

Developing Conceptual Understanding of Number

Applications

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Applications 1

Vocabulary

Notes

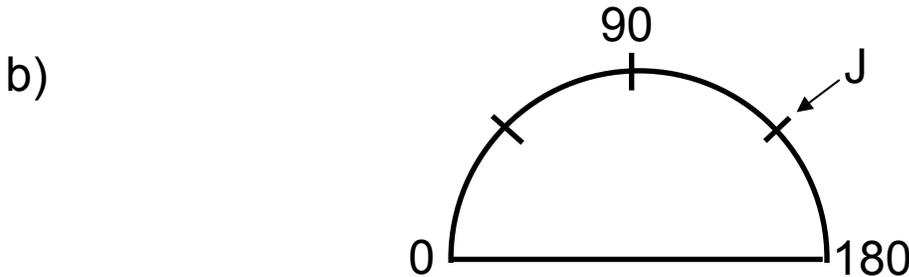
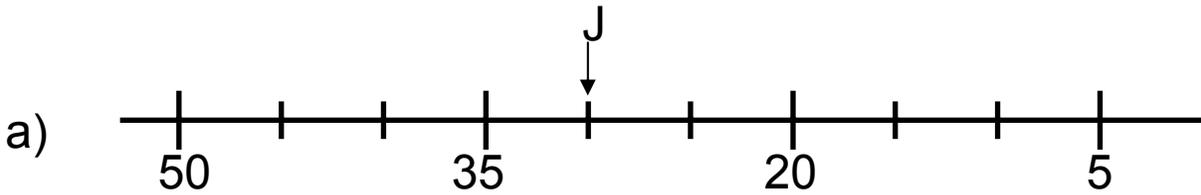
- Application sets focus on checking and reinforcing understanding. Activities in these sets are not intended to repeat previous ones, but rather to encourage students to demonstrate their understanding in a slightly different context.

Answers

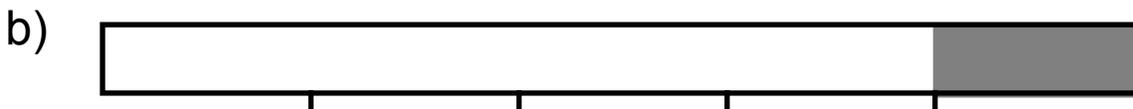
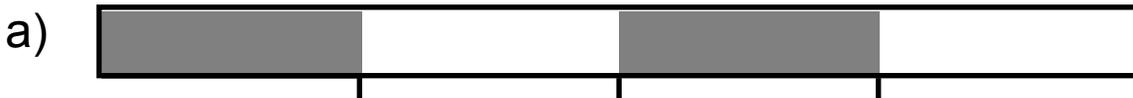
- a) 30
 - b) 135
- a) $\frac{2}{4}$ or $\frac{1}{2}$, 0.50, 50%
 - b) $\frac{1}{5}$, 0.2, 20%
- 06999

Applications 1

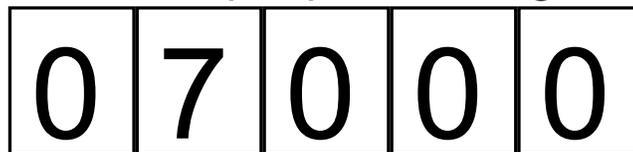
1. For each number line, state a possible value for J.



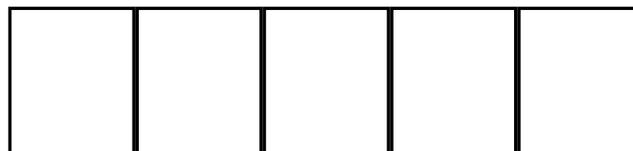
2. What part of each rectangle is shaded? Give your answer as a fraction, a decimal and a percent.



3. The meter below counts people entering a baseball stadium.



Show the meter just before the last person had entered.



Applications 2

Vocabulary

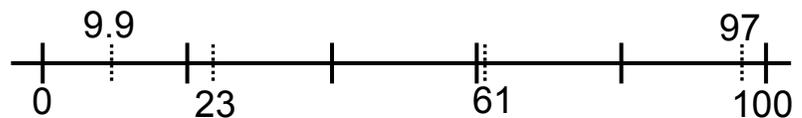
Notes

- For #5, students should not go to the smallest interval because it is not necessary to know that the smallest interval is 0.1. They might be better to think of this as a ruler and consider the larger intervals first.

Answers

1. 400

2.



3. 19

4. Possible Answers:

- More than 50% is shaded.
- Less than half is unshaded.
- Approximately 60% is shaded.
- Approximately $\frac{6}{10}$ is shaded.
- Approximately 0.4 is unshaded.
- Less than $\frac{2}{3}$ is shaded.
- ...

5. 37.5

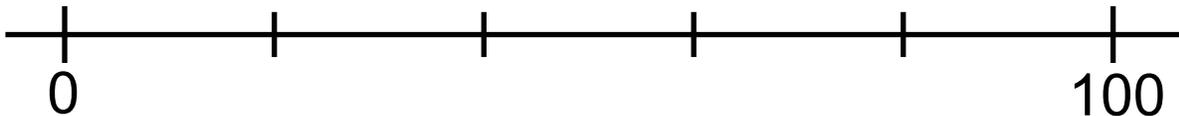
Applications 2

1. The numbers shown below are part of a Gattegno chart. What number would be at C?

A	3	4
20	B	40
200	300	C
2000	3000	D

2. Place the following numbers on the number line below.

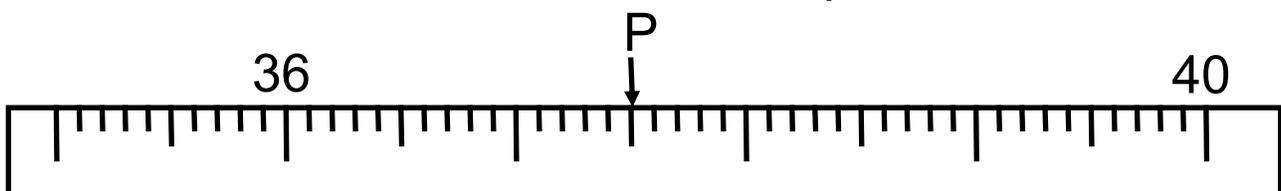
23, 9.9, 61, 97



3. The number 12 is halfway between 5 and what number?
4. Describe the relationship shown in this picture in at least four different ways.



