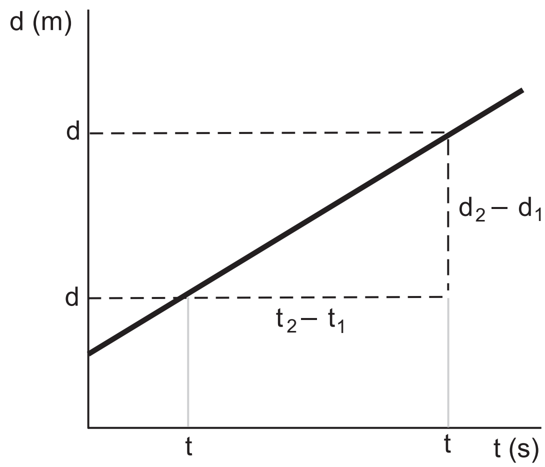
APPENDIX 1: MECHANICS

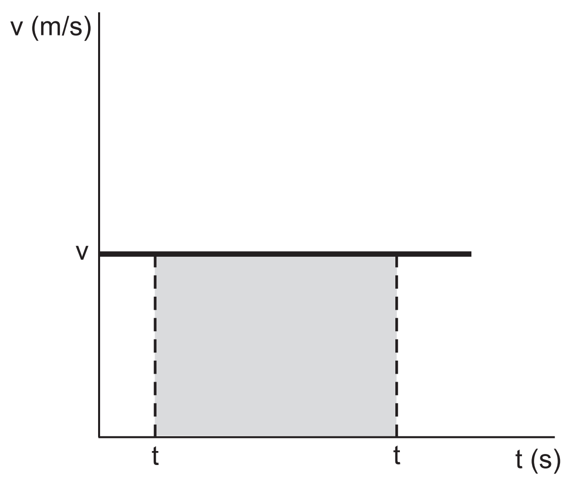
Appendix 1.1: Derivations for Constant Motion

Derivations for Constant Motion

The *position-time* graph for constant motion is a straight oblique line.



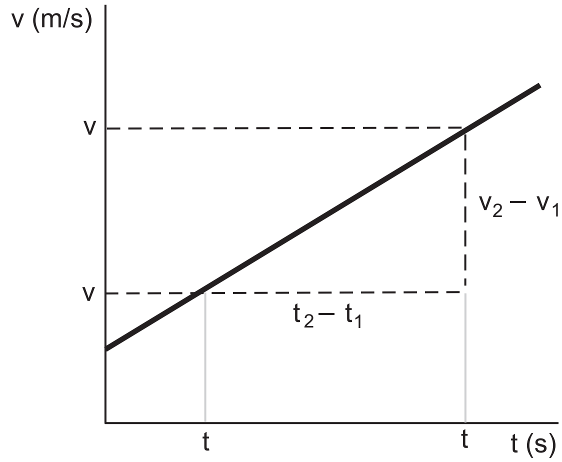
As demonstrated in Senior 3 Physics, to derive average velocity, $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$, find the slope of the position-time graph by the slope formula.



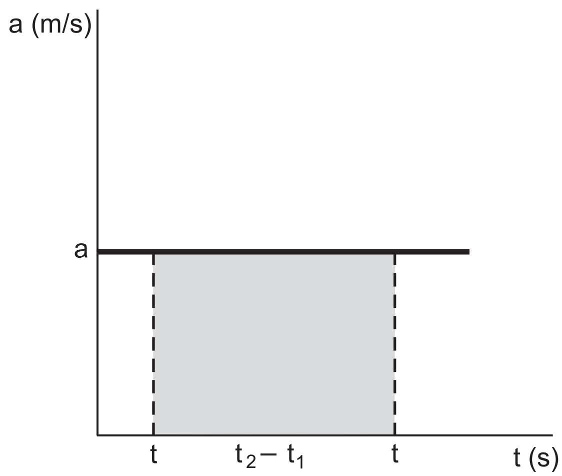
The velocity-time graph for constant motion is a horizontal line. The area between the line and the horizontal axis is a rectangle (Area = length x width) and corresponds to the displacement $\Delta \vec{d} = \vec{v}\Delta t$.

Derivations for Accelerated Motion

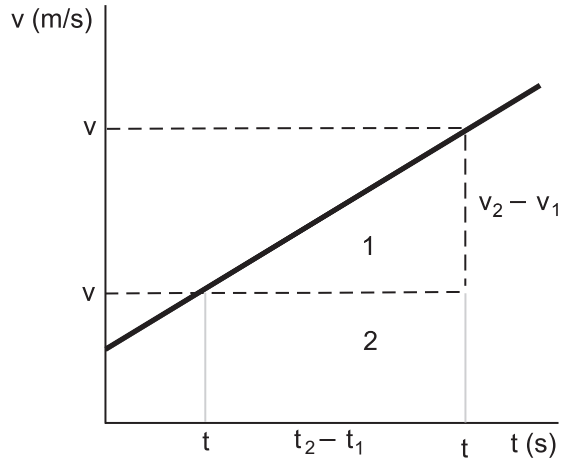
The *velocity-time* graph for accelerated motion is a straight oblique line.



As demonstrated in Senior 3 Physics, to derive average acceleration, $\bar{a}_{ave} = \frac{\overline{\Delta v}}{\Delta t}$, find the slope of the velocity-time graph by the slope formula. The slope of the velocity-time graph gives $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$.



The area between the line and the horizontal axis of the velocity-time graph is a trapezoid and corresponds to the displacement of the object for that time interval. The trapezoid can be divided into a rectangle and a triangle.



Then the total area and displacement is the sum of these areas.

