Grade 10 Introduction to Applied and Pre-Calculus Mathematics: Solidifying Prior Learning

Support Document

Manitoba

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Schools are encouraged to share this resource with parents, guardians, and communities, as appropriate.

Any websites referenced in this resource are subject to change without notice. Educators are advised to preview and evaluate websites and online resources before recommending them for student use.

This resource is also available on the Manitoba Education website at www.edu.gov.mb.ca/k12/cur/math/supports.html.

While the department is committed to making its publications as accessible as possible, some of the mathematical formulas, equations, and graphics in this document are not fully accessible at this time.

Available in alternate formats upon request.

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Development Team Members	Ashley Cook	Elm Creek School Prairie Rose School Division
	Whitney Kreller-Lamont	Neelin High School Brandon School Division
	Thomas Locke	Mathematics Consultant Winnipeg School Division
	Ralph Mason	Faculty of Education University of Manitoba
	Blaine McIntosh	Grant Park High School Winnipeg School Division
	Kelly Scallion	R.D. Parker Collegiate School District of Mystery Lake
	Wanda Stockford	Miami School Prairie Rose School Division
	Sacha Wilson	West Kildonan Collegiate Seven Oaks School Division
Manitoba Education Staff	Peter Andres Middle Years Math Consultant	Early Childhood and Development Unit Instruction, Curriculum and Assessment Branch
	Kari Bergmuller Senior Years Math Consultant (since September 2019)	Early Childhood and Development Unit Instruction, Curriculum and Assessment Branch
	Louise Boissonneault Coordinator	Document Production Services Unit Inclusion Support Branch
	Ian Donnelly Senior Years Math Consultant (until June 2019)	Early Childhood and Development Unit Instruction, Curriculum and Assessment Branch
	Lynn Harrison Desktop Publisher	Document Production Services Unit Inclusion Support Branch
	Grant Moore Publications Editor	Document Production Services Unit Inclusion Support Branch
	Diana Turner Acting Coordinator	Early Childhood and Development Unit Instruction, Curriculum and Assessment Branch

Introduction

Introduction

Purpose of Document

This document is intended to support teachers of Grade 10 Introduction to Applied and Pre-Calculus (IAP) Mathematics students. Its purpose is to help teachers facilitate students' revisiting (or spiralling) outcomes across multiple grade levels by connecting student prior knowledge with current grade-level content. The content provides opportunities for Grade 10 IAP students to become more proficient in problem solving, and to build their knowledge of and make multiple connections to their prior learning. Revisiting outcomes will help students solidify their understanding of important mathematical concepts and procedures across prior grades to enable them to achieve at higher levels with current grade-level mathematics content.

For some students, the content of the document may provide some students new opportunities and experiences to understand more deeply the mathematical concepts they encountered in prior grades. The experience gained by students working through the content of this support document will aid their successful transition into Grade 11 mathematics.

Background

Feedback received from post-secondary institutions in Manitoba has highlighted the need for support to ease students' transition from secondary to post-secondary school. Teachers have shared a similar concern when talking about student transition within the public school system. Specifically, some students have difficulty applying and connecting content from prior grades to the content currently under study. By design, the Western and Northern Canadian Protocol (WNCP)–approved resources for Manitoba curricula do not include content related to prior learning. As a result, unless teachers find supplemental material, students may not have enough experience with some concepts to see how they connect to prior learning and to develop a deep understanding. Used alongside the approved text resources, this support document gives students an opportunity to revisit prior learning, build experiences, and facilitate a deeper understanding of grade-level concepts.

Rationale

National Council of Teachers of Mathematics (NCTM) research supports the idea of students being given the opportunity to revisit (or spiral) and connect concepts across multiple grade levels to solidify their understanding. Anthony and Walshaw identify "making connections" as one of the strategies for effective pedagogy in mathematics. "Tasks that require students to make multiple connections within and across topics help them appreciate the interconnectedness of different mathematical ideas and the relationships that exist between mathematics and real life" (p. 15). Teachers want their students to have good conceptual understanding of mathematics topics and be fluent with procedures. "Fluency builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems" (NCTM, p. 42). Students need foundational experiences that give meaning and understanding of procedures.

> In moving to fluency, students also need opportunities to rehearse or practice strategies and procedures to solidify their knowledge. …Students need opportunities to practice on a moderate number of carefully selected problems after they have established a strong conceptual foundation and the ability to explain the mathematical basis for a strategy or procedure (Pashler et al.; Rohrer, Rohrer and Taylor) (NCTM, p. 45).

There is a need for practice. A moderate amount of problems should be carefully selected to stress concept development and reveal possible misconceptions.

> If teachers want students to be proficient in problem solving, students must be given opportunities to practice problem solving. If strong deductive reasoning is a goal, student work must include tasks that require such reasoning. And, of course, if competence in procedures is an objective, the curriculum must include attention to such procedures. (Grouws and Cebulla, p. 17)

> To increase opportunities for invention, teachers should frequently use non-routine problems, periodically introduce a lesson involving a new skill by posing it as a problem to be solved, and regularly allow students to build new knowledge based on their intuitive knowledge and informed procedures. (Grouws and Cebulla, p. 18)

The questions created for this document will encourage students' continuing development of number sense, will give them opportunities to practise and deepen their prior learning, and will provide non-routine problems to encourage students to think and communicate about mathematics.

Beliefs about Student and Mathematics Learning

Students are curious, active learners with individual interests, abilities, needs, and career goals. They come to school with varying knowledge, life experiences, expectations, and backgrounds. A key component in developing mathematical literacy in students is making connections to these backgrounds, experiences, goals, and aspirations. Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences.

This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals, and a variety of pedagogical and assessment approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial, and symbolic representations of mathematics.

Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem-solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

The learning environment should value, respect, and address all students' experiences and ways of thinking so that students are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Assessment *for* learning, assessment *as* learning, and assessment *of* learning are all critical to helping students learn mathematics. A variety of evidence and a variety of assessment approaches should be used in the mathematics classroom.

First Nations, Métis, and Inuit Perspectives

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas and have varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural, and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning and learn mathematics best when it is contextualized and not taught as discrete content.

Many First Nations, Métis, and Inuit students come from cultural environments where learning takes place through active, hands-on participation. Traditionally, little or no emphasis was placed upon the written word. Oral communication, along with practical applications and experiences, is important to student learning and understanding.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students.

The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region and strive to achieve higher levels of multicultural education (Banks and Banks).

Affective Domain

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, support risk taking, and provide opportunities for success help students to develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, to participate willingly in classroom activities, to persist in challenging situations, and to engage in reflective practices.

Teachers, students, and parents need to recognize the relationship between the affective and cognitive domains and to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Autonomous and responsible learners are engaged in ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Goals for Students

The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between mathematical knowledge and skills and their applications
- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world

Students who have met these goals

- gain an understanding and appreciation of the role of mathematics in society
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical problem solving
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history

Mathematical Processes

The seven mathematical processes are critical aspects of learning, doing, and understanding mathematics. Students must encounter these processes regularly in a mathematics program in order to achieve the goals of mathematics education. *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* incorporates the following interrelated mathematical processes. It is intended that they permeate the teaching and learning of mathematics. Students are expected to

- use communication in order to learn and express their understanding
- make connections among mathematical ideas, other concepts in mathematics, everyday experiences, and other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technology as a tool for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

All seven processes should be infused in the teaching and learning of mathematics. Students will encounter these processes through the content of this support document. For a detailed description of each mathematical process, refer to *Grades 9 to 12 Mathematics: Curriculum Framework of Outcomes* (available on the Manitoba Education website at www.edu.gov.mb.ca/k12/cur/math/framework_9-12/index.html).

Document Details

How to Use the Document

The pages of the document are organized according to the specific learning outcomes in the Grade 10 Introduction to Applied and Pre-Calculus (IAP) Mathematics curriculum framework. For example, page R–9 refers to outcome 10I.R.9, which is “Solve problems that involve systems of linear equations in two variables, graphically and algebraically.” Rather than providing questions that address the outcome specified on the page, the questions are **focused on prior learning** that students should solidify and revisit before or during the process of learning the content of the Grade 10 IAP outcome. Teachers have access to text resources as sources of questions related directly to the outcome; the questions in this document are a source of questions that connect prior learning to the Grade 10 IAP outcomes.

The pages of this document are not intended to be used in a prescribed order. The pages can be used in any order to match the sequence of instruction of the Grade 10 IAP topics determined by the teacher. The pages are ready to be used as they are, but it is not intended that students be assigned and complete all questions on a page. Rather, the specific questions and the number of questions that teachers assign will differ depending on the varying needs of their students.

Since the questions solidify prior learning and are not questions directly related to an outcome, a teacher may decide to use questions from a page that refer to an outcome different from the one students are currently working on. For example, students may be learning content for outcome 10I.R.9 but a teacher may choose to have students do questions from page R-4. Since the questions involve prior learning, the content that students require to solve the problems will be accessible. Teachers need to make thoughtful choices about the specific questions that are used and the timing of their use to best meet the needs of their students.

Teachers also need to make a decision about the frequency of use of these questions. A teacher may choose to use one page per week in a semester course or more intermittently, depending on the needs of the students. It is not necessary for every student to do the same questions on each page. Additionally, teachers need to consider whether a question will be solved individually or collaboratively in small groups or as a whole class. Many of the questions ask students to explain or justify their thinking, so teachers may need to facilitate the discussion between students. Teachers will need to gauge the amount of time allotted to this work to strike an appropriate balance so that reinforcing and connecting prior learning does not overshadow the development of understanding of outcomes of the Grade 10 IAP curriculum.

Solutions are provided for each question. A "Teacher Note" icon **(TN)** identifies questions with some further information listed, along with the solution. The notes may give optional instructions to accompany a question, an indication of discussion topics for follow-up to a question, or suggestions as to how the question could be adjusted to make it accessible at different levels.

Finally, the questions in the document may serve as models for teachers to look for or create other questions. One type of question used regularly is the open question. When creating an open question, it is usually best to provide thoughtful constraints on the openness to encourage students to reason and analyze within those constraints. There are several different types of questions illustrated in the document and they are described in the next section.

Types of Questions

The pages are organized with questions for developing number sense at the top of each page. These questions are designed to help students continue to build the number sense making that they began in Early Years and continued in Middle Years mathematics. The other questions on the page relate to the prior learning required to achieve the outcome as given at the top of the page (and referred to by the page number). There is a variety of types of questions in the document:

- Questions to practise a strategy or a skill previously learned by students
- Open questions designed to promote student thinking and dialogue, such as
  - open-ended questions with multiple solutions
  - open-middle questions with single answers but multiple paths or strategies to get to the answers
- Questions to support conceptual understanding of concepts previously learned by students
- Questions to develop students' problem-solving skills (whether a question is a novel problem will depend on the experience of each student)

Foundational experiences make up a separate section of this document. The content of this section is based on the work of Dr. Ralph Mason and has been adapted for use here. The foundational experiences show possibilities for teachers to provide experiences for students that lay the foundation for deeper learning of concepts. The implementation of the foundational experiences will require an extended period of time, ranging from one to multiple class periods. Teacher planning will involve creating an appropriate sequence of questions and anticipating the responses (and questions) that students will have to guide the development of their understanding.

Foundational Experiences

Foundational Experiences

For everything we learn, we rely on prior experiences to help us understand, connect, and remember concepts and skills. When foundational experiences precede formal instruction for a mathematics topic, students can access those experiences in their memories to make sense of the mathematical ideas underlying the formal terminology, symbols, and arithmetic.

During instruction for foundational experiences, students have opportunities to reason, problem solve, estimate, visualize, communicate, use technology, and connect ideas—that is, opportunities to experience mathematics through the seven mathematical processes of the curriculum. Foundational experiences should be accessible and appealing, and they should sustain engagement over time.

Students may not always be interested in mathematical concepts and skills, but they are more likely to be interested when they are actively engaged with them. If students find the experience is worth talking about (e.g., with peers, their teacher, and others), then they are demonstrating their interest while building their foundational understanding of the content.

Patterned Mental Math

The intention of the "patterned mental math" style of questions is to have students notice patterns when doing a series of mental math problems that the teacher has carefully selected and sequenced. The patterns may be practice for some students and foundational experiences for others, depending on the focus of the patterned mental math and the prior experience of the students. This style of mental math lends itself to explore a variety of arithmetic and algebra topic areas. An experienced teacher will recognize topics for which this format would be helpful to students.

The process begins with students receiving a blank template. See page 18 for samples. The teacher orally provides mental math problems for students to write down and solve using the spaces on the template. The first three groups of questions are prepared ahead of time and given orally by the teacher to the students. The fourth group on the template is reserved for students to follow the structure they notice in the first three groups of questions and to make up their own questions and solutions for parts (a) to (d). The fourth group will allow students to follow and test the pattern.

Radical Square Hunt

This experience is intended for students to use areas of squares to explore irrational numbers as support for IAP outcome A2. This will be a foundational experience for students learning about irrational (radical) numbers and their representations. For those familiar with square roots, the experience will be an opportunity to solidify and connect their knowledge of area of squares and representations of radical numbers.

Powering Up and Down

This section is intended to describe students' visual and tactile foundational experiences that allow them to explore and experience powers and exponential growth as support for IAP outcome A3. Some students may have worked through some of these foundational experiences in Grade 9 in preparation for their work on powers with integral bases (outcomes 9N1 and 9N2). These experiences will be foundational for the development of their understanding of powers and exponential growth (or decay).

Patterned Mental Math

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding common factors and trinomial factors*

1. The following series of questions is support for IAP outcome A5. The questions are intended to help students gain a deeper understanding of factoring a difference of squares while purposefully doing mental math. A student would have created the completed template (shown below) after being given oral instructions from the teacher, such as the following:

   "In section 1a, write the expression '5 subtract 1' and write the result."

   "In section 1b, write the expression and the result of 5 plus 1."

   "In section 1c, write the product of the answers from a and b."

   "In section 1d, write the expression and the result of '5 squared subtract 1 squared'."

   "In section 2a, write the expression and the result of '10 subtract 1'."

   "In section 2b, write the expression and the result of '10 plus 1'."

   "In section 2c, write the product of the answers from a and b."

   "In section 2d, write the expression and the result of '10 squared subtract 1 squared'."

   "In section 3a, write the expression and the result of '–5 subtract 1'."

   "In section 3b, write the expression and the result of '–5 plus 1'."

   "In section 3c, write the product of the answers from a and b."

   "In section 3d, write the expression and the result of '–5 squared subtract 1 squared'."

   "In section 4, using the structure of questions 1, 2, and 3 above, make up your own."

   "What do you notice? What do you wonder? Write three things in the space at the bottom. Compare what you wrote with what your neighbour wrote."

Patterned Mental Math

Date Monday

Name Pat

1a $5 - 1 = 4$	2a $10 - 1 = 9$
1b $5 + 1 = 6$	2b $10 + 1 = 11$
1c product $4 \times 6 = 24$	2c $9 \times 11 = 99$
1d $5^2 - 1^2 = 24$	2d $10^2 - 1^2 = 99$

3a $-5 - 1 = -6$	4a $2 - 1 = 1$
3b $-5 + 1 = -4$	4b $2 + 1 = 3$
3c $(-6)(-4) = 24$	4c $1 \times 3 = 3$
3d $(-5)^2 - (1)^2 = 24$	4d $2^2 - 1^2 = 3$

The product of the difference and sum is the same value as the difference of squares.
The pattern works when adding and subtracting 1 even when starting with a negative number.
Could I add or subtract something other than 1 and still follow the pattern?

This may be a foundational experience for students who have not yet come to understand formal algebra as generalized arithmetic. This experience may help them make a general rule. For students with algebra experience, the teacher may ask the class (or some students), "Can you show that the pattern works for all cases, generally?"

As a next step, students could be asked to make a square that is 5×5 (using tiles, graph paper, etc.) and subtract a square of size 1×1. They should demonstrate (by rearranging pieces) and explain why, when the square is subtracted, the area is the same as a 4×6 rectangle.

Patterned Mental Math

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Multiplication of polynomial expressions concretely, pictorially, symbolically*

2. The following series of questions is support for IAP outcome A4 to have students do purposeful mental math to connect their understanding of the distributive property to multiplication of polynomial expressions. A teacher could begin with the following oral instructions to explore another pattern while doing mental math.

   "In section 1a, write the expression '7 plus 1' and write the result."

   "In section 1b, write the expression and the result of '7 times the answer from a'."

   "In section 1c, write the product of '7 times 7'."

   "In section 1d, write the sum of 7 and the answer from c."

   "In section 2a, write the expression and the result of '6 plus 1'."

   "In section 2b, write the expression and the result of '6 times the answer from a'."

   "In section 2c, write the product of '6 times 6'."

   "In section 2d, write the sum of 6 and the answer from c."

   The instructions and pattern would proceed, as with the first example, through the group 3 questions given by the teacher and the group 4 questions where students use the pattern structure to make up their own questions. This is followed by three statements of "notice and wonder." This example could help some students explore and articulate the distributive property.

Patterned Mental Math Date ____________

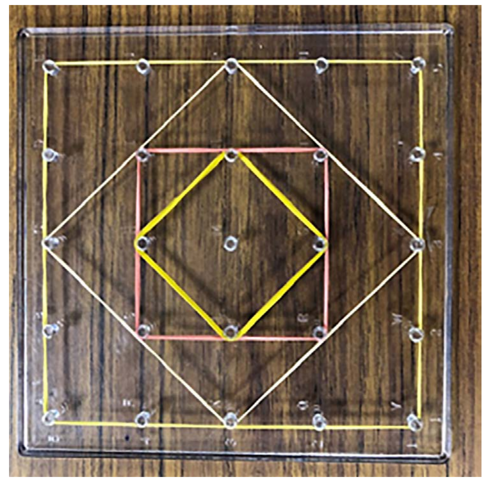
Name _______________

1a		2a
1b		2b
1c		2c
1d		2d

3a		4a
3b		4b
3c		4c
3d		4d

Radical Square Hunt

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding irrational numbers: representing, identifying, simplifying, ordering*



You can have students use geoboards and elastics to create various squares where the smallest square on the geoboard (1 × 1) represents an area of 1 unit. Alternatively, you can use grid paper where the smallest square (1 cm × 1 cm or possibly ¼ inch × ¼ inch) represents an area of 1 unit. Initially, work within a 4 × 4 graph paper grid or geoboard (see page 21).

Ask students, “What whole number areas can you represent by creating squares on the geoboard (or graph paper with vertices at grid intersection points)?”

As squares of different sizes are created, encourage students to count the areas on the geoboard by physically pointing at each 1 unit^2 area. Students should write the multiplication statement for length times width equals area in ways that are meaningful to them. For example, students could write the following:

For the large yellow square above $4 \times 4 = 16$; $\sqrt{16} \times \sqrt{16} = 16$

For the tan-coloured square above $\sqrt{8} \times \sqrt{8} = 8$; $2\sqrt{2} \times 2\sqrt{2} = 8$

Initially, with a geoboard, students may only create squares with areas of 1, 4, 9, and 16. If so, ask the students what other whole number areas can be represented with squares on the geoboard. Or ask, “Can a square with an area of 8 be created?” Some students will probably start to create squares using oblique lines as sides (rather than vertical and horizontal). If necessary, ask students to draw the diagonal of the square with area of 4 to get them thinking about lengths other than horizontal and vertical. The teacher will need to pay attention to what students are saying and what they are trying in order to determine the next good question to ask to probe their thinking. Note: On a 4 × 4 grid, the squares that can be created have whole number areas of 1, 2, 4, 5, 8, 9, 10, and 16—it is not necessary that all students find all of these areas.

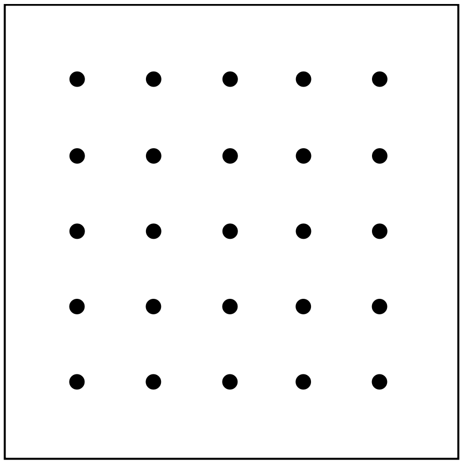
Teachers can help students to make use of their foundational experiences within the above activity. The following is an example for outcome 10I.A.2: “Demonstrate an understanding of irrational numbers by: representing, identifying, and simplifying irrational numbers; ordering irrational numbers.” By referring to the students’ foundational experiences with the radical square hunt, teachers can help students to link their understandings of the Pythagorean theorem with the mathematics of the new outcome.

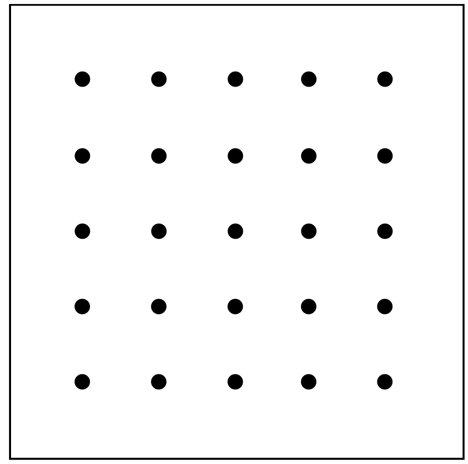
As a next step, students could use the Pythagorean theorem to explore the relationship between the areas and the lengths of the sides of the squares they have created. They could be asked, "Compare the lengths of sides of the squares with areas of 2 and 8." To extend the examples, students could use graph paper to work with a larger 5×5 grid. Now, they could be asked, "Create a square with an area of 18." [Note: Squares with areas of 13, 17, and 25 are also possible.] "What do you notice and wonder when you compare the lengths of the sides of the squares with areas 2, 8, and 18?" "Imagine an even larger grid. Are there other square areas that could be related?"

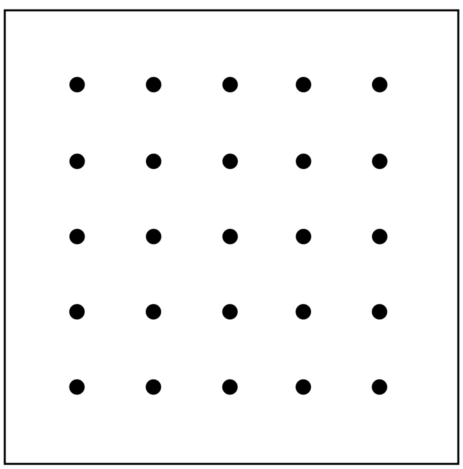
The next step would have students notice, think about, and express some of the square side-length relationships (that is, a square with an area of 2 has a side of $\sqrt{2}$; a square with an area of 8 has a side of $\sqrt{8}$, which is the same length as $2\sqrt{2}$ or three times the length of the square with side $\sqrt{2}$). Similarly, a square with an area of 18 has a side of $\sqrt{18}$, which is the same length as $3\sqrt{2}$ (that is, three times the length of the square with side $\sqrt{2}$). They may notice the pattern $1\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}$ and wonder what is the next area of square in this pattern. They may wonder what other square areas can be related this way, or they may notice $\sqrt{5}$ and $\sqrt{20}$ have this same relationship. With encouragement, students can inquire about and express such relationships collaboratively.

