Solutions by Strand

M–1

Solidifying Prior Learning Solutions

1. Pairs of numbers (a, b) are (1, 2), (2, 4), (3, 6), (4, 8)…. The value of b is twice the value of a.

2. a) One of the possible solutions using the integers 1, 2, 3, 4, 5, 6.

$$\frac{[6]}{[1]} - \frac{[2]}{[4]} - \frac{[3]}{[5]} \text{ is a little larger than } \frac{[6]}{[1]} - \frac{[2]}{[5]} - \frac{[3]}{[4]}.$$

It is the largest because we start with the largest fraction possible by using the biggest numerator (6) and the smallest denominator (1). Then, subtract the smallest fractions possible created by using the small remaining integers (2 and 3) in the numerator and the large remaining integers (4 and 5) in the denominator.

Another possible solution using the integers 4, 5, 6, 7, 8, 9:

$$\frac{[9]}{[4]} - \frac{[5]}{[7]} - \frac{[6]}{[8]} \text{ is a little larger than } \frac{[9]}{[4]} - \frac{[5]}{[8]} - \frac{[6]}{[7]}.$$

Similar justification as above.

(TN) Student work on this question provides an opportunity to remind students about the process of subtracting fractions. Students could be asked to compare their answers with each other to see which numbers and their arrangements yield the largest result. Student groups could be asked to articulate a process to determine the largest result, given a set of any 6 numbers.

b) One of the possible numbers "close" to zero using the integers 1, 2, 3, 4, 5, 6:

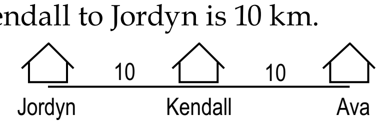
$$\frac{[3]}{[1]} - \frac{[4]}{[6]} - \frac{[5]}{[2]} \text{ which is a little closer to zero than } \frac{[6]}{[3]} - \frac{[4]}{[2]} - \frac{[1]}{[5]}$$

One of the possible numbers "close" to zero using the integers 4, 5, 6, 7, 8, 9:

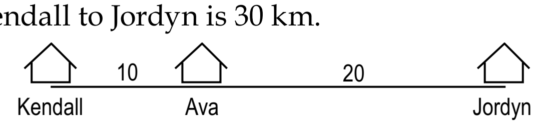
$$\frac{[9]}{[5]} - \frac{[8]}{[6]} - \frac{[4]}{[7]} \text{ which is a little closer to zero than } \frac{[9]}{[4]} - \frac{[8]}{[6]} - \frac{[5]}{[7]}$$

3. The answers can be any real number ranging from a minimum of 10 km to a maximum of 30 km. Three possible answers and accompanying diagrams are shown:

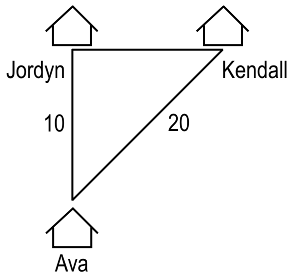
a) Kendall to Jordyn is 10 km.



b) Kendall to Jordyn is 30 km.

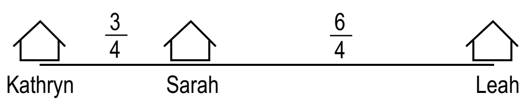


c) Kendall to Jordyn is $\sqrt{300}$ or 17.3 km.

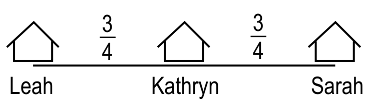


4. The answers can be any real number ranging from a minimum of $\frac{3}{4}$ mile to a maximum of $2\frac{1}{4}$ miles. These two possible answers are shown; the other answers do not have Kathryn, Sarah, and Leah all living along the same line.

a) Kathryn to Leah is $\frac{9}{4} = 2\frac{1}{4}$ mi.



b) Kathryn to Leah is $\frac{3}{4}$ mi.



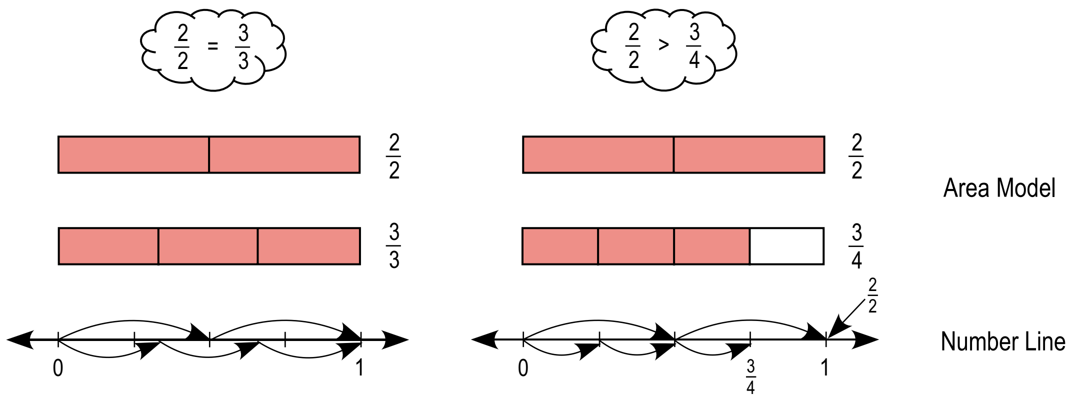
5. The perimeter of ΔADC is 25.04 m. The area of ΔABD is $\frac{6 \times 8}{2} = 24$ m^2, so the area of ΔABC is $24 + 0.50(24) = 36$ m^2. The height of ΔABC is 8 and the area is 36, so the length of base BC is 9 m. Therefore the length of DC is 3 m. By the Pythagorean theorem, AD is 10. By the Pythagorean theorem, the length of AC is 12.04 m. Therefore, the perimeter of ΔADC is 3 + 10 + 12.04.

M–2

Solidifying Prior Learning Solutions

1. Many solutions are possible. Here is one example.

   If $a = 2$, then $b > 3$. For example, $a = 2$ and $b = 4$



   Other possible answers:

a	2	2	3	3	6	6
b	4	5	5	6	10	11

2. a) She was alive for 64 404 000 minutes. Assumptions: Born and died at the same time of the day. There are 31 leap years in her lifetime. 122 years × 365 days/year = 44530 days; 31 extra days (leap years); and 164 days.

   Total time: 44530 + 31 + 164 = 44725 days or 64 404 000 minutes.

   b) Millionth minute, not quite 2 years old. 1 000 000 ÷ 60 ÷ 24 = $694.\overline{4}$ days or 1.901 years.

   Billionth minute, 1901 years old. 1 000 000 000 ÷ 60 ÷ 24 = $694444.\overline{4}$ days or 1901 years.

   (TN) Students need to discuss how they will handle leap years. Alternatives include the following: they could ignore them, they could count them, or they could assume 365.25 days per year to include leap years.

3. Jaswinder's statement:

   a) True for $\frac{6}{9} < \frac{9}{10}$. The fraction with numerator and denominator differing by 3 is less than the fraction with numerator and denominator differing by only 1.

   b) Not true for $\frac{4}{5} < \frac{90}{100}$. The fraction with numerator and denominator differing by 1 is less than the fraction with numerator and denominator differing by 10.

4. The diameter is 5.04 cm.

   $\pi \times r^2 = 20$

   $$r = \sqrt{\frac{20}{\pi}} = 2.52\text{ cm},\ \text{so the diameter is 5.04 cm.}$$

5. The large pizza is a better deal. Compare the areas with the given diameters.

   18″ pizza area = $\pi(9)^2 = 254.47$ in.^{2}

   12″ pizza area = $\pi(6)^2 = 113.10$ in.^{2}

   Two medium pizzas have an area of 226.20 in.^{2}, which is not as big as a large pizza.

   (TN) Discuss assumptions about what the measurement refers to with a 12″ or an 18″ pizza (i.e., **diameter**, radius, circumference, area). Discuss ways to compare the pizzas to determine the better deal (i.e., **area**, number of slices, radius, diameter, circumference).

6. a) She wanted B and C. An enlargement needs to keep the same ratio for width and length, so the picture does not get distorted. The pictures with the same ratios for width and length are 10 × 12 and 8 × 9.6. They both have a ratio of width to height as $0.833\overline{3} : 1$.

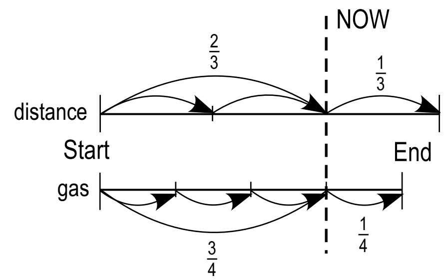
   $$0.83333 = \frac{10}{12} = \frac{8}{9.6}$$

   b) The length of the enlarged photograph is 43.2 cm.

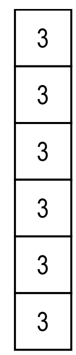
M–3

Solidifying Prior Learning Solutions

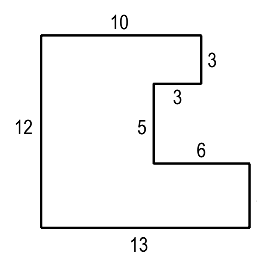
1. Gilles will run out of gas. He has driven two-thirds of the trip and used more than two-thirds of a tank of gas. He used three-quarters of a tank of gas and $\frac{3}{4} > \frac{2}{3}$. An assumption is that Gilles will continue to use the same number of litres per kilometre for the last part of the trip.



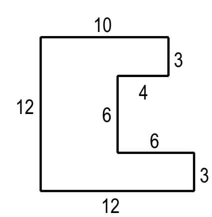
2. There were 18 cookies in the back at the start. Working backwards, there were
   - 3 cookies left for the sixth person
   - 6 before the fifth person ate half
   - 9 before the fourth person ate one-third
   - 12 before the third person ate one-fourth
   - 15 before the second person ate one-fifth
   - 18 before the first person ate one-sixth



3. The missing lengths need to be estimated. Estimates will vary within the constraints.
   a) For these measurements, the area is 117 units^2.
   b) For these measurements, the perimeter is 56 units.
   c) The area can be determined by dividing the shape into three horizontal rectangles with areas of 30, 35, and 52.



   a) For these measurements, the area is 102 units^2.
   b) For these measurements, the perimeter is 56 units.
   c) The area can be determined by subtracting the area of the L-shaped corner (6 + 36) from the 12 × 12 rectangle.



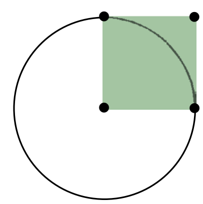
(TN) Given the constraints, the unknown horizontal sides must sum to 16. The unknown vertical sides must sum to 12. If students estimate and calculate within the constraints, the perimeter will always be 56 units. Although they may choose to work with them, students are not restricted to whole number sides. This question could be a lead in to a discussion of domain and range (e.g., “What are the possible numbers?”).

4. Two different solutions:

The volume could be 47.15 inches^3. Let the circumference of the cylinder be the full paper width of 8.5 inches. The relationship $C = \pi(d)$ means the diameter of the top and bottom circles must be less than or equal to 2.7056… inches (dividing: $8.5 \div \pi$). In fractions of inches, the diameter could be $2\frac{11}{16}$ inches. That means the height of the cylinder can be $8\frac{5}{16}\left(\text{subtracting: } 11 - 2\frac{11}{16}\right)$. The volume is calculated as $\pi \times \left(1\frac{11}{32}\right)^2 \times 8\frac{5}{16}$.

Alternatively, the volume could be 48.11 inches^3. Let the circumference of the cylinder be the full paper length of 11 inches, and then the diameter is less than or equal to 3.5014… inches. Let the diameter be 3.5 inches, making the height of the cylinder 5 inches. The volume is $\pi \times (1.75)^2 \times 5$.

5. Just over 3 squares will fit into the circle (exactly π squares will fit).



(TN) Students could cut up the 4 squares drawn on graph paper to determine that just over 3 squares fit in the circle. If they use graph paper, they will be able to make a more precise estimate (more than 3 and one-tenth, or close to 3 and fourteen-hundredths). Then they should connect what they see with the cut-out areas to the algebraic formulas for the areas. The ratio of the area of the circle to the area of the square is $\pi r^2 : r^2$.

6. The circumference is generally longer than the height of a coffee cup. Use your fingers to compare heights and circumferences; most people will be unable to wrap their fingers around the circumference of a cup but will be able to stretch their fingers beyond its height. The distance around the rim (circumference) is about 3 (or exactly π) times greater than the distance across the cup (diameter).

(TN) Rather than providing rulers, teachers could allow students to only use their hands or other referents.

