

INTRODUCTION

Purpose of This Document

Grade 6 Mathematics: Support Document for Teachers provides various suggestions for instruction, assessment strategies, and learning resources that promote the meaningful engagement of mathematics learners in Grade 6. The document is intended to be used by teachers as they work with students in achieving the learning outcomes and achievement indicators identified in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes* (2013) (Manitoba Education).

Background

Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes is based on *The Common Curriculum Framework for K–9 Mathematics*, which resulted from ongoing collaboration with the Western and Northern Canadian Protocol (WNCP). In its work, WNCP emphasizes

- common educational goals
- the ability to collaborate and achieve common goals
- high standards in education
- planning an array of educational activities
- removing obstacles to accessibility for individual learners
- optimum use of limited educational resources

The growing effects of technology and the need for technology-related skills have become more apparent in the last half century. Mathematics and problem-solving skills are becoming more valued as we move from an industrial to an informational society. As a result of this trend, mathematics literacy has become increasingly important. Making connections between mathematical study and daily life, business, industry, government, and environmental thinking is imperative. The Kindergarten to Grade 12 mathematics curriculum is designed to support and promote the understanding that mathematics is

- a way of learning about our world
- part of our daily lives
- both quantitative and geometric in nature

Beliefs about Students and Mathematics Learning

The Kindergarten to Grade 8 mathematics curriculum is designed with the understanding that students have unique interests, abilities, and needs. As a result, it is imperative to make connections to all students' prior knowledge, experiences, and backgrounds.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with unique knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem-solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Conceptual understanding: comprehending mathematical concepts, relations, and operations to build new knowledge. (Kilpatrick, Swafford, and Findell 5)

Procedural thinking: carrying out procedures flexibly, accurately, efficiently, and appropriately.

Problem solving: engaging in understanding and resolving problem situations where a method or solution is not immediately obvious. (OECD 12)

First Nations, Métis, and Inuit Perspectives

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural, and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning, and learn mathematics best when it is contextualized and not taught as discrete content.

Many First Nations, Métis, and Inuit students come from cultural environments where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks).

Affective Domain

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom learning activities, persist in challenging situations, and engage in reflective practices.

Teachers, students, and parents* need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward reaching these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessment of personal goals.

* In this document, the term *parents* refers to both parents and guardians and is used with the recognition that in some cases only one parent may be involved in a child's education.

Middle Years Education

Middle Years education is defined as the education provided for young adolescents in Grades 5, 6, 7, and 8. Middle Years learners are in a period of rapid physical, emotional, social, moral, and cognitive development.

Socialization is very important to Middle Years students, and collaborative learning, positive role models, approval of significant adults in their lives, and a sense of community and belonging greatly enhance adolescents' engagement in learning and commitment to school. It is important to provide students with an engaging and social environment within which to explore mathematics and to construct meaning.

Adolescence is a time of rapid brain development when concrete thinking progresses to abstract thinking. Although higher-order thinking and problem-solving abilities develop during the Middle Years, concrete, exploratory, and experiential learning is most engaging to adolescents.

Middle Years students seek to establish their independence and are most engaged when their learning experiences provide them with a voice and choice. Personal goal setting, co-construction of assessment criteria, and participation in assessment, evaluation, and reporting help adolescents take ownership of their learning. Clear, descriptive, and timely feedback can provide important information to the mathematics student. Asking open-ended questions, accepting multiple solutions, and having students develop personal strategies will help students to develop their mathematical independence.

Adolescents who see the connections between themselves and their learning, and between the learning inside the classroom and life outside the classroom, are more motivated and engaged in their learning than those who do not observe these connections.

Adolescents thrive on challenges in their learning, but their sensitivity at this age makes them prone to discouragement if the challenges seem unattainable. Differentiated instruction allows teachers to tailor learning challenges to adolescents' individual needs, strengths, and interests. It is important to focus instruction on where students are and to see every contribution as valuable.

