Grade 12
Pre-Calculus Mathematics
Achievement Test

# Booklet 2

June 2015

Manitoba

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Available in alternate formats upon request.

# Instructions

## Selected Response Questions

- There are 9 questions each worth one mark.
- Calculators are **not** allowed for this part of the test.
- You may use the spaces beside each question for rough work.
- Provide only one answer per question.
- There is no penalty for guessing.
- Record your answers on the sheet provided.

## Constructed Response Questions

- There are 20 questions worth a total of 50 marks.
- Calculators are **not** allowed for this part of the test.
- For full marks, your answer must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear, and well organized.
- Write each solution in the space provided.

No marks will be awarded for work done on this page.

## Question 14 — 1 mark

There are 2 types of pencils, 3 colours of highlighters, and 5 styles of pens.

If you must select one of each to form a set, how many different sets of writing instruments are possible?

a) 10

b) 11

c) 25

d) 30

## Question 15 — 1 mark

The point $P(\theta)$ lies on the unit circle. What are the coordinates of the point $P$ if $\theta = 120^\circ$?

a) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

b) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

c) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

d) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

## Question 16 — 1 mark

Identify the function that has a domain of $\{x|x \geq 7\}$ and a range of $\{y|y \geq 0\}$.

a) $f(x) = \sqrt{x} + 7$

b) $f(x) = \sqrt{x} - 7$

c) $f(x) = \sqrt{x+7}$

d) $f(x) = \sqrt{x-7}$

## Question 17 — 1 mark

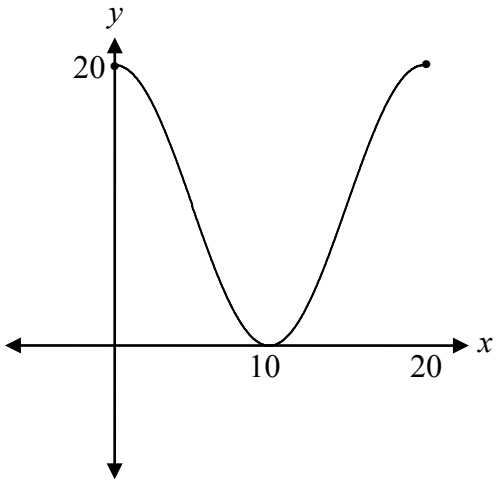
Using $y = -10\cos[\mathrm{B}(x - \mathrm{C})] + \mathrm{D}$, the value of C that corresponds to the following graph is:

a) 5

b) 10

c) 15

d) 20



## Question 18 — 1 mark

Given the following row of Pascal's Triangle, identify the binomial expansion with these coefficients.

1 5 10 10 5 1

a) $(x+y)^4$

b) $(x+y)^5$

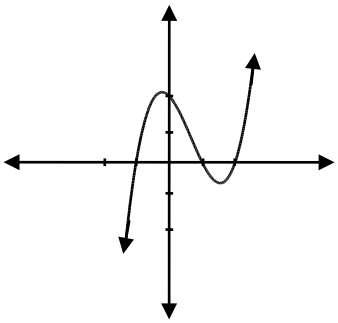
c) $(x+y)^6$

d) $(x+y)^7$

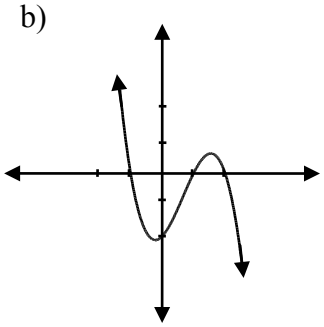
## Question 19 — 1 mark

Identify the graph of the function $f(x) = -(x-2)(x-1)^2(x+1)$.