7. Here are three of the many possible answers:

$4 \times 30 \times 21 = 2520$ cm^3

$7 \times 24 \times 15 = 2520$ cm^3

$2 \times 13 \times 96 = 2496$ cm^3

8. Dylan could

   a) be building a rectangular prism (box), a rectangle-based pyramid, a prism with triangular ends and some square faces, or a polyhedron with some square faces

   b) **not** be building a cylinder (unless the shape shown can be bent around a curved end), a sphere, cone, or a regular polygon (other than a cube if the shape shown is a square)

9. If the width is represented by x, then the length is $2x$ and the height is $4x$. Algebraic expressions describing features of the box include the following:

If		$\boxed{w = x}$ $l = 2x$ $h = 4x$	$w = \frac{n}{2}$ $\boxed{l = n}$ $h = 2n$	$w = \frac{H}{4}$ $l = \frac{H}{2}$ $\boxed{h = H}$
then	Area of Base =	$2x^2$	$\frac{n^2}{2}$	$\frac{H^2}{8}$
	Surface Area =	$28x^2$	$7n^2$	$\frac{7H^2}{4}$
	Volume =	$8x^3$	n^3	$\frac{H^3}{8}$

(TN)₁ Students could be asked to compare these different expressions and describe what they notice.

(TN)₂ Students who are ready may be asked to find expressions for diagonals. For the first example above, the expression for diagonal length is $\sqrt{x^2 + (2x)^2 + (4x)^2} = x\sqrt{21}$ and, for the second example, the expression for diagonal length is $n\frac{\sqrt{21}}{2}$.

10. Cylinder with radius, r, and a height of 4. Algebraic expressions describing features include the following:

   Area of the circular base = πr^2

   Volume = $4\pi r^2$

   Surface Area = $\left[2\left(\pi r^2\right) + 2\pi r(4)\right]$ **or** $\left[2\pi r^2 + 8\pi r\right]$ **or** $\left[2\pi r(r + 4)\right]$

11. The table entries show cubes of increasing size.

Side Length	Surface Area	Volume	SA : Vol ratio	SA : Vol ratio Comparison
1	6 x 1=6	1^3 = 1	6 : 1	6 : 1
2	6 x 4 = 24	2^3 = 8	24 : 8	6 : 2
3	6 x 9 = 54	3^3 = 27	54 : 27	6 : 3
4	6 x 16 = 96	4^3 = 64	96 : 64	6 : 4
5	6 x 25 = 150	5^3 = 125	150 : 125	6 : 5

a) The next side length of 6 cm will have a $\frac{\text{SA}}{\text{Vol}}$ ratio of $\frac{6}{6}$ or 1.

b) The ratio of $\frac{\text{SA}}{\text{Vol}}$ is greater than 1 for side lengths less than 6 and is less than 1 for side lengths greater than 6.

c) The ratio of SA : Vol when side length is 100 is 60000 : 1000000 = 674 : 100.

d) The ratio of SA : Vol is 6 : n when side length n is $\frac{\text{SA}}{\text{Vol}} = \frac{6n^2}{n^3} = \frac{6}{n}$.

e) The ratio of SA : Vol for a cylinder with height equal to r is 4 : r.

12. First, you need to decide on a way to determine what it means to have a "better fit." One possibility is to compare the difference in surface area between the hole and the peg, given the circles are the same size. The smaller the difference is the "better fit." The following solution proceeds with this meaning of "better fit." You may use a different meaning or make a comparison, given the squares are the same size.

The first diagram shows a round peg in a square hole. The radius of the circle is r and each side of the square is $2r$. The area difference, hole − peg, is $(2r)^2 - \pi r^2 = (4 - \pi)r^2 = (0.858...)r^2$. The second diagram shows a square peg in a round hole. The radius of the circle is r. The diameter of the square is $2r$, which means each side is $\sqrt{2}r$ (using the Pythagorean theorem). The area difference, hole − peg, is $\pi r^2 - 2r^2 = (\pi - 2)r^2 = (1.141...)r^2$. Using the described meaning of "better fit", the round peg in a square hole is a better fit.

(TN) Students may come up with different ways of determining a "better fit." For example, instead, students may keep the square size the same for comparison purposes. Then, the area difference (round hole − square peg) is $\pi\left(\frac{\sqrt{2}}{2}s\right)^2 - s^2 = \left(\frac{\pi}{2} - 1\right)s^2 = (0.570...)s^2$. The area difference (square hole − round peg) is $s^2 - \pi\left(\frac{s}{2}\right)^2 = \left(1 - \frac{\pi}{4}\right)s^2 = (0.214...)s^2$. Again, the round peg in a square hole is a better fit.

M–4 Solidifying Prior Learning Solutions

1. There are multiple solutions including:

   $\frac{17}{68} = 0.25$ $\quad\quad$ $\frac{36}{48} = 0.75$ $\quad\quad$ $\frac{72}{16} = 4.50$

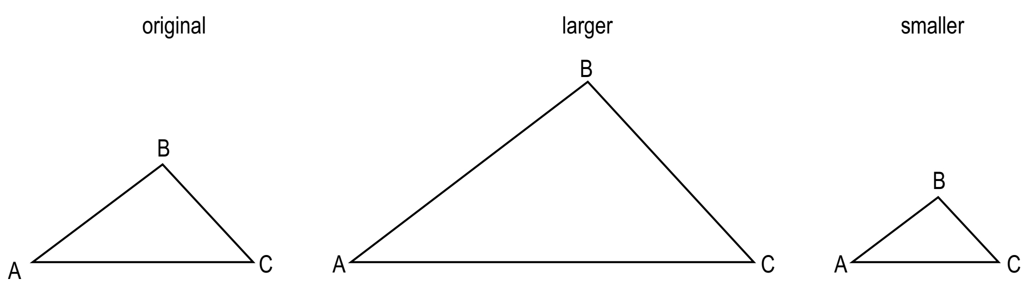
2. The fewest number of people surveyed is 125.

   93.6% could be 93.6 out of 100, but it is not possible to have 0.6 people. It could be 936 out of 1000, but that does not represent the fewest number. Simplify the fraction $\frac{936}{1000} = \frac{117}{125}$.

   There could be as little as 117 of 125 who completed the survey.

3. The hose length is 46.5 m (distance EC). Use the Pythagorean theorem to find AB is 15. Then, use the Pythagorean theorem with ΔABC to find BC is 36. Use the Pythagorean theorem again with ΔECD to find $EC = \sqrt{15^2 + 44^2} = 46.5$ m.

4. The larger triangle has all angles the same measure as the original. The smaller triangle also has all angles the same measure as the original. In the example shown, the larger triangle has all sides that are 2 times the original triangle. The smaller triangle has all sides that are two-thirds of the original triangle.



(TN) Some students may be directed to draw right triangles, since they may be easier to draw as enlargements or reductions. Some students may draw similar shapes by using the fact that the angles must be the same rather than noticing that relationship after drawing the triangles. A cut-out of the original triangle could be used to create an enlargement or reduction by using the angles in the cut-out to trace the angles for an enlarged (or reduced) triangle. It could be suggested to students that they draw the triangles on grid paper. They could then use the grid lines to ensure the shapes are enlargements or reductions.

5. a) The area of ΔAKL is 72 units^2. You are given the square area, which is 64, so each side of square BCDE is 8 units. Let the height of ΔAKL be h. Compare the areas where $\Delta ACD = 2BCDE$, then $8(8 + h) \div 2 = 2(64)$, so $h = 24$ units. Since $h = 24$ and you are given $KL = 6$, then the area of ΔAKL is $6 \times 24 \div 2 = 72$ units^2.

   b) The area of trapezoid KCDL is 126 units^2. The area of square BCDE is 144, so sides of the square are 12 units and the area of ΔACD is 288. Let the height of ΔAKL be h as in part (a) above. $\Delta ACD = 2BCDE$, and then $12(12 + h) \div 2 = 2(144)$, so $h = 36$ units.

   The area of trapezoid KCDL can be found in two ways. First, area ΔACD – area ΔAKL and, second, using the formula $\left(\dfrac{CD + KL}{2}\right) \times$ height.

   First calculation: the trapezoid area is $\Delta ACD - \Delta AKL$, $288 - KL \times 36 \div 2 = 288 - 18KL$.

   Second calculation: the trapezoid area is $\left(\dfrac{12 + KL}{2}\right) \times 12 = 72 + 6KL$.

   Set the first and second expressions equal to each other and solve for KL.

   $288 - 18KL = 72 + 6KL$, so $KL = 9$ units.

   The area of the trapezoid is $72 + 6(KL) = 126$ units^2.

(TN) It is important to let students think and work through this problem. Other solution methods are possible (and may even be preferred). For example, for part (a), a student could find the area of $\Delta CBK = \Delta DEL = 1 \times 8 \div 2 = 4$ and subtract known areas to find the area ΔAKL is 72. For part (b), similar triangles could be used after drawing the altitude from A to the midpoint of CD (label it F). The height, AF, can be found to be 48 units using the area of ΔACD, which is $12 \times AF \div 2 = 288$. $\Delta AFD \sim \Delta DEF$, so $\dfrac{AF}{FD} = \dfrac{DE}{EL}$, so $EL = 1.5$ units.

The trapezoid area can be found by subtracting areas $144 - \Delta DEL - \Delta CBK = 144 - 9 - 9 = 126$ units^2.

A–1 *Solidifying Prior Learning Solutions*

1. Some of the possible answers:

	Similarities	Differences
2^4 and 4^2	■ both use same digits (2 and 4) ■ both are powers ■ both evaluate to 16	■ bases and exponents are switched
3^2 and 2^3	■ both use same digits (2 and 3) ■ both are powers	■ bases and exponents are switched ■ evaluate to different values (9 and 8)

2. GCF
   a) Factors of 32: {1,2,4,8,16,32}; Factors of 8: {1,2,4,8}, GCF is 8
   b) Factors of 24: {1,2,3,4,6,8,12,24}; Factors of 18: {1,2,3,6,9,18}, GCF is 6

3. LCM
   a) Multiples of 32: 32, 64, 96, 128… Multiples of 8: 8, 16, 24, 32, 40, 48… LCM is 32
   b) Multiples of 12: 12, 24, 36, 48… Multiples of 18: 18,36,54,72… LCM is 36

4. Some of the possible answers:
   1×1 tiles: $112 \times 84 = 9408$ tiles
   2×2 tiles: $56 \times 42 = 2352$ tiles
   4×4 tiles: $28 \times 21 = 588$ tiles
   7×7 tiles: $16 \times 12 = 192$ tiles
   14×14 tiles: $8 \times 6 = 48$ tiles
   28×28 tiles: $4 \times 3 = 12$ tiles
   The fewest number of square tiles Sara could use is 12.

   (TN) Students could be led to connect GCF and the solution to this problem.
   GCF (112, 84) = 28, so 28 cm by 28 cm is the largest size tile that evenly fits both dimensions.

5. Divisibility
   a) The possible answers are {60, 120, 180, 240, 300, 360, 400…}. The smallest possible is 60.

   $$\underbrace{(2)\overbrace{(3)(2)}^{6}}_{4}(5) = 60$$ (4 is included because $2 \times 2 = 4$. 6 is included because $2 \times 3 = 6$.)

   b) The smallest number divisible by all of the numbers from 1 to 20 is {232 792 560}. It is the product of 5, 7, 9, 11, 13, 16, 17, 19.

6. If there are 20 students and 20 lockers, then lockers 1, 4, 9, and 16 are left open and 12, 18, and 20 are touched the most at 6 times each.

   [TN] Students could be asked to extend this problem to 100 students and 100 lockers; then, lockers 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 are left open.

   The ones left open are square numbers because square numbers have an odd number of factors (i.e., there are 5 factors of 16, {1,2,4,8,16}). All other numbers (including primes) have pairs of factors (i.e., There are 2 factors of 7, {1,7}, and 6 factors of 12, {1,2,3,4,6,12}). The locker numbers touched most often have the greatest number of factors.

7. Sets

   a) Some possible responses are as follows:

      Notice:

      "They are all square numbers."

      "The differences between the numbers increases by two each time."

      "The numbers alternate between even and odd."

      Wonder:

      "What else could be included in the set? 225?"

      "Why does it start at 36 and end at 121?"

   b) These numbers are all 1 less than Set A. The difference between the numbers also increases by two each time and are the same as Set A. The numbers also alternate between even and odd.

   c) These numbers are all 16 less than Set A. The difference between the numbers also increases by two each time and is the same as Set A. The numbers also alternate between even and odd.

Solidifying Prior Learning Solutions

1. Some of the possible answers:

   a) $\frac{14}{10}, \frac{141}{100}, \frac{1414}{1000}, \frac{...}{...}, \frac{14}{10}, \frac{13}{10}, \frac{12}{10}, \frac{11}{10}, \frac{7}{5}, \frac{6}{5}, \frac{4}{3}$

   b) $\frac{17}{10}, \frac{173}{100}, \frac{1732}{1000}, \frac{...}{...}, \frac{16}{10}, \frac{8}{5}$

   c) $\frac{22}{10}, \frac{223}{100}, \frac{2236}{1000}, \frac{...}{...}, \frac{22}{10}, \frac{21}{10}, \frac{11}{5}, \frac{17}{8}$

2. Some of the possible answers:

   $0 + 1 + 4 = 2 + 3$

   $4 - (2 + 1) = 3^0$

   $3 \div (2 + 1) = 4^0$

3. The hypotenuse of the third triangle (the leg of the fourth triangle) has a hypotenuse of 2 units (10 cm).

   The hypotenuse of the eighth triangle (the leg of the ninth triangle) has a hypotenuse of 3 units (15 cm).

