Final Practice Exam
Answer Key
Instructions

The final examination will be weighted as follows:

- Modules 1–8 100%

Time allowed: 3 hours

**Note:** You are allowed to bring the following to the exam: pens/pencils (2 or 3 of each), blank paper, a ruler, a scientific calculator, and your Final Exam Resource Sheet. Your Final Exam Resource Sheet must be handed in with the exam.

Show all calculations and formulas used. Use all decimal places in your calculations and round the final answers to the correct number of decimal places. Include units where appropriate. Clearly state your final answer.
General Marking Principles

- Concepts learned in Grade 12 are worth 1 mark each. Concepts learned earlier (unless they are retaught under the curriculum, e.g., absolute value, reciprocals) are worth 0.5 mark each.
- Some errors are deducted only once (e.g., not putting arrow heads on graphs).
- Errors are followed through (e.g., if an arithmetic error is made in the first line, it is still possible for the student to receive nearly full marks).
- Many types of communication errors receive a 0.5 mark deduction, but 0.5 mark is the maximum communication error deduction for the entire exam.
Answer all questions to the best of your ability. Show all your work.

Long-Answer Questions (100 marks)

1. Given that \( f(x) = \frac{1}{x} \) and \( g(x) = x^3 + 6x - 3 \), find the following. (2 marks)

(Module 2, Lesson 5)

a) \( f(f(x)) \)

Answer:

\[
f(f(x)) = \frac{1}{\frac{1}{x}} = x
\]

b) \( g(f(-2)) \)

Answer:

\[
g(f(-2)) = g\left(\frac{1}{2}\right) = \frac{1}{8}
\]
2. Divide, using long division or synthetic division, and write in the form given by the division algorithm. (2 marks) (Module 4, Lesson 2)

\((-x^3 - 4x^2 + 7x + 4) ÷ (x + 3)\)

**Answer:**

\[
\begin{array}{c|cccc}
-3 & -1 & -4 & 7 & 4 \\
 & 3 & 3 & -30 \\
\hline
 & -1 & -1 & 10 & -26 \\
\end{array}
\]

\(\therefore -x^3 - 4x^2 + 7x + 4 = (-x^2 - x + 10)(x + 3) - 26\)  

(0.5 mark for set up)

(1 mark for division)

(0.5 mark for division statement)

3. Rewrite \(2 \log x + \frac{1}{2} \log 5 - \frac{1}{3} \log (x + 2)\) as a single logarithm statement. (3 marks)  
(Module 7, Lesson 2)

**Answer:**

\[
\log x^2 + \log \sqrt{5} - \log \left(\sqrt[3]{x + 2}\right) = \log \frac{x^2 \sqrt{5}}{\sqrt[3]{x + 2}}
\]

(1 mark for product law)

(1 mark for quotient law)

(1 mark for power law)
4. Given the sketch of \( f(x) \) drawn below, sketch the following functions.

\[ y = f(x - 2) \]  \hspace{1cm} (1 mark) (Module 2, Lesson 1)

Answer:
b) \( y = |f(x)| + 2 \) \hspace{1cm} (2 marks) (Module 2, Lesson 5)

Answer:
5. Using the sketch of $f(x)$, sketch the following.

\[ y = -\frac{1}{2} f(x) \quad (2 \text{ marks}) \quad \text{(Module 3, Lesson 1)} \]

**Answer:**

[Diagram showing the graph of $y = -\frac{1}{2} f(x)$]
b) \( y = f(-2x) \)  \( (2 \text{ marks}) \) (Module 3, Lesson 2)

Answer:

![Graph of \( f(-2x) \)]

\( f(-2x) \)

\( \left( \frac{5}{2}, 3 \right) \)

\( 2 \)

\( 2 \)

\( x \)

\( y \)

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c) \( y = f^{-1}(x) \)  \( (1 \text{ mark}) \) (Module 3, Lesson 3)

Answer:

![Graph of \( f^{-1}(x) \)]

\( f^{-1}(x) \)

\( 2 \)

\( 2 \)

\( -2 \)

\( x \)

