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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre (formerly the Manitoba Text Book Bureau). Order online at www.mtbb.mb.ca.

This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.
DESCRIPTION

Time: 3 hours

Numbers and Marks by Question Type

<table>
<thead>
<tr>
<th></th>
<th>Selected Response</th>
<th>Constructed Response</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Booklet 1*</td>
<td>—</td>
<td>18</td>
<td>37</td>
</tr>
<tr>
<td>Booklet 2</td>
<td>9</td>
<td>19</td>
<td>53</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>37</td>
<td>90</td>
</tr>
</tbody>
</table>

* The first 4 questions in Booklet 1 require a calculator.
  You will have access to your calculator for the first 45 minutes of the test.

GENERAL DIRECTIONS

- Read all instructions carefully.
- The perforated pages can be removed from the test booklet. No marks will be given for work done on these pages.
- The blank pages at the back of each booklet may be used as scrap paper, but must not be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- After 45 minutes, put away your calculator. Even though you may not have finished Booklet 1, Booklet 2 will be distributed at this time. You may choose to continue working on Booklet 1 or start working on Booklet 2, but you will no longer have access to your calculator.
Instructions

- There are 18 questions worth a total of 37 marks.
- Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- A calculator icon appears next to the questions that require a calculator.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.
Formula Sheet

\[ s = \theta r \]

\[ \sin^2 \theta + \cos^2 \theta = 1 \]

\[ \tan^2 \theta + 1 = \sec^2 \theta \]

\[ 1 + \cot^2 \theta = \csc^2 \theta \]

\[ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \]

\[ \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \]

\[ \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \]

\[ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \]

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \]

\[ \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \]

\[ \sin 2\alpha = 2 \sin \alpha \cos \alpha \]

\[ \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \]

\[ \cos 2\alpha = 1 - 2 \sin^2 \alpha \]

\[ \cos 2\alpha = 2 \cos^2 \alpha - 1 \]

\[ \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \]

\[ \log_a (MN) = \log_a M + \log_a N \]

\[ \log_a \left( \frac{M}{N} \right) = \log_a M - \log_a N \]

\[ \log_a (M^n) = n \log_a M \]

\[ P(n, r) = \frac{n!}{(n-r)!} \]

\[ C(n, r) = \frac{n!}{r!(n-r)!} \]

\[ t_{k+1} = nC_k a^{n-k} b^k \]

For \( ax^2 + bx + c = 0 \),

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
Some questions may contain directing words such as explain, identify, and justify. These words are defined below.

**Describe**: Use words to provide the process or to report details of the response.

**Determine**: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

**Evaluate**: Find the numerical value.

**Explain**: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

**Identify/Indicate**: Recognize and select the answer by stating or circling it.

**Justify**: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

**Sketch the graph**: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

**Solve**: Give a solution for a problem or determine the value(s) of a variable.

**State**: Give an answer without an explanation or justification.

**Verify**: Establish the truth of a statement by substitution or comparison.
No marks will be awarded for work done on this page.
No marks will be awarded for work done on this page.
There are 24 different movies Kiandra can download to her computer. Determine the number of ways she can select 15 movies.
Given $\theta = 40^\circ$,

a) convert $\theta$ to radians.

b) determine the coterminal angles of $\theta$ where $\theta \in \mathbb{R}$.
Peter invests $560 per month at an annual interest rate of 4.2%, compounded monthly. Determine how many monthly investments he will need to make to obtain at least $500 000.

Express your answer as a whole number.

Use the formula:

\[
FV = \frac{R \left[ (1 + i)^n - 1 \right]}{i}
\]

where

- \( FV \) = the future value
- \( R \) = the investment amount each period
- \( i \) = the annual interest rate
- \( n \) = the number of investments
- \( \frac{i}{m} \) = the number of compounding periods per year

\[
FV = \frac{\left( 1 + \frac{0.042}{12} \right)^n - 1}{\frac{0.042}{12}} \cdot 560
\]

Solve for \( n \) to determine the number of months needed.
Ishmael has 4 dogs, 5 cats, and 3 horses. If he arranges all of them in a row, determine how many ways they can be arranged if each type of animal must be grouped together.

Note: A calculator is not required for the remaining test questions.
Question 5

Solve the following equation algebraically over the interval \( 0 \leq \theta \leq 2\pi \).

\[ 2\cos^2 \theta + 9\cos \theta - 5 = 0 \]
Question 6  

2 marks

Determine which term contains \( \frac{1}{x^6} \) in the binomial expansion of \( \left( \frac{2}{x^3} + 3x^2 \right)^7 \).
Determine the radius of a circle which has an arc length of 5 cm with a central angle of 3 radians.
Tyler incorrectly sketched the angle $\theta = -\frac{7\pi}{4}$ in standard position.

Describe his error.
Question 9

Given the identity \( \sec \theta + \cos \theta = \frac{2 - \sin^2 \theta}{\cos \theta} \),

a) determine the non-permissible values of \( \theta \), over the interval \( 0 \leq \theta \leq 2\pi \).

b) prove the identity for all permissible values of \( \theta \).
Expand using the laws of logarithms.

$$\log\left(\frac{a}{b^4}\right)$$
State the equation of \( g(x) \) in terms of \( f(x) \).

\[
g(x) = \underline{\quad} \]

\[
g(x) = \underline{\quad} \]
Question 12  

1 mark

Explain why the inverse of the graph of $y = f(x)$ is not a function.
Solve the following equation algebraically:

$$\log(x^2 + 5) - \log(x^2 + 1) = \log 3$$
Describe the transformations used to obtain the graph of the function \( y = 5f(x + 1) \) from the graph of \( y = f(x) \).
Question 15

Solve algebraically:

\[ \binom{n}{2} = 3n + 4 \]
Given the graph of \( f(x) \), sketch the graph of \( y = \frac{1}{2} f(x - 1) \).
Question 17

Explain why \( f(x) = (x + 2)^{\frac{1}{2}} (x - 1)^{\frac{1}{2}} \) is not a polynomial function.
Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $h(x) = (f \cdot g)(x)$. 

\begin{itemize}
  \item The graph of $f(x)$.
  \item The graph of $g(x)$.
\end{itemize}
No marks will be awarded for work done on this page.
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