by using concrete or virtual models
- problem solving involving shapes
- classifying shapes based on their properties

Terminology

3-D Terms:

3-D objects: An object that has length, width, and height; also called a solid object (e.g., prism, pyramid, cylinder, cone).

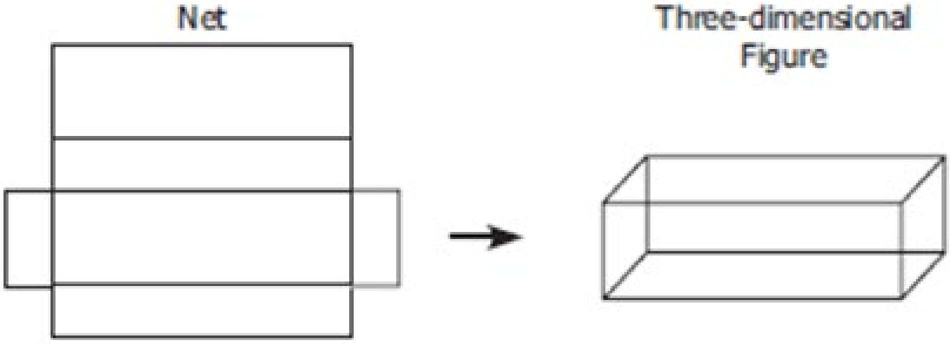
Base: A particular side or face of a geometric figure.

Congruent: Two figures that have the same shape and size.

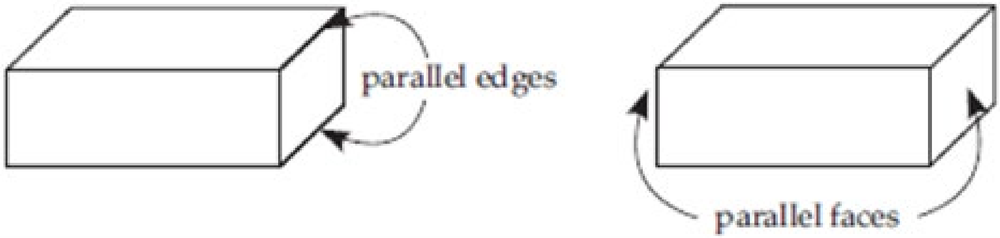
Edge: The line segment where two plane faces of a solid figure meet.

Face: A flat surface of a solid.

Net: The 2-D set of polygons of which a 3-D object is composed.



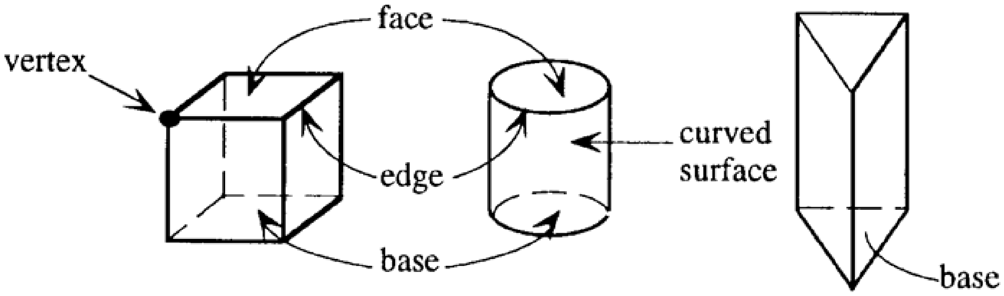
Parallel: Faces or edges of a 3-D object that never intersect; they are equidistant (equal distance) from each other.



Prism: A 3-D figure (solid) that has two congruent and parallel faces that are polygons (the bases); the remaining faces are parallelograms.

Pyramid: A polyhedron whose base is a polygon and whose lateral faces are triangles that share a common vertex.

Vertex, Vertices: The common point where three or more edges of a 3-D solid meet. Note: A cone has an **apex,** but it is often referred to as a vertex.



Note: Cones and spheres have curved surfaces (i.e., a cone has one face and one curved surface).

2-D Terms:

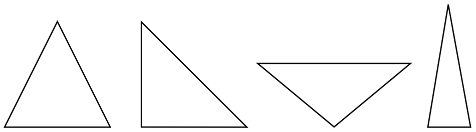
Polygon: A closed plane figure formed by three or more line segments.

Regular polygon: A polygon in which all sides and all angles are congruent.

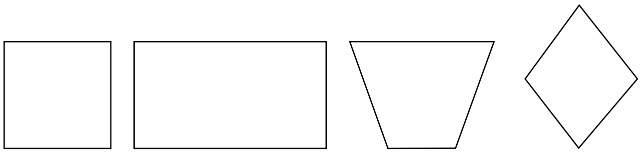
Irregular polygon: A polygon whose sides and angles are not all congruent.

Polygon Names: Note: Regular polygons are shown first.

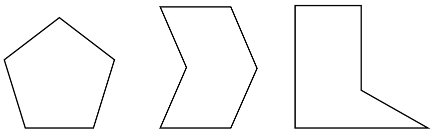
3 sides—triangle



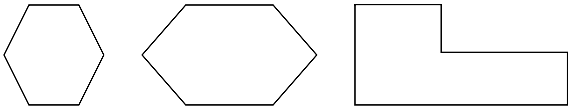
4 sides—quadrilateral



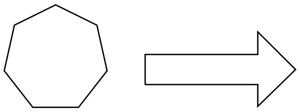
5 sides—pentagon



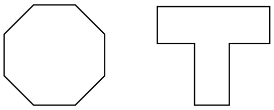
6 sides—hexagon



7 sides—heptagon



8 sides—octagon



MATHEMATICAL LANGUAGE

cube
sphere
cone
cylinder
prism
pyramid
face
edge
vertex
vertices
curved surface
skeleton
polygon
regular polygon
irregular polygon
triangle
quadrilateral
pentagon
hexagon
heptagon
octagon
three-dimensional
two-dimensional
attribute
property
sides
shape
object

Assessing Prior Knowledge: Journal/Notebook Entry

- How are 2-D shapes and 3-D objects related?
- Draw five different polygons and name them.
- Draw five different hexagons.

The student

☐ identifies how 2-D shapes and 3-D objects are related (3-D objects are made up of 2-D shapes. 3-D objects get their names from their 2-D shaped bases.)