5. What number does the letter P represent?



Applications 3

Vocabulary

Notes

- Application sets focus on checking and reinforcing understanding. Activities in these sets are not intended to repeat previous ones, but rather to encourage students to demonstrate their understanding in a slightly different context.
- For 3a), note that tax is not included since this is a garage sale, but if tax were added, the answer would not change.

Answers

1. 3, 9, 15, 21, 27
2. $7214 \div 70$ is larger than 100 since it is approximately 103. I know that $391.3 - 296.5$ is less than 100 since $391 - 291$ is 100 and I am subtracting more than 291. Since 191×0.5 is taking half of 191, and half of 200 is 100, this is also less than 100.
3. a) Greater than \$1.00.
b) Possible Answers:
 - 39¢ rounds up to 40¢ ; 52¢ rounds down to 50¢ ; 17¢ rounds down to 15¢ . So, $40 + 50 + 15 = 105$. However, I rounded up 1¢ , and rounded down by 2¢ twice, making a total difference of 3¢ down. Therefore the total is $105 + 3$ or 108¢ or \$1.08.
 - Round 39¢ up to 40¢ and add 52¢ to get 92¢ . To go from 39¢ to 40¢ , I used 1¢ from the 17¢ , leaving 16¢ to be added to 92¢ . It takes 8¢ of the 16¢ to get to \$1.00, so the answer is \$1.08.
 - Round $39 \rightarrow 40$, $52 \rightarrow 50$, $17 \rightarrow 20$.
 $40 + 50 + 20 = 110$; $110 - 1 + 2 - 3 = 108$.
 - ...
4. 125 km

Applications 3

1. List the five smallest whole numbers that are odd and multiples of 3.
2. Without finding the exact answer, which answer is larger than 100? Explain how you know.

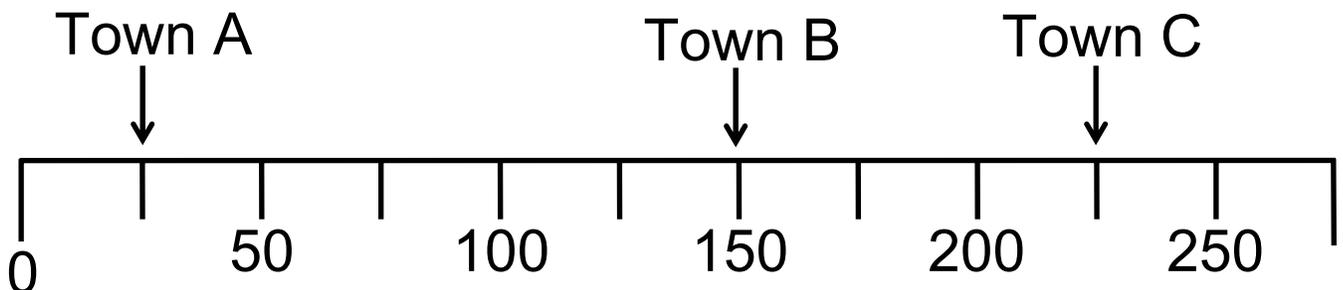
$$7214 \div 70$$

$$391.3 - 296.5$$

$$191 \times 0.5$$

3. You go to a garage sale. You buy items for 39¢, 52¢, and 17¢.
 - a) Estimate whether the total cost for the 3 items is less than, greater than, or equal to \$1.00.
 - b) By rounding, show how you could find the actual cost of the three items.

4. According to the scale below, how many kilometres is it from Town A to Town B?



Applications 4

Vocabulary

Notes

Answers

1. 614, 641, 623, 632, 650, 605
2. 48
3. a) 1600 (twice as much)
b) 832 (Since $25 \times 32 = 800$, 26×32 is one more multiple of 32, so add 32 to 800 to get 832.)
4. 15°C
5. Possible Answers:
 - The intervals are 5°C each. Therefore, the difference is 3 intervals or 3×5 or 15°C .
 - The high temperature is 25°C and the low temperature is 10°C . Therefore, the difference is 15°C .
 - ...

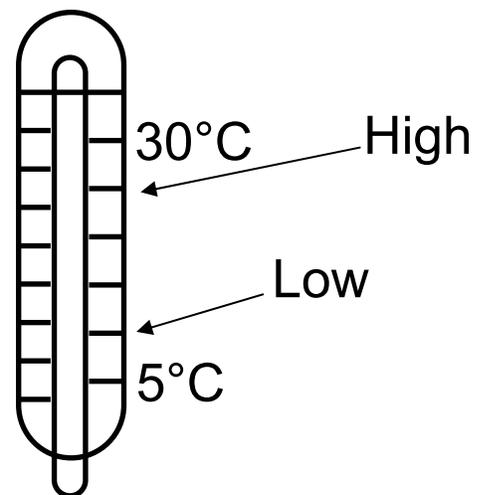
Applications 4

- List all possible 3-digit numbers that have a 6 in the hundreds place and whose digits have a sum of 11.
- Find the smallest whole number that can be used to make the following statement true:

$$53 + \square > 100$$

- If you know that $25 \times 32 = 800$, what is the exact value of:
 - 25×64 ? Explain.
 - 26×32 ? Explain.

- Using the thermometer shown, find the difference in temperature between the low and high.



- Explain two ways to find the answer in 4.

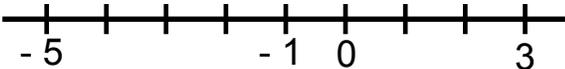
Applications 5

Vocabulary

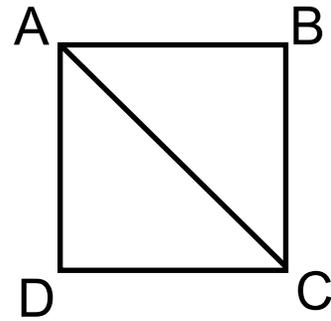
Notes

- Application sets focus on checking and reinforcing understanding. Activities in these sets are not intended to repeat previous ones, but rather to encourage students to demonstrate their understanding in a slightly different context.
- For #1, although right triangles were not covered in the Geometry Set, students still could use right triangle as an answer.
- For #3, students could say “calculation”. However, they should be encouraged to use estimation.
- For #6, students need to read carefully so they represent the non-shaded portion, not the shaded portion.

