Grade 11 Pre-Calculus Mathematics (30S)

A Course for Independent Study

Field Validation Version
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ACKNOWLEDGEMENTS

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<td>Megan Hudson</td>
<td>Carolyn Wilkinson</td>
</tr>
<tr>
<td>Instructional Design Assistant</td>
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<td>Winnipeg</td>
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<td>Carole Bilyk</td>
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<td>Project Leader</td>
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<td>Louise Boissonneault</td>
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<td>Coordinator</td>
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<tr>
<td>Ian Donnelly</td>
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<tr>
<td>Consultant (since February 2012)</td>
<td></td>
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<tr>
<td>Lynn Harrison</td>
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<td>Desktop Publisher</td>
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<td>Myrna Klassen</td>
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<td>Consultant</td>
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<td>Gilles Landry</td>
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<td>Philippe Leclercq</td>
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GRADE 11 PRE-CALCULUS MATHEMATICS (30S)

Introduction
Course Content

Welcome to Grade 11 Pre-Calculus Mathematics! This course is a continuation of the concepts you have studied in previous years, as well as an introduction to new topics. It builds upon the pre-calculus topics you were introduced to in Grade 10 Introduction to Applied and Pre-Calculus Mathematics. You will put to use many of the skills that you have already learned to solve problems and learn new skills along the way. This course helps you develop the skills, ideas, and confidence you will need to continue studying math in the future.

Problem solving, communication, reasoning, and mental math are some of the themes you will discover in each module. You will engage in a variety of activities that promote the connections between symbolic math ideas and the world around you.

There are three main areas that you will be exploring: Number, Patterns and Relations, and Shape and Space.

This course is divided into eight modules, organized as follows:

- Module 1: Sequences and Series
- Module 2: Factoring and Rational Expressions
- Module 3: Quadratic Functions
- Module 4: Solving Quadratic and Rational Equations
- Module 5: Radicals
- Module 6: Systems of Equations and Inequalities
- Module 7: Trigonometry
- Module 8: Absolute Value and Reciprocal Functions

There are two online resources for this course.

- Glossary
- Graph Paper

The online resources can be found at <www.edu.gov.mb.ca/k12/dl/downloads/index.html>. If you do not have access to the Internet, contact the ISO office at 1-800-465-9915 to get a copy of these online resources.
What Will You Need?

Please note that you do not need a textbook to complete this course. All of the content is included with this package.

Required Resources

The only required resources for this course are a scientific calculator and graph paper. Graph paper is available as one of the online resources found on the downloads page.

Optional Resources

- A graphing calculator, a freeware graphing program, or a graphing calculator app. None of these are required when writing either your midterm or final examination.

- Access to a computer with spreadsheet and graphing capabilities will be an advantage, but not a requirement. Use of the Internet may be suggested as a resource in some places, but if you do not have access to an online computer, you will still be able to complete the related learning activities and assignments.

- Access to a photocopier would be helpful because it would let you make a copy of your assignments before you send them to your tutor/marker. That way, if you and your tutor/marker want to discuss an assignment, you would each have a copy to refer to.

Resource Sheet

When you write your midterm and final examinations, you will be allowed to take an Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½” by 11”, with both sides in your handwriting or typewritten. It is to be submitted with your exam. The Exam Resource Sheet is not worth any marks.

Creating your own resource sheet an excellent way to review. It also provides you with a convenient reference and quick summary of the important facts of each module. Each student is asked to complete a resource sheet for each module to help with studying and reviewing.

The lesson summaries are written for you to use as a guide, as are the module summaries at the end of each module. Refer to these when you create your own resource sheet. Also, refer to the online glossary (found on the downloads page) to check the information on your resource sheet.
After you complete each module’s resource sheet, you should summarize the sheets from all of the modules to prepare your Exam Resource Sheet. When preparing your Midterm Exam Resource Sheet, remember that your midterm examination is based on Modules 1 to 4. When preparing your Final Exam Resource Sheet, remember that your final examination is based on the entire course, Modules 1 to 8.

How Will You Know How You’re Doing?

You will know how well you are learning by your successful completion of the following course components:

Learning Activities

Each learning activity has two parts—Part A has BrainPower questions and Part B has questions related to the content in the lesson

Part A: BrainPower

The BrainPower questions are provided as a warm-up activity for you before trying the other questions. Each question should be completed quickly and without using a calculator, and most should be completed without using pencil and paper to write out the steps. Some of the questions will relate directly to content of the course. Some of the questions will review content from previous courses—content that you need to be able to answer efficiently.

Being able to do these questions in a few minutes will be helpful to you as you continue with your studies in mathematics. If you are finding it is taking you longer to do the questions, you can try one of the following:

- work with your learning partner to find more efficient strategies for completing the questions
- ask your tutor/marker for help with the questions
- search online for websites that help you practice the computations so you can become more efficient at completing the questions.

None of the assignment questions or exam questions will require you to do the calculations quickly or without a calculator. However, it is for your benefit to complete the questions as they will help you in the course. Also, being able to successfully complete the BrainPower exercises will help build your confidence in mathematics. BrainPower questions are like a warm-up you would do before competing in a sporting event.
Part B: Course Content Questions

One of the easiest and fastest ways to find out how much you have learned is to complete Part B of the learning activities. These have been designed to let you assess yourself by comparing your answers with the answer keys at the end of each module. There is at least one learning activity in each lesson. You will need a notebook or loose-leaf pages to write your answers.

Make sure you complete each learning activity. Besides giving you instant feedback, they will help you practice what you have learned and prepare you to successfully complete hand-in assignments and exams. Many of the questions on the exams will be similar to the questions in the learning activities. If you were able to answer them correctly, you are likely to do well on your exams. If you did not answer them correctly, you need to go back to the lesson and review the instructions and examples. Don't skip ahead without learning. If you do, you will be wasting your time, and you won't be able to complete later lessons. If you have difficulty answering the questions in the learning activity, ask your learning partner to help you, or contact your tutor/marker.

Do not mail learning activities to your tutor/marker.