☐ correctly draws and names polygons

☐ is able to draw five different hexagons and to recognize that a hexagon can be regular or irregular

- **Fill an outline with 2-D shapes (e.g., tangram pieces, pentominoes, or polygons).**
- **Reproduce 2-D shapes from drawings, real objects (e.g., tables, houses, letters of the alphabet), or attributes on geoboards.**

Suggestions for Instruction

- **Puzzle Outlines:** Different manipulatives can be used to fill in shapes of different puzzles. Sandra Ball and Carole Fullerton have created different puzzles for pattern blocks, tangrams, pentominoes, and Cuisenaire rods. Find their puzzles by downloading a free copy of *Daily Math Investigations: Meaningful Math Routines: Alternatives to Calendar Designed to Keep Your Students Engaged, Thinking and Reasoning Mathematically!* from the following site: https://mindfull.wordpress.com/free-downloads-2/daily-math-investigations-k-3/.

  Questions that can be used during the learning experiences include the following:

  - What did you notice about the puzzle?
  - What strategy did you use?
  - Which shapes were used to fill in the puzzle?
  - Could other shapes fit into the puzzle?

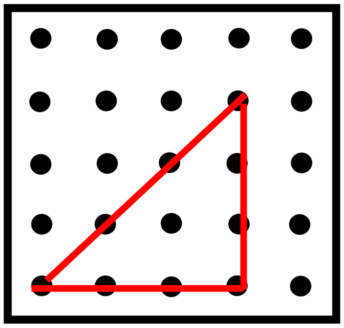
- There are many ready-made commercial resources that contain puzzles of pattern blocks, tangrams, and pentominoes.
  - **Pattern Block Puzzles:** Use pattern blocks to fill in shapes of puzzles. Have students match blocks to the outlines. Encourage them complete the puzzles in more than one way. Discuss with the students the relationships between the pieces. Have students complete the same puzzle outline using the greatest number of pattern blocks and the smallest number of pattern blocks.
  - **Tangram Puzzles:** Use a complete set of seven tangram pieces and puzzles to get students to match pieces to the outlines. Encourage them to complete the puzzles in different ways. Find more tangram puzzles at the following websites:
    - Tangram Channel: www.tangram-channel.com/tangram-puzzles/
    - Activity Village: www.activityvillage.co.uk/tangrams
    - NRICH: http://nrich.maths.org/public/leg.php?code=5044
  - **Pentomino Puzzles:** Use a complete set of 12 pentomino shapes and puzzles of the shapes. Have students match pieces to the outlines.
  - **Cuisenaire Matching Tasks:** Use Cuisenaire rods and puzzles of pictures created from the rods. Have students match rods to the outlines. Encourage them to complete the puzzles in more than one way. Have students calculate the value of the puzzle as an extension activity and make connections to number operations and patterns.

- **Making Puzzles:** Have children create their own puzzles with different pattern blocks, tangrams, pentominoes, and Cuisenaire rods. Have children outline their pictures and exchange with a partner. Each partner will attempt to fill in the other's puzzle.

- **Grandfather Tang's Story:** Read *Grandfather Tang's Story: A Tale Told with Tangrams* by Anne Tompert, illustrated by Robert Andrew Parker, to the students. The story uses tangrams to illustrate the story a grandfather tells his granddaughter. Have students recreate the tangram creatures found throughout the book. Have students assemble the tangram pieces to replicate shapes from the book and trace around the outside of the figures. Have them fill in the shape with black marker. The tracings can be placed in a math centre.

BLM 4.SS.4.1

- **Geoboard Copy:** Create 2-D shapes on a geoboard or show different pictures of objects. Have students copy the shape with their own geoboards. Start with using one band and then progress the activity by using more bands to create complex shapes. Have students create different shapes on BLM 4.SS.4.1, and create a centre using the cards (Van de Walle, Karp, Lovin, and Bay-Williams 357).



Assessing Understanding (2-D Shapes): Paper-and-Pencil Task or Interview

Seven Ways to Make a Hexagon: Have students find as many of the seven combinations as they can to cover the outline of a hexagon pattern block and record how they know they have all the shapes.

The student

☐ uses the correct math vocabulary
☐ records solutions accurately
☐ applies an understanding of patterns to solve the problem

- **Reproduce a structure using 3-D objects (e.g., cubes, 3-D pentominoes).**

Suggestions for Instruction

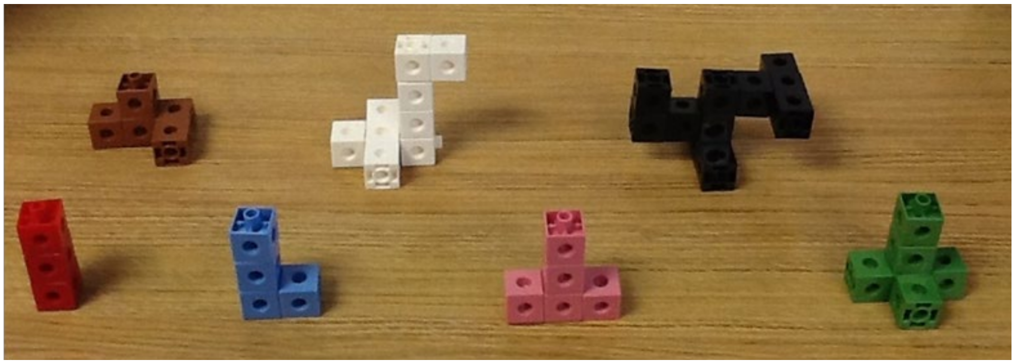
When students combine, separate, and transform shapes, they are investigating relationships among the shapes. When students reproduce an object, they need to think about how the whole object looks and about how the parts relate to each other and to the whole.

In producing a 3-D object, students must do the following:

- "see" the component parts of the whole 3-D object
- produce each component part (in the correct shape and size, either full or scaled)
- put the component parts together in the correct relationship to each other and in the correct proximity and orientation of the whole 3-D object

As stated in *First Steps in Mathematics: Geometry and Space,* "Students need to analyze the component parts that form the object—their shape, size, and placement, considering how the components fit and hold together. They will need to learn from their mistakes by observing what goes wrong when insufficient attention is paid to details of shape, size, and placement" (Western Australian Minister for Education 73).

- **Building Objects:** Show different pictures of objects such as vehicles, playground equipment, bikes, and buildings. In pairs, have students build the pictures using cubes.
- **Build the Structure:** Ask students to build a structure using between four and six blocks. Have students sit in pairs with a file folder separating each of the students' structures. Have one student explain his/her structure while the other recreates the speaker's structure without looking. Encourage students to use correct math vocabulary.
- **3-D Pentominoes Build:** Show students different 3-D pentominoes and have them reproduce them using cubes. *3-D Pentomino Puzzle* gives different representations for 3-D pentominoes and is available at the following address: https://www.learningresources.com/text/pdf/2240_TG.pdf.
- **Cube Challenge:** Show students different arrangements of structures made from cubes, one at a time. The picture below shows examples. Ask students to build the structure. Observe to see if the students build the structures in sections and if they are able to recognize when their structure is different than the arrangement being displayed. This learning experience provides an opportunity for students to discover that objects are the same despite differences in their orientations.



