

Grade 12  
Pre-Calculus Mathematics  
Achievement Test

# **Booklet 1**

June 2017

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While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.


# Grade 12 Pre-Calculus Mathematics Achievement Test

## DESCRIPTION

**Time: 3 hours**

**Numbers and Marks by Question Type**

	Selected Response	Constructed Response	Marks
<b>Booklet 1*</b>	–	17	33
<b>Booklet 2</b>	10	22	57
<b>Total</b>	10	39	<b>90</b>


\* The first 5 questions in *Booklet 1* require a calculator.   
You will have access to your calculator for the first 45 minutes of the test.

## GENERAL DIRECTIONS

- Read all instructions carefully.
- The perforated pages can be removed from the test booklet. No marks will be given for work done on these pages.
- The blank pages at the back of each booklet may be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- After 45 minutes, put away your calculator. Even though you may not have finished *Booklet 1*, *Booklet 2* will be distributed at this time. You may choose to continue working on *Booklet 1* or start working on *Booklet 2*, but you will no longer have access to your calculator.

# Instructions

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- There are 17 questions worth a total of 33 marks.
- Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- A calculator icon  appears next to the questions that require a calculator.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.

# Formula Sheet

$$s = \theta r$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\log_a (MN) = \log_a M + \log_a N$$

$$\log_a \left( \frac{M}{N} \right) = \log_a M - \log_a N$$

$$\log_a (M^n) = n \log_a M$$

$$P(n, r) \text{ or } {}_n P_r = \frac{n!}{(n-r)!}$$

$$C(n, r) \text{ or } {}_n C_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_n C_k a^{n-k} b^k$$

For  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Terminology Sheet

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Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are defined below.

**Describe:** Use words to provide the process or to report details of the response.

**Determine:** Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

**Evaluate:** Find the numerical value.

**Explain:** Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

**Identify/Indicate:** Recognize and select the answer by stating or circling it.

**Justify:** Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

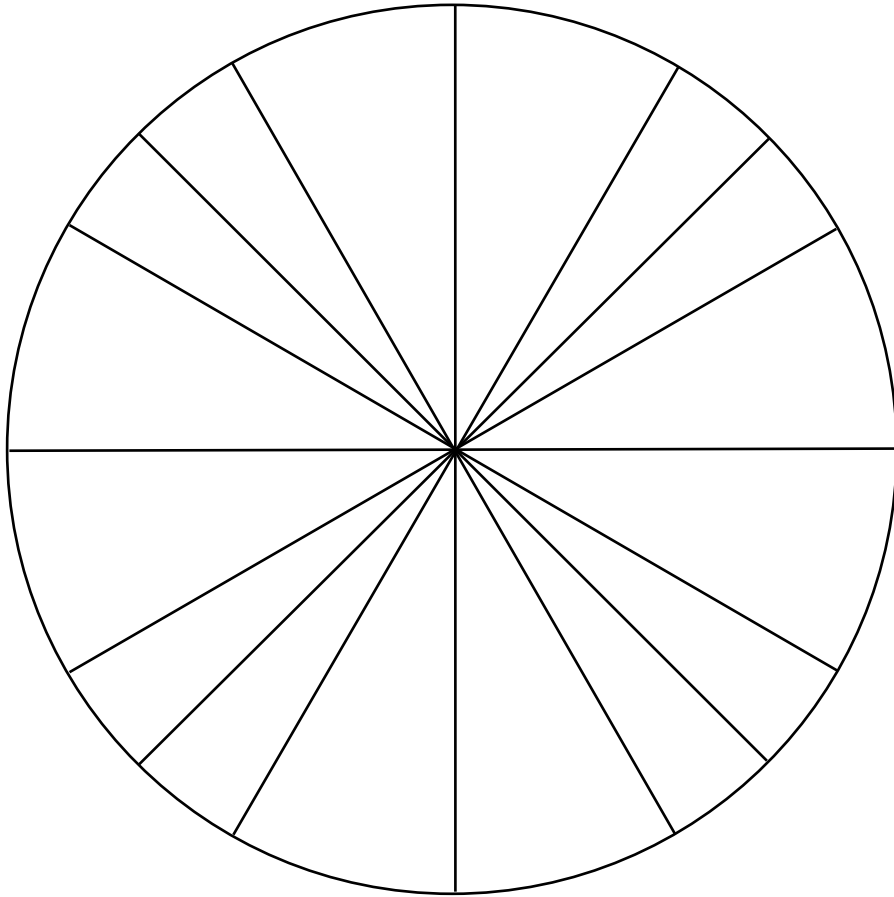
**Sketch the graph:** Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