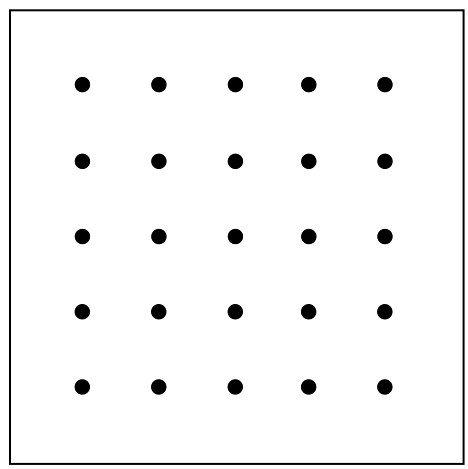
This record sheet is for students to keep track of their work with elastics on a geoboard.

Radical Square Hunt









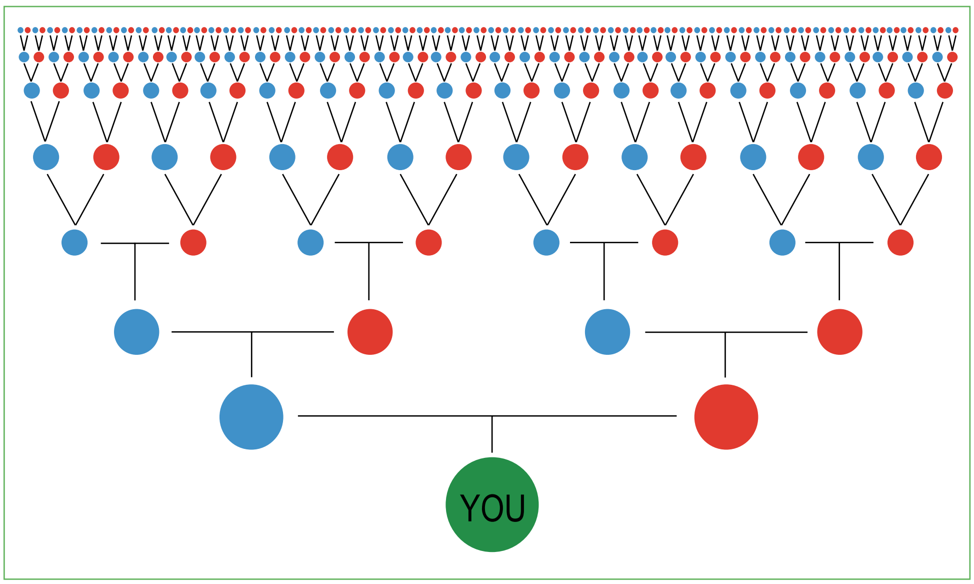
Powering Up and Down

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding powers with integral and rational exponents*

Students can engage in these foundational experiences with the concept of powers without relying on formal symbolic representations of powers, these activities can precede formal instruction with the topic.

The following list of experiences adds to the depth of student understanding of powers and exponential growth.

1. **Family Tree:** How many direct ancestors did you have when Treaty 1 was signed in 1871? How many ancestors did you have when Columbus landed on the East Coast of North America? How many direct ancestors do you have that are alive today? How can you represent the patterns within your ancestry with objects? …with drawings? …with numbers? …with words?



2. **Shampoo Sales:** In the 1980s, advertisements for a shampoo company used the math of doubling to show how quickly a good idea can be spread. After trying the shampoo, the model said, "I told two friends and they told two friends, and so on, and so on, and so on." With each level of telling two friends, the number of images of the model doubled. What number of images do you expect to be displayed in the advertisement at each successive stage? When does the display of images get out of hand? How can you better represent the array of images so that you can show many iterations? What does this pattern suggest about the power of online communication?

   Here is one of the many variations on the advertisement: www.youtube.com/watch?v=brC_jK6stBs (CTB).

3. **Ebola Virus:** In the chart below, which sections of the pattern of contagion approach doubling? What questions might you be able to explore with doubling if the monthly doubling of new cases did model the spread of Ebola? What information about this disease and its treatment might help with your thinking?

Ebola in Guinea, Liberia, and Sierra Leone		
Date (Month)	New Cases per Month	Deaths per Month (40%)
2014-03-31	120	48
2014-04-30	114	45
2014-05-31	75	30
2014-06-30	290	116
2014-07-31	723	289
2014-08-31	1730	692
2014-09-30	3501	1400
2014-10-31	6987	2794
		(CDC)

4. **Number Line:** Create a number line on a length of paper, counting by centimetres. Use one-centimetre grid paper to cut out lengths or rectangles, doubling each time. Attach the cut-outs to the number line. Describe the pattern. Describe the length of the number line you'll need to keep going.

5. **Hundreds Board:** Start with a photocopy of a hundreds board with numbers, and have a few copies of a blank hundreds board (without numbers) on hand. Glue squares of coloured paper onto the hundreds board to show the positions of the doubling numbers {1, 2, 4, 8, …}. Aim to show the first ten or so. Describe some of your thinking as you experience the power of doubling in this activity.

6. **Climbing the Doubling Ladder:** Have students create a doubling ladder as a table of values (see below). Just as in counting, you start at zero; when you start doubling, you start at one. Students will notice that doubling starting at zero is not very interesting. With the exception of students who have an interest and are ready for further exploration, it is intended that students go up the doubling ladder, starting at the "seed" value on the ground at step 0. Building a ladder with five to eight steps is usually sufficient for students to be able to construct mental images of the ladder steps that go beyond what they have recorded on paper.

Steps	Doubling Value
...	...
3	8 = 1 x 2 x 2 x 2
2	4 = 1 x 2 x 2
1	2 = 1 x 2
seed	1
...	...

Have students individually create a table as above and fill in the steps and the doubling values above the seed.

In groups of 2 or 3, instruct them to move up and down the steps of the doubling ladder and keep track of the doubling value for each of the following tasks.

A. Start at the ground and count up 4 steps.
   - What is the doubling value?
   - Count up 8 steps. What is the doubling value?
   - How many steps do you need to go up before reaching a 3-digit number? What is its value?
   - How many steps do you need to go up before reaching a 4-digit number? What is its value?

B. Start at the ground and count up 3 steps.
   - Start at step 1 and count up 3 steps.
   - Start at step 3 and count up 3 steps.
   - Describe and explain any patterns that you notice.

C. Start at step 4 and count down 3 steps (record the starting value and the resulting doubling value).
   - Start at step 4 and go down 3 steps.
   - Start at step 10 and go down 3 steps.
   - Start at step 5 and go down 3 steps.
   - Describe and explain any patterns that you notice.

D. Start at step 3, then up, up, down, down, down.
- Start anywhere, then up, up, down, down, down.
- Describe and explain any patterns that you notice.

After each section of tasks, students should be given an opportunity to share with the group the patterns they notice. Teachers might want to have some groups talk to others, as needed, to be sure that all students have an opportunity to see the patterns developing. After the experience, it is worthwhile to debrief as a whole class about what they noticed in general and what they wondered about. Some extensions and possible wonderings after this experience include the following:*

"What doubling values appear if the steps of the ladder go below ground?"

"What numbers appear if it is a tripling ladder instead of a doubling ladder?"

7. **Staying on the Doubling Ladder:** This experience is intended to help students develop a strong sense of the numbers on the doubling ladder and how powers interact with the four basic operations. Have students individually create a table, as shown below, and fill in the steps and the doubling values above the seed. Alternatively, students may use a doubling ladder that they have previously created.

Steps	Doubling Value
...	...
3	8 = 1 x 2 x 2 x 2
2	4 = 1 x 2 x 2
1	2 = 1 x 2
seed	1
...	...

Students could be asked to work in groups of 2 or 3 as they work on the following tasks.

* The following articles describe Manitoba teachers and students engaging in these activities:

Slivinski, Peter, Steven Erickson, and Ralph T. Mason. "Mathematics 9: Designing Foundational Experiences for the Hardest Topics." *MERN Journal*, vol. 11, 2015, pp. 50–57.

Mason, Ralph T., and Steven Erickson. "Foundational Experiences for Secondary Mathematics: A New Approach to Curriculum?" *MERN Journal*, vol. 11, 2015, pp. 17–20. Available online at http://mbtrc.org/data/documents/Journal-V11.pdf.

Pick two numbers on the ladder. Determine if you can perform the operation (each one separately) and obtain a result that is also a number on the doubling ladder. Is it always true? Sometimes? If so, under what circumstances?

- Add them.
- Subtract them.
- Multiply them.
- Divide them.

Debrief with students about the patterns they see. They may ask if they can pick the same number twice (as in 4 + 4). You may wish to encourage such exploration.

8. **Undoubling on the Doubling Ladder:** By cutting rectangles from grid paper, students can easily make rectangles to represent the values on the doubling ladder, up to 64. Lead them in undoubling, starting with 64 and either folding or cutting to undouble. When they get down to a rectangle of area 1, undoubling will make values below the seed. As they proceed, have students record these "undoubling numbers" on their ladder. To help students visualize the results of undoubling, invite them to cut out a "magnified seed," a ten-by-ten square to represent 1. With the magnified seed, students can make fraction and decimal representations for several layers of the doubling ladder below the seed level of 1.

   Extension: Repeat activity with a new seed. Are any observations and descriptions of patterns still true? Why or why not?

Questions by Strand

Solidifying Prior Learning M–1

Support for Strand:	*Measurement*
Prior Learning for:	*Problems involving measurement using SI and imperial units, estimation strategies, or measurement strategies.*

1. Find several possible values for a and b that make the equation true: $\frac{1}{a} = \frac{2}{b}$. Describe what you notice about a and b.

2. Select 6 different positive integers less than 10. Place each number in one of the boxes. **(TN)**

$$\frac{[\square]}{[\square]} - \frac{[\square]}{[\square]} - \frac{[\square]}{[\square]}$$

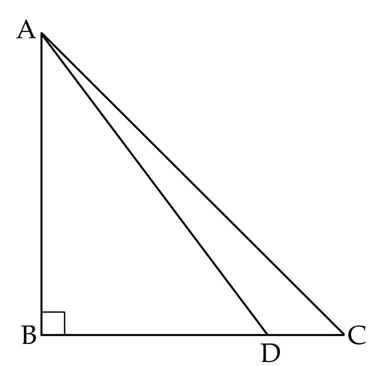
   a) Arrange 6 integers so the difference is a large number. How large of a difference is possible?*

   b) Arrange 6 integers to find a difference that is close to zero. How small of a difference is possible?

3. Ava lives 10 kilometres from Kendall and 20 kilometres from Jordyn.
   How far is it from Kendall's house to Jordyn's house?
   Provide at least two possible answers and diagrams to support the answers.

4. Sarah lives $\frac{3}{4}$ of a mile from Kathryn and twice that distance from Leah.
   How far is it from Kathryn's house to Leah's house?
   Provide at least two possible answers and diagrams to support the answers.

5. ΔABC is a right triangle (as shown, not to scale) with $\angle ABC = 90°$, BD = 6 m, AB = 8 m. The area of ΔABC is 50% greater than the area of ΔABD.
   Determine the perimeter of ΔADC.



* Based on a problem by Robert Kaplinsky and Ellen Metzger. Available at www.openmiddle.com/adding-mixed-numbers-3/.

Solidifying Prior Learning

M–2

Support for Strand:	*Measurement*
Prior Learning for:	*Apply proportional reasoning when converting between SI and imperial units of measure.*

1. Find three pairs of positive integers for a and b that make this inequality true: $\left[\frac{2}{a}\right] > \left[\frac{3}{b}\right]$.

   Explain your thinking using an area model or number line.

2. There are 365 days in a year (one more for a leap year), 24 hours in a day, 60 minutes in each hour, and 60 seconds in each minute. **(TN)**
   a) According to the Guinness Book of World Records, the oldest human to ever live was 122 years and 164 days old at the time of her death. Jeanne Louise Calment was born on February 21, 1875, and died at a nursing home in Arles in Southern France on August 4, 1997. How many minutes was this person alive? (What assumptions have you made?)
   b) How old would you be when you celebrate your millionth minute? Billionth minute?

3. Jaswinder says, "If a numerator and a denominator of one fraction are closer together than the numerator and denominator of a second fraction, the first fraction is bigger."
   a) Write a pair of fractions that support his statement.
   b) Write a pair of fractions that show that his statement is not always true.

4. The area of a circle is 20 cm^2. What is the diameter?

5. What is the better deal? One large 18″ pizza for $25 or two medium 12″ pizzas for $25? **(TN)**

6. Your mother asks you to go to her desk and get your school picture and its enlargement. When you get to her desk, you find five pictures in various sizes:
   i) 9 cm × 10 cm
   ii) 10 cm × 12 cm
   iii) 8 cm × 9.6 cm
   iv) 6 cm × 8 cm
   v) 5 cm × 6.5 cm
   a) Which two could be a picture and an enlargement without distortion?
   b) She wants to print an enlarged picture with a width of 36 cm. Find the length of this enlargement.

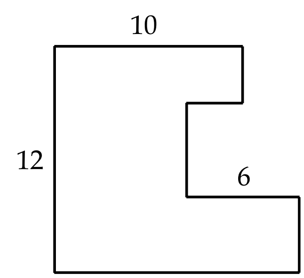
Solidifying Prior Learning M–3

Support for Strand:	*Measurement*
Prior Learning for:	*Solve problems involving surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres using SI and imperial units.*

1. Gilles has driven $\frac{2}{3}$ of the trip distance in his car. He started with a full tank and his tank is now $\frac{1}{4}$ full. Will he likely run out of gas? How do you know? What assumptions are you making?

2. Six friends shared a bag of cookies. The first person ate $\frac{1}{6}$ of the cookies, the next ate $\frac{1}{5}$ of what remained, and the next ate $\frac{1}{4}$ of what remained. The fourth ate $\frac{1}{3}$ of what remained before the next person ate $\frac{1}{2}$ of what remained, which left three cookies for the sixth friend. How many cookies did the bag contain at the start?

3. Consider the following shape (not drawn to scale): (TN)



   a) Calculate a possible answer for the area.
   b) Calculate a possible answer for the perimeter.
   c) Compare your answers to a classmate. Explain how you arrived at your answers.

4. Using one piece of 8½ × 11 paper, construct a cylinder with two ends and a large volume. Assume that no overlap is needed to join the edges.

5. Construct a circle of radius, *r* (any size you want), and then cut out several squares of length *r* (same length as the circle radius). Cut up the squares to fit them inside the circle. How many squares of side length *r* will fit into a circle of radius *r*? (TN)

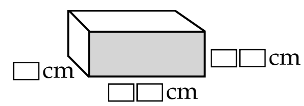
6. Check a variety of different coffee cups. (TN)
   a) Which is greater: the height of a coffee cup or the distance around the rim?
   b) How does the distance across the cup compare to the distance around the rim?
   c) What do you notice? What do you wonder?

Solidifying Prior Learning

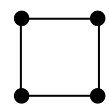
M–3

Support for Strand:	*Measurement*
Prior Learning for:	*Solve problems involving surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres using SI and imperial units. (continued)*

7. Using the numbers from 1 to 9 (at most once each), find the length, width, and height of the rectangular prism (as shown) so that the volume is close to 2500 cm^3.



8. Dylan starts building a 3-D figure as shown:
   a) What figure could Dylan be building?
   b) What figure could Dylan not be building?



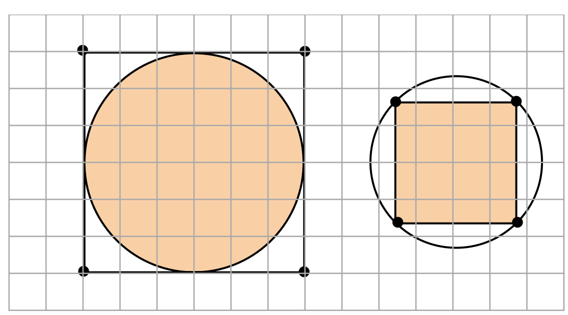
9. A rectangular prism (box) has a length that is twice the width and a height that is twice the length. What algebraic expressions can describe features of this box? **(TN)**

10. A cylinder has an unknown radius and a height of four units.

    What algebraic expressions can describe features of this cylinder?

11. Using the model of a cube and starting with a 1 cm × 1 cm × 1 cm cube, what is the ratio of the surface area to the volume (SA : Vol)? Consider growing cubes of various sizes. As the side length of the cube increases, what do you notice about the SA : Vol ratio? Some questions to consider:
    a) Is there a side length where the ratio is exactly 1?
    b) When is the SA : Vol ratio greater than 1? When is it less than 1?
    c) What is ratio of SA : Vol for a cube with a side length 100?
    d) Show algebraically that the ratio of the SA : Vol for a cube with a side length of n is $6 : n$.
    e) What is the SA : Vol ratio for cylinders with height equal to the radius—that is, with radius r and height r?

12. Which is a better fit: a round peg in a square hole or a square peg in a round hole? **(TN)**

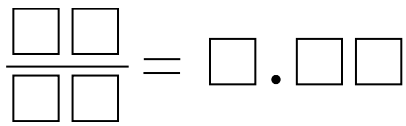


Solidifying Prior Learning

M–4

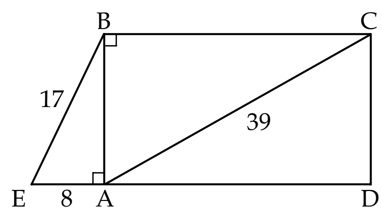
Support for Strand:	*Measurement*
Prior Learning for:	*Develop and apply primary trigonometric ratios to solve right angle triangles.*

1. Using digits from 0 through 9, at most one time each, fill in each of the boxes so that the fraction equals the decimal.*



2. What is the fewest number of people surveyed if exactly 93.6% of people completed a survey?**

3. Francie owns a garden plot. A diagram of this garden with rectangle ABCD and right triangle ABE is shown. She knows the length of AC is 39 m, the length of side EA is 8 m, and the length of side EB is 17 m. There is a water tap at C. How long must a hose be to ensure Francie can reach any location in the garden? (Calculate to the nearest tenth of a metre.)



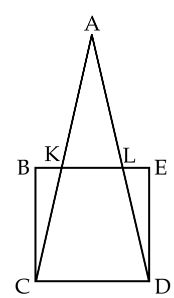
4. Draw a triangle. **(TN)**

   Keeping the shape of the triangle the same, make a much bigger version and a much smaller version of it. Measure the sides and angles of both of your new triangles. What do you notice and wonder as you compare the measures of the angles and the sides?

5. The area of ΔACD is twice the area of the square BCDE. AC and AD intersect BE at K and L, respectively. **(TN)**

   a) The measure of KL is 6 cm. The area of square BCDE is 64 cm^2. Determine the area of the top triangle ΔAKL.

   b) Suppose the measure of KL is unknown. If the area of the square is 144 cm^2, determine the area of trapezoid KCDL.



* Based on a problem by Gisele Garcia. Available at www.openmiddle.com/sum-of-fractions-closest-to-10/.

**Based on a problem by Robert Kaplinsky. Available at www.openmiddle.com/interpretting-percentages/.

Solidifying Prior Learning

A–1

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding factors of whole numbers: primes, GCF, LCM, roots*

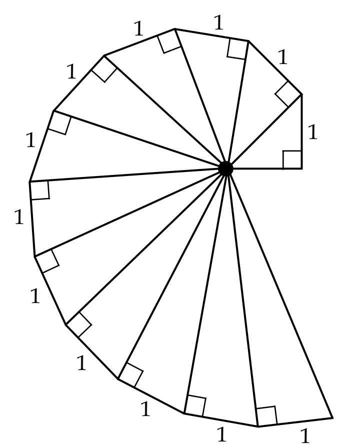
1. How are these pairs of powers the same? How are they different?
   a) 2^4 4^2 b) 3^2 2^3
2. Greatest Common Factor (GCF):
   a) List the factors of 32. List the factors of 8. What is the GCF of 32 and 8?
   b) List the factors of 24. List the factors of 18. What is the GCF of 24 and 18?
3. Least Common Multiple (LCM):
   a) List some multiples of 32. List some multiples of 8. What is the LCM of 32 and 8?
   b) List some multiples of 12. List some multiples of 8. What is the LCM of 12 and 8?
4. Sara has an unlimited supply of square tiles. Sara has 1 cm × 1cm tiles, 2 cm × 2 cm tiles, 3 cm × 3 cm tiles, and so on, up to 100 cm × 100 cm tiles. (The side length of every tile is a whole number.) A rectangular tabletop with an 84 cm by 112 cm surface is to be completely covered using tiles that are all the same size and no cuts can be made. How many tiles might Sara use to cover the tabletop? Could she use fewer tiles? What is the least number of tiles Sara could use? Provide a diagram. **(TN)**
5. The number 720 is divisible by all the numbers from 1 to 6, since 1 × 2 × 3 × 4 × 5 × 6 = 720.
   a) Find a smaller number that is also divisible by the numbers from 1 to 6. Is it the smallest? Compare your answer with other classmates. Compare your strategies.
   b) Find a number that is divisible by all the numbers from 1 to 20. Can you find one that is smaller? Compare your answer with other classmates. Compare your strategies.
6. A school hallway has 20 lockers assigned to 20 students that are numbered 1, 2, 3 … 19, 20. Their teacher says, "We will play a game." The first student is to go down the hall and open all the lockers. The second student is to start with the second locker and close every second locker. The third student is to start with the third locker and change every third locker (i.e., open a closed locker or close an open one). The fourth student changes every fourth locker, and so on. The process continues until all 20 students have participated. Which lockers are open at the end of the game? Which lockers were touched the most? **(TN)**
7. Set A is {36, 49, 64, 81, 100, 121}.
   a) What do you notice and wonder about this set of numbers?
   b) Set B is {35, 48, 63, 80, 99, 120}. How do these numbers relate to Set A?
   c) Set C is {20, 33, 48, 65, 84, 105}. What does this set have in common with Sets A and B?

Solidifying Prior Learning

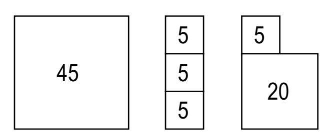
A–2

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding irrational numbers: representing, identifying, simplifying, ordering*

1. Replace x with a rational number to make each of these statements true:
   a) $1 < x < \sqrt{2}$
   b) $\frac{3}{2} < x < \sqrt{3}$
   c) $2 < x < \sqrt{5}$
   d) Do you notice any patterns that help you find more fractions that work?
2. Use all of the digits 0 to 4 (only once each) to create mathematical equations (i.e., $1^2 + 3 = 4 + 0$ will work BUT $(3)(2) + 1 - 4 = 3$ will not work).
3. Use a ruler to draw the first 10 right triangles of the **Spiral of Theodorus** (let 1 unit = 5 cm). The first right triangle has two legs. Each leg measures 1 unit (5 cm). Draw the next right triangle using the hypotenuse of the previous triangle as the first leg with a second leg of 1 unit (5 cm). Repeat the process. Through measurement, determine an approximate value of the lengths. Which hypotenuse lengths are whole units? Predict which triangle will be the next one to have a whole number hypotenuse.



4. The number inside each square represents its area in cm^2. The area of each square is a natural number. The squares are not drawn to scale. **(TN)**
   a) Should the heights of the stacks of squares be equal if they were drawn to scale? Justify your answer.
   b) Find stacks of squares that are the same height as a square with area 72 cm^2.