   The hypotenuse of triangles 3, 8, 15, and 24 will be whole number units (multiples of 5 cm).

   The legs of triangles 4, 9, 16, and 25 will be whole number units (multiples of 5 cm).

4. Squares:

   a) Yes, the stacks of squares are the same height. Find the lengths of sides using a calculator:

   $\sqrt{45} \doteq 6.708$

   $\sqrt{5} \doteq 2.236$

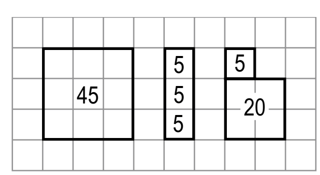
   $(2.236)(3) \doteq 6.708$

   $\sqrt{20} \doteq 4.472$

   $\sqrt{20} + \sqrt{5} \doteq 4.472 + 2.236 \doteq 6.708$

   OR

   Find the height using a visual model with $\square$ = 5.



b) Some possible answers for squares that have the same height as a square with area of 72: (3 squares with an area of 8), (1 square with an area of 8 and 1 square with an area of 32).

(TN) For part (a), simplifying the radical $\sqrt{45} = 3\sqrt{5} = \sqrt{5} + \sqrt{20}$ is a justification that is at the level of the end of Grade 10, so it **does not yet need to be shown this way**. Other answers may compare lengths and/or areas by dividing larger squares into arrays of smaller squares.

5. Repeating fractions:

a) As a fraction, 0.3333… is $\frac{3}{9}, \frac{1}{3}$.

Solve the equation:
$$\begin{aligned} 10x &= 3 + x \\ 9x &= 3 \\ x &= \frac{3}{9} = \frac{1}{3} \end{aligned}$$

b) Create and solve equations:

i) Let x = 0.636363…
$$\begin{aligned} 100x &= 63.636363... \\ 100x &= 63 + x \\ 99x &= 63 \\ x &= \frac{63}{99} = \frac{21}{33} \end{aligned}$$

ii) Let x = 5.454545…
$$\begin{aligned} 100x &= 545.4545... \\ 100x &= 540 + x \\ 99x &= 540 \\ x &= \frac{540}{99} = \frac{60}{11} \end{aligned}$$

A–3

Solidifying Prior Learning Solutions

1. $2^2 = 4$

   $3^2 = 9 = 4 + 5$

   $4^2 = 16$

   $5^2 = 25 = 12 + 13$

   The sum of two consecutive numbers is always odd. Squaring an even number is always even. So, the pattern is not possible with the square of even numbers. Squaring an odd number is always odd, and any odd number can be written as the sum of two consecutive numbers (i.e., begin with any odd number, and the first consecutive number is half of the even number before the odd number in question). This pattern always works with the square of an odd number.

2. One possible answer is:

   $\sqrt{9}$ $\quad$ $\sqrt{16}$ $\quad$ $\sqrt{2+7}$ $\quad$ $\sqrt{4}+5$ $\quad$ $\sqrt[2]{8}$

3. 2 000 000 000

   Multiplying powers of 10 is easy to do mentally. The product of each pair of 2s and 5s is 10. Determine the number of pairs of 2s and 5s.

   $$\left(5^4\right)\left(20^5\right) = (5)^4\,(5\cdot 2\cdot 2)^5 = 5^4\cdot 5^5\cdot 2^5\cdot 2^5 = 5^9\cdot 2^9\cdot 2 = 2\cdot 10^9$$

   There are many other pairs of numbers. Here are two examples:

   $$\left(35^2\right)\left(2^2\right) = (5\cdot 7)^2\cdot 2^2 = 5^2\cdot 7^2\cdot 2^2 = (70)^2 = 4900$$

   $$\left(2^6\right)\left(5^8\right) = 2^6\cdot 5^6\cdot 5^2 = (2\cdot 5)^6\cdot 5^2 = 10^6\cdot 25 = 25000000$$

4. Expressions in the form

   a) Some possible answers are: $[0][6]^{[7]} = [0][8]^{[4]}$, $[2][4]^{[5]} = [4][2]^{[9]}$, $[9][3]^{[7]} = [3][9]^{[4]}$

   b) Some possible answers are: $[1][5]^{[3]}$, $[4][3]^{[5]}$, $[8][2]^{[6]}$

5. Answers will vary depending on the interpretation of "large" result. Some answers are $(1)\left(4^5\right) = 1024$, $(1)\left(5^4\right) = 625$, or $(2)\left(3^5\right) = 486$.

A–4 Solidifying Prior Learning Solutions

1. Area model:

   a) 26 × 14 is:

	10	1
10	200	60
4	80	24

	50	1
10	500	10
2	100	2

   b) Product: 200 + 60 + 80 + 24 = 364

   c) One possible answer is 51 × 12 = 612

2. Area model:

	$\boxed{3x}$	7
$2x$	$6x^2$	$\boxed{14x}$
$\boxed{2}$	$\boxed{6x}$	14

   The product represented is $(3x + 7)(2x + 2)$.

   The product is equivalent to $6x^2 + 14x + 6x + 14$ or $6x^2 + 20x + 14$.

3. Area model:

	$6x^2y^2$	$5x^2y$
3	$18x^3y^2$	$15x^2y$
$5x^3y$	$30x^6y^3$	$25x^5y^2$

$$\left[6x^3y^2 + 5x^2y\right]\left[3 + 5x^3y\right] = 18x^3y^2 + 15x^2y + 30x^6y^3 + 25x^5y^2$$

4. Rectangular prisms:

   a) Surface area is 78 m²: $\left[2(5)(3) + 2(3)(3) + 2(5)(3)\right]$

   Volume is 45 m³: (5)(3)(3)

   b) $\left[2(5-x)(2x-1)\right] + \left[2(5-x)(2x+1)\right] + \left[2(2x+1)(2x-1)\right] = 40x - 2$

   c) $(2x+1)(2x-1)(5-x) = -4x^3 + 20x^2 - x - 5$

d) A height of $(5 - x)$ is possible if $x < 5$.

A length of $(2x + 1)$ is possible if $x > \frac{-1}{2}$.

A width of $(2x - 1)$ is possible if $x > \frac{1}{2}$.

Therefore, x must be less than 5 but greater than $\frac{1}{2}$.

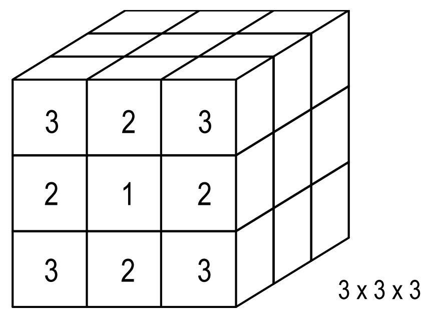
The only possible whole number values of x are {1, 2, 3, 4}.

5. Two of the possible answers:

a) $[3x + 8][x + 3] = 3x^2 + 17x + 24$

b) $[3x + 12][x + 2] = 3x^2 + 18x + 24$

6. Painted cubes:



a) Construct or draw a 3 × 3 × 3 cube made from 27 smaller cubes.

b) Painted orange

i) Number painted on 1 face: 6 (one on each large cube face)

ii) Number painted on 2 faces: 12 (4 on each of front, middle, and back)

iii) Number painted on 3 faces: 8 (the corners of large cube)

iv) Number painted on 4 faces: 0

v) Number painted on zero faces: 1 (the very centre of the large cube)

c) This chart shows the number of faces painted for the other cube sizes.

Painted Cube	1 Face Painted	2 Faces Painted	3 Faces Painted	4 Faces Painted	0 Faces Painted
3 x 3 x 3	6	12	8	0	1
4 x 4 x 4	24	24	8	0	8
5 x 5 x 5	54	36	8	0	27

Some things students may notice:

- "There are never more than 3 faces painted."
- "There are always the same number of cubes with 3 faces painted."
- "The number of cubes with 2 faces goes up by 12 each time the cube dimension increases."

Students may wonder:

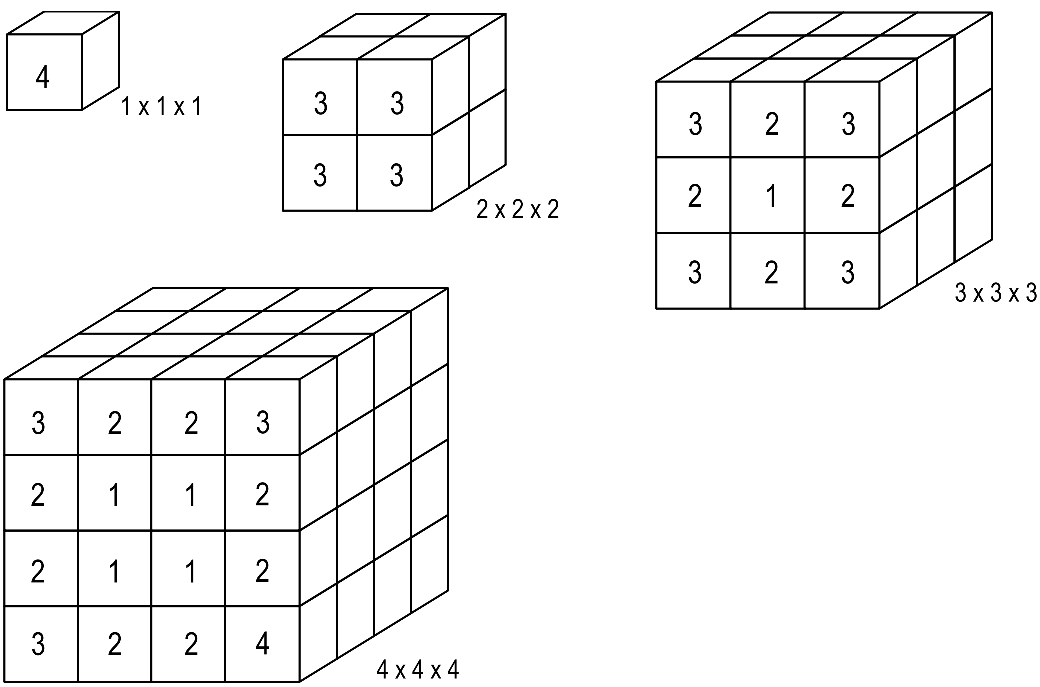
- "Does a $1 \times 1 \times 1$ cube or a $2 \times 2 \times 2$ cube fit the same pattern?"
- "Is there a general rule that can describe the number of cubes with each number of faces painted?"

d) The general $n \times n \times n$ cube:

Painted Cube	1 Face Painted	2 Faces Painted	3 Faces Painted	4 Faces Painted	0 Faces Painted
$n \times n \times n$	$6(n-2)^2$	$12(n-2)$	8	0	$(n-2)^3$

The algebraic expressions for the general case may look different but should simplify to equal the ones in the table. For an $n \times n \times n$ cube, the number of cubes with the following features:

- 1 face painted on each of the 6 faces is $(n - 2)(n - 2)$.
- 2 faces painted on each of the 12 edges is $(n - 2)$.
- 3 faces painted on each of the 8 corners.
- 4 faces painted is 0. There are never more than 3 faces painted (a $1 \times 1 \times 1$ cube is the exception).
- 0 faces painted on the cube inside all of the surface cubes with dimensions $(n - 2)(n - 2)(n - 2)$.



A–5

Solidifying Prior Learning Solutions

1. Name the two numbers that have the following characteristics:
   a) The possible answers are 1, 24 or 2, 12 or 3, 8 or 4, 6 or −1, −24 or −2, −12 or −3, −8 or −4, −6.
   b) Some of the possible answers are 0, −13 or −1, −12 or −5, −8 or 1, −14 or −3, 16 or ….
   c) 8 and 3
   d) −3 and −5
2. The sums are: 19, 11, 9, −19, −11, −9 for the expressions: 1 + 18, 2 + 9, 3 + 6, −1 + (−18), −2 + (−9), and −3 + (−6).
3. The differences are: −11, −4, −1, 1, 4, 11 for the expressions: 1 − 12, 2 − 6, 3 − 4, 4 − 3, 6 − 2, and 12 − 1.
4. −18 since the product matches the expressions: −18 × 1, −9 × 2, −6 × 3, −3 × 6, −2 × 9, and −1 × 18.
5. $(3x + 5)(2x + 2)$ is equivalent to the trinomial $6x^2 + 16x + 10$.
6. Equations:
   a) $4x + 8 = \boxed{4}(x + 2)$
   b) $\boxed{6}x^2 + 12x = \boxed{6}(x + 2)$
   c) $\left(2x^2 + \boxed{6}x + 3\right)\left(\boxed{2}x + \boxed{4}\right) = 4x^3 + 20x^2 + 30x + 12$
   d) $12x + 9 = \boxed{3}\left(\boxed{4x} + \boxed{3}\right)$
7. Binomial products and trinomial expressions:
   a) Coefficients are 4 or 8.