Answers

1. Possible Answers:
 - $\triangle ABC$ is an isosceles triangle because two sides (AB and BC) are equal.
 - $\triangle ABC$ is a right triangle because in a square, all angles are right angles. So $\angle B = 90^\circ$.
 - ...
2. a) 10.606667
b) 11.858537
c) 200.3620
3. Possible Answers:
 - After rounding, I know that five times four is 20 and 20 divided by 2 is 10. Therefore, the answer is about 10.
 - ...
4. 
5. Possible Answers:
 - You will get more than \$7.00 in change because $20 - 13 = 7$ and the purchase is less than \$13.
 - You will get more than \$7.00 in change because $7.00 + 12.35 = 19.35$ which is less than 20.00.
 - ...
6. 62.5% ; $\frac{5}{8}$; 0.625

Applications 5



1. Given that ABCD is a square.
What type of triangle is $\triangle ABC$?
How do you know?

2. Without finding the answer, place the decimal point in the correct position to make a true statement.

- a) $5.16 \times 3.7 \div 1.8 = 10606667$
b) $48.62 \div 6.15 \times 1.5 = 11858537$
c) $49.1 \times 3.92 + 7.89 = 2003620$

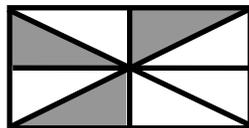
3. Explain how you found the answer in 2a)

4. Arrange the following numbers on a number line.

3 -5 0 -1

5. At a store, your total bill is \$12.35. You pay with a twenty dollar gift certificate. Will you get more or less than \$7.00 in change? How do you know?

6. Give a percent, fraction and a decimal representation for the non-shaded part of the figure.



Applications 6

Vocabulary

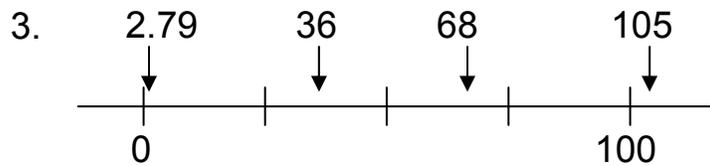
Notes

- For #3, if students have difficulty placing the numbers in the proper place on the number line, encourage them to label the interval values.

Answers

1. 135°

2. a) =
b) <
c) <



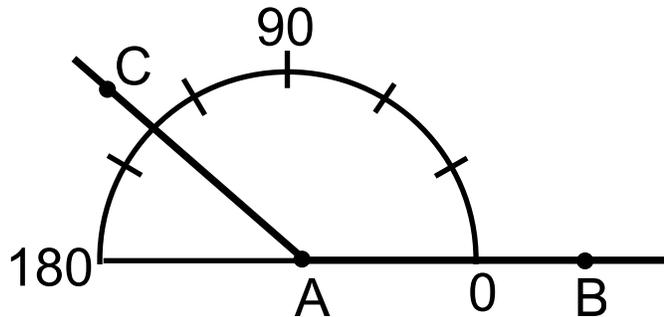
4. a) $\frac{2}{4}$

b) Possible Answers:

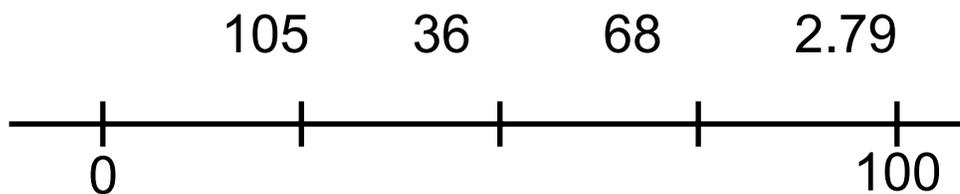
$$\frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{4}{3}, \frac{5}{3}, \frac{5}{4}$$

Applications 6

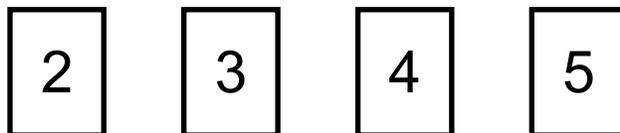
1. How large is $\angle CAB$?



2. Complete each statement with $>$, $=$ or $<$ to make a true statement.
- a) 24×0.5 ___ $24 \div 2$
- b) $20 \div 6$ ___ $20 \div 5.9$
- c) 95×0.99 ___ 95
3. Place the following numbers on the number line below.



4. Suppose you have the following number cards:



Use two cards to make a fraction of the form $\frac{\square}{\square}$ that satisfies the following:

- a) equal to $\frac{1}{2}$
- b) >1

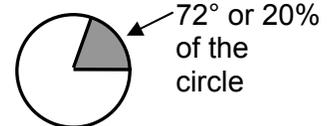
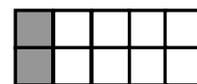
Applications 7

Vocabulary

Notes

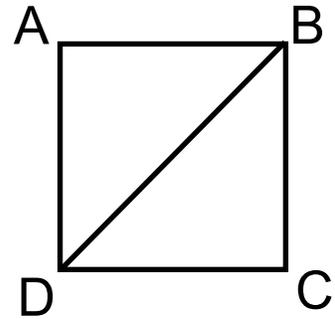
Answers

1. $\angle ABD = 45^\circ$. This is because $\triangle ABD$ is a right triangle which is also isosceles. One angle is 90° and the two equal angles add up to 90° which makes each angle equal to 45° .
2. $60 > 56 > 45 > 44$
3. 3, 4
4. Since the intervals are each 50, place 60 between the first two marks after 0 (which represent 50 and 100) but much closer to the first mark.
5. Possible Answers:
 - 0.20
 - $\frac{20}{100}$
 - $\frac{1}{5}$
 - $10\% + 10\%$
 - $20 \div 100$
 - twenty percent
 - one-fifth
 - ...