\( y \)
6. Write and simplify the fifth term of the expansion of \((x + 1)^8\). (2 marks) (Module 1, Lesson 5)

Answer:

\[
t_5 = \binom{8}{4} x^4 (1)^4 = 70x^4
\]

\(n = 8\)
\(k = 4\) 
1 mark for \(\binom{8}{4}\)
\(t_5 = \binom{8}{4}(x)^4(1)^4\)  
1 mark for consistent factors
\(t_5 = 70x^4\)

7. There are 9 boys and 11 girls in a Grade 12 English class. In how many ways can 5 students be chosen for a group project if the group must have 3 female members and 2 male members? (3 marks) (Module 1, Lesson 4)

Answer:

The committee must have 3 female members and 2 male members.

\[
\binom{9}{2} \binom{11}{3} = 5940
\]

There are 5940 possible group combinations for this project.

\(\binom{9}{2} \times \binom{11}{3}\)  
1 mark for \(\binom{9}{2}\)
5940 ways  
1 mark for \(\binom{11}{3}\) 
1 mark for multiplying the cases
8. Convert $1265^\circ$ to radians. Write the exact answer. (1 mark) (Module 5, Lesson 1)

Answer:

$$\left(1265^\circ\right) \left(\frac{\pi}{180}\right) = \frac{1265\pi}{180} = \frac{253\pi}{36}$$

9. You know that $\sin \alpha = -\frac{2}{7}$ and $\pi < \alpha < \frac{3\pi}{2}$. You also know that $P(\beta)$ is in Quadrant IV and $\cos \beta = \frac{4}{5}$. Find $\sin(\alpha + \beta)$. (4 marks) (Module 6, Lesson 4)

Answer:

The angle that satisfies these requirements is located in Quadrant III.

$\sin \alpha = -\frac{2}{7}$ and $\pi < \alpha < \frac{3\pi}{2}$ means $P(\alpha)$ is in Quadrant III

$\therefore \cos \alpha < 0$

$\sin^2 \alpha + \cos^2 \alpha = 1$

$$\left(-\frac{2}{7}\right)^2 + \cos^2 \alpha = 1$$

$$\cos^2 \alpha = \frac{45}{49}$$

$$\cos \alpha = -\frac{3\sqrt{5}}{7}$$

$\cos \beta = \frac{4}{5}$ and $P(\beta)$ is in Quadrant IV means $\sin \beta < 0$

$\sin^2 \beta + \cos^2 \beta = 1$

$\sin^2 \beta + \left(\frac{4}{5}\right)^2 = 1$

$$\sin^2 \beta = \frac{9}{25}$$

$$\sin \beta = -\frac{3}{5}$$

$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$$= \left(-\frac{2}{7}\right) \left(\frac{4}{5}\right) + \left(-\frac{3\sqrt{5}}{7}\right) \left(-\frac{3}{5}\right) = -\frac{8}{35} + \frac{9\sqrt{5}}{35} = -\frac{8 + 9\sqrt{5}}{35}$$
10. Consider the function \( f(x) = -(x - 1)(x + 3)(x - 7) \).
   a) Determine the end behaviour of the function. (1 mark) (Module 4, Lesson 1)
   \[ \text{Answer:} \]
   The graph points up to the left and down on the right. In other words, the graph begins in Quadrant II and ends in Quadrant IV.

   b) Find all \( x \)- and \( y \)-intercept(s). (2 marks) (Module 4, Lesson 1)
   \[ \text{Answer:} \]
   \( x = \{1, -3, 7\} \)
   \( y = -21 \)

   c) Sketch the function. (2 marks) (Module 4, Lesson 4)
   \[ \text{Answer:} \]
11. Sketch the function $g(x) = 9 - 3^x$ and state its range, $x$-intercept, and equation of asymptote. (4 marks) (Module 7, Lesson 1)

**Answer:**

- **Range:** $(-\infty, 9)$  
  (0.5 mark)
- **$x$-intercept:** 2  
  (1 mark)
- **Equation of asymptote:** $y = 9$  
  (0.5 mark)
- **Graph**  
  (2 marks)

12. Sketch the function $f(x) = \log_3 x - 2$. (2 marks) (Module 7, Lesson 4)

**Answer:**
13. Graph the following function using transformations. State the domain and range of the function. (4 marks) (Module 8, Lesson 1)

\[ g(x) = 2\sqrt{x} + 4 \]

Answer:

Diagram showing a graph of the function with labeled points and axes.