Mathematics Education Goals for Students

The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between mathematical knowledge and skills and their applications
- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world

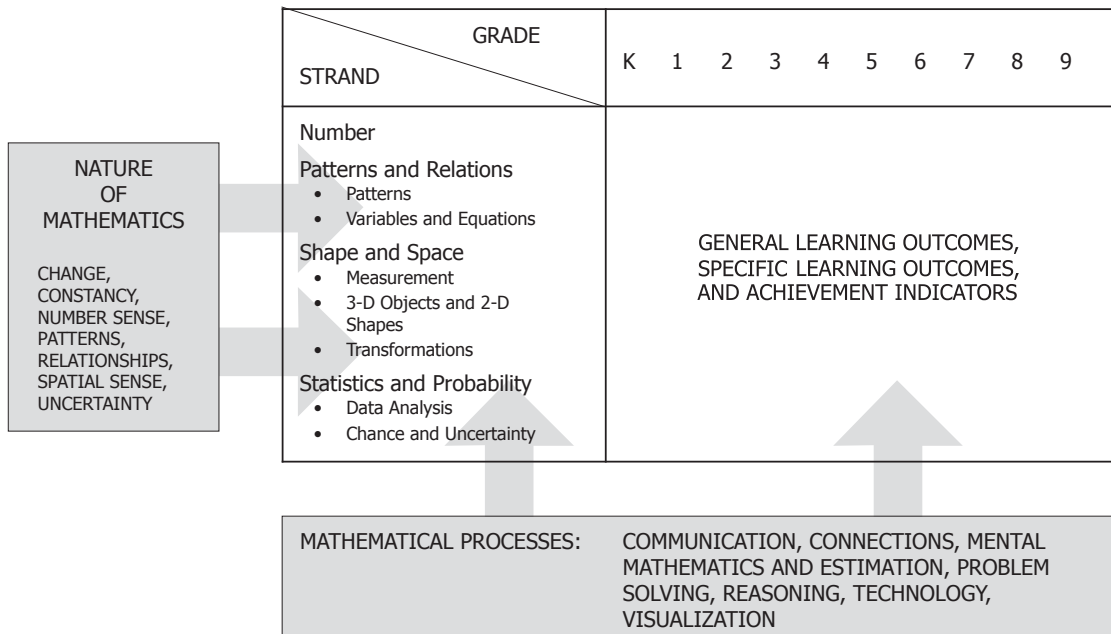
Mathematics education must prepare students to use mathematics to think critically about the world.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

CONCEPTUAL FRAMEWORK FOR KINDERGARTEN TO GRADE 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

There are critical components that students must encounter in mathematics to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The common curriculum framework incorporates these seven interrelated mathematical processes, which are intended to permeate teaching and learning:

- **Communication [C]:** Students communicate daily (orally, through diagrams and pictures, and by writing) about their mathematics learning. They need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. This enables them to reflect, to validate, and to clarify their thinking. Journals and learning logs can be used as a record of student interpretations of mathematical meanings and ideas.
- **Connections [CN]:** Mathematics should be viewed as an integrated whole, rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—concrete, pictorial, and symbolic (the symbolic mode consists of oral and written word symbols as well as mathematical symbols). The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas.
- **Mental Mathematics and Estimation [ME]:** The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills. Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense.
- **Problem Solving [PS]:** Students are exposed to a wide variety of problems in all areas of mathematics. They explore a variety of methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problems.
- **Reasoning [R]:** Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.
- **Technology [T]:** The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.
- **Visualization [V]:** Mental images help students to develop concepts and to understand procedures. Students clarify their understanding of mathematical ideas through images and explanations.

These processes are outlined in detail in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes* (2013).

Strands

The learning outcomes in the Manitoba curriculum framework are organized into four strands across Kindergarten to Grade 9. Some strands are further subdivided into substrands. There is one general learning outcome per substrand across Kindergarten to Grade 9.

The strands and substrands, including the general learning outcome for each, follow.

Number

- Develop number sense.