Applications 7

1. Given ABCD is a square.
What is the size of $\angle ABD$?
How do you know?



2. Use the symbol $>$ or $<$ to arrange the answers to the following in descending order.

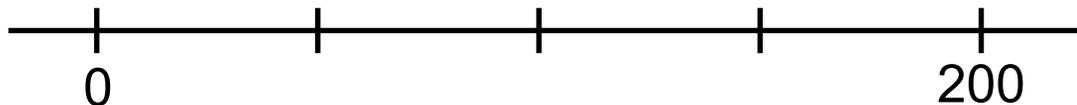
$$4.5 \times 10$$

$$50\% \text{ of } 120$$

$$24 + 32$$

$$88 \times 0.5$$

3. What two numbers have a product of 12 and a sum of 7?
4. Explain where to place the number 60 on the number line below.



5. Represent 20% in eight different ways. Use symbols, operations, diagrams and words. Use at least one of each.

Applications 8

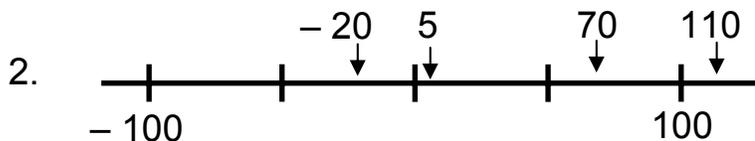
Vocabulary

Notes

- For 4a), 75° is the only answer. The only isosceles triangles with a 30° angle are a 30° - 30° - 120° triangle or a 30° - 75° - 75° triangle. Since all angles are acute, the 30° - 30° - 120° triangle is not possible.
- For #5, all possible answers are between 8 and 10.

Answers

1. 6 and 2



3. Possible Answers:

- Tina's sweater is more expensive because she started out with more money than George but ended up with less money.
- Find the cost of each item to show the sweater is more expensive:
Sweater: $95 - 43 = \$52.00$
Pants: $90 - 45 = \$45.00$
- ...

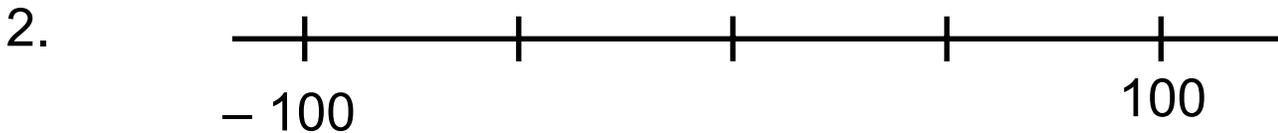
4. a) 75°
b) RS is the shortest side because it is opposite the smallest angle.

5. Possible Answers:

- 9
- 9.7
- 8.124
- ...

Applications 8

1. What two numbers have a product of 12 and a difference of 4?



Place the numbers -20 , 5 , 110 , and 70 on the number line above.

3. George and Tina go shopping. George starts out with \$90 and Tina starts out with \$95. Tina buys a sweater and has \$43 left. George buys pants and has \$45 left. What was more expensive: Tina's sweater or George's pants? Explain how you know.



\$?



\$43 left

\$90



\$?



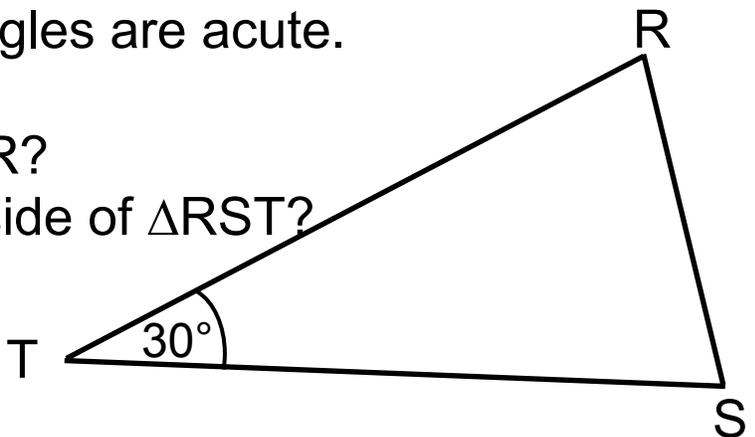
\$45 left

4. $\triangle RST$ is isosceles. All angles are acute.

a) What is the size of $\angle R$?

b) What is the shortest side of $\triangle RST$?

How do you know?



5. Name a number that is between 20×0.5 and $16 \div 2$.

Applications 9

Vocabulary

- inserting

Notes

- Application sets focus on checking and reinforcing understanding. Activities in these sets are not intended to repeat previous ones, but rather to encourage students to demonstrate their understanding in a slightly different context.
- For 2b), if students are experiencing difficulty, have them consider the x -coordinates of the four points and then the y -coordinates.
- In general, the word “coordinates” can be used in two different ways. If you are asked for the coordinates of a point, you are expected to give the ordered pair containing an x -coordinate and a y -coordinate. If you are considering two or more points, the word coordinates may refer to just the x -coordinates of all points or just the y -coordinates.

Answers

1. a) $>$
b) $>$
c) $=$
d) $<$
e) $>$
f) $<$

2. a) Possible Answers:

- $c = e$
- c is positive
- $c > a$ because a is negative
- ...