$\Delta \vec{d} =$ area of triangle + area of rectangle

$$\Delta \vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\left(\vec{v}_2 - \vec{v}_1\right)\Delta t$$

But $\vec{v}_2 - \vec{v}_1 = \vec{a}\Delta t$, so substitute $\vec{a}\Delta t$ for $\vec{v}_2 - \vec{v}_1$, giving

$$\Delta \vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\left(\vec{a}\Delta t\right)\Delta t$$

and

$$\Delta \vec{d} = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}\Delta t^2$$

The final equation is derived by eliminating Δt from two equations.

The displacement can be found using the average velocity for the interval ($\Delta d = v_{ave}\Delta t$).

Therefore:

1. $\Delta \vec{d} = \left(\dfrac{\vec{v}_1 + \vec{v}_2}{2}\right)\Delta t$

   and

2. $\vec{v}_2 + \vec{v}_1 = \vec{a}\Delta t$

Ignore vector notation, since the equations involve the products of vectors.

Solve (1) for Δt to obtain

3. $\Delta t = \dfrac{2\Delta d}{v_1 + v_2}$

Substitute this expression for Δt in (2)

$$a\frac{2\Delta d}{v_1 + v_2} = v_2 - v_1$$

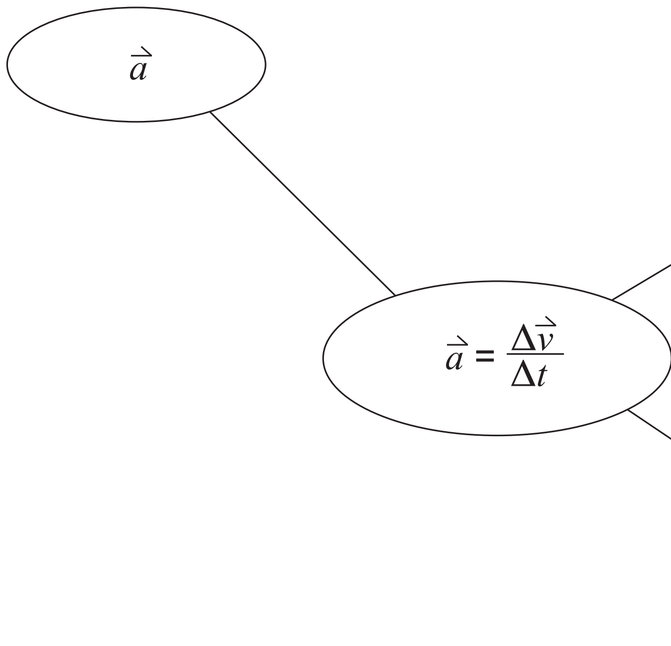
Multiply across and

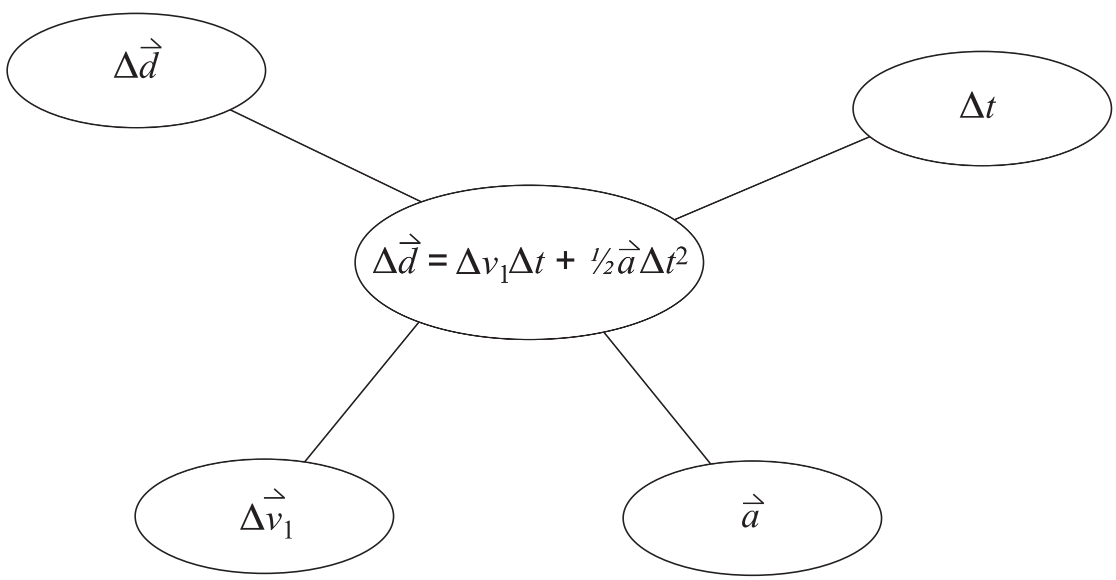
$$2a\Delta d = (v_2 - v_1)(v_2 + v_1)$$

Therefore, $2a\Delta d = v_2^2 - v_1^2$

Or, $v_2^2 - v_1^2 = 2a\Delta d$

Appendix 1.2: Category Concept Map





Appendix 1.3: Kinematics Problem Set

Sample Problems

Problem 1: A dragster accelerates from rest, covering a 400.0 m distance in 8.00 seconds. (The acceleration is constant.)

a) Calculate the average acceleration during this time.

b) Calculate the final velocity of the dragster.

c) Calculate the average velocity of the dragster.

d) Calculate the velocity 4.00 seconds after the dragster began to move.

e) Compare the displacement of the dragster during the first 4.00 seconds and the last 4.00 seconds of the trip. Account for the difference.

