

Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

June 2024

Grade 12 pre-calculus mathematics achievement test.
Booklet 1. June 2024

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Manitoba Education and Early Childhood Learning
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Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time Required to Complete the Test: 3 hours

Additional Time Allowed: 30 minutes

Numbers and Marks by Question Type

	Selected Response	Constructed Response	Marks
Booklet 1*	—	16	34
Booklet 2	10	21	56
Total	10	37	90

* The first 5 questions with the symbol  in *Booklet 1* require a scientific calculator. You will have access to your calculator for the first 45 minutes of the test.

Note that diagrams and graphs provided in the test booklets may not be drawn to scale.

DIRECTIONS

- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited. Please turn off your cell phone and all other such devices.

No marks will be awarded for work done on this page.

Formula Sheet

$$s = \theta r$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\log_a (MN) = \log_a M + \log_a N$$

$$\log_a \left(\frac{M}{N} \right) = \log_a M - \log_a N$$

$$\log_a (M^n) = n \log_a M$$

$$P(n, r) \text{ or } {}_n P_r = \frac{n!}{(n-r)!}$$

$$C(n, r) \text{ or } {}_n C_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_n C_k a^{n-k} b^k$$

$$\text{Given } ax^2 + bx + c = 0,$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are defined below.

Describe: Use words to provide the process or to report details of the response.

Determine: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

Identify: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

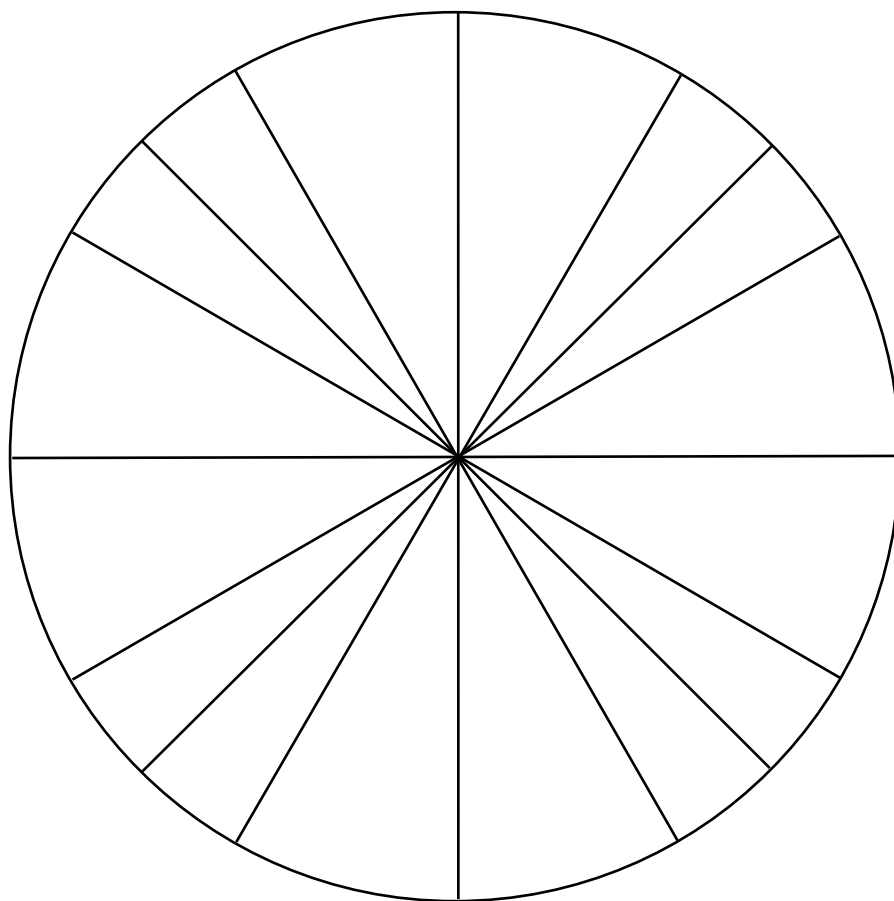
Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Solve: Give a solution for a problem or determine the value(s) of a variable.