Domain: \( \{ x \mid x \geq 0 \} \) (0.5 mark)

Range: \( \{ y \mid y \geq 4 \} \) (0.5 mark)

(1 mark for correct radical shape)

(1 mark for vertical shift)

(1 mark for vertical stretch)
14. Given the following graph of \( f(x) \), graph \( \sqrt{f(x)} \). (4 marks) (Module 8, Lesson 2)

Answer:

(1 mark for invariant points at \( f(x) = 0 \) and \( f(x) = 1 \))
(1 mark for restricting domain to \( f(x) \geq 0 \))
(1 mark for \( \sqrt{f(x)} > f(x) \) when \( 0 < f(x) < 1 \))
(1 mark for endpoint at \( (-1, 2) \))
15. Graph the following function. Pay attention to whether the graph should have a point of discontinuity or a vertical asymptote. (4 marks) (Module 8, Lesson 5)

\[ y = \frac{4x}{x - 1} \]

**Answer:**
Non-permissible value at \( x = 1 \)
The non-permissible value at \( x = 1 \) represents a vertical asymptote.
Horizontal asymptote at \( y = 4 \).

Find points on either side of \( x = 1 \).

\[ x = 2 \]
\[ y = \frac{4(2)}{2 - 1} = 8 \]

Point: \((2, 8)\)

\[ x = 0 \]
\[ y = \frac{4 \cdot 0}{0 - 1} = 0 \]

Point: \((0, 0)\)
16. Graph the following function. Pay attention to whether the graph should have a point of discontinuity or a vertical asymptote. (5 marks) (Module 8, Lesson 5)

\[ y = \frac{x + 1}{x^2 - 4x - 5} \]

Answer:

\[ y = \frac{x + 1}{(x + 1)(x - 5)} \]

Non-permissible values at \( x = -1 \) and \( x = +5 \).

Since \( x + 1 \) is also in the numerator, \( x = -1 \) represents a point of discontinuity, which occurs at \( y = -1 \).

\[ y = \frac{1}{(-1) - 5} = -\frac{1}{6} \]

\( \therefore \) point of discontinuity is \( (-1, -\frac{1}{6}) \).

\( x = 5 \) is vertical asymptote.

The horizontal asymptote is \( y = 0 \), since the degree of the denominator is larger than the numerator.

Graph \( y = \frac{1}{x - 5} \) with a hole at \( (-1, -\frac{1}{6}) \).

Points on each side of asymptote:

\( x = 6 \quad y = 1 \)
\( x = 4 \quad y = -1 \)

(1 mark for vertical asymptote)
(1 mark for horizontal asymptote)
(1 mark for point \( (-1, -\frac{1}{6}) \) as a point of discontinuity)
(1 mark for graph right of vertical asymptote)
(1 mark for graph left of vertical asymptote)
17. Solve for the variable in the equation $\log_2 64 = x$. (1 mark) (Module 7, Lesson 2)

Answer:

\[
\left(\sqrt{2}\right)^x = 64
\]

\[
\frac{1}{2}^x = 2^6
\]

\[
\frac{1}{2} x = 6
\]

\[
x = 12
\]

18. Solve the following equations. Your answer should be exact, whenever possible. Otherwise, round to two decimal places.

a) $5(3^x) = e^{x-1}$ (4 marks) (Module 7, Lesson 5)

Answer:

\[
\left(5 \cdot 3^x\right) = \ln \left(e^{x-1}\right)
\]

(0.5 mark for applying logs to both sides)

\[
\ln 5 + x \ln 3 = (x-1) \ln e
\]

(1 mark for Product Log Law; 1 mark for Power Log Law)

\[
\ln 5 + x \ln 3 = x \ln e - \ln e
\]

(0.5 mark for collecting like terms)

\[
x \ln 3 - x \ ln e = -\ln e - \ln 5
\]