Assignments

Lesson assignments are located throughout the modules, and include questions similar to the questions in the learning activities of previous lessons. The assignments have space provided for you to write your answers on the question sheets. You need to show all your steps as you work out your solutions, and make sure your answers are clear (include units, where appropriate). There is no answer key for hand-in assignments because your tutor/marker will correct these assignments and then return them to you. These assignments make up 55 percent of your final mark. You must complete each assignment in order to receive a final mark in this course. You will mail or email these assignments to the tutor/marker along with the appropriate cover page once you complete each module.
Midterm and Final Examinations

The course includes a midterm examination and a final examination. You will write both exams under supervision. Both exams have a time limit of 2.5 hours. The midterm exam is based on Modules 1 to 4 and is worth 20 percent of the final course mark. You should write it when you have completed Module 4. In order to do well on the midterm exam, you should review all of the work that you have completed from Modules 1 to 4, including all learning activities and assignments. As a student, you can use your Midterm Exam Resource Sheet to bring any formulas you have not memorized into the exam with you. You will be required to bring the following supplies to the midterm exam: pencils (2 or 3 of each), blank paper, a ruler, a scientific or graphing calculator, and your Midterm Exam Resource Sheet.

The final exam is cumulative, so it is based on Modules 1 to 8. It is worth 25% of the final mark of the course. The exam content will consist of 20% from Modules 1 to 4 and 80% from Modules 5 to 8. You will write it when you have completed Module 8. In order to do well on the final exam, you should review all of the work that you have completed from Modules 1 to 8, including all learning activities and assignments. You can use your Final Exam Resource Sheet to bring any formulas you have not memorized into the exam with you. **Formulas are not provided on the exam.** You will be required to bring the following supplies to the final exam: pencils (2 or 3 of each), blank paper, a ruler, a scientific or graphing calculator, and your Final Exam Resource Sheet.

You are responsible for applying for the exams and making arrangements to have the exams sent to your proctor from the Independent Study Option office. Before you finish Module 4, you will need to make arrangements to write your midterm exam. Before you finish Module 8, you will need to make arrangements to write your final exam. When you write your midterm and final exams, you will be supervised by a proctor. Contact the Independent Study Option office (referred to as ISO) at 1-800-465-9915 if you need help arranging this.
To write your examinations, you need to make the following arrangements:

- **If you are attending school**, ask your school’s Independent Study Option (ISO) school facilitator to request your examination. Do this at least **three weeks before** you are ready to write your examination. For more information on examination procedures, please contact your ISO school facilitator or visit the distance learning website at <www.edu.gov.mb.ca/k12/dl/iso/index.html>.

- **If you are not attending school**, check the **Examination Request Form** for options available to you. The form was mailed to you with this course. **Three weeks before** you are ready to write the examination, fill in the Examination Request Form and mail, fax, or email it to
  
  ISO Office  
  555 Main Street  
  Winkler, MB R6W 1C4  
  Fax: 204-325-1719  
  Toll-Free Telephone: 1-800-465-9915  
  Email: distance.learning@gov.mb.ca

**Practice Examinations and Answer Keys**

To help you succeed in your midterm and final examinations, you are encouraged to write the practice exams that are found at <www.edu.gov.mb.ca/k12/dl/downloads/index.html>. These exams are very similar to the actual exams that you will be writing. They also include an answer key, so that you can check your answers when you have finished writing them. This will give you the confidence you need to do well on your exams. If you do not have access to the Internet, contact the ISO office at 1-800-465-9915 to get a copy of the practice exams.

**What If You Need Help?**

Here are two people who can help you be successful in your course.

**Your Tutor/Marker**

Tutor/markers are experienced educators who tutor independent students and mark assignments and examinations. When you are having difficulty with something in this course, be sure to contact your tutor.marker, who is there to help you. Your tutor/marker’s name and contact information were sent to you with this course. You can also obtain this information in the **Who Is My Tutor/Marker** section of the distance learning website at <www.edu.gov.mb.ca/k12/dl/iso/assistance.html>.
The first learning activity and assignment for this course will involve contacting your tutor/marker.

Your Learning Partner

Another person who can help you with your course is a learning partner. A learning partner is someone you choose who will help you learn. It may be someone who knows something about math, but it doesn't have to be. A learning partner could be someone else who is taking this course, a teacher, parent, sibling, or a friend, or anybody else who can help you. Most importantly, a learning partner should be someone you feel comfortable with and who will support you as you work through this course.

Your learning partner can help you keep on schedule, check your work, help you make sense of assignments, read your course with you, or look at your learning activities and respond to them. You may even study for your exam with your learning partner.

One of the best ways that your learning partner can help you is by reviewing your midterm and final practice exams with you. These are found at <www.edu.gov.mb.ca/k12/dl/downloads/index.html>, along with their answer keys. Your learning partner can administer your practice exam, check your answers with you, and then help you learn the things that you missed.

How Much Time Will You Need?

Learning through independent study has several advantages over learning in the classroom. You are in charge of how you learn and can choose how quickly you will complete the course. You don't have to wait for your teacher or classmates, and you can work as quickly as you want. You can also complete as many lessons at a time as you want. Read the next few pages to get an idea of how to pace yourself. You have one full year from the date of your registration to complete this course, but the pace at which you complete the course is up to you.
Chart A: Semester 1

Here is a suggested timeline that you can follow if you start your course in September and need to complete it by the end of January.

<table>
<thead>
<tr>
<th>Module</th>
<th>Completion Date</th>
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<tbody>
<tr>
<td>Module 1</td>
<td>Mid-September</td>
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<td>Module 2</td>
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<td>Module 3</td>
<td>Mid-October</td>
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<td>Module 4 and Midterm Exam</td>
<td>Early November</td>
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<td>Module 5</td>
<td>Mid-November</td>
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<td>Module 6</td>
<td>Early December</td>
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<td>Module 7</td>
<td>Mid-December</td>
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<tr>
<td>Module 8 and Final Exam</td>
<td>Mid-January</td>
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Chart B: Semester 2

Here is a suggested timeline that you can follow if you start your course in January and need to complete it by June.

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<td>Module 1</td>
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<td>Module 2</td>
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<td>Module 3</td>
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<td>Module 4 and Midterm Exam</td>
<td>Late March</td>
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<td>Module 5</td>
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<td>Module 6</td>
<td>Late April</td>
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<td>Module 7</td>
<td>Mid-May</td>
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<tr>
<td>Module 8 and Final Exam</td>
<td>Late May</td>
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Chart C: Full School Year (Not Semestered)

Here is a suggested timeline that you can follow if you have registered for this course in September and would like to complete it by June.

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<td>Module 7</td>
<td>Mid-April</td>
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<tr>
<td>Module 8 and Final Exam</td>
<td>Late May</td>
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Do not wait until the last minute to complete your work, since your tutor/marker may not be available to mark it immediately. Make sure that you leave enough time for your work to travel through the mail, as that might take over a week. It may also take a few weeks for your tutor/marker to mark everything and send the marks to your school.