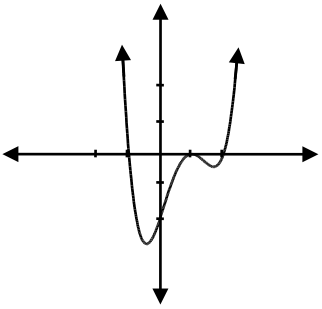
a)



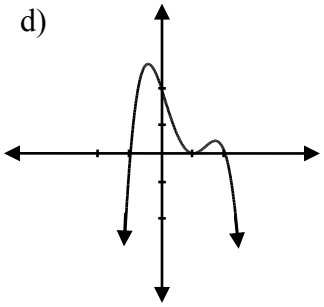
b)



c)



d)



## Question 20 — 1 mark

Determine the value of $\log_9\left(\log_3 27\right)$.

a) $\frac{1}{3}$

b) $\frac{1}{2}$

c) $2$

d) $3$

## Question 21 — 1 mark

If $\left(x,\ y\right)$ is a point on the graph of $y = f\left(x\right)$, identify the coordinates of this point on the graph of $g\left(x\right) = f\left(2x\right) + 5$.

a) $\left(\frac{x}{2},\ y + 5\right)$

b) $\left(2x,\ y + 5\right)$

c) $\left(\frac{x}{2},\ y - 5\right)$

d) $\left(\frac{x}{2} - 5,\ y\right)$

## Question 22 — 1 mark

Identify an equivalent expression for $1 + \log_2 5$.

a) $\log_2 5$

b) $\log_2 7$

c) $\log_2 10$

d) $\log_2 11$

## Question 23 — 4 marks — 114

Solve:

$$2\log_4 x - \log_4 (x+3) = 1$$

## Question 24 — 3 marks — 115

The following transformations are applied to $f(x)$, resulting in a new function, $g(x)$.

- reflection over the *y*-axis
- horizontal translation of 3 units to the right
- vertical translation of 4 units down

Write the equation of $g(x)$ in terms of $f(x)$.

$g(x) =$ _____

## Question 25 — a) 3 marks b) 1 mark

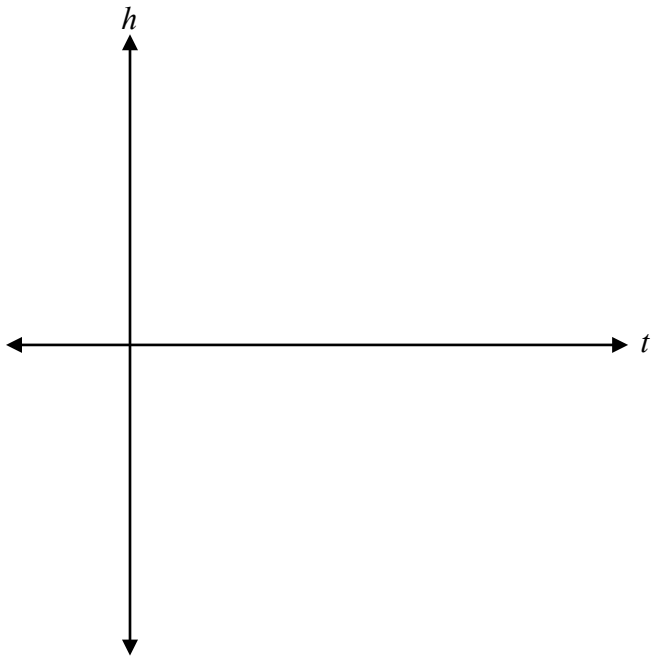
116
117

The height of a bicycle pedal as the bicycle is moving at a constant speed can be represented by the following function:

$$h(t) = 15\cos\frac{2\pi}{5}t + 30$$

where $h$ is the height of the pedal above the ground, in cm, and $t$ is the time, in seconds.

a) Sketch a graph of at least one period of this function, where $t \geq 0$.