5. Let $x = 0.33333\ldots$, then $10x = 3.33333\ldots$. This is the same as $10x = 3 + 0.33333\ldots$ and the same as $10x = 3 + x$.
   a) Use the equation to write 0.33333… as a fraction.
   b) Describe how you can represent the following repeating decimals as fractions (rational numbers).
      i) 0.636363…
      ii) 5.454545…

Solidifying Prior Learning

A–3

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding powers with integral and rational exponents*

1. Take any integer greater than 1 and square it. Find two consecutive integers whose sum equals that square number (e.g., 11 squared is 121. Consecutive integers with a sum of 121 are 60 and 61.). For which integers is this possible? For which integers is this not possible? Explain.

2. Replace each box in the radical expressions below with one of the digits from 1 to 9 (used once each) to produce an integer result.*

   $\sqrt{\square}$ $\quad\sqrt{\square\square}$ $\quad\sqrt{\square+\square}$ $\quad\sqrt{\square}+\square$ $\quad\sqrt[\square]{\square}$

3. Without a calculator, determine the value of $\left(5^4\right)\left(20^5\right)$. Rewrite the expression in a different form to help perform the calculation. Why does rewriting help? Write another pair of numbers that will be easier to multiply after rewriting in a similar way.

4. Using digits from 0 to 9 (at most, once each), create an expression by filling in the blanks $(\square)(\square)^{\square}$.

   a) Create two expressions in this form that are equivalent. Are other equivalent expressions possible?

   b) Create two or more expressions with values between 100 and 1000.

5. Find three different positive integers with a sum of 10. Place each number in one of the blanks in the expression $(\square)(\square)^{\square}$ to create a large result. Compare your results with classmates' results.

* Based on a problem by Owen Kaplinsky. Available at www.openmiddle.com/tag/5-nf-1/.

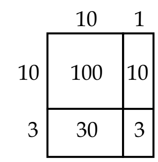
Solidifying Prior Learning

A–4

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Multiplication of polynomial expressions concretely, pictorially, symbolically*

1. The product of 11×13 can be represented using an area model as shown on the right. $(11)(13) = (10 + 1)(10 + 3)$

   The product is $100 + 30 + 10 + 3 = 143$.

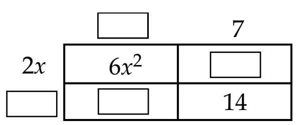


   a) Use the area model to represent the product of 26×14.

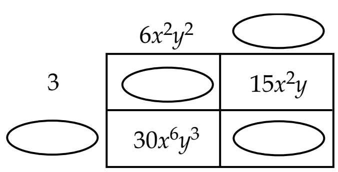
   b) What is the product?

   c) Use an area model to illustrate a product of two 2-digit integers that is between 500 and 700.

2. Fill in the missing monomials and then write the product that is represented by this area model:



3. Fill in the missing monomials and then write the product that is represented by this area model:



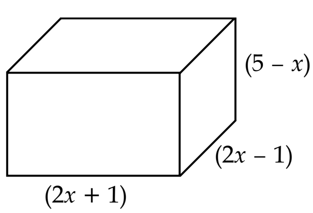
4. Rectangular prisms (boxes):

   a) Determine the surface area and volume of a rectangular prism (box) with a width of 5 metres, a depth of 3 metres, and a height of 3 metres.

   b) Write an expression for the surface area of the rectangular prism shown.

   c) Write an expression for the volume of the rectangular prism shown.

   d) What are the possible whole number values for x?

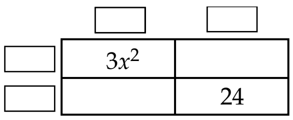


Solidifying Prior Learning

A–4

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Multiplication of polynomial expressions concretely, pictorially, symbolically (continued)*

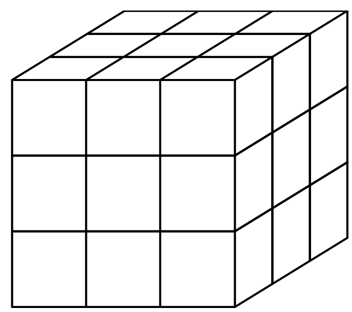
5. Using positive integer coefficients, what might be in the missing parts and the grid?



Write the equation represented:

(______________)(______________) = $3x^2$ + ______________ + ______________ + 24

6. The painted cube:
   a) Construct (or sketch) a 3 × 3 × 3 cube that is made from 1 × 1 × 1 cubes. How many smaller 1 × 1 × 1 cubes are needed?
   b) Imagine the outside of the large 3 × 3 × 3 cube is painted orange. *x*
      i) How many small cubes have only 1 orange face?
      ii) How many small cubes have 2 faces covered in orange paint?
      iii) How many small cubes have 3 faces covered in orange paint?
      iv) How many small cubes have 4 faces covered in orange paint?
      v) How many cubes have zero faces covered in orange paint?



   c) Determine the number of sides painted for the same questions regarding a 4 × 4 × 4 cube, then a 5 × 5 × 5 cube, and, finally, a 6 × 6 × 6 cube. What do you notice or wonder?
   d) How would you answer the questions regarding a $N \times N \times N$ cube? Justify your reasoning.

Solidifying Prior Learning

A–5

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding common factors and trinomial factors*

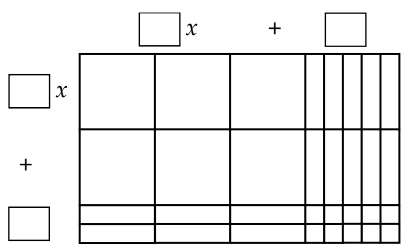
1. Name two integers that
   a) have a product of 24
   b) have a sum of −13
   c) have a product of 24 and a sum of 11
   d) have a product of 15 and a sum of −8

2. The product of two integers is 18; what are all the possible sums of the two integers?

3. The product of two integers is 12; what are all the possible differences of the two integers?

4. The following list is the sums of pairs of integers that share the same product: −17, −7, −3, 3, 7, 17. What is the product? Justify your answer.

5. Write the product of binomials and the equivalent trinomial represented by the area model.



6. Replace the boxes with numbers to make the equation true:
   a) $4x + 8 = \square(x + 2)$
   b) $\square x^2 + 12x = \square(x + 2)$
   c) $\left(2x^2 + \square x + 3\right)\left(\square x + \square\right) = 4x^3 + 20x^2 + 30x + 12$
   d) $12x + 9 = \square\left(\square + \square\right)$

7. Replace each box with the operations + and −. What are the possible coefficients for the linear (middle) term of the trinomial product? What do you notice or wonder?
   a) $\left(n \square 2\right)(n + 6)$
   b) $(x - 3)\left(x \square 3\right)$
   c) $\left(a \square 5\right)\left(a \square 4\right)$
   d) $(2b + 3)\left(b \square 5\right)$

Solidifying Prior Learning

A–5

Support for Strand:	*Algebra and Number*
Prior Learning for:	*Understanding common factors and trinomial factors (continued)*

8. Find two numbers that
   a) have a product of 36 and a sum of -15
   b) have a product of -18 and a sum of -7

9. Use algebra tiles to represent the product, and then write the trinomial product symbolically:
   a) $(2x+3)(x+2)$
   b) $(2x+1)(x+4)$
   c) $2(x+3)(x+4)$

10. Use algebra tiles to represent the product of two binomials resulting in the trinomial product in as many ways as possible, using different numbers of x-tiles or unit tiles as appropriate. Then write the binomial products symbolically.
   a) $(\_\_\_\_)(\_\_\_\_) = x^2 + \square x + 6$
   b) $(\_\_\_\_)(\_\_\_\_) = x^2 + \square x + 7$
   c) $(\_\_\_\_)(\_\_\_\_) = x^2 + \square x + 9$

11. Use algebra tiles to represent each trinomial and whether it can be written as a product of binomial factors. **(TN)**
   a) $x^2 + 8x + 8$
   b) $x^2 + 7x + 12$
   c) $x^2 + 1$

Solidifying Prior Learning

R–1

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Interpret and explain the relationships among data, graphs, and contexts*

1. In the addition shown, P, Q, and R each represent a single digit and the sum is 2009. Find the value of the sum of digits P + Q + R.*

```
  PQP
+ RQQQ
 2009
```

2. Examine the table of values for white sturgeon fish in Manitoba.

White Sturgeon Fish	
Age	**Weight (pounds)**
26	116
30	148
33	172
154	400

   a) Graph the ordered pairs of data. Why should these points **not** be connected?

   b) Using your graph, predict the weight of a 10-year-old sturgeon.

   c) Using your graph, predict the age of a 200-pound sturgeon.

3. There is a strong relationship between ______________ and ______________.

   a) Fill in the blanks with phrases that make sense (e.g., "the weather" and "number of beachgoers").

   b) Create a graph that could represent the relationship.

   c) Show another version of your graph of these same variables using data values that hide the relationship.

4. Given an algebraic equation, create a story or picture(s). Be sure to define the variables.

   For example: A delivery of building materials will cost $50 plus $0.25 for each kilometre driven from the store.

   $C = 50 + 0.25d$
   C = cost ($)
   d = distance (km)

   a) $c = 12 + 2f$

   b) $y = 25x$

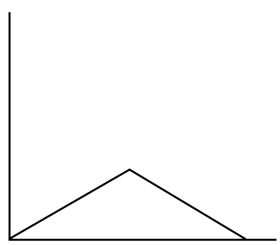
   c) $b = a + 12$

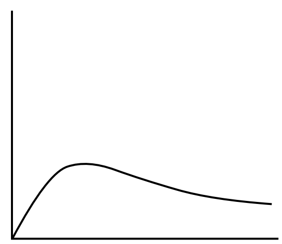
* Based on a problem by CEMC. Available at www.cemc.uwaterloo.ca/contests/past_contests/2009/2009Gauss8Contest.pdf.

Solidifying Prior Learning R–1

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Interpret and explain the relationships among data, graphs, and contexts (continued)*

5. Choose one of the graphs below. Label the axes. Describe the pattern using a story, table, or rule. **(TN)**

a) 

b) 

6. David read 40 pages of a novel on day one, 37 pages on day two, and 34 pages on day three. This pattern of daily reading continued until David finished his book.
   a) Predict how long this pattern could continue. Explain your thinking.
   b) What was the total number of pages David read in one week?
   c) David started a new book. It is 150 pages. He maintained the same reading pattern and finished the book on a Tuesday. What day of the week did he start reading his book?

Solidifying Prior Learning

R–2

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Demonstrate an understanding of relations and functions*

Developing Number Sense

1. Which one doesn't belong? Find a reason and explain why each number does not belong in the set.*

$\frac{1}{20}$	$\frac{20}{25}$
$\frac{2}{3}$	$\frac{5}{4}$

2. Enzo has two piles of buttons. In each pile there are red buttons and blue buttons. In one pile, the ratio of the number of red to the number of blue buttons is 1 : 2. In the second pile the ratio of the number of red to the number of blue buttons is 3 : 5. If Enzo has a total of 20 red buttons, what might the total number of blue buttons be?

3. Grace plants flowers (x) around her tomato plants (•) to protect them from bugs. **(TN)**

N = 1	N = 2	N = 3
x x x x • x x x x	x x x x x • • x x • • x x x x x	x x x x x x • • • x x • • • x x • • • x x x x x x

a) Describe how you see the pattern growing. Extend the pattern by drawing the next two shapes.

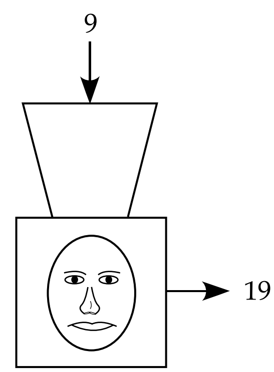
b) For a large value of N, describe how to find the number of tomato plants (•) and the total number of plants (x + •).

4. Bobo the function robot takes a number as input (x) and gives out another number as output (y). You put in a 9 and you get back 19.

a) Use words to describe three different possible rules that Bobo could be using.

b) Represent each rule with an equation.

c) Create a rule that involves more than one operation.



* Based on a problem by Hélène Matte. Available at www.wodb.ca/numbers.html.

Solidifying Prior Learning

R–3

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Demonstrate an understanding of slope: rise & run, lines, range of change....*

1. Which one doesn't belong? Find a reason and explain why each number does not belong in the set.*

33%	$\frac{1}{3}$
$\frac{5}{3}$	$0.\overline{6}$

2. For each question, use digits from 1 to 9 (without repeats) to make the value of each expression close to 1 (but not ≥ 1).** **(TN)**

a) $\frac{\square}{\square} + \frac{\square}{\square}$

b) $\frac{\square}{\square} - \frac{\square}{\square}$

c) $\frac{\square}{\square} \times \frac{\square}{\square}$

d) $\frac{\square}{\square} \div \frac{\square}{\square}$

3. Complete the products in the tables below. What do you notice? What do you wonder?

a)

Expression	Product
(4)(5)	
(3)(5)	
(2)(5)	
(1)(5)	
(0)(5)	
(−1)(5)	
(−2)(5)	

b)

Expression	Product
(4)(−5)	
(3)(−5)	
(2)(−5)	
(1)(−5)	
(0)(−5)	
(−1)(−5)	
(−2)(−5)	

4. Draw a square with vertices at E(6,1) and G(2,1).
   a) What is the length of each side? What is the perimeter? What is the area?
   b) Sketch another possible square using E and G as vertices. What are the characteristics of the second square?

* Based on a problem by Erick Lee. Available at www.wodb.ca/numbers.html.

**Based on a problem by Owen Kaplinsky. Available at www.openmiddle.com/tag/5-nf-1/.

Solidifying Prior Learning

R–3

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Demonstrate an understanding of slope: rise & run, lines, range of change.... (continued)*

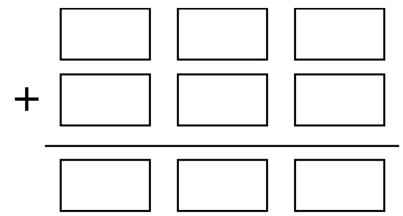
5. Draw a rectangle at W(–2,4) and Y(2,1).
   a) Draw a rectangle using these points. What are the coordinates of the other vertices? What is the area?
   b) Sketch another possible rectangle using W and Y as vertices. What are the characteristics of the second rectangle?

6. Draw a line segment from point A(1, 1) to point B(5, 4). **(TN)**
   a) What are some points that would be on the line extended through AB? Describe the pattern.
   b) Draw another line segment from point X(6, 2) to point Y(14, 6). What are some points that would be on the line extended through XY? Describe the pattern.

Solidifying Prior Learning R–4

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Describe and represent linear relations, using words, tables, graphs, equations....*

1. Use all of the digits from 1 to 9 once each to complete the three-digit sum. Is there another possibility?*



2. Create a rule for a decreasing pattern over six days.

Day						
Value						

   a) Describe a possible context for the decreasing pattern. What will your numbers represent?

   b) Show the first six days of your pattern in a table of values.

   c) Describe the pattern in words.

   d) Represent the pattern as a graph.

3. The table of values displays a pattern/relationship between x- and y-values.

   a) Find two or more ordered pairs (x, y) that can fill the boxes.

   b) Explain how these points fit the pattern.

   c) If you graphed the points, would they all lie on the same line? Explain.

x	y
4	16
☐	☐
12	44

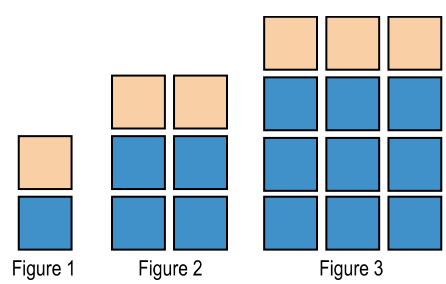
4. Given the pattern of blocks:

   a) Describe the pattern(s) you see.

   b) How many blocks will be in Figure 4? Figure 5?

   c) How many blocks are required to build Figure 10? Explain your thinking.

   d) Your friend says, "The total number of blocks for each figure does not increase in linear fashion." What do you think your friend means?



* Based on a problem by Owen Kaplinsky. Available at www.openmiddle.com/tag/5-nf-1/.

Solidifying Prior Learning

R–4

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Describe and represent linear relations, using words, tables, graphs, equations.... (continued)*
Developing Number Sense	

5. Given the pattern of blocks:

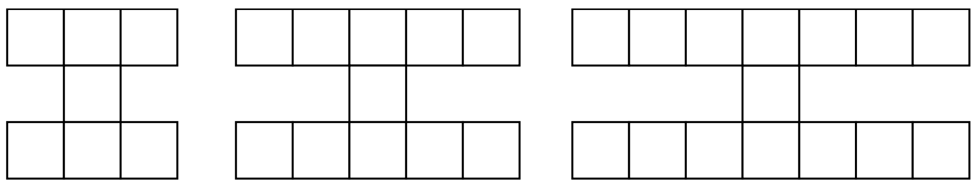


Figure 1 Figure 2 Figure 3

   a) Describe the pattern(s) you see.
   b) How many blocks will be in Figure 4? Figure 5?
   c) How many blocks are required to build Figure 10? Explain your thinking.
   d) Which figure will be closest to having 1000 blocks? Explain.
   e) Your friend says, "The number of blocks for each figure increases in linear fashion." What does your friend mean?

6. At Mathville Middle School, 30 boys and 20 girls entered a math contest. Certificates were awarded to 30% of the boys and 40% of the girls. What percentage of all of the participating students received certificates? Explain the process of your calculation.

7. Without a calculator:
   a) Which expression represents the smallest value when x is a number between 0 and 1? Explain your reasoning.
      i) x
      ii) x^2
      iii) $\frac{1}{x}$
      iv) $2x$
      v) $\sqrt{x}$

* Based on a problem by CEMC. Available at www.cemc.uwaterloo.ca/contests/past_contests/2009/2009Gauss8Contest.pdf.

Solidifying Prior Learning

R–4

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Describe and represent linear relations, using words, tables, graphs, equations.... (continued)*

b) Which expression represents the smallest value when x is a number between 1 and 4? Explain your reasoning.*

i) x

ii) x^2

iii) $\frac{1}{x}$

iv) $2x$

v) $\sqrt{x}$

8. Choose a babysitting fee for each hour of babysitting. As a babysitter, you charge a flat fee of $5 for snacks in addition to the hourly fee.

a) Graph the relationship between the number of hours babysitting and the total amount earned. Label the axes with what they represent.

b) Would you describe the relationship as linear? Explain.

c) Could your graph of this scenario include negative numbers?

d) The parents just told you they are going to give you a $20 bonus before you start so you can buy dinner. How does this change your graph?

* Based on a problem by CEMC. Available at www.cemc.uwaterloo.ca/contests/past_contests/2009/2009Gauss8Contest.pdf.

Solidifying Prior Learning

R–5

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Characteristics of graphs of linear relations, intercepts, slope, domain, and range*
Developing Number Sense	

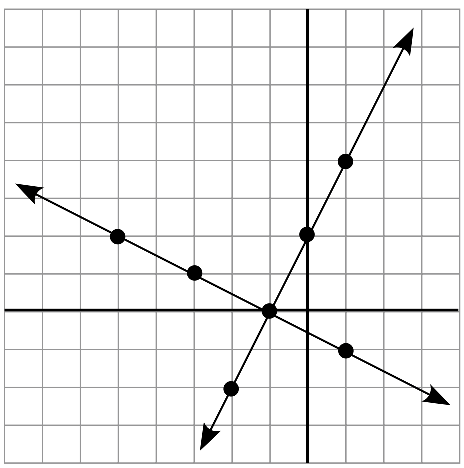
1. Which of the following numbers is closest to 1? Explain.*

   a) $\frac{11}{10}$ b) $\frac{111}{100}$ c) 1.101 d) $\frac{1111}{1000}$ e) 1.011

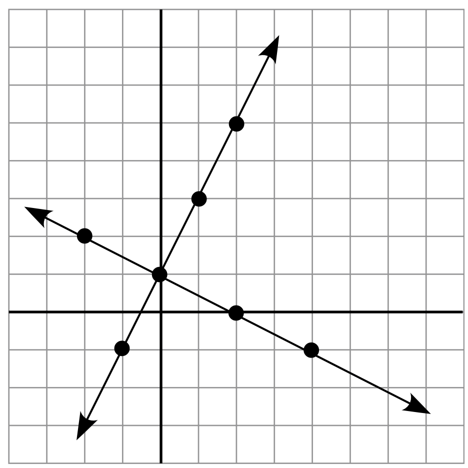
2. One car is travelling at 60 km/h and a motorcycle is travelling at 16 m/s. Which vehicle is travelling faster? How do you know?

3. How are these graphs similar? How are these graphs different?

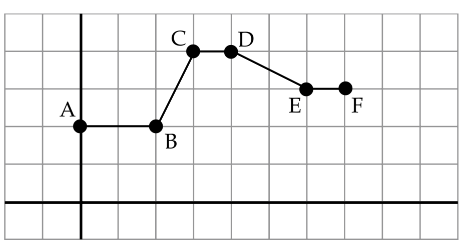
Graph A



Graph B



4. What could this graph represent? Label the axes. Tell the story.



* Based on a problem by CEMC. Available at www.cemc.uwaterloo.ca/contests/past_contests/2011/2011PascalSolution.pdf.

Solidifying Prior Learning R–5

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Characteristics of graphs of linear relations, intercepts, slope, domain, and range (continued)*

5. Your school table tennis team wants to buy team clothing. They have decided to purchase t-shirts costing $10 and hats costing $15 (tax is included). The team only has $300 to spend.
   a) What different combinations of t-shirts and hats can the team buy with the entire $300?
   b) What is the smallest number of hats that they could buy? What is the largest number of hats that they can buy?
   c) In one sentence, describe a pattern that helps describe all of the possible combinations.

6. Create tables of values for each of the equations. What similarities and differences do you notice? Predict what each of the graphs will look like with a rough sketch. **(TN)**

   Check your predictions by creating a graph (with paper and pencil or technology such as at www.desmos.com).
   a) $y = -3x + 5$
   b) $y = -x + 5$
   c) $y = \frac{1}{3}x + 5$
   d) $y = x + 5$

Solidifying Prior Learning R–6

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Relate linear relations as slope-intercept form, general form, and point-slope form*

1. Write each of the following phrases as algebraic expressions:
   a) Add a number and the square of the number.
   b) Subtract 10 from a number.
   c) Add 1 to a number and then double the result.
   d) Double a number and then add 1.
   e) Subtract a number from 12 and then divide the result by 4.
   f) Add 2 to a number, then triple the result and add one more.

2. Find the value of the missing number in the equation: $-2(1 + 5) + 3(6 + \square) = 3$

3. Create two equations with an unknown, with both multiplication and addition (or subtraction), with brackets, and with solutions that are negative numbers. Challenge others to solve them. Create an equation that
   a) uses only integers with an integer solution
   b) includes fractions with an integer solution

4. Given the equation $-3x + 4 = 5x + 2$:
   a) Without solving, rewrite an equation with the same solution that has all terms on one side of the equal sign, leaving zero on the other.
   b) Compare your work with others. What is the same? What is different?

5. Create an equation.
   a) The original equation needs to meet all of the following conditions:
      i) contains one variable
      ii) includes at least 4 terms (2 of which are constants)
      iii) uses a different integer for each term's coefficient
   b) Rearrange your equation so that all terms are on one side with zero on the other so that you have a positive coefficient in front of your variable. Check that the solution is the same for the original and the rearranged equations.

Solidifying Prior Learning

R–6

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Relate linear relations as slope-intercept form, general form, and point-slope form (continued)*

6. Fill in the blanks with numbers to establish a pattern rule:

   "Start at ____________ and add/subtract ____________ for each new term."

   a) Show your pattern in a table of values, giving the pattern value and the term number.

