Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

January 2020

Manitoba

Grade 12 pre-calculus mathematics achievement test.
Booklet 1. January 2020

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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre.

Order online at www.manitobalrc.ca.

This resource will also be available on the Manitoba Education website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time Required to Complete the Test: 3 hours

Additional Time Allowed: 30 minutes

Numbers and Marks by Question Type

	Selected Response	Constructed Response	Marks
Booklet 1*	—	14	34
Booklet 2	8	21	56
Total	8	35	**90**

* The first 4 questions with the symbol  in *Booklet 1* require a scientific calculator. You will have access to your calculator for the first 45 minutes of the test.

Note that diagrams and graphs provided in the test booklets may not be drawn to scale.

DIRECTIONS

- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited. Please turn off your cell phone and all other such devices.

No marks will be awarded for work done on this page.

Formula Sheet

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n\log_a M$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k}b^k$$

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are defined below.

Describe: Use words to provide the process or to report details of the response.

Determine: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

Identify: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Solve: Give a solution for a problem or determine the value(s) of a variable.

State: Give an answer without an explanation or justification.

Verify: Establish the truth of a statement by substitution or comparison.

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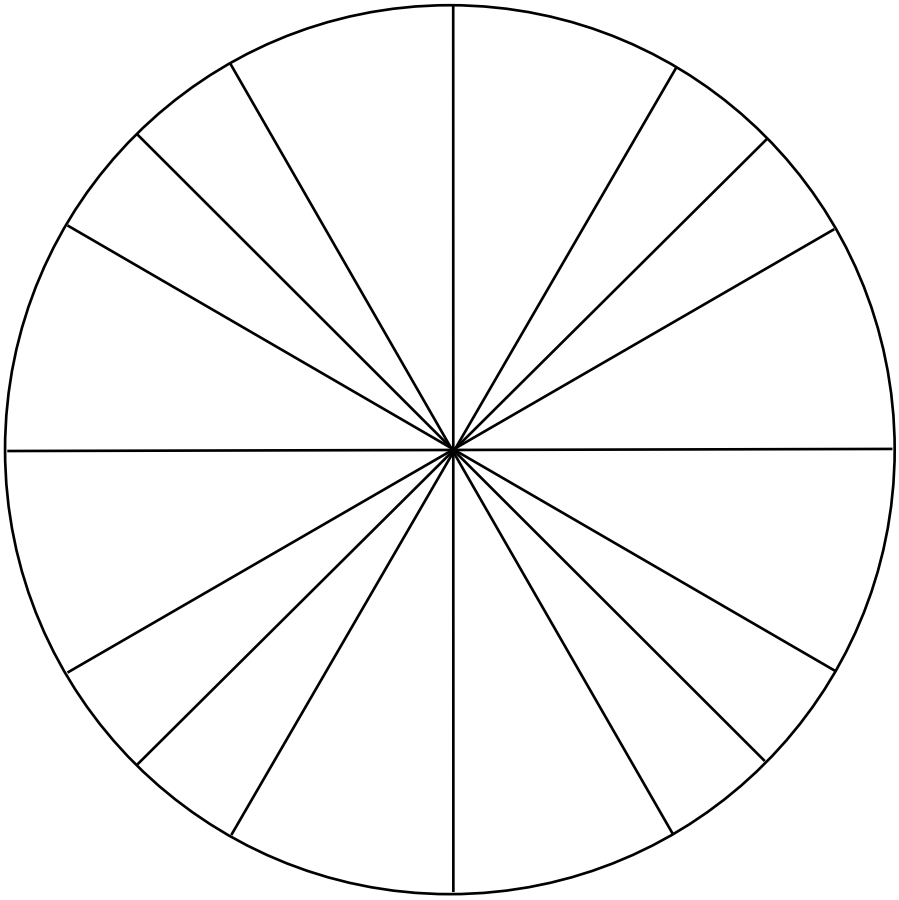