      $(n-2)(n+6) = n^2 + 4n - 12$ or $(n+2)(n+6) = n^2 + 8n + 12$
   b) Coefficients are −6 or 0.

      $(x-3)(x-3) = x^2 - 6x + 9$ or $(x-3)(x+3) = x^2 - 9$
   c) Coefficients are −1, 1, −9 or 9.

      $(a-5)(a+4) = a^2 - a - 20$ or $(a+5)(a-4) = a^2 + a - 20$

      or $(a-5)(a-4) = a^2 - 9a + 20$ or $(a+5)(a+4) = a^2 + 9a + 20$
   d) Coefficients are 13 or −7.

      $(2b+3)(b+5) = 2b^2 + 13b + 15$ or $(2b+3)(b-5) = 2b^2 - 7b - 15$

Students may notice the following:

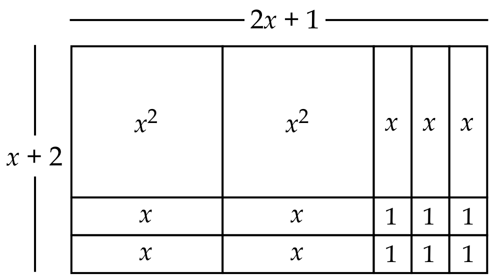
- "In the first three, the coefficients of the middle term are the sum of the constants in the binomials."
- "The variables in the first three have binomial factors with coefficients of 1."

They may wonder:

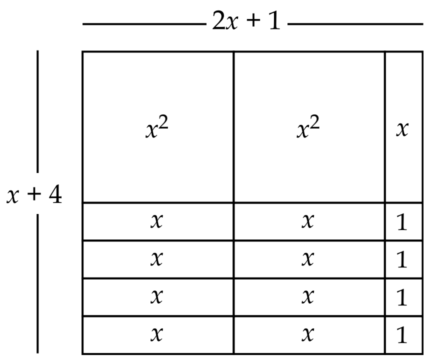
- "Why is the fourth one different?"
- "Is there a pattern that fits the fourth one?"

8. Find two numbers:
   a) −3 and −12
   b) −9 and 2

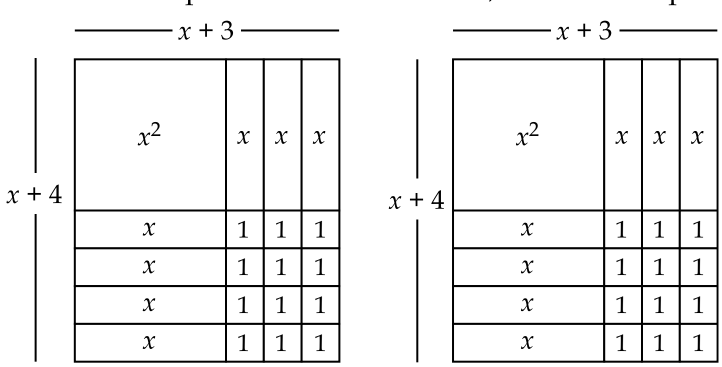
9. Representing the product of binomials:
   a) This model represents $2x^2 + 7x + 6$, which is the product of $(2x + 3)(x + 2)$.



   b) This model represents $2x^2 + 9x + 4$, which is the product of $(2x + 1)(x + 4)$.

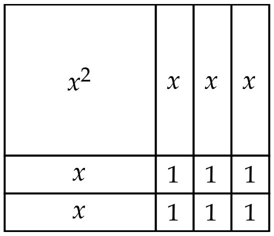


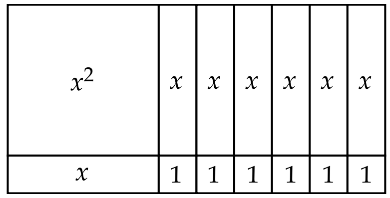
   c) This model represents $2x^2 + 14x + 24$, which is the product of $2(x + 3)(x + 4)$.



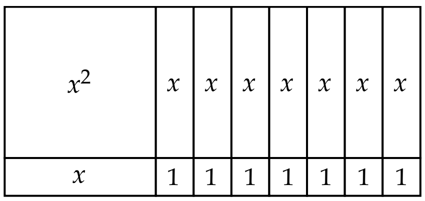
10. Representing the factors of trinomials:

a) There are two ways to use one x^2 tile and 6 unit tiles to create rectangles. They represent the products $(x + 3)(x + 2)$ and $(x + 6)(x + 1)$.

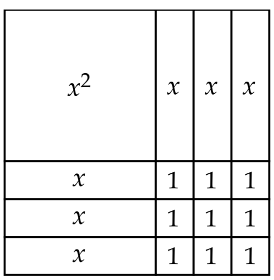


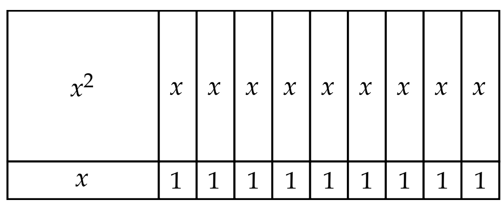


b) There is one way to use one x^2 tile and 7 unit tiles to create a rectangle. It represents the product $(x + 7)(x + 1)$.



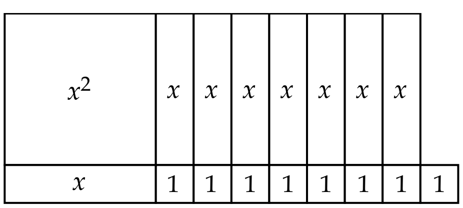
c) There are two ways to use one x^2 tile and 9 unit tiles to create rectangles (one is a square). They represent the products $(x + 3)^2$ and $(x + 9)(x + 1)$.

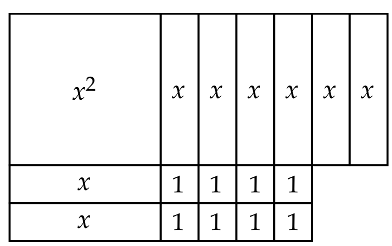




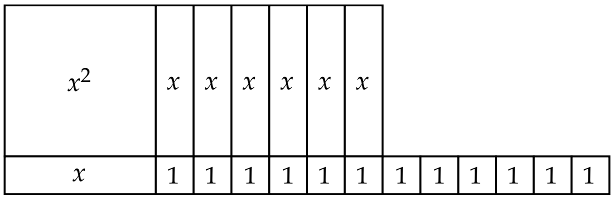
11. Use algebra tiles to show the following:

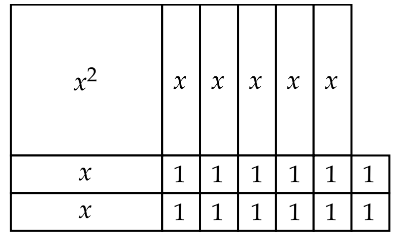
a) The trinomial cannot be written as binomial factors. The 8 unit tiles can only be arranged as rectangles in the two ways shown (1 × 8 or 2 × 4) and the 8 x-tiles cannot be arranged with them to make a rectangle area.

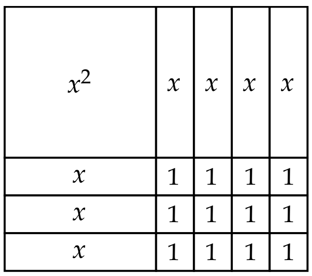




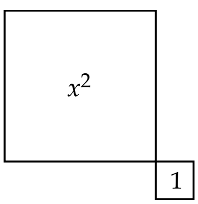
b) The trinomial can be written as binomial factors. The 12 unit tiles can be arranged as rectangles in the three ways shown (1×12, 2×6, or 3×4). The 1×12 and 2×6 arrangements do not make a rectangle with the 7 x-tiles, but the 3×4 arrangement of unit tiles does. The trinomial can be written as $(x + 4)(x + 3)$.







c) The expression cannot be written as binomial factors. The 1 unit tile can only be arranged in the one way shown (1×1). There are no x-tiles to make a rectangle area (2 x-tiles are needed).



[TN] You may want to talk to students about what the rectangular arrangements show. For each of the examples above:

a) The representation shows that the trinomial $x^2 + 8x + 8$ cannot be factored but could be written as $(x + 7)(x + 1) + 1$ or as $(x + 4)(x + 2) + 2x$.

b) The representation shows that the trinomial $x^2 + 7x + 12$ could be written as $(x + 6)(x + 1) + 6$, as $(x + 5)(x + 2) + 2$, or as $(x + 4)(x + 3)$, which is a factored form.

c) The representation shows that the expression cannot be written in any other form.

Solidifying Prior Learning Solutions

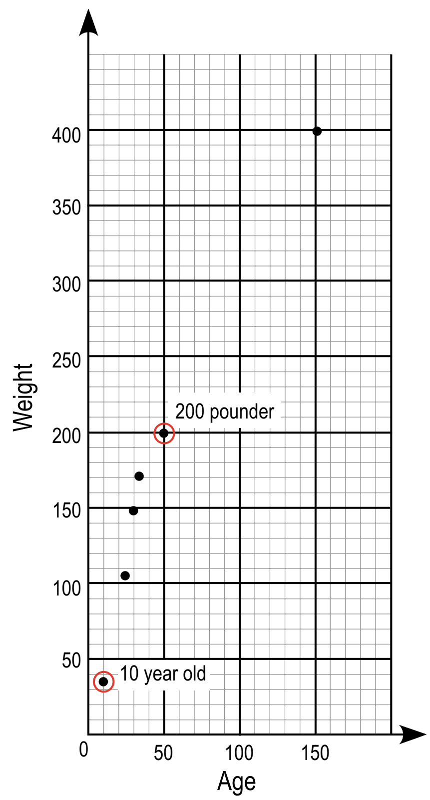
1. P + Q + R = 10 (P = 4, Q = 5, R = 1)

   Some possible reasoning:

   P + Q must have a ones digit of 9 (ones column). Q + Q must have a ones digit of zero. However, Q cannot be zero because column one requires Q + P = 9, so Q must be 5.

2. Graph and predict:

White Sturgeon Fish	
Age	Weight (pounds)
26	116
30	148
33	172
154	400



   a) Age, in the first column, is the independent variable on the *x*-axis and weight is the dependent variable on the *y*-axis.

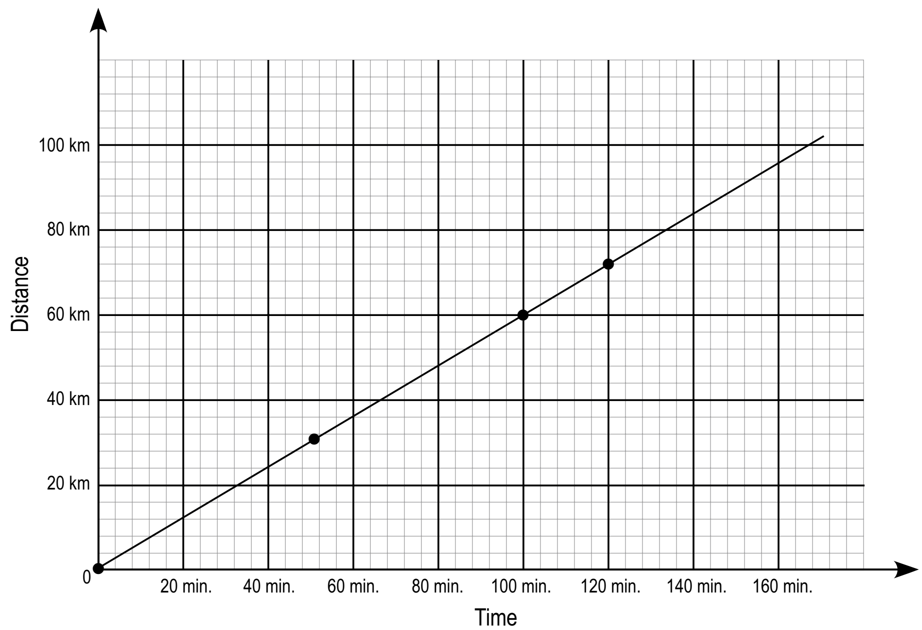
   b) According to this data, a 10-year-old sturgeon could be between 35 and 45 pounds.

   c) According to this data, a 200-pound sturgeon might be 40 to 50 years old.