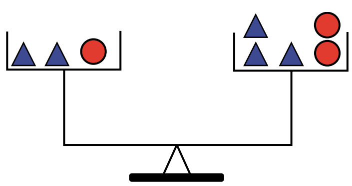
   b) Plot the pattern as a set of points (e.g., term number, pattern value).

   c) Establish a second rule by changing ONLY the start number. Create a new table of values. Plot the new set of points on the same graph with the pattern from the first rule.

   d) Establish a third rule by changing ONLY the number you add/subtract each time. Create a new table of values. Plot the new set of points on the same graph with the patterns from the first and second rule.

   e) What do you notice or wonder about the data in the tables and the graphs associated with the three rules?

7. Find the relationship between the ▲ and the ● by determining the integer values the ▲ and ● might represent to keep the balance. Find two pairs of integers for the triangle and circle. Describe the general relationship between the value of the triangle and the value of the circle.

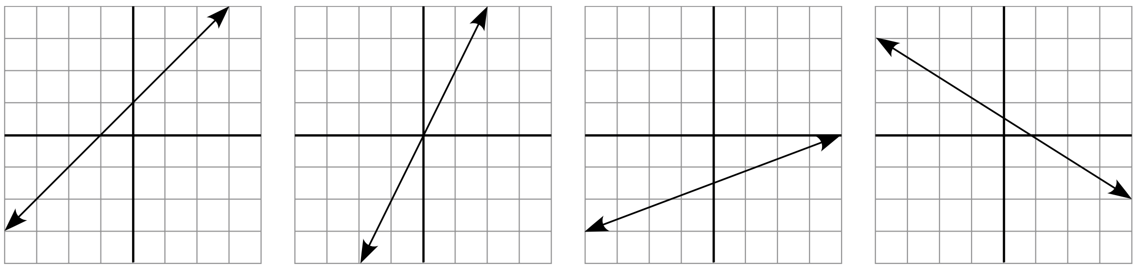


Solidifying Prior Learning

R–6

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Relate linear relations as slope-intercept form, general form, and point-slope form (continued)*

8. Which one doesn't belong? Find a reason and explain why each graph does not belong in the set.*



9. Joti is saving up to buy tickets to a concert. She has $50 saved already and is earning $15 per week to add to her savings.
   a) Model her savings over time in two different ways (e.g., words, graph, table, equation).
   b) Joti expects the ticket to cost between $120 and $280. How long will it take Joti to save enough money?
   c) Joti has other priorities and she would like to consider an alternative. She will use only $20 of her original savings and $10 of her weekly earnings. How long will it take her to save enough money for the ticket? How do your models change to represent this alternative?

* Based on a problem by Mary Bourassa. Available at https://wodb.ca/graphs.html.

Solidifying Prior Learning

R–7

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Determine the equation of a linear relation from a graph, a point and a slope, two points, a point and the equation of a parallel or perpendicular line, and a scatterplot*

1. For each set of numbers, calculate the mean (average). For each set, represent the values and the mean on a number line. Considering all the cases below, what do you notice (as an observation) or wonder (question)?
   a) 2, 3, 4
   b) −8, −6, −4
   c) −3, −1, 1, 3, 5, 7
   d) −7, 1
   e) −10, 10, 30
2. Calculate the sum:
   a) 9 + 16
   b) 36 + 64
   c) 144 + 25

   How are these expressions alike? Create other expressions that share this property.
3. Using a Cartesian plane:
   a) Draw a polygon with the conditions that it has to have
      i) vertices in three or more quadrants
      ii) at least one pair of perpendicular sides
      iii) at least one pair of parallel sides
   b) Using only verbal communication, instruct a partner to draw an identical polygon. Choose one of the vertices as a starting point. This is the only point you can identify using coordinates. Provide verbal instructions (with no hand waving) to have your partner create a drawing of the polygon.
   c) Evaluate the communication by comparing the drawing to the original polygon.

Solidifying Prior Learning

R–7

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Determine the equation of a linear relation from a graph, a point and a slope, two points, a point and the equation of a parallel or perpendicular line, and a scatterplot (continued)*

4. The following data shows the number of birds counted in an area on some days over a period of a few weeks in the spring. **(TN)**

Day	3	5	9	11	13	15	21
Bird Count	1	5	6	8	10	14	12

a) Plot the data points on a graph.

b) Draw a line that you think best represents the data trend.

c) Compare your line to a classmate's. How are they the same or different? Justify why you think one line is a better representation of the trend of the data.

5. The weights of the triangles on the equal arm balances are unknown. The weight of each marble (circle) is 7 g. For each scenario, do the following:

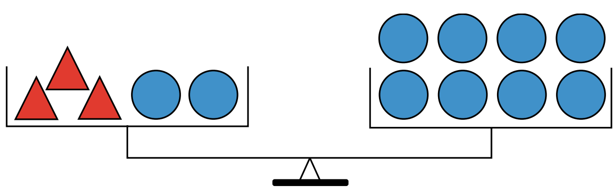
a) Determine the triangle weight without using algebra.

b) Describe, in words, the steps you took to determine the triangle weight.

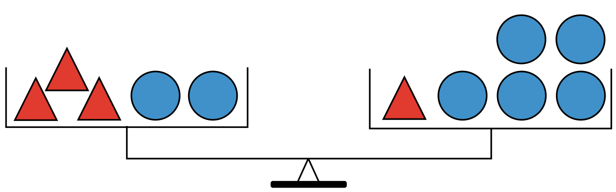
c) Write an equation to represent the scenario and solve the equation algebraically.

d) How does your description in words connect with your algebraic steps?

i)



ii)



Solidifying Prior Learning

R–7

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Determine the equation of a linear relation from a graph, a point and a slope, two points, a point and the equation of a parallel or perpendicular line, and a scatterplot (continued)*

6. The following steps illustrate the use of the distributive property to mentally calculate 15×13:

   $15(10 + 3)$

   $(15 \times 10) + (15 \times 3)$

   $150 + 45$

   195

   a) What is the distributive property? Describe it in words, algebraically, or with an area model.

   b) Use the distributive property to write equivalent expressions.

      i) 7×14 is the same as (____)(____ + ____).

      ii) $(6)(5x + 2)$ is the same as (____ + ____).

      iii) $120y - 84$ is the same as (____)(____ − ____).

7. On a coordinate plane, draw the graph of a line that goes through a point in the second quadrant and intersects the x-axis at 45°. Draw a second line that is perpendicular and passes through the origin. **(TN)**

Solidifying Prior Learning R–8

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Represent a linear function using function notation*

1. Use the distributive property to mentally determine the product of each expression below. Explain the thinking you used to arrive at your answer. **(TN)**
   a) 12×16
   b) 26×8
   c) 10.5×12

2. It's Waffle Wednesday and John makes waffles for his friends. The recipe calls for $1\frac{1}{2}$ cups of flour and 3 teaspoons of baking powder. Based on experience, John knows the recipe won't quite be enough to feed everyone and doubling the recipe is too much. He decides to add one-third more to all of the ingredients because he thinks it is easier than adding one-half more. How much flour and how much baking powder should he put in the mixing bowl? Do you agree or disagree with John? Why?

3. Notation is important for communication. Choose one imperial unit and one metric unit, and then write all the notations you know for the related units of length, area, and volume.

4. Use the input and output values of the function machine, g, as shown below, to determine the missing output and input values. Describe what the function machine, g, is doing with the input. **(TN)**

Input		Output
1		5
2	→	8
3	g	11
5		
18		
		98

Solidifying Prior Learning R–8

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Represent a linear function using function notation (continued)*

5. Temperature in the USA is reported in °F (Fahrenheit) and temperature in Canada is reported in °C (Celsius). You can input degrees Celsius into a function machine, T, and output degrees Fahrenheit. The operations of the function machine are as follows: divide by 5, multiply by 9, and then add 32.

   a) Write out the operations of T algebraically.

   b) Outside the thermometer reads +40°C. What output value will machine T show?

   c) Outside, the thermometer reads −40°C. What output value will machine T show?

   d) Your oven reads 350°F. What input value in °C gives an output value of 350°F?

Solidifying Prior Learning

R–9

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving systems of linear equations in two variables, graphically and algebraically*

1. Sansa is 5 years older than Arya. At least one of them is a teenager and the sum of their ages is a prime number. How old might they be?
2. Consider the exchange rate of a USA dollar (USD) and a Canadian dollar (CAD).
   a) When a USA dollar is worth about $1\frac{1}{2}$ times a Canadian dollar, a Canadian dollar is worth $\frac{2}{3}$ of a USA dollar. How are these two fractions related?
   b) If a USA dollar is worth about $1\frac{1}{4}$ times a Canadian dollar, what fraction of a USA dollar would a Canadian dollar be worth?
3. The sum of the ages of Bonnie, her sister, her mother, and her father is currently 89. In relation to Bonnie, her sister is 3 years younger and her mother is 3 times as old. Bonnie's father is 4 years older than her mother. Represent their ages using algebraic expressions on a number line. Is this enough information to determine their ages? Explain.
4. The school track team plans to order hoodies for each team member and needs to decide which company to go with.
   a) Company A charges an initial fee of $350 to print the school logo and $20 per hoodie.
   b) Company B charges an initial fee of $200 to print the school logo and $25 per hoodie.

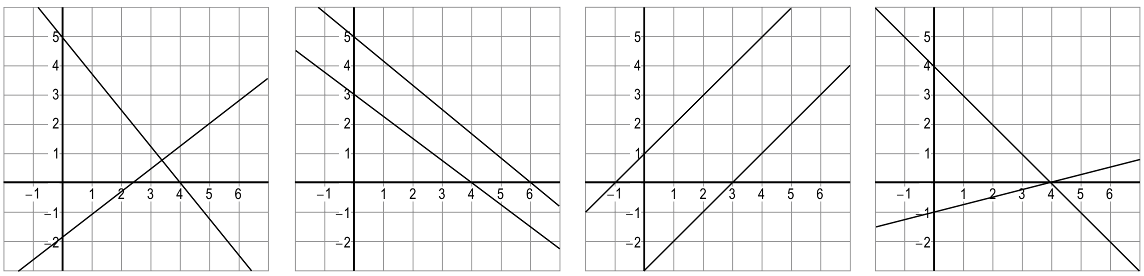
   Which company would you recommend they choose? Explain your answer.
5. Start with the point (3, 5) on a graph.
   a) Use a ruler to draw two lines that both cross through this point.
   b) Write down several attributes of each of the two lines you just drew. What is the same? What is different?

Solidifying Prior Learning

R–9

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving systems of linear equations in two variables, graphically and algebraically (continued)*

6. Which one doesn't belong? Find a reason and explain why each one does not belong in the set.*



* Based on a problem by Kyle Ramstad. Available at https://wodb.ca/graphs.html.

Solidifying Prior Learning R–10

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving the distance between two points and the midpoint of a line segment*

1. Using all of the digits from 1 to 9, complete the following to make a true statement.*

$$\frac{\square}{\square} > \frac{\square\square}{\square} = \frac{\square}{\square} > \frac{\square}{\square}$$

2. If we add two integers, A and B, and get A + B = –1, what do we know about the integers? Demonstrate your answer on a number line.

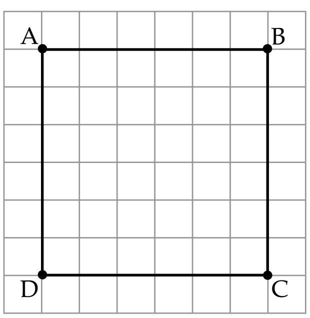
3. My age is 35 years old. How old is someone half my age? How old is someone half your age? What is the age of a person halfway between your age and my age? **(TN)**

4. Mr. Bowe is 42 years old. His age is halfway between your age and Ms. Carlyle. How old is Ms. Carlyle?

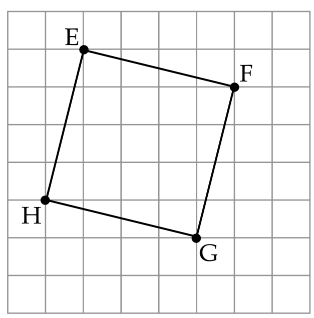
5. Liam is 9 years old. His age is halfway between you and your cousin Sara. How old is Sara? Explain your thinking.

6. Consider the squares in the diagrams below. Describe where lines need to be drawn to divide each square into 4 smaller squares. Is there more than one place to draw the lines?

a)



b)



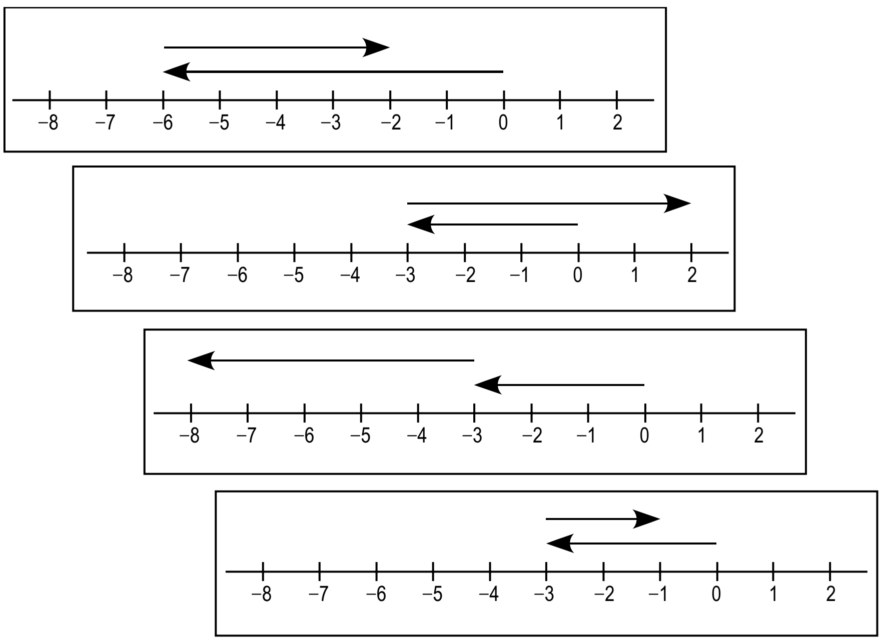
* Based on a problem by Owen Kaplinsky. Available at www.openmiddle.com/tag/5-nf-1/.

Solidifying Prior Learning

R–10

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving the distance between two points and the midpoint of a line segment (continued)*

7. Consider the four number lines shown:*
   a) Which one does not belong? Find a reason and explain how each one does not belong.
   b) What expression does each number line represent?



* Based on a problem by Nicole Paris. Available at https://wodb.ca/numbers.html.

Solidifying Prior Learning

R–10

Support for Strand:	*Relations and Functions*
Prior Learning for:	*Solve problems involving the distance between two points and the midpoint of a line segment (continued)*

8. Use the distributive property to mentally determine the quotient of 280 ÷ 8. Explain the thinking you used to arrive at your answer.

9. a) The arithmetic mean of 1, 3, 5, 7, 9 is the same as the median. Explain why.
   b) What two integers can be added to this list that will not change the mean value? How many pairs can you find?

10. The vertices of the base of an isosceles triangle are at A(1, 3) and B(7, 3).
    a) Where might the third vertex, C, be? Where else might C be?
    b) Draw the triangle on a coordinate plane.
    c) Determine the area and perimeter of your triangle.

11. A triangle is formed by joining vertices at A(3, 0), B(10, 0), and C(0, 6). **(TN)**
    a) Plot ΔABC.
    b) Find the lengths of AB, AC, and BC (exact and to the nearest thousandth).
    c) Plot point D on AC at (1.5, 3). Show that the length of CD equals half of AC.
    d) Draw a line through D parallel to AB and label the line's intersection with BC, E.
    e) What do you notice or wonder?

Solutions by Strand

M–1

Solidifying Prior Learning Solutions

1. Pairs of numbers (a, b) are (1, 2), (2, 4), (3, 6), (4, 8)…. The value of b is twice the value of a.

2. a) One of the possible solutions using the integers 1, 2, 3, 4, 5, 6.

$$\frac{[6]}{[1]} - \frac{[2]}{[4]} - \frac{[3]}{[5]} \text{ is a little larger than } \frac{[6]}{[1]} - \frac{[2]}{[5]} - \frac{[3]}{[4]}.$$

It is the largest because we start with the largest fraction possible by using the biggest numerator (6) and the smallest denominator (1). Then, subtract the smallest fractions possible created by using the small remaining integers (2 and 3) in the numerator and the large remaining integers (4 and 5) in the denominator.

Another possible solution using the integers 4, 5, 6, 7, 8, 9:

$$\frac{[9]}{[4]} - \frac{[5]}{[7]} - \frac{[6]}{[8]} \text{ is a little larger than } \frac{[9]}{[4]} - \frac{[5]}{[8]} - \frac{[6]}{[7]}.$$

Similar justification as above.

(TN) Student work on this question provides an opportunity to remind students about the process of subtracting fractions. Students could be asked to compare their answers with each other to see which numbers and their arrangements yield the largest result. Student groups could be asked to articulate a process to determine the largest result, given a set of any 6 numbers.

b) One of the possible numbers "close" to zero using the integers 1, 2, 3, 4, 5, 6:

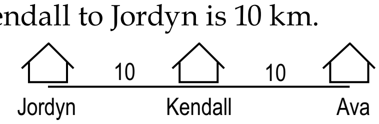
$$\frac{[3]}{[1]} - \frac{[4]}{[6]} - \frac{[5]}{[2]} \text{ which is a little closer to zero than } \frac{[6]}{[3]} - \frac{[4]}{[2]} - \frac{[1]}{[5]}$$

One of the possible numbers "close" to zero using the integers 4, 5, 6, 7, 8, 9:

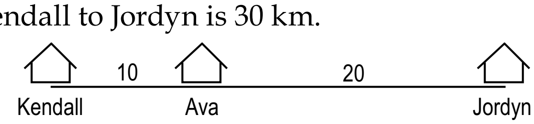
$$\frac{[9]}{[5]} - \frac{[8]}{[6]} - \frac{[4]}{[7]} \text{ which is a little closer to zero than } \frac{[9]}{[4]} - \frac{[8]}{[6]} - \frac{[5]}{[7]}$$

3. The answers can be any real number ranging from a minimum of 10 km to a maximum of 30 km. Three possible answers and accompanying diagrams are shown:

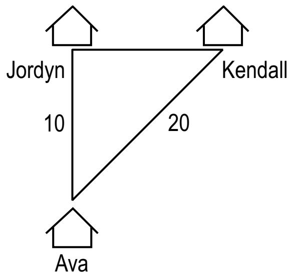
a) Kendall to Jordyn is 10 km.



b) Kendall to Jordyn is 30 km.

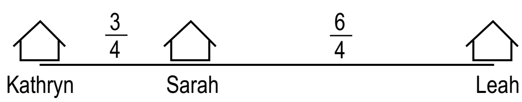


c) Kendall to Jordyn is $\sqrt{300}$ or 17.3 km.

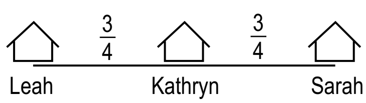


4. The answers can be any real number ranging from a minimum of $\frac{3}{4}$ mile to a maximum of $2\frac{1}{4}$ miles. These two possible answers are shown; the other answers do not have Kathryn, Sarah, and Leah all living along the same line.

a) Kathryn to Leah is $\frac{9}{4} = 2\frac{1}{4}$ mi.



b) Kathryn to Leah is $\frac{3}{4}$ mi.

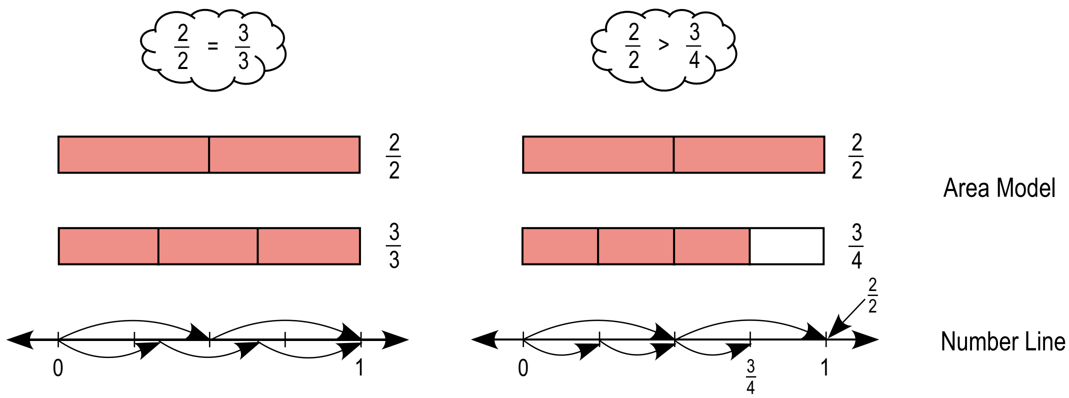


5. The perimeter of ΔADC is 25.04 m. The area of ΔABD is $\frac{6 \times 8}{2} = 24$ m^2, so the area of ΔABC is $24 + 0.50(24) = 36$ m^2. The height of ΔABC is 8 and the area is 36, so the length of base BC is 9 m. Therefore the length of DC is 3 m. By the Pythagorean theorem, AD is 10. By the Pythagorean theorem, the length of AC is 12.04 m. Therefore, the perimeter of ΔADC is 3 + 10 + 12.04.

M–2 Solidifying Prior Learning Solutions

1. Many solutions are possible. Here is one example.

   If $a = 2$, then $b > 3$. For example, $a = 2$ and $b = 4$



   Other possible answers:

a	2	2	3	3	6	6
b	4	5	5	6	10	11

2. a) She was alive for 64 404 000 minutes. Assumptions: Born and died at the same time of the day. There are 31 leap years in her lifetime. 122 years × 365 days/year = 44530 days; 31 extra days (leap years); and 164 days.

   Total time: 44530 + 31 + 164 = 44725 days or 64 404 000 minutes.

   b) Millionth minute, not quite 2 years old. $1\ 000\ 000 \div 60 \div 24 = 694.\overline{4}$ days or 1.901 years.

   Billionth minute, 1901 years old. $1\ 000\ 000\ 000 \div 60 \div 24 = 694444.\overline{4}$ days or 1901 years.

   (TN) Students need to discuss how they will handle leap years. Alternatives include the following: they could ignore them, they could count them, or they could assume 365.25 days per year to include leap years.

3. Jaswinder’s statement:

a) True for $\frac{6}{9} < \frac{9}{10}$. The fraction with numerator and denominator differing by 3 is less than the fraction with numerator and denominator differing by only 1.

b) Not true for $\frac{4}{5} < \frac{90}{100}$. The fraction with numerator and denominator differing by 1 is less than the fraction with numerator and denominator differing by 10.

4. The diameter is 5.04 cm.

$\pi \times r^2 = 20$

$r = \sqrt{\frac{20}{\pi}} = 2.52 \text{ cm},$ so the diameter is 5.04 cm.

5. The large pizza is a better deal. Compare the areas with the given diameters.

18″ pizza area = $\pi(9)^2 = 254.47$ in.^{2}

12″ pizza area = $\pi(6)^2 = 113.10$ in.^{2}

Two medium pizzas have an area of 226.20 in.^{2}, which is not as big as a large pizza.

(TN) Discuss assumptions about what the measurement refers to with a 12″ or an 18″ pizza (i.e., **diameter**, radius, circumference, area). Discuss ways to compare the pizzas to determine the better deal (i.e., **area**, number of slices, radius, diameter, circumference).