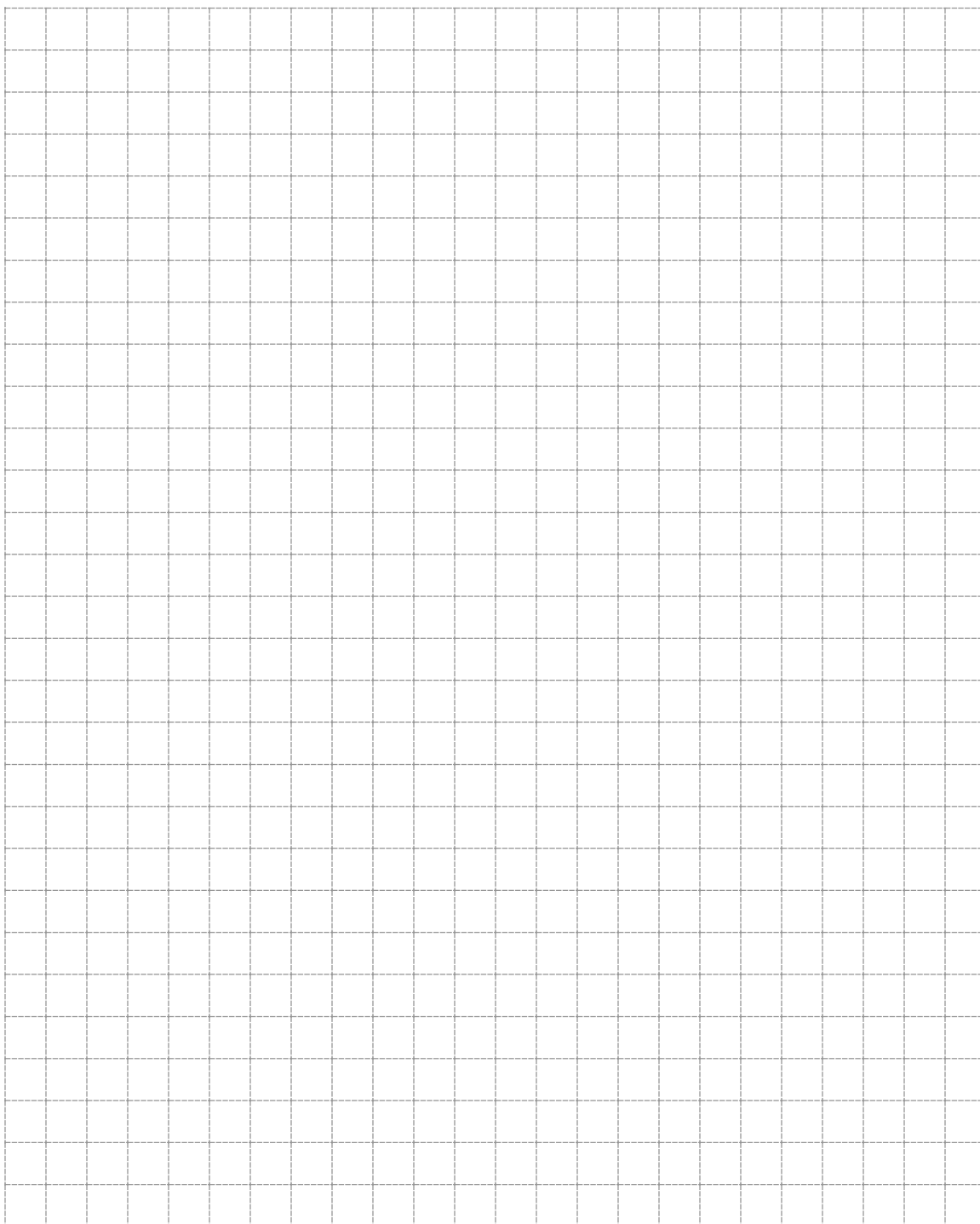
State: Give an answer without an explanation or justification.