If you need this course to graduate this school year, remember to schedule and complete your final exam by June 1.

When Do You Send in Your Assignments?

In this course, you have the choice of either mailing or emailing your assignments. You will submit your assignments at the end of each module.

- Each time you mail something, you must include the print version of the applicable Cover Sheet (found at the end of this Introduction).
- Each time you email something, you must include the electronic version of the applicable Cover Sheet (found at <www.edu.gov.mb.ca/k12/dl/downloads/index.html>).

Complete the information at the top of each Cover Sheet before mailing or emailing it along with your assignments.
Mailing Your Assignments

If you choose to mail your completed assignments, please photocopy all the materials first so that you will have a copy of your work in case your package goes missing. You will need to place the applicable module Cover Sheet and assignment(s) in an envelope, and address it to

ISO Tutor/Marker
555 Main Street
Winkler MB R6W 1C4

Your tutor/marker will mark your work and return it to you by mail.

Emailing Your Assignments

If you choose to email your completed assignments, make sure you have saved electronic copies before you send them. That way, you can refer to your assignments when you discuss them with your tutor/marker. Also, if the original assignments are lost, you will be able to resubmit them.

To email your completed assignments, you will first need to do one of the following:

- **If you are attending school**, please ask your ISO school facilitator (the person who signed your ISO Registration/Admission Form) for permission to email your assignments and to determine your school’s procedure for emailing assignments. Contact your tutor/marker to confirm that the course material can be marked electronically.

- **If you are not attending school**, please obtain permission directly from your tutor/marker to submit your assignments electronically. Also, please confirm that the course material can be marked electronically.

Saving and Submitting Your Assignments

1. Save your work as described below.

   **File Names**
   When saving your work, identify the assignment number(s) in the file name (e.g., Assignment 1.1.doc).

   **File Types**
   Save your work in the following file types (as applicable):
   - **Written work**: Microsoft Word files (doc) or RTF files
   - **Scanned work**: PDF files (save multiple pages on one file)
The Independent Study Option office cannot access
- file extensions other than those listed above
- cloud servers for file storage/sharing (e.g., SkyDrive)

File Size
- The total size of attached files per email must not exceed 5 MB.
- If files are larger than 5 MB, you must submit them as compressed files. If you are not familiar with this process, please go online to <www.wikihow.com/> to learn how to compress files with your operating system.

2. Use the following format to compose your email.

   To: distance.learning@gov.mb.ca
   cc: [Your ISO school facilitator’s email address if you attend school]
   Subject: [Your Name] Grade 11 Pre-Calculus Mathematics
   Attachment(s): Module 1 Cover Sheet, Assignment 1.1.doc,
   Assignment 1.2.doc, and Assignment 1.3.doc,
   Message: Module 1 Cover Sheet, Assignment 1.1,
   Assignment 1.2, and Assignment 1.3
   Tutor/marker: ________________________________
   School: ________________________________

   The subject line of your email must clearly indicate your name and the course name. The assignment number(s) must be identified in the file names and in the body of the email message, as shown in the sample provided.

3. Attach your files to the email.

4. Email your assignments to <distance.learning@gov.mb.ca>. Do not email your assignments directly to your tutor/marker. Emails sent directly to the tutor/marker will be returned unread.

   Your tutor/marker will mark your work and return it to you by email.

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The Independent Study Option office does not provide technical support for hardware-related issues. If troubleshooting is required, consult a professional computer technician.
What Are the Guide Graphics For?

Graphics have been placed inside the margins of the course to identify a specific task. Each graphic has a specific purpose to guide you. A description of each graphic is described below:

**Lesson Introduction:** The introduction sets the stage for the lesson. It may draw upon prior knowledge or briefly describe the organization of the lesson. It also lists the outcomes for the lesson. Outcomes describe what you will learn.

**Learning Activity:** Complete this learning activity to help you review or practice what you have learned and prepare for your assignment and exam. You will **not** send learning activities to your tutor/marker.

**Assignment:** This is an assignment that you complete and send to your tutor/marker. You will be sending in your assignments at the end of every module.

**Mail-in:** Indicates when it is time to mail in your assignments.

**Email:** It is now time to email your completed assignment(s) to your tutor/marker for assessment.

**Tutor/Marker:** Indicates when the tutor/marker is referenced in helping the student.

**Learning Partner:** Indicates when the student may seek help from their learning partner.

**Resource Sheet:** Indicates material that may be valuable to include on your resource sheet.

**Examination:** It is time to write your midterm or final examination.

**Note:** Take note of and remember this important information or reminder.
Math Goals

In Module 1, the first learning activity involves you having a conversation with your tutor/marker. Having this conversation with your tutor/marker has two important purposes. First, it introduces you to a very valuable resource - your tutor/marker. He or she is available for you to answer questions, explain concepts, and guide you through this course. You can discuss your math learning and progress. Feel free to contact your tutor/marker by phone or email at anytime during this course.

The second important purpose of this assignment is to get you thinking about your math goals. You may have a future career in mind and this course is getting you one step closer to it by completing a prerequisite for a future required course. There may be specific skills or topics you are interested in learning about and they are covered in this course.

If you are unsure of your math goals or why they are important, consider this:

- goals give you a sense of direction and purpose in taking this course
- goals help motivate you to learn and do your best, even when it’s tough
- when you accomplish your goals, there is a great sense of achievement and success

Good goals need to be realistic, specific, and they should reflect what is important to you. They should give you direction and take you further down the path from where you have been to where you want to go.

Goals can be long term or short term, but they are the pathway that takes you from where you were/are, closer to where you want to go.

Getting Started

Take some time right now to skim through the course material, locate your cover sheets, and familiarize yourself with how the course is organized. Get ready to learn!
GRADE 11 PRE-CALCULUS
MATHEMATICS (30S)

Module 1
Sequences and Series
Module 1: Sequences and Series

Introduction

In previous math courses, you have dealt with patterns in the context of graphs. In this module, you will be dealing with patterns a little differently. Patterns show up all around you in the natural world. The petals in flowers very often appear in a pattern. Pine cones form according to a pattern called a Fibonacci sequence.

Patterns also appear in lists of numbers. Whenever a pattern appears in a list of numbers, it is called a sequence. Sequences have many applications. An automobile will depreciate in value according to a sequence. Real estate and other investments will earn interest or gain in value according to a specific sequence. Loans will be paid according to a predetermined sequence. In this module, you will study two different types of sequences: arithmetic and geometric. You will also study two different types of series: arithmetic and geometric. Series are used when you are trying to find the sum of a large, even infinite, list of numbers. In this module you will learn when it is possible to find the sum of different types of series.