Patterns and Relations

- Patterns
 - Use patterns to describe the world and solve problems.
- Variables and Equations
 - Represent algebraic expressions in multiple ways.

Shape and Space

- Measurement
 - Use direct and indirect measure to solve problems.
- 3-D Objects and 2-D Shapes
 - Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
- Transformations
 - Describe and analyze position and motion of objects and shapes.

Statistics and Probability

- Data Analysis
 - Collect, display, and analyze data to solve problems.
- Chance and Uncertainty
 - Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Learning Outcomes and Achievement Indicators

The Manitoba curriculum framework is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators:

- **General learning outcomes** are overarching statements about what students are expected to learn in each strand/substrand. The general learning outcome for each strand/substrand is the same throughout the grades from Kindergarten to Grade 9.
- **Specific learning outcomes** are statements that identify the specific skills, understanding, and knowledge students are required to attain by the end of a given grade.
- **Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success.

In this document, the word *including* indicates that any ensuing items **must be addressed** to meet the learning outcome fully. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are **not requirements that must be addressed** to meet the learning outcome fully.

Summary

The conceptual framework for Kindergarten to Grade 9 mathematics describes the nature of mathematics, the mathematical processes, and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Learning activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes, and lead students to an understanding of the nature of mathematics through specific knowledge, skills, and attitudes among and between strands. *Grade 6 Mathematics: Support Document for Teachers* is meant to support teachers to create meaningful learning activities that focus on formative assessment and student engagement.

ASSESSMENT

Authentic assessment and feedback are a driving force for the suggestions for assessment in this document. The purposes of the suggested assessment activities and strategies are to parallel those found in *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning* (Manitoba Education, Citizenship and Youth). These include the following:

- assessing *for, as, and of* learning
- enhancing student learning
- assessing students effectively, efficiently, and fairly
- providing educators with a starting point for reflection, deliberation, discussion, and learning

Assessment *for* learning is designed to give teachers information to modify and differentiate teaching and learning activities. It acknowledges that individual students learn in idiosyncratic ways, but it also recognizes that there are predictable patterns and pathways that many students follow. It requires careful design on the part of teachers so that they use the resulting information to determine not only what students know, but also to gain insights into how, when, and whether students apply what they know. Teachers can also use this information to streamline and target instruction and resources, and to provide feedback to students to help them advance their learning.

Assessment *as* learning is a process of developing and supporting metacognition for students. It focuses on the role of the student as the critical connector between assessment and learning. When students are active, engaged, and critical assessors, they make sense of information, relate it to prior knowledge, and use it for new learning. This is the regulatory process in metacognition. It occurs when students monitor their own learning and use the feedback from this monitoring to make adjustments, adaptations, and even major changes in what they understand. It requires that teachers help students develop, practise, and become comfortable with reflection, and with a critical analysis of their own learning.

Assessment *of* learning is summative in nature and is used to confirm what students know and can do, to demonstrate whether they have achieved the curriculum learning outcomes, and, occasionally, to show how they are placed in relation to others. Teachers concentrate on ensuring that they have used assessment to provide accurate and sound statements of students' proficiency so that the recipients of the information can use the information to make reasonable and defensible decisions.