Verify: Establish the truth of a statement by substitution or comparison.

No marks will be awarded for work done on this page.



No marks will be awarded for work done on this page.



Question 1



1 mark 101

A soccer team consists of 14 players. Determine the number of ways a captain and an assistant captain can be selected.

Question 2



2 marks 102

The pH of a person's blood can be found using the formula

$$\text{pH} = 6.1 + \log\left(\frac{B}{C}\right)$$

where B represents the concentration of bicarbonate in the blood, in mEq/L

C represents the concentration of carbonic acid in the blood, in mEq/L

Kansas has a blood pH of 7.41. In her blood sample, the concentration of carbonic acid is 1.41 mEq/L. Determine the concentration of bicarbonate in her blood.

Express the answer correct to one decimal place.

Question 3

3 marks 103

Solve $(2 \tan x - 1)(\tan x + 1) = 0$ where $x \in \mathbb{R}$.

Question 4



3 marks 104

Determine and simplify the 7th term in the binomial expansion of $\left(\frac{2}{x^2} + 3x\right)^9$.

Question 5



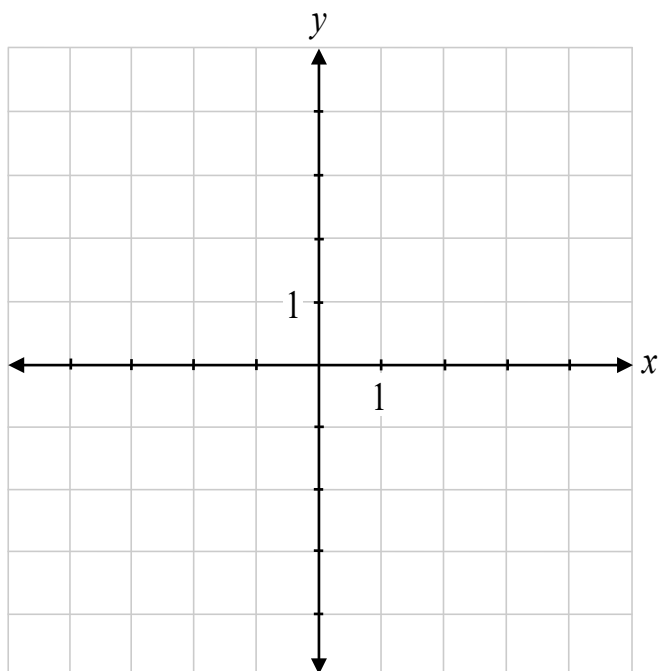
2 marks 105

A winch, with a diameter of 14 cm, is used to pull a boat out of the water. As the winch rotates, it pulls in a cable. Determine the length of cable that is pulled in if the winch rotates 526° .



Note: A calculator is not required for the remaining test questions.

Sketch the graph of $f(x) = \sqrt{\frac{1}{3}x} - 1$.



Question 7

3 marks 107

Given $y = f(x)$, determine the equation of the resulting function, $g(x)$, after the following transformations:

- reflection over the y -axis
- vertical stretch by a factor of 3
- horizontal translation 2 units left