**Solve:** Give a solution for a problem or determine the value(s) of a variable.

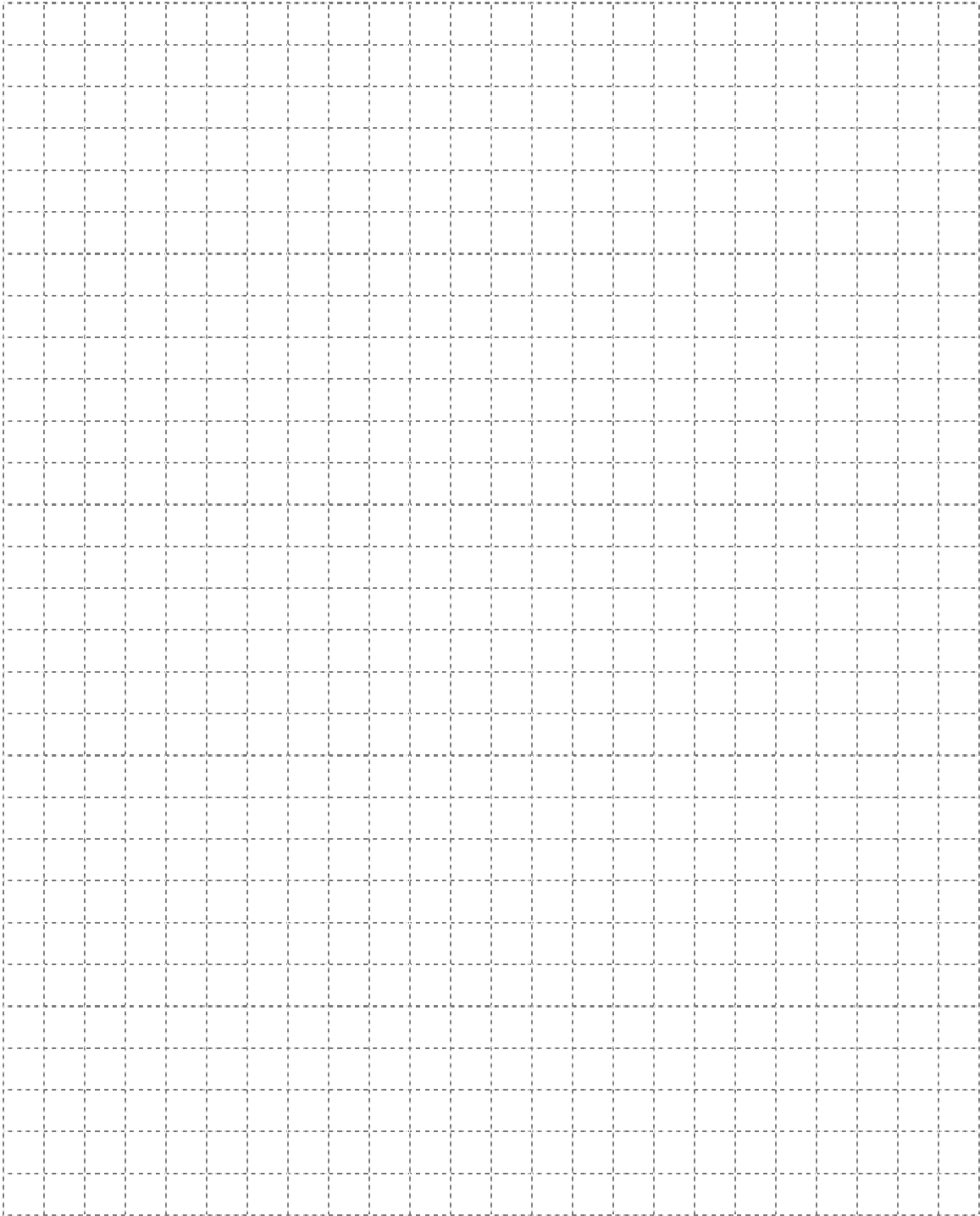
**State:** Give an answer without an explanation or justification.