Overview of Planning Assessment			
	Assessment <i>for</i> Learning	Assessment <i>as</i> Learning	Assessment <i>of</i> Learning
Why Assess?	<ul style="list-style-type: none"> ■ to enable teachers to determine next steps in advancing student learning 	<ul style="list-style-type: none"> ■ to guide and provide opportunities for each student to monitor and critically reflect on his or her learning and identify next steps 	<ul style="list-style-type: none"> ■ to certify or inform parents or others of the student's proficiency in relation to curriculum learning outcomes
Assess What?	<ul style="list-style-type: none"> ■ each student's progress and learning needs in relation to the curriculum outcomes 	<ul style="list-style-type: none"> ■ each student's thinking about his or her learning, what strategies he or she uses to support or challenge that learning, and the mechanisms he or she uses to adjust and advance his or her learning 	<ul style="list-style-type: none"> ■ the extent to which each student can apply the key concepts, knowledge, skills, and attitudes related to the curriculum outcomes
What Methods?	<ul style="list-style-type: none"> ■ a range of methods in different modes that make a student's skills and understanding visible 	<ul style="list-style-type: none"> ■ a range of methods in different modes that elicit the student's learning and metacognitive processes 	<ul style="list-style-type: none"> ■ a range of methods in different modes that assess both product and process
Ensuring Quality	<ul style="list-style-type: none"> ■ accuracy and consistency of observations and interpretations of student learning ■ clear, detailed learning expectations ■ accurate, detailed notes for descriptive feedback to each student 	<ul style="list-style-type: none"> ■ accuracy and consistency of a student's self-reflection, self-monitoring, and self-adjustment ■ engagement of the student in considering and challenging his or her thinking ■ the student records his or her own learning 	<ul style="list-style-type: none"> ■ accuracy, consistency, and fairness of judgments based on high-quality information ■ clear, detailed learning expectations ■ fair and accurate summative reporting
Using the Information	<ul style="list-style-type: none"> ■ provide each student with accurate descriptive feedback to further his or her learning ■ differentiate instruction by continually checking where each student is in relation to the curriculum outcomes ■ provide parents or guardians with descriptive feedback about student learning and ideas for support 	<ul style="list-style-type: none"> ■ provide each student with accurate, descriptive feedback that will help him or her develop independent learning habits ■ have each student focus on the task and his or her learning (not on getting the right answer) ■ provide each student with ideas for adjusting, rethinking, and articulating his or her learning ■ provide the conditions for the teacher and student to discuss alternatives ■ the student reports about his or her own learning 	<ul style="list-style-type: none"> ■ indicate each student's level of learning ■ provide the foundation for discussions on placement or promotion ■ report fair, accurate, and detailed information that can be used to decide the next steps in a student's learning

Source: Manitoba Education, Citizenship and Youth. *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2006, 85.

INSTRUCTIONAL FOCUS

The Manitoba mathematics curriculum framework is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of learning outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Integration of the mathematical processes within each strand is expected.
- Problem solving, conceptual understanding, reasoning, making connections, and procedural thinking are vital to increasing mathematical fluency, and must be integrated throughout the program.
- Concepts should be introduced using manipulatives and gradually developed from the concrete to the pictorial to the symbolic.
- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Use educational resources by adapting to the context, experiences, and interests of students.
- Collaborate with teachers at other grade levels to ensure the continuity of learning of all students.
- Familiarize yourself with exemplary practices supported by pedagogical research in continuous professional learning.
- Provide students with several opportunities to communicate mathematical concepts and to discuss them in their own words.

“Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students” (Ontario 24).

DOCUMENT ORGANIZATION AND FORMAT

This document consists of the following sections:

- **Introduction:** The Introduction provides information on the purpose and development of this document, discusses characteristics of and goals for Middle Years learners, and addresses Aboriginal perspectives. It also gives an overview of the following:
 - **Conceptual Framework for Kindergarten to Grade 9 Mathematics:** This framework provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.
 - **Assessment:** This section provides an overview of planning for assessment in mathematics, including assessment *for, as, and of* learning.
 - **Instructional Focus:** This discussion focuses on the need to integrate mathematics learning outcomes and processes across the four strands to make learning experiences meaningful for students.
 - **Document Organization and Format:** This overview outlines the main sections of the document and explains the various components that comprise the various sections.
- **Number:** This section corresponds to and supports the Number strand for Grade 6 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Patterns and Relations:** This section corresponds to and supports the Patterns and Variables and Equations substrands of the Patterns and Relations strand for Grade 6 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Shape and Space:** This section corresponds to and supports the Measurement, 3-D Objects and 2-D Shapes, and Transformations substrands of the Shape and Space strand for Grade 6 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Statistics and Probability:** This section corresponds to and supports the Data Analysis and Chance and Uncertainty substrands of the Statistics and Probability strand for Grade 6 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Blackline Masters (BLMs):** Blackline masters are provided to support student learning. They are available in *Microsoft Word* format so that teachers can alter them to meet students' needs. They are also available in *Adobe PDF* format.
- **Bibliography:** The bibliography lists the sources consulted and cited in the development of this document.