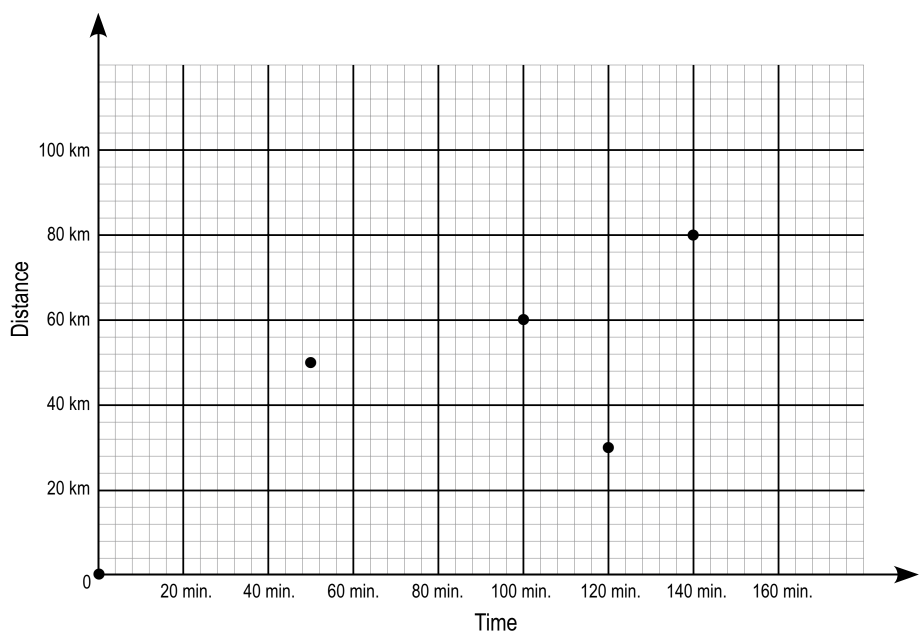
3. Graphs showing a strong relationship will have data points that are all close to a line (or other predictable curve).

   a) Two examples are "distance travelled on a road trip" and "time" or "average temperature in Thompson" and "month."

   b) This graph shows a strong relationship between "distance travelled" and "time." The speed was constant.



   c) This graph shows a weak relationship between "distance travelled" and "time." The speed and direction of travel was not consistent throughout the trip (even going backwards between 100 and 120 minutes).



4. Some of the possible answers:

   a) C = 12 + 2f

   Possible scenario: Renting skates

   Cost of skate rental (C) = $12 basic fee plus $2 per hour ($f$).

   b) $Y = 25x$

   Possible scenario: Distance travelled on a bike

   (Y) = rate (speed) of 25km/h multiplied by number of hours travelled (x)

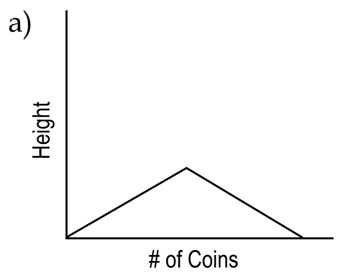
   c) $b = 12 - a$

   Possible scenario: Cookies left

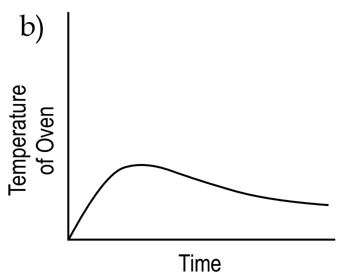
   The number of cookies left in the package (b) = the number of cookies originally in the package (12) subtracting the number you have eaten (a).

5. Some of the possible answers:

   a) Stacking and unstacking coins



   b) Graphing the temperature of the oven as it heats up, reaches maximum temperature, and gradually begins to cool



(TN) Other examples of this kind of question can be found by searching the Internet for “graphing stories” initiated by Dan Meyer. Follow this link: www.graphingstories.com.

6. David's reading:

a) This pattern could continue until day 14, when David reads 1 page. According to the pattern, he would read zero pages (or negative pages) after that. The day can be found using the relationship from the table, pages read is 40, 37, 34, etc. The relationship could be written as pages = $40 - 3(d - 1)$.

b) David reads 217 pages in one week.

Day	Total Pages	
1	40	Friday
2	40 + 37	Saturday
3	40 + 37 + 34	Sunday
4	40 + 37 + 34 + 31	Monday
5	40 + 37 + 34 + 31 + 28 = 170	Tuesday
6	40 + 37 + 34 + 31 + 28 + 25	
7	40 + 37 + 34 + 31 + 28 + 25 + 22 = 217	

c) David would begin reading on Friday in order to follow the same pattern and to finish a 150-page book on a Tuesday (reading only 8 pages on the last day). See the table above.

R–2 Solidifying Prior Learning Solutions

1. Some possible answers are:

$\frac{1}{20}$: the only unit fraction (numerator of 1)	$\frac{20}{25}$: the only two-digit numerator
$\frac{2}{3}$: the only repeating decimal	$\frac{5}{4}$: the only value greater than 1

2. The total number of blue buttons is any number from 34 to 39.

   One solution involves writing the ratios for Pile 1 as Red : Blue is $n:2n$ and for Pile 2 as Red : Blue is $3m:5m$ for positive integers n and m. Knowing the total number of red is 20, solve the equation $n + 3m = 20$. There are 6 solutions for (n, m), namely (2, 6), (5, 5), (8, 4), (11, 3), (14, 2), and (17, 1). The number of blue can be found by substituting the 6 possible pairs for n and m into the equation $2n + 5m$.

3. Grace's garden:

   a) Garden shapes

N = 4	N = 5
x x x x x x	x x x x x x x
x • • • • x	x • • • • • x
x • • • • x	x • • • • • x
x • • • • x	x • • • • • x
x • • • • x	x • • • • • x
x x x x x x	x x x x x x x

Shape #	# of Flower Plants	# of Tomato Plants	Total Number of Plants
1	8	1	9
2	12	4	16
3	16	9	25
4	20	16	36
5	24	25	49

   b) The number of flowers can be found by adding 4 corner flowers and then N flowers on each of the 4 sides, so $4 + 4N$. The total number of plants is a square, $(N + 2)^2$, which could be written as $N^2 + 4N + 4$. The total number of plants could also be found by adding the N^2 tomatoes in the middle and the $4 + 4N$ flowers around the outside.

(TN) For more opportunities to examine patterns with questions like this, visit www.youcubed.org/tasks (Stanford Graduate School of Education) or www.visualpatterns.org (Nguyen).

4. Some answers are as follows:

   a) "double the input number and then add one"

      "add ten to the input number"

      "twenty less one-ninth of the input number"

   b) $(2x + 1 = y)$

      $(x + 10 = y)$

   c) $4(x+1) \div 2 - 1$ or

      $\left(10\left(\sqrt{x}+1\right)-2\right) \div 2$

      $(x \div 3) \times 7 - 2$

R–3

Solidifying Prior Learning Solutions

1. Some possible answers are as follows:

33%: the only value displayed as a percent	$\frac{1}{3}$: the only unit fraction
$\frac{5}{3}$: the only number greater than 100%	$0.\overline{6}$: the only repeating decimal number

2. Some answers are as follows:

a) $\frac{\boxed{1}}{\boxed{2}} + \frac{\boxed{3}}{\boxed{7}} = \frac{13}{14}, \frac{\boxed{1}}{\boxed{2}} + \frac{\boxed{4}}{\boxed{9}} = \frac{17}{18}, \frac{\boxed{7}}{\boxed{8}} + \frac{\boxed{1}}{\boxed{9}} = \frac{71}{72}$

b) $\frac{\boxed{3}}{\boxed{2}} - \frac{\boxed{5}}{\boxed{8}} = \frac{14}{16}, \frac{\boxed{8}}{\boxed{5}} - \frac{\boxed{2}}{\boxed{3}} = \frac{14}{15}, \frac{\boxed{9}}{\boxed{6}} - \frac{\boxed{4}}{\boxed{7}} = \frac{39}{42}$

c) $\left(\frac{\boxed{4}}{\boxed{1}}\right)\left(\frac{\boxed{2}}{\boxed{9}}\right) = \frac{8}{9}, \left(\frac{\boxed{5}}{\boxed{2}}\right)\left(\frac{\boxed{3}}{\boxed{8}}\right) = \frac{15}{16}$

d) $\frac{\boxed{1}}{\boxed{2}} \div \frac{\boxed{4}}{\boxed{7}} = \frac{7}{8}, \frac{\boxed{4}}{\boxed{3}} \div \frac{\boxed{9}}{\boxed{6}} = \frac{8}{9}, \frac{\boxed{5}}{\boxed{2}} \div \frac{\boxed{8}}{\boxed{3}} = \frac{15}{16}$

(TN) Students could be encouraged to describe any patterns they see and compare results with each other to determine which expressions evaluate to numbers closer to 1. Additionally, other restrictions could be added to make this task easier or more difficult. For example, it may be easier to find expressions that include (rather than exclude) equal to 1. It may be more challenging to insist that a certain digit (such as 4) must be included in each expression.

Variations on this challenge include making the values as large or as small as possible.

3. Products:

a)

Expression	Product
(4)(5)	20
(3)(5)	15
(2)(5)	10
(1)(5)	5
(0)(5)	0
(−1)(5)	−5
(−2)(5)	−10

b)

Expression	Product
(4)(−5)	−20
(3)(−5)	−25
(2)(−5)	−10
(1)(−5)	−5
(0)(−5)	0
(−1)(−5)	5
(−2)(−5)	10

Some things you may notice: the products in (a) decrease but the products in (b) increase. Both tables have a product with a constant difference of 5. Some things you may wonder: How would the pattern change if the first factor began with negative values? What real-world scenario might the products in the tables represent? How can we describe the meaning of a negative multiplied by a negative?

4. Square:

   a) One possible answer is a side is 4 units long going from E to G. The perimeter would be 16 linear units and the area would be 16 square units. A different answer has the points E and G as diagonally opposite vertices, which means each side of the square is $\sqrt{2}$ units long. The perimeter is $4\sqrt{2}$ or 5.656 units and the area is 2 square units.

   b) Squares with vertical and horizontal sides could be drawn by adding points A and C at (2, 5) and (6, 5) OR at (2, −3) and (6, −3). Alternately, points for a square with sides at 45° to the horizontal or vertical have points A and C at (4, 3) and (4, −1).

5. Rectangle—the three possible answers are as follows:

   a) Rectangle with W and Y as diagonally opposite vertices has other vertices at A(−2, 1) and B(2, 4) Area = 12 square units.

   b) Two possible rectangles could be drawn with adjacent vertices at W and Y.

      i) A(−5, 0) and B(-1, −3) Area = 25 square units

      ii) A(1, 8) and B(5, 5) Area = 25 square units

6. Line segment:

   a) Some answers are (−3, −2), (9, 7), or (3, 2.5).

      Each y-value increases by 3 when each x-value increases by 4.

   b) Some answers are (−2, −2), (22, 10), (10, 4) or (8, 3).

      Each y-value increases by 4 when each x-value increases by 8. You could also say that each y-value increases by 1 when each x-value increases by 2.

(TN) *Marbleslides* and other activities from Desmos allow students to explore slope and other characteristics of equations. Find the activities at https://teacher.desmos.com.

R–4 Solidifying Prior Learning Solutions

1. Some answers are as follows:

$$\begin{array}{r} 1\;3\;8 \\ +\,6\;5\;4 \\ \hline 7\;9\;2 \end{array} \qquad \begin{array}{r} 6\;3\;4 \\ +\,1\;5\;8 \\ \hline 7\;9\;2 \end{array} \qquad \begin{array}{r} 2\;4\;1 \\ +\,5\;9\;6 \\ \hline 8\;3\;7 \end{array}$$

   (TN) There are many solutions. Providing students a hint by filling in one of the rows or columns may help vary the level of challenge.