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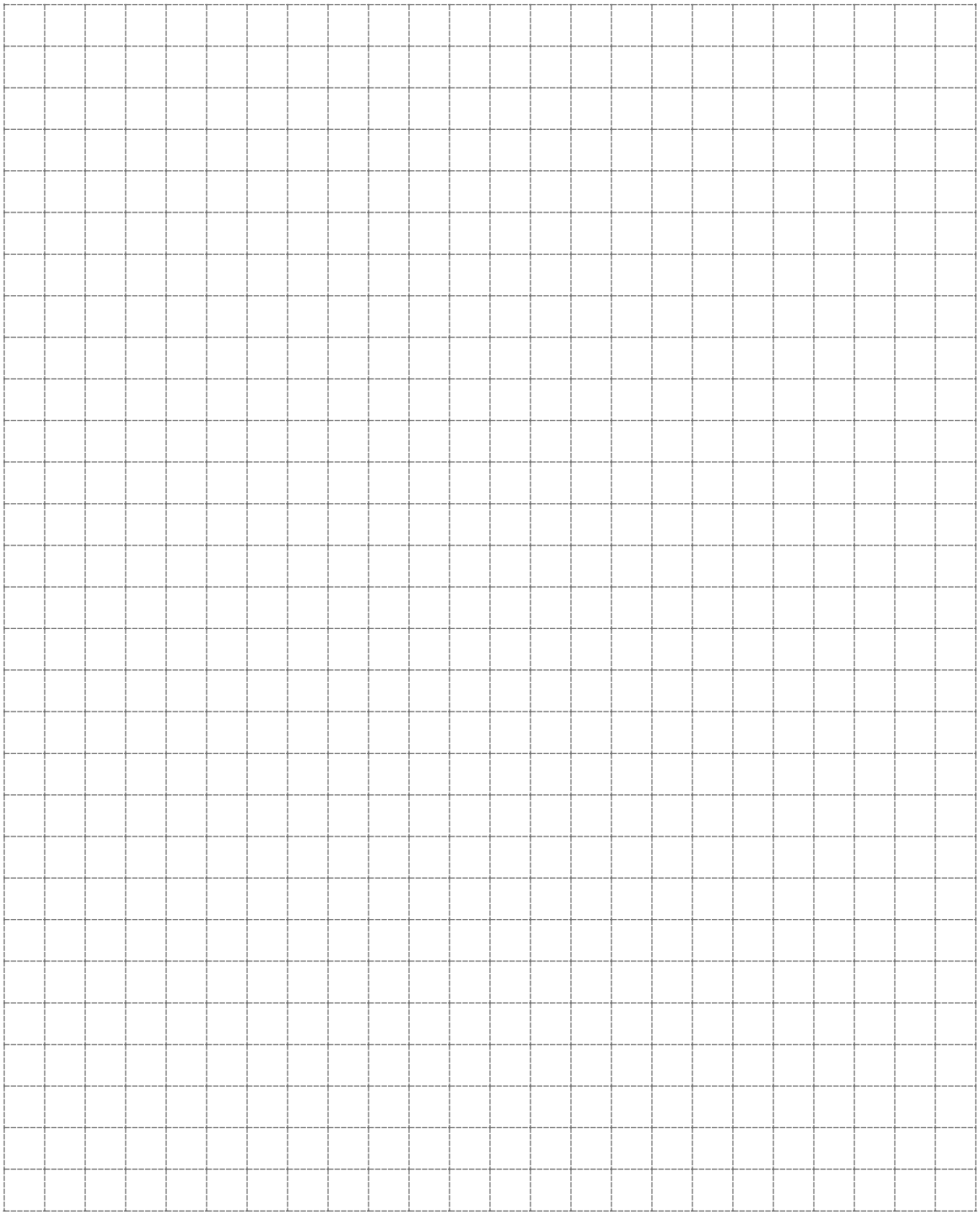


Question 1  3 marks 101

Determine and simplify the 8th term in the binomial expansion of $\left(x-\frac{2}{x^3}\right)^{10}$.

Question 2  2 marks 102

The temperature of hot chocolate is recorded as it cools down. The data shows that the temperature cools according to the equation:

$$T = 82(0.87)^t + 16$$

where T is the temperature of the hot chocolate, in degrees Celsius after t minutes and

t is the time, in minutes, after the hot chocolate is made.

In order to avoid burns, hot chocolate should not be served at a temperature higher than 71°C. Determine, algebraically, the amount of time it will take the hot chocolate to reach this temperature.

Question 3

3 marks 103

Solve for θ, algebraically, over the interval $[0, 2\pi]$.

$$3\sin^2\theta + 6\sin\theta + 2 = 0$$

Question 4  3 marks 104

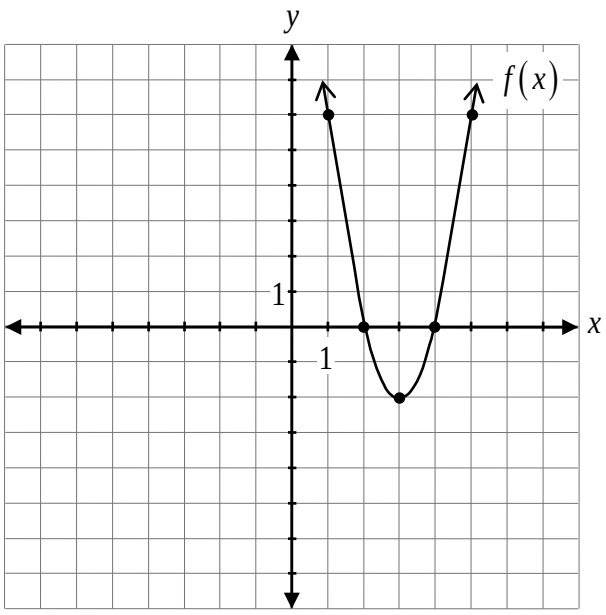
A committee of 4 people is to be selected from 6 high school students and 5 middle years students.