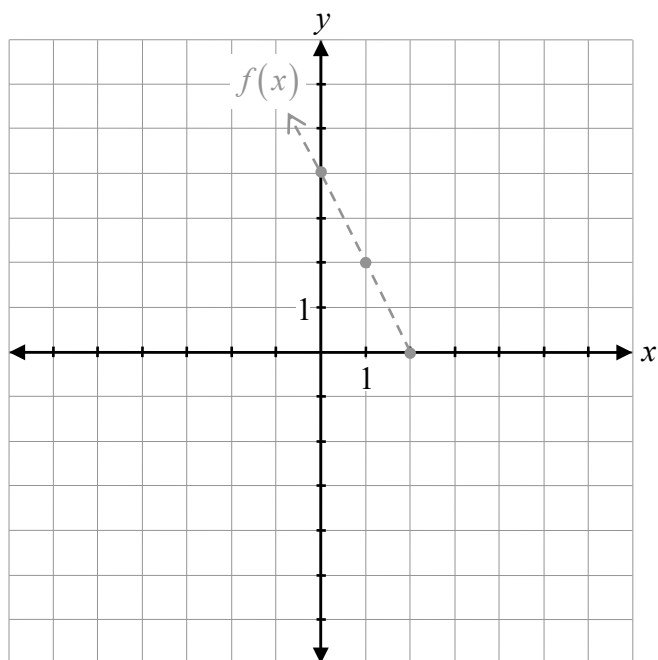
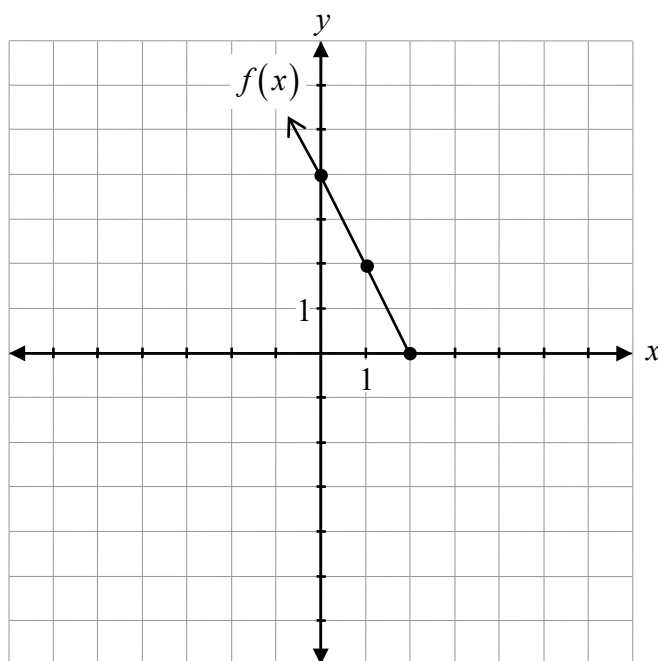
$g(x) =$ _____

Explain why the graph of $y = \frac{6x+7}{3x+5}$ has a horizontal asymptote at $y = 2$.

Question 9

3 marks 109

Given the graph of $y = f(x)$, sketch the graph of $y = -\sqrt{f(x)} + 2$.



The graph of $f(x)$ has already been drawn for your reference. No marks will be awarded for the graph of $f(x)$.

Question 10

3 marks 110

Using the laws of logarithms, completely expand the given expression.

$$\log\left(\frac{A}{\sqrt[3]{B} \cdot C^4}\right)$$

Question 11

3 marks 111

Prove the following identity for all permissible values of θ .