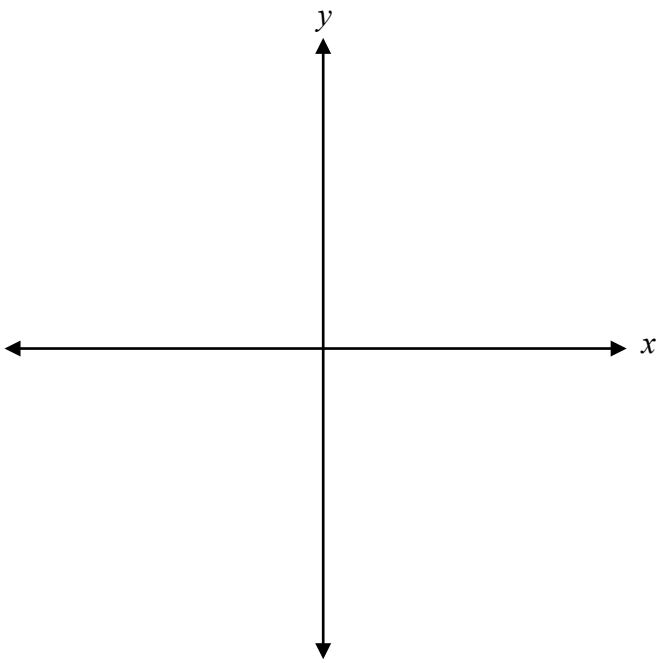



b) Determine the height of the bicycle pedal at 7.5 seconds.

## Question 26 — 4 marks — 118

Sketch the graph of the function $f(x)$ and determine the $y$-intercept.

$$f(x) = \frac{4}{(x-2)(x+2)}$$




$y$-intercept:_____

## Question 27 — 1 mark — 119

Kim solved the following logarithmic equation:

$$\log_2\left(-\frac{x}{3}\right) = \log_2(x-4)$$

$$-\frac{x}{3} = x - 4$$

$$-x = 3x - 12$$

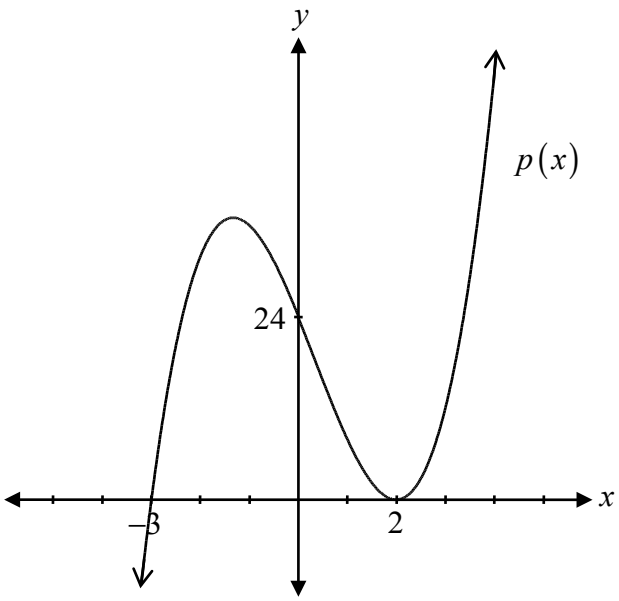
$$-4x = -12$$

~~$x = 3$~~

Explain why $x = 3$ is an extraneous solution.

## Question 28 — 3 marks — 120

Determine the equation of the polynomial function represented by the graph.




$p(x) =$ _____

## Question 29 — 1 mark — 121

Determine the coterminal angles with $\frac{2\pi}{3}$ over the interval $[-2\pi, 4\pi]$.

## Question 30 — a) 1 mark b) 1 mark [122 123]

a) Solve the following equation:

$$0 = \sqrt{4x - 8} - 2$$

b) Explain how your answer in a) is related to the graph of $y = \sqrt{4x - 8} - 2$.

## Question 31 — 3 marks — 124

Determine the exact value of $\sin\frac{13\pi}{12}$.

## Question 32 — a) 2 marks b) 1 mark [125 126]

Given the functions $f(x) = x + 2$ and $g(x) = \dfrac{1}{x-5}$:

a) Determine the equation of the composite function $f(g(x))$ and its domain.