Solution:

Identify:

$$\vec{v}_1 = 0 \text{ m/s}$$

$$\Delta\vec{d} = 400.0 \text{ m [right]}$$

$$\Delta t = 8.00 \text{ s}$$

$$\vec{a}_{\text{ave}} = ?$$

The average acceleration can be found by relating the displacement, initial velocity, and the time interval.

$$\Delta\vec{d} = \vec{v}_1\,\Delta t + \frac{1}{2}\vec{a}\,\Delta t^2$$

Answer:

$$400 \text{ m [right]} = (0 \text{ m/s})(8.00 \text{ s}) + \frac{1}{2}\left(\vec{a} \text{ m/s}^2\right)(8.00\ s)^2$$

$$400 \text{ m [right]} = 0 \text{ m} + 32.0\vec{a} \text{ m/s}^2\left(\text{s}^2\right)$$

$$\vec{a} = 12.5 \text{ m/s}^2 \text{ [right]}$$

$$\vec{v}_1 = 0 \text{ m/s}$$

$$\Delta\vec{d} = 400.0 \text{ m [right]}$$

$$\Delta t = 8.00 \text{ s}$$

$$\vec{a}_{ave} = 12.5 \text{ m/s}^2 \text{ [right]}$$

Since errors due to significant figures propagate to the next calculations, the given information should be used wherever possible. In this case, relate displacement, time interval, and the initial velocity with the final velocity.

$$\Delta\vec{d}\left(\frac{\vec{v}_1+\vec{v}_2}{2}\right)\Delta t$$

Answer:

$$400.0\text{ m }[\text{right}]=\left(\frac{0+\vec{v}_2}{2}\right)8.00\text{ s}$$

$$\frac{400.0\text{ m }[\text{right}]}{8.00\text{ s}}=\frac{\vec{v}_2}{2}$$

$$50.0\text{ m/s }[\text{right}]=\frac{\vec{v}_2}{2}$$

$$\vec{v}_2=100\text{ m/s }[\text{right}]$$

Look back and check. Check your answer with $\vec{v}_2=\vec{v}_1+\vec{a}\,\Delta t$.

$$\vec{v}_1=0\text{ m/s}$$

$$\Delta\vec{d}=400.0\text{ m }[\text{right}]$$

$$\Delta t=8.00\text{ s}$$

$$\vec{a}_{ave}=12.5\text{ m/s}^2\ [\text{right}]$$

$$\vec{a}_{ave}=?$$

Since the acceleration is constant, the initial and final velocities can simply be averaged.

$$\left(\frac{\vec{v}_1+\vec{v}_2}{2}\right)=\vec{v}_{\text{ave}}$$

Answer:

$$\left(\frac{0\text{ m/s}+100\text{ m/s }[\text{right}]}{2}\right)=\vec{v}_{\text{ave}}$$

$$\vec{v}_{\text{ave}}=50.0\text{ m/s }[\text{right}]$$

Look back and check. Check your answer using $\Delta\vec{d}=\left(\frac{\vec{v}_1+\vec{v}_2}{2}\right)\Delta t$.

$\vec{v}_1 = 0 \text{ m/s}$

$\Delta\vec{d} = 400.0 \text{ m [right]}$

$\Delta t = 4.00 \text{ s}$

$\vec{a}_{ave} = 12.5 \text{ m/s}^2 \text{ [right]}$

$\vec{v}_2 \text{ at } 4.00 \text{ s} = ?$

In this case, relate acceleration, time interval, and the initial velocity with the final velocity.

$$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$$

Answer:

$\vec{v}_2 = 0 \text{ m/s} + \left(12.5 \text{ m/s}^2 \text{ [right]}\right)4.00 \text{ s}$

$\vec{v}_2 = 50.0 \text{ m/s [right]}$

Look back and check. Double check the calculations.

$$\vec{v}_1 = 0 \text{ m/s}$$

Part I

Time interval 0 s to 4 s

$\Delta t = 4.00 \text{ s}$

$\vec{a}_{ave} = 12.5 \text{ m/s}^2 \text{ [right]}$

$\vec{v}_2 \; at \; 4.00 \text{ s} = 50.0 \text{ m/s [right]}$

$\Delta\vec{d} \text{ at } 4.00 \text{ s} = ?$

The displacement can be determined using initial velocity, final velocity, and the time interval.

$$\Delta\vec{d} = \left(\frac{\vec{v}_1 + \vec{v}_1}{2}\right)\Delta t$$

Answer:

$\Delta\vec{d} = \left(\frac{0 \text{ m/s} + 50.0 \text{ m/s [right]}}{2}\right)4.00 \text{ s}$

$\Delta\vec{d} = \left(25.0 \text{ m/s [right]}\right)\left(4.00 \text{ s}\right)$

$\Delta\vec{d} = 100 \text{ m [right]}$

Look back and check using $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$.

Part II

Time interval 4.00 s to 8.00 s

During the interval between 4.00 s and 8.00 s, the dragster travels from a position of 100 m [right] to a position 400 m [right] from the starting line. The dragster has travelled 300 m [right] during the last 4.00 s of the trip.

The dragster travelled 100 m [right] during the first 4.00 s. During these 4.00 s, the dragster was accelerating from rest and was moving with a small average velocity. During the second 4.00 s interval, the dragster was travelling at a much larger average velocity. Thus, for equal time intervals, the dragster will travel a greater distance during the time interval with the larger average velocity. Students have a tendency to assume the velocity is constant.

Problem 2: A car is travelling along a street. It accelerates from rest at 3.00 m/s^2 for 4.50 seconds. The car then travels at constant velocity for 12.0 seconds. At this time, the driver spies an amber light at the next intersection, steps on the brake, and brings the car to rest in 3.00 seconds.

a) Calculate the final velocity for the first interval.

b) Calculate the displacement for the first interval.

c) Calculate the displacement for the second interval.

d) Calculate the acceleration during the third interval.

e) Calculate the displacement for the third interval.

f) Determine the total displacement for the entire time interval.

