Developing Conceptual Understanding of Number

Set I: Pre-Algebra Patterns

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Pre-Algebra Patterns 1

Vocabulary

Notes

- For 3b), “rit” has not been included because it is not an English word. “Rat” has been included because there is no indication that an “a” cannot be replaced with another “a”.

Answers

1. a) 7
   b) 256
   c) each number is double the number in the previous step.

2. a) 8
   b) 10

3. a) bat
   b) ret, rot, rut or rat

Pre-Algebra Patterns 1

1. The following chart reveals a number pattern.

Step	1	2	3	4	5	6	7	8
Number	1	2	4	8	16	32	64	128

a) What is the first step where the number is greater than 50?

b) If the pattern continues, what is the number at step 9?

c) Describe the pattern in the "number" row of the chart.

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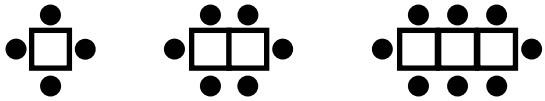
2. The following diagrams show a pattern in the number of seats arranged around an increasing number of small tables.



What is the value of the missing number in each of the following charts?

a)

Number of Small Tables	1	2	3
Number of Seats	4	6	

b)

Number of Small Tables	Number of Seats
1	4
2	6
3	8
4	

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3. Consider the following chart:

top	lip	men	bet	B
tap	lap	man	A	rat

a) Give a word to replace A in the chart.

b) Give two possible words to replace B in the chart.

Pre-Algebra Patterns 2

Vocabulary

Notes

- It is important that students always indicate what the variable represents.

Answers

1. Possible Answers:
   - Place the object in term 3 on a row of 7 blocks
   - Add 7 blocks underneath the object in term 3
   - …

2. a) 3:30
   b) BONG BING BING BING

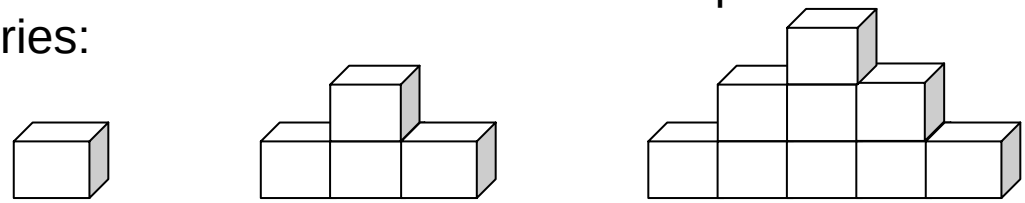
3. a) Possible Answers:
      - the number of cats in the school yard after 3 ran away.
      - …
   b) Possible Answers:
      - the number of cats caught by the pound if half the cats escaped.
      - …

4. Let *a* be Carole's age. Then her mother's age is $2a + 10$.

5. a) 9
   b) Let the term number be *t*. Then, the number of squares is $2t - 1$.

Pre-Algebra Patterns 2

1. Describe how to build the fourth shape in the following series:



1 2 3

2. A clock goes BONG for every hour and BING for every 15 minutes.
   For example BONG BONG BING represents 2:15.
   a) What time is it if the clock goes BONG BONG BONG BING BING?
   b) What would you hear at 1:45?

3. If c represents the number of cats in the school yard, what situation could each of the following represent?
   a) $c - 3$
   b) $\frac{c}{2}$

4. When you double Carole's age and add 10, you get her mother's age. Write a mathematical expression that shows the mother's age. Tell what your variable represents.

5. The following chart reveals the number of squares in a pattern.

Term Number	1	2	3	4
Number of Squares	1	3	5	7

   a) How many squares would exist for term number 5?
   b) Write a mathematical expression that would allow you to determine the number of squares for any term.

Pre-Algebra Patterns 3

Vocabulary

Notes

- For 1b), it is acceptable if students come up with a name that only satisfies the two patterns they identified in part a).

Answers

1. a) Possible Answers:
   - pattern is alphabetical in order
   - the number of letters in each name increases by 1 each time
   - the names alternate girl then boy then girl, etc.

   b) Eleanor, Emmalou, Eveline or Ellymae are suitable answers because they satisfy all three conditions in part a. (ie. girl's name starting with "E" and having 7 letters)

2. a)

Term Number	1	2	3	4
Number of Small Squares	3	5	7	9

   b) You could find the number of small squares in term 7 by:
   - Drawing all the figures and counting squares for figure 7
   - Extending the chart in part a).
   - Creating an algebraic expression as in part c) and substituting 7 for the variable
   - …

   c) Total small squares is $2t + 1$ where t = term number.

Pre-Algebra Patterns 3

1. There are several possible patterns in the following list of names:

   Ann, Brad, Carol, Daniel, _____ , _____

   a) Describe two of the patterns you found.
   b) What is a possible fifth term in the list of names? Why?