- **Identify and name common attributes of rectangular prisms from sets of rectangular prisms.**
- **Identify and name common attributes of triangular prisms from sets of triangular prisms.**
- **Identify examples of rectangular and triangular prisms found in the environment.**
- **Construct and describe a model of rectangular and triangular prisms using materials such as pattern blocks or modelling clay.**

Suggestions for Instruction

A prism is a 3-D figure (solid) that has two congruent and parallel faces that are polygons (the bases); the remaining faces are parallelograms. The bases take the shape of any polygon. Prisms can be classified in terms of the shape of their base. In Grade 4 it is not expected to use all the formal geometric language. However, it is good to seize opportunities to model correct language. For example, the words *congruent* and *parallel* can be introduced to students. In Grade 4, students will be describing and constructing rectangular and triangular prisms and identifying examples of them in the environment.

Prisms can be built by vertically or horizontally piling different blocks together. Have students stack pattern blocks or attribute blocks to make rectangular prisms and triangular prisms. Discuss the attributes of the objects. Then have students construct the prisms using modelling clay.

- **Prisms:** Provide students with a variety of rectangular and triangular prisms. Square prisms fall into the category of rectangular prisms because a square is a rectangle. Provide many variations of each type of object to the students. Students need to see narrow, wide, tall, and short prisms so they can recognize any type of prism. Have students sort the prisms. Engage in a conversation about their attributes. Have students compare the prisms. Comparisons can lead to discussions about differences and similarities. Students can make anchor charts of the prisms, recording the following 3-D attributes:
    - square or triangle faces
    - number of faces or edges
    - identical (congruent) faces
    - number of vertices

- **Wanted Posters:** Have students create wanted posters of a rectangular or triangular prism. Have them list the attributes and name environmental objects that look like the object.
- **Prisms in the Environment:** Have students find examples of prisms in the classroom, school, playground, community, and their homes. If possible, have students take pictures of the prisms and then they can be made into an individual or a class book. Each picture should be accompanied by a description of where the prism was found along with the correct name for the prism.

- **Construct rectangular prisms from their nets.**
- **Construct triangular prisms from their nets.**

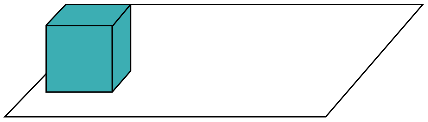
Suggestions for Instruction

- **Skeletons:** Make a skeleton of a rectangular and triangular prism. A skeleton is an open 3-D figure constructed from materials (e.g., straws, pipe cleaners). Constructing a skeleton before constructing nets can help students see the vertices and edges that make up a prism.
- **Creating a Net by Rolling and Tracing:** A net is the 2-D set of polygons of which a 3-D object is composed.

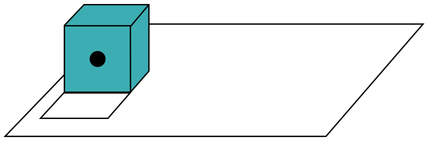
  A net can be folded and assembled to recreate a 3-D object. Nets focus on the faces that make up a 3-D object and how they fit together. Students must come to realize there may be different nets that make a single object. The faces of a net can be connected in various ways to make an object. Ensure that the drawing is correct by having the students cut the tracing to see if they can make a prism. Marian Small outlines an activity in which students roll an object and trace its faces to create a net (Small, *Grades K–3* 72).

Creating a Net by Rolling and Tracing

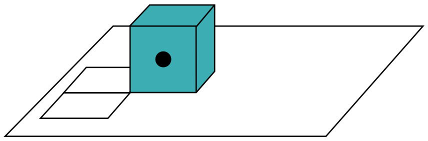
Step 1: Trace one face and mark it with a dot.



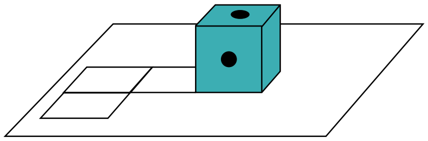
Step 2: Roll onto another face and trace it, then mark it with a dot.



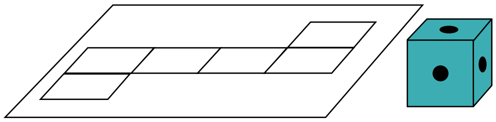
Step 3: Roll onto another face and trace it, then mark it with a dot.



Step 4: Roll onto another face and trace it, then mark it with a dot.



Step 5 and 6: Continue rolling, tracing, and marking faces until you have traced all 6 faces. Make sure that the arrangement of rectangles will form a net. Cut out the prism to see if it makes the 3-D object.



BLM 4.SS.5.1

- **Nets:** Provide nets of rectangular and triangular prisms, BLM 4.SS.5.1, for students and ask them to identify the 3-D object. Have them verify by folding.
- **Air Nets:** NRICH (University of Cambridge) presents videos at http://nrich.maths.org/6307 showing nets being created from Polydron pieces. Watch the videos about rectangular and triangular prisms. Ask students to do the following:
  - Predict what object the net will create.
  - Predict whether or not the net will fold into a 3-D object.

Assessing Understanding: Paper-and-Pencil Task or Interview

Ask students, “How are rectangular and triangular prisms alike?”

The student is able to explain that

☐ both rectangular and triangular prisms are 3-D objects

☐ both rectangular and triangular prisms have two polygon faces (bases) that are the same size and shape (congruent) as their opposite sides

☐ all non-base faces of prisms are rectangles that touch the bases.

Grade 4: Shape and Space (Geometry) (4.SS.6)

Enduring Understandings:

Geometric shapes and objects can be classified by attributes.

Objects can be described and compared using geometric attributes.

Any 2-D shape or 3-D object can be created by combining and dissecting other shapes.

A shape or picture is symmetrical if it can be divided into two congruent halves, each part a reflection of the other.

Essential Questions:

What are the characteristics of symmetrical shapes?

How do you identify lines of symmetry?