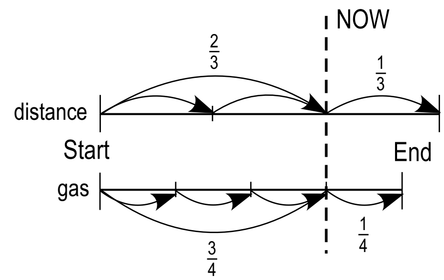
6. a) She wanted B and C. An enlargement needs to keep the same ratio for width and length, so the picture does not get distorted. The pictures with the same ratios for width and length are 10 × 12 and 8 × 9.6. They both have a ratio of width to height as $0.833\overline{3} : 1$.

$$0.83333 = \frac{10}{12} = \frac{8}{9.6}$$

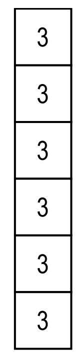
b) The length of the enlarged photograph is 43.2 cm.

Solidifying Prior Learning Solutions

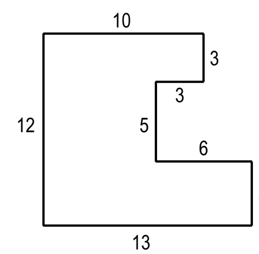
1. Gilles will run out of gas. He has driven two-thirds of the trip and used more than two-thirds of a tank of gas. He used three-quarters of a tank of gas and $\frac{3}{4} > \frac{2}{3}$. An assumption is that Gilles will continue to use the same number of litres per kilometre for the last part of the trip.



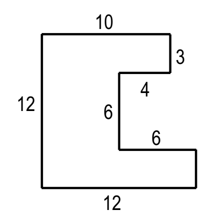
2. There were 18 cookies in the back at the start. Working backwards, there were
   - 3 cookies left for the sixth person
   - 6 before the fifth person ate half
   - 9 before the fourth person ate one-third
   - 12 before the third person ate one-fourth
   - 15 before the second person ate one-fifth
   - 18 before the first person ate one-sixth



3. The missing lengths need to be estimated. Estimates will vary within the constraints.
   a) For these measurements, the area is 117 units^2.
   b) For these measurements, the perimeter is 56 units.
   c) The area can be determined by dividing the shape into three horizontal rectangles with areas of 30, 35, and 52.



   a) For these measurements, the area is 102 units^2.
   b) For these measurements, the perimeter is 56 units.
   c) The area can be determined by subtracting the area of the L-shaped corner (6 + 36) from the 12 × 12 rectangle.



(TN) Given the constraints, the unknown horizontal sides must sum to 16. The unknown vertical sides must sum to 12. If students estimate and calculate within the constraints, the perimeter will always be 56 units. Although they may choose to work with them, students are not restricted to whole number sides. This question could be a lead in to a discussion of domain and range (e.g., "What are the possible numbers?").

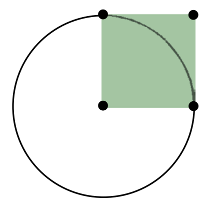
4. Two different solutions:

The volume could be 47.15 inches^3. Let the circumference of the cylinder be the full paper width of 8.5 inches. The relationship $C = \pi(d)$ means the diameter of the top and bottom circles must be less than or equal to 2.7056… inches (dividing: $8.5 \div \pi$). In fractions of inches, the diameter could be $2\frac{11}{16}$ inches. That means the height of the cylinder can be $8\frac{5}{16}\left(\text{subtracting: } 11 - 2\frac{11}{16}\right)$. The volume is calculated as

$$\pi \times \left(1\frac{11}{32}\right)^2 \times 8\frac{5}{16}.$$

Alternatively, the volume could be 48.11 inches^3. Let the circumference of the cylinder be the full paper length of 11 inches, and then the diameter is less than or equal to 3.5014… inches. Let the diameter be 3.5 inches, making the height of the cylinder 5 inches. The volume is $\pi \times (1.75)^2 \times 5$.

5. Just over 3 squares will fit into the circle (exactly π squares will fit).



(TN) Students could cut up the 4 squares drawn on graph paper to determine that just over 3 squares fit in the circle. If they use graph paper, they will be able to make a more precise estimate (more than 3 and one-tenth, or close to 3 and fourteen-hundredths). Then they should connect what they see with the cut-out areas to the algebraic formulas for the areas. The ratio of the area of the circle to the area of the square is $\pi r^2 : r^2$.

6. The circumference is generally longer than the height of a coffee cup. Use your fingers to compare heights and circumferences; most people will be unable to wrap their fingers around the circumference of a cup but will be able to stretch their fingers beyond its height. The distance around the rim (circumference) is about 3 (or exactly π) times greater than the distance across the cup (diameter).

(TN) Rather than providing rulers, teachers could allow students to only use their hands or other referents.

7. Here are three of the many possible answers:

$4 \times 30 \times 21 = 2520$ cm^3

$7 \times 24 \times 15 = 2520$ cm^3

$2 \times 13 \times 96 = 2496$ cm^3

8. Dylan could
   a) be building a rectangular prism (box), a rectangle-based pyramid, a prism with triangular ends and some square faces, or a polyhedron with some square faces
   b) **not** be building a cylinder (unless the shape shown can be bent around a curved end), a sphere, cone, or a regular polygon (other than a cube if the shape shown is a square)

9. If the width is represented by x, then the length is $2x$ and the height is $4x$. Algebraic expressions describing features of the box include the following:

If		$w = x$ $l = 2x$ $h = 4x$	$w = \frac{n}{2}$ $l = n$ $h = 2n$	$w = \frac{H}{4}$ $l = \frac{H}{2}$ $h = H$
then	Area of Base =	$2x^2$	$\frac{n^2}{2}$	$\frac{H^2}{8}$
	Surface Area =	$28x^2$	$7n^2$	$\frac{7H^2}{4}$
	Volume =	$8x^3$	n^3	$\frac{H^3}{8}$

(TN)₁ Students could be asked to compare these different expressions and describe what they notice.

(TN)₂ Students who are ready may be asked to find expressions for diagonals. For the first example above, the expression for diagonal length is $\sqrt{x^2 + (2x)^2 + (4x)^2} = x\sqrt{21}$ and, for the second example, the expression for diagonal length is $n\frac{\sqrt{21}}{2}$.

10. Cylinder with radius, r, and a height of 4. Algebraic expressions describing features include the following:

Area of the circular base = πr^2

Volume = $4\pi r^2$

Surface Area = $\left[2\left(\pi r^2\right) + 2\pi r(4)\right]$ **or** $\left[2\pi r^2 + 8\pi r\right]$ **or** $\left[2\pi r(r + 4)\right]$

11. The table entries show cubes of increasing size.

Side Length	Surface Area	Volume	SA : Vol ratio	SA : Vol ratio Comparison
1	6 x 1=6	$1^3 = 1$	6 : 1	6 : 1
2	6 x 4 = 24	$2^3 = 8$	24 : 8	6 : 2
3	6 x 9 = 54	$3^3 = 27$	54 : 27	6 : 3
4	6 x 16 = 96	$4^3 = 64$	96 : 64	6 : 4
5	6 x 25 = 150	$5^3 = 125$	150 : 125	6 : 5

a) The next side length of 6 cm will have a $\frac{\text{SA}}{\text{Vol}}$ ratio of $\frac{6}{6}$ or 1.

b) The ratio of $\frac{\text{SA}}{\text{Vol}}$ is greater than 1 for side lengths less than 6 and is less than 1 for side lengths greater than 6.

c) The ratio of SA : Vol when side length is 100 is 60000 : 1000000 = 674 : 100.

d) The ratio of SA : Vol is 6 : n when side length n is $\frac{\text{SA}}{\text{Vol}} = \frac{6n^2}{n^3} = \frac{6}{n}$.

e) The ratio of SA : Vol for a cylinder with height equal to r is 4 : r.

12. First, you need to decide on a way to determine what it means to have a "better fit." One possibility is to compare the difference in surface area between the hole and the peg, given the circles are the same size. The smaller the difference is the "better fit." The following solution proceeds with this meaning of "better fit." You may use a different meaning or make a comparison, given the squares are the same size.

The first diagram shows a round peg in a square hole. The radius of the circle is r and each side of the square is $2r$. The area difference, hole − peg, is $(2r)^2 - \pi r^2 = (4 - \pi)r^2 = (0.858...)r^2$. The second diagram shows a square peg in a round hole. The radius of the circle is r. The diameter of the square is $2r$, which means each side is $\sqrt{2}r$ (using the Pythagorean theorem). The area difference, hole − peg, is $\pi r^2 - 2r^2 = (\pi - 2)r^2 = (1.141...)r^2$. Using the described meaning of "better fit", the round peg in a square hole is a better fit.

(TN) Students may come up with different ways of determining a "better fit." For example, instead, students may keep the square size the same for comparison purposes. Then, the area difference (round hole − square peg) is $\pi\left(\frac{\sqrt{2}}{2}s\right)^2 - s^2 = \left(\frac{\pi}{2} - 1\right)s^2 = (0.570...)s^2$. The area difference (square hole − round peg) is $s^2 - \pi\left(\frac{s}{2}\right)^2 = \left(1 - \frac{\pi}{4}\right)s^2 = (0.214...)s^2$. Again, the round peg in a square hole is a better fit.

M–4

Solidifying Prior Learning Solutions

1. There are multiple solutions including:

   $\frac{17}{68} = 0.25$ $\frac{36}{48} = 0.75$ $\frac{72}{16} = 4.50$

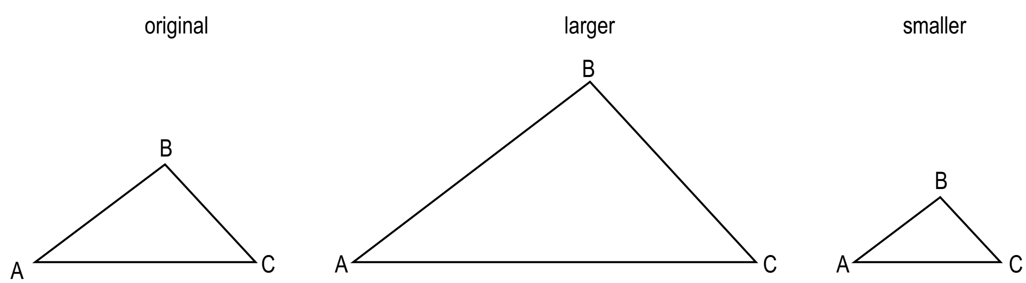
2. The fewest number of people surveyed is 125.

   93.6% could be 93.6 out of 100, but it is not possible to have 0.6 people. It could be 936 out of 1000, but that does not represent the fewest number. Simplify the fraction $\frac{936}{1000} = \frac{117}{125}$.

   There could be as little as 117 of 125 who completed the survey.

3. The hose length is 46.5 m (distance EC). Use the Pythagorean theorem to find AB is 15. Then, use the Pythagorean theorem with ΔABC to find BC is 36. Use the Pythagorean theorem again with ΔECD to find $EC = \sqrt{15^2 + 44^2} = 46.5$ m.

4. The larger triangle has all angles the same measure as the original. The smaller triangle also has all angles the same measure as the original. In the example shown, the larger triangle has all sides that are 2 times the original triangle. The smaller triangle has all sides that are two-thirds of the original triangle.



(TN) Some students may be directed to draw right triangles, since they may be easier to draw as enlargements or reductions. Some students may draw similar shapes by using the fact that the angles must be the same rather than noticing that relationship after drawing the triangles. A cut-out of the original triangle could be used to create an enlargement or reduction by using the angles in the cut-out to trace the angles for an enlarged (or reduced) triangle. It could be suggested to students that they draw the triangles on grid paper. They could then use the grid lines to ensure the shapes are enlargements or reductions.

5. a) The area of ΔAKL is 72 units^2. You are given the square area, which is 64, so each side of square BCDE is 8 units. Let the height of ΔAKL be h. Compare the areas where ΔACD = 2BCDE, then $8(8 + h) \div 2 = 2(64)$, so $h = 24$ units. Since $h = 24$ and you are given KL = 6, then the area of ΔAKL is $6 \times 24 \div 2 = 72$ units^2.

   b) The area of trapezoid KCDL is 126 units^2. The area of square BCDE is 144, so sides of the square are 12 units and the area of ΔACD is 288. Let the height of ΔAKL be h as in part (a) above. ΔACD = 2BCDE, and then $12(12 + h) \div 2 = 2(144)$, so $h = 36$ units.

   The area of trapezoid KCDL can be found in two ways. First, area ΔACD – area ΔAKL and, second, using the formula $\left(\frac{CD + KL}{2}\right) \times$ height.

   First calculation: the trapezoid area is ΔACD − ΔAKL, $288 - KL \times 36 \div 2 = 288 - 18KL$.

   Second calculation: the trapezoid area is $\left(\frac{12 + KL}{2}\right) \times 12 = 72 + 6KL$.

   Set the first and second expressions equal to each other and solve for KL.

   $288 - 18KL = 72 + 6KL$, so KL = 9 units.

   The area of the trapezoid is $72 + 6(KL) = 126$ units^2.

(TN) It is important to let students think and work through this problem. Other solution methods are possible (and may even be preferred). For example, for part (a), a student could find the area of ΔCBK = ΔDEL = $1 \times 8 \div 2 = 4$ and subtract known areas to find the area ΔAKL is 72. For part (b), similar triangles could be used after drawing the altitude from A to the midpoint of CD (label it F). The height, AF, can be found to be 48 units using the area of ΔACD, which is $12 \times AF \div 2 = 288$. ΔAFD ~ ΔDEF, so $\frac{AF}{FD} = \frac{DE}{EL}$, so EL = 1.5 units.

The trapezoid area can be found by subtracting areas 144 − ΔDEL − ΔCBK = 144 − 9 − 9 = 126 units^2.

A–1

Solidifying Prior Learning Solutions

1. Some of the possible answers:

	Similarities	Differences
2^4 and 4^2	■ both use same digits (2 and 4) ■ both are powers ■ both evaluate to 16	■ bases and exponents are switched
3^2 and 2^3	■ both use same digits (2 and 3) ■ both are powers	■ bases and exponents are switched ■ evaluate to different values (9 and 8)

2. GCF

a) Factors of 32: {1,2,4,8,16,32}; Factors of 8: {1,2,4,8}, GCF is 8

b) Factors of 24: {1,2,3,4,6,8,12,24}; Factors of 18: {1,2,3,6,9,18}, GCF is 6

3. LCM

a) Multiples of 32: 32, 64, 96, 128… Multiples of 8: 8, 16, 24, 32, 40, 48… LCM is 32

b) Multiples of 12: 12, 24, 36, 48… Multiples of 18: 18,36,54,72… LCM is 36

4. Some of the possible answers:

1×1 tiles: $112 \times 84 = 9408$ tiles

2×2 tiles: $56 \times 42 = 2352$ tiles

4×4 tiles: $28 \times 21 = 588$ tiles

7×7 tiles: $16 \times 12 = 192$ tiles

14×14 tiles: $8 \times 6 = 48$ tiles

28×28 tiles: $4 \times 3 = 12$ tiles

The fewest number of square tiles Sara could use is 12.

(TN) Students could be led to connect GCF and the solution to this problem.
GCF (112, 84) = 28, so 28 cm by 28 cm is the largest size tile that evenly fits both dimensions.

5. Divisibility

a) The possible answers are {60, 120, 180, 240, 300, 360, 400…}. The smallest possible is 60.

$\underbrace{(2)\overbrace{(3)(2)}^{6}}_{4}(5) = 60$ (4 is included because $2 \times 2 = 4$. 6 is included because $2 \times 3 = 6$.)

b) The smallest number divisible by all of the numbers from 1 to 20 is {232 792 560}. It is the product of 5, 7, 9, 11, 13, 16, 17, 19.

6. If there are 20 students and 20 lockers, then lockers 1, 4, 9, and 16 are left open and 12, 18, and 20 are touched the most at 6 times each.

   (TN) Students could be asked to extend this problem to 100 students and 100 lockers; then, lockers 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 are left open.

   The ones left open are square numbers because square numbers have an odd number of factors (i.e., there are 5 factors of 16, {1,2,4,8,16}). All other numbers (including primes) have pairs of factors (i.e., There are 2 factors of 7, {1,7}, and 6 factors of 12, {1,2,3,4,6,12}). The locker numbers touched most often have the greatest number of factors.

7. Sets

   a) Some possible responses are as follows:

      Notice:

      "They are all square numbers."

      "The differences between the numbers increases by two each time."

      "The numbers alternate between even and odd."

      Wonder:

      "What else could be included in the set? 225?"

      "Why does it start at 36 and end at 121?"

   b) These numbers are all 1 less than Set A. The difference between the numbers also increases by two each time and are the same as Set A. The numbers also alternate between even and odd.

   c) These numbers are all 16 less than Set A. The difference between the numbers also increases by two each time and is the same as Set A. The numbers also alternate between even and odd.

A–2 Solidifying Prior Learning Solutions

1. Some of the possible answers:

   a) $\frac{14}{10}, \frac{141}{100}, \frac{1414}{1000}, \frac{...}{...}, \frac{14}{10}, \frac{13}{10}, \frac{12}{10}, \frac{11}{10}, \frac{7}{5}, \frac{6}{5}, \frac{4}{3}$

   b) $\frac{17}{10}, \frac{173}{100}, \frac{1732}{1000}, \frac{...}{...}, \frac{16}{10}, \frac{8}{5}$

   c) $\frac{22}{10}, \frac{223}{100}, \frac{2236}{1000}, \frac{...}{...}, \frac{22}{10}, \frac{21}{10}, \frac{11}{5}, \frac{17}{8}$

2. Some of the possible answers:

   $0 + 1 + 4 = 2 + 3$

   $4 - (2 + 1) = 3^0$

   $3 \div (2 + 1) = 4^0$

3. The hypotenuse of the third triangle (the leg of the fourth triangle) has a hypotenuse of 2 units (10 cm).

   The hypotenuse of the eighth triangle (the leg of the ninth triangle) has a hypotenuse of 3 units (15 cm).

   The hypotenuse of triangles 3, 8, 15, and 24 will be whole number units (multiples of 5 cm).

   The legs of triangles 4, 9, 16, and 25 will be whole number units (multiples of 5 cm).

4. Squares:

   a) Yes, the stacks of squares are the same height. Find the lengths of sides using a calculator:

   $\sqrt{45} \doteq 6.708$

   $\sqrt{5} \doteq 2.236$

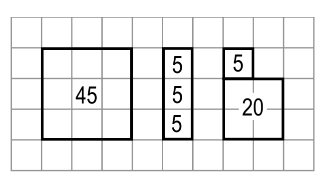
   $(2.236)(3) \doteq 6.708$

   $\sqrt{20} \doteq 4.472$

   $\sqrt{20} + \sqrt{5} \doteq 4.472 + 2.236 \doteq 6.708$

   OR

   Find the height using a visual model with □ = 5.



b) Some possible answers for squares that have the same height as a square with area of 72: (3 squares with an area of 8), (1 square with an area of 8 and 1 square with an area of 32).

[TN] For part (a), simplifying the radical $\sqrt{45} = 3\sqrt{5} = \sqrt{5} + \sqrt{20}$ is a justification that is at the level of the end of Grade 10, so it **does not yet need to be shown this way**. Other answers may compare lengths and/or areas by dividing larger squares into arrays of smaller squares.

5. Repeating fractions:

a) As a fraction, 0.3333… is $\frac{3}{9}, \frac{1}{3}$.

Solve the equation: $10x = 3 + x$

$$9x = 3$$

$$x = \frac{3}{9} = \frac{1}{3}$$

b) Create and solve equations:

i) Let $x = 0.636363\ldots$

$$100x = 63.636363...$$

$$100x = 63 + x$$

$$99x = 63$$

$$x = \frac{63}{99} = \frac{21}{33}$$

ii) Let $x = 5.454545\ldots$

$$100x = 545.4545...$$

$$100x = 540 + x$$

$$99x = 540$$

$$x = \frac{540}{99} = \frac{60}{11}$$

A–3

Solidifying Prior Learning Solutions

1. $2^2 = 4$

   $3^2 = 9 = 4 + 5$

   $4^2 = 16$

   $5^2 = 25 = 12 + 13$

   The sum of two consecutive numbers is always odd. Squaring an even number is always even. So, the pattern is not possible with the square of even numbers. Squaring an odd number is always odd, and any odd number can be written as the sum of two consecutive numbers (i.e., begin with any odd number, and the first consecutive number is half of the even number before the odd number in question). This pattern always works with the square of an odd number.

2. One possible answer is:

   $\sqrt{9}$ $\sqrt{16}$ $\sqrt{2+7}$ $\sqrt{4}+5$ $\sqrt[2]{8}$

3. 2 000 000 000

   Multiplying powers of 10 is easy to do mentally. The product of each pair of 2s and 5s is 10. Determine the number of pairs of 2s and 5s.

   $$\left(5^4\right)\left(20^5\right) = (5)^4(5\cdot2\cdot2)^5 = 5^4\cdot5^5\cdot2^5\cdot2^5 = 5^9\cdot2^9\cdot2 = 2\cdot10^9$$

   There are many other pairs of numbers. Here are two examples:

   $$\left(35^2\right)\left(2^2\right) = (5\cdot7)^2\cdot2^2 = 5^2\cdot7^2\cdot2^2 = (70)^2 = 4900$$

   $$\left(2^6\right)\left(5^8\right) = 2^6\cdot5^6\cdot5^2 = (2\cdot5)^6\cdot5^2 = 10^6\cdot25 = 25000000$$

4. Expressions in the form

   a) Some possible answers are: $[0][6]^{[7]} = [0][8]^{[4]}$, $[2][4]^{[5]} = [4][2]^{[9]}$, $[9][3]^{[7]} = [3][9]^{[4]}$

   b) Some possible answers are: $[1][5]^{[3]}$, $[4][3]^{[5]}$, $[8][2]^{[6]}$

5. Answers will vary depending on the interpretation of “large” result. Some answers are $(1)\left(4^5\right) = 1024$, $(1)\left(5^4\right) = 625$, or $(2)\left(3^5\right) = 486$.

A–4 Solidifying Prior Learning Solutions

1. Area model:

   a) 26×14 is:

	10	1
10	200	60
4	80	24

	50	1
10	500	10
2	100	2

   b) Product: $200 + 60 + 80 + 24 = 364$

   c) One possible answer is $51 \times 12 = 612$

2. Area model:

	$3x$	7
$2x$	$6x^2$	$14x$
2	$6x$	14

   The product represented is $(3x + 7)(2x + 2)$.

   The product is equivalent to $6x^2 + 14x + 6x + 14$ or $6x^2 + 20x + 14$.

3. Area model:

	$6x^2y^2$	$5x^2y$
3	$18x^3y^2$	$15x^2y$
$5x^3y$	$30x^6y^3$	$25x^5y^2$

$$\left[6x^3y^2 + 5x^2y\right]\left[3 + 5x^3y\right] = 18x^3y^2 + 15x^2y + 30x^6y^3 + 25x^5y^2$$

4. Rectangular prisms:

   a) Surface area is 78 m^2: $\left[2(5)(3) + 2(3)(3) + 2(5)(3)\right]$

   Volume is 45 m^3: $(5)(3)(3)$

   b) $\left[2(5 - x)(2x - 1)\right] + \left[2(5 - x)(2x + 1)\right] + \left[2(2x + 1)(2x - 1)\right] = 40x - 2$

   c) $(2x + 1)(2x - 1)(5 - x) = -4x^3 + 20x^2 - x - 5$

d) A height of $(5 - x)$ is possible if $x < 5$.

