



Introduction



## INTRODUCTION

*Mental Math: Grade 8 Mathematics* is a complement to the Grade 8 Mathematics curriculum. This document is intended for use in helping students to develop strategies for performing mental calculations and estimations more flexibly, efficiently, and accurately.

### Why Mental Mathematics and Estimation?

Mental math and estimation are essential components of the Manitoba Kindergarten to Grade 8 Mathematics curriculum and are one of the seven processes to be thoroughly integrated into the teaching and learning of Grade 8 Mathematics learning outcomes. Manitoba's teachers are asked to report student success in mental math and estimation on provincial student report cards, as well as on provincial assessments. This guide provides possible methods for both teaching and assessing these skills.

The Grade 8 Mathematics curriculum states:

“Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids. It improves computational fluency by developing efficiency, accuracy, and flexibility.” (Manitoba Education and Advanced Learning, 2013, 12)

Mental calculation requires the use of knowledge of numbers and mathematical operations. Mental calculation is at the root of the estimation process, and it allows individuals to determine whether results obtained with a calculator or a pen and paper are reasonable.

“Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgements and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to know which strategy to use and how to use it.” (Manitoba Education and Advanced Learning, 2013, 12)

Students who have experience with mental math and estimation develop the ability to work flexibly with numbers. Practising mental calculation and estimation is important in the development of number sense and it is foundational in gaining a clear understanding of place value, mathematical operations, and basic numeracy.

Mental calculation and estimation are very practical skills that can be used to do quick calculations at times when a pencil or a calculator is unavailable. Using mental calculation strategies can also eliminate some steps in written calculations and help simplify processes as students progress through the grades. Teachers should provide opportunities for their students to use mental math and estimation on a regular basis and encourage their students to find examples of the usefulness of mental calculation in their lives, such as when shopping, travelling, playing sports, or doing other everyday activities.

## Strategies

Through a thorough implementation of the Manitoba Kindergarten to Grade 8 curriculum, Grade 8 students will have learned many of the mental math and estimation strategies outlined in this document in previous grades. Students may not have used these methods for several years, however, or they may not have seen how the strategies they learned connect to their current studies.

Students may also not realize that the strategies that are most effective for mental calculation are often not the same strategies that are most effective for written calculation. These are two very different skills that require two very different processes. In order to encourage flexibility, efficiency, and accuracy, it is important that this distinction is made.

## Document Features

This document includes three main sections: this introduction, a section describing strategies, and a series of mental mathematics questions organized by *learning targets*.

*Learning targets* are groups of related outcomes derived from *Glance Across the Grades – Manitoba 2015*. In the same way, this document consists of thirteen (13) learning targets related to the Grade 8 Mathematics learning outcomes.

Learning Target (as used in Glance Across the Grades)	Specific Learning Outcomes	Number of Pages
Operations with Whole Numbers (Multiplication/Division)	8.N.1, 8.N.7	12
Representation of Rational Numbers	8.N.4	8
Operations with Rational Numbers	8.N.2, 8.N.3, 8.N.5, 8.N.6, 8.N.8	11
Patterning and Algebraic Thinking	8.PR.1	4
Algebraic Representations with Equations	8.PR.2	5
Length	8.SS.1	3
Area	8.SS.3	3
Volume (Capacity)	8.SS.4	3
Identifying, Sorting, Comparing, and Constructing	8.SS.2, 8.SS.5	3
Position and Motion	8.SS.6	2
Collection, Organization, and Analysis of Data	8.SP.1	2
Probability	8.SP.2	2

The learning target and strategies of focus are identified on each page. The questions on each page are divided into three different categories:

- Six **prior learning questions** from earlier grade levels (These are meant for reflection and practice, as well as for formative assessment.)
- Four **Grade 8 questions** related to the learning target of study
- Two **other questions** for teachers to insert their own questions

The answers to the questions are provided in the column on the extreme right-hand side of each page. The flexible and efficient use of strategies should always remain the focus of these mental math activities. It is critical that students reflect on their processes more than on the correctness of their answers in order for them to further develop their mental math abilities.