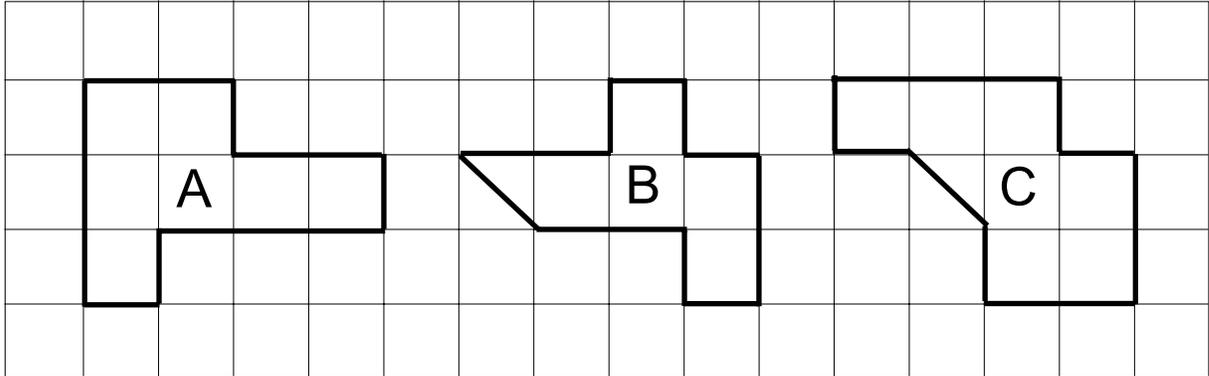
- b) Possible Answers

- $d = b$ because P and Q are horizontal points
- $a = g$ because P and S are vertical points.
- $h = f$ because S and R the same distance from the horizontal axis.
- The x -coordinates of points Q and R (c and e respectively) are equal because the points are the same distance from the vertical axis.

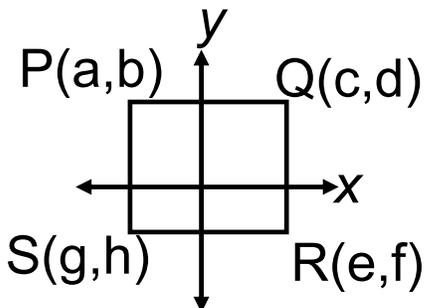
(Explanations may be different.)

Applications 9

1. Consider shapes A, B, and C. Make each statement true by inserting one of the symbols $<$, $>$, or $=$ in the box.



- | | | | |
|----|----------------|----------------------|----------------|
| a) | Perimeter of A | <input type="text"/> | Perimeter of B |
| b) | Area of A | <input type="text"/> | Area of B |
| c) | Perimeter of B | <input type="text"/> | Perimeter of C |
| d) | Area of B | <input type="text"/> | Area of C |
| e) | Perimeter of A | <input type="text"/> | Perimeter of C |
| f) | Area of A | <input type="text"/> | Area of C |

2.  Rectangle PQRS with PQ a horizontal line is shown on the diagram on the left.

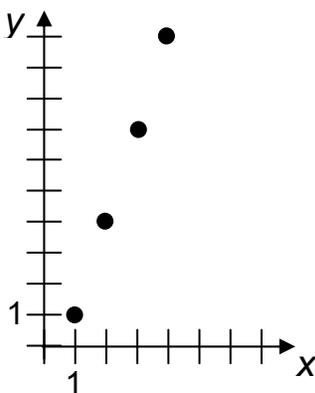
- List two things you know about c .
- What coordinates are equal? How do you know?

Applications 10

Vocabulary

Notes

Graph for 3c)



Answers

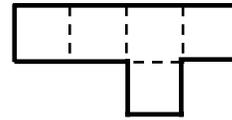
- 81 cm^2
 - 9 cm
 - 108 cm
- 5°F
 - Possible Answers:
 - Between 5°C and 10°C
 - Closer to 5°C than 10°C
 - ...
- Possible Answers:
 - The number of circles increases by 3.
 - A row of 3 circles is added each time.
 - ...
 -

Term Number	1	2	3	4
Number of Circles	1	4	7	10

- (see left for graph)

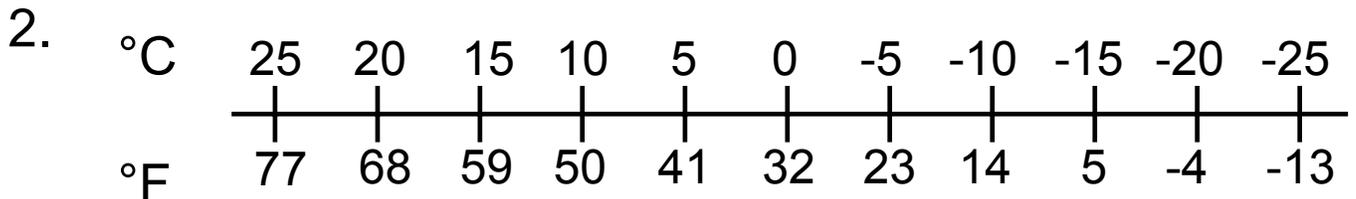
Applications 10

1. Consider the following figure comprised of 5 squares.



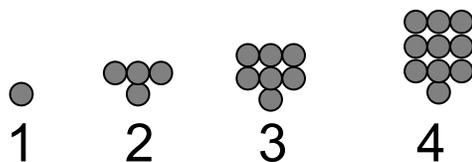
It has a total area of 405 cm^2 .

- a) What is the area of one square?
- b) What is the length of a side for one square?
- c) What is the perimeter of the figure?



- a) What is -15°C in $^{\circ}\text{F}$?
- b) If the temperature in Grand Forks is reported as 45°F , what do you know about the temperature in $^{\circ}\text{C}$?

3. The following diagrams show a pattern:



- a) Describe the pattern in words.
- b) Construct a chart showing the term number and the increasing number of small circles in the pattern.
- c) If x is the term number and y is the number of circles, draw a graph showing the pattern.

Applications 11

Vocabulary

Notes

- For #2, if you calculate $4216 \div 248$, $D = 1$, $E = 7$ and $G = 7$. Students are not expected to find each of the digits.

Answers

1. 36 and 1

2. a) No.

Possible Explanations:

- Although $2 \times 8 = 16$, 2×258 is a 3-digit number, not a 4-digit number as shown in the question.

• ...

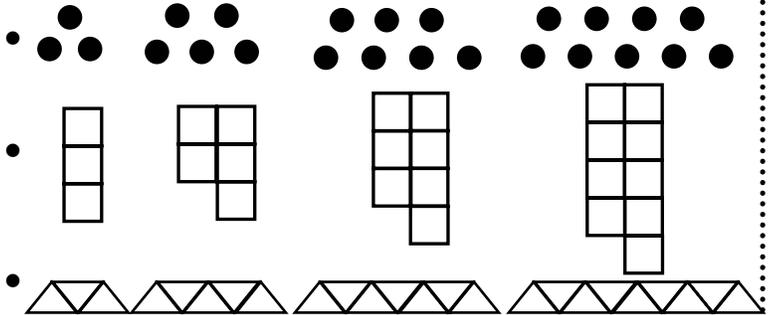
b) $D = 1$

Possible Explanations:

- If $D = 0$, the product is approximately 2000. If $D = 2$, the product is approximately 5000. So, $D = 1$ for the product to be between 2000 and 4000.