$f(g(x)) =$ _____

domain: _____

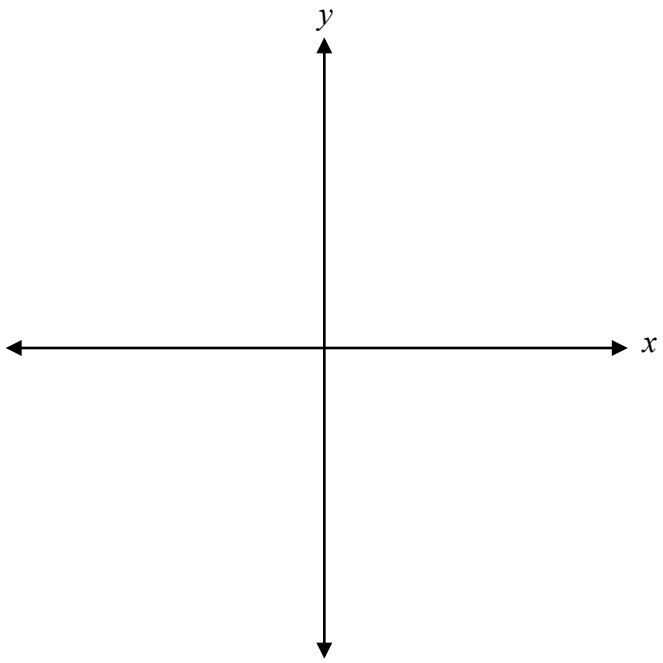
b) Determine the $x$-intercept and $y$-intercept of $f(g(x))$.

$x$-intercept: _____

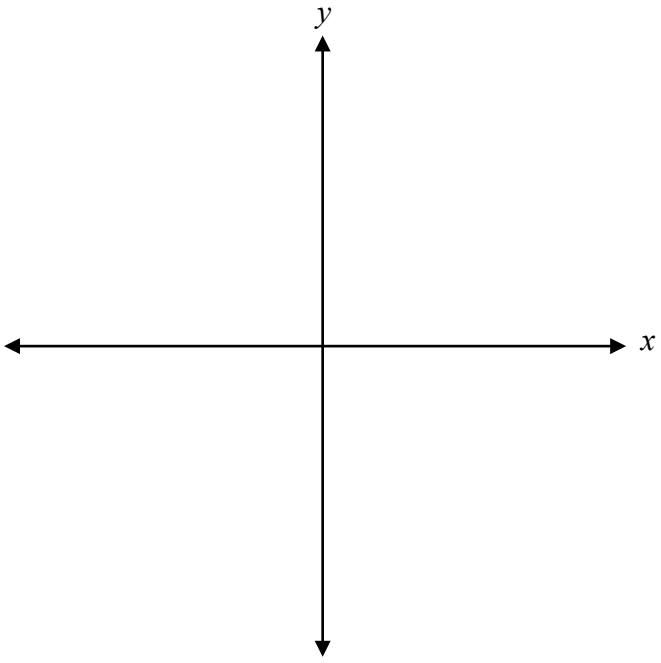
$y$-intercept: _____

## Question 33 — a) 2 marks b) 1 mark [127 128]

a) Sketch the graph of $f(x) = \log_5(x-1)$.




b) Sketch the graph of $f^{-1}(x)$.

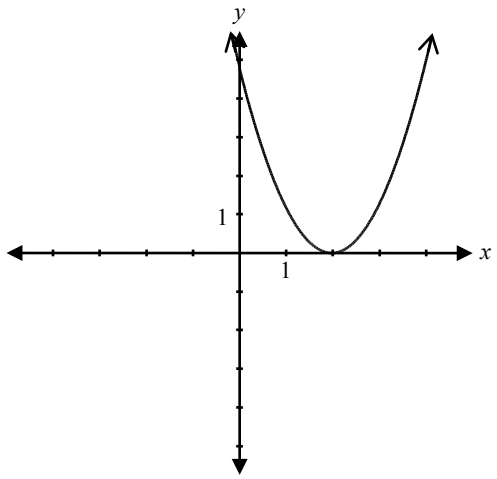


## Question 34 — 1 mark — 129

Explain what the graph of a rational function looks like near a vertical asymptote.

## Question 35 — 1 mark — 130

Given the graph of $f(x) = (x-2)^2$,




determine one possible restriction for the domain of $f(x)$ so that its inverse is a function.

Domain:_____

## Question 36 — 2 marks — 131

Over the interval $[0,\ 2\pi]$, determine the non-permissible values of $\theta$ in the expression $\csc\theta(\cos\theta+1)$.

## Question 37 — 1 mark — 132

Explain why ${}_3C_8$ is undefined.