Assignments in Module 1

To obtain credit for Module 1, you will need to send the following three assignments to your tutor/marker. Your evaluation for this module is based on these assignments.

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<th>Lesson</th>
<th>Assignment Number</th>
<th>Assignment Title</th>
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<tbody>
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<td>Arithmetic Sequences and Series</td>
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<td>4</td>
<td>Assignment 1.2</td>
<td>Geometric Sequences and Series</td>
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Resource Sheet

When you write your midterm exam, you are encouraged to take a Midterm Exam Resource Sheet with you into the exam. This sheet will be one letter-sized page, 8½” by 11”, with both sides in your handwriting or typewritten. You will submit it with your exam, but you do not receive any marks for it.

Many students have found that preparing a resource sheet is an excellent way to review. It provides you with a summary of the important facts of each module. You should complete a resource sheet for each module to help with your studying and reviewing. Lesson summaries and module summaries are included for you to use as a guide.

You may use the list of instructions provided below to help you with preparing your resource sheet for the material in Module 1. On this sheet, you should record math terms and definitions, formulas, sample questions, or a list of places where you often make mistakes. You should also identify special areas that require extra attention or review by writing the page numbers.

After you have completed each module’s resource sheet, you may summarize the sheets from Modules 1, 2, 3, and 4 to prepare your Midterm Exam Resource Sheet. The midterm exam for this course is based on Modules 1 to 4. Your final exam is based on Modules 1 to 8.

Resource Sheet for Module 1

1. List all the important math terms, and define them if necessary.
2. List all the formulas and perhaps a sample problem that shows how the formula is used.
3. If necessary, write the solutions to some problems, showing in detail how you did the calculations.
4. Copy any questions that represent the key points of the lesson, and perhaps include the solutions as well.
5. Identify the problems you found most difficult, and copy the page numbers onto the resource sheet so that you can review them before writing the exam. You may also copy the problems and the solutions onto your resource sheet, and later write them onto your Midterm Exam Resource Sheet.
6. Write any comments, ideas, shortcuts, or other reminders that may be helpful during an exam.
Lesson Focus

In this lesson, you will

- learn how to identify arithmetic sequences
- learn how to find the first term, the common difference, the number of terms, and the value of a specific term in an arithmetic sequence
- learn how arithmetic sequences are related to linear functions
- learn how to write a recursive formula
- learn how to derive a rule for finding the general term of an arithmetic sequence

Lesson Introduction

When you were younger, you were probably asked to count by ones from one to increasingly larger numbers. You were also probably asked to count by odd numbers, by even numbers, by fives, by tens, or by other values. These are all examples of arithmetic sequences.

Arithmetic sequences appear often in real-life situations. Hockey games and the formation of pyramids are another two examples where arithmetic sequences occur. Throughout this lesson, you will learn all about arithmetic sequences and their many applications.

Arithmetic Sequences

A sequence is an ordered list of terms. The general form of a sequence is: $t_1, t_2, t_3, t_4, \ldots$ where each numerical subscript denotes the term in the sequence. This means that if you wanted to find the fifth term in a sequence, you would be looking for $t_5$. Also, $t_{12}$ denotes the twelfth term in the sequence.

Sequences of numbers that follow the pattern of adding the same number (either positive or negative) to one term to get the next term are called arithmetic sequences. In other words, an arithmetic sequence is a list of numbers in which each succeeding term is found by adding or subtracting the same value from the previous term.

Include all definitions on your resource sheet.
Example 1
Which of these sequences are arithmetic sequences?

a) 2, 5, 8, 11, . . .
b) 1, 2, 4, 7, 11, . . .
c) 7, 3, –1, –5, . . .
d) 1, 1, 2, 3, 5, . . .

Solution

a) In this sequence, if you add ‘3’ to the first term, you will get the second term. This works for any pair of consecutive terms (or terms that follow each other) in this sequence. Therefore, this is an arithmetic sequence.

b) In this sequence, if you add ‘1’ to the first term, you will get the second term. However, to get to the third term, you need to add ‘2’ to the second term. This is not an arithmetic sequence because you are not adding the same value to each term to get the next term.

c) In this sequence, if you add ‘–4’ or subtract ‘4’ from each term, you will get the next term. Therefore, this is an arithmetic sequence.

d) In this sequence, the first terms are the same. The rest of the terms are the sum of the previous two terms. However, the same number is not added to every term to get the next term. Therefore, this is not an arithmetic sequence. It is the Fibonacci sequence.

In an arithmetic sequence, the number that is added to each term to get the next term is called the **common difference**. Include this definition on your resource sheet.

Example 2
Find the common difference for each of these arithmetic sequences.

a) 5, 9, 13, 17, . . .
b) 1, 0, –1, –2, . . .
c) 2, 4, 6, 8, . . .

Solution

a) The number that is added to each term to get the next term is ‘4’. Therefore, the common difference of this arithmetic sequence is 4.

b) The number that is added to each term to get the next term is ‘–1’. Therefore, the common difference of this arithmetic sequence is –1.

c) The number that is added to each term to get the next term is ‘2’. Therefore, the common difference of this arithmetic sequence is 2.
Arithmetic Sequences and Linear Functions

Given a sequence such as 1, 6, 11, 16 ... it may be easy for you to find the 10th or even the 20th term by just adding 5, the common difference, to each term until you have a sequence with 20 terms. What happens when you need to find the 600th term \((t_{600})\)?

Calculating the 600th term in this way would take an extremely long time. In order to find terms that are increasingly farther away from the first term in a sequence, a formula can be used. For example, look at the arithmetic sequence:

\[2, 5, 8, 11, \ldots\]

Each term in the sequence is 3 more than the preceding term:

\[2, 2 + 3 = 5, 5 + 3 = 8, 8 + 3 = 11, \ldots\]

You can denote this pattern by stating that:

\[t_1 = 2 \quad \text{(where } t_1 \text{ is the first term)}\]
\[t_2 = t_1 + 3 \quad \text{(where } t_2 \text{ is the second term)}\]
\[t_3 = t_2 + 3\]

In general \(t_n = t_{n-1} + 3, n \geq 2\) where the \(n\)th term is 3 more than the previous term. The variable, \(n\), is used as a counter to count the number of terms. Its domain is \(\{1, 2, 3, 4, \ldots\}\), which is the set of natural numbers.