Problem 2 requires that the student separate the motion into appropriate intervals for which the acceleration is constant. Each section is treated separately. The final velocity for one interval becomes the initial velocity of the next interval.

Students are often intimidated by problems that require many steps in their solution. Problems can be presented separated into smaller steps at the beginning of this section. As the students become more proficient at solving problems, more sophisticated questions can be asked.

Appendix 1.4: Inclined Planes

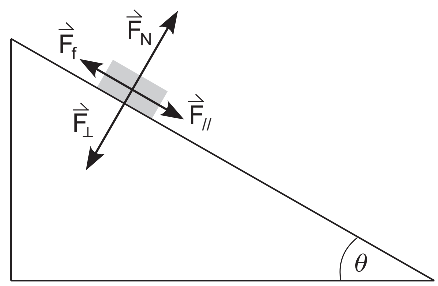
Illustrative Example 1

Students should solve problems for objects on inclined planes. For example:

A person in a wheelchair is travelling up an inclined sidewalk. The coefficient of friction is 0.11, and the mass of the person and the wheelchair is 65.0 kg. The degree of incline is 7°. Can the person rest comfortably on the inclined sidewalk, or will this person roll down the incline? Calculate the acceleration if the person cannot rest comfortably on the inclined sidewalk. Support your answer mathematically.

Solution:

Step 1: Draw a free-body diagram of the situation.



Step 2: Calculate the components of $\vec{F}_g$.

$$\vec{F}_{gx} = m\vec{g}\sin\theta$$

$$\vec{F}_{gx} = (65.0 \text{ kg})(9.8 \text{ m/s}^2)\sin 7.0^o$$

$$\vec{F}_{gx} = 77.63 \text{ N [down the incline]}$$

$$\vec{F}_{gx} = m\vec{g}\cos\theta$$

$$\vec{F}_{gx} = (65.0 \text{ kg})(9.8 \text{ m/s}^2)\cos 7.0^o$$

$$\vec{F}_{gx} = 632.3 \text{ N [perpendicular to the incline]} = \vec{F}_N \text{ (normal force)}$$

Step 3: Calculate the friction.

$$\vec{F}_f = \mu\vec{F}_N$$

$$\vec{F}_f = (0.11)(632.3 \text{ N})$$

$$\vec{F}_f = 69.5 \text{ N}$$

Step 4: Calculate the net force.

$$\vec{F}_{net} = \vec{F}_{gx} + \vec{F}_{f}$$
$$\vec{F}_{net} = 77.63 \text{ N} + (-69.5 \text{ N})$$
$$\vec{F}_{net} = 8.08 \text{ N [down the incline]}$$

Therefore, the person will not rest comfortably and will start moving down the inclined sidewalk.

Step 5: Substitute in the equation,

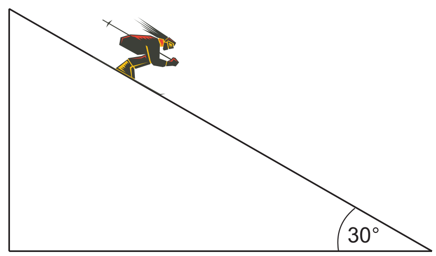
$$\vec{a} = \frac{\vec{F}_{net}}{m}$$
$$\vec{a} = \frac{8.08 \text{ N}}{65.0 \text{ kg}}$$
$$\vec{a} = 0.124 \text{ m/s}^2$$

Illustrative Example 2

A skier of mass 75.0 kg is skiing down a ski run that is inclined at an angle of 30.0°. The coefficient of kinetic friction between the skis and the snow is 0.150. Calculate $\vec{F}_{net}$, the acceleration of the skier, the speed the skier obtains after 8.00 s, and the distance traveled during the 8.00 s.

Solution

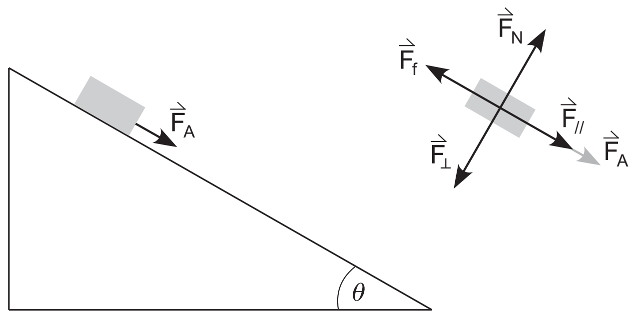
Step 1: Draw a picture of the situation and label the diagram.



Step 2: List the quantities needed for the Dynamics section and the quantities needed for the Kinematics section.

