Grade 11 Applied Mathematics (30S)

Grade 11 Applied Mathematics (30S) is intended for students considering post-secondary studies who do not require a study of theoretical calculus. It is context-driven and promotes the learning of numerical and geometrical problem-solving techniques as they relate to the world around us. It builds upon the foundation knowledge and skills from Grade 10 Introduction to Applied and Pre-Calculus Mathematics and builds a foundation for Grade 12 Applied Mathematics.

Primary goals of Applied Mathematics are to have students develop critical-thinking skills through problem solving and model real-world situations mathematically to make predictions.

These goals may be attained in a number of ways. Students may collect data in experiments and activities and then develop mathematical concepts by analyzing that data. They are encouraged to learn and demonstrate effective communication skills through a variety of media. Students are expected to become proficient in both oral and written communication skills.

Applied Mathematics is designed to promote student flexibility and responsibility. Flexibility is encouraged by having students work on non-routine problems and projects. Responsibility is encouraged as students work individually and in cooperative groups to explore connections with other mathematical areas, school subjects, and real-life applications.

Technology is an integral part of both learning and assessment in Applied Mathematics. Graphing calculators, spreadsheets, or other computer software will be used by students for mathematical explorations, modelling, and problem solving.

Assessment of Grade 11 Applied Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in Grade 11 Applied Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The Grade 11 Applied Mathematics course includes the following topics: measurement, geometry, logical reasoning, statistics, and relations and functions. Additionally, students will complete a Mathematics Research Project.

Learning outcomes from various topic areas may be combined in any order when considering learning activities. Two possible teaching sequences and suggested hours are presented in the following tables. The time suggested includes time for instructional and assessment time.

Possibility 1			Possibility 2		
Unit	Learning Outcomes	Suggested Hours	Unit	Learning Outcomes	Suggested Hours
Quadratic Functions	R2, M1, L2	20	Problem Solving	M1, L2	5
Scale	M2, M3, M1, L2	15	Quadratic Functions	R2	15
Proofs	G1, G2, L1, L2	10	Research Project	RP1	10
Statistics	S1, S2, L2	20	Proofs	G1, G2, L1, L2	10
Research Project	RP1, L2	10	Statistics	S1, S2, L2	20
Systems of Inequalities	R1, M1, L2	20	Systems of Inequalities	R1, L2	20
Trigonometry	G3, G2, L1, L2	15	Trigonometry	G3, L1, L2	15
			Scale	M2, M3, L2	15
	Total	**110**		**Total**	**110**

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 11 Applied Mathematics

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Measurement		**General Learning Outcome:** Develop spatial sense and proportional reasoning.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11A.M.1.	Solve problems that involve the application of rates. [CN, PS, R, T]	■ Interpret rates in a context, such as the arts, commerce, the environment, medicine, or recreation. ■ Solve a rate problem that requires the isolation of a variable. ■ Determine and compare rates and unit rates. ■ Make and justify a decision, using rates. ■ Interpret a graph that represents a rate. ■ Draw a graph to represent a rate. ■ Explain, using examples, the relationship between the slope of a graph and a rate. ■ Describe a context for a rate or unit rate. ■ Identify and explain factors that influence a rate in a context. ■ Solve a contextual problem that involves rates or unit rates.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Measurement *(continued)*		**General Learning Outcome:** Develop spatial sense and proportional reasoning.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11A.M.2.	Solve problems that involve scale diagrams, using proportional reasoning. [CN, PS, R, T, V]	■ Provide examples where scale diagrams are used to model a 2-D shape or a 3-D object. ■ Determine, using proportional reasoning, the scale factor, given one dimension of a 2-D shape or a 3-D object and its representation. ■ Determine, using proportional reasoning, an unknown dimension of a 2-D shape or a 3-D object, given a scale diagram or a model. ■ Draw, with or without technology, a scale diagram of a 2-D shape according to a specified scale factor (enlargement or reduction). ■ Solve a contextual problem that involves scale diagrams.
11A.M.3.	Demonstrate an understanding of the relationships among scale factors, areas, surface areas, and volumes of similar 2-D shapes and 3-D objects. [C, CN, PS, R, T, V]	■ Determine the area of a 2-D shape, given the scale diagram, and justify the reasonableness of the result. ■ Determine the surface area and volume of a 3-D object, given the scale diagram, and justify the reasonableness of the result. ■ Explain, using examples, the effect of a change in the scale factor on the area of a 2-D shape. ■ Explain, using examples, the effect of a change in the scale factor on the surface area of a 3-D object. ■ Explain, using examples, the effect of a change in the scale factor on the volume of a 3-D object. ■ Explain, using examples, the relationships among scale factor, area of a 2-D shape, surface area of a 3-D object, and volume of a 3-D object. ■ Solve a spatial problem that requires the manipulation of formulas. ■ Solve a contextual problem that involves the relationships among scale factors, areas, and volumes.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Geometry	**General Learning Outcome:** Develop spatial sense.