The definition for this arithmetic sequence consists of two statements:

\[
\begin{cases}
  t_1 = 2 \\
  t_n = t_{n-1} + 3, n \geq 2
\end{cases}
\]

In this case, \(n\) must be 2 or bigger because \(t_1\) is already defined.

Both parts of the statement are required.

The definition states

a) the value of the first term is 2 as shown by the first statement, \(t_1 = 2\)

b) any other term after the first is 3 more than the previous term as shown by the second statement, \(t_n = t_{n-1} + 3, \text{ when } n \geq 2\)

Note that 7, 10, 13, 16, ... is another sequence in which each term after the first is 3 more than the previous term. However, it is a different sequence from the sequence 2, 5, 8, 11, ... because the first terms are different.
You can denote the arithmetic sequence 7, 10, 13, 16, \ldots by
\[
\begin{align*}
t_1 &= 7 \\
t_n &= t_{n-1} + 3, \ n \geq 2
\end{align*}
\]

Also note that it is important to distinguish between the term number and the value of the term.

**Example 3**

Write the first five terms of the arithmetic sequence defined by
\[
\begin{align*}
t_1 &= 4 \\
t_n &= t_{n-1} + 5, \ n \geq 2
\end{align*}
\]

**Solution**

Evaluating the formula can easily be organized by using a chart. The term number, \(n\), is recorded in the first column; the value of the term is calculated in the second column.

\[
\begin{align*}
t_1 &= 4 & \text{(from the first statement)} \\
t_2 &= t_{2-1} + 5 & \text{(from the second statement)} \\
&= t_1 + 5 & \text{simplify} \\
&= 4 + 5 & \text{substitute} \\
&= 9 & \text{simplify} \\
t_3 &= t_{3-1} + 5 \\
&= t_2 + 5 \\
&= 9 + 5 \\
&= 14 \\
t_4 &= t_{4-1} + 5 \\
&= t_3 + 5 \\
&= 14 + 5 \\
&= 19 \\
t_5 &= t_{5-1} + 5 \\
&= t_4 + 5 \\
&= 19 + 5 \\
&= 24
\end{align*}
\]

Therefore, the first five terms are 4, 9, 14, 19, 24.
In Example 3, after the first term, the succeeding terms are defined by the value of the preceding term and the common difference. If after the first term, or the first few terms, the remaining terms are defined in terms of the value(s) of the preceding term(s), the definition is called a **recursive definition** or recursive formula. It may be helpful for you to include this definition on your resource sheet.

**Example 4**

Write a recursive formula for the arithmetic sequence 7, 11, 15, 19, . . ., and graph it using both graph paper and a graphing calculator (if you have one).

**Solution**

\[ t_1 = 7 \]

Since each term is 4 more than the previous term, it follows that the common difference is 4 and \( t_2 = t_1 + 4, \) \( t_3 = t_2 + 4, \) . . . Therefore, the recursive formula is

\[
\begin{align*}
    t_1 &= 7 \\
    t_n &= t_{n-1} + 4, \, n \geq 2
\end{align*}
\]

The points on the graph will be the set of ordered pairs where the first coordinates are the term numbers and the second coordinates are the corresponding term values. The coordinates are (1, 7), (2, 11), (3, 15), (4, 19), . . .

The graph is shown below. Notice that the points are not joined. The data is discrete, not continuous. Also note that there is no easy way to indicate the pattern of dots, which continues indefinitely. The arrow attached to (4, 19) points in only one direction.
On a graphing calculator, use the sequence mode to graph the sequence using the recursion formula:

\[
\begin{align*}
    t_1 &= 7 \\
    t_n &= t_{n-1} + 4, \quad n \geq 2
\end{align*}
\]

and the first point in the sequence as \((1, 7)\).

Using the Seq mode, enter your information into Y=.

```
Plot1 Plot2 Plot3
nMin=1
\cdot u(n)\cdot (u(n-1)) + 4
\cdot u(nMin)=7
\cdot u(n)=
```

Then set your window.

```
WINDOW
nMin=1
nMax=10
PlotStart=1
PlotStep=1
Xmin=-1
Xmax=10
\downarrow Xscl=1
Ymin=-1
Ymax=50
Yscl=5
```

Finally, press GRAPH.

If you need instruction on how to use your graphing calculator for sequences, you might try searching the Internet for help. One useful site is <www.mathbits.com>.
From these examples, we could generalize the recursion formula for an arithmetic sequence to be:

\[
\begin{align*}
  t_1 &= \text{first term} \\
  t_n &= t_{n-1} + d, \ n \geq 2
\end{align*}
\]

where \(d\) is the common difference.

**Example 5**

Find \(t_7\) and \(t_{100}\) of the arithmetic sequence 50, 42, 34, 26, . . .

**Solution**

The common difference is –8 and the recursive definition is:

\[
\begin{align*}
  t_1 &= 50 \\
  t_n &= t_{n-1} - 8, \ n \geq 2
\end{align*}
\]

Listing the terms from \(t_2\) to \(t_7\), you get

\[
\begin{align*}
  t_2 &= 50 - 8 = 42 \\
  t_3 &= 42 - 8 = 34 \\
  t_4 &= 34 - 8 = 26 \\
  t_5 &= 26 - 8 = 18 \\
  t_6 &= 18 - 8 = 10 \\
  t_7 &= 10 - 8 = 2
\end{align*}
\]

Therefore, \(t_7 = 2\).

When you are using a recursive formula or definition, you need to know \(t_{n-1}\) in order to find \(t_n\). You need to know \(t_{99}\) in order to find \(t_{100}\). Using recursion would be time consuming, as it would take a long time to list all the terms up to \(t_{100}\). There is a different method to solve this problem.

It is easier to find \(t_{100}\) by relating the arithmetic sequence to a linear function. The answer is shown after Example 7.

A **linear function** is a function whose graph is an oblique or horizontal line. If the **domain** of a linear function is the set of **natural numbers**, the **range** is called an **arithmetic sequence**. Recall that the set of **natural numbers** is \(N = \{1, 2, 3, 4, \ldots \}\).

Include these definitions on your resource sheet.
Example 6
Which of the following rules defines a linear function? Graph the linear function(s).

a) \( y = \frac{1}{x-2} \)

b) \( y = 3x + 5 \)

c) \( y = \sqrt{x-1} \)

d) \( y = x^2 + 1 \)

e) \( y = 4 - x \)

Solution
The only rules that produce straight line graphs are \( y = 3x + 5 \) and \( y = -x + 4 \). Hence (b) and (e) are linear functions. The graphs of the two linear functions are shown below. Notice their domains are the real numbers and the points are connected.

b) \( y = 3x + 5 \)
slope = 3
\( y \)-intercept = 5
Note: A straight line equation is of the form $y = mx + b$ where $m$ is the slope of the line and $b$ is the $y$-intercept. In example 6, part (e) above, the equation $y = 4 - x$ can be rearranged to the form $y = -x + 4$.