**Verify:** Establish the truth of a statement by substitution or comparison.


No marks will be awarded for work done on this page.



No marks will be awarded for work done on this page.

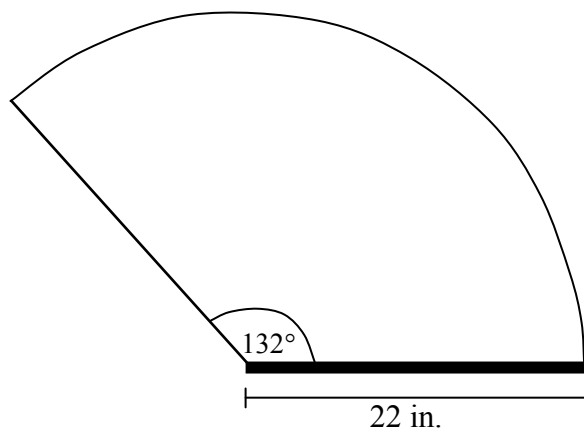





Question 1 

2 marks 101

A section of a car windshield is cleaned by a wiper as shown in the diagram below. The arm of the wiper is 22 inches, and it rotates through a central angle of  $132^\circ$ . Determine the length of the arc that is created by the tip of the wiper.



Question 2 

2 marks 102

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There are 20 boys and 11 girls who can be selected to be on a team.

Determine the number of ways that 7 boys and 5 girls can be selected for this team.

### Question 3

2 marks 103

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A water filtration system which removes impurities from a sample of water can be modelled by

$$P = 0.25(0.55)^n, \text{ where:}$$

$P$  = the percentage of impurities remaining, in decimal form

$n$  = the number of filters

Determine, algebraically, how many filters are required so that less than 1% of the impurities remain in the water sample. Express your answer as a whole number.

Question 4 

3 marks 104

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In the binomial expansion of  $\left(x^2 - \frac{2}{y}\right)^8$ , determine the middle term in simplified form.

Solve the following equation algebraically over the interval  $[0, 2\pi]$ .

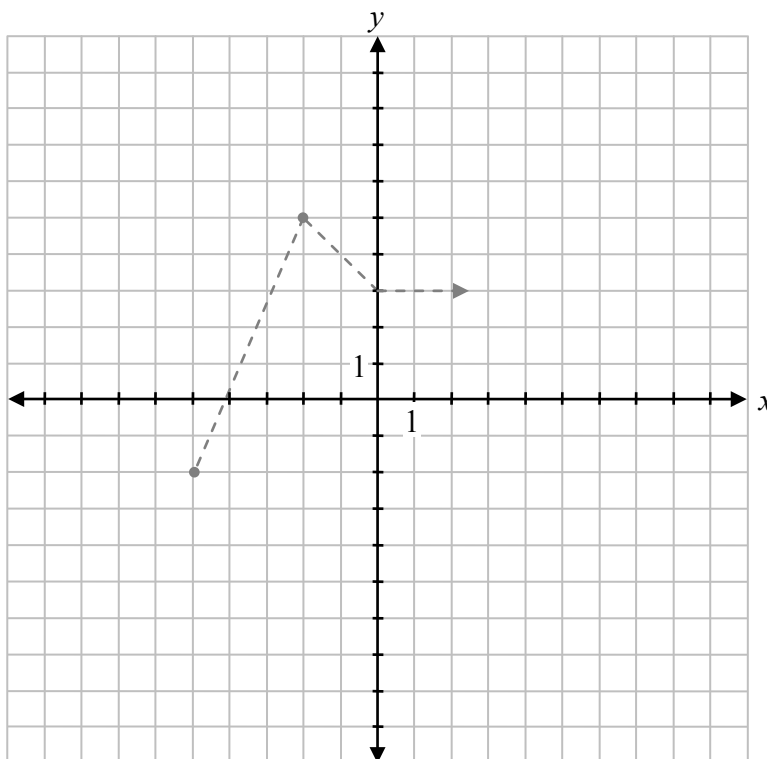
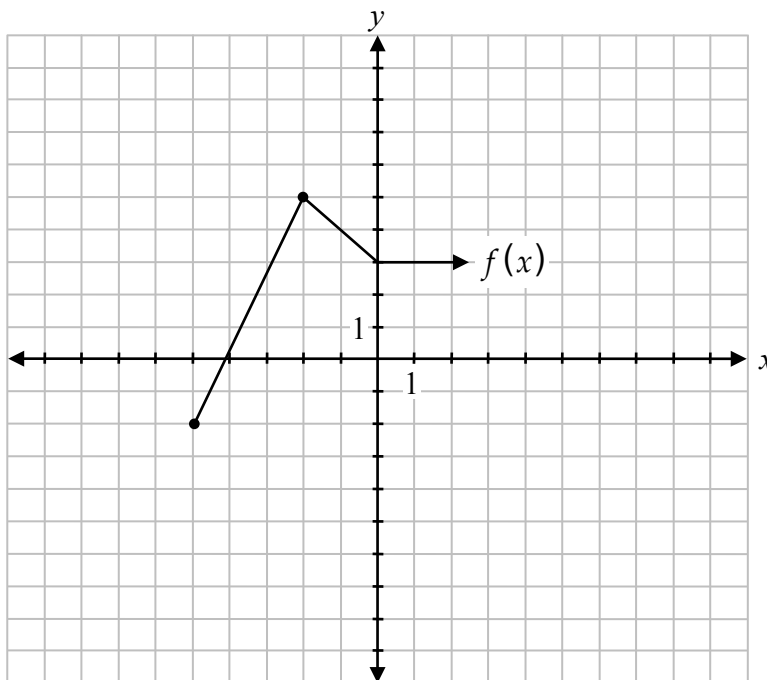
$$6 \sin^2 \theta + \sin \theta - 1 = 0$$