	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11A.G.1.	Derive proofs that involve the properties of angles and triangles. [CN, R, T, V] It is intended that deductive reasoning be limited to direct proof.	■ Generalize, using inductive reasoning, the relationships between pairs of angles formed by transversals and parallel lines, with or without technology. ■ Prove, using deductive reasoning, properties of angles formed by transversals and parallel lines, including the sum of the angles in a triangle. ■ Generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides (n) in a polygon, with or without technology. ■ Identify and correct errors in a proof of a property involving angles. ■ Verify, with examples, that if lines are not parallel, the angle properties do not apply.
11A.G.2.	Solve problems that involve the properties of angles and triangles. [CN, PS, T, V]	■ Determine the measures of angles in a diagram that involves parallel lines, angles, and triangles, and justify the reasoning. ■ Identify and correct errors in a solution to a problem that involves the measure of angles. ■ Solve a contextual problem that involves angles or triangles. ■ Construct parallel lines, using only a compass or a protractor, and explain the strategy used. ■ Determine whether lines are parallel, given the measure of an angle at each intersection formed by the lines and a transversal.
11A.G.3.	Solve problems that involve the cosine law and the sine law, including the ambiguous case. [CN, PS, R, T]	■ Sketch a diagram to represent a problem that involves the cosine law or sine law. ■ Explain the steps in a proof of the sine law or cosine law. ■ Solve a problem involving the cosine law that requires the manipulation of a formula. ■ Explain, concretely, pictorially, or symbolically, whether zero, one, or two triangles exist, given two sides and a non-included angle. ■ Solve a problem involving the sine law that requires the manipulation of a formula. ■ Solve a contextual problem that involves the cosine law or the sine law.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Logical Reasoning	**General Learning Outcome:** Develop logical reasoning.

	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11A.L.1.	Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems. [C, CN, PS, R, T] It is intended that this learning outcome be integrated throughout the course.	■ Make conjectures by observing patterns and identifying properties, and justify the reasoning. ■ Explain why inductive reasoning may lead to a false conjecture. ■ Compare, using examples, inductive and deductive reasoning. ■ Provide and explain a counter-example to disprove a conjecture. ■ Prove algebraic and number relationships, such as divisibility rules, number properties, mental mathematics strategies, or algebraic number tricks. ■ Prove a conjecture, using deductive reasoning (not limited to two column proofs). ■ Determine whether an argument is valid, and justify the reasoning. ■ Identify errors in a proof. ■ Solve a contextual problem involving inductive or deductive reasoning.

11A.L.2.	Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, T, V]
	
	It is intended that this learning outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction, and similar puzzles and games.
		■ Determine, explain, and verify a strategy to solve a puzzle or to win a game such as
		  ■ guess and check
		  ■ look for a pattern
		  ■ make a systematic list
		  ■ draw or model
		  ■ eliminate possibilities
		  ■ simplify the original problem
		  ■ work backward
		  ■ develop alternative approaches
		■ Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.
		■ Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Grade 11 Applied Mathematics