Example 7
Write the first five terms and find the common difference for each arithmetic sequence defined by the given function whose domain is the natural numbers:

a) $f(x) = 2x + 5$

b) $g(x) = -3x + 10$
Solution

Since the domain of an arithmetic sequence is the set of natural numbers, \( N = \{1, 2, 3, 4, \ldots \} \), the first five terms are as follows.

a) \( f(x) = 2x + 5 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( f(1) = 2(1) + 5 = 2 + 5 = 7 )</td>
</tr>
<tr>
<td>2</td>
<td>( f(2) = 2(2) + 5 = 4 + 5 = 9 )</td>
</tr>
<tr>
<td>3</td>
<td>( f(3) = 2(3) + 5 = 6 + 5 = 11 )</td>
</tr>
<tr>
<td>4</td>
<td>( f(4) = 2(4) + 5 = 8 + 5 = 13 )</td>
</tr>
<tr>
<td>5</td>
<td>( f(5) = 2(5) + 5 = 10 + 5 = 15 )</td>
</tr>
</tbody>
</table>

The first five terms are 7, 9, 11, 13, 15.

The common difference is 2.

b) \( g(x) = -3x + 10 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( g(1) = -3(1) + 10 = -3 + 10 = 7 )</td>
</tr>
<tr>
<td>2</td>
<td>( g(2) = -3(2) + 10 = -6 + 10 = 4 )</td>
</tr>
<tr>
<td>3</td>
<td>( g(3) = -3(3) + 10 = -9 + 10 = 1 )</td>
</tr>
<tr>
<td>4</td>
<td>( g(4) = -3(4) + 10 = -12 + 10 = -2 )</td>
</tr>
<tr>
<td>5</td>
<td>( g(5) = -3(5) + 10 = -15 + 10 = -5 )</td>
</tr>
</tbody>
</table>

The first five terms are 7, 4, 1, \(-2\), \(-5\).

The common difference is \(-3\).

If you compare the common difference of each arithmetic sequence to its defining linear function, you may notice that the common difference has the same value as the slope. In other words, the slope of the line is the increase in term value as you proceed from one term to the next.

Note: It will be very helpful for you to remember that common difference = slope in an arithmetic sequence.
Now that you know how linear functions are related to arithmetic sequences, you can go back and find $t_{100}$ in Example 5. You found the following pieces of information about the arithmetic sequence in Example 5.

$$
\begin{align*}
t_1 &= 50 \\
t_2 &= 42 \\
t_3 &= 34 \\
t_4 &= 26 \\
t_5 &= 18 \\
t_6 &= 10 \\
t_7 &= 2
\end{align*}
$$

The ordered pairs for the first seven terms of the function are:

$$(1, 50), (2, 42), (3, 34), (4, 26), (5, 18), (6, 10), (7, 2)$$

As every arithmetic sequence is a linear function, this set of points belongs to a straight line. Therefore, you can use the slope to find a defining linear function for this line to help you find $t_{100}$. Recall that the formula for slope given two points $(x_1, y_1)$ and $(x_2, y_2)$ on a line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The equation for a line given its slope, $m$, and a point, $(x_1, y_1)$, on the line is:

$$y - y_1 = m(x - x_1)$$

You can use any two points on the line to find its slope and equation.

If you use the first two points on the line, $(1, 50)$ and $(2, 42)$, then the slope is:

$$m = \frac{42 - 50}{2 - 1} = \frac{-8}{1} = -8$$

The defining linear function is:

$$y - y_1 = m(x - x_1)$$

$$y - 50 = -8(x - 1)$$

$$y = -8x + 8 + 50$$

$$y = -8x + 58$$

The defining function is $f(x) = -8x + 58$, whose slope is the common difference for the sequence. The domain of $f(x)$ is the set of natural numbers.
Now, it is easier to find \( t_{100} \). To find this value, replace \( x \) with 100.

\[
f(100) = -8(100) + 58
\]

\[
f(100) = -800 + 58
\]

\[
f(100) = -742
\]

Therefore, \( t_{100} = -742 \).

From this example, you can see that \( f(x) \) represents the term value whereas \( x \) represents the number of the term.

**Example 8**

What is the defining linear function that produces the following first four terms of an arithmetic sequence?

a) \( f(x) \): 1, 3, 5, 7, . . .

b) \( g(x) \): 2, 4, 6, 8, . . .

**Solution**

a) Since the domain of the function is the set of natural numbers, the ordered pairs of the function are \((1, 1), (2, 3), (3, 5), (4, 7), \ldots\)

As each set of points belongs to a straight line, use the slope to find its equation.

Using \( m = \frac{y_2 - y_1}{x_2 - x_1} \) and \( y - y_1 = m(x - x_1) \), then, \( m = \frac{3 - 1}{2 - 1} = \frac{2}{1} = 2 \).

The equation is: \( y - 1 = 2(x - 1) \)

\[
y = 2x - 2 + 1
\]

\[
y = 2x - 1
\]

The defining function is \( f(x) = 2x - 1 \).

b) The ordered pairs are \((1, 2), (2, 4), (3, 6), (4, 8), \ldots\)

\[
m = \frac{4 - 2}{2 - 1} = \frac{2}{1} = 2
\]

The equation is: \( y - 2 = 2(x - 1) \)

\[
y = 2x - 2 + 2
\]

\[
y = 2x
\]

The defining function is \( g(x) = 2x \).
General Term Formula

In general, the arithmetic sequence $t_1, t_2, t_3, \ldots$ can be written as $t_1, t_1 + d, t_1 + 2d, \ldots$ where $d$ is the common difference between consecutive terms. As we have seen, $d$ is the slope of the defining linear function. Notice the pattern for the number of $d$s in each term. In the first term, no $d$s; in the second term, one $d$; in the third term, two $d$s. Thus, there is always one less $d$ than the term number. In general, $t_n = t_1 + (n - 1)d$. This formula is called the general term of an arithmetic sequence. It may be helpful for you to include this explanation and formula on your resource sheet.

**Example 9**

Use the formula $t_n = t_1 + (n - 1)d$ to find $t_{10}$, $t_{21}$, and $t_{1000}$ of the arithmetic sequence 30, 36, 42, 48, . . . .