**Note: A calculator is not required for the remaining test questions.**

Question 6

2 marks 106

Given the graph of  $y = f(x)$ , sketch the graph of  $y = f(-x + 4)$ .

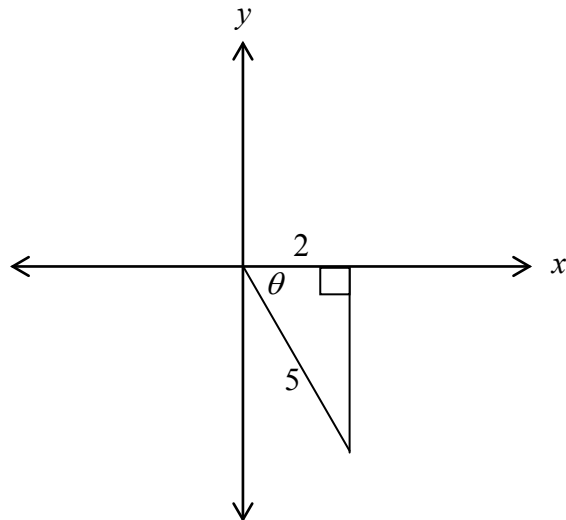


The graph of  $f(x)$  has already been drawn for your reference. No marks will be awarded for the graph of  $f(x)$ .

Question 7

2 marks 107

Given the following triangle, determine  $\csc \theta$ .



## Question 8

3 marks 108

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Solve algebraically.