Guide to Components and Icons

Each of the sections supporting the strands of the Grade 6 Mathematics curriculum includes the components and icons described below.

Enduring Understanding(s):

These statements summarize the core idea of the particular learning outcome(s). Each statement provides a conceptual foundation for the learning outcome. It can be used as a pivotal starting point in integrating other mathematics learning outcomes or other subject concepts. The integration of concepts, skills, and strands remains of utmost importance.

General Learning Outcome(s):

General learning outcomes (GLOs) are overarching statements about what students are expected to learn in each strand/substrand. The GLO for each strand/substrand is the same throughout Kindergarten to Grade 8.

SPECIFIC LEARNING OUTCOME(S):

ACHIEVEMENT INDICATORS:

Specific learning outcome (SLO) statements define what students are expected to achieve by the end of the grade.

A code is used to identify each SLO by grade and strand, as shown in the following example:

6.N.1 The first number refers to the grade (Grade 6).
↑ ↑ ↑
| | |
— The letter(s) refer to the strand (Number).
|
— The last number indicates the SLO number.
[C, CN, ME, PS, R, T, V]

Each SLO is followed by a list indicating the applicable mathematical processes.

Achievement indicators are examples of a representative list of the depth, breadth, and expectations for the learning outcome. The indicators may be used to determine whether students understand the particular learning outcome. These achievement indicators will be addressed through the learning activities that follow.

PRIOR KNOWLEDGE

Prior knowledge is identified to give teachers a reference to what students may have experienced previously. Teachers should assess students' prior knowledge before planning instruction.

RELATED KNOWLEDGE

Related knowledge is identified to indicate the connections among the Grade 6 mathematics learning outcomes.

BACKGROUND INFORMATION

Background information is provided to give teachers knowledge about specific concepts and skills related to the particular learning outcome(s).

MATHEMATICAL LANGUAGE

Lists of terms students will encounter while achieving particular learning outcomes are provided. These terms can be placed on mathematics word walls or used in a classroom mathematics dictionary. *Kindergarten to Grade 8 Mathematics Glossary: Support Document for Teachers* (Manitoba Education, Citizenship and Youth) provides teachers with an understanding of key terms found in Kindergarten to Grade 8 mathematics. The glossary is available on the Manitoba Education and Advanced Learning website at www.edu.gov.mb.ca/k12/cur/math/supports.html.

LEARNING EXPERIENCES

Suggested instructional strategies and assessment ideas are provided for the specific learning outcomes and achievement indicators. In general, learning activities and teaching strategies related to specific learning outcomes are developed individually, except in cases where it seems more logical to develop two or more learning outcomes together. Suggestions for assessment include information that can be used to assess students' progress in their understanding of a particular learning outcome or learning experience.



Assessing Prior Knowledge

Suggestions are provided to assess students' prior knowledge and to help direct instruction.

Observation Checklist

Checklists are provided for observing students' responses during lessons.

Suggestions for Instruction

- **Achievement indicators appropriate to particular learning experiences are listed.**

The instructional suggestions include the following:

- **Materials/Resources:** Outlines the resources required for a learning activity.
- **Organization:** Suggests groupings (individual, pairs, small group, and/or whole class).
- **Procedure:** Outlines detailed steps for implementing suggestions for instruction.

Some learning activities make use of BLMs, which are found in the Blackline Masters section in *Microsoft Word* and *Adobe PDF* formats.

PUTTING THE PIECES TOGETHER



Putting the Pieces Together tasks, found at the end of the learning outcomes, consist of a variety of assessment strategies. They may assess one or more learning outcomes across one or more strands and may make cross-curricular connections.