Strand: Statistics		**General Learning Outcome:** Develop statistical reasoning.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11A.S.1.	Demonstrate an understanding of normal distribution, including ■ standard deviation ■ z-scores [CN, PS, T, V]	■ Explain, using examples, the meaning of standard deviation. ■ Calculate, using technology, the population standard deviation of a data set. ■ Explain, using examples, the properties of a normal curve, including the mean, median, mode, standard deviation, symmetry, and area under the curve. ■ Determine whether a data set approximates a normal distribution, and explain the reasoning. ■ Compare the properties of two or more normally distributed data sets. ■ Explain, using examples that represent multiple perspectives, the application of standard deviation for making decisions in situations, such as warranties, insurance, or opinion polls. ■ Solve a contextual problem that involves the interpretation of standard deviation. ■ Determine, with or without technology, and explain the z-score for a value in a normally distributed data set. ■ Solve a contextual problem that involves normal distribution.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Statistics *(continued)*	**General Learning Outcome:** Develop statistical reasoning.
Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11A.S.2. Interpret statistical data, using ■ confidence intervals ■ confidence levels ■ margin of error [C, CN, R, T] It is intended that the focus of this learning outcome be on interpretation of data rather than on statistical calculations.	■ Explain, using examples, how confidence levels, margin of error, and confidence intervals may vary depending on the size of the random sample. ■ Explain, using examples, the significance of a confidence interval, margin of error, or confidence level. ■ Make inferences about a population from sample data, using confidence intervals, and explain the reasoning. ■ Provide examples from print or electronic media in which confidence intervals and confidence levels are used to support a particular position. ■ Interpret and explain confidence intervals and margin of error, using examples found in print or electronic media. ■ Support a position by analyzing statistical data presented in the media.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Grade 11 Applied Mathematics

Strand: Relations and Functions		**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11A.R.1.	Model and solve problems that involve systems of linear inequalities in two variables. [CN, PS, T, V]	■ Model a problem, using a system of linear inequalities in two variables. ■ Graph the boundary line between two half planes for each inequality in a system of linear inequalities, and justify the choice of solid or broken lines. ■ Determine and explain the solution region that satisfies a linear inequality, using a variety of strategies when given a boundary line. ■ Determine, graphically, the solution region for a system of linear inequalities, and verify the solution. ■ Explain, using examples, the significance of the shaded region in the graphical solution of a system of linear inequalities. ■ Solve an optimization problem, using linear programming.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Strand: Relations and Functions *(continued)*	**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.

	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*
11A.R.2.	Demonstrate an understanding of the characteristics of quadratic functions, including ■ vertex ■ intercepts ■ domain and range ■ axis of symmetry [CN, PS, T, V] It is intended that completion of the square not be required.	■ Determine, with technology, the intercepts of the graph of a quadratic function or the roots of the corresponding quadratic equation. ■ Explain the relationships among the roots of an equation, the zeros of the corresponding function, and the *x*-intercepts of the graph of the function. ■ Explain, using examples, why the graph of a quadratic function may have zero, one, or two *x* intercepts. ■ Express a quadratic equation in factored form, using the zeros of a corresponding function or the *x*-intercepts of its graph. ■ Determine, with technology, the coordinates of the vertex of the graph of a quadratic function. ■ Determine the equation of the axis of symmetry of the graph of a quadratic function, given the *x*-intercepts of the graph. ■ Determine the coordinates of the vertex of the graph of a quadratic function, given the equation of the function and the axis of symmetry, and determine whether the *y*-coordinate of the vertex is a maximum or a minimum. ■ Determine the domain and range of a quadratic function. ■ Sketch the graph of a quadratic function. ■ Solve, with technology, a contextual problem involving data that is best represented by graphs of quadratic functions, and explain the reasoning. ■ Solve a contextual problem that involves the characteristics of a quadratic function.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Grade 11 Applied Mathematics

Strand: Mathematics Research Project		**General Learning Outcome:** Develop an appreciation of the role of mathematics in society.
	Specific Learning Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11A.RP.1.	Research and give a presentation on a historical event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]	■ Collect primary or secondary data (statistical or informational) related to the topic.
		■ Assess the accuracy, reliability, and relevance of the primary or secondary data collected by
		  ■ identifying examples of bias and points of view
		  ■ identifying and describing the data collection methods
		  ■ determining whether the data is relevant
		  ■ determining whether the data is consistent with information obtained from other sources on the same topic
		■ Interpret data, using statistical methods if applicable.
		■ Identify controversial issues, if any, and present multiple sides of the issues with supporting data.
		■ Organize and present the research project, with or without technology.