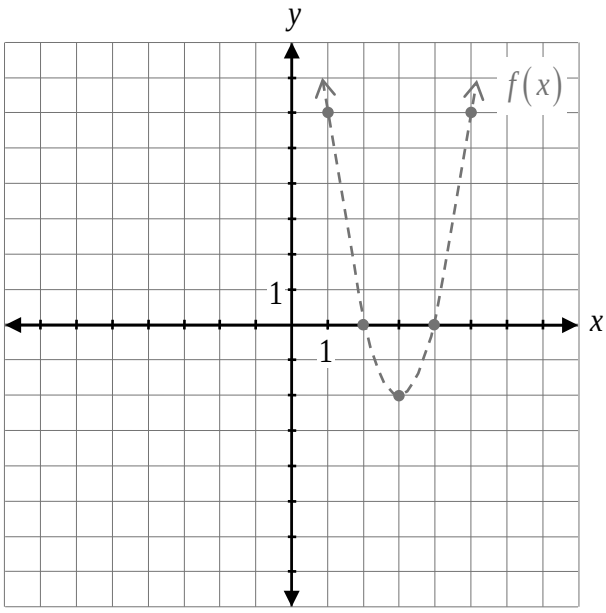
Determine the number of possible committees if the committee must have at least 3 high school students.

Note: A calculator is not required for the remaining test questions.

Question 5 — 2 marks — 105

Given the graph of $y = f(x)$, sketch the graph of $y = -|f(x)|$.





The graph of $f(x)$ has already been drawn for your reference.

No marks will be awarded for the graph of $f(x)$.

Question 6 — 3 marks — 106

Describe the transformations used to obtain the graph of the function $y = 4f\left(\frac{x}{2}\right) - 3$ from the graph of $y = f(x)$.

Question 7 — 2 marks — 107

If $\log 4 = m$ and $\log 3 = n$, express $\log 48$ in terms of m and n.

Question 8 — 2 marks — 108

State the domain and range of the radical function, $f(x) = -3\sqrt{x-8}+1$.

Domain:_____

Range: _____

Question 9 — 1 mark — 109

Explain why the graph of $y = \tan x$ does not have an amplitude.

Question 10 — 3 marks — 110

Prove the following identity for all permissible values of θ.

$$\frac{\sec\theta - \sin^2\theta\sec\theta}{\tan\theta\sin\theta} = \csc^2\theta - 1$$

Left-Hand Side	Right-Hand Side

Question 11 — 2 marks — 111

Given that $x = 3$ is one of the zeros of $p(x) = x^3 - 7x - 6,$ express $p(x)$ in completely factored form.

$p(x) =$ _____

Question 12 — 3 marks — 112

Solve, algebraically.