Dynamics	Kinematics
mass = 75.0 kg	$\vec{v}_1 = 0$ m/s
$\theta = 30.0°$	$\Delta t = 8.00$ s
$\vec{F}_g = m\vec{g}$	
components of $\vec{F}_g : \vec{F}_N, \vec{F}_{\parallel}$	

Free-body Diagram



Step 3: Solve for various forces.

$$\vec{F}_N = m\vec{g}\cos\theta = (75.0 \text{ kg})(9.8 \text{ m/s}^2)$$

$$\vec{F}_N = 636.5 \text{ N [perpendicular to the incline]}$$

$$\vec{F}_{\parallel} = m\vec{g}\sin\theta = (75.0 \text{ kg})(9.8 \text{ m/s}^2)\sin 30.0^o$$

$$\vec{F}_{\parallel} = 367.5 \text{ N [down the incline]}$$

$$\vec{F}_f = \mu\vec{F}_N$$

$$\vec{F}_f = 0.150(636.5 \text{ N})$$

$$\vec{F}_f = 95.45 \text{ N}$$

$$\vec{F}_{net} = \vec{F}_{parallel} + \vec{F}_N + \vec{F}_\perp + \vec{F}_f$$

$$\vec{F}_{net} = 367.5 \text{ N [down the incline]} + 636.5 \text{ N [out of surface]} + 636.5 \text{ N [into surface]} + 95.45 \text{ N [up the incline]}$$

$$\vec{F}_{net} = 272 \text{ N [down the incline]}$$

This answers part (a). To solve for the acceleration, part (b), use:

$$\vec{\mathrm{F}}_{\mathrm{net}} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{net}}{m = 272 \text{ N} / 75.0 \text{ kg}}$$

$$\vec{a} = 3.63 \text{ m/s}^2 \; [\text{down the incline}]$$

Step 4: Now that we know the acceleration, we can now solve for

$$\vec{v}_{\mathrm{f}} = \vec{v}_{\mathrm{i}} + \vec{a}\Delta t$$

$$\vec{v}_{\mathrm{f}} = 0 \text{ m/s} + (3.63)(8.00)$$

$$\vec{v}_{\mathrm{f}} = 29.0 \text{ m/s} \; [\text{down the incline}]$$

This solves for part (c).

Step 5: Solve for the distance travelled.

$$\Delta\vec{d} = \left(\frac{\vec{v}_1 + \vec{v}_2}{2}\right)\Delta t$$

$$\Delta\vec{d} = \frac{0 \text{ m/s} + 29.0 \text{ m/s}}{2}(8.00 \text{ s})$$

$$\Delta\vec{d} = 116 \text{ m} \; [\text{down the incline}]$$

Appendix 1.5: Momentum and Impulse

Illustrative Example

Bullet being fired from a rifle: A gun of mass 3.00 kg fires a bullet with a mass of 19.4 g with a muzzle velocity of 549 m/s. Calculate:

1. the recoil velocity of the gun
2. the impulse applied to the bullet
3. the impulse applied to the gun
4. the average force acting on the bullet if it travelled the length of the barrel distance during 3.60 x 10^{-3} s.

Solution:

1. Draw a set of coordinates and show the initial and final states.

Initial

$$m_g = 3.00 \text{ kg}$$
$$m_b = 19.4 \text{ g} = 1.94\times10^{-2} \text{ kg}$$
$$\vec{v}_{1g} = 0 \text{ m/s}$$

Final

$$\vec{v}_{2g} = ?$$
$$\vec{v}_{2b} = 549 \text{ m/s [right]}$$

Since this example illustrates conservation of momentum in one dimension, the vector directions can be labelled right/left or using +/– signs.

$$\vec{P}_{\text{total initial}} = \vec{P}_{\text{total final}}$$
$$\vec{P}_{1g} + \vec{P}_{1b} = \vec{P}_{2g} + \vec{P}_{2b}$$
$$m_g \vec{v}_{1g} + m_b \vec{v}_{1b} = m_b \vec{v}_{2g} + m_b \vec{v}_{2b}$$

So, substituting the values into the above equation, we get

$$(3.00 \text{ kg})(0 \text{ m/s}) + (1.94\times10^{-2} \text{ kg})(0 \text{ m/s}) = (3.00 \text{ kg})\vec{v}_{2g} + (1.94\times10^{-2} \text{ kg})(549 \text{ m/s [ri}$$
$$0 \text{ kg} \cdot \text{m/s} = (3.00 \text{ kg})\vec{v}_{2g} + 10.6 \text{ kg} \cdot \text{m/s [right]}$$
$$(3.00 \text{ kg})\vec{v}_{2g} = 10.6 \text{ kg} \cdot \text{m/s [right]}$$
$$\vec{v}_{2g} = 3.53 \text{ kg} \cdot \text{m/s [right]}$$
$$\text{or } \vec{v}_{2g} = 3.53 \text{ kg} \cdot \text{m/s [left]}$$

2. Impulse applied to the bullet equals the change in momentum of the bullet.

$$\text{Impulse on bullet} = m_b \Delta \vec{v}_b = m_b\left(\vec{v}_{2b} - \vec{v}_{1b}\right)$$
$$= \left(1.94 \times 10^{-2}\ \text{kg}\right)\left(549\ \text{m/s} - 0\ \text{m/s}\right)$$
$$= 10.6\ \text{kg} \cdot \text{m/s}\ [\text{right}]$$
$$= 10.6\ \text{N} \cdot \text{s}\ [\text{right}]$$

3. Impulse applied to the gun is equal to, but opposite in direction to, the impulse of the bullet.

   Impulse on gun = 10.6 kg m/s [left]
   = 10.6 N s [left]

d) To calculate the average force, substitute the values into

$$\vec{F}_{ave} = \frac{\Delta \vec{p}_b}{\Delta t}$$
$$\vec{F}_{ave} = \frac{10.6\ \text{kg} \cdot \text{m/s}\ [\text{right}]}{3.60 \times 10^{-3}\ \text{s}}$$
$$\vec{F}_{ave} = 2.94 \times 10^{3}\ \text{N}\ [\text{right}]$$

Appendix 1.6: Collisions in Two Dimensions

Illustrative Example

A curling rock of 18.8 kg is sliding at 1.45 m/s [E]. It collides with a stationary target rock in a glancing fashion. The final velocity of the target rock is 1.00 m/s [30.0 S of E].