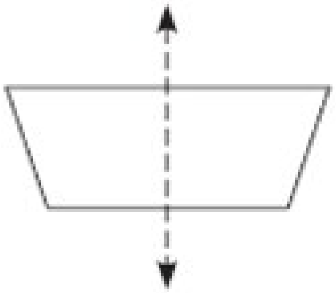
Are some shapes more symmetrical than others?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.SS.6 Demonstrate an understanding of line symmetry by ■ identifying symmetrical 2-D shapes ■ creating symmetrical 2-D shapes ■ drawing one or more lines of symmetry in a 2-D shape [C, CN, V]	➛ Identify the characteristics of symmetrical and non-symmetrical 2-D shapes. ➛ Sort a set of 2-D shapes as symmetrical and non-symmetrical. ➛ Complete a symmetrical 2-D shape, half the shape, and its line of symmetry. ➛ Identify lines of symmetry of a set of 2-D shapes, and explain why each shape is symmetrical. ➛ Determine whether or not a 2-D shape is symmetrical by using a Mira or by folding and superimposing. ➛ Create a symmetrical shape with or without manipulatives. ➛ Provide examples of symmetrical shapes found in the environment, and identify the line(s) of symmetry. ➛ Sort a set of 2-D shapes as those that have no lines of symmetry, one line of symmetry, or more than one line of symmetry.

PRIOR KNOWLEDGE

Students will be drawing on their previous knowledge of 2-D shapes to assist them in their understanding of symmetry.

BACKGROUND INFORMATION

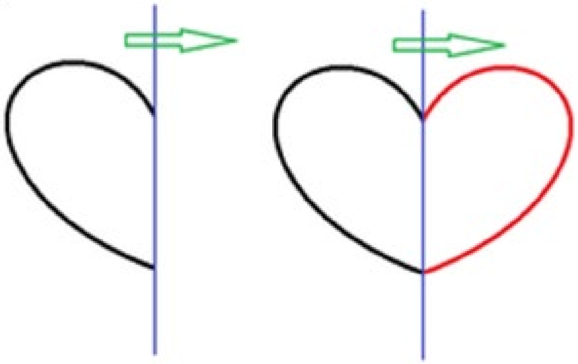


Symmetry is the property of having the same size and shape across a dividing line or around a point. If a shape or image can be folded so that the two halves match exactly, then the shape or figure has line symmetry. The fold line is actually a line of symmetry or line of reflection.

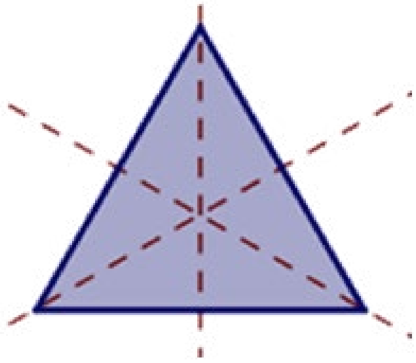
A **line of symmetry** is a line that divides a figure into two congruent parts so that they can be matched by folding the shape in half. The two parts are mirror images of each other.

A shape is **symmetrical** when two sides of a shape are balanced about a line or point, when two sides of a shape are mirror images, or when a shape has one or more lines of symmetry.

Students will be exploring the attribute of line symmetry with 2-D shapes and figures. Folding a shape to see if one half exactly matches the other is a way to find out if a shape has symmetry. If it has symmetry the shape is symmetrical. Two-dimensional symmetrical shapes can be divided along one or more lines of symmetry.



1 line of symmetry



3 lines of symmetry

Students will find that the more sides there on a regular polygon, the more lines of symmetry there will be. Also, the number of lines of symmetry on a regular polygon is equal to the number of vertices of that polygon.

LEARNING EXPERIENCES

- **Identify the characteristics of symmetrical and non-symmetrical 2-D shapes.**
- **Sort a set of 2-D shapes as symmetrical and non-symmetrical.**
- **Complete a symmetrical 2-D shape, half the shape, and its line of symmetry.**
- **Identify lines of symmetry of a set of 2-D shapes, and explain why each shape is symmetrical.**
- **Determine whether or not a 2-D shape is symmetrical by using a Mira or by folding and superimposing.**
- **Create a symmetrical shape with or without manipulatives.**
- **Provide examples of symmetrical shapes found in the environment, and identify the line(s) of symmetry.**
- **Sort a set of 2-D shapes as those that have no lines of symmetry, one line of symmetry, or more than one line of symmetry.**

Suggestions for Instruction

- Provide students opportunities to explore symmetry through creative explorations.
  - Have students fold a paper in half, open and drop paint on one half, then fold to "print." Have students unfold the paper and discuss the attributes of the image.
  - Fold a paper in half, open and draw half a shape with heavy-black crayon lines. Other dark crayons may work as well. Refold and press firmly on the crayon lines to transfer them to the other half. Have students open and retrace the lighter "print" if needed.
  - Questions to explore with the students:
    - What do you notice about the reflection?
    - What do you notice about images?
    - How are the images alike? Is there a relationship between the two images?
- **Let's Fly a Kite:** Read the story, *Let's Fly a Kite* by Stuart J. Murphy, illustrated by Brian Floca. Then demonstrate the use of a plastic mirror to show how students can check for symmetry without cutting or folding the pictures in the book.

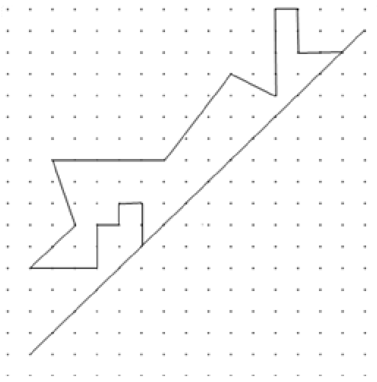
BLM
4.SS.6.1

- **Mirrors and Miras:** Supply mirrors or Miras for students to sort upper-case alphabet letters (BLM 4.SS.6.1) into two groups—symmetrical and not symmetrical.
- **Extension:** Have students sort the letters into three groups—no lines of symmetry, one line of symmetry, more than one line symmetry.

- **Symmetry through the Year:** During holiday times, have students cut holiday symbols such as hearts, shamrocks, and snowflakes by folding paper and cutting symmetrical figures. The symbols can be reused for classroom decorations.
- **Complete the Picture:** Provide students with pictures that show half of the image. Have them place a Mira along a line of symmetry. Students will use the reflection to draw the other half of the picture. Have students test the accuracy by reflecting the drawn half onto the original side. Ask students for another way to check the reflection (cut out the shape and fold).

BLM
4.SS.6.2

- **Picture Creation:** Using dot paper (BLM 4.SS.6.2), have students draw a line (horizontally, vertically, or diagonally) though several dots. Have students make a design or picture completely on one side of the line. Have students make a mirror image of their design on the other side of the line. Students can even exchange designs and make the mirror image of each other's picture. Have students check their work with a mirror.



- Have students discuss examples of symmetrical shapes in their everyday life.