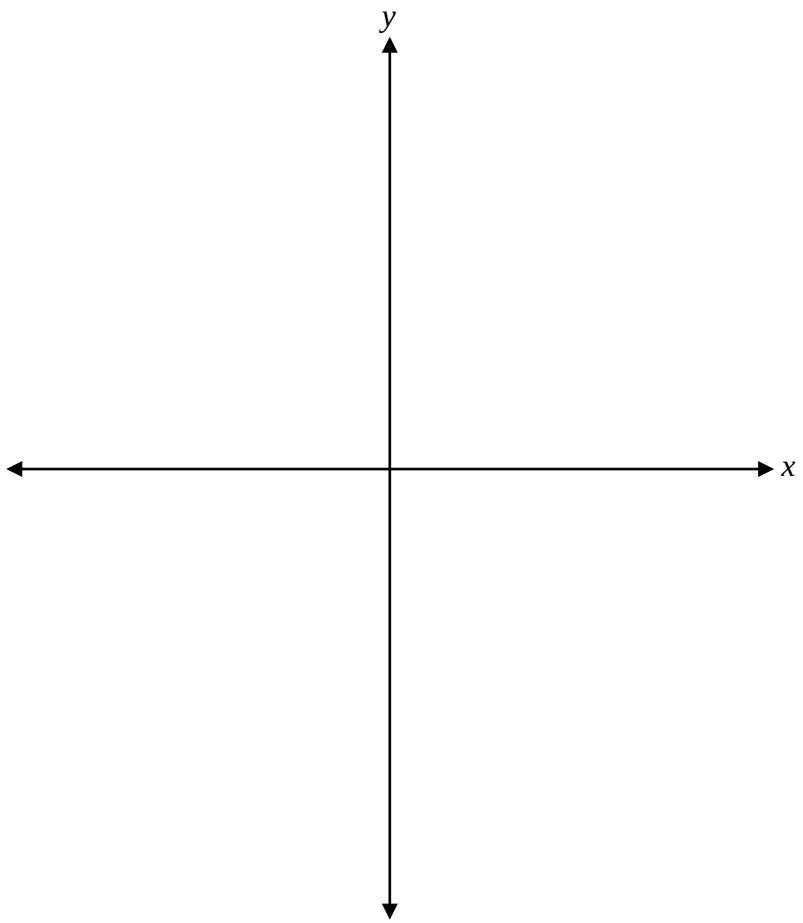
$${}_{n+1}P_2 = 6$$

Question 13 — 2 marks — 113

Express $\log(3x-1) - \log x + \log 9$ as a single logarithm.

Question 14 — 3 marks — 114

Sketch the graph of $P(x) = -(x-2)^2(x+3)$.



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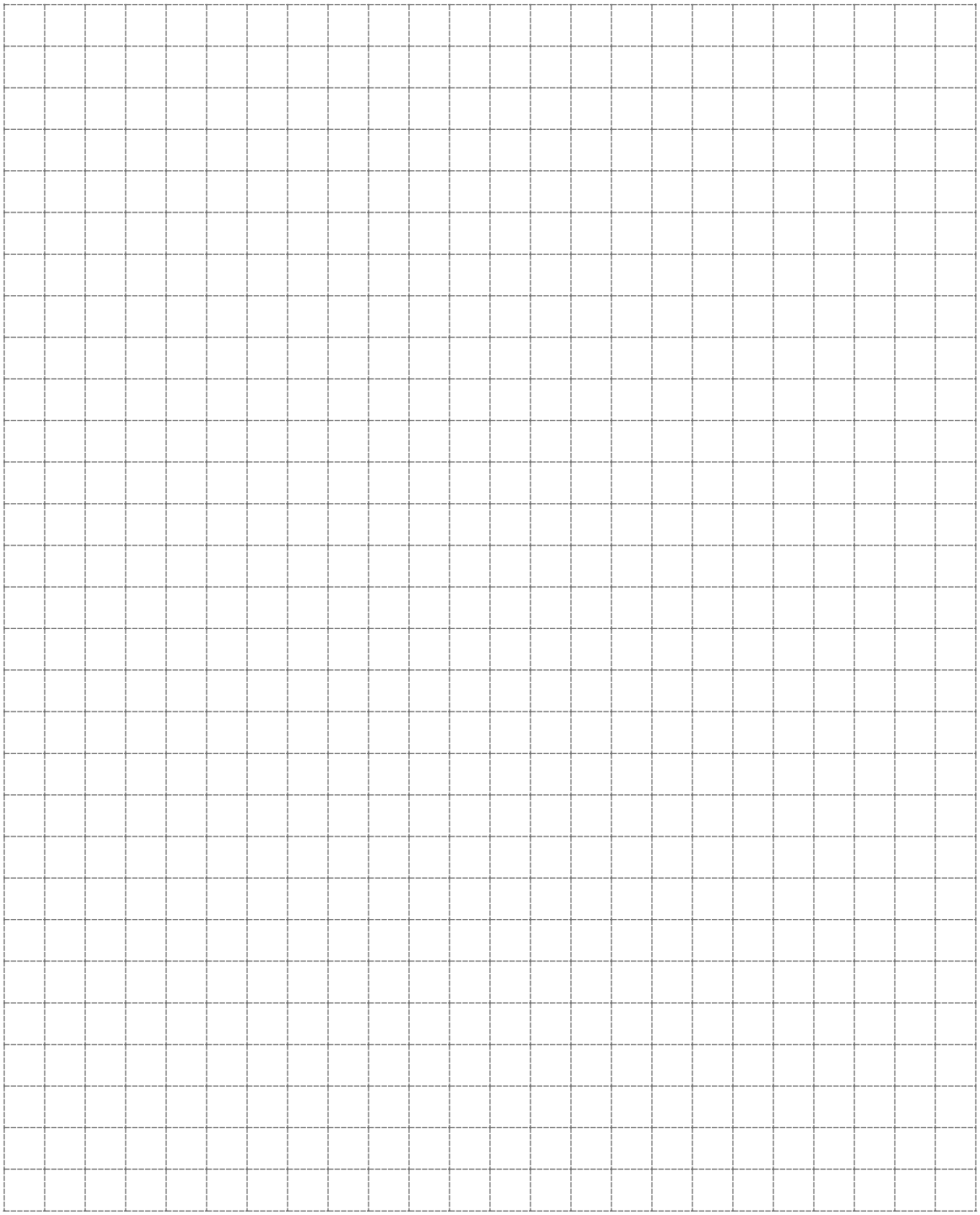

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