## Question 38 — 3 marks — 133

Solve:

$$ {}_{n}P_{3} = 48(n-1) $$

## Question 39 — a) 2 marks b) 1 mark c) 1 mark

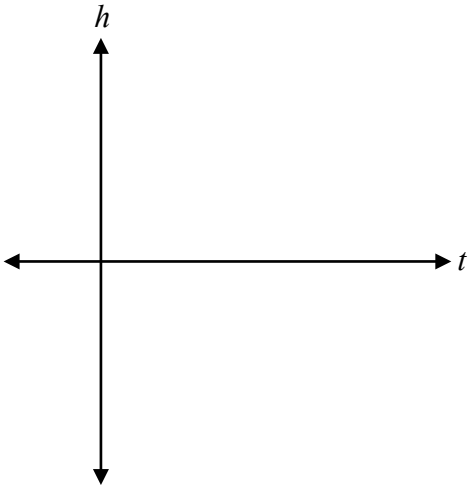
134 135 136

Christine dives off a diving board.

Her dive is modelled by the function $h(t) = t^3 - 3t^2 - t + 3$, where $h$ is her height in metres, relative to the water surface and $t$ is the time in seconds after diving off the diving board.

a) Given that $(t+1)$ is a factor for the function $h(t)$, determine the other factors.

b) Sketch the graph of the function $h(t)$ for the time interval $t = 0$ to $t = 3$.




c) Determine how many seconds Christine is underwater.

## Question 40 — 3 marks — 137

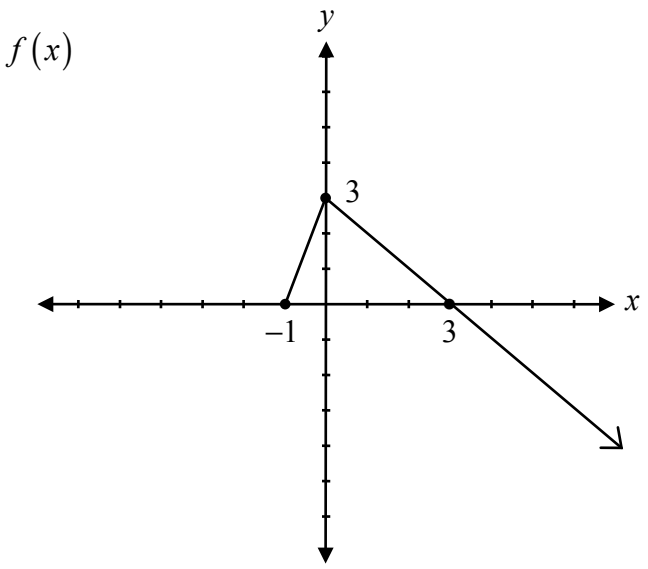
Prove the identity for all permissible values of $x$.

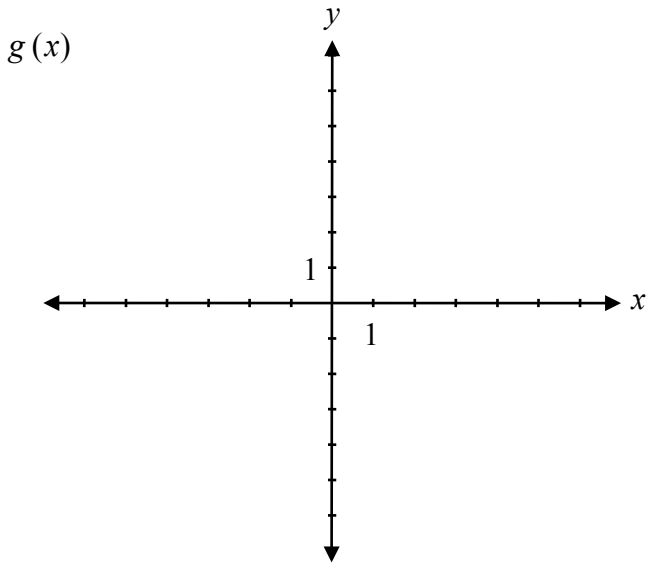
$$\sec x + \tan x = \frac{\cos x}{1 - \sin x}$$

| Left-Hand Side | Right-Hand Side |
| --- | --- |
| | |

## Question 41 — 2 marks — 138

Given the graph of $f(x)$, sketch the graph of the function $g(x) = -|f(x)|$.