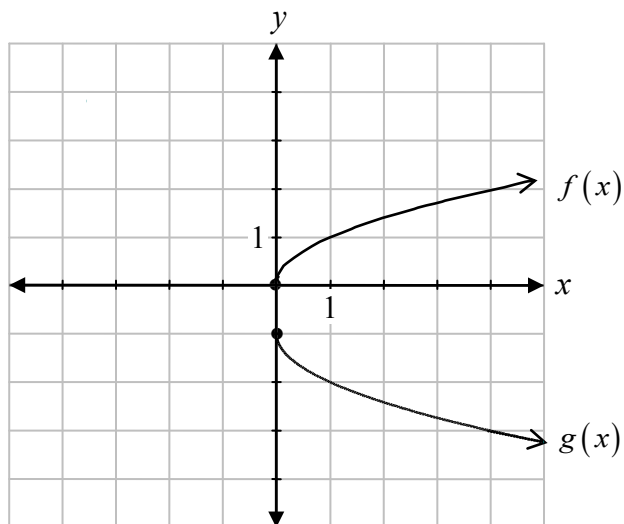
$${}_n P_2 = 9n$$



Question 9

2 marks 109

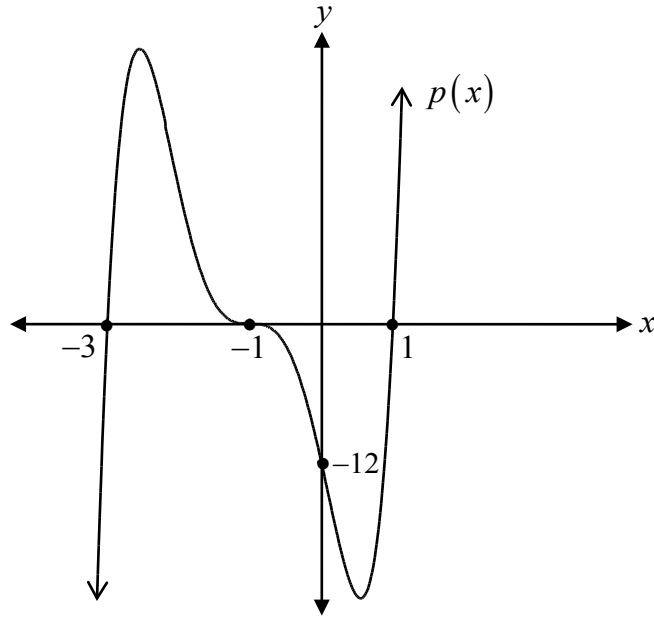
Describe the transformations applied to the graph of  $f(x)$  to obtain the graph of  $g(x)$ .



Question 10

2 marks 110

Determine, algebraically, the value of the leading coefficient of the graph of the polynomial function,  $p(x)$ .



## Question 11

1 mark <sup>111</sup>

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Frank, Liam, Chan, and Thao are going to a movie.

Determine the number of ways they can sit in a row of four chairs, if Frank and Chan must sit beside each other.

## Question 12

2 marks 112

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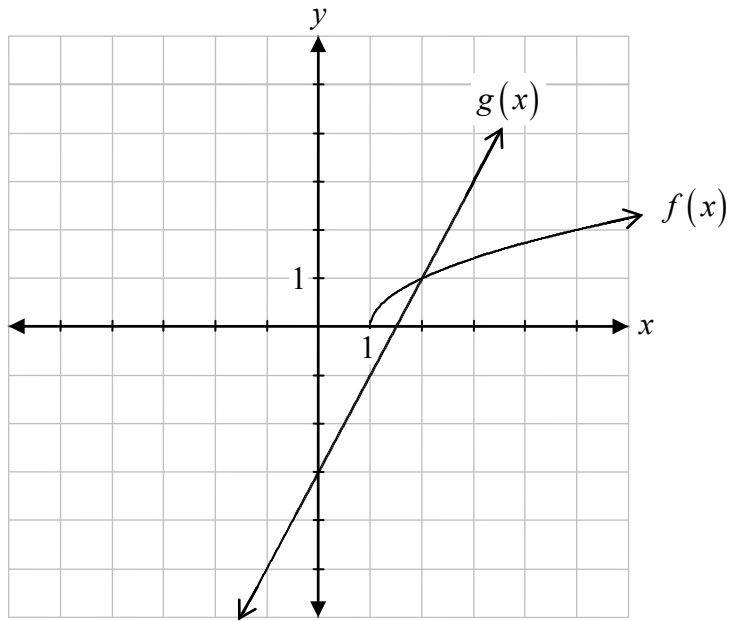
Determine, algebraically, if  $f(x) = \frac{1}{x+5}$  and  $g(x) = \frac{1}{x-5}$  are inverses of each other.

Justify your answer.

Question 13

1 mark 113

Using the graphs of  $y = f(x)$  and  $y = g(x)$ , solve  $f(x) = g(x)$ .



## Question 14

1 mark 114

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An angle in standard position measures  $\frac{3\pi}{4}$ .

Determine in which quadrant the terminal arm of this angle is located after a rotation of 3 radians.

Justify your answer.

Prove the following identity for all permissible values of  $\theta$ .

$$\frac{\sin 2\theta}{1 - \cos 2\theta} = \cot \theta$$

Left-Hand Side	Right-Hand Side

Question 16

1 mark 116

If the range of  $y = f(x)$  is  $-3 \leq y \leq 6$ , determine the range of  $y = 2f(3x)$ .