**Solution**

$t_1 = 30$ and $d = t_2 - t_1 = 36 - 30 = 6$ or

$d = t_3 - t_2 = 42 - 36 = 6$ or

$d = t_4 - t_3 = 48 - 42 = 6$

Therefore,

$t_{10} = t_1 + (10 - 1)d = 30 + 9(6) = 30 + 54 = 84$

$t_{21} = t_1 + (21 - 1)d = 30 + 20(6) = 30 + 120 = 150$

$t_{1000} = t_1 + (1000 - 1)d = 30 + 999(6) = 30 + 5994 = 6024$

The general term formula $t_n = t_1 + (n - 1)d$ can be used to solve questions about an arithmetic sequence such as finding the unknown values of $t_n$, $t_1$, $n$, or $d$.

**Example 10**

How many terms are in the arithmetic sequence 26, 28, 30, . . . , 1062?

**Solution**

Using the general term formula, $t_n = t_1 + (n - 1)d$, you have $t_n = 1062$, $t_1 = 26$, and $d = 2$. Now substitute these values into the formula and solve for $n$.

$1062 = 26 + (n - 1)(2)$

$1062 = 26 + 2n - 2$

$1062 = 24 + 2n$

$1038 = 2n$

$519 = n$

Therefore, 1062 = $t_{519}$. Hence, there are 519 terms in the sequence.
You could also solve Example 10 by finding the defining linear function, using the points (1, 26) and (2, 28).

\[ m = \frac{28 - 26}{2 - 1} = 2 \]

\[ y - 26 = 2(x - 1) \]

\[ y = 2x - 2 + 26 \]

\[ y = 2x + 24 \text{ or} \]

\[ f(x) = 2x + 24 \]

Since you know the value of the term to be 1062, you know \( f(x) \). Just substitute and solve for \( x \), its corresponding term number.

Therefore, \( 1062 = 2x + 24 \)

\[ 1038 = 2x \]

\[ 519 = x \]

Hence, 519 terms are in the sequence.

Make sure you complete the following learning activity to practice what you have just learned in this lesson. Recall that learning activities have two parts. The first part is a BrainPower section that is intended as a warm-up activity to prepare your mental muscles for Part B. Part B contains questions that allow you to practice the material you have just learned in the lesson. Learning activities do not need to be handed in. However, completing every learning activity will give you an idea of what types of questions will be asked on upcoming assignments and the exams.
Learning Activity 1.1

Complete the following, and check your answers in the learning activity answer keys found at the end of this module.

**Part A: BrainPower**

The BrainPower questions are provided as a warm-up activity for your brain before trying the questions in Part B. Try to complete each question quickly, without the use of a calculator and without writing many steps on paper.

1. At the end of a poker game, Alysha had $62. During the game she had won $250, lost $175, won $35, and lost $55. How much money did Alysha have at the start, in dollars?

2. Complete the pattern: 1, 2, 5, 10, 17, ___, ___, ____

3. Evaluate when $z = -1$: $(3z + 1)(z - 5)$

4. Evaluate: $\left(\frac{8}{16}\right)(12)$

5. A clothing store is selling shirts for $30.00. They decided to raise the price of the shirt by 20%. A few days later they decided to raise the price another 20%. What is the new price of the shirt, in dollars?

Use the following right-angled triangle to answer the next three questions. Do not simplify your answer.

6. Identify the sine ratio for $\theta$.

7. Identify the cosine ratio for $\theta$.

8. Identify the tangent ratio for $\theta$.  

*continued*
Learning Activity 1.1 (continued)

Part B: Arithmetic Sequences

Remember, these questions are similar to the ones that will be on your assignments and exams. So, if you were able to answer them correctly, you are likely to do well on your assignments and exams. If you did not answer them correctly, you need to go back to the lesson and learn them.

1. Which of the following sequences are arithmetic?
   a) 2, 5, 8, 11, . . .
   b) -5, -1, 3, 7, . . .
   c) 2, 5, 7, 10, . . .
   d) 2, $2 \frac{1}{2}$, 1, $1 \frac{1}{2}$, 0, . . .

2. Write a recursive formula for each sequence. Indicate whether the sequence is arithmetic or not.
   a) 10, 7, 4, 1, . . .
   b) 2, 4, 8, 16, . . .
   c) 30, 41, 52, 63, . . .
   d) 3, -6, 12, -24, . . .
   e) 5, 15, 45, 135, . . .

3. Write the first four terms of each sequence.
   a) \[
   \begin{align*}
   t_1 &= 3 \\
   t_n &= t_{n-1} + 5, \ n \geq 2
   \end{align*}
   \]
   b) \[
   \begin{align*}
   t_1 &= 10 \\
   t_n &= t_{n-1} - 2, \ n \geq 2
   \end{align*}
   \]
   c) \[
   \begin{align*}
   t_1 &= 3.7 \\
   t_n &= t_{n-1} + 1.3, \ n \geq 2
   \end{align*}
   \]
   d) \[
   \begin{align*}
   t_1 &= 1 \\
   t_2 &= 1 \\
   t_n &= t_{n-1} + t_{n-2}, \ n \geq 3
   \end{align*}
   \]

continued
Learning Activity 1.1 (continued)

\[
\begin{align*}
&\begin{cases}
  t_1 = 3 \\
  t_2 = -1 \\
  t_n = t_{n-1} + t_{n-2}, \ n \geq 3
\end{cases}
\end{align*}
\]

e) \quad \begin{align*}
  t_1 &= 3 \\
  t_2 &= -1 \\
  t_n &= t_{n-1} + t_{n-2}, \ n \geq 3
\end{align*}

4. Write the first three terms of the arithmetic sequence defined by the linear function.
   a) \( f(x) = 2x + 4 \)
   b) \( g(x) = 5x - 2 \)
   c) \( h(x) = -3x + 2 \)
   d) \( m(x) = x - 5 \)

5. Three terms of an arithmetic sequence (not necessarily the first three terms) are 27, 30, 33. Write two different linear functions that may have produced this sequence. The domain is the set of natural numbers.