2. Many answers are possible. Here is one example:

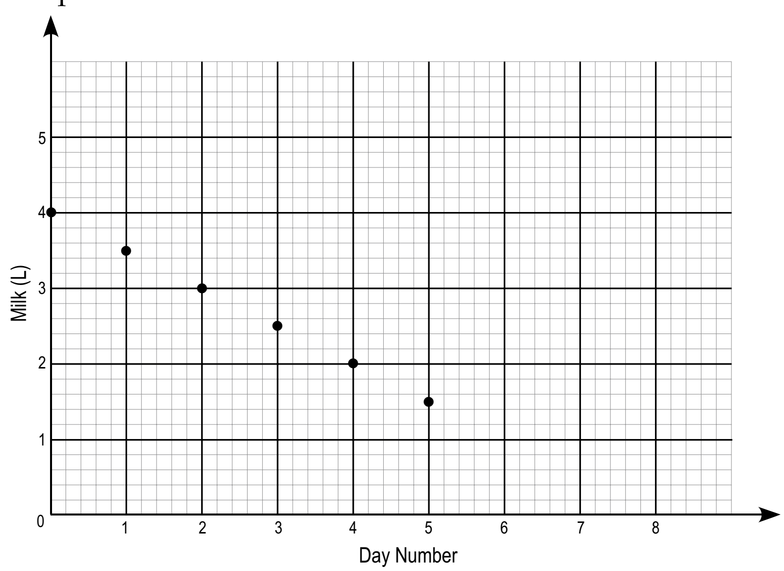
   a) A context could be the number of litres of milk left in a household refrigerator. Day zero is the day the milk was purchased. "Day" represents the number of days after purchase. The "Value" represents the approximate number of litres in the fridge at the beginning of each day.

   b) Table of values:

Day	0	1	2	3	4	5
Litres of Milk	4	3.5	3	2.5	2	1.5

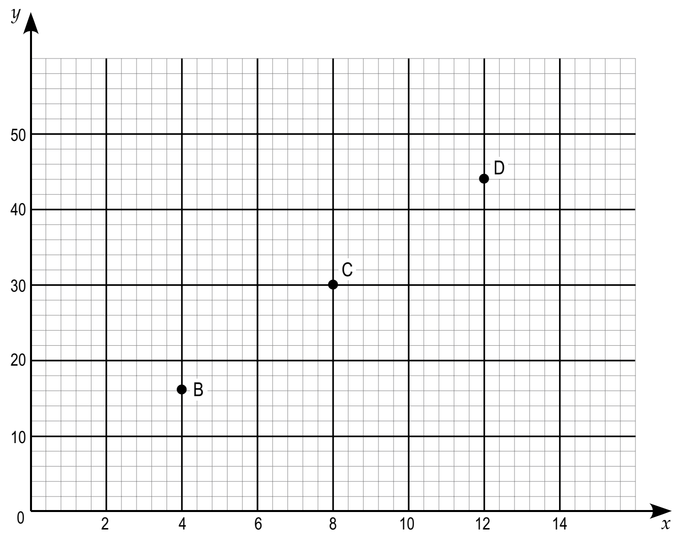
   c) The pattern begins with 4 litres of milk in the fridge and decreases by a constant of 500 mL each day. The pattern will change when more milk is purchased or the milk runs out.

   d) Graph:



3. Some answers are as follows:

a)

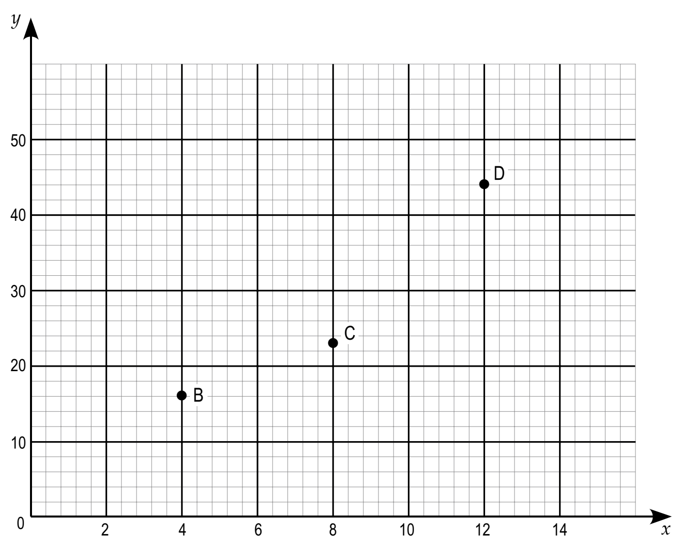


i) (8, 30)

ii) This fits a pattern because the *x*-values go up by 4, (4, 8, 12) and the *y*-values go up by 14 (16, 30, 44).

iii) These values are all on a straight line when graphed, since the *y*-value increases by 14 each time the *x*-value increases by 4.

b)



i) (8, 23)

ii) This fits a pattern because the *x*-values go up by 4 (4, 8, 12) and the *y*-values go up by increasing values, by 7, and then by 21 (16, 23, 44).

iii) These values do not fit on the same line when graphed. The *y*-values increase by different amounts each time, as the *x*-values increase by a constant 4.

4. Pattern:
   a) There are many ways to describe this pattern. Here are a couple of examples. "The pattern has squares of increasing size with an additional top layer of blocks." Or, "The pattern shows rectangles of increasing size where the height is always one more than the base."
   b) Figure 4 will have 20 blocks: a 4 × 4 square of blocks with a 1 × 4 row on top.
   Figure 5 will have 30 blocks: a 5 × 5 square of blocks with a 1 × 5 row on top.
   c) Figure 10 will have 110 blocks; following the same pattern, there will be a 10 × 10 square of blocks with a 1 × 10 row on top.
   d) The friend means that a graph showing the figure number versus the number of blocks would not show a set of points that all lie along the same line. The data values to be graphed are shown in the table. The x-values go up by a constant value of 1, but the y-values go up by different amounts each time (by 4 then 6, then 8, then 10), so they will not lie on the same line.

Figure number	1	2	3	4	5
Number of blocks	2	6	12	20	30

5. Pattern:
   a) There are many ways to describe this pattern. Here are a couple of examples. "The pattern shows two rectangular sections of blocks of increasing length sandwiching one block in the middle." Or, "The pattern shows a vertical pillar of 3 blocks with 4 blocks added each time, 2 at the top (left and right) and 2 at the bottom (left and right)."
   b) Figure 4 will have 19 blocks. There are 15 in figure 3 and there will be 4 more in the next figure. Figure 5 will have 23 blocks. There are 19 in figure 4 and there will be 4 more in the next figure.
   c) Figure 10 will have 43 blocks. Figure 1 has a central pillar of 3 blocks with 4 added to the top (L&R) and bottom (L&R). Figure 2 has a central pillar of 3 blocks with 2 sets of 4 added. Figure 3 has a central pillar of 3 with 3 sets of 4 added. So, figure 10 will have a central pillar of 3 blocks with 10 sets of 4 added for a total of 43.
   d) The figure closest to 1000 blocks is figure 249. There will be a central pillar of 3 blocks and 249 sets of 4 for a total of 999 blocks.
   e) The friend means that a graph showing the figure number versus the number of blocks would show a set of points that all lie along the same line. The data values to be graphed are shown in the table. The x-values go up by a constant value (of 1) and the y-values also go up a constant value (of 4) each time, so the data points will lie on the same line.

Figure number	1	2	3	4	5
Number of blocks	7	11	15	19	23

6. 34%

   10% of the boys is 3, so 30% is 9.

   10% of the girls is 2, so 40% is 8.

   There are 17 students (9 boys + 8 girls) out of a total of 50 who received certificates.

   $$\frac{17}{50} = \frac{17 \times 2}{50 \times 2} = \frac{34}{100}$$

   Therefore, 34%.

7. Mental calculations:

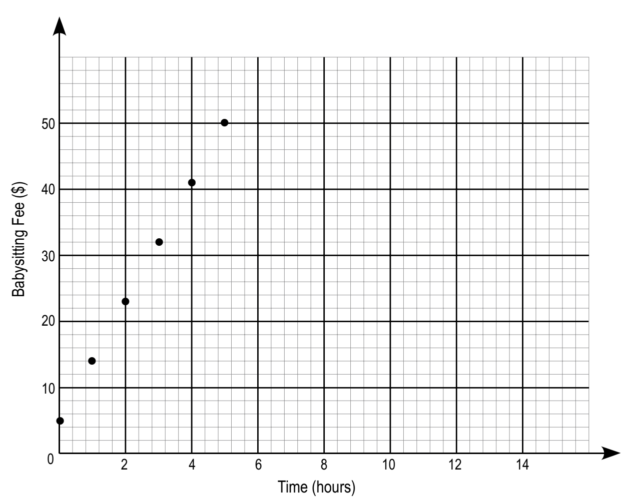
   a) x^2 has the smallest value when x is between 0 and 1. The table shows the values using benchmarks of 0, $\frac{1}{2}$, and 1. The smallest value is x^2 when $x = \frac{1}{2}$.

Expression	x is close to 0	$x = \frac{1}{2}$	$x = 1$
x	~0	$\frac{1}{2}$	1
x^2	~0	$\frac{1}{4}$	1
$\frac{1}{2}$	very large	2	1
$2x$	~0	1	2
$\sqrt{x}$	~0	~0.7 since (0.7)(0.7) = 0.49	1

b) $\frac{1}{x}$ has the smallest value when x is between 1 and 4. The table shows the values using benchmarks of 1, $1\frac{1}{2}$, 2, and 4. The smallest value is $\frac{1}{x}$ when $x = 4$.

Expression	$x = 1$	$x = 1\frac{1}{2} = \frac{3}{2}$	$x = 2$	$x = 4$
x	1	$1\frac{1}{2}$	2	4
x^2	1	$\frac{9}{4} = 2\frac{1}{4}$	4	16
$\frac{1}{x}$	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{4}$
$2x$	2	3	4	8
$\sqrt{x}$	1	~1.2 since (1.2)(1.2) = 1.44	~1.4 since (1.4)(1.4) = 1.96	2

8. Babysitting.
   a) The graph shows the total babysitting fee with a rate of \$9 per hour.
   b) It is a linear relationship because the points all lie along the same line.
   c) No negative numbers will be included because you cannot earn a negative amount of money or work a negative number of hours.
   d) All the points will be shifted up 20 units. The graph will start at 0 hours and \$25, the next points will be at 1 hour and \$34, and then 2 hours and \$43, etc.



R–5

Solidifying Prior Learning Solutions

1. 1.011 is closest to 1. Compare after converting all answers to thousandths. The numbers in the same order are $\frac{1100}{1000}, \frac{1110}{1000}, \frac{1101}{1000}, \frac{1111}{1000}, \frac{1011}{1000}$.

2. The car is travelling a little faster. One way to know is to determine how far both vehicles travel in a fixed time.
   - The car travels 60 km in 60 min. That means the car travels 1 km in 1 minute or 1 km in 60 seconds. That is the same as travelling 1 km in 60 seconds.
   - The motorcycle travels 960 m in 60 seconds or 0.960 km in 60 seconds.

3. Some possible answers are as follows:

Similarities	Differences
Both graphs have two lines that intersect.	Graph A lines cross on the *x*-axis (negative value). Graph B lines cross on the *y*-axis (positive value).
Both graphs show crossing at right angles.	Graph A, the decreasing line goes into quadrant I. Graph B, the decreasing line is not in quadrant I.
Both graphs have one line that increases and one that decreases from left to right.	

4. One possible answer is as follows:

   As a graph of distance (*y*-axis) versus time (*x*-axis), this graph could tell the story of a cat and a stationary ball. At the start (AB), the cat remains at a fixed distance from the ball for a certain time. Then (BC) the cat rapidly moves further away from the ball, and then (CD) it remains stationary for a short time before (DE) slowly crawling back to the ball and stopping (EF) at a distance slightly further away than it was originally.

5. Possible answers include the following:

   a) Hat and t-shirt combinations

Hats ($15)	T-Shirts ($10)	Total (max. $300)
20	0	15(20) + 10(0) = 300
18	3	15(18) + 10(3) = 300
16	6	15(16) + 10(6) = 300
14	9	15(14) + 10(9) = 300
…	…	…
0	30	15(0) + 10(30) = 300

b) 0 is the smallest number of hats they could buy.

20 is the largest number of hats they could buy.

c) The team can purchase 20 hats for \$300 and, for every 2 hats less, they can purchase 3 shirts. They should purchase an even number of hats and a number of T-shirts that is divisible by 3.

6. Tables of values are shown for the equations. Many similarities and differences may be noticed.

a)

x	$-3x + 5$
−2	11
−1	8
0	5
1	2
2	−1

b)

x	$-x + 5$
−2	7
−1	6
0	5
1	4
2	3

c)

x	$x + 5$
−2	3
−1	4
0	5
1	6
2	7

d)

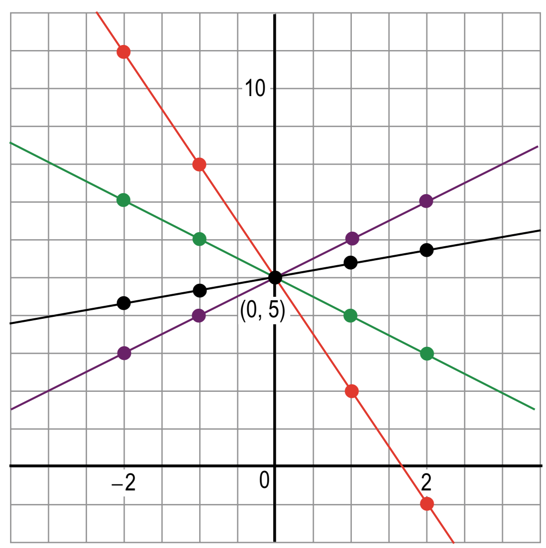
x	$\frac{1}{3}x + 5$
−2	4.3333333
−1	4.6666667
0	5
1	5.3333333
2	5.6666667

Some examples follow:

Similarities in the tables are that the chosen x-values are all the same. Also, each table of values includes the point (0, 5). Each equation has a constant of 5 added to the x-term.

Differences in the tables are the y-values. Equation (d) has y-values that are rational numbers represented by repeating decimals. The y-values in equations a) and (b) are decreasing and the y-values in equations c) and d) are increasing. The equations for (a) and (b) have negative coefficients for the x-terms, unlike equations (c) and (d).

The graphs of the equations
(a screenshot from desmos.com):



(TN) *Activity Builders* from Desmos can be used to further explore algebraic ideas on graphs. The link here is for a "Marbleslides" activity to explore transformations of linear equations: https://teacher.desmos.com/activitybuilder/custom/566b31734e38e1e21a10aac8.