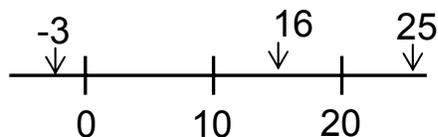
• ...

3. Possible Answers:



• ...

4.



Applications 11

1. What two numbers have a product of 36 and a sum of 37?
2. Use the partially completed 3-digit by 2-digit multiplication to answer the questions that follow.

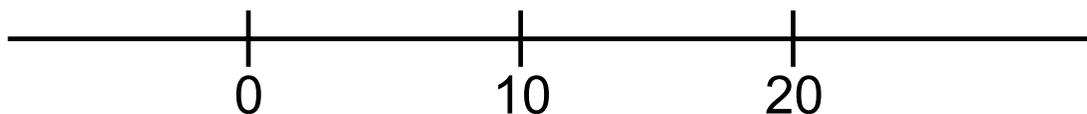
$$\begin{array}{r}
 \\
 \\
 \\
 \times \boxed{D} \boxed{E} \\
 \hline
 1 \boxed{G} \\
 2 \\
 \hline
 4
 \end{array}$$

Note: \boxed{D} , \boxed{E} , and \boxed{G} each represent a single digit.

- a) Can E be 2? Explain.
 - b) What is the value of D? Why?
3. Given the following chart, create a set of diagrams showing the pattern in the chart.

Figure	1	2	3	4
Number	3	5	7	9

4. Place the following numbers on the number line below:
16, 25, -3



Applications 12

Vocabulary

- quotient

Notes

- For #2, a number with a zero in the thousands place would not normally be considered a 4-digit number.
- For #2, students may find it easier to find all 12 possibilities if they organize their answers.
- For #4, have a blank Cartesian plane available for students to use.
- For #4, it is not possible to order the x-coordinates since you are not sure what order the points are in.

Answers

1. Possible Answers:
 - The quotient is less than 7.
 - The quotient is not a whole number.
 - The quotient is positive.
 - ...
2. 6060, 1560, 5160, 2460, 4260, 3360, 4062, 1362, 3162, 2262, 4062, 2064, 1164
3. Possible Answers:
 - If there isn't a gas station between here and my destination, I will likely run out of gas because I have $\frac{1}{3}$ of the distance to travel but I only have $\frac{1}{4}$ of a tank of gas.
 - ...
4. Possible Answers:
 - The y-coordinate of B is the same as the y-coordinate of A and C. The x-coordinate of B is between the x-coordinates of A and C.
 - ...

Applications 12

1. It is known that $35 \div 5 = 7$. What can you say for sure about the quotient of 35 and 5.1?
2. List all the 4-digit numbers that are even, have a 6 in the tens place, and whose digits add to 12.
3. You have driven $\frac{2}{3}$ of the trip distance in your car.
You started with a full tank and your tank is now $\frac{1}{4}$ full. Will you likely run out of gas? Explain.



4. Three points A, B and C lie on a horizontal line. B is between A and C. What can you say for sure about the coordinates of B?

Applications 13

Vocabulary

Notes

Answers

- yes
 - no
 - yes
- They will arrive at the same time. Victor will arrive in 10 minutes. Since Sharon's sedan travels 50 km in 25 minutes, it travels 10 km in 5 minutes or 20 km in 10 minutes.

- Possible Answers:

-

x	y
1	1
2	4
3	9
4	16
...	...

-



(Consider only the black squares as a part of the pattern.)

- This pattern is the set of square numbers.
- $y = x^2$
- ...

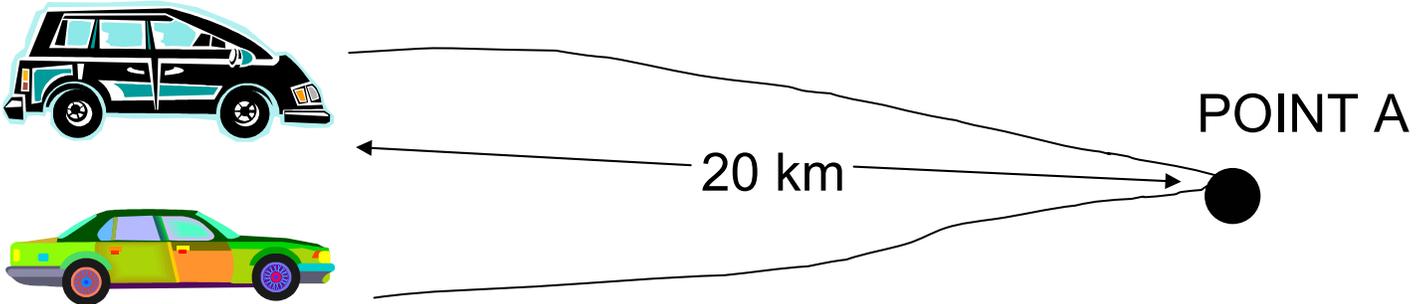
Applications 13

1. Given $6 < \square < 11$.

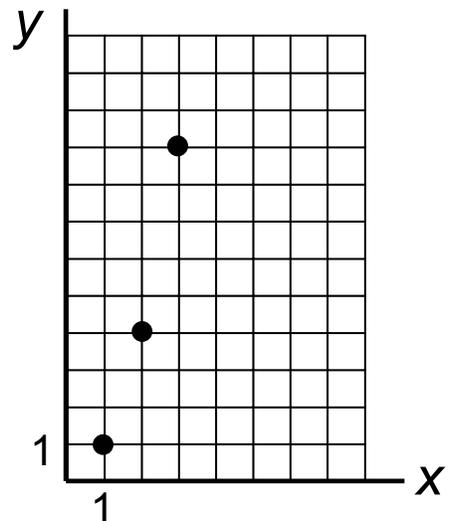
Tell whether each number could go in the box.