Assessing Understanding: Paper-and-Pencil Task or Interview

Ask students to identify and record two 2-D shapes that have line symmetry. Have them show where the lines of symmetry are. Have them explain why the shapes are symmetrical.

The student

☐ makes symmetrical shapes

☐ justifies why the shapes are symmetrical

☐ identifies the lines of symmetry

BLM
4.SS.6.3

Have students fill out BLM 4.SS.6.3, *The Frayer Model,* to show their understanding of line symmetry.

Ask students, "How would you define symmetry to someone who did not know what it is?"

The student

☐ states that you can fold the shape on top of itself, or a shape can be folded so that the two halves match exactly, or gives a similar response

☐ gives examples and non-examples of symmetrical shapes or figures

GRADE 4 MATHEMATICS

Statistics and Probability (Data Analysis)

Grade 4: Statistics and Probability (Data Analysis) (4.SP.1, 4.SP.2)

Enduring Understandings:

Graphs are data displays that quickly reveal information about data.

Data can be collected and organized in a variety of ways.

Data can be used to answer questions.

It is not only important to be able to read and construct graphs, but it is important to be able to draw conclusions about the data.

Essential Questions:

How can data be recorded?

What kind of information can we get from different types of graphs?

How can data be organized and interpreted to answer questions and draw conclusions?

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
4.SP.1 Demonstrate an understanding of many-to-one correspondence. [C, R, T, V]	➛ Compare graphs in which different intervals or correspondences are used, and explain why the interval or correspondence was used. ➛ Compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences, and explain how they are the same and different. ➛ Explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence. ➛ Find examples of graphs in which many-to-one correspondence is used in print and electronic media, such as newspapers, magazines, and the Internet, and describe the correspondence used.

Specific Learning Outcome(s):	**Achievement Indicators:**
4.SP.2 Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions. [C, PS, R, V]	➛ Identify an interval and correspondence for displaying a set of data in a graph, and justify the choice. ➛ Create and label (with categories, title, and legend) a pictograph to display a set of data using many-to-one correspondence, and justify the choice of correspondence used. ➛ Create and label (with axes and title) a bar graph to display a set of data using many-to-one correspondence, and justify the choice of interval used. ➛ Answer a question using a graph in which data is displayed using many-to-one correspondence.

PRIOR KNOWLEDGE

Students may have

- formulated questions and collected data using concrete objects, tallies, check marks, charts, or lists
- constructed and interpreted concrete graphs, pictographs, and bar graphs to solve problems
- collected and organized first-hand data

BACKGROUND INFORMATION

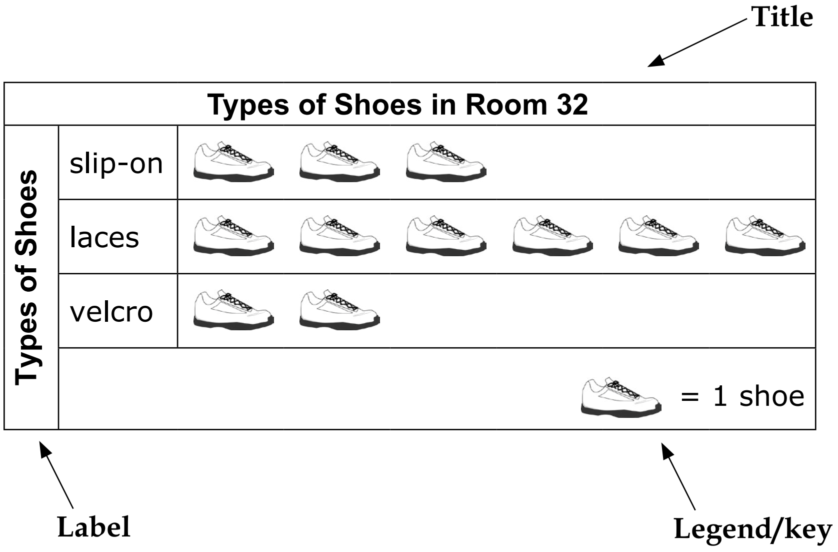
Displaying data enables problems to be solved or information to be communicated. Graphs make data more accessible and easier to interpret. Reading data from a graph should reveal some sort of information at a glance.

Prior to Grade 4, students may have had opportunities to collect data, organize data using tally charts, and display data in pictographs and bar graphs. In Grade 4, students will be comparing, finding, constructing, and interpreting pictographs and bar graphs involving **many-to-one correspondence.** At this level, students may discover that the data they collect is too large to display in a graph using a one-to-one correspondence. Many-to-one correspondence is a representation of many objects by one object or interval in a graph. For example, in a pictograph, one happy face can represent 5 people, and in a bar graph, one rectangle on the graph paper can represent 10 years.

An **interval** is the distance or difference between two numbers or quantities. In graphing, the interval of numbers on one or both axes needs to have equal numerical spacing.

Pictographs use visual items to represent data. A **pictograph** uses uniform, representative pictures to depict quantities of objects or be the same size and shape to avoid misleading the audience.

Example of a pictograph:



Pictographs need to have a title, labels, and pictures. Legends/keys are needed when the pictures or symbols are used to represent more than one quantity (many-to-one correspondence).

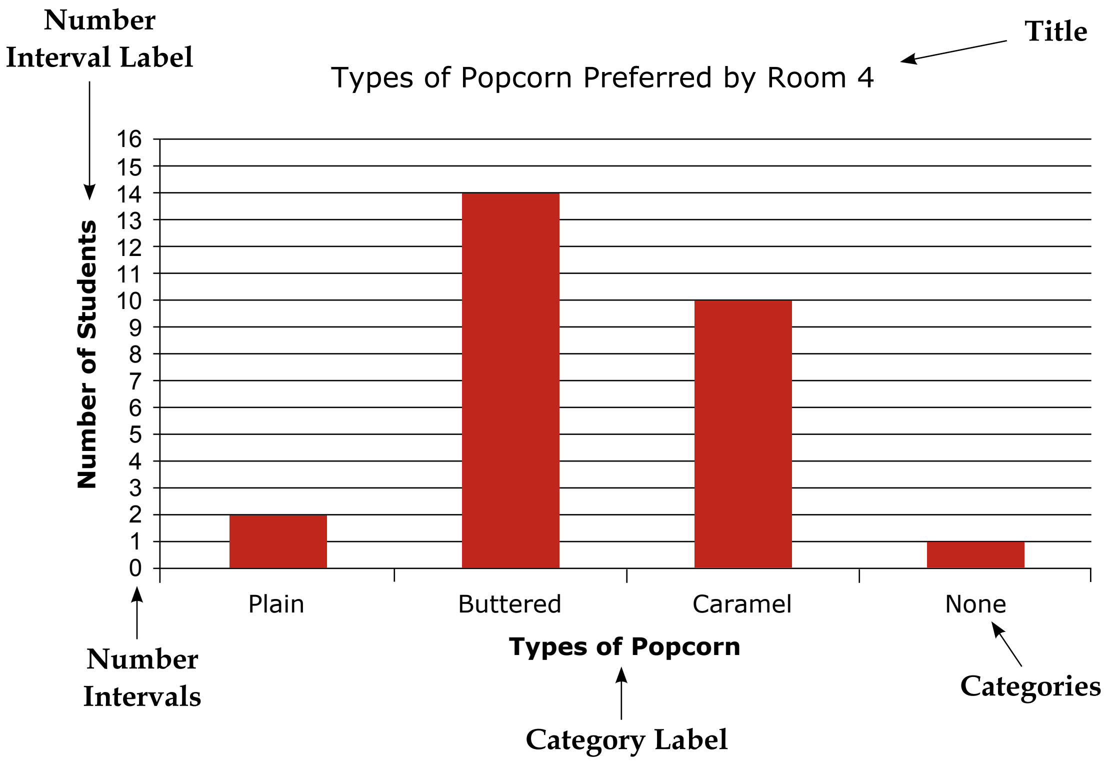
Note: The legend of a pictograph may be called its key. Other resources call it a scale or scale statement.

