Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet I

January 2024

Grade 12 pre-calculus mathematics achievement test. Booklet 1. January 2024

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Available in alternate formats upon request.

## Grade 12 Pre-Calculus Mathematics Achievement Test

## DESCRIPTION

Time Required to Complete the Test: 3 hours
Additional Time Allowed: $\mathbf{3 0}$ minutes
Numbers and Marks by Question Type

|  | Selected <br> Response | Constructed <br> Response | Marks |
| :---: | :---: | :---: | :---: |
| Booklet $\mathbf{1}^{*}$ | - | $\mathbf{1 5}$ | $\mathbf{3 3}$ |
| Booklet 2 | 8 | 24 | 57 |
| Total | 8 | 39 | $\mathbf{9 0}$ |

* The first 5 questions with the symbol 排活 in Booklet 1 require a scientific calculator. You will have access to your calculator for the first 45 minutes of the test.

Note that diagrams and graphs provided in the test booklets may not be drawn to scale.

## DIRECTIONS

- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited. Please turn off your cell phone and all other such devices.

No marks will be awarded for work done on this page.

## Formula Sheet

$s=\theta r$
$\sin ^{2} \theta+\cos ^{2} \theta=1$
$\tan ^{2} \theta+1=\sec ^{2} \theta$
$1+\cot ^{2} \theta=\csc ^{2} \theta$
$\sin (\alpha-\beta)=\sin \alpha \cos \beta-\cos \alpha \sin \beta$
$\cos (\alpha-\beta)=\cos \alpha \cos \beta+\sin \alpha \sin \beta$
$\tan (\alpha-\beta)=\frac{\tan \alpha-\tan \beta}{1+\tan \alpha \tan \beta}$
$\sin (\alpha+\beta)=\sin \alpha \cos \beta+\cos \alpha \sin \beta$
$\cos (\alpha+\beta)=\cos \alpha \cos \beta-\sin \alpha \sin \beta$
$\tan (\alpha+\beta)=\frac{\tan \alpha+\tan \beta}{1-\tan \alpha \tan \beta}$
$\sin 2 \alpha=2 \sin \alpha \cos \alpha$
$\cos 2 \alpha=\cos ^{2} \alpha-\sin ^{2} \alpha$
$\cos 2 \alpha=1-2 \sin ^{2} \alpha$
$\cos 2 \alpha=2 \cos ^{2} \alpha-1$
$\tan 2 \alpha=\frac{2 \tan \alpha}{1-\tan ^{2} \alpha}$
$\log _{a}(M N)=\log _{a} M+\log _{a} N$
$\log _{a}\left(\frac{M}{N}\right)=\log _{a} M-\log _{a} N$
$\log _{a}\left(M^{n}\right)=n \log _{a} M$
$P(n, r)$ or ${ }_{n} P_{r}=\frac{n!}{(n-r)!}$
$C(n, r)$ or ${ }_{n} C_{r}=\frac{n!}{r!(n-r)!}$
$t_{k+1}={ }_{n} C_{k} a^{n-k} b^{k}$

Given $a x^{2}+b x+c=0$,
$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

## Terminology Sheet

Some questions may contain directing words such as explain, identify, and justify. These words are defined below.

Describe: Use words to provide the process or to report details of the response.
Determine: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

Evaluate: Find the numerical value.
Explain: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

Identify: Recognize and select the answer by stating or circling it.
Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Solve: Give a solution for a problem or determine the value(s) of a variable.
State: Give an answer without an explanation or justification.
Verify: Establish the truth of a statement by substitution or comparison.

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A pendulum that is 35 cm long swings through an angle of $50^{\circ}$. Determine the length of the arc through which the pendulum swings.


Solve algebraically, where $0 \leq \theta \leq 2 \pi$.

$$
2 \cos ^{2} \theta=\sin ^{2} \theta-2 \cos \theta
$$

Determine the number of arrangements of the letters in the word ATTENTION which begin with the letter A.

## Question 4

3 marks
Solve for $x$, algebraically.

$$
e^{2 x+1}=5^{x}
$$

There are 10 teachers and 17 students who would like to attend a field trip.
Determine the number of ways that 3 teachers and 9 students can be selected given that Mr. Jones and Mrs. Carol, two of the teachers, must be selected to attend the field trip.

## Note: A calculator is not required for the remaining test questions.

Given the graph of $f(x)$, sketch the graph of $y=-f(2 x)$.



The graph of $f(x)$ has already been drawn for your reference.

No marks will be awarded for the graph of $f(x)$

There are 5 roads between Anneville and Berrybourg, and 2 roads between Berrybourg and Carriton.

Determine how many ways Blake can travel from Anneville to Carriton and back to Anneville, given the following conditions:

- he must travel through Berrybourg in both directions
- he cannot use the same road twice

Determine which term contains $x^{0}$ in the binomial expansion of $\left(x^{2}+\frac{1}{x}\right)^{6}$.

Given $f(x)=x^{3}+1$, determine the equation of $f^{-1}(x)$.

Guillermo was asked to determine the number of ways to select a president, a vice president, and a treasurer from a group of 11 people.

His solution: ${ }_{11} C_{3}$.
Explain why he should have used a permutation instead of a combination.

Prove the following identity for all permissible values of $x$.

$$
\frac{\csc ^{2} x \sec x}{\tan x+\cot x}=\csc x
$$

| Left-Hand Side | Right-Hand Side |
| :--- | :--- |

Determine the value of $x$, algebraically.

$$
5 \log _{a} 2-\frac{1}{4} \log _{a} 16=\log _{a} x
$$

Tamara must determine the factors of $x^{4}-13 x+2 x^{3}-14 x^{2}+24$.
Explain why the coefficients Tamara used to set up her synthetic division are not written correctly.


Determine the equation of the graph of $g(x)$ in terms of $f(x)$.

$g(x)=$ $\qquad$

## Question 15

Expand, using the laws of logarithms.

$$
\log _{2}\left[\frac{(x-1)(x-2)}{x}\right]
$$

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