Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 2

June 2019
Grade 12 pre-calculus mathematics achievement test.
Booklet 2. June 2019

This resource is available in print and electronic formats.
ISBN: 978-0-7711-7798-9 (print)
ISBN: 978-0-7711-7799-6 (pdf)

1. Mathematics—Examinations, questions, etc.
2. Educational tests and measurements—Manitoba.
3. Mathematics—Study and teaching (Secondary)—Manitoba.
4. Pre-calculus—Study and teaching (Secondary)—Manitoba.
I. Manitoba. Manitoba Education and Training.
510.76

Copyright © 2019, the Government of Manitoba, represented by the Minister of Education and Training.

Manitoba Education and Training
Winnipeg, Manitoba, Canada

Permission is hereby given to reproduce this resource for non-profit educational purposes provided the source is cited.

After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre. Order online at www.manitobairc.ca.

This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.
DESCRIPTION

Time Required to Complete the Test: 3 hours
Additional Time Allowed: 30 minutes

<table>
<thead>
<tr>
<th>Numbers and Marks by Question Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Selected Response</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td><strong>Booklet 1</strong>*</td>
</tr>
<tr>
<td><strong>Booklet 2</strong></td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>

* The first 4 questions with the symbol in Booklet 1 require a scientific calculator. You will have access to your calculator for the first 45 minutes of the test.

Note that diagrams and graphs provided in the test booklets may not be drawn to scale.

DIRECTIONS

Selected Response Questions

- Calculators are not allowed for this part of the test.
- You may use the spaces beside each question for rough work.
- Provide only one answer per question.
- There is no penalty for guessing.
- Record your answers on the sheet provided.

Constructed Response Questions

- Calculators are not allowed for this part of the test.
- For full marks, your answer must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear, and well organized.
- Write each solution in the space provided.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited. Please turn off your cell phone and all other such devices.
No marks will be awarded for work done on this page.
Question 18  
1 mark

The range of \( y = f(x) \) is \(-6 \leq y \leq 12\). The range of the transformed function \( y = af(x) \) is \(-2 \leq y \leq 4\). Identify the value of \( a \).

a) \(-3\)
b) \(-\frac{1}{3}\)
c) \(\frac{1}{3}\)
d) 3

Question 19  
1 mark

Identify the expression which is equivalent to \( 3 \log y - \frac{1}{2} \log x + \log z \).

a) \(\log \left( \frac{y^3}{\sqrt{xz}} \right)\)
b) \(\log \left( \frac{y^3 z}{\sqrt{x}} \right)\)
c) \(\log \left( \frac{y^3}{x^2 z} \right)\)
d) \(\log \left( \frac{y^3 z}{x^2} \right)\)

Question 20  
1 mark

Identify the measure of the angle \(-\frac{2\pi}{9}\) in degrees.

a) \(-400^\circ\)
b) \(-40^\circ\)
c) \(40^\circ\)
d) \(320^\circ\)
**Question 21**  
1 mark

If \( y = f(x) \) has a domain of \([2, 5]\) and a range of \([6,10]\), identify the domain of \( y = f^{-1}(x) \).

a) \( \left[ \frac{1}{2}, \frac{1}{5} \right] \)

b) \([-5, -2]\)

c) \([-10, -6]\)

d) \([6,10]\)

**Question 22**  
1 mark

Identify which of the following is a polynomial function.

a) \( p(x) = -\frac{1}{2}(x + 2)^3(x - 3) \)

b) \( p(x) = 2x^2 + x - 3 \)

c) \( p(x) = 3x^{-4} + x^2 - 6 \)

d) \( p(x) = 2^x + 3 \)

**Question 23**  
1 mark

Identify the total number of terms in the expansion of \((x - y)^9\).

a) 8

b) 9

c) 10

d) 11
Question 24 1 mark

Identify the exact value of \(2 \cos^2 (15^\circ) - 1\).

a) 1
b) \(\frac{1}{2}\)
c) \(\frac{\sqrt{3}}{2}\)
d) \(\sqrt{3}\)

Question 25 1 mark

The zeros of the function \(y = f(x)\) are \(x = -2\) and \(x = 3\). Identify the zeros of the function \(g(x) = 2f(x - 4)\).