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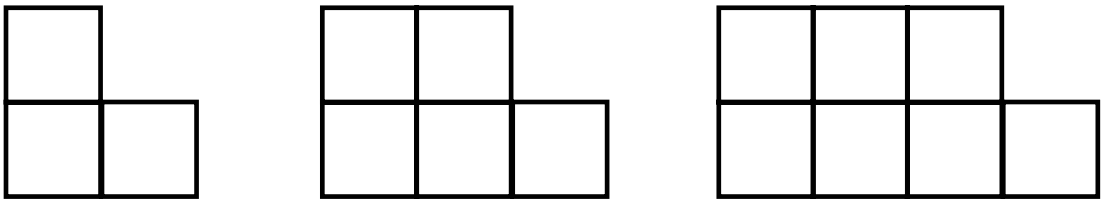
2. 

   a) Construct a chart showing the term number and the increasing number of small squares in the pattern above.
   b) How could you find the number of small squares in the seventh term?
   c) For the pattern shown above, write an algebraic expression showing the total number of small squares where *t* = the term number.

Pre-Algebra Patterns 4

Vocabulary

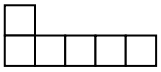
- rule
- sequence
- regular die

Notes

- For #3, have students discuss the patterns they found to justify their picture for term 4.
  Some possible patterns:
  - Filled in Boxes:
    9, 21, 33, …
  - Size of figures: 3 x 3; 5 x 5; 7 x 7; so the next one is 9 x 9. All figures have the perimeter and the diagonals filled in.
  - Filled in Boxes: $3^2 - 0^2$; $5^2 - 2^2$; $7^2 - 4^2$; so the next one is $9^2 - 6^2$.
- For #4, have dice available for students to confirm that the sum of the opposite sides is always 7.

Answers

1. a) 11 triangles
   b) The number of triangles is one more than the term number.

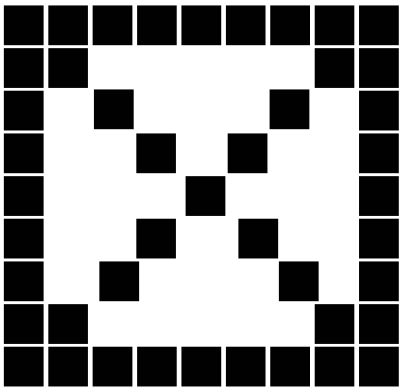
2. a) 

   b)

Term Number	1	2	3	4	5
Number of Small Squares	3	4	5	6	7

3. Possible Answer:

   33 + 12 = 45 squares should be shaded. All squares on the diagonal of the large square are shaded.



4. a) 7
   b) Sum of the hidden numbers is 6 + 7 + 7 = 20 assuming you can see the numbers on 4 sides of each die. [6 is the number on the bottom of the top die]

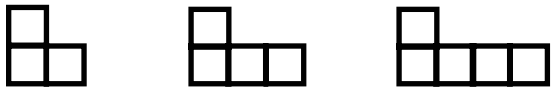
Pre-Algebra Patterns 4

1. The number of triangles in a pattern is shown in the following chart:

Term Number	1	2	3	4
Number of Triangles	2	3	4	5

   a) Find the number of triangles for term number 10.
   b) Give a rule describing the number of triangles for any term.

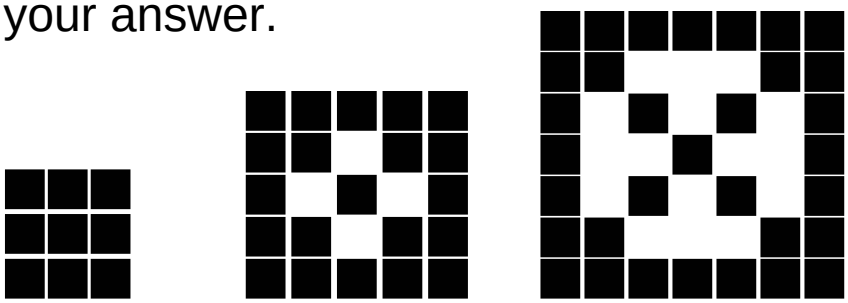
2. The number of small squares is increasing in the following pattern:



Term 1 Term 2 Term 3

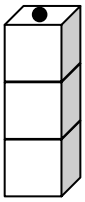
   a) Draw a picture showing Term 4 in the pattern.
   b) Make a table showing the number of small squares for each of the first 5 terms.

3. Build or draw the fourth term in the following sequence. Justify your answer.



#1 #2 #3

4. Three dice are stacked on top of each other. The number on the top of the highest die is 1.
   a) What is the sum of the numbers on opposite sides of a regular die?
   b) What is the sum of the hidden numbers for your stack of 3 dice? Explain.



