Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

January 2019

Manitoba

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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre. Order online at www.manitobalrc.ca.

This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time: 3 hours

Numbers and Marks by Question Type

	Selected Response	Constructed Response	Marks
Booklet 1*	—	17	33
Booklet 2	10	22	57
Total	10	39	**90**

* The first 5 questions in *Booklet 1* require a calculator.  You will have access to your calculator for the first 45 minutes of the test.

Note that diagrams and graphs provided in the test booklets may not be drawn to scale.

DIRECTIONS

- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited. Please turn off your cell phone and all other such devices.

No marks will be awarded for work done on this page.

Formula Sheet

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n\log_a M$$

$P(n,r)$ or ${}_nP_r = \dfrac{n!}{(n-r)!}$

$C(n,r)$ or ${}_nC_r = \dfrac{n!}{r!(n-r)!}$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are defined below.

Describe: Use words to provide the process or to report details of the response.

Determine: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

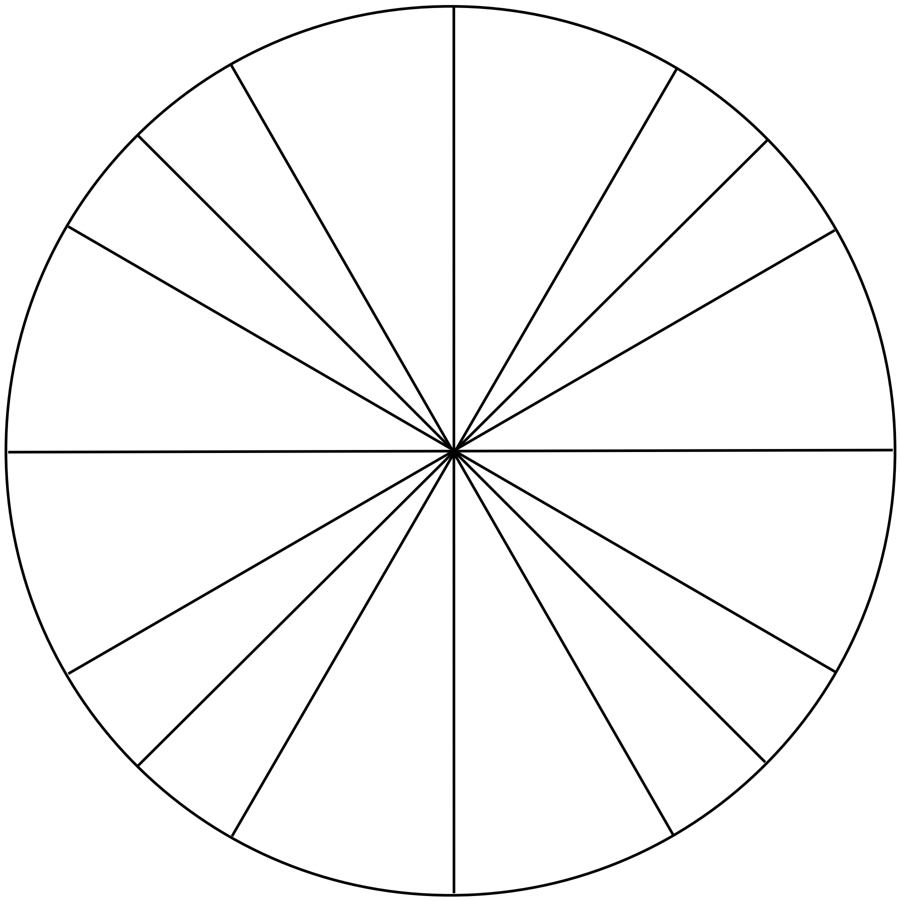
Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Solve: Give a solution for a problem or determine the value(s) of a variable.

State: Give an answer without an explanation or justification.

Verify: Establish the truth of a statement by substitution or comparison.

No marks will be awarded for work done on this page.



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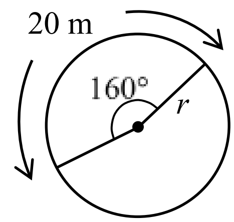
Question 1 2 marks 101

Determine the length of the radius, r, given an arc length of 20 metres and a central angle of 160°.



Question 2 — 1 mark 102

There are eight cars parked in a row. Determine the number of possible arrangements of these eight cars if Mrs. Jones must always park in the third spot and Mr. Rodriguez must always park in the last spot.

Question 3 3 marks 103

Bill wins \$1 300 000 in a lottery and invests the entire amount at an annual interest rate of 2.5% compounded quarterly. He will withdraw \$10 000 at the end of every three months.

Determine, algebraically, the total number of withdrawals, including the partial amount, that Bill can make until there is no money left. Express your answer as a whole number.

Use the formula:

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

where PV = the present value deposited

R = the amount of each withdrawal

n = the number of equal withdrawals

$i = \dfrac{\text{the annual interest rate (in decimal form)}}{\text{the number of compounding periods}}$

Question 4 — 3 marks 104

Determine and simplify the 12th term in the binomial expansion of $\left(x^3 - \frac{1}{2x^2}\right)^{12}$.