Marian Small (Small, *Grades 4–8,* 180) suggests the following important points about pictographs with scales:

- The scale must be clearly stated in the legend/key.
- The same symbol should be used throughout the graph. This symbol may or may not reflect the context of the data.
- The first symbol in each category starts at the same level or baseline.
- The symbols in each category are equally spaced.
- The symbol chosen should allow for partial symbols that are easy to interpret.
- Pictographs can be vertical or horizontal.
- It is important to include labels and a concise but meaningful title to help the reader understand the graph.

A **bar graph** is a graph that uses horizontal or vertical bars to display data.

Example:



A bar graph needs the following labels:

- title
- categories
- category label
- number intervals (Note: Numbers are labelled on the line not the space.)
- number interval label

Generally the data graphed at the elementary level is discrete data (data attained by counting in whole numbers). In this case, there are always spaces left between the bars.

Marian Small (Small, *Grades 4–8,* 183) suggests the following important points about bar graphs:

- A grid square is used to represent the same quantity throughout.
- Bars should be separated to indicate that they represent discrete data.
- If there are no pieces of data for a category or interval, a space can be left where the bar would be, although it is not required.

- Both axes should be labelled. Each bar should have a label, which might be a discrete topic category, a discrete number category, or a numerical interval category. The other axis, the scale axis, is labelled numerically.
- Axes headings should be used for clarity.
- It is important to include a concise but meaningful title to help the reader understand the graph.

Bar graphs compare the frequency of discrete data. Data are displayed using a number of rectangles (bars) that are the same width. Each bar represents one of the categories that the data have been sorted into. The bars are displayed either horizontally or vertically with a space between them. The height or length of a bar represents the number of observations in that category. The numbers on the y-axis of a vertical bar graph or the x-axis of horizontal bar graph are called the scale.

To get students ready to interpret data on graphs and use them as a problem-solving tool, have them gather meaningful data and learn how to construct graphs. Students will learn that graphs must be clearly labelled and titled so that they communicate information concisely and visually. Interpreting graphs can help students become aware of the features of a graph that can lead to understanding and making sense of the data displayed. Curcio (2001) identifies three levels of graph comprehension: reading the data, reading between the data, and reading beyond the data.

Reading the Data

This level of comprehension requires a literal reading of the graph. The reader simply “lifts” data explicitly stated in the graph, or the information found in the graph title and axes labels, from the graph. There is no interpretation at this level. Reading that requires this type of comprehension is a very low-level cognitive task.

Reading between the Data

This level of comprehension includes the interpretation and integration of the data in the graph. It requires the ability to compare quantities (e.g., greater than, tallest, smallest) and the use of other mathematical concepts and skills (e.g., addition, subtraction, multiplication, division) that allow the reader to combine and integrate data and identify the mathematical relationships expressed in the graph.

Reading beyond the Data

This level of comprehension requires the reader to predict or infer from the data by tapping existing schemata (i.e., background knowledge, knowledge in memory) for information that is neither explicitly nor implicitly stated in the graph. Whereas reading between the data might require the reader to make an inference that is based on the data presented in the graph, reading beyond the data requires that the inference be made on the basis of information in the reader’s head, not in the graph.

MATHEMATICAL LANGUAGE

categories
label
title
data
tallies
match
more
less
same amount as
most
least
bar graph
compare
pictograph
survey
interval
list
axes
many-to-one correspondence
legend

Assessing Prior Knowledge

Present students with the following pictograph.

Favourite Apple Colours					
Red	🍎	🍎	🍎	🍎	🍎
Green	🍎				
Yellow	🍎	🍎	🍎		

Ask the following questions:

- What does the pictograph show? How do you know?
- What does this tell about the colours of apples?
- Which colour of apple is liked more—yellow or green? How do you know?
- How many people were surveyed? How do you know?

Have students then create a bar graph from the data in the pictograph.

The student

- ☐ describes the data represented in the pictograph
- ☐ answers questions pertaining to the pictograph
- ☐ uses mathematical language correctly
- ☐ identifies an interval and correspondence for displaying a set of data in a graph, and justifies the choice
- ☐ creates a bar graph correctly, including labelling the title and axes

- **Compare graphs in which different intervals or correspondences are used, and explain why the interval or correspondence was used.**
- **Compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences, and explain how they are the same and different.**
- **Explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence.**
- **Find examples of graphs in which many-to-one correspondence is used in print and electronic media, such as newspapers, magazines, and the Internet, and describe the correspondence used.**

Suggestions for Instruction

BLMs
4.SP.1.1
4.SP.1.2

- **Many-to-One Correspondence:** Provide students with several pictographs and bar graphs (examples are provided in BLMs 4.SP.1.1 and 4.SP.1.2) that show the same data using different scales and intervals.

  (Note: BLM 4.SP.1.1 shows one-half a symbol. A discussion may need to occur to ensure that students understand the meaning of one-half of a symbol.)

  Sample questions that can be asked include the following:

  - Why do they look different?
  - Do the pictographs/bar graphs show the same data?
  - How could you arrange the intervals differently on the bar graph?
  - Why do we use different ways to represent numbers in the legend/key in a pictograph?
  - What conclusions might a person make by looking at these graphs?