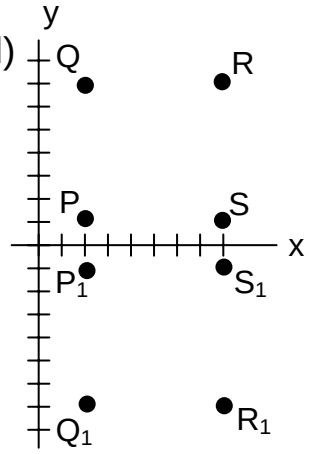
Pre-Algebra Patterns 5

Vocabulary

Notes

- For #3, students should discuss the relationship of the term number to "*x*" and the relationship of the number of triangles to "*y*".
- For #4, this is a review for Set H.
- Points on the *x*-axis have a *y*-coordinate of 0. Similarly, points on the *y*-axis have an *x*-coordinate of 0.

4. d)



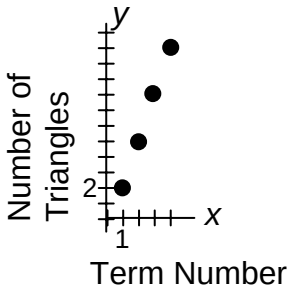
Answers

1. a) $n + 3$
   b) $t - 2$
   c) $2n - 1$ where n = your age

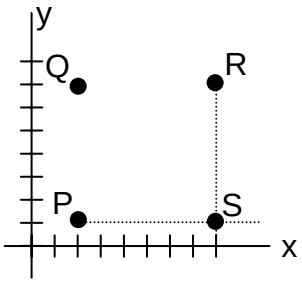
2.

Term Number	1	2	3	4
Number of People	4	7	10	13

3. a) 11 triangles
   b)



4. a) (0, -3)
   b) (1, 4) and (1, -4)
   c) S = (8, 1)



   d) P_1=(2, -1) ; Q_1 = (2, -7) ; R_1 = (8, -7) ; S_1 = (8, -1) [See sketch on the left]

Pre-Algebra Patterns 5

1. Write a mathematical expression for each of the following:
   a) a number, *n*, increased by 3.
   b) The number of birds, *t*, in a nest after 2 flew away.
   c) Your brother's age if he is twice your age decreased by 1.

2. Complete the following chart using the rule "the number of people is one more than triple the term number".

Term Number	1	2	3	4
Number of People				

3. The following chart reveals the number of triangles in an increasing pattern.

Term Number	1	2	3	4
Number of Triangles	2	5	8	

   a) How many triangles occur for term number 4?
   b) If *x* is the term number and *y* is the number of triangles, draw a graph showing the pattern in the table.

4. a) Find the coordinates of all points on the *y*-axis and 3 units below the *x*-axis.
   b) Find the coordinates of all points 4 units from the *x*-axis and 1 unit right of the *y*-axis.
   c) PQRS is a square with P = $(2,1)$, Q = $(2,7)$, and R = $(8,7)$. Find S.
   d) If PQRS is reflected in the *x*-axis to get square $P_1Q_1R_1S_1$, find the coordinates of P_1,Q_1,R_1 and S_1.

Pre-Algebra Patterns 6

Vocabulary

- generate

Notes

- For #3, this is exactly the same question as #3 on 8I-4.
- For #3, have students discuss the patterns they found to justify their picture for term 4. Some possible patterns:
  - Filled in Boxes: 9, 21, 33, …
  - Size of figures: 3 x 3; 5 x 5; 7 x 7; so the next one is 9 x 9. All figures have the perimeter and the diagonals filled in.
  - Filled in Boxes: $3^2 - 0^2$; $5^2 - 2^2$; $7^2 - 4^2$; so the next one is $9^2 - 6^2$.
- For #4, have dice available for students to confirm that the sum of the opposite sides is always 7.

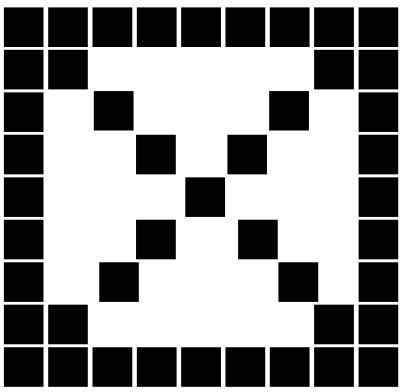
Answers

1. a) 21 triangles
   b) The number of triangles is 1 more than 2 times the term number.

2. a) 18 small squares
   b) $n^2 + 2$ where n is the term number.

3. Possible Answer:

33 + 12 = 45 squares should be shaded. All squares on the diagonal of the large square are shaded.