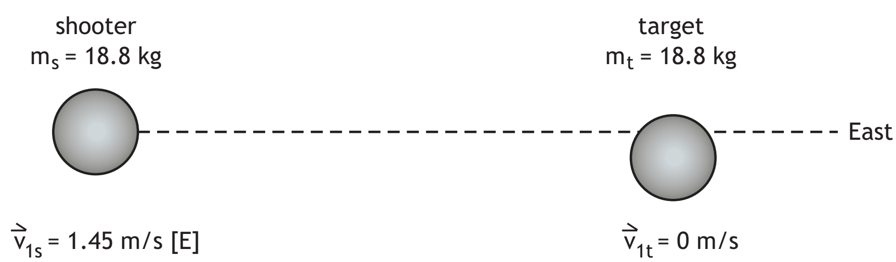
Calculate:

1. the total initial momentum and the total final momentum
2. the final momentum of the first rock
3. the final velocity of the first rock (the shooter)
4. the change in momentum of the first rock
5. describe the motion of the centre of mass
6. calculate the impulse applied to the first rock (the shooter)

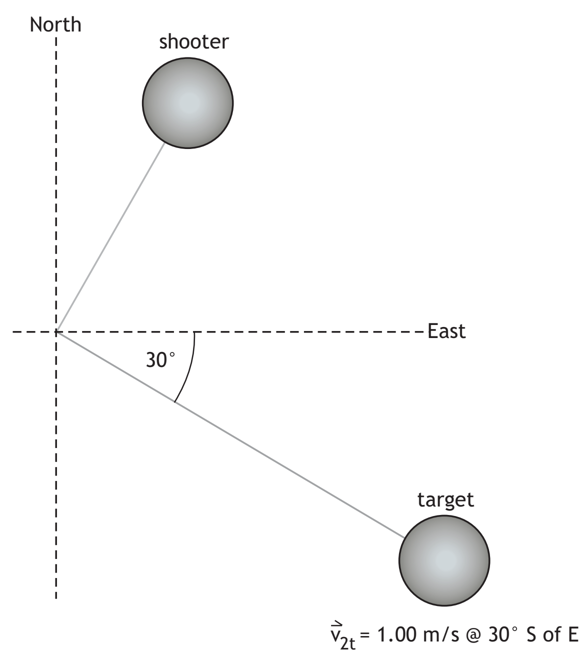
Solution:

a) Draw a set of coordinate axes and draw a diagram of the situation.

Before collision



After collision



$$\begin{aligned}\vec{P}_{\text{initial total}} &= \vec{P}_{\text{initial shooter}} + \vec{P}_{\text{initial target}}\\ &= m_S \vec{v}_{1S} + m_T \vec{v}_{1T}\\ &= (18.8\text{ kg})(1.45\text{ m/s [E]}) + (18.8\text{ kg})(0\text{ m/s})\\ &= 27.3\text{ kg}\cdot\text{m/s [E]}\end{aligned}$$

Since momentum is conserved, the total final momentum is the same.

b) To calculate the final momentum of the first rock, we substitute the values obtained in Part (a) into the following equation:

$$\vec{P}_{\text{initial total}} = \vec{P}_{\text{final total}}$$

$$\vec{P}_{\text{initial shooter}} + \vec{P}_{\text{initial target}} = \vec{P}_{\text{final shooter}} + \vec{P}_{\text{final target}}$$

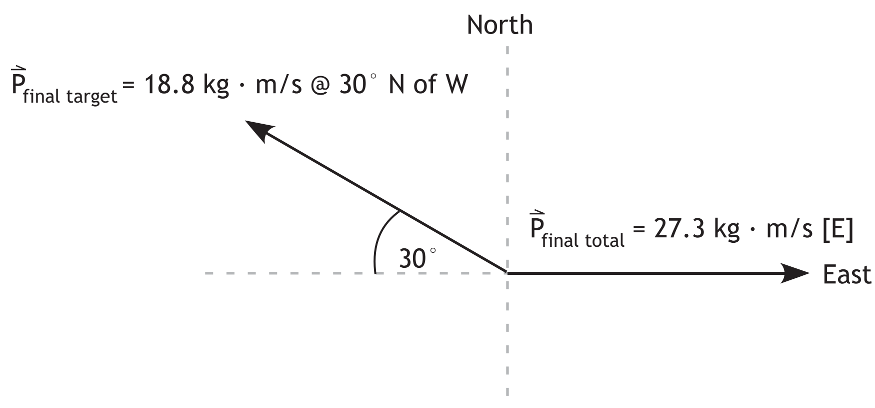
$$27.3\text{ kg}\cdot\text{m/s [E]} = \vec{P}_{\text{final shooter}} + m_{\text{target}}\,\vec{v}_{\text{final target}}$$

$$27.3\text{ kg}\cdot\text{m/s [E]} = \vec{P}_{\text{final shooter}} + (18.8\text{ m/s})(1.00\text{ m/s})[30.0^{\circ}\text{ S of E}]$$

$$27.3\text{ kg}\cdot\text{m/s [E]} = \vec{P}_{\text{final shooter}} + 18.8\text{ kg}\cdot\text{m/s }[30.0^{\circ}\text{ S of E}]$$

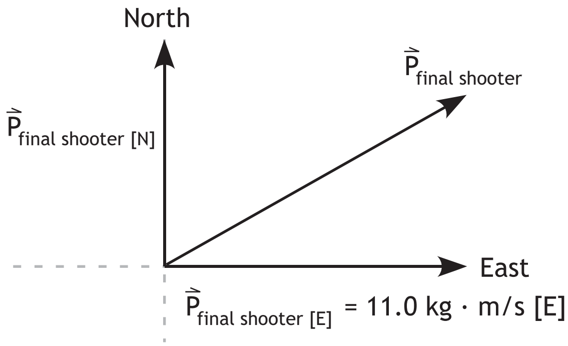
$$27.3\text{ kg}\cdot\text{m/s[E]} - 18.8\ kg\cdot\text{m/s }[30.0^{\circ}\text{ S of E}] = \vec{P}_{\text{final shooter}}$$

$$27.3\text{ kg}\cdot\text{m/s[E]} + 18.8\ kg\cdot\text{m/s }[30.0^{\circ}\text{ N of W}] = \vec{P}_{\text{final shooter}}$$