A length of $(2x + 1)$ is possible if $x > \frac{-1}{2}$.

A width of $(2x - 1)$ is possible if $x > \frac{1}{2}$.

Therefore, x must be less than 5 but greater than $\frac{1}{2}$.

The only possible whole number values of x are {1, 2, 3, 4}.

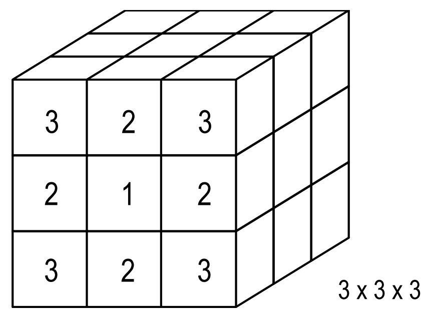
5. Two of the possible answers:

a) $[3x + 8][x + 3] = 3x^2 + 17x + 24$

b) $[3x + 12][x + 2] = 3x^2 + 18x + 24$

6. Painted cubes:

a) Construct or draw a 3 × 3 × 3 cube made from 27 smaller cubes.



b) Painted orange

i) Number painted on 1 face: 6 (one on each large cube face)

ii) Number painted on 2 faces: 12 (4 on each of front, middle, and back)

iii) Number painted on 3 faces: 8 (the corners of large cube)

iv) Number painted on 4 faces: 0

v) Number painted on zero faces: 1 (the very centre of the large cube)

c) This chart shows the number of faces painted for the other cube sizes.

Painted Cube	1 Face Painted	2 Faces Painted	3 Faces Painted	4 Faces Painted	0 Faces Painted
3 x 3 x 3	6	12	8	0	1
4 x 4 x 4	24	24	8	0	8
5 x 5 x 5	54	36	8	0	27

Some things students may notice:

- "There are never more than 3 faces painted."
- "There are always the same number of cubes with 3 faces painted."
- "The number of cubes with 2 faces goes up by 12 each time the cube dimension increases."

Students may wonder:

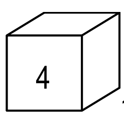
- "Does a $1 \times 1 \times 1$ cube or a $2 \times 2 \times 2$ cube fit the same pattern?"
- "Is there a general rule that can describe the number of cubes with each number of faces painted?"

d) The general $n \times n \times n$ cube:

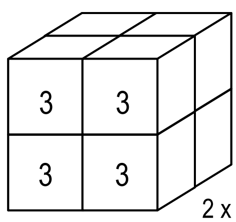
Painted Cube	1 Face Painted	2 Faces Painted	3 Faces Painted	4 Faces Painted	0 Faces Painted
$n \times n \times n$	$6(n - 2)^2$	$12(n - 2)$	8	0	$(n - 2)^3$

The algebraic expressions for the general case may look different but should simplify to equal the ones in the table. For an $n \times n \times n$ cube, the number of cubes with the following features:

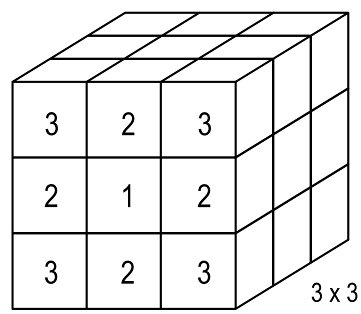
- 1 face painted on each of the 6 faces is $(n - 2)(n - 2)$.
- 2 faces painted on each of the 12 edges is $(n - 2)$.
- 3 faces painted on each of the 8 corners.
- 4 faces painted is 0. There are never more than 3 faces painted (a $1 \times 1 \times 1$ cube is the exception).
- 0 faces painted on the cube inside all of the surface cubes with dimensions $(n - 2)(n - 2)(n - 2)$.



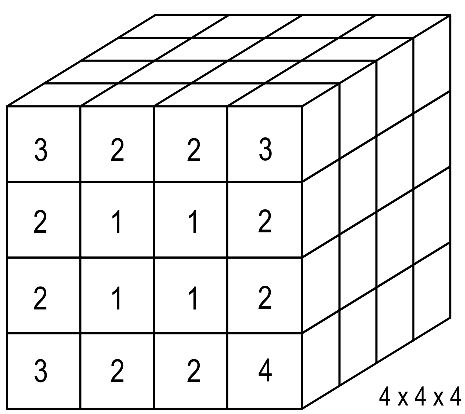
1 x 1 x 1



2 x 2 x 2



3 x 3 x 3



4 x 4 x 4

A–5

Solidifying Prior Learning Solutions

1. Name the two numbers that have the following characteristics:
   a) The possible answers are 1, 24 or 2, 12 or 3, 8 or 4, 6 or −1, −24 or −2, −12 or −3, −8 or −4, −6.
   b) Some of the possible answers are 0, −13 or −1, −12 or −5, −8 or 1, −14 or −3, 16 or ….
   c) 8 and 3
   d) −3 and −5
2. The sums are: 19, 11, 9, −19, −11, −9 for the expressions: 1 + 18, 2 + 9, 3 + 6, −1 + (−18), −2 + (−9), and −3 + (−6).
3. The differences are: −11, −4, −1, 1, 4, 11 for the expressions: 1 − 12, 2 − 6, 3 − 4, 4 − 3, 6 − 2, and 12 − 1.
4. −18 since the product matches the expressions: −18 × 1, −9 × 2, −6 × 3, −3 × 6, −2 × 9, and −1 × 18.
5. $(3x + 5)(2x + 2)$ is equivalent to the trinomial $6x^2 + 16x + 10$.
6. Equations:
   a) $4x + 8 = \boxed{4}(x + 2)$
   b) $\boxed{6}x^2 + 12x = \boxed{6}(x + 2)$
   c) $(2x^2 + \boxed{6}x + 3)(\boxed{2}x + \boxed{4}) = 4x^3 + 20x^2 + 30x + 12$
   d) $12x + 9 = \boxed{3}(\boxed{4x} + \boxed{3})$
7. Binomial products and trinomial expressions:
   a) Coefficients are 4 or 8.
      $(n - 2)(n + 6) = n^2 + 4n - 12$ or $(n + 2)(n + 6) = n^2 + 8n + 12$
   b) Coefficients are −6 or 0.
      $(x - 3)(x - 3) = x^2 - 6x + 9$ or $(x - 3)(x + 3) = x^2 - 9$
   c) Coefficients are −1, 1, −9 or 9.
      $(a - 5)(a + 4) = a^2 - a - 20$ or $(a + 5)(a - 4) = a^2 + a - 20$
      or $(a - 5)(a - 4) = a^2 - 9a + 20$ or $(a + 5)(a + 4) = a^2 + 9a + 20$
   d) Coefficients are 13 or −7.
      $(2b + 3)(b + 5) = 2b^2 + 13b + 15$ or $(2b + 3)(b - 5) = 2b^2 - 7b - 15$

Students may notice the following:

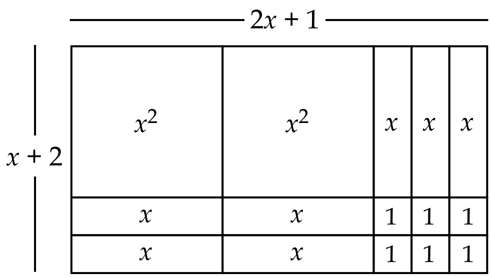
- "In the first three, the coefficients of the middle term are the sum of the constants in the binomials."
- "The variables in the first three have binomial factors with coefficients of 1."

They may wonder:

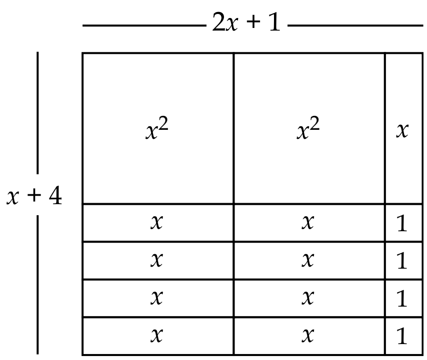
- "Why is the fourth one different?"
- "Is there a pattern that fits the fourth one?"

8. Find two numbers:
   a) −3 and −12
   b) −9 and 2

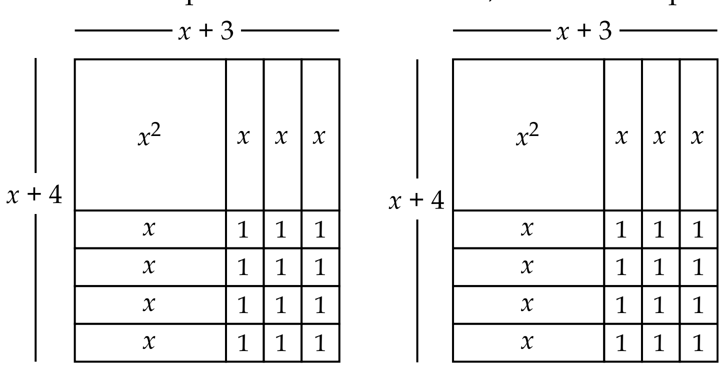
9. Representing the product of binomials:
   a) This model represents $2x^2 + 7x + 6$, which is the product of $(2x + 3)(x + 2)$.



   b) This model represents $2x^2 + 9x + 4$, which is the product of $(2x + 1)(x + 4)$.

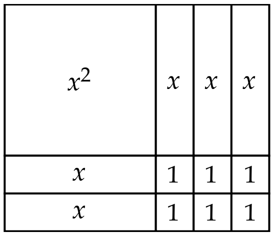


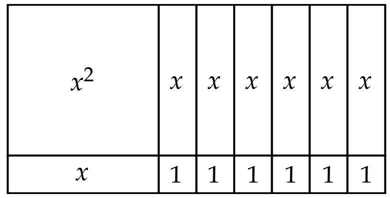
   c) This model represents $2x^2 + 14x + 24$, which is the product of $2(x + 3)(x + 4)$.



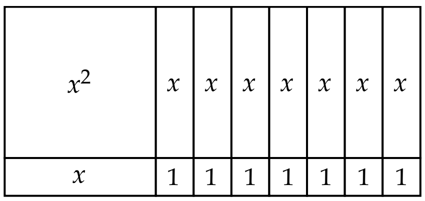
10. Representing the factors of trinomials:

a) There are two ways to use one x^2 tile and 6 unit tiles to create rectangles. They represent the products $(x + 3)(x + 2)$ and $(x + 6)(x + 1)$.

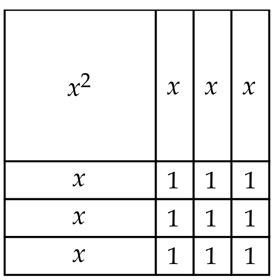


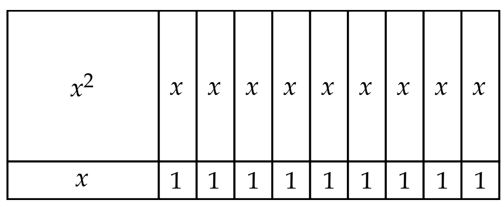


b) There is one way to use one x^2 tile and 7 unit tiles to create a rectangle. It represents the product $(x + 7)(x + 1)$.



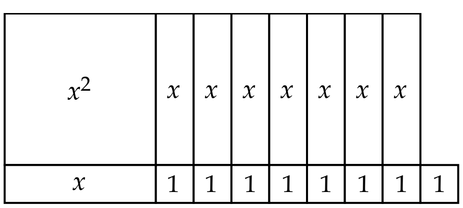
c) There are two ways to use one x^2 tile and 9 unit tiles to create rectangles (one is a square). They represent the products $(x + 3)^2$ and $(x + 9)(x + 1)$.

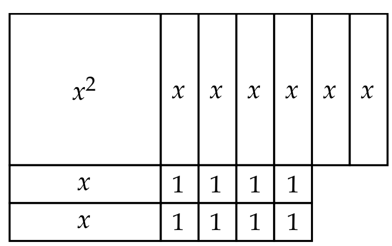




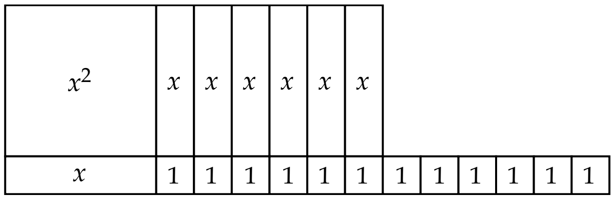
11. Use algebra tiles to show the following:

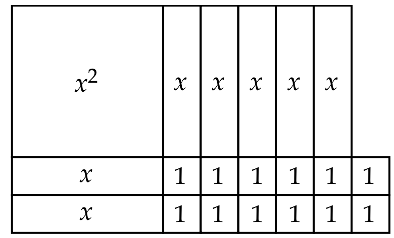
a) The trinomial cannot be written as binomial factors. The 8 unit tiles can only be arranged as rectangles in the two ways shown (1 × 8 or 2 × 4) and the 8 x-tiles cannot be arranged with them to make a rectangle area.

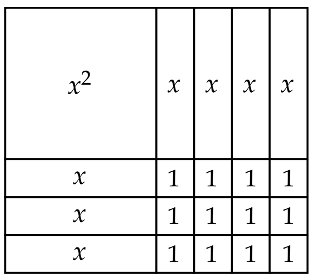




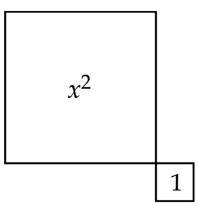
b) The trinomial can be written as binomial factors. The 12 unit tiles can be arranged as rectangles in the three ways shown (1 × 12, 2 × 6, or 3 × 4). The 1 × 12 and 2 × 6 arrangements do not make a rectangle with the 7 x-tiles, but the 3 × 4 arrangement of unit tiles does. The trinomial can be written as $(x + 4)(x + 3)$.







c) The expression cannot be written as binomial factors. The 1 unit tile can only be arranged in the one way shown (1 × 1). There are no x-tiles to make a rectangle area (2 x-tiles are needed).



[TN] You may want to talk to students about what the rectangular arrangements show. For each of the examples above:

a) The representation shows that the trinomial $x^2 + 8x + 8$ cannot be factored but could be written as $(x + 7)(x + 1) + 1$ or as $(x + 4)(x + 2) + 2x$.

b) The representation shows that the trinomial $x^2 + 7x + 12$ could be written as $(x + 6)(x + 1) + 6$, as $(x + 5)(x + 2) + 2$, or as $(x + 4)(x + 3)$, which is a factored form.

c) The representation shows that the expression cannot be written in any other form.

R–1

Solidifying Prior Learning Solutions

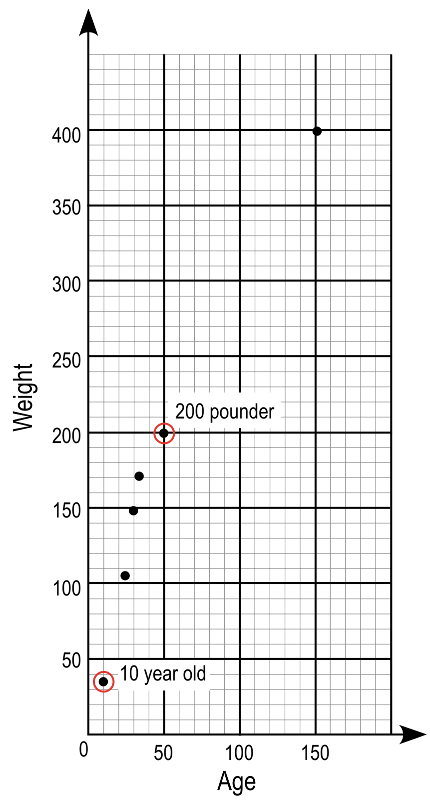
1. P + Q + R = 10 (P = 4, Q = 5, R = 1)

   Some possible reasoning:

   P + Q must have a ones digit of 9 (ones column). Q + Q must have a ones digit of zero. However, Q cannot be zero because column one requires Q + P = 9, so Q must be 5.

2. Graph and predict:

White Sturgeon Fish	
Age	Weight (pounds)
26	116
30	148
33	172
154	400



a) Age, in the first column, is the independent variable on the x-axis and weight is the dependent variable on the y-axis.

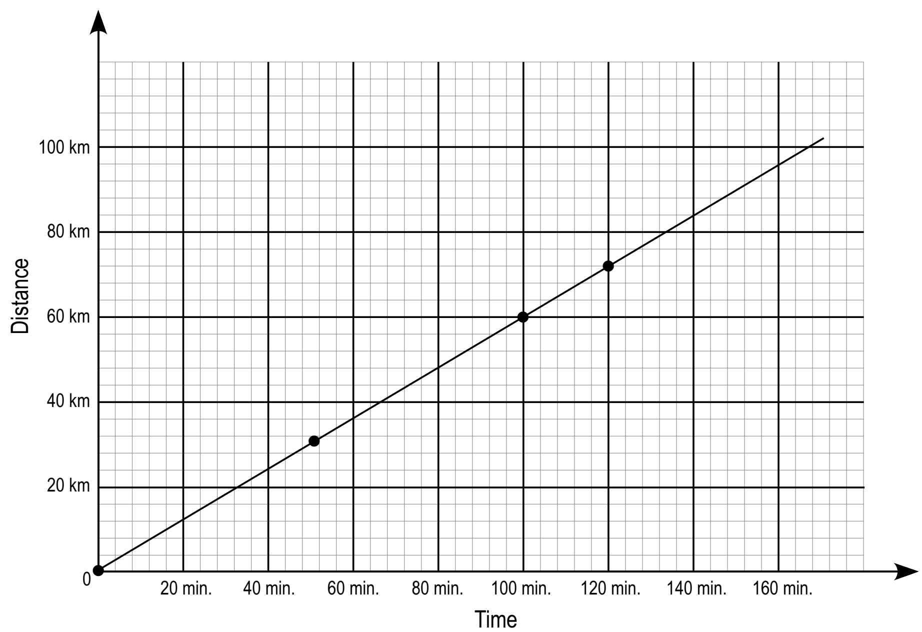
b) According to this data, a 10-year-old sturgeon could be between 35 and 45 pounds.

c) According to this data, a 200-pound sturgeon might be 40 to 50 years old.

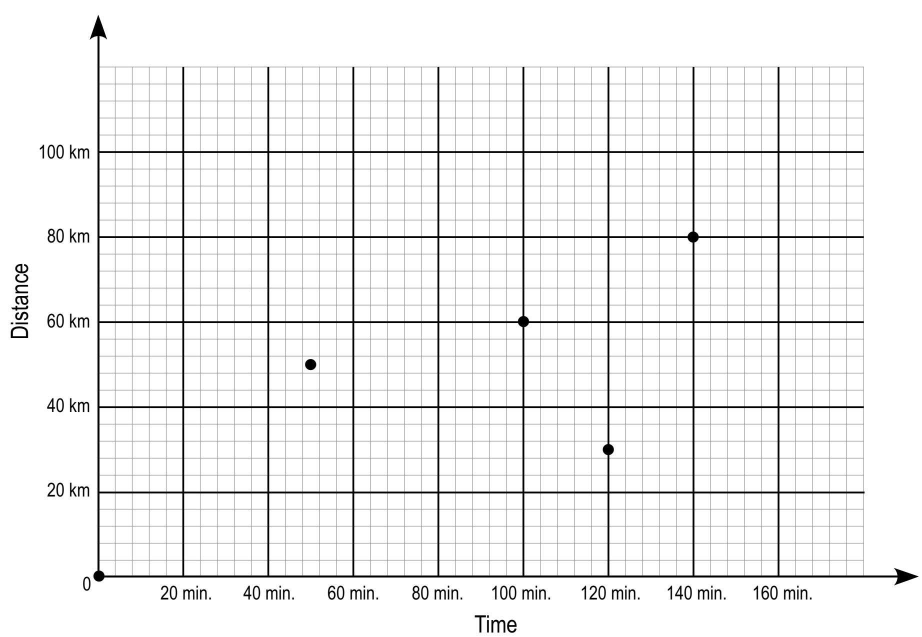
3. Graphs showing a strong relationship will have data points that are all close to a line (or other predictable curve).

   a) Two examples are "distance travelled on a road trip" and "time" or "average temperature in Thompson" and "month."

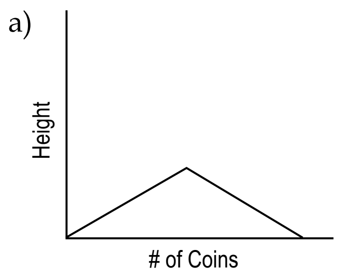
   b) This graph shows a strong relationship between "distance travelled" and "time." The speed was constant.



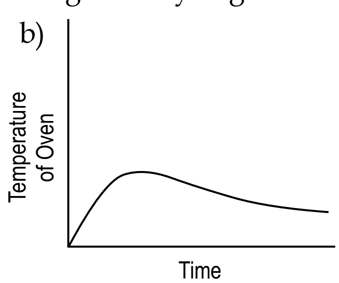
   c) This graph shows a weak relationship between "distance travelled" and "time." The speed and direction of travel was not consistent throughout the trip (even going backwards between 100 and 120 minutes).



4. Some of the possible answers:
   a) C = 12 + 2f
      Possible scenario: Renting skates
      Cost of skate rental (C) = $12 basic fee plus $2 per hour ($f$).
   b) $Y = 25x$
      Possible scenario: Distance travelled on a bike
      (Y) = rate (speed) of 25km/h multiplied by number of hours travelled (x)
   c) $b = 12 - a$
      Possible scenario: Cookies left
      The number of cookies left in the package (b) = the number of cookies originally in the package (12) subtracting the number you have eaten (a).
5. Some of the possible answers:
   a) Stacking and unstacking coins



   b) Graphing the temperature of the oven as it heats up, reaches maximum temperature, and gradually begins to cool



(TN) Other examples of this kind of question can be found by searching the Internet for "graphing stories" initiated by Dan Meyer. Follow this link: www.graphingstories.com.

6. David's reading:

a) This pattern could continue until day 14, when David reads 1 page. According to the pattern, he would read zero pages (or negative pages) after that. The day can be found using the relationship from the table, pages read is 40, 37, 34, etc. The relationship could be written as pages = $40 - 3(d - 1)$.

b) David reads 217 pages in one week.

Day	Total Pages	
1	40	Friday
2	40 + 37	Saturday
3	40 + 37 + 34	Sunday
4	40 + 37 + 34 + 31	Monday
5	40 + 37 + 34 + 31 + 28 = 170	Tuesday
6	40 + 37 + 34 + 31 + 28 + 25	
7	40 + 37 + 34 + 31 + 28 + 25 + 22 = 217	

c) David would begin reading on Friday in order to follow the same pattern and to finish a 150-page book on a Tuesday (reading only 8 pages on the last day). See the table above.

R–2

Solidifying Prior Learning Solutions

1. Some possible answers are:

$\frac{1}{20}$: the only unit fraction (numerator of 1)	$\frac{20}{25}$: the only two-digit numerator
$\frac{2}{3}$: the only repeating decimal	$\frac{5}{4}$: the only value greater than 1

2. The total number of blue buttons is any number from 34 to 39.

   One solution involves writing the ratios for Pile 1 as Red : Blue is $n:2n$ and for Pile 2 as Red : Blue is $3m:5m$ for positive integers n and m. Knowing the total number of red is 20, solve the equation $n + 3m = 20$. There are 6 solutions for (n, m), namely (2, 6), (5, 5), (8, 4), (11, 3), (14, 2), and (17, 1). The number of blue can be found by substituting the 6 possible pairs for n and m into the equation $2n + 5m$.