A blank template is also provided in a section titled Reproducible Sheets. Teachers may use it to prepare additional question sheets.

A file in Word format is available in the Mathematics Group on the Maple (Manitoba Professional Learning Environment) site at [www.mapleforem.ca](http://www.mapleforem.ca). It is provided to enable teachers to add or modify questions to suit the needs of their students.

## Methodology

Teachers should ensure that mental calculation exercises are short in duration because they require significant sustained concentration. Although these calculations should be done within a certain period of time, it is important that teachers avoid emphasizing speed. Although speed may be a factor in basic recall, it is not the primary goal of developing the mental computational methods targeted in this document. Ensuring that mental math and estimation occur regularly in a classroom will encourage students to see these skills as important in their daily lives and to build their abilities. Possible methods to do this in a class include the following:

- Incorporating Number Talks: A five- to ten-minute class conversation crafted around a mental math problem and the flexible methods students use to arrive at an answer (Parrish)
- Projecting portions of this program at the front of the class, visually or orally, and discussing the strategies used afterwards
- Making use of a template such as the one provided on page 13 to encourage communication among students about strategies
- Student journaling
- Teacher facilitation of discussion and small group work
- Presentation of portions of this program as a game with competing teams
- Student creation of mental math questions
- Questioning, reflecting, and discussing methods used
- Discussing errors in the process that led to an incorrect answer

The primary goal of this program is to improve student use of mental math strategies. Through communication about the strategies and the methods being used in class, students can self-assess their own strategy choice and become more comfortable with trying new processes. Students should be encouraged to continually look for, and practice, the processes they discover to be the most efficient and accurate.

## Assessment

Exercises from this document can be used as assessments *for, as, and of* learning.

- *Assessment for learning (Prior Learning Questions section)*: The first six questions on each page should serve as formative assessment. Results from this can be used to pinpoint student learning gaps from previous years and direct future instruction. These prior learning sections are exclusively for use as formative assessment. They cannot factor into summative assessments because their material deals exclusively with outcomes from previous grades.
- *Assessment as learning*: This takes place through quality student self-assessment and reflection. Discussions and reflections help to solidify student understanding. Without this effort, it is unlikely that students will grow in their mental math and estimation abilities. Quality feedback is critical to improved student success.
- *Assessment of learning*: This can take place through the grade-specific sections of the question pages, as well as through any related discussions and activities that students take part in related to these grade-specific sections. Teachers can use these results both formatively and summatively.

Mental math and estimation skills can be assessed in a variety of ways. Although written products give teachers some insight into student thinking, both observations and conversations can provide additional information on student ability in mental math and estimation, and should be used as an additional source of assessment where possible. Rubric use, with a focus on communication skills related to mental math and estimation, can be a method of formatively assessing student conversations and making classroom observations a part of summative data.

Student discussion is an effective way for students to present and self-assess their own thinking. Students are required to be clear and concise when explaining their reasoning to others, and in turn they are given the opportunity to learn new approaches from other students and the teacher. These exchanges about the strategies and results will also allow the teacher to identify the difficulties encountered by some. Subsequently, the teacher can help students discover new, relevant, useful, and important strategies.

# Report Cards

The following is the draft version of Manitoba Education and Training’s *Manitoba Report Card Grade Scale Mathematics Achievement Profiles* for mental mathematics and estimation

This profile should be used throughout assessment processes, as well as when assigning a mark to the mental math section of report cards. Students are expected to connect and apply mental math and estimation strategies with skills and knowledge, as well as to communicate mental mathematics and estimation strategies concretely, orally, and written in the form of pictures/diagrams, words, symbols/numbers, graphs, and/or charts.