$$\cot \theta - \tan \theta = \frac{2 \cos 2\theta}{\sin 2\theta}$$

Left-Hand Side	Right-Hand Side

Question 12

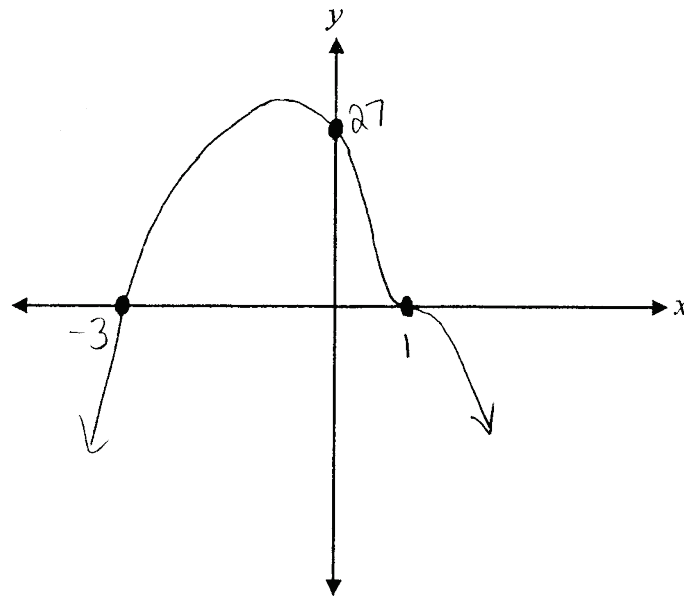
3 marks 112

Express $p(x) = -2x^3 + x^2 + 13x + 6$ in completely factored form.

$p(x) =$ _____

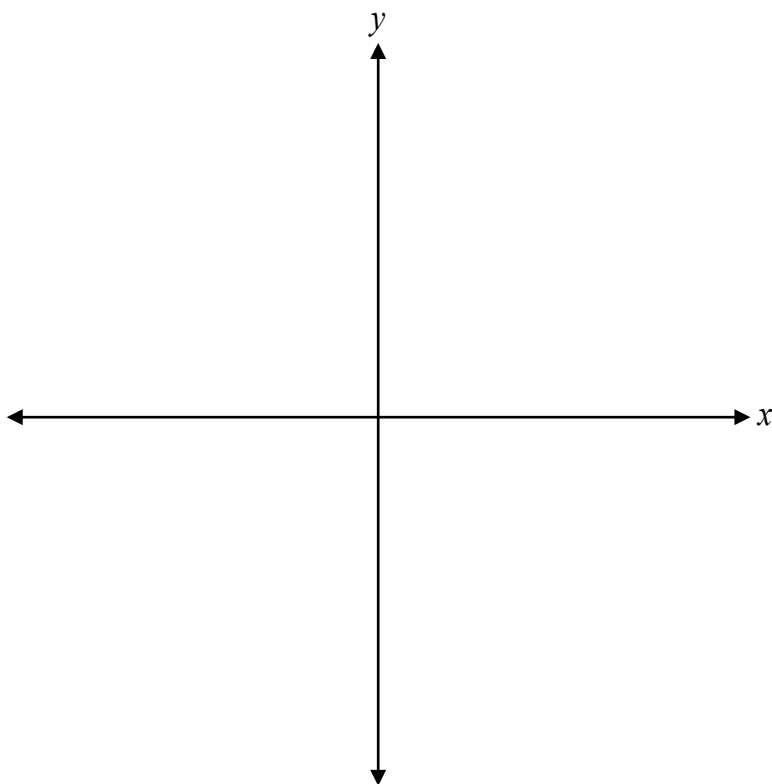
Kaitlyn was asked to sketch the graph of the polynomial function, $p(x) = -(x-1)(x+3)^3$.

Kaitlyn's graph:

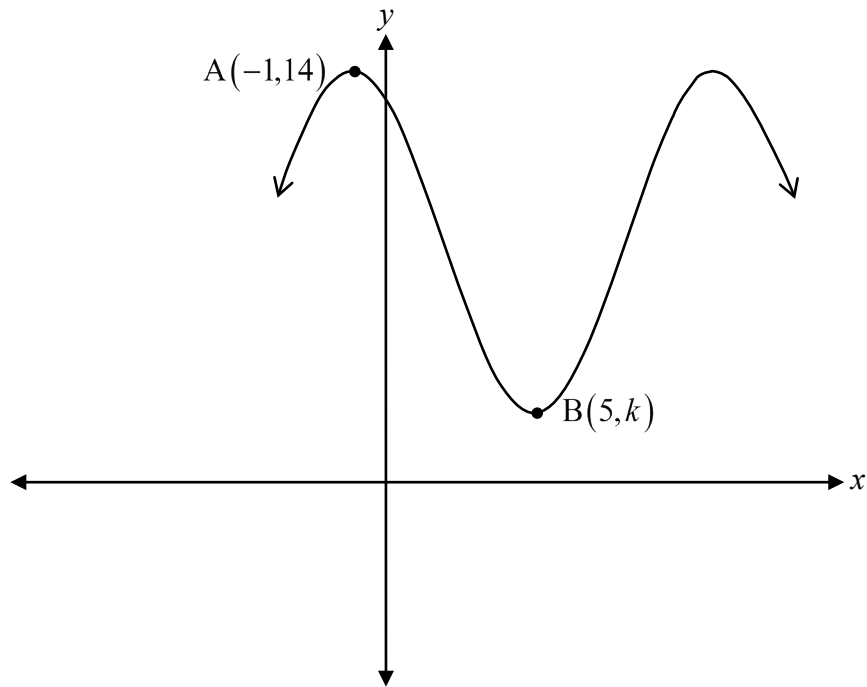


Describe an error on her graph.

Sketch the angle of -5 radians in standard position.



The graph of a sinusoidal function is sketched below. The point A is a maximum and the point B is a minimum. State the value of k if the amplitude of the function is 6.

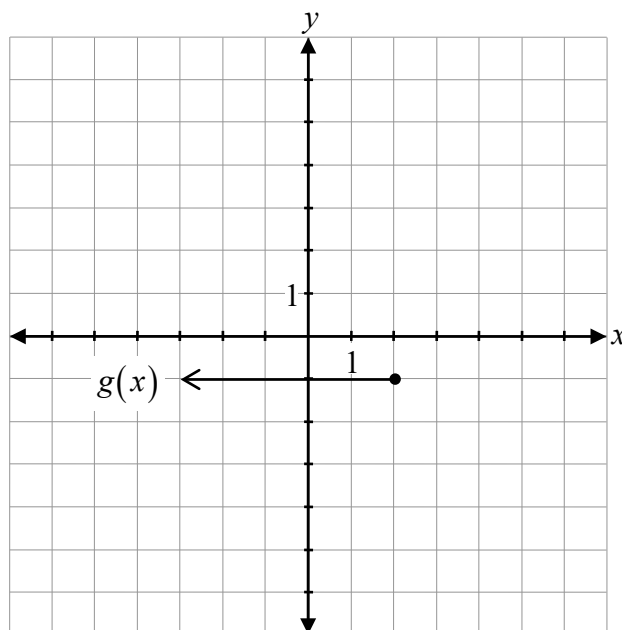
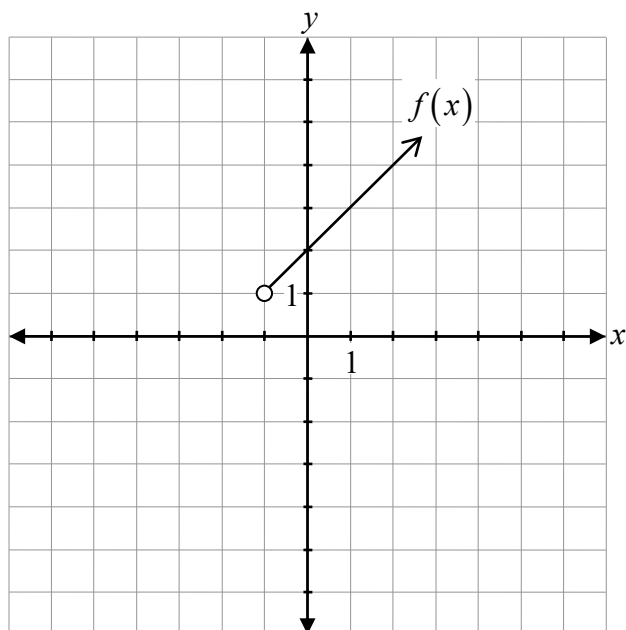