3. Grace's garden:

   a) Garden shapes

N = 4	N = 5
x x x x x x x • • • • x x • • • • x x • • • • x x • • • • x x x x x x x	x x x x x x x x • • • • • x x • • • • • x x • • • • • x x • • • • • x x x x x x x x

Shape #	# of Flower Plants	# of Tomato Plants	Total Number of Plants
1	8	1	9
2	12	4	16
3	16	9	25
4	20	16	36
5	24	25	49

   b) The number of flowers can be found by adding 4 corner flowers and then N flowers on each of the 4 sides, so $4 + 4N$. The total number of plants is a square, $(N + 2)^2$, which could be written as $N^2 + 4N + 4$. The total number of plants could also be found by adding the N^2 tomatoes in the middle and the $4 + 4N$ flowers around the outside.

(TN) For more opportunities to examine patterns with questions like this, visit www.youcubed.org/tasks (Stanford Graduate School of Education) or www.visualpatterns.org (Nguyen).

4. Some answers are as follows:

   a) "double the input number and then add one"
      "add ten to the input number"
      "twenty less one-ninth of the input number"

   b) $(2x + 1 = y)$
      $(x + 10 = y)$

   c) $4(x+1) \div 2 - 1$ or
      $\left(10\left(\sqrt{x}+1\right)-2\right) \div 2$
      $(x \div 3) \times 7 - 2$

R–3

Solidifying Prior Learning Solutions

1. Some possible answers are as follows:

33%: the only value displayed as a percent	$\frac{1}{3}$: the only unit fraction
$\frac{5}{3}$: the only number greater than 100%	$0.\overline{6}$: the only repeating decimal number

2. Some answers are as follows:

a) $\frac{\boxed{1}}{\boxed{2}} + \frac{\boxed{3}}{\boxed{7}} = \frac{13}{14}, \frac{\boxed{1}}{\boxed{2}} + \frac{\boxed{4}}{\boxed{9}} = \frac{17}{18}, \frac{\boxed{7}}{\boxed{8}} + \frac{\boxed{1}}{\boxed{9}} = \frac{71}{72}$

b) $\frac{\boxed{3}}{\boxed{2}} - \frac{\boxed{5}}{\boxed{8}} = \frac{14}{16}, \frac{\boxed{8}}{\boxed{5}} - \frac{\boxed{2}}{\boxed{3}} = \frac{14}{15}, \frac{\boxed{9}}{\boxed{6}} - \frac{\boxed{4}}{\boxed{7}} = \frac{39}{42}$

c) $\left(\frac{\boxed{4}}{\boxed{1}}\right)\left(\frac{\boxed{2}}{\boxed{9}}\right) = \frac{8}{9}, \left(\frac{\boxed{5}}{\boxed{2}}\right)\left(\frac{\boxed{3}}{\boxed{8}}\right) = \frac{15}{16}$

d) $\frac{\boxed{1}}{\boxed{2}} \div \frac{\boxed{4}}{\boxed{7}} = \frac{7}{8}, \frac{\boxed{4}}{\boxed{3}} \div \frac{\boxed{9}}{\boxed{6}} = \frac{8}{9}, \frac{\boxed{5}}{\boxed{2}} \div \frac{\boxed{8}}{\boxed{3}} = \frac{15}{16}$

(TN) Students could be encouraged to describe any patterns they see and compare results with each other to determine which expressions evaluate to numbers closer to 1. Additionally, other restrictions could be added to make this task easier or more difficult. For example, it may be easier to find expressions that include (rather than exclude) equal to 1. It may be more challenging to insist that a certain digit (such as 4) must be included in each expression.

Variations on this challenge include making the values as large or as small as possible.

3. Products:

a)

Expression	Product
(4)(5)	20
(3)(5)	15
(2)(5)	10
(1)(5)	5
(0)(5)	0
(−1)(5)	−5
(−2)(5)	−10

b)

Expression	Product
(4)(−5)	−20
(3)(−5)	−25
(2)(−5)	−10
(1)(−5)	−5
(0)(−5)	0
(−1)(−5)	5
(−2)(−5)	10

Some things you may notice: the products in (a) decrease but the products in (b) increase. Both tables have a product with a constant difference of 5. Some things you may wonder: How would the pattern change if the first factor began with negative values? What real-world scenario might the products in the tables represent? How can we describe the meaning of a negative multiplied by a negative?

4. Square:
   a) One possible answer is a side is 4 units long going from E to G. The perimeter would be 16 linear units and the area would be 16 square units. A different answer has the points E and G as diagonally opposite vertices, which means each side of the square is $\sqrt{2}$ units long. The perimeter is $4\sqrt{2}$ or 5.656 units and the area is 2 square units.
   b) Squares with vertical and horizontal sides could be drawn by adding points A and C at (2, 5) and (6, 5) OR at (2, −3) and (6, −3). Alternately, points for a square with sides at 45° to the horizontal or vertical have points A and C at (4, 3) and (4, −1).
5. Rectangle—the three possible answers are as follows:
   a) Rectangle with W and Y as diagonally opposite vertices has other vertices at A(−2, 1) and B(2, 4) Area = 12 square units.
   b) Two possible rectangles could be drawn with adjacent vertices at W and Y.
      i) A(−5, 0) and B(-1, −3) Area = 25 square units
      ii) A(1, 8) and B(5, 5) Area = 25 square units
6. Line segment:
   a) Some answers are (−3, −2), (9, 7), or (3, 2.5).

      Each y-value increases by 3 when each x-value increases by 4.
   b) Some answers are (−2, −2), (22, 10), (10, 4) or (8, 3).

      Each y-value increases by 4 when each x-value increases by 8. You could also say that each y-value increases by 1 when each x-value increases by 2.

(TN) *Marbleslides* and other activities from Desmos allow students to explore slope and other characteristics of equations. Find the activities at https://teacher.desmos.com.

R–4 Solidifying Prior Learning Solutions

1. Some answers are as follows:

$\begin{array}{r} 1\ 3\ 8 \\ +\ 6\ 5\ 4 \\ \hline 7\ 9\ 2 \end{array}$	$\begin{array}{r} 6\ 3\ 4 \\ +\ 1\ 5\ 8 \\ \hline 7\ 9\ 2 \end{array}$	$\begin{array}{r} 2\ 4\ 1 \\ +\ 5\ 9\ 6 \\ \hline 8\ 3\ 7 \end{array}$

(TN) There are many solutions. Providing students a hint by filling in one of the rows or columns may help vary the level of challenge.

2. Many answers are possible. Here is one example:

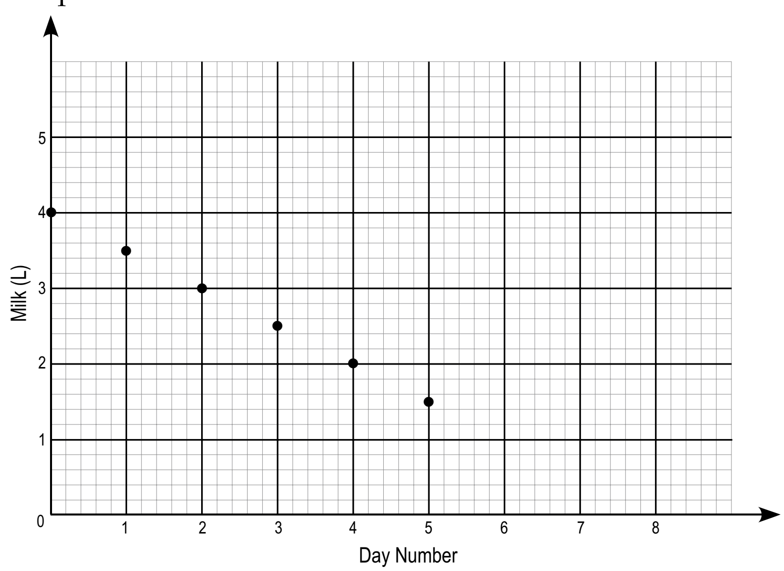
   a) A context could be the number of litres of milk left in a household refrigerator. Day zero is the day the milk was purchased. "Day" represents the number of days after purchase. The "Value" represents the approximate number of litres in the fridge at the beginning of each day.

   b) Table of values:

Day	0	1	2	3	4	5
Litres of Milk	4	3.5	3	2.5	2	1.5

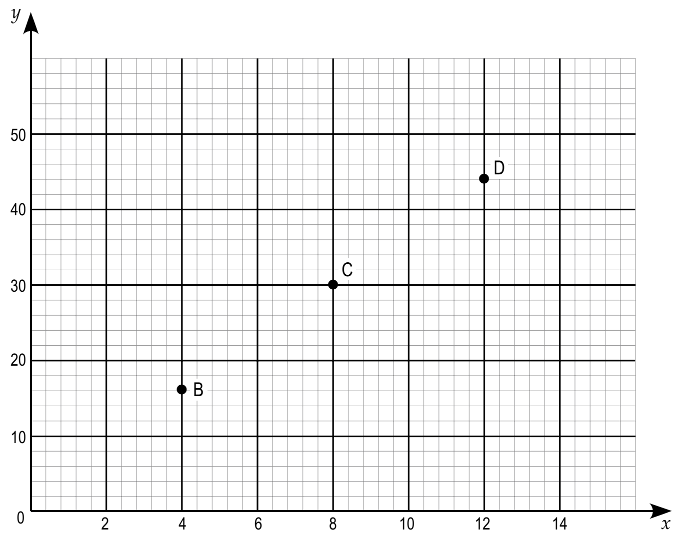
   c) The pattern begins with 4 litres of milk in the fridge and decreases by a constant of 500 mL each day. The pattern will change when more milk is purchased or the milk runs out.

   d) Graph:



3. Some answers are as follows:

a)

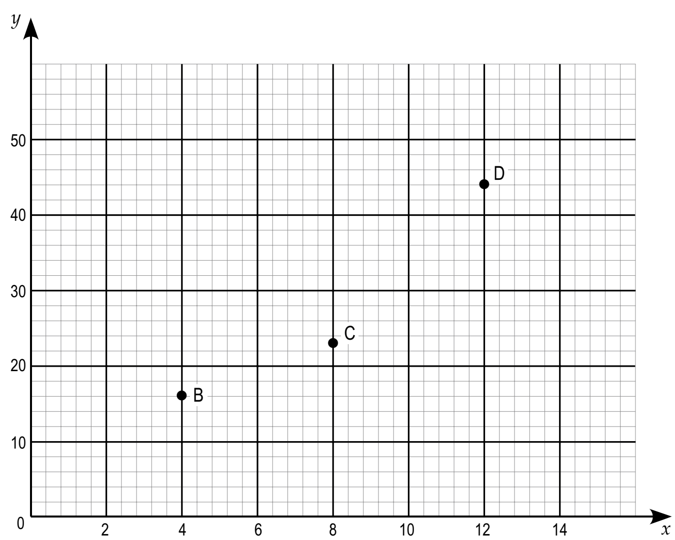


i) (8, 30)

ii) This fits a pattern because the *x*-values go up by 4, (4, 8, 12) and the *y*-values go up by 14 (16, 30, 44).

iii) These values are all on a straight line when graphed, since the *y*-value increases by 14 each time the *x*-value increases by 4.

b)



i) (8, 23)

ii) This fits a pattern because the *x*-values go up by 4 (4, 8, 12) and the *y*-values go up by increasing values, by 7, and then by 21 (16, 23, 44).

iii) These values do not fit on the same line when graphed. The *y*-values increase by different amounts each time, as the *x*-values increase by a constant 4.

4. Pattern:
   a) There are many ways to describe this pattern. Here are a couple of examples. “The pattern has squares of increasing size with an additional top layer of blocks.” Or, “The pattern shows rectangles of increasing size where the height is always one more than the base.”
   b) Figure 4 will have 20 blocks: a 4 × 4 square of blocks with a 1 × 4 row on top.
      Figure 5 will have 30 blocks: a 5 × 5 square of blocks with a 1 × 5 row on top.
   c) Figure 10 will have 110 blocks; following the same pattern, there will be a 10 × 10 square of blocks with a 1 × 10 row on top.
   d) The friend means that a graph showing the figure number versus the number of blocks would not show a set of points that all lie along the same line. The data values to be graphed are shown in the table. The x-values go up by a constant value of 1, but the y-values go up by different amounts each time (by 4 then 6, then 8, then 10), so they will not lie on the same line.

Figure number	1	2	3	4	5
Number of blocks	2	6	12	20	30

5. Pattern:
   a) There are many ways to describe this pattern. Here are a couple of examples. “The pattern shows two rectangular sections of blocks of increasing length sandwiching one block in the middle.” Or, “The pattern shows a vertical pillar of 3 blocks with 4 blocks added each time, 2 at the top (left and right) and 2 at the bottom (left and right).”
   b) Figure 4 will have 19 blocks. There are 15 in figure 3 and there will be 4 more in the next figure. Figure 5 will have 23 blocks. There are 19 in figure 4 and there will be 4 more in the next figure.
   c) Figure 10 will have 43 blocks. Figure 1 has a central pillar of 3 blocks with 4 added to the top (L&R) and bottom (L&R). Figure 2 has a central pillar of 3 blocks with 2 sets of 4 added. Figure 3 has a central pillar of 3 with 3 sets of 4 added. So, figure 10 will have a central pillar of 3 blocks with 10 sets of 4 added for a total of 43.
   d) The figure closest to 1000 blocks is figure 249. There will be a central pillar of 3 blocks and 249 sets of 4 for a total of 999 blocks.
   e) The friend means that a graph showing the figure number versus the number of blocks would show a set of points that all lie along the same line. The data values to be graphed are shown in the table. The x-values go up by a constant value (of 1) and the y-values also go up a constant value (of 4) each time, so the data points will lie on the same line.

Figure number	1	2	3	4	5
Number of blocks	7	11	15	19	23

6. 34%

   10% of the boys is 3, so 30% is 9.

   10% of the girls is 2, so 40% is 8.

   There are 17 students (9 boys + 8 girls) out of a total of 50 who received certificates.

   $$\frac{17}{50} = \frac{17 \times 2}{50 \times 2} = \frac{34}{100}$$

   Therefore, 34%.

7. Mental calculations:

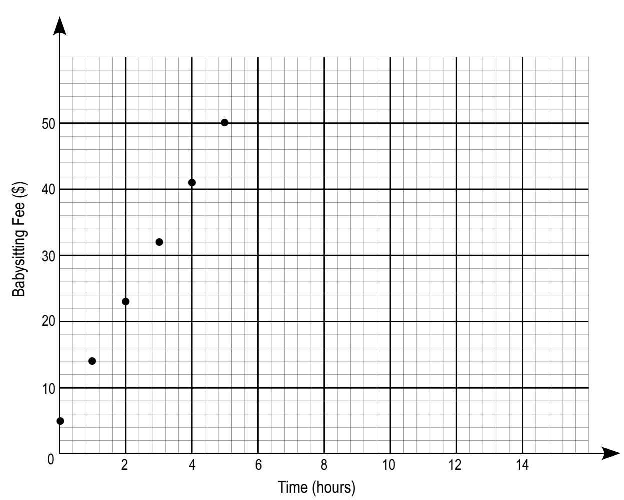
   a) x^2 has the smallest value when x is between 0 and 1. The table shows the values using benchmarks of 0, $\frac{1}{2}$, and 1. The smallest value is x^2 when $x = \frac{1}{2}$.

Expression	x is close to 0	$x = \frac{1}{2}$	$x = 1$
x	~0	$\frac{1}{2}$	1
x^2	~0	$\frac{1}{4}$	1
$\frac{1}{2}$	very large	2	1
$2x$	~0	1	2
$\sqrt{x}$	~0	~0.7 since (0.7)(0.7) = 0.49	1

b) $\frac{1}{x}$ has the smallest value when x is between 1 and 4. The table shows the values using benchmarks of 1, $1\frac{1}{2}$, 2, and 4. The smallest value is $\frac{1}{x}$ when $x = 4$.

Expression	$x = 1$	$x = 1\frac{1}{2} = \frac{3}{2}$	$x = 2$	$x = 4$
x	1	$1\frac{1}{2}$	2	4
x^2	1	$\frac{9}{4} = 2\frac{1}{4}$	4	16
$\frac{1}{x}$	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{4}$
$2x$	2	3	4	8
$\sqrt{x}$	1	~1.2 since (1.2)(1.2) = 1.44	~1.4 since (1.4)(1.4) = 1.96	2

8. Babysitting.
   a) The graph shows the total babysitting fee with a rate of $9 per hour.
   b) It is a linear relationship because the points all lie along the same line.
   c) No negative numbers will be included because you cannot earn a negative amount of money or work a negative number of hours.
   d) All the points will be shifted up 20 units. The graph will start at 0 hours and $25, the next points will be at 1 hour and $34, and then 2 hours and $43, etc.



R–5

Solidifying Prior Learning Solutions

1. 1.011 is closest to 1. Compare after converting all answers to thousandths. The numbers in the same order are $\frac{1100}{1000}, \frac{1110}{1000}, \frac{1101}{1000}, \frac{1111}{1000}, \frac{1011}{1000}$.

2. The car is travelling a little faster. One way to know is to determine how far both vehicles travel in a fixed time.
   - The car travels 60 km in 60 min. That means the car travels 1 km in 1 minute or 1 km in 60 seconds. That is the same as travelling 1 km in 60 seconds.
   - The motorcycle travels 960 m in 60 seconds or 0.960 km in 60 seconds.

3. Some possible answers are as follows:

Similarities	Differences
Both graphs have two lines that intersect.	Graph A lines cross on the *x*-axis (negative value). Graph B lines cross on the *y*-axis (positive value).
Both graphs show crossing at right angles.	Graph A, the decreasing line goes into quadrant I. Graph B, the decreasing line is not in quadrant I.
Both graphs have one line that increases and one that decreases from left to right.	

4. One possible answer is as follows:

   As a graph of distance (*y*-axis) versus time (*x*-axis), this graph could tell the story of a cat and a stationary ball. At the start (AB), the cat remains at a fixed distance from the ball for a certain time. Then (BC) the cat rapidly moves further away from the ball, and then (CD) it remains stationary for a short time before (DE) slowly crawling back to the ball and stopping (EF) at a distance slightly further away than it was originally.

5. Possible answers include the following:

   a) Hat and t-shirt combinations

Hats ($15)	T-Shirts ($10)	Total (max. $300)
20	0	15(20) + 10(0) = 300
18	3	15(18) + 10(3) = 300
16	6	15(16) + 10(6) = 300
14	9	15(14) + 10(9) = 300
…	…	…
0	30	15(0) + 10(30) = 300

b) 0 is the smallest number of hats they could buy.

20 is the largest number of hats they could buy.

c) The team can purchase 20 hats for $300 and, for every 2 hats less, they can purchase 3 shirts. They should purchase an even number of hats and a number of T-shirts that is divisible by 3.

6. Tables of values are shown for the equations. Many similarities and differences may be noticed.

a) N

x	$-3x + 5$
–2	11
–1	8
0	5
1	2
2	–1

b) N

x	$-x + 5$
–2	7
–1	6
0	5
1	4
2	3

c) N

x	$x + 5$
–2	3
–1	4
0	5
1	6
2	7

d) N

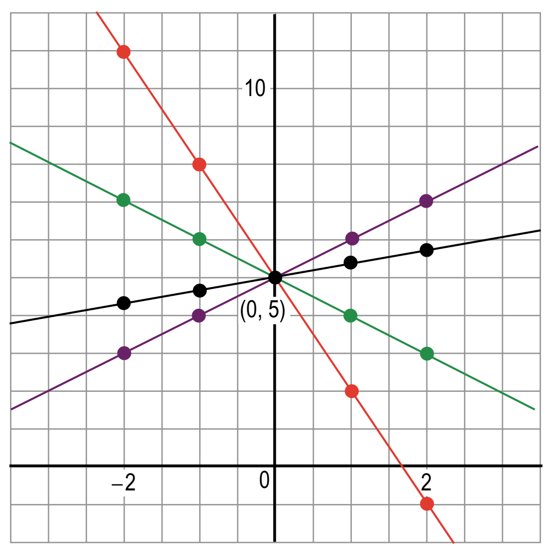
x	$\frac{1}{3}x + 5$
–2	4.3333333
–1	4.6666667
0	5
1	5.3333333
2	5.6666667

Some examples follow:

Similarities in the tables are that the chosen x-values are all the same. Also, each table of values includes the point (0, 5). Each equation has a constant of 5 added to the x-term.

Differences in the tables are the y-values. Equation (d) has y-values that are rational numbers represented by repeating decimals. The y-values in equations a) and (b) are decreasing and the y-values in equations c) and d) are increasing. The equations for (a) and (b) have negative coefficients for the x-terms, unlike equations (c) and (d).

The graphs of the equations
(a screenshot from desmos.com):



(TN) *Activity Builders* from Desmos can be used to further explore algebraic ideas on graphs. The link here is for a "Marbleslides" activity to explore transformations of linear equations: https://teacher.desmos.com/activitybuilder/custom/566b31734e38e1e21a10aac8.

R–6 Solidifying Prior Learning Solutions

1. Possible expressions:

   a) $n + n^2$

   b) $x - 10$

   c) $2(a + 1)$

   d) $2n + 1$

   e) $\frac{12 - y}{4}$

   f) $3(n + 2) + 1$

2. Missing number is −1.

3. Many answers are possible. Here are examples:

   a) $3(\square + 1) - 5 = -4 \times 7 + 11; \square = -5$

   $14 + 3(2n + 10) = -2 + 4; n = -7$

   b) $\frac{1}{2}(\square + 3) + 5 = \frac{1}{3}(14 - 2); \square = -5$

   $5 - \frac{2}{5}(x - 3) = \frac{3}{2}(-7 + 13); x = -7$

4. a) $0 = 8x - 2$ or $2 - 8x = 0$

   b) The equations have the same terms but may be written in a different order. The signs may be opposite, but they should have one negative term and the other positive.

5. Many answers are possible. Here are examples:

a) Original Equation	b) Rearranged Equation	Solution
$3x - 5 = 4x + 6$	$0 = x + 11$	$x = -11$ for both
$7x + 6 = -3x + 21$	$10k - 15 = 0$	$k = 1.5$ for both
$2(3n + 7) + 1 = 4(n + 5) - 6$	$2n + 1 = 0$	$n = -0.5$ for both

6. Many answers are possible. Here is one example:

"Start at __3__ and add __2__ for each new term."