Question 17

1 mark 117

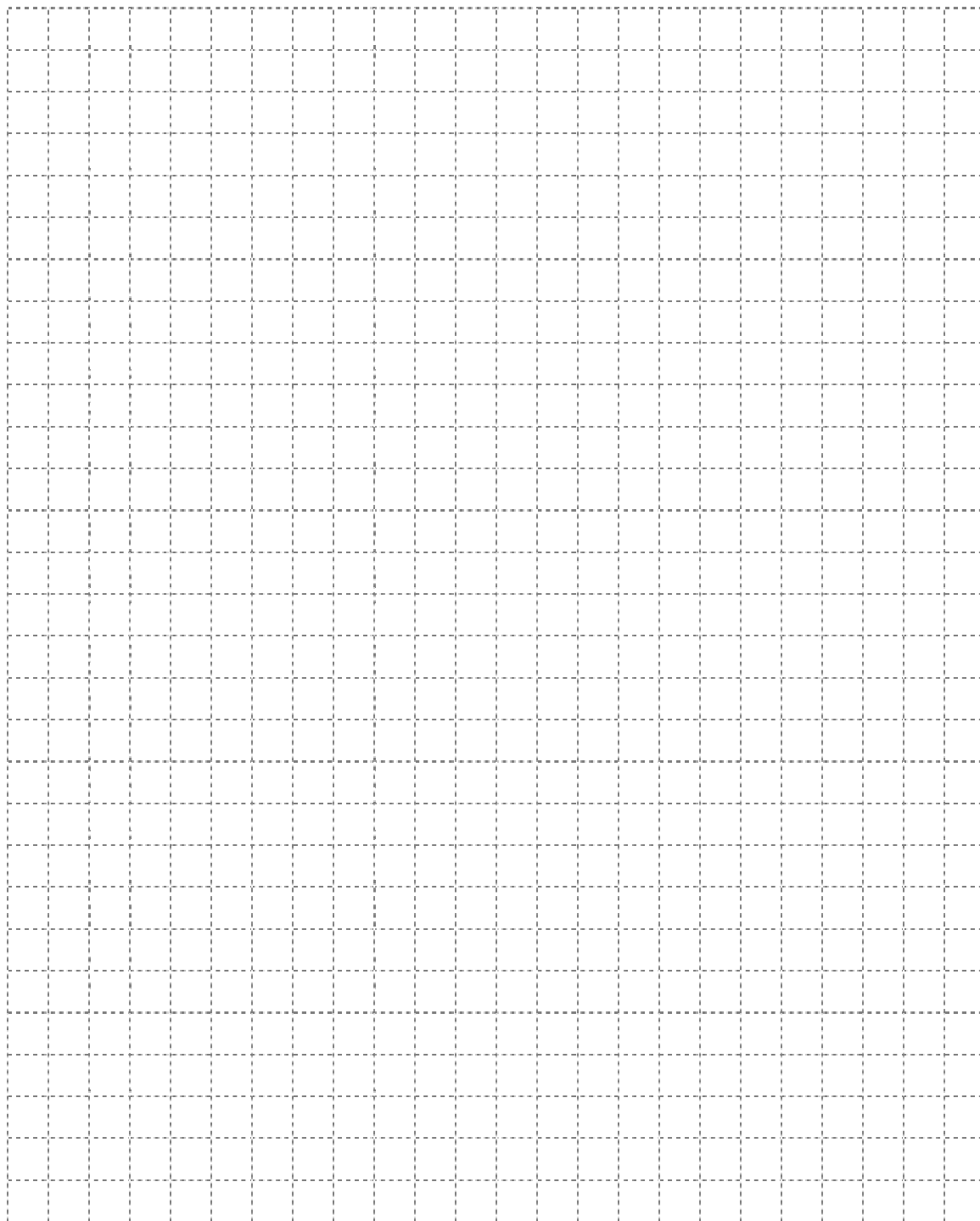
Maurice incorrectly solved the equation,  $\sin \theta + 1 = 0$ , over the interval  $[0^\circ, 360^\circ]$ .

$$\begin{aligned}\sin \theta + 1 &= 0 \\ \sin \theta &= -1 \\ \sin \theta &= 270^\circ\end{aligned}$$

Describe his error.



No marks will be awarded for work done on this page.



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