a) \(x = -6\) and \(x = -1\)
b) \(x = 2\) and \(x = 7\)
c) \(x = -4\) and \(x = 6\)
d) \(x = 0\) and \(x = 10\)

Question 26 1 mark

Identify the value of \(\log_4 \left(\frac{1}{4}\right)\).

a) \(-16\)
b) \(-1\)
c) 1

d) 16
Question 27

Sketch the graph of at least one period of the function \( y = -\cos\left( x + \frac{\pi}{4} \right) + 3 \).
Question 28

Justify that \((x - 5)\) is not a possible factor of the function \(P(x) = x^3 - 3x^2 - 4x + 12\).
Question 29

Sketch the graph of \( f(x) = \frac{6}{(x + 2)(x - 3)} \) and state the \( y \)-intercept.

\[ y \text{-intercept: } \] 

[Diagram of a graph with grid lines and axes labeled \( x \) and \( y \). The graph shows the function \( f(x) \) and includes a \( y \)-intercept at \( (0, y) \).]

\( y \)-intercept: ____________________________
Determine how many 3-digit odd numbers less than 300 are possible using the digits 1, 2, 3, 4, 5, 6 if repetition is not allowed.
Given that \( \cos \alpha = -\frac{5}{13} \) and \( \sin \beta = \frac{2}{3} \), where \( \alpha \) and \( \beta \) terminate in the same quadrant, determine the exact value of \( \cos(\alpha - \beta) \).
Given the graph of \( y = 4^x \), sketch the graph of \( y = 2(4)^{x-3} + 1 \).

The graph of \( f(x) \) has already been drawn for your reference. No marks will be awarded for the graph of \( f(x) \).
Question 33

Determine the coterminal angle of $\frac{\pi}{5}$ over the interval $[-2\pi, 0]$.

Question 34

State the domain of the graph of $y = \log(x - 4) - 8$. 
Question 35

Given the graph of \( y = 5 \sin \left[ 2 \left( x + \frac{\pi}{4} \right) \right] - 3 \), determine the exact value of the \( x \)-coordinate in the point \( P \).
Verify that the following equation is true for $x = \frac{5\pi}{6}$.

\[ \frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x} \]

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
</table>
Given that \((x + 1)\) is one of the factors of \(P(x) = x^3 - x^2 + kx - 8\), determine the value of \(k\).
Given the function $f(x) = \sqrt{x}$, describe how to use transformations to determine the domain of the function $g(x) = f(x + 2) + 1$. 
Given the graph of \( y = f(x) \), state the equation of the vertical asymptote of \( y = \frac{1}{f(x)} \).
Question 40 2 marks

Solve, algebraically.

\[ 16^x = 64^{2x-1} \]
Question 41

Given one of the factors of \( P(x) = x^3 + 2x^2 - 5x - 6 \) is \((x + 3)\), express \( P(x) \) in completely factored form.

\[
P(x) = ______________________
\]
Question 42

Sketch the graph of \( p(x) = 3(x + 1)^2 (x - 2)^2 \).
Question 43

Given that \( f(x) = x^2 - 4 \) and \( g(x) = \sqrt{x} \), determine \( f(g(x)) \) and state its domain.

\[ f(g(x)) = \]
Determine a possible equation of the function $f(x)$.

\[ f(x) = \frac{1}{x - 1} \]
Question 45

Explain why the graph of \( y = \log_2 x \) does not have a \( y \)-intercept.
Evaluate.

\[ \sin^2 \left(-\frac{\pi}{3}\right) + \cos \left(\frac{17\pi}{6}\right) \sec \left(\frac{\pi}{6}\right) \]
Question 47

1 mark

Determine the coordinates of the point of discontinuity (hole) on the graph of \( y = \frac{x^2 - 3x}{x} \).
Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $h(x) = f(x) + g(x)$. 

\[ h(x) = f(x) + g(x) \]
Given that $\csc \theta = \frac{4}{\sqrt{7}}$ and $\cos \theta > 0$, determine the exact value of $\tan \theta$. 
No marks will be awarded for work done on this page.