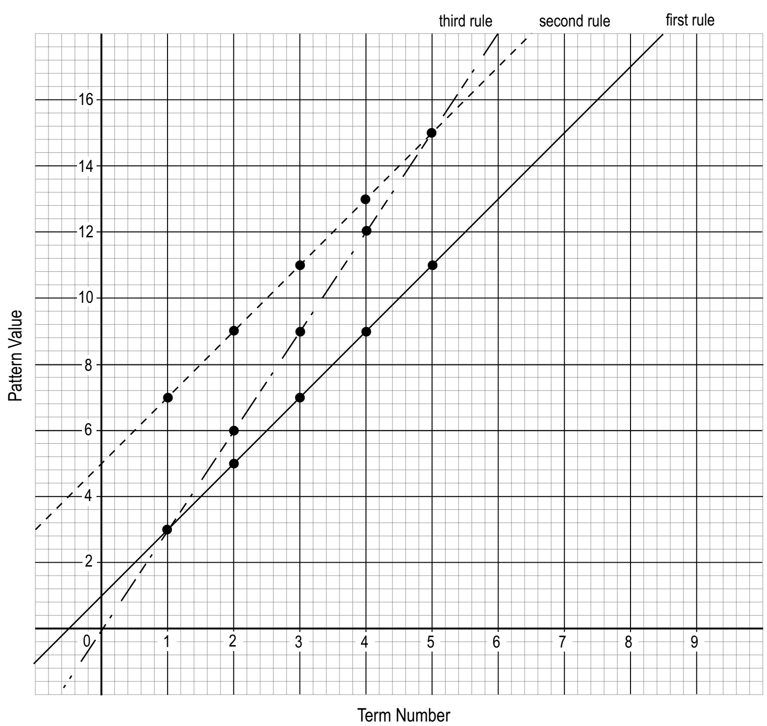
	Start at	Add
1st rule	3	2
2nd rule	7	2
3rd rule	3	2

Term 1	Term 2	Term 3	Term 4	Term 5
3	5	7	9	11
7	9	11	13	15
3	6	9	12	15

(TN) Students may notice that the pattern values of the second rule can be obtained by adding a constant (in this case, 4) to the pattern values of the first rule.

They may say that the pattern values increase at the same rate. They may notice the pattern values increase at a faster (or different) rate in the third rule than in the first rule or second rule.

They may notice the graph of the line going through the points of the first rule is parallel to the line going through the points of the second rule (the line for the third rule is not parallel).



7. The triangle and circle could both be equal to zero. Among other possibilities, the triangle could be 2 and the circle −2. Balance (equality) can be maintained by removing 2 triangles and 1 circle from both sides, leaving nothing (zero) on the left and a triangle and circle on the right. In general, to have a result of zero, the triangle and the circle must both have the same numerical value, with one being positive and the other negative.

8. Many answers are possible, but they just need a good explanation. Here are some examples. Each graph doesn't belong because

   - the first graph is the only one that crosses the x-axis at a negative x-value
   - the second graph is the only one that passes through the origin (0, 0)
   - the third graph is the only one that crosses the y-axis at a negative y-value
   - the fourth graph is the only one that goes down from left to right

9. Joti's savings:

   a) Show two of the following models:

   Words: Joti initially has $50; after week 1, she has $65; after week 2, she has $80. Continue this pattern of growth of savings each week.

   Graph:

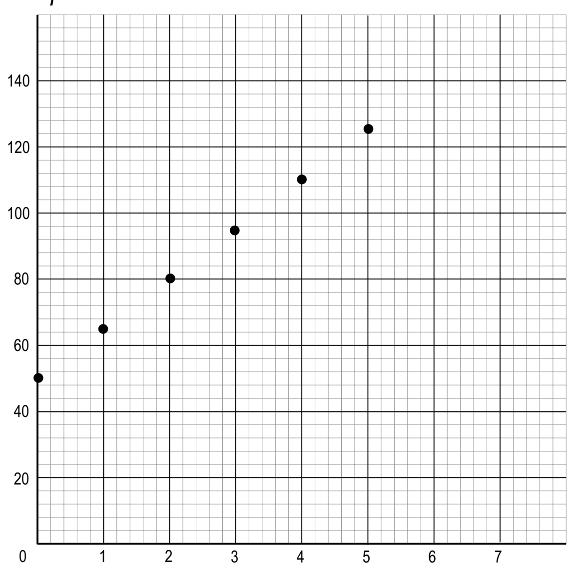


   Table:

Week	Savings
0	50
1	65
2	80
3	95
4	110
5	125

   Equation: $S = 50 + 15n$, where S is number of dollars saved, n is number of weeks

   b) Between 5 and 16 weeks, depending on the ticket price. It will take at least 5 weeks to save $120 and at least 16 weeks to save $280.

   c) Between 10 and 26 weeks, depending on the ticket price. It will take 10 weeks to save $120 and 26 weeks to save $280 at the new rate.

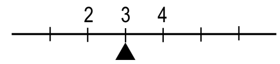
   Describe changes for the two models used:

   - Words and table have updated numbers.
   - The graph starts lower on the y-axis and goes up more slowly each week.
   - The equation changes to $S = 20 + 10n$.

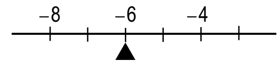
R–7 Solidifying Prior Learning Solutions

1. Mean calculations and number line representations:

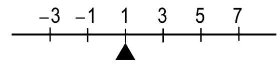
   a) $(2 + 3 + 4) \div 3 = 3$

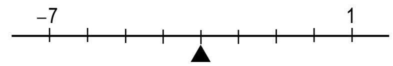
   b) $(-8 - 6 - 4) \div 3 = -18 \div 3 = -6$

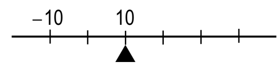
   c) $(-3 - 1 + 1 + 3 + 5 + 7) \div 6 = 12 \div 6 = 2$

   d) $(-7 + 1) \div 2 = -3$

   e) $(-10 + 10 + 30) \div 3 = 30 \div 3 = 10$

   Notice: All sets of numbers are in increasing order. All the sets of numbers go up by consistent intervals (1s, 2s, 8s, or 20s). If there is an odd set of numbers, the mean is one of the numbers. If there is an even set of numbers, the mean is in the middle between two of the numbers.

2. Sums are as follows:

   a) 25

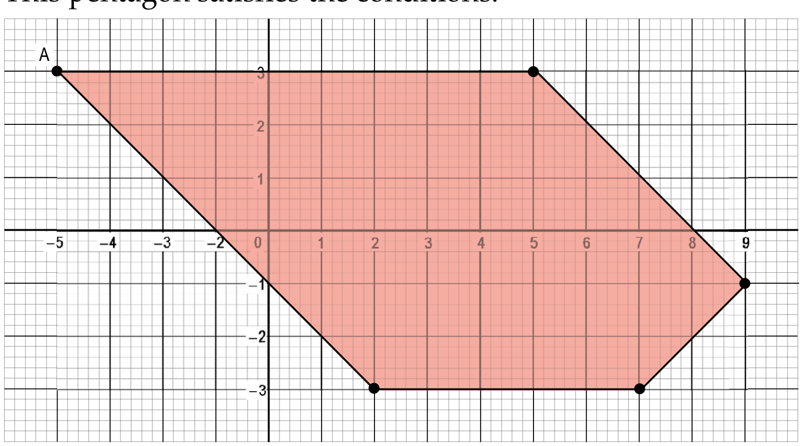
   b) 100

   c) 169

   All the expressions are the sum of perfect squares and the sum is also a perfect square. Many other expressions are possible (collectively called Pythagorean triples). Some examples are 81 + 144 (= 15^2) or 1 + 0 (= 1^2) or 64 + 225 (= 17^2).

3. Many answers are possible. Here is a sample:

   a) This pentagon satisfies the conditions.

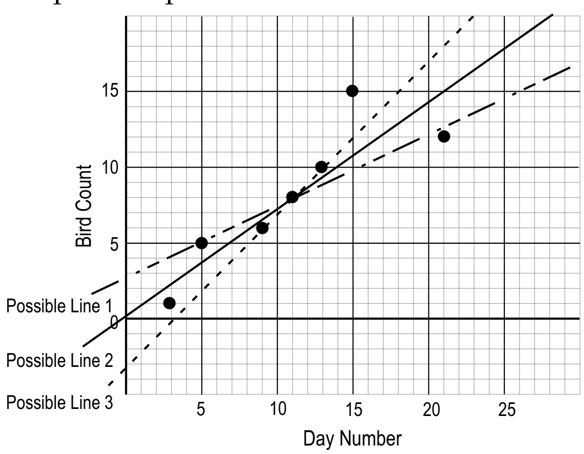
   

b) Sample start of a description: "To draw the pentagon, start at (−4, 3) move horizontally 9 units right to next vertex. Then turn 45° clockwise and draw a line down and to the right, moving a horizontal distance of four units and a vertical distance of four units to the next vertex.…"

c) Based on the result, consider how communication could be improved.

4. Spring Bird Count versus Number of Days:

a) Graph the 7 points.



b) Several different lines are possible. Three are shown.

c) The three example lines are all increasing (going up L to R). They all go through a point near the middle (11, 7) or (11, 8). They are not parallel. They go up at different angles. They go through different numbers of points.

i) Justification for Line 1: It is close to three points with a pair of points above and a pair below it.

ii) Justification for Line 2: It is through one point with other pairs of points that are the same distance away from the line on either side of the line.

iii) Justification for Line 3: It passes through 3 points [the down side is that there are 3 other points above the line with only 1 below the line.

(TN) You might want to have some students explore the plotting of a "median-median line." It is a way of determining the linear trend of a scatterplot by finding the medians of three groups of data points and drawing a fit line using the three resulting median points (without any calculation). In the example above, "possible Line 2" is the fit line that would be the result of using the median-median fit line process.

5. The equation could be as follows:

a) i) Δ = 14 g

ii) Δ = 10.5 g

b) i) Take off two marbles on each side, leaving 3 triangles on the left and 6 marbles on the right. That means each triangle must be the same as two marbles or 14 g.

ii) Take off one triangle and two marbles from each side, leaving 2 triangles on the left and 3 marbles on the right. That means each triangle must be the same as $1\frac{1}{2}$ marbles or $10\frac{1}{2}$ g.

c) i) Solve $3n + 2 = 8$, where n is the number of marbles equivalent to a Δ. Or, solve , where n is the number of grams equivalent to a Δ.

ii) Solve the equation: $3t + 2 = t + 5$, where t is the number of marbles equivalent to a Δ. Or solve $3t + 14 = t + 35$, where t is the number of grams equivalent to a Δ.

d) The algebraic steps could be similar. For example, when using the first equation, $3n + 2 = 8$, first subtract 2 on the left and right (which is the same as taking two marbles off each side). Then divide both sides by 3 (same as proportional reasoning that 3 triangles balances 6 marbles, so 1 triangle balances 2 marbles). The result is $n = 2$ marbles with a total weight of 14 g.

6. Distributive property:

a) The distributive property could be described as, "The result of multiplying two parts of a sum separately and then adding their products is the same result as adding the two parts first and then finding the product." Algebraically, $a(b + c) = ab + ac$.

Using an area model,

	b	*c*
a	*ab*	*ac*

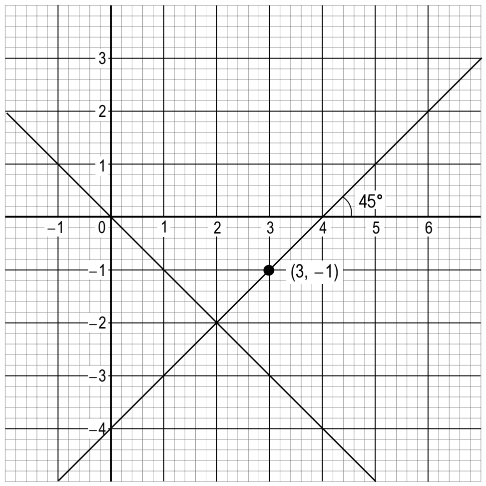
b) Note: Other equivalent answers are possible for each of the following:

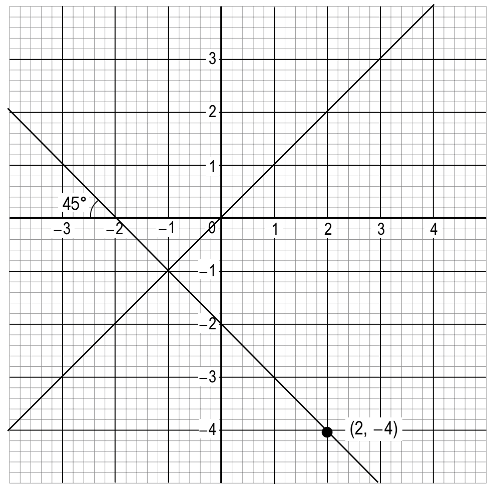
i) 7×14 is the same as $(7)(10 + 4)$ or $(7)(11 + 3)$ or...

ii) $(6)(5x + 2)$ is the same as $30x + 12$

iii) $120y - 84$ is the same as $(6)(20y - 14)$ or $(12)(10y - 7)$ or...

7. Many answers are possible. Here are two possibilities starting at (3, −1) and (2, −4), respectively.





(TN) You may want to have students compare their graphs with each other. Given the criteria, there are only two possibilities for the second line, regardless of the starting point. They are illustrated above (through the origin sloping down at 45° or up at 45°).

R–8

Solidifying Prior Learning Solutions

1. Many answers are possible. Here are two examples for (a) 12×16:
   i) Decompose 16 into $10 + 6$, multiply 12×10, and then add 120 and 72 to get 192. Alternatively, decompose 16 into $20 - 4$, multiply 12×20, and 12×4, then subtract 240 and 48 to get 192.
   ii) Decompose 12 into $10 + 2$, multiply 16×10 and 16×2, then add 160 and 32 to get 192.

   (TN) You may want to do number talks as a regular routine with your students to help them develop their mental math ability and number sense. Number talks can be done with different operations and number systems. To get an idea of the process, search Youtube for "Jo Boaler Number Talks 18×5."

2. Use 2 cups flour and 4 teaspoons baking powder. One-third of 3 teaspoons is 1 teaspoon, so he needs a total of 4 teaspoons of baking powder. Similarly, $1\frac{1}{2}$ can be represented as $\frac{3}{2}$ and one-third of 3 halves is 1 half, so he needs a total of 4 halves—that is, adding $\frac{1}{2}$ cup of flour to the recipe is a total of 2 cups of flour.

3. Other units may be chosen:
   - Notations for length include centimetre, cm; inch, in., ".
   - Notations for area include square centimetre, cm^2; square inch, sq. in., $in.^2$.
   - Notations for volume include cubic centimetre, cm^3, c.c.; cubic inch, cu. in., $in.^3$.

4. The missing values (two outputs and one input), in order vertically, are 17, 56, and 32. The function machine, g, is multiplying by 3 and then adding 2.

   (TN) Teachers may want to guide students in the determination of an output rule. If the input values go up by a constant value, it is informative to find the differences between output values. In this case, the input values go up by 1 and the differences between output values is consistently 3 ($8 - 5 = 3$ and $11 - 8 = 3$). This means that for an increase of 1 in input, the output goes up 3 times. The function involves multiplying the input by 3 and then adding a constant value. In this example, the constant added each time is 2.

5. Machine, T:
   a) This may be written in a variety of ways using one or more variables. For example, Machine T, for input c, $9\left(\frac{c}{5}\right) + 32 \rightarrow$ output.
   b) 104°F
   c) −40°F
   d) $176.\overline{6}$°C. Work in reverse starting with output of 350°F, $(350 - 32) \div 9 \times 5$.

R–9 Solidifying Prior Learning Solutions

1. Some answers include 9 and 14, 12 and 17, or 13 and 18.

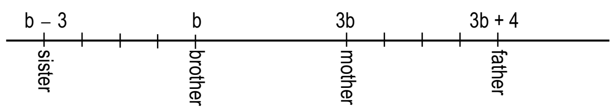
2. Exchange rate:

   a) They are reciprocals of each other. $1\frac{1}{2}$ is the same as $\frac{3}{2}$, which is the reciprocal of $\frac{2}{3}$.

   Using ratios, CAD : USD is 3 : 2, and the ratio of USD : CAD is 2 : 3.

   b) $\frac{4}{5}$ of a U.S. dollar. Since $1\frac{1}{4}$ is the same as $\frac{5}{4}$ and the reciprocal is $\frac{4}{5}$.

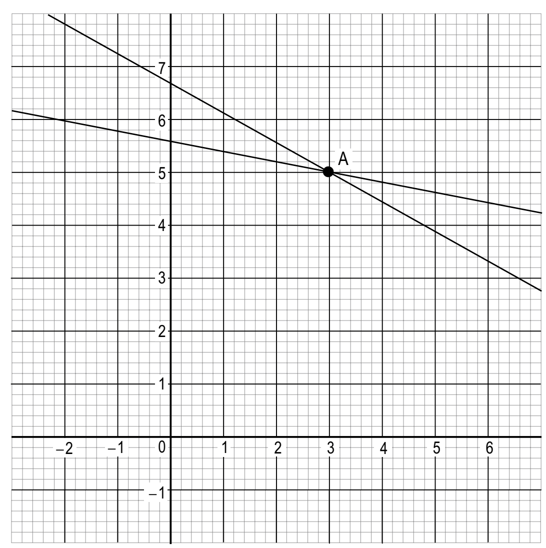
3. Bonnie's family:



   This is enough information. Create and solve the linear equation with the sum of all four terms above set equal to 89.

4. Company B is better for a smaller track team. The recommendation depends on how many people are on the track team. If there are fewer than 30 people, recommend Company B because it will be cheaper; if there are more than 30 people, recommend Company A because it will be cheaper.

5. Answers will vary depending on the lines drawn. Here are responses based on the graphs of the lines shown:



   The two lines have the point (3, 5) in common and no other point is on both lines. The slopes of both of the lines are decreasing (from left to right). The *x*-intercepts of both graphs are greater than 10. The *y*-intercept of one line is close to 7 and the *y*-intercept of the other is close to 6.

6. Answers can vary significantly. Here is an example why each one does not belong:
   - The first graph shows two lines with one point in common in the first quadrant.
   - The second graph shows no point on either line with both negative *x*- and *y*-values.
   - The third graph shows two parallel lines.
   - The fourth graph shows two lines with the same *x*-intercept.

R–10

Solidifying Prior Learning Solutions

1. Many answers are possible. Here is one example:

   $\frac{9}{2} > \frac{14}{7} = \frac{6}{3} > \frac{8}{5}$

2. Many answers are possible. Here are two examples: A and B have opposite signs; on a number line, the distance of the negative number from 0 is one more than the distance of the positive number from zero.

3. Answers vary according to "your" age. In this sample solution, your age is 16 and my age is 35, so then half my age is $17\frac{1}{2}$. If your age is 16, then half your age is 8. Someone who is halfway between 16 and 35 is $25\frac{1}{2}$.

   (TN) Students may be encouraged to notice that the sum of half the two ages $\left(17\frac{1}{2} + 8\right)$ is the same as halfway between the two ages. They could be asked to justify algebraically that relationship works for any two ages.

4. Answers vary according to "your" age. In this sample solution, your age is 16 so Ms. Carlyle must be 68 years old. You are 26 years younger and Ms. Carlyle is 26 years older than Mr. Bowe.

5. Answers vary according to "your" age. In this sample solution, your age is 16 so Sara is 2. Liam is 7 years younger than your age of 16, so Sara is 7 years younger than Liam.

6. Squares:

   a) The square side lengths are 6 units. Plot a point 3 units to the right of A and another 3 units to the right of D, and then draw a vertical line through the points. Plot a point 3 units below A and another 3 units below B, and then draw a horizontal line through the points.

   b) From point E to F is 4 units right and 1 unit down. Plot a point 2 units to the right of E and $\frac{1}{2}$ unit down, then plot another point 2 units to the right of H and $\frac{1}{2}$ unit down, and then draw a line through the points. Plot a point 2 units below E and $\frac{1}{2}$ unit left, then plot another point 2 units below F and $\frac{1}{2}$ unit left, and then draw a line through the points.

   There is no other place to draw the lines to make 4 squares.

7. WODB:

   a) Many different answers are possible, but they just need a good explanation. Here are some examples describing why each one does not belong:

      "First number line shows a first step going from zero to −5 (the others go to −3)."

      "Second number line results in a positive number after both steps."

      "Third number line shows both steps going left (that is, both negative)."

      "Fourth number line shows a result that is only 1 unit away from zero."

   b) The expressions represented by the number line are as follows:

      - first number line is −6 + 4
      - second number line is −3 + 5
      - third number line is −3 − 5
      - fourth number line is −3 + 2

8. Decompose 280 into friendly multiples of 8, such as 240 and 40. Using an area model, the answer is 35.

	30 +	5
8	240	40

   Other multiples of 280 are possible, such as:

   (160 + 80 + 40) ÷ 8
   = 20 + 10 + 5
   = 35

9. a) The median is 5. $\frac{1+9+3+7+5}{5} = 5$ (mean)

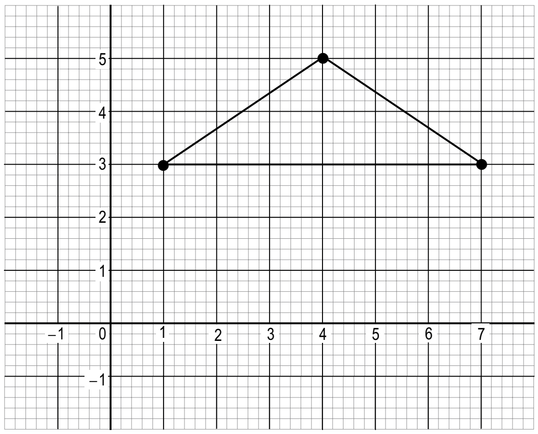
   b) Many answers exist.

10. Isosceles triangle:

a) One possible answer is (4, 5). Other possibilities include (4, 8) or (4, −1). In general, the vertex may be at any location with an *x*-coordinate of 4 as long as the y-coordinate is not 3.

b) Graph of triangle with third vertex at (4, 5).

c) The triangle shown has an area of 6 units^2 and a perimeter of $6 + 2\sqrt{13} = 13.211$ units.



11. Triangle ABC:

a) Plot ΔABC.

b) AB is 7 units.

AC is $\sqrt{45} = 6.708$

BC is $\sqrt{136} = 11.662$

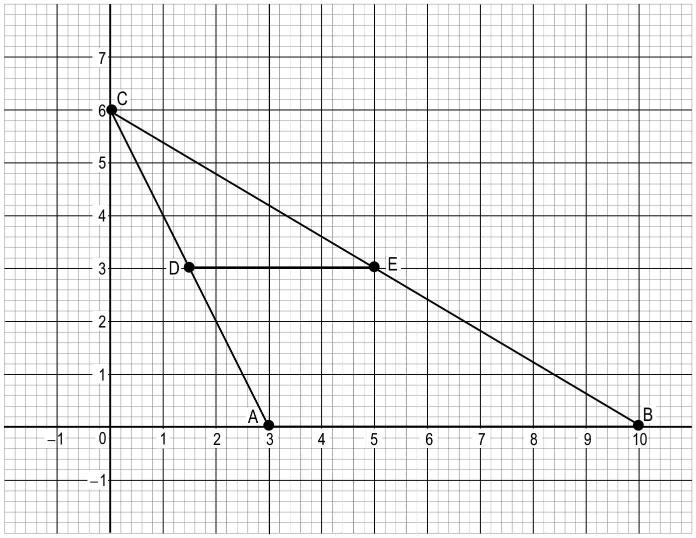
c) CD is $\sqrt{11.25} = 3.354$

$\frac{1}{2}\text{AC} = 0.5 \times 6.708 = 3.354$

d) Plot parallel line, DE.

e) A wide variety of answers are possible; here are some examples. Notice how the length of DE is 3.5 units or the length of DE is half of AB.

Wonder: "Is EB half of CB?"



(TN) Some students may notice (or you may want to point out) that ΔABC ~ ΔDEC because the parallel lines create congruent corresponding angles. Use this question to have students revisit the idea of similar triangles. Since the triangles are similar and CD is half of AC, then it is true that the other corresponding sides are in the same proportion $\left(\text{i.e., DE} = \frac{1}{2}\text{AB and CE} = \frac{1}{2}\text{CB}\right)$.

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