Question 16

1 mark

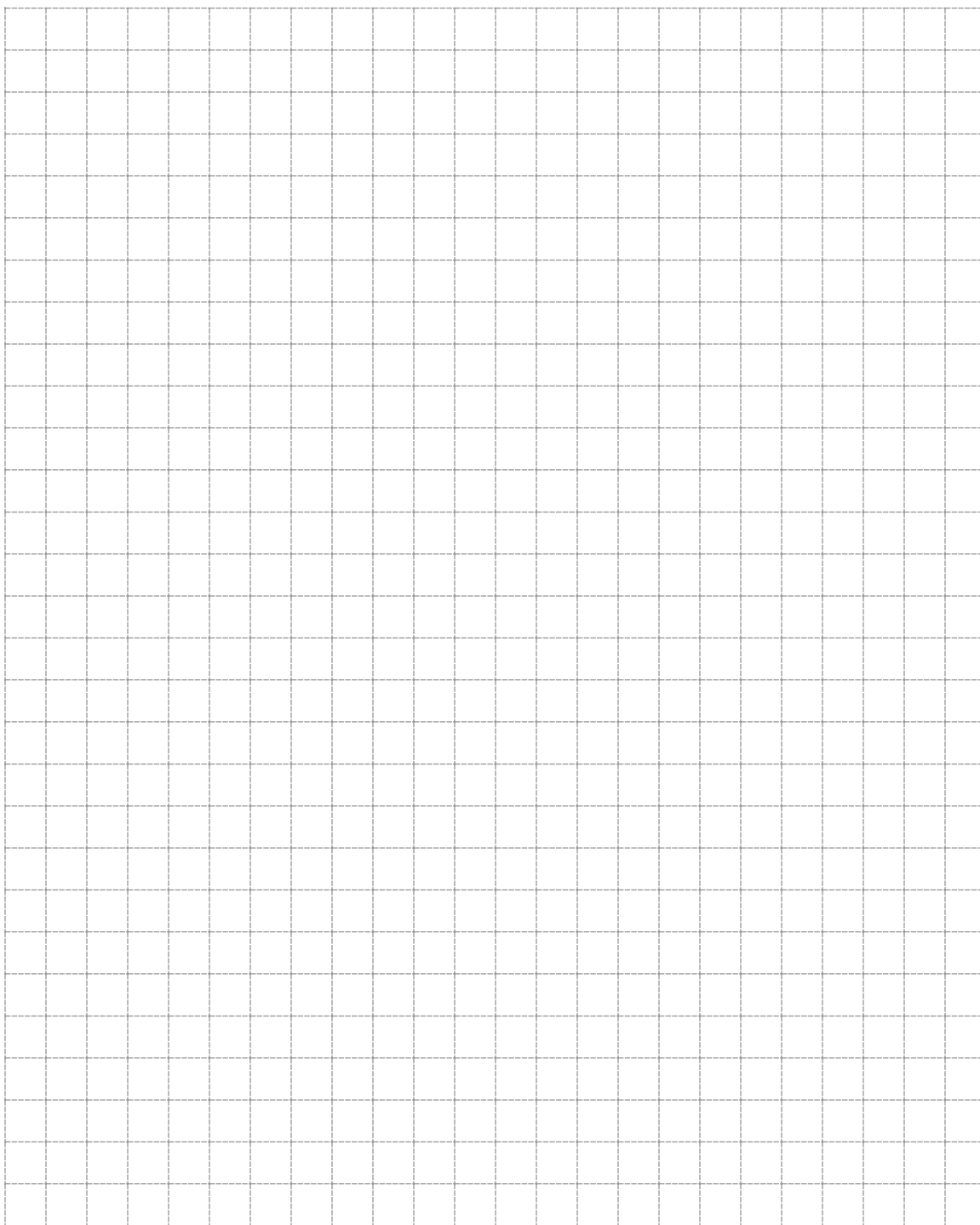
116

Given the graphs of $f(x)$ and $g(x)$, state the domain of $(f + g)(x)$.



Domain: _____

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