4. a) 34 (6 + 7 + 7 + 7 + 7)
   b) 71 (1 + 14 + 14 + 14 + 14 + 14)

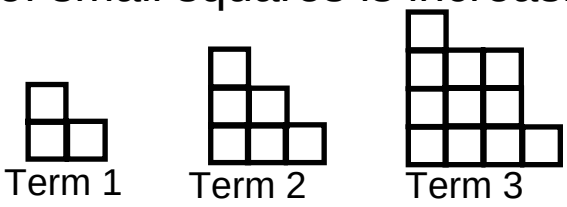
Pre-Algebra Patterns 6

1. The number of triangles in a pattern is shown in the following chart:

Term Number	1	2	3	4
Number of Triangles	3	5	7	9

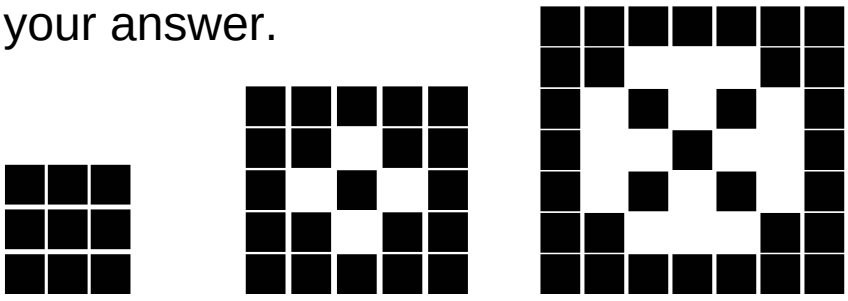
   c) Find the number of triangles for term number 10.
   d) Give a rule describing the number of triangles for any term.

2. The number of small squares is increasing in the following pattern:



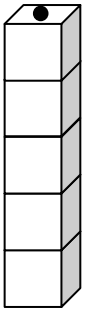
   c) How many small squares are needed to generate Term 4?
   d) Give an expression describing the number of small squares for any term.

3. Build or draw the fourth term in the following sequence. Justify your answer.



4. Five dice are stacked on top of each other. The number on the very top is 1.

   e) What is the sum of the hidden numbers?
   f) What is the sum of all the numbers showing?



Pre-Algebra Patterns 7

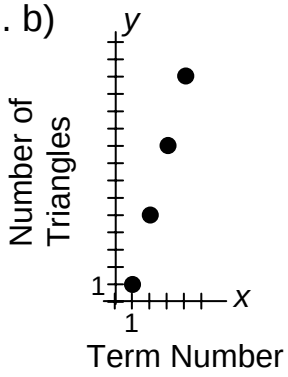
Vocabulary

- quadrant

Notes

- For #2, the algebraic expression is $4(n + 1)$ where n is the term number.

3. b)



Answers

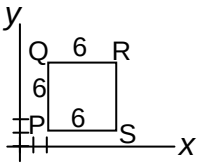
1. a) $t - 5$
   b) $c + 12$ where c is the number of candies before you add the dozen.
   c) $\frac{n}{2} + 10$ or $\frac{1}{2}n + 10$ where n is your age

2.

Term Number	1	2	3	…	8
Number of Small Squares	8	12	16		36

3. a) $y = 4x - 3$
   b) See sketch on the left.

4. a) (0,3) and (0,-3)
   b) (1,4); (1, -4); (-1,4); (-1, -4)
   c)



R = (8,7) and S = (8,1)

   d) $P_2 = (-2, -1)$; $Q_2 = (-2, -7)$; $R_2 = (-8, -7)$ and $S_2 = (-8,-1)$

Pre-Algebra Patterns 7

1. Write a mathematical expression for each of the following:
   a) a number, *t*, decreased by 5.
   b) the number of candies in a bowl after you add a dozen.
   c) your brother's age if he is ten years older than half your age.

2. Complete the following chart using the rule "the number of small squares is the product of 4 and the sum of the term number and 1".

Term Number	1	2	3	…	8
Number of Small Squares					

3. The following chart reveals the number of triangles in an increasing pattern.

Term Number	1	2	3	4
Number of Triangles	1	5	9	13

   d) If *x* is the term number, write a mathematical expression for the number of triangles.
   e) If *x* is the term number and *y* is the number of triangles, draw a graph showing the pattern in the chart.

4. a) Find the coordinates for all points on the *y*-axis and 3 units from the *x*-axis.
   b) Find the coordinates for all points 4 units from the *x*-axis and 1 unit from the *y*-axis.
   f) PQRS is a square with P = $(2,1)$ and Q = $(2,7)$. Find R and S if they are in the same quadrant as P and Q.
   d) If PQRS is reflected in the *x*-axis and the new square is then reflected in the y-axis to get square $P_2Q_2R_2S_2$, find the coordinates of P_2, Q_2, R_2 and S_2.