- a) 9.032
- b) 5.98
- c) 10.9999

2. Victor's van travels at a rate of 20 km every 10 minutes. Sharon's sedan travels at a rate of 50 km every 25 minutes. If both vehicles start at the same time, will Sharon's sedan reach Point A, 20 km away, before, at the same time as, or after Victor's van? Explain your reasoning.



3. The graph shows a pattern. Express the pattern three different ways. You may use charts, diagrams, words, or symbols.



Applications 14

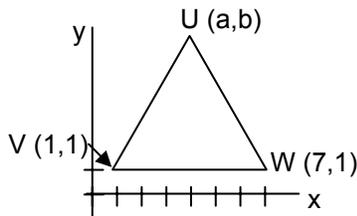
Vocabulary

- equilateral triangle

Notes

- For #3, u represents units and u^2 represents square units.

2. a) Possible Diagram

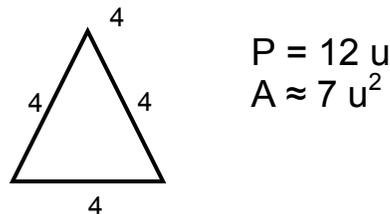
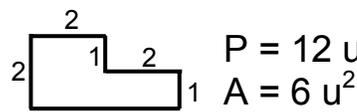
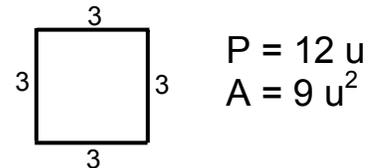
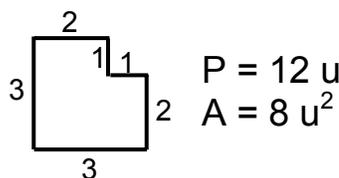


Answers

1. $0.65, \frac{5}{8}, 0.624, 62\%$

2. b) Possible Answers (see diagram at left):
- | | |
|------------------------|------------------------|
| x-coordinate of U: | y-coordinate of U: |
| • positive ($a > 0$) | • positive ($b > 0$) |
| • $a > 1$ | • $b < 7$ |
| • $a < 7$ | • between 6 and 7 |
| • has a value of 4 | • $b > 1$ |
| • ... | • ... |

3. Possible Answers:



4. $P = 40$ because the meter has intervals of 20 and P is at the second interval larger than 0. $R = 100$ because it is two intervals less than 140.

Applications 14

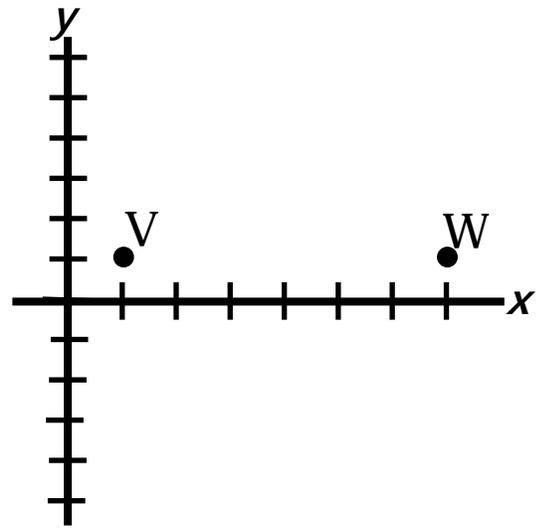
1. Arrange from largest to smallest:

$$0.624, 62\%, \frac{5}{8}, 0.65$$

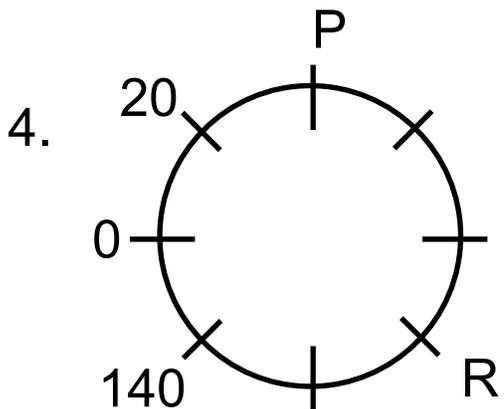
2. Consider equilateral triangle UVW with vertices at V(1,1) and W(7,1).

a) Sketch a possible equilateral triangle UVW .

b) From your sketch, what can you say for sure about the coordinates of U?



3. Draw four shapes, each with a perimeter of 12 but with a different area.



On the meter, some of the numbers have washed off. What numbers should be written at P and R? Explain.

Applications 15

Vocabulary

Notes

- For #3, there is only one rectangle with horizontal and vertical sides. However, there are an infinite number of possible rectangles with W and P as vertices.
- For #3, students may want to use a coordinate grid to help them.

Answers

1. No, $\$40 + \$25 + \$40 = \105 . $\$38$ is $\$2$ less than $\$40$, $\$27$ is $\$2$ more than $\$25$ and $\$41$ is 1 more than $\$40$, so the total is actually $\$106$.
2. a) iii)2
b) $\frac{12}{13}$ is close to 1 and so is $\frac{7}{8}$. So, if I add these two fractions, my total will be close to 2 but slightly less than 2.
3. Possible Answers:
 - (4,3) and (-2, 1)
 - (0,9) and (6,7)
 - (-1, 0) and (3, 4)
 - ...

Applications 15

7. a) Is \$100 enough to buy all three items? How can you tell?

\$38



\$27



\$41



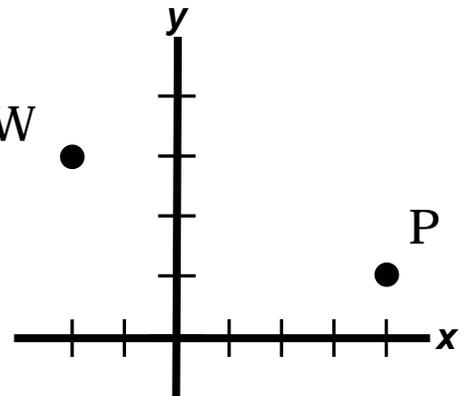
b) By rounding, show how you can find the actual cost for the 3 items before tax.