6. Find an equation of the linear function that produces the following first three terms of an arithmetic sequence.
   a) \( 5, 8, 11, \ldots \)
   b) \( 6, 6\frac{1}{2}, 7, \ldots \)
   c) \( 10, 8, 6, \ldots \)
   d) \( 2, 2\frac{1}{10}, 2\frac{1}{5}, \ldots \)

7. Find the 50th term of each of the following arithmetic sequences.
   a) \( f(x): 7, 11, 15, \ldots \)
   b) \( g(x): 10, 15, 20, \ldots \)
   c) \( h(x): 4\frac{1}{2}, 5, 5\frac{1}{2}, \ldots \)

8. The fifth term of an arithmetic sequence is –15 and the ninth term is –3. Find the value of the hundredth term.
9. A pile of bricks is arranged in rows. The numbers of bricks in the rows form an arithmetic sequence. One such pile of bricks with 3 rows is shown below:

```
  X28 X74 X68
  X69 X72 X64
  X20 X72 X6F
```

(First row)
(second row)
(third row)

In another pile of bricks, there are 45 bricks in the fifth row and 33 bricks in the eleventh row.

a) How many bricks are in the first row?
b) How many bricks are in the \( n \)th row?
c) What is the maximum number of rows of bricks possible?

10. Find which term of the sequence 20, 23, 26, \ldots \) is the value 245,

a) by using the general term formula
b) by using the defining linear function

11. Find the term indicated in each arithmetic sequence by using the general term formula.

a) 5, 3, 1, \ldots \( t_{20} \)
b) 8, 11, 14, \ldots \( t_{21} \)
c) 6, 3, 0, -3, \ldots \( t_{57} \)
d) 18, 17, 16, 15, \ldots \( t_{132} \)
e) 7, 8.3, 9.6, 10.9, \ldots \( t_{172} \)

12. Joe bought a painting for $1800. After seven years, this painting sold for $10,550. If the painting appreciated arithmetically, determine the annual amount of appreciation.

13. For the arithmetic sequence 7, 11, 15, 19, \ldots \) find \( t_{29} \).

14. Give an example of an arithmetic sequence, and then

a) prove your sequence is an arithmetic sequence
b) write the defining linear function of your sequence
c) write your arithmetic sequence using the general term formula

15. Graph the arithmetic sequence 7, 4, 1, -2, \ldots , for the first six terms

a) on graph paper (A reproducible copy of a sheet of graph paper is included in Appendix B found on the downloads page.)
b) on the graphing calculator
Now that you have completed your first lesson and your first learning activity in Grade 11 Pre-Calculus, it will be helpful for you to contact your tutor/marker. Part of Learning Activity 1.2 will be to do so. As mentioned in the introduction to this course, your tutor/marker is available if you need help learning any of the material. Also, your tutor/marker will be marking all of your assignments and exams. Your tutor/marker’s contact information is on the cover sheet that came with this course. If you are unable to find your cover sheet, phone the Independent Study Option at 1-800-465-9915 and someone will provide you with the information you need.

Learning Activity 1.2

This learning activity is the only one that doesn’t include a BrainPower section, although it still has two parts.

Part A: Contacting Your Tutor/Marker

Your first task is to contact your tutor/marker by phone.

Fill in the following blanks using information provided with your course:

My tutor/marker’s name is _____________________________________

I can phone my tutor/marker at 1-866-____________________________

My tutor/marker’s email address is ______________________________

Be ready to discuss the following topics and the reasons for your answers with your tutor/marker and learning partner. If you like, make some notes before you call so that you feel prepared. Feel free to add any other questions or comments that you may have.

1. I am taking this course by distance education because:

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

continued
Learning Activity 1.2 (continued)

2. What I like about math and can do mathematically is (include favourite topic, skill, where you use math, etc.):
   
   
   
   
   
3. What I dislike about math or have difficulty doing is:
   
   
   
   
   
4. Previous math experiences that influence the way I feel about math are:
   
   
   
   
   
5. The next math course I would like to take is:
   
   
   
   
   
continued
Learning Activity 1.2 (continued)

6. What I am hoping this course will help me accomplish and learn for the future:

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

7. What I am doing and how I organize things to help me succeed in this course is:

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

During your phone conversation, jot down a sentence or two about what you and your tutor/marker talk about, in the spaces above. For example, if you are taking this course because it doesn’t fit into your schedule at school or because you travel with your basketball team and would miss a large number of classes, state that in the space below question 1.

continued
Learning Activity 1.2 (continued)

Part B: Your Math Pathway

Use the answers to the questions from the conversation with your tutor/marker as a starting point to fill in the following diagram. In the Math History box, jot down point-form notes about your prior experience and knowledge about math (questions 2, 3, and 4). In the Math Destination box, jot down what completing this course will help you accomplish in the future (questions 5 and 6).

In the Pathway box, write down what you will need to do to move along the pathway from your History to your Destination.

<table>
<thead>
<tr>
<th>Math History</th>
<th>Pathway</th>
<th>Math Destination</th>
</tr>
</thead>
</table>

For example, your destination might be receiving at least 75% in this course so you feel confident going into Grade 12 Pre-Calculus Mathematics so that you can take nursing after you graduate. Your destination could also be that you need to learn how to solve equations, and are trying to determine what will help you accomplish this. It may mean figuring out how you best learn and study math. It may mean setting up a schedule so you complete the assignments on time. You may need to find your calculator manual and figure out how to use it, set up regular appointments with your learning partner, research a topic on the Internet, or read a textbook about a certain math concept or skill. Your pathway is unique to you.

continued
Learning Activity 1.2 (continued)

As you move through this course and work on achieving your goals, self-assessment is important for you to determine whether you are getting closer to your destination. It helps you determine whether the steps along your pathway are taking you in the right direction. You will need to periodically ask yourself: Am I doing my assignments? Are my note-taking skills improving? How often have I contacted my tutor/marker or worked with my learning partner? Have I found useful homework websites if necessary? Is my schedule working? What do I need to change or adjust so I can get to my destination?

Repeatedly going through this cycle of looking at where you have been, where you want to go, and where you currently are is recommended. At any time, you may want to revise your goals or set new ones as you evaluate your own progress and learning.

- Look back/history. Reflect on what you know, how far you have come.
- Look around/pathway. Assess if you are achieving your goals, determine if new learning or understanding has occurred, and check your progress.
- Look forward/destination. Determine what you want to know, set goals.

Each time you go through these steps, you will become better at mathematics!

It is important that you keep this diagram handy, as you will revisit it at other points in this course.

Lesson Summary

In this lesson, you learned about arithmetic sequences and how they are formed. You also looked at how arithmetic sequences are related to linear functions. From these linear functions, you noticed that the slope of a linear function was equal to the common difference of an arithmetic sequence. In the next lesson, you will be building your knowledge about arithmetic sequences and learning about arithmetic series.