Adding the vector components of the vector diagram shown, we obtain the following data:

	E – W	N – S
$\vec{P}_{\text{final total}}$	$27.3\ \text{kg} \cdot \text{m/s}\ [\text{E}]$	$0\ \text{kg} \cdot \text{m/s}$
$\vec{P}_{\text{final Tl}}$	$16.3\ \text{kg} \cdot \text{m/s}\ [\text{W}]$	$9.4\ \text{kg} \cdot \text{m/s}\ [\text{N}]$
$\vec{P}_{\text{final s}}$	$11.0\ \text{kg} \cdot \text{m/s}\ [\text{E}]$	$9.4\ \text{kg} \cdot \text{m/s}\ [\text{N}]$



To solve for $\vec{P}_{\text{final s}}$, we combine the components from the diagram.

The resultant vector is 14.5 kg · m/s [40.5 N of E].

c) To solve for the velocity of the shooter rock, substitute values into the following equation:

$$\vec{P}_{\text{final s}} = m_s \vec{v}_{2s}$$

$$14.5\ \text{kg} \cdot \text{m/s}\left[40.5^{\circ}\ \text{N of E}\right] = 18.8\ kg\ \vec{v}_{2s}$$

$$\vec{v}_{2s} = 0.771\ \text{m/s}\ 40.5^{\circ}\ \text{N of E}$$

d) The change in momentum of the shooter can be found by taking the difference between the final momentum and the initial momentum for the shooter. In this case, it would require a subtraction of vector quantities. However, the change in momentum of the shooter is equal but opposite to the change in momentum of the target. This can be easily calculated since the initial momentum of the target stone is 0 kg · m/s.

$$\begin{aligned}\Delta \vec{p}_s &= -\Delta \vec{p}_t = -\left(\vec{p}_{2t} - \vec{p}_{1T}\right)\\ &= -\left[18.8\ \text{kg} \cdot \text{m/s}\left(30.0^{\circ}\ \text{S of E}\right) - 0\right]\\ &= 18.8\ \text{kg} \cdot \text{m/s}\left[30.0^{\circ}\ \text{N of W}\right]\end{aligned}$$

e) The system of the two rocks has a centre of mass midway between them. As the rocks move, so does this centre of mass. The total momentum of the system can be thought of as the momentum of the centre of mass. The relationship for momentum can be expressed as

$$\vec{P}_{\text{total}} = m_{\text{total}}\,\vec{v}_{\text{centre of mass}}$$

$$27.3\text{ kg}\cdot\text{m/s}\,[\text{E}] = (18.8\text{ kg} + 18.8\text{ kg})\left(\vec{v}_{\text{centre of mass}}\right)$$

$$\vec{v}_{\text{centre of mass}} = 0.726\text{ m/s}\,[\text{E}]$$

f) Again, the impulse applied to the shooter rock causes a change in its momentum. We have already calculated the change in momentum for the shooter, so impulse equals 18.8 N · s[30.0 N of E].

Appendix 1.7: Projectiles

Illustrative Examples

1. An object is launched horizontally off a 100 m cliff at a speed of 10 m/s. Determine the range of the projectile from the base of the cliff.

   Solution:

   Determine the time in air using $d_y = v_y t + ½gt^2$ where

   $d = –100$ m

   $v_y = 0$ m/s, and

   $g = –9.81$ m/s^2 (Time = 4.52 s)

   Determine the range using $d_x = v_x t$. (Range = 42.5 m).