- **Tiger Math:** Read the book *Tiger Math: Learning to Graph from a Baby Tiger* by Ann Whitehead Nagda and Cindy Bickel. The story is about an orphaned baby tiger who is hand-raised at a zoo. Different kinds of graphs including pictographs and bar graphs track a variety of data. The format of the book keeps all the graphing on the left side of the book and the story of the tiger on right side. Have a discussion with the students about the implication of the scale used on page 8. The book allows for conversations about why graphs are used and are important.

- Have students find examples of pictographs and bar graphs in which many-to-one correspondence is used and have students describe and justify the scale chosen.

  Guided questions to the discussion can include the following:

  - What does each symbol represent?
  - How many does one symbol represent?
  - Why do you think this scale representation is appropriate? Would you use another representation and why?
  - How does the legend affect the appearance of a pictograph?
  - How do the intervals affect the appearance of a bar graph?

- **Create and label (with categories, title, and legend) a pictograph to display a set of data using many-to-one correspondence, and justify the choice of correspondence used.**
- **Answer a question using a graph in which data is displayed using many-to-one correspondence.**

Suggestions for Instruction

Note: When students are constructing pictographs and bars graphs, many-to-one correspondence should be the focus. If the data is less than 20, one-to-one correspondence is appropriate. For larger numbers, intervals of 2, 4, 5, 10, 25, 100, or 1000 should be used based on the data being graphed. As students construct their graphs, data collected should be of a larger data amount. Students should be able to discuss their data displays and be able to explain why they chose their scale. It is important for students to ensure that the intervals in their graphs are consistent. When creating pictographs, students make decisions about what symbol to use based on the data being used. Discussions are needed to help students create their own pictographs, and they should be prepared to justify their symbol choice.

- **Creating a Pictograph:** Have pairs of students place 40 two-colour counters in a bag. Have each group dump the counters out of the bag and record on a chart how many of one colour is seen. Have the students do five trials and record each result of how many times the one colour is shown. Have students graph the data in a pictograph using a symbol that represents more than one counter. Ensure that the pictograph is appropriately labelled. Have each group discuss how they decided on the legend. Ask students if they needed to use fractions of symbol in their pictographs. Ask students if they could have used a legend so that there would be no fraction.

- **Constructing a Pictograph:** Present the following information to students:

  A taste test at an ice cream store had these results:

  - 80 people liked chocolate
  - 60 people liked vanilla
  - 20 people liked butterscotch
  - 30 people like strawberry

  Have students construct pictographs based on the data. They must decide on which many-to-one correspondence to use. When the graphs have been created, have students share and discuss them. A discussion can follow on the

  - symbols chosen to represent the ice cream
  - legends used
  - advantages and disadvantages of their symbols and legends

  Repeat the activity as needed using other data involving large numbers. Ideas include the following:

  - size of community and habitat populations (connections to social studies and science)
  - measurements (connections to science)

Assessing Understanding

Develop criteria with the students about how to assess the construction of a pictograph. Possible criteria may include the following:

- appropriate title
- legend/key presented in order to show how much each symbol represents
- same symbol throughout graph
- symbols start at the same level and are equally spaced
- appropriate labels

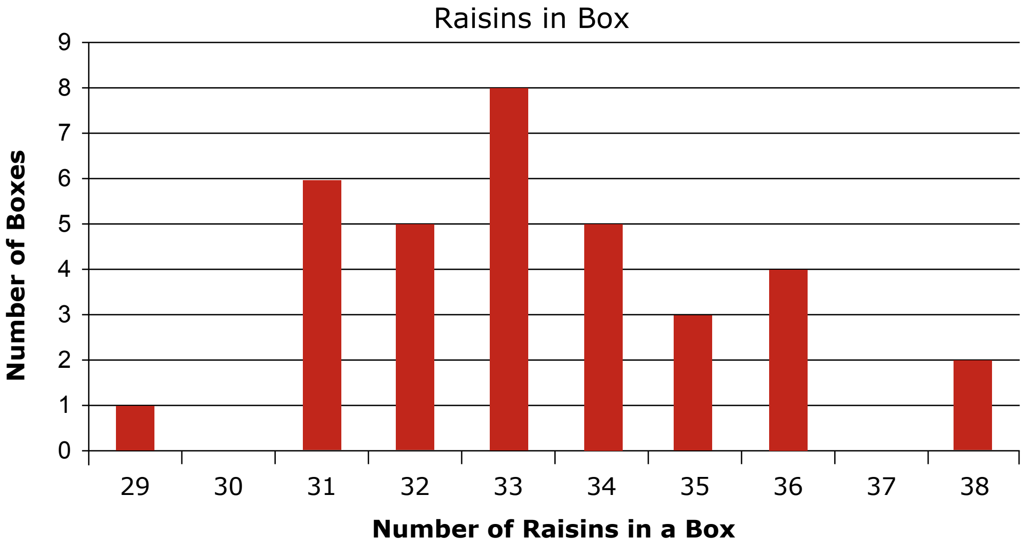
- **Create and label (with axes and title) a bar graph to display a set of data using many-to-one correspondence, and justify the choice of interval used.**
- **Answer a question using a graph in which data is displayed using many-to-one correspondence.**