R–6

Solidifying Prior Learning Solutions

1. Possible expressions:
   a) $n + n^2$
   b) $x - 10$
   c) $2(a + 1)$
   d) $2n + 1$
   e) $\dfrac{12 - y}{4}$
   f) $3(n + 2) + 1$

2. Missing number is −1.

3. Many answers are possible. Here are examples:
   a) $3(\square + 1) - 5 = -4 \times 7 + 11;\ \square = -5$

   $14 + 3(2n + 10) = -2 + 4;\ n = -7$

   b) $\frac{1}{2}(\square + 3) + 5 = \frac{1}{3}(14 - 2);\ \square = -5$

   $5 - \frac{2}{5}(x - 3) = \frac{3}{2}(-7 + 13);\ x = -7$

4. a) $0 = 8x - 2$ or $2 - 8x = 0$
   b) The equations have the same terms but may be written in a different order. The signs may be opposite, but they should have one negative term and the other positive.

5. Many answers are possible. Here are examples:

a) Original Equation	b) Rearranged Equation	Solution
$3x - 5 = 4x + 6$	$0 = x + 11$	$x = -11$ for both
$7x + 6 = -3x + 21$	$10k - 15 = 0$	$k = 1.5$ for both
$2(3n + 7) + 1 = 4(n + 5) - 6$	$2n + 1 = 0$	$n = -0.5$ for both

6. Many answers are possible. Here is one example:

   "Start at __3__ and add __2__ for each new term."

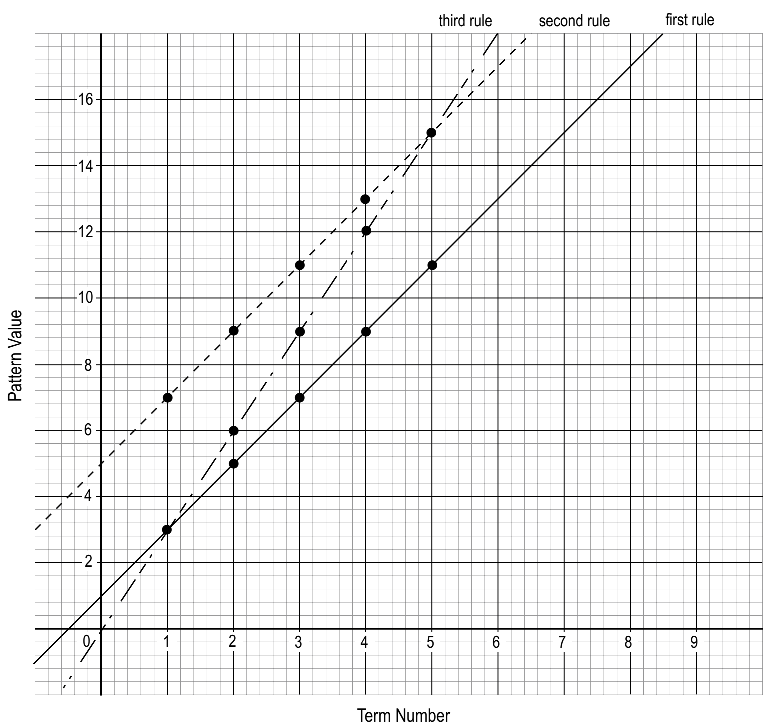
	Start at	Add
1st rule	3	2
2nd rule	7	2
3rd rule	3	2

Term 1	Term 2	Term 3	Term 4	Term 5
3	5	7	9	11
7	9	11	13	15
3	6	9	12	15

   (TN) Students may notice that the pattern values of the second rule can be obtained by adding a constant (in this case, 4) to the pattern values of the first rule.

   They may say that the pattern values increase at the same rate. They may notice the pattern values increase at a faster (or different) rate in the third rule than in the first rule or second rule.

   They may notice the graph of the line going through the points of the first rule is parallel to the line going through the points of the second rule (the line for the third rule is not parallel).



7. The triangle and circle could both be equal to zero. Among other possibilities, the triangle could be 2 and the circle −2. Balance (equality) can be maintained by removing 2 triangles and 1 circle from both sides, leaving nothing (zero) on the left and a triangle and circle on the right. In general, to have a result of zero, the triangle and the circle must both have the same numerical value, with one being positive and the other negative.

8. Many answers are possible, but they just need a good explanation. Here are some examples. Each graph doesn't belong because
    - the first graph is the only one that crosses the *x*-axis at a negative *x*-value
    - the second graph is the only one that passes through the origin (0, 0)
    - the third graph is the only one that crosses the *y*-axis at a negative *y*-value
    - the fourth graph is the only one that goes down from left to right
9. Joti's savings:
    a) Show two of the following models:

    Words: Joti initially has \$50; after week 1, she has \$65; after week 2, she has \$80. Continue this pattern of growth of savings each week.

    Graph:

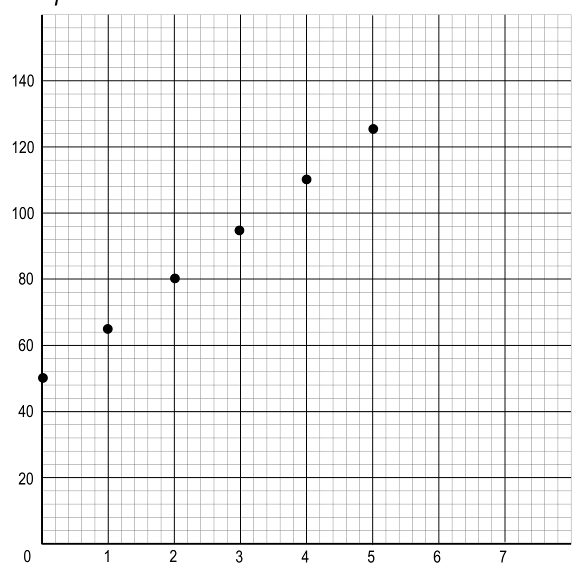


    Table:

Week	Savings
0	50
1	65
2	80
3	95
4	110
5	125

    Equation: $S = 50 + 15n$, where S is number of dollars saved, n is number of weeks

    b) Between 5 and 16 weeks, depending on the ticket price. It will take at least 5 weeks to save \$120 and at least 16 weeks to save \$280.

    c) Between 10 and 26 weeks, depending on the ticket price. It will take 10 weeks to save \$120 and 26 weeks to save \$280 at the new rate.

    Describe changes for the two models used:
    - Words and table have updated numbers.
    - The graph starts lower on the *y*-axis and goes up more slowly each week.
    - The equation changes to $S = 20 + 10n$.

R–7 Solidifying Prior Learning Solutions

1. Mean calculations and number line representations:

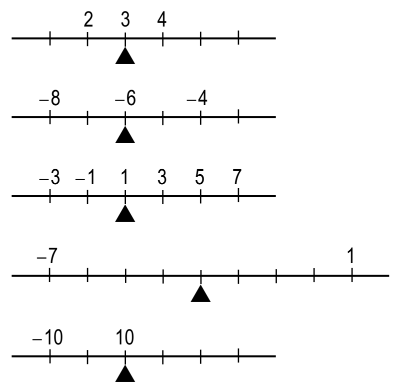
   a) $(2 + 3 + 4) \div 3 = 3$

   b) $(-8 - 6 - 4) \div 3 = -18 \div 3 = -6$

   c) $(-3 - 1 + 1 + 3 + 5 + 7) \div 6 = 12 \div 6 = 2$

   d) $(-7 + 1) \div 2 = -3$

   e) $(-10 + 10 + 30) \div 3 = 30 \div 3 = 10$



   Notice: All sets of numbers are in increasing order. All the sets of numbers go up by consistent intervals (1s, 2s, 8s, or 20s). If there is an odd set of numbers, the mean is one of the numbers. If there is an even set of numbers, the mean is in the middle between two of the numbers.

2. Sums are as follows:

   a) 25

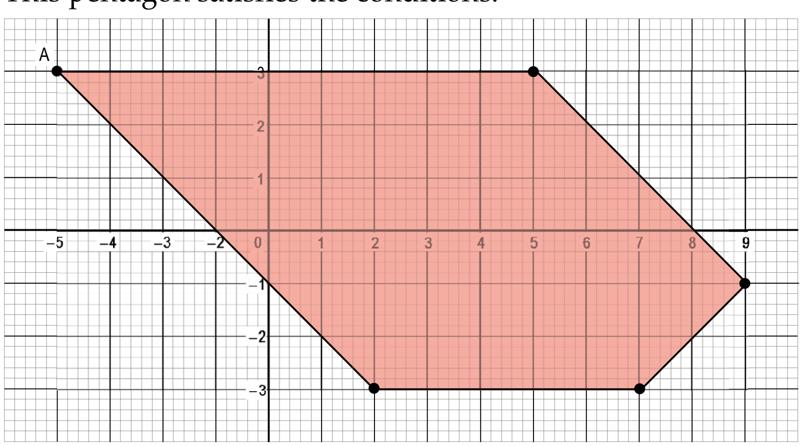
   b) 100

   c) 169

   All the expressions are the sum of perfect squares and the sum is also a perfect square. Many other expressions are possible (collectively called Pythagorean triples). Some examples are 81 + 144 (= 15^2) or 1 + 0 (= 1^2) or 64 + 225 (= 17^2).

3. Many answers are possible. Here is a sample:

   a) This pentagon satisfies the conditions.

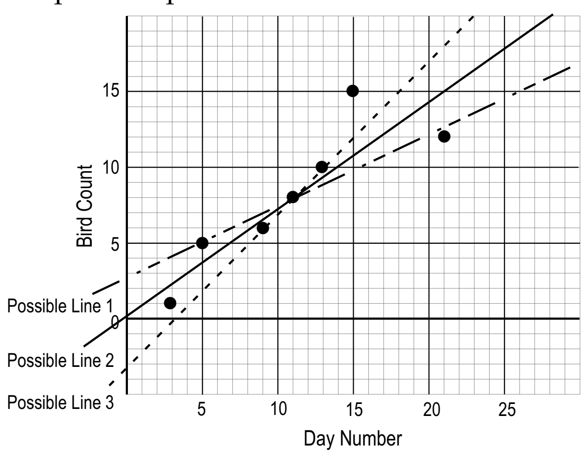


b) Sample start of a description: "To draw the pentagon, start at (−4, 3) move horizontally 9 units right to next vertex. Then turn 45° clockwise and draw a line down and to the right, moving a horizontal distance of four units and a vertical distance of four units to the next vertex...."

c) Based on the result, consider how communication could be improved.

4. Spring Bird Count versus Number of Days:

a) Graph the 7 points.



b) Several different lines are possible. Three are shown.

c) The three example lines are all increasing (going up L to R). They all go through a point near the middle (11, 7) or (11, 8). They are not parallel. They go up at different angles. They go through different numbers of points.

i) Justification for Line 1: It is close to three points with a pair of points above and a pair below it.

ii) Justification for Line 2: It is through one point with other pairs of points that are the same distance away from the line on either side of the line.

iii) Justification for Line 3: It passes through 3 points [the down side is that there are 3 other points above the line with only 1 below the line.

(TN) You might want to have some students explore the plotting of a "median-median line." It is a way of determining the linear trend of a scatterplot by finding the medians of three groups of data points and drawing a fit line using the three resulting median points (without any calculation). In the example above, "possible Line 2" is the fit line that would be the result of using the median-median fit line process.

5. The equation could be as follows:

   a) i) $\Delta = 14$ g

      ii) $\Delta = 10.5$ g

   b) i) Take off two marbles on each side, leaving 3 triangles on the left and 6 marbles on the right. That means each triangle must be the same as two marbles or 14 g.

      ii) Take off one triangle and two marbles from each side, leaving 2 triangles on the left and 3 marbles on the right. That means each triangle must be the same as $1\frac{1}{2}$ marbles or $10\frac{1}{2}$ g.

   c) i) Solve $3n + 2 = 8$, where n is the number of marbles equivalent to a Δ. Or, solve , where n is the number of grams equivalent to a Δ.

      ii) Solve the equation: $3t + 2 = t + 5$, where t is the number of marbles equivalent to a Δ. Or solve $3t + 14 = t + 35$, where t is the number of grams equivalent to a Δ.

   d) The algebraic steps could be similar. For example, when using the first equation, $3n + 2 = 8$, first subtract 2 on the left and right (which is the same as taking two marbles off each side). Then divide both sides by 3 (same as proportional reasoning that 3 triangles balances 6 marbles, so 1 triangle balances 2 marbles). The result is $n = 2$ marbles with a total weight of 14 g.

6. Distributive property:

   a) The distributive property could be described as, "The result of multiplying two parts of a sum separately and then adding their products is the same result as adding the two parts first and then finding the product." Algebraically, $a(b + c) = ab + ac$.