2. An object is launched at 30° above the horizontal with a speed of 15 m/s. Determine:

   a) total time in air

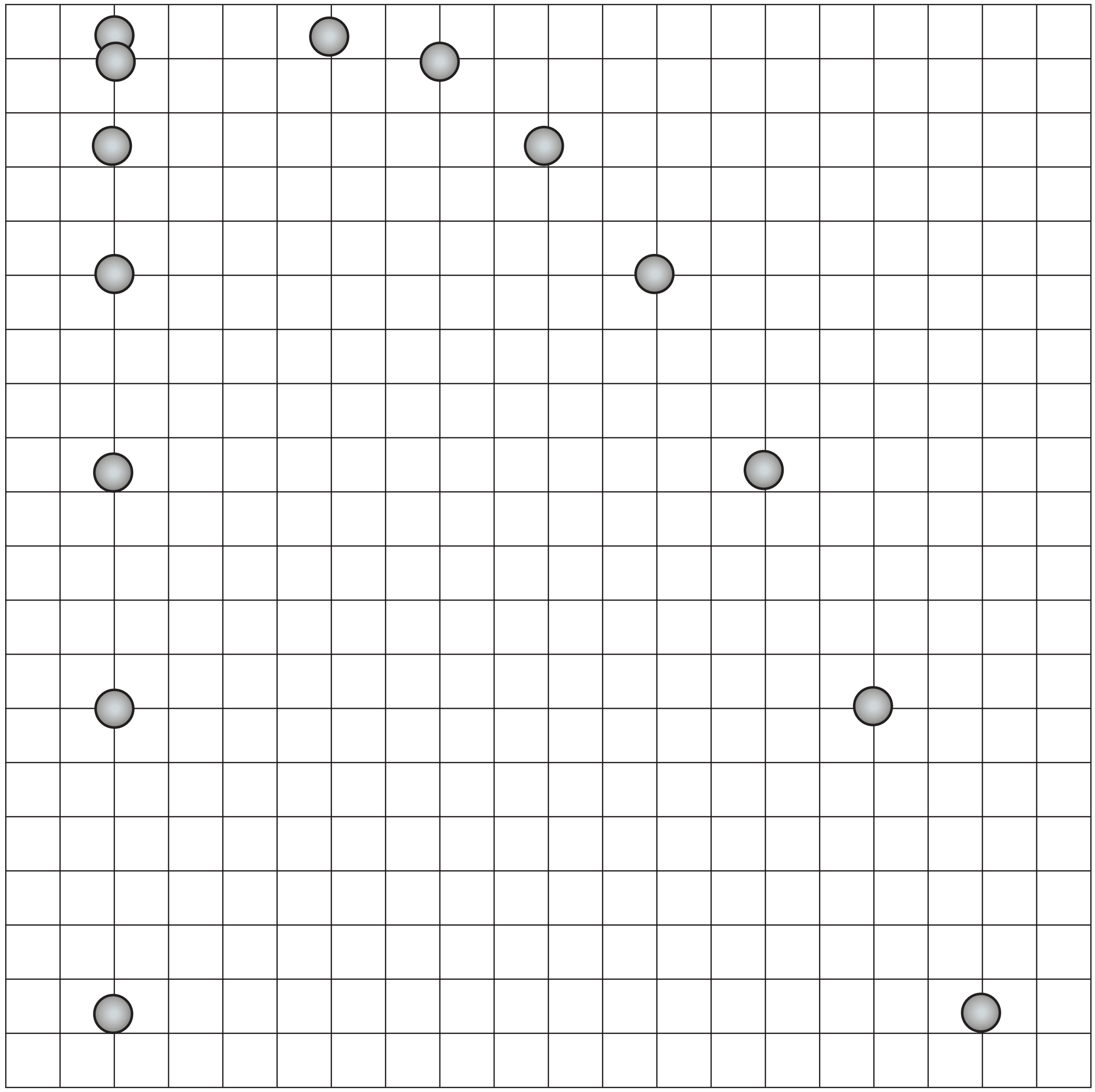
   b) maximum height

   c) range

   Solution:

   Calculate the components v_{1y} (+7.5 m/s) and v_{1x} (+13.0). Determine the time in air using $d_y = v_{1y}t + ½\ at^2$ where $d = 0$ m ($t = 1.53$ s). Determine the maximum height using $d_y = v_{avg}t$ where t is half the total time ($d_y = 2.87$ m). Determine the range using $d_x = v_x t$ where t is the total time in the air ($d_x = 19.9$ m).

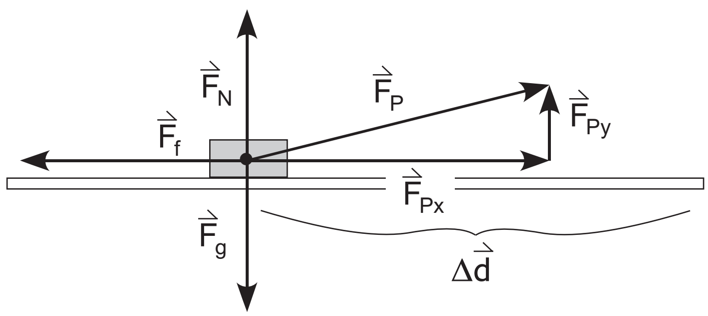
   Note: The above solution is only a suggested method to resolve the problem. Various textbooks may have alternative approaches.



Appendix 1.8: Force-Work Relationships

Illustrative Examples

1. A 20.0 kg crate is pulled 50.0 m along a horizontal floor by a constant force exerted by a person,
   F = 1.00 x 102 N, which acts at a 20.0° angle to the horizontal. The floor is rough and exerts a friction force F_f = 15.0 N. Determine the work done by each force acting on the crate, and the net work done on the crate.



In this case, the force of gravity and the normal force do no work since the crate is not displaced in their direction.

Solution A:

Add up the work done by each object.

Work done by the Person (W_p)

Work = $F_p \Delta d \cos\theta$

W_p = (1.00 x 102 N)(50.0 m) (cos 20.0°) = +4.70 x 10^3 J

Work done by Friction (W_f)

Work = $F_f \Delta d \cos\theta$

W_f = (15.0 N)(50.0 m)(cos 180.0°) = –7.50 x 10^2 J

(The negative sign means work is done, which opposes the motion, and energy is lost by the crate.)

Net Work Done:

Since work is a scalar, we can take the algebraic sum of the work done by each force.

$W_{net} = W_p + W_f$ = (+4.70 x 10^3 J) + (–7.50 x 10^2 J) = +3950 J

Solution B:

Find the net force on the object in the direction of motion and multiply by the displacement.

$W_{net} = F_{net}\Delta d \cos\theta$

where $F_{net} = F_p \cos\theta + F_f = 100 \cos 20 + -15 = 79$ N

$W_{net} = (50 \text{ m})\ 79$

$W_{net} = +3950$ J

Appendix 1.9: Centrifuge Demonstration

This demonstration consists of two water-filled bottles containing ping pong balls and golf balls sitting on a rotating platform. The ping pong balls are hollow and float, and the golf balls are solid and sink. Each ball is attached by a thread to either the top or the bottom of the jar. When the apparatus is rotated, the ping pong balls tilt in toward the centre of rotation and the golf balls tilt outwards away from the centre of rotation.