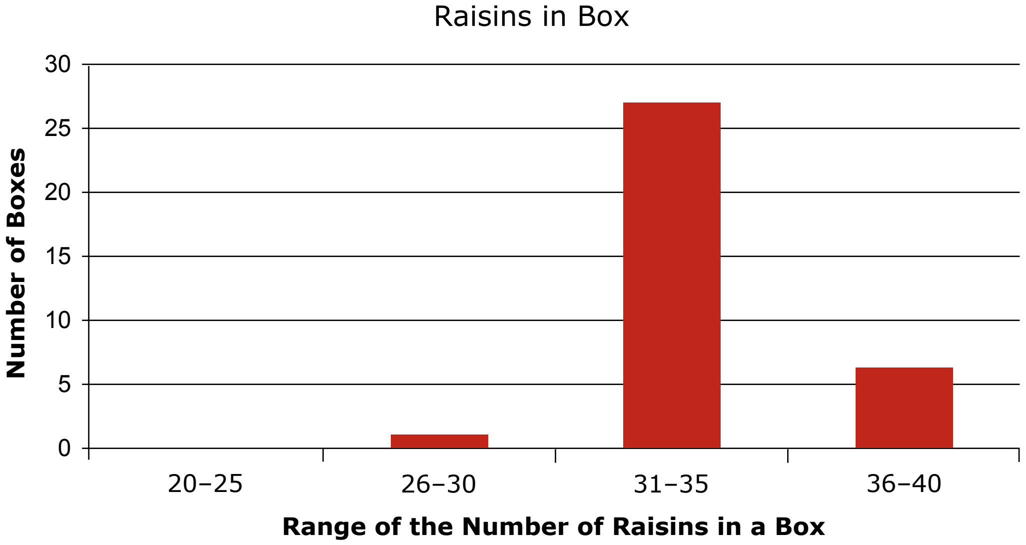
Suggestions for Instruction

- **Constructing a Bar Graph:** Have students conduct school-wide investigations to find out what additional types of equipment students would like to see in their playground. Have students come up with choices such as curved slide, basketball hoop, swinging bridge, rock wall, or tunnel.
  - Students survey each classroom by preparing blank tickets for the survey choice.
  - When the data has been collected, guide the students in organizing the data by asking "What would be an easy way to count the tickets?" Students may suggest grouping in 10s or 5s. Have the students justify their choices. Discuss the problem of units left over.
  - Have students in pairs make bar graphs using intervals of their choice. Students discuss their choice.
  - Have each group present their graphs to the rest of the class, discussing the following questions:
    - What three things do you know from looking at the data?
    - What type of equipment was most popular?
    - Why did you choose the interval for your graph?
    - Is a bar graph a good graph to use to represent this data to the rest of the school?
- **Graphing Raisins:** Provide each student with a small box of raisins. Have students estimate and justify the number of raisins in the box. Have students count and record the number of raisins in each box. Have the class pool the results and discuss how to display the information. Discuss the possibility of grouping the data (counts between 15–20, 21–26, etc.). Have the students create a bar graph with the data. Create a bar graph that shows the grouping of the data. Discuss what data can be derived from the graph. Discuss the advantages and disadvantages of each graph. Ask students the impact of using intervals of different sizes to represent the data. Have students answer the following questions about their bar graph:
  - Which count occurred most frequently?
  - Which count occurred the fewest number of times?

- Which count is the lowest?
- Which count is the highest?
- How can we determine which is the best graph form to use to display our data?
- How do you think another brand of raisins would compare?
- What other questions come to mind when looking at the data on the graph?
- Who do you think would be interested in this data?

Examples:





- Have students find pictographs and bar graphs from magazines and newspapers or use previously collected data from students. In groups, discuss and analyze the graphs. Have students identify general statements that can be verified by the data presented in the graphs. Students' analyses of the graphs should focus on drawing conclusions and encourage questions about the data.

Assessing Understanding

Develop criteria with the students about how to assess the construction of a bar graph. Possible criteria may include the following:

- both axes labelled
- categories labelled
- numbers labelled on the lines
- spaces between bars
- bars filled in correctly

PUTTING THE PIECES TOGETHER

Food Labels

Organization:

Groups of 2 or 3

Materials:

A collection of food labels from different foods (students can choose a food category such as cereal, crackers, beverages, etc.).

Context:

BLMs
4.SP.2.1

The class wants to raise money for a field trip. The class decided to sell snacks at recess time. The principal told the class the snacks must be healthy and told the class to prepare a presentation with graphs showing data about the food they want to sell. The presentation must include the following:

- a pictograph or bar graph that represents many-to-one correspondence
- an analysis of the data from the food labels
- a statement about why the snacks will be good choice to sell to the students at the school

Show and discuss how to read a food label, using labels such as the one shown in BLM 4.SP.2.1. Information about food labels can be found at https://www.canada.ca/en/health-canada/services/understanding-food-labels/nutrition-facts-tables.html. This discussion can match outcomes from the Grade 4 Physical Education/Health Education curriculum.

Nutrition Facts
Valeur nutritive

Per 37 crackers (20 g) / pour 37 craquelins (20 g)

Amount Teneur	% Daily Value % valeur quotidienne
Calories / Calories 90	
Fat / Lipides 3.5 g	5 %
Saturated / saturés 1 g + Trans / trans 0 g	5 %
Cholesterol / Cholestérol 5 mg	
Sodium / Sodium 170 mg	7 %
Carbohydrate / Glucides 13 g	4 %
Fibre / Fibres 1 g	4 %
Sugars / Sucres 0 g	
Protein / Protéines 2 g	
Vitamin A / Vitamine A	0 %
Vitamin C / Vitamine C	0 %
Calcium / Calcium	2 %
Iron / Fer	6 %

INGREDIENTS: ENRICHED WHEAT FLOUR, CHEDDAR CHEESE (MILK, BACTERIAL CULTURE, SALT, MICROBIAL ENZYME, ANNATTO), VEGETABLE OIL (CANOLA AND/OR SUNFLOWER), SALT, YEAST, SUGAR, AUTOLYZED YEAST, SEASONING (CONTAINS CELERY, ONION POWDER), BAKING POWDER, AMMONIUM BICARBONATE.

INGRÉDIENTS : FARINE DE BLÉ ENRICHE, CHEDDAR (LAIT, CULTURE BACTÉRIENNE, SEL, ENZYME MICROBIEN, ROCOU), HUILE VÉGÉTALE (CANOLA ET/OU TOURNESOL), SEL, LEVURE, SUCRE, LEVURE AUTOLYSÉE, ASSAISONNEMENT (CONTIENT DU CÉLÉRI, POUDRE D'OIGNON), POUDRE À PÂTE, BICARBONATE D'AMMONIUM.

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Brainstorm with the class different categories of food they could sell (crackers, beverages, granola bars, etc.). Give each group a category and have them research the nutritional facts of food within the category. Each group must decide on 4 specific food items that are healthy to sell. Groups must present their information to the whole class. The presentation must include the following:

- a pictograph or bar graph that represents many-to-one correspondence
- an analysis of the data from the food labels
- a statement about why the snacks will be a good choice to sell to the students at the school

Get the students to vote on what food items would be good to sell at school. Have students make a graph of the votes. Have students answer the following questions about the vote by using the information gathered from each group.

- What was the favourite food of the class?
- Do you think it would sell to other students in the school?
- What are the top three favourite food choices in the class?
- Do you think the top three food choices are healthy? Why?
- Do you think the food will sell?

A further discussion can be held with the class that can lead to drawing conclusions about the data collected. Questions include the following:

- What impact on your eating habits may result from hearing the data presented?
- Will the data presented encourage you to look at food labels?
- Do words and images on packages mislead people about what is actually in the food?
- How can nutritional information on labels help you find out how nutritional the food is?

Look for the following:

☐ Graphs were completed correctly.
☐ Data were represented correctly.
☐ Data were summarized correctly.
☐ Students were able to compare quantities in the graph.
☐ Conclusions were valid.

Notes

GRADE 4 MATHEMATICS

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