      Using an area model,

	b	*c*
a	*ab*	*ac*

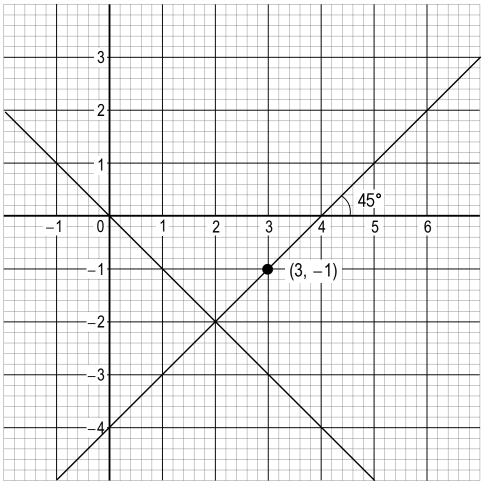
   b) Note: Other equivalent answers are possible for each of the following:

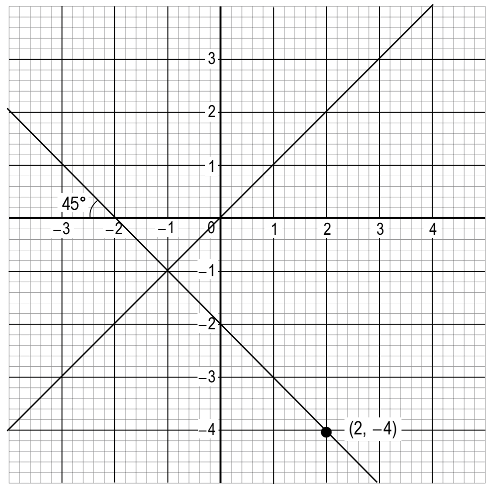
      i) 7×14 is the same as $(7)(10 + 4)$ or $(7)(11 + 3)$ or...

      ii) $(6)(5x + 2)$ is the same as $30x + 12$

      iii) $120y - 84$ is the same as $(6)(20y - 14)$ or $(12)(10y - 7)$ or...

7. Many answers are possible. Here are two possibilities starting at (3, −1) and (2, −4), respectively.





(TN) You may want to have students compare their graphs with each other. Given the criteria, there are only two possibilities for the second line, regardless of the starting point. They are illustrated above (through the origin sloping down at 45° or up at 45°).

R–8

Solidifying Prior Learning Solutions

1. Many answers are possible. Here are two examples for (a) 12×16:
   i) Decompose 16 into $10 + 6$, multiply 12×10, and then add 120 and 72 to get 192. Alternatively, decompose 16 into $20 - 4$, multiply 12×20, and 12×4, then subtract 240 and 48 to get 192.
   ii) Decompose 12 into $10 + 2$, multiply 16×10 and 16×2, then add 160 and 32 to get 192.

   (TN) You may want to do number talks as a regular routine with your students to help them develop their mental math ability and number sense. Number talks can be done with different operations and number systems. To get an idea of the process, search Youtube for "Jo Boaler Number Talks 18×5."

2. Use 2 cups flour and 4 teaspoons baking powder. One-third of 3 teaspoons is 1 teaspoon, so he needs a total of 4 teaspoons of baking powder. Similarly, $1\frac{1}{2}$ can be represented as $\frac{3}{2}$ and one-third of 3 halves is 1 half, so he needs a total of 4 halves—that is, adding $\frac{1}{2}$ cup of flour to the recipe is a total of 2 cups of flour.

3. Other units may be chosen:
   - Notations for length include centimetre, cm; inch, in., ".
   - Notations for area include square centimetre, cm^2; square inch, sq. in., $in.^2$.
   - Notations for volume include cubic centimetre, cm^3, c.c.; cubic inch, cu. in., $in.^3$.

4. The missing values (two outputs and one input), in order vertically, are 17, 56, and 32. The function machine, g, is multiplying by 3 and then adding 2.

   (TN) Teachers may want to guide students in the determination of an output rule. If the input values go up by a constant value, it is informative to find the differences between output values. In this case, the input values go up by 1 and the differences between output values is consistently 3 ($8 - 5 = 3$ and $11 - 8 = 3$). This means that for an increase of 1 in input, the output goes up 3 times. The function involves multiplying the input by 3 and then adding a constant value. In this example, the constant added each time is 2.

5. Machine, T:
   a) This may be written in a variety of ways using one or more variables. For example, Machine T, for input c, $9\left(\frac{c}{5}\right) + 32 \rightarrow$ output.
   b) 104°F
   c) −40°F
   d) $176.\overline{6}$°C. Work in reverse starting with output of 350°F, $(350 - 32) \div 9 \times 5$.

R–9 Solidifying Prior Learning Solutions

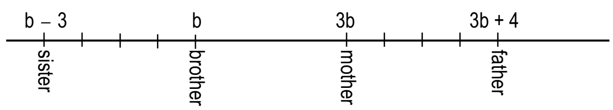
1. Some answers include 9 and 14, 12 and 17, or 13 and 18.
2. Exchange rate:

   a) They are reciprocals of each other. $1\frac{1}{2}$ is the same as $\frac{3}{2}$, which is the reciprocal of $\frac{2}{3}$.

   Using ratios, CAD : USD is 3 : 2, and the ratio of USD : CAD is 2 : 3.

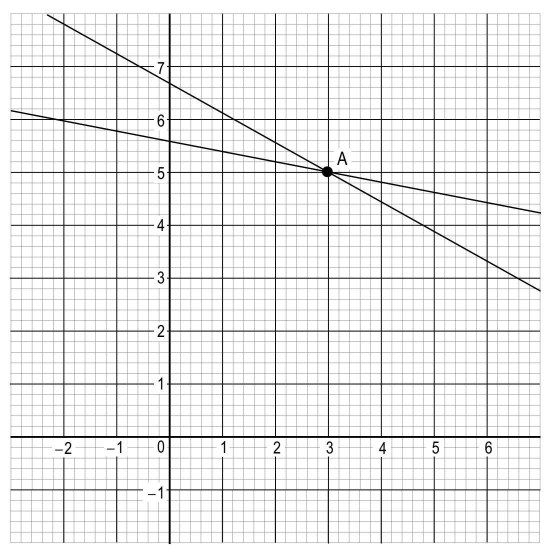
   b) $\frac{4}{5}$ of a U.S. dollar. Since $1\frac{1}{4}$ is the same as $\frac{5}{4}$ and the reciprocal is $\frac{4}{5}$.

3. Bonnie's family:



   This is enough information. Create and solve the linear equation with the sum of all four terms above set equal to 89.

4. Company B is better for a smaller track team. The recommendation depends on how many people are on the track team. If there are fewer than 30 people, recommend Company B because it will be cheaper; if there are more than 30 people, recommend Company A because it will be cheaper.
5. Answers will vary depending on the lines drawn. Here are responses based on the graphs of the lines shown:



   The two lines have the point (3, 5) in common and no other point is on both lines. The slopes of both of the lines are decreasing (from left to right). The x-intercepts of both graphs are greater than 10. The y-intercept of one line is close to 7 and the y-intercept of the other is close to 6.

6. Answers can vary significantly. Here is an example why each one does not belong:
   - The first graph shows two lines with one point in common in the first quadrant.
   - The second graph shows no point on either line with both negative x- and y-values.
   - The third graph shows two parallel lines.
   - The fourth graph shows two lines with the same x-intercept.

R–10

Solidifying Prior Learning Solutions

1. Many answers are possible. Here is one example:

   $$\frac{9}{2} > \frac{14}{7} = \frac{6}{3} > \frac{8}{5}$$

2. Many answers are possible. Here are two examples: A and B have opposite signs; on a number line, the distance of the negative number from 0 is one more than the distance of the positive number from zero.

3. Answers vary according to "your" age. In this sample solution, your age is 16 and my age is 35, so then half my age is $17\frac{1}{2}$. If your age is 16, then half your age is 8. Someone who is halfway between 16 and 35 is $25\frac{1}{2}$.

   (TN) Students may be encouraged to notice that the sum of half the two ages $\left(17\frac{1}{2} + 8\right)$ is the same as halfway between the two ages. They could be asked to justify algebraically that relationship works for any two ages.

4. Answers vary according to "your" age. In this sample solution, your age is 16 so Ms. Carlyle must be 68 years old. You are 26 years younger and Ms. Carlyle is 26 years older than Mr. Bowe.

5. Answers vary according to "your" age. In this sample solution, your age is 16 so Sara is 2. Liam is 7 years younger than your age of 16, so Sara is 7 years younger than Liam.

6. Squares:
   a) The square side lengths are 6 units. Plot a point 3 units to the right of A and another 3 units to the right of D, and then draw a vertical line through the points. Plot a point 3 units below A and another 3 units below B, and then draw a horizontal line through the points.
   b) From point E to F is 4 units right and 1 unit down. Plot a point 2 units to the right of E and $\frac{1}{2}$ unit down, then plot another point 2 units to the right of H and $\frac{1}{2}$ unit down, and then draw a line through the points. Plot a point 2 units below E and $\frac{1}{2}$ unit left, then plot another point 2 units below F and $\frac{1}{2}$ unit left, and then draw a line through the points.

   There is no other place to draw the lines to make 4 squares.

7. WODB:
   a) Many different answers are possible, but they just need a good explanation. Here are some examples describing why each one does not belong:

      "First number line shows a first step going from zero to -5 (the others go to -3)."

      "Second number line results in a positive number after both steps."

      "Third number line shows both steps going left (that is, both negative)."

      "Fourth number line shows a result that is only 1 unit away from zero."

   b) The expressions represented by the number line are as follows:
      - first number line is $-6 + 4$
      - second number line is $-3 + 5$
      - third number line is $-3 - 5$
      - fourth number line is $-3 + 2$

8. Decompose 280 into friendly multiples of 8, such as 240 and 40. Using an area model, the answer is 35.

	30 +	5
8	240	40

Other multiples of 280 are possible, such as:

$(160 + 80 + 40) \div 8$
$= 20 + 10 + 5$
$= 35$

9. a) The median is 5. $\dfrac{1+9+3+7+5}{5} = 5$ (mean)

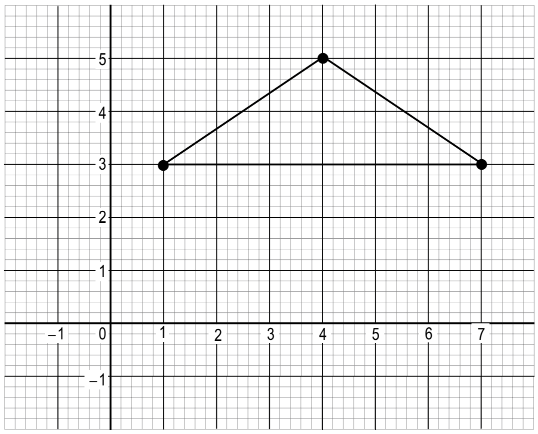
   b) Many answers exist.

10. Isosceles triangle:

a) One possible answer is (4, 5). Other possibilities include (4, 8) or (4, −1). In general, the vertex may be at any location with an *x*-coordinate of 4 as long as the y-coordinate is not 3.

b) Graph of triangle with third vertex at (4, 5).

c) The triangle shown has an area of 6 units^2 and a perimeter of $6 + 2\sqrt{13} = 13.211$ units.



11. Triangle ABC:

a) Plot ΔABC.

b) AB is 7 units.

AC is $\sqrt{45} = 6.708$

BC is $\sqrt{136} = 11.662$

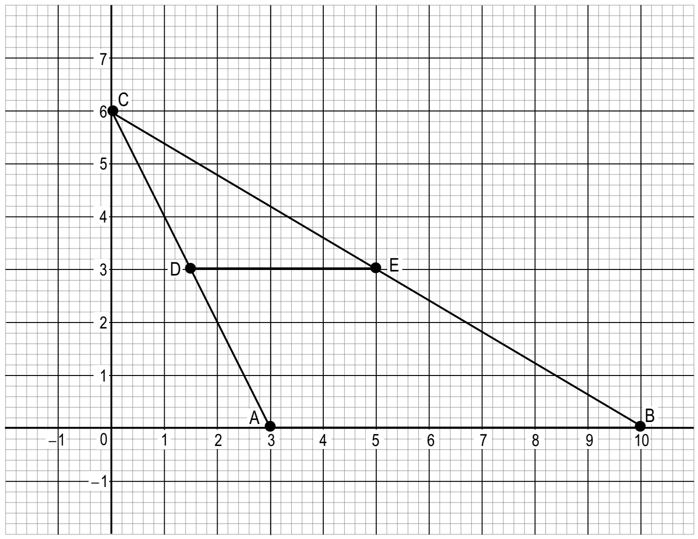
c) CD is $\sqrt{11.25} = 3.354$

$\frac{1}{2}\text{AC} = 0.5 \times 6.708 = 3.354$

d) Plot parallel line, DE.

e) A wide variety of answers are possible; here are some examples. Notice how the length of DE is 3.5 units or the length of DE is half of AB.

Wonder: "Is EB half of CB?"



(TN) Some students may notice (or you may want to point out) that $\Delta ABC \sim \Delta DEC$ because the parallel lines create congruent corresponding angles. Use this question to have students revisit the idea of similar triangles. Since the triangles are similar and CD is half of AC, then it is true that the other corresponding sides are in the same proportion $\left(\text{i.e., } DE = \frac{1}{2}AB \text{ and } CE = \frac{1}{2}CB\right)$.