(0.5 mark for isolating for $x$)

\[
x(\ln 3 - 1) = -1 - \ln 5
\]

(0.5 mark for correct evaluation of a quotient of logs)

\[
x = \frac{-1 - \ln 5}{\ln 3 - 1}
\]

\[
x = \frac{-2.609437912}{0.098612288}
\]

\[
x = -26.46
\]
b) \( \log_2 (x - 4) + \log_2 (x - 3) = 1 \)  

\[ \log_2 [(x - 4)(x - 3)] = 1 \]  

\[ x^2 - 7x + 12 = 2^1 \]  

\[ x^2 - 7x + 10 = 0 \]  

\[ (x - 5)(x - 2) = 0 \]  

\[ x = 2 \quad x = 5 \]  

\[ x = 2 \text{ is extraneous} \]  

\[ \therefore x = 5 \]  

19. A bacteria culture is growing according to the formula \( y = 1000e^{0.6t} \), where \( t \) is the time in days.

a) Determine the number of bacteria after 7 days. (2 marks) (Module 7, Lesson 7)

\[ y = 1000e^{0.6(7)} = 1000e^{4.2} = 66,686 \text{ bacteria} \]

b) How long will it take the bacteria culture to triple in size? (3 marks) (Module 7, Lesson 7)

\[ 3x = xe^{0.6t} \]

\[ 3 = e^{0.6t} \]

\[ \ln 3 = \ln e^{0.6t} \]

\[ \ln 3 = 0.6t (\ln e) \]

\[ t = \frac{\ln 3}{0.6} = 1.83 \text{ days} \]
20. Determine all of the angles that are coterminal with $\frac{2\pi}{3}$ over the domain $[-2\pi, 4\pi]$.

(2 marks) (Module 5, Lesson 1)

Answer:

\[
\frac{2\pi}{3} - \frac{6\pi}{3} = \frac{-4\pi}{3}
\]

\[
\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}
\]

The two angles that are coterminal with $\frac{2\pi}{3}$ over the domain $[-2\pi, 4\pi]$ are

\[-\frac{4\pi}{3} \text{ and } \frac{8\pi}{3}.
\]

21. Determine all of the angles that are coterminal with $-261^\circ$ in general form. (1 mark) (Module 5, Lesson 1)

Answer:

$-261^\circ + 360^\circ n$, $n \in I$
22. Determine the exact value of each of the following expressions. (2 marks) (Module 5, Lesson 4)

a) \( \sec\left( -\frac{2\pi}{3} \right) \)

Answer:
\[
\sec\left( -\frac{2\pi}{3} \right) = \sec\left( \frac{4\pi}{3} \right) = \frac{1}{\cos\left( \frac{4\pi}{3} \right)} = \frac{1}{-\frac{1}{2}} = -2
\]

b) \( \cot(630^\circ) \)

Answer:
\[
cot(630^\circ) = \cot(270^\circ) = \frac{\cos(270^\circ)}{\sin(270^\circ)} = \frac{0}{-1} = 0
\]
23. Sketch the following curve. State the amplitude, phase shift, period, domain, and range of the function. (5 marks) (Module 5, Lesson 6)

\[ y = -\frac{1}{2} \sin \left( \frac{\pi}{2} x \right) - 1 \]

Answer:

Amplitude: \( \frac{1}{2} \)

Phase Shift: none

Period: \( \frac{2\pi}{\frac{\pi}{2}} = 2\pi \left( \frac{2}{\pi} \right) = 4 \)

Domain: \((-\infty, \infty)\)

Range: \([-1.5, -0.5]\)
24 Find the equation of the following graph as a cosine function. *(3 marks)*
(Module 5, Lesson 6)

\[ g(x) = 3 \cos \left( x - \frac{\pi}{2} \right) - 2 \]

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**Answer:**

Here is one possible answer for a trigonometric equation of the form

\[ y = A \cos (B(x - C)) + D \] (the value of \( C \) could be \(-\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \text{etc.}\):)

\[ g(x) = 3 \cos \left( x - \frac{\pi}{2} \right) - 2 \]