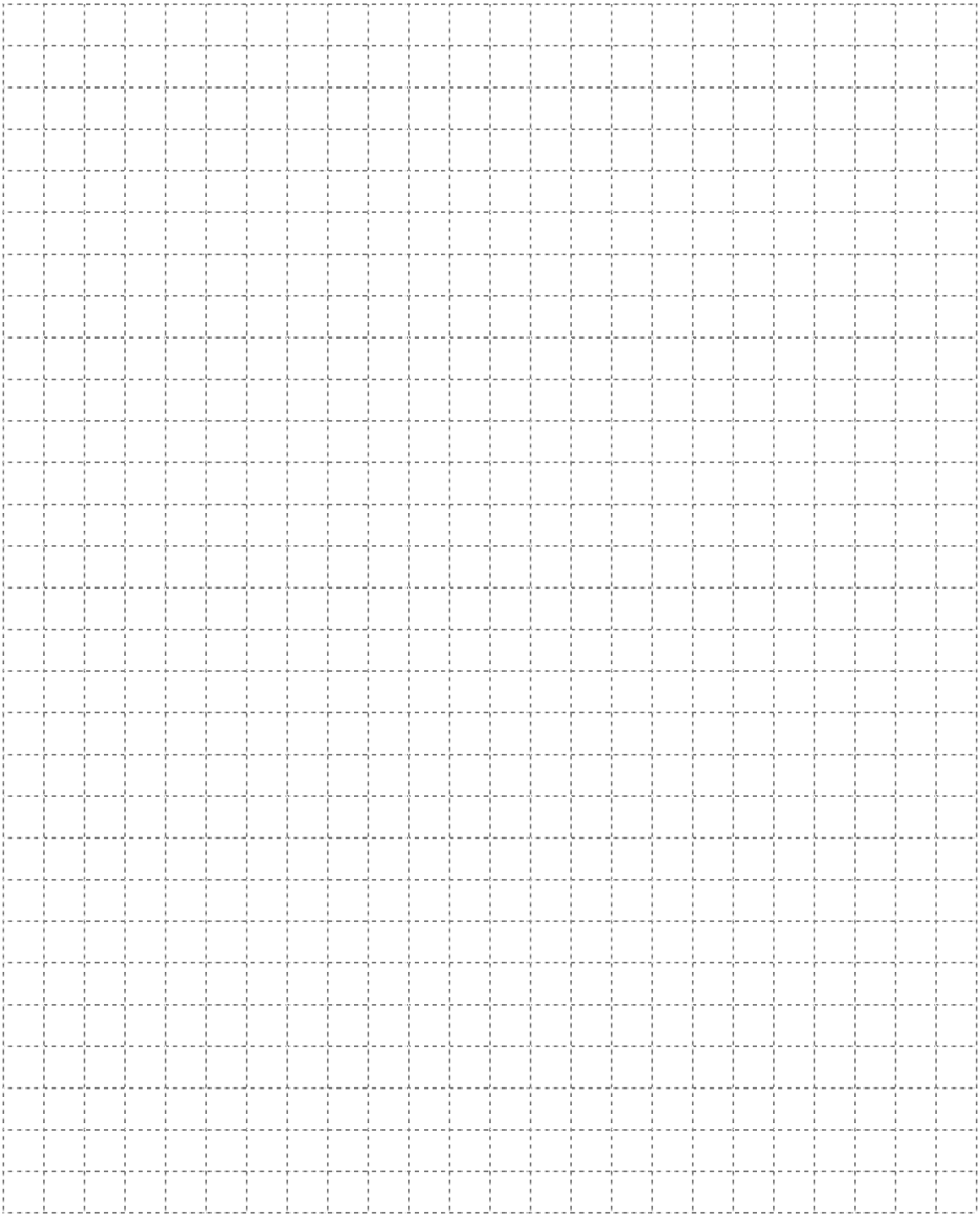


## Question 42 — 2 marks — 139

Given that $\cot\theta = -\frac{2}{5}$, where $\theta$ is in Quadrant IV, determine the exact value of $\sin\theta$.

