GRADE 12 PRE-CALCULUS MATHEMATICS (40S)

Midterm Practice Exam
Instructions

The midterm examination will be weighted as follows:

- Modules 1–4: 100%

Time allowed: 3 hours

**Note:** You are allowed to bring the following to the exam: pens/pencils (2 or 3 of each), blank paper, a ruler, a scientific calculator, and your Midterm Exam Resource Sheet. Your Midterm Exam Resource Sheet must be handed in with the exam. You will receive your Midterm Exam Resource Sheet back from your tutor/marker with the next module work that is submitted for marking.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer.
General Marking Principles

- Concepts learned in Grade 12 are worth 1 mark each. Concepts learned earlier (unless they are retaught under the curriculum, e.g., absolute value, reciprocals) are worth 0.5 mark each.
- Some errors are deducted only once (e.g., not putting arrowheads on graphs).
- Errors are followed through (e.g., if an arithmetic error is made in the first line, it is still possible for the student to receive nearly full marks).
- Many types of communication errors receive a 0.5 mark deduction, but 0.5 mark is the maximum communication error deduction for the entire exam.
Name: _________________________________

Answer all questions to the best of your ability. Show all your work.

Long-Answer Questions (100 marks)

1. Given the sketch of $f(x)$ drawn below, show each transformation algebraically and graphically. State the domain and range of each function.

![Graph of f(x)](image)

a) $y = \frac{1}{2} f(x) - 2 \quad (3 \text{ marks})$
b) \( y = f\left(\frac{1}{2}(x + 4)\right) \) (3 marks)
2. The \( y \)-intercept of the function \( g(x) \) is 4. What would the new \( y \)-intercept be for each of the following? \((3 \times 1 \text{ mark each} = 3 \text{ marks})\)

a) \( y = -2g(x) \)

b) \( y = g(x) + 1 \)

c) \( y = g(3x) \)

d) \( y = g(-x) \)
3. Use the graph of the function drawn below to sketch \( y = \frac{1}{f(x)} \). (2 marks)
4. Using the sketch of \( f(x) \), sketch the following.

\[ y = -3f(x) \quad (2 \text{ marks}) \]

Name: ________________________________
b) \( y = f(-2x) \)  \hspace{1cm} (2 \text{ marks})

c) \( y = f^{-1}(x) \)  \hspace{1cm} (2 \text{ marks})
5. In how many ways can you order 13 songs in a playlist if
   a) there are no restrictions? (1 mark)

   b) your favourite song must be first? (2 marks)

6. How many distinct ways can 4 green cups, 2 blue cups, and 1 red cup be arranged on a shelf? (2 marks)
7. Four men and five women are on a parent council committee. In how many ways can a five-member subcommittee be formed if the women must have a majority on this subcommittee? (4 marks)

8. Evaluate each of the following using factorial notation. Show your work. (4 × 2 marks each = 8 marks)
   a) \( _3P_2 \)
   
   b) \( _5P_2 \)
   
   c) \( _7C_3 \)
   
   d) \( _6C_2 \)
9. Solve without using a calculator. (3 marks)

\( n+3 \binom{2}{2} = 20 \)
10. Given the following row of Pascal’s Triangle, determine the next row. (1 mark)

<p>| | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>55</td>
<td>165</td>
<td>330</td>
<td>462</td>
<td>462</td>
<td>330</td>
<td>165</td>
<td>55</td>
<td>11</td>
<td>1</td>
</tr>
</tbody>
</table>

11. Write and simplify the last term of the expansion of \( \left(2 + \frac{2}{x^2}\right)^4 \). (3 marks)
12. Expand and simplify \((2x - 3)^4\) using the Binomial Theorem. (4 marks)
13. For the function \( y = x^3 - 3x^2 - x + 3 \), find the following:
   
   a) the zeros of the function if you know that \((x + 1)\) is a factor of the polynomial \((4 \text{ marks})\)

   b) left-right behaviour \((1 \text{ mark})\)

   c) the \(y\)-intercept of the function \((1 \text{ mark})\)
d) the sketch of the graph of the function (2 marks)
14. Graph a quartic that has roots of $-1$ and $+2$ and a root with a multiplicity 2 at $+1$. The function equation has a leading coefficient of $-3$. (3 marks)
15. If \( f(x) = \frac{1}{x - 3} \) and \( g(x) = |x| \), determine the equation and graph of \( h(x) = g(f(x)) \).

State the domain and range. (5 marks)
16. Consider the graphs of \( f(x) \) and \( g(x) \) below. Use these graphs to sketch \( (g - f)(x) \).

(3 marks)
17. The following graph represents a transformation of \( f(x) = |x| \). Write an expression for the new absolute value function. (2 marks)
18. The graph of a function $f(x)$ is drawn below.

![Graph of $f(x)$]

a) Reflect the graph of $f(x)$ in the line $y = x$ to achieve the graph of $g(x)$. (1 mark)
b) Write the equation of the new function $g(x)$ in terms of $f(x)$. (1 mark)

c) Reflect the graph of $f(x)$ in the $x$-axis to achieve the graph of $h(x)$. (1 mark)

d) Write the equation of the new function $h(x)$ in terms of $f(x)$. (1 mark)
19. A function contains the ordered pairs \((-1, 0), (0, 6), \) and \((-3, 7)\). What are the corresponding ordered pairs if this function is reflected through the following lines? 
\(3 \times 1 \text{ mark each} = 3 \text{ marks}\)

a) \(y = 0 \) (x-axis)

b) \(x = 0 \) (y-axis)

c) \(y = x\)
20. For each of the following relations, determine if they are one-to-one functions. Explain your reasoning. (2 \times 1 \text{ mark each} = 2 \text{ marks})

a) 

b)
21. Show algebraically that the functions \( f \) and \( g \) are inverses of each other. (2 marks)

\[
f(x) = \sqrt{4x - 1} \\
g(x) = \frac{x^2 + 1}{4}
\]
22. Find $f^{-1}(x)$ algebraically. Graph $f^{-1}(x)$. Consider the domain and range of $f^{-1}(x)$. (3 marks)

$$f(x) = (x - 6)^2, \quad x \leq 6$$
23. Given that \( f(x) = |2x - 1| \) and \( g(x) = x^3 - 1 \), find the following.

(3 \times 1 \text{ mark each} = 3 \text{ marks})

a) \( f(f(x)) \)

b) \( g(g(-1)) \)

c) \( f(g(2)) \)

24. Given: \( f(x) = x^3 \) and \( g(x) = x - 2 \)

a) Determine \( f(g(x)) \) and describe the graph of \( f(g(x)) \) in terms of a transformation of \( f(x) \).

(2 \text{ marks})

b) Determine \( g(f(x)) \) and describe the graph of \( g(f(x)) \) in terms of a transformation of \( f(x) \).

(2 \text{ marks})
25. For each of the following polynomials, determine whether it is divisible by $x - 2$. Show your work. (2 × 2 marks each = 4 marks)
   a) $f(x) = -x^4 + x^3 - 8x^2 + 6$
   b) $g(x) = -x^3 + x^2 - 5x + 14$

26. Divide, using long division or synthetic division, and write in the form given by the division algorithm. (3 marks)
   $(2x^3 - 4x^2 - 12x - 14) \div (x - 4)$
27. Factor \( g(x) = x^4 + 2x^3 - 20x^2 - 66x - 45 \) completely. (5 marks)