(1 mark for correct value of \( A \))

(1 mark for correct value of \( C \))

(1 mark for correct value of \( D \))
25. Solve the following equations over the indicated intervals. Provide exact answers wherever possible. Write answers as exact values.

a) \(2 \sin^2 \theta + 3 \cos \theta = 3\), where \(-2\pi \leq \theta \leq 0\)  

Answer:

\[2 \left(1 - \cos^2 \theta\right) + 3 \cos \theta = 3\]  
(1 mark for identity)

\[2 - 2 \cos^2 \theta + 3 \cos \theta - 3 = 0\]  
(1 mark for: distribute collect like terms multiply both sides by \(-2\))

\[-2 \cos^2 \theta + 3 \cos \theta - 1 = 0\]

\[2 \cos^2 \theta - 3 \cos \theta + 1 = 0\]

\[(2 \cos \theta - 1)(\cos \theta - 1) = 0\]  
(1 mark for: factor solve)

\[\cos \theta = \frac{1}{2}\]

\[\cos \theta = 1\]

Related: \(\theta = \frac{\pi}{3}\) or \(\theta = 0\)  
(0.5 mark)

Final answers:  
\(\theta = \frac{-\pi}{3}\)  
\(\theta = 0\)  
(1.5 marks)

\[\theta = \frac{-5\pi}{3}\]

b) \(6 \sin^2 x + 5 \sin x + 1 = 0\), where \(0 \leq x < 2\pi\).  

Answer:

\[(3 \sin x + 1)(2 \sin x + 1) = 0\]

\[\sin x = -\frac{1}{3}\]

\[\sin x = -\frac{1}{2}\]

Related arc \(= \sin^{-1} 0.3398\) and \(x = \frac{7\pi}{6}, \frac{11\pi}{6}\)

\[x = \pi + 0.3398, 2\pi - 0.3398, \frac{7\pi}{6}, \frac{11\pi}{6}\]

\[x = 3.48, 5.94, \frac{7\pi}{6}, \frac{11\pi}{6}\]
26 Explain the difference between a trigonometric identity and a trigonometric equation.  
(1 mark) (Module 6, Lesson 2)

Answer:
A trigonometric equation is only true for certain values of the variable, while a trigonometric identity is true for all values of the variable that are in its domain.

27 Consider the equation \( \sin(2x) = -\cos\left(2\left(x + \frac{\pi}{4}\right)\right) \).

a) Graph \( y = \sin(2x) \) and \( y = -\cos\left(2\left(x + \frac{\pi}{4}\right)\right) \) on the same coordinate grid. (4 marks)  
(Module 5, Lesson 6 and Module 6, Lesson 5)

Answer:

b) Using the graph you created in (b), do you believe this demonstrates an identity?  
(1 mark) (Module 6, Lesson 5)

Answer:
Yes, because the two graphs are identical.
28. Prove the identity \( \frac{2 \sin x}{\sin 2x} = \sec x \). \((2 \text{ marks}) \) (Module 6, Lesson 5)

**Answer:**

\[
\text{LHS} = \frac{2 \sin x}{\sin (2x)}
\]

\[
= \frac{2 \sin x}{2 \sin x \cos x}
\]

\[
= \frac{1}{\cos x}
\]

\[
= \sec x
\]

\[
= \text{RHS}
\]

29. Prove the identity \( \cos 2 \theta + 2 \sin^2 \theta = 1 \). \((2 \text{ marks}) \) (Module 6, Lesson 5)

**Answer:**

\[
\text{LHS} = \cos 2 \theta + 2 \sin^2 \theta
\]

\[
= 1 - 2 \sin^2 \theta + 2 \sin^2 \theta
\]

\[
= 1
\]

\[
= \text{RHS}
\]

30. Find the exact value of \( \frac{\tan 80^\circ + \tan 55^\circ}{1 - \tan 80^\circ \cdot \tan 55^\circ} \). \((1 \text{ mark}) \) (Module 6, Lesson 4)

**Answer:**

\[
\tan (80^\circ + 55^\circ) = \tan 135^\circ
\]

\[
= -1
\]