2. a) Without calculating an exact answer, choose the best estimate for $\frac{12}{13} + \frac{7}{8}$. Explain your choice.

- i) 21
- ii) 19
- iii) 2
- iv) 1

b) Is your estimate greater than or less than the exact answer? Explain.

2. Consider points W and P as shown. Draw 3 rectangles which have W and P as two of the vertices.

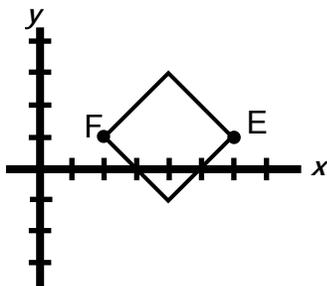


Applications 16

Vocabulary

Notes

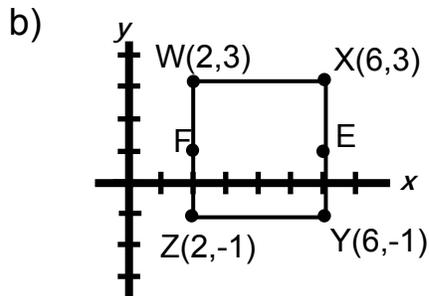
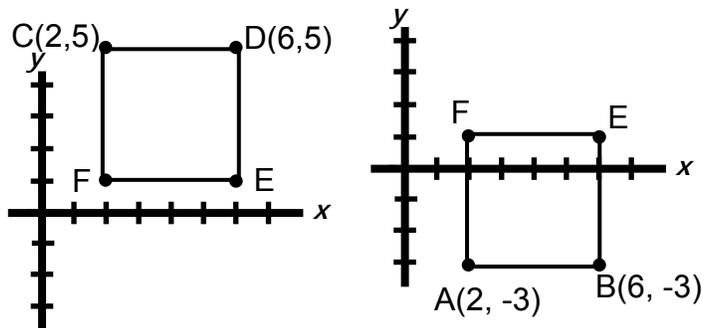
- For 1b), the possible pairs of numbers are 24, 1; 12, 2; 8, 3; 6, 4; -24, -1; -12, -2; -8, -3; -6, -4. Note the difference of 24 and 1 is the same as the difference of -24 and -1.
- For 1b), a student might argue that the difference between 3 and 8 is either 5 or -5.
- For 3a), the following diagram is possible, although the square would be called CEDF or AEBF and not CDEF or ABEF, respectively.



Answers

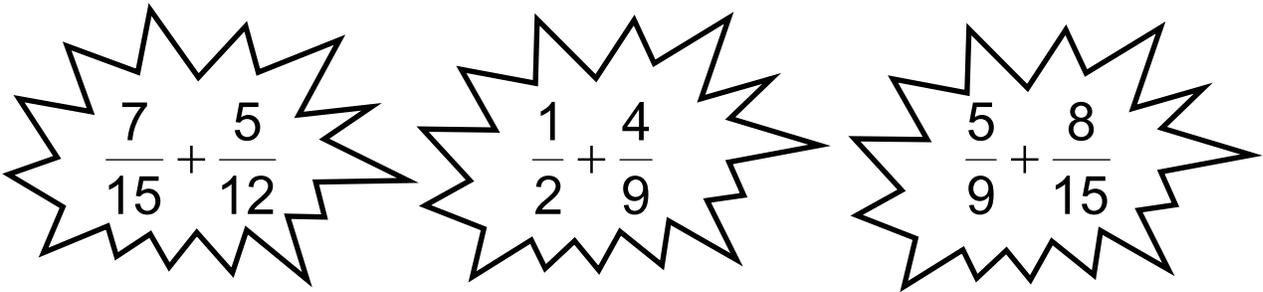
- 6 and 8
 - 23, 10, 5, 2
- $\frac{5}{9} + \frac{8}{15}$ is more than 1 because both $\frac{5}{9}$ and $\frac{8}{15}$ are more than $\frac{1}{2}$.

- Possible Answers:



Applications 16

- What two numbers have a product of 48 and a difference of 2?
 - If you know the product of two numbers is 24, what are all the possible differences between the two numbers?
- Which of the following sums is more than 1? Explain how you know.

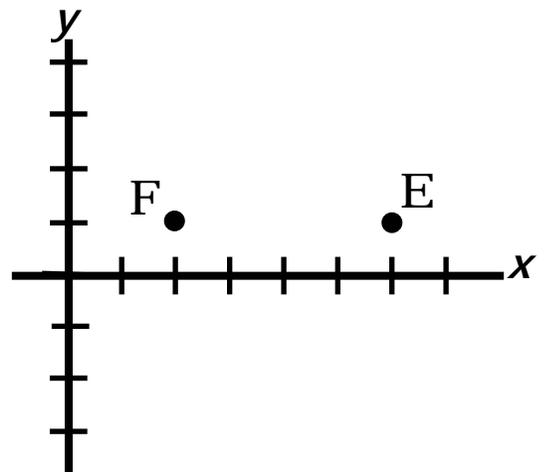


Three starburst shapes containing the following fraction addition problems:

$$\frac{7}{15} + \frac{5}{12}$$
$$\frac{1}{2} + \frac{4}{9}$$
$$\frac{5}{9} + \frac{8}{15}$$

- Consider squares with vertices at $E(6,1)$ and $F(2,1)$.

- Sketch two possible squares $CDEF$ and $ABEF$.
- Find the coordinates for a new square $WXYZ$ where the points E and F are midpoints of sides XY and WZ .



Applications Question Bank

8. If we know that $35 \div 5 = 7$, what can you say about the quotient of 35 and 5.1?

(E)

9. Which answer is smallest? Why?

a) $1639 \div 27$

b) $1600 \div 27$

c) $1550 \div 27$

(E)

10. Which quotient is biggest? Why?

a) $400 \div 24$

b) $400 \div 25$

c) $400 \div 26$

(E)

11. Is \$100 enough to buy all three items? How can you tell?

\$36



\$23



\$41



(E)