Question 5 3 marks 105

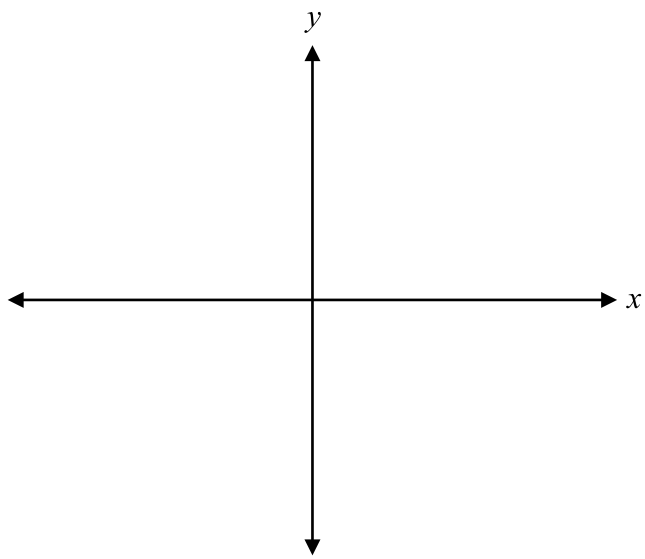
Solve, algebraically.

$$e^{2x-3} = 7^{x+1}$$

Note: A calculator is not required for the remaining test questions.

Question 6 — 1 mark 106

Sketch the angle $\frac{7\pi}{3}$ in standard position.



Question 7 3 marks

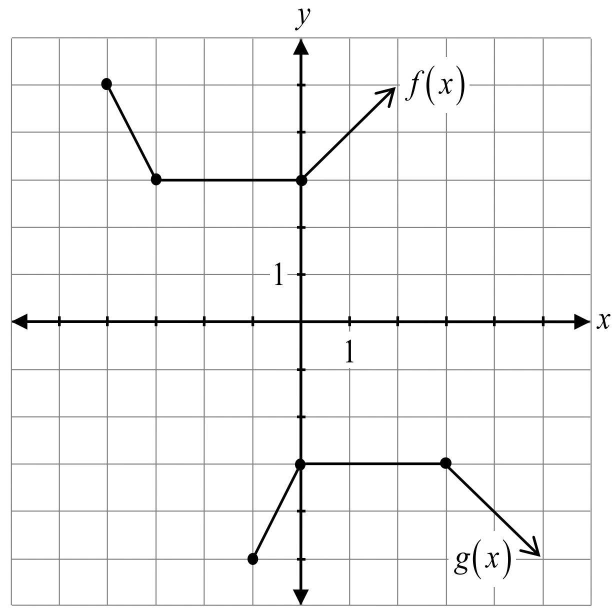
Determine, algebraically, all of the zeros of the polynomial function $P(x) = x^4 - 5x^3 - 4x^2 + 20x$.

Question 8 1 mark 108

Justify why four of the terms in the binomial expansion of $(-x+y)^6$ are positive.

Question 9 2 marks 109

Determine the equation of $g(x)$ in terms of $f(x)$.



$g(x) =$ ______________________________

Question 10 — 3 marks — 110

Prove the identity for all permissible values of x.

$$\frac{\sin x + \tan x}{\cot x + \csc x} = \frac{\sin^2 x}{\cos x}$$

Left-Hand Side	Right-Hand Side

Question 11 1 mark 111

Given that the point $(-2,1)$ is on the graph of $y = f(x)$, describe how the coordinates of the corresponding point on the graph of $y = f(4x)$ are different.

Question 12 — 3 marks 112

Using the laws of logarithms, completely expand the expression:

$$\log\left(\frac{x^2\sqrt{y}}{w-1}\right)$$

Question 13 — 1 mark 113

Given $\sec\theta = -\frac{5}{4}$ and $\tan\theta > 0,$ state the quadrant in which θ terminates.

Justify your answer.

Question 14 2 marks 114

State an equation of a rational function that has a vertical asymptote at $x = -8$ and a horizontal asymptote at $y = 9$.

Question 15 1 mark 115

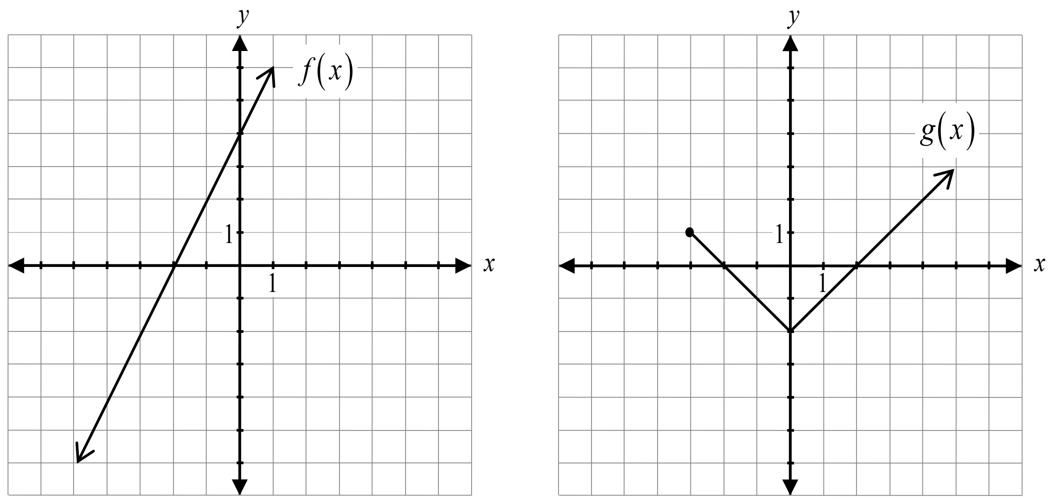
Given the point $(5, 1)$, state the coordinates of the corresponding point after a reflection across the line $y = x$.

Question 16 1 mark 116

Simplify.

$$\frac{(n-13)!}{(n-12)!}$$

Given the graphs of $y = f(x)$ and $y = g(x)$,



a) determine the value of $f\left(g(2)\right)$.

b) determine the value of $(g - f)(-3)$.

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