Manitoba Report Card Grade Scale Mathematics Achievement Profiles—Grades 1 to 8 (DRAFT April 2017)					
Subject Category: Mental Mathematics and Estimation					
Category Indicator	Extent to which the student is meeting grade-level <sup>1</sup> learning outcomes across the provincial report card grading scale				
	Not demonstrated (ND)	Limited (1)	Basic (2)	Good (3)	Very good to excellent (4)
Connects and applies mental math and estimation strategies with skills and knowledge	Does not yet demonstrate the required understanding and application of concepts and skills.	Requires considerable, ongoing teacher support and/or direction to <ul style="list-style-type: none"> <li>— connect knowledge of mental math and/or estimation to the understanding of number properties and relationships, and</li> <li>— use mental math and/or estimation strategies and/or must work through the strategies on paper or with concrete objects.</li> </ul> May have computational errors.	Requires occasional teacher or peer support to connect knowledge of mental math and/or estimation to the understanding of number properties and relationships. Chooses and uses modelled and familiar mental math and/or estimation strategies. May have computational errors.	Accurately connects knowledge of mental math and/or estimation to the understanding of number properties and relationships. Uses mental math and/or estimation strategies accurately.	Flexibly and efficiently connects knowledge of mental math and estimation to the understanding of number properties and relationships. Efficiently and flexibly chooses and uses mental math and estimation strategies.
Communicates mental mathematics and estimation strategies <ul style="list-style-type: none"> <li>■ Concretely</li> <li>■ Orally</li> <li>■ Written                             <ul style="list-style-type: none"> <li>o Pictorial/Diagrams</li> <li>o Words</li> <li>o Symbolic/Numbers</li> <li>o Graphs/Charts</li> </ul> </li> </ul>	Has persistent learning difficulties which hinder or prevent acquisition of knowledge and skills.	Uses very little mathematical vocabulary. Requires considerable, ongoing teacher support to describe mental math and estimation strategies used for computation.	Requires occasional teacher or peer support to use mathematical vocabulary and describe the mental math and estimation strategies used for computation.	May require occasional prompting for clarification. Uses mathematical vocabulary to explain the mental math and estimation strategies used for computation.	Uses a broad range of mathematical vocabulary clearly, completely, and precisely to explain the mental math and estimation strategies used for computation.

<sup>1</sup> As developmentally appropriate for the time of year towards attaining end-of-grade academic outcomes or academic outcomes described in an individual education plan. References in the table to “assistance,” etc. do not refer to adaptations defined as “a change in the teaching process, materials, assignments or pupil products to assist a pupil to achieve the expected learning outcomes.” ([www.ed.gov.mb.ca/k12/specedu/programming/adaptation.html](http://www.ed.gov.mb.ca/k12/specedu/programming/adaptation.html))

## Grade 8 Mental Math Outcomes

In the Manitoba Grade 8 Mathematics curriculum, the learning outcomes listed below are the only ones that include a specific [ME] designation. This designation means that when addressing the following outcomes, teachers **must** integrate mental math and estimation as a process.

- 8.N.2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, **ME**, R, T]
- 8.N.6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. [C, CN, **ME**, PS]
- 8.N.8. Solve problems involving positive rational numbers. [C, CN, **ME**, PS, R, T, V]
- 8.PR.1. Graph and analyze two-variable linear relations. [C, **ME**, PS, R, T, V]

Teachers **may** integrate and assess mental math and estimation into all learning outcomes, however, and this document is meant to provide support in doing so.

“Mental mathematics is a combination of strategies that enhances flexible thinking and number sense. Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Estimation is often used to make mathematical judgments and to develop useful, efficient strategies for dealing with situations in daily life. Strategies in mental mathematics and estimation enable students to calculate mentally without the use of external aids. In the process, they improve their computational fluency—developing efficiency, accuracy, and flexibility.”

—Manitoba Education and Advanced Learning (2015)

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## Notes