No marks will be awarded for work done on this page.



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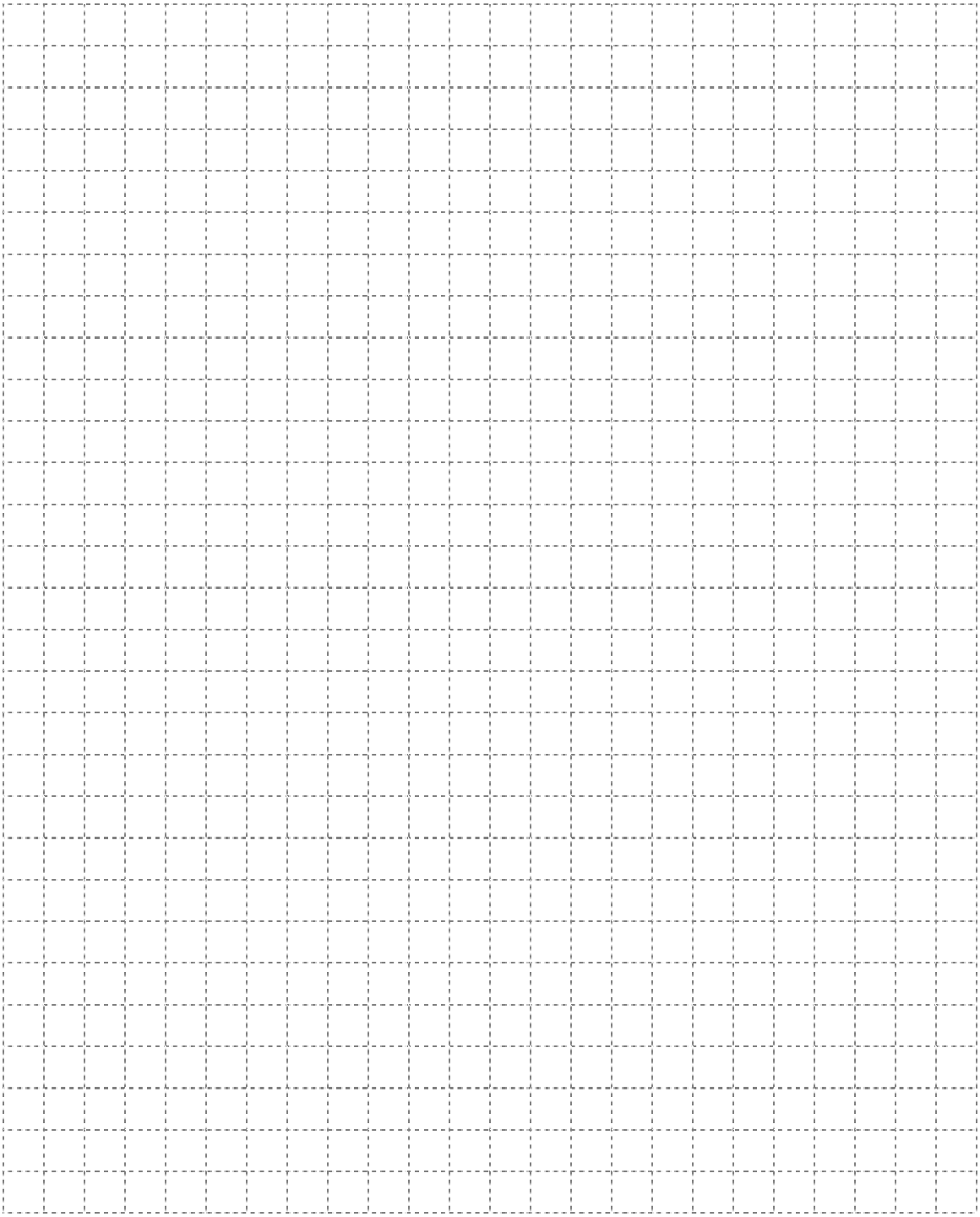

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