Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

June 2026

Manitoba

Grade 12 Pre-Calculus Mathematics Achievement Test:
Marking Guide (June 2026)

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Manitoba Education and Early Childhood Learning
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This resource will be available on the Manitoba Education and Early Childhood Learning website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Contents

General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Early Childhood Learning in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth one to five marks each, and selected response questions are worth one mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. The appendix provides examples of such irregularities as well as procedures to follow to report irregularities.

If a *Scoring Sheet* is marked with "0" only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Early Childhood Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION

Shade in the circles below for a maximum total deduction of 5 marks (½ mark deduction per error).
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).

Example: Marks assigned to the student

Marks Awarded	Booklet 1 25	Selected Response 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**Maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

Question 1 T1

Jack rolled a water bottle from one edge of his desk to the other. The desk measures 60 cm from edge to edge. The water bottle has a diameter of 7 cm. Determine the angle that the water bottle rotated, in degrees.

Solution

$s = \theta r$

$\theta = \dfrac{s}{r}$

$\theta = \dfrac{60}{3.5}$ 1 mark for substitution

$\theta = 17.142\,857...$

$\theta = (17.142\,857...)\left(\dfrac{180^\circ}{\pi}\right)$ 1 mark for conversion

$\theta = 982.213\,363...^\circ$

$\theta = 982.213^\circ$

2 marks

Exemplar 1

$S = \theta r$ Water Bottle

$r = 3.5cm$

$S = 60cm$

$\theta = \frac{S}{r}$

$\theta = \frac{60cm}{3.5cm} = 17.14285°$

1 out of 2

+ 1 mark for substitution

Exemplar 2

$S = \theta r$

$\theta = \frac{S}{r}$

$\theta = \frac{60}{7}$

$\theta = 8.571428°$

½ out of 2

+ 1 mark for substitution
– ½ mark for procedural error (substituting diameter instead of radius)

Exemplar 3

$S = \theta r$ $S = 60cm$

$60 = \theta(3.5)$ $\theta =$

$\theta = 17.143$ $r = 3.5$

$\theta = 17.143\left(\frac{180}{\pi}\right)$

$\theta = 982.213$ ← E5

2 out of 2

award full marks
E5 (units of measure omitted in final answer)

Question 2 P3

There are 15 dogs and 12 cats in an animal shelter. Determine the number of ways that three dogs and two cats can be selected if Scout, one of the dogs, must be selected.

Solution

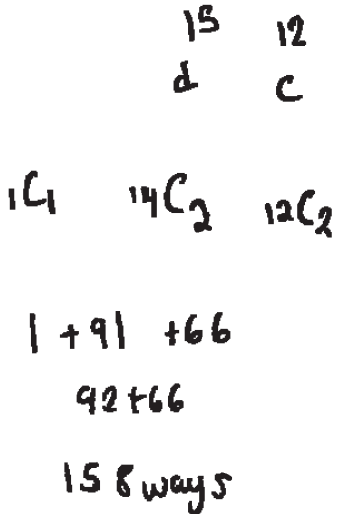
${}_1C_1 \bullet {}_{14}C_2 \bullet {}_{12}C_2$	1 mark for ${}_{14}C_2$
6006	½ mark for ${}_{12}C_2$
	½ mark for product of combinations

2 marks

Note:

${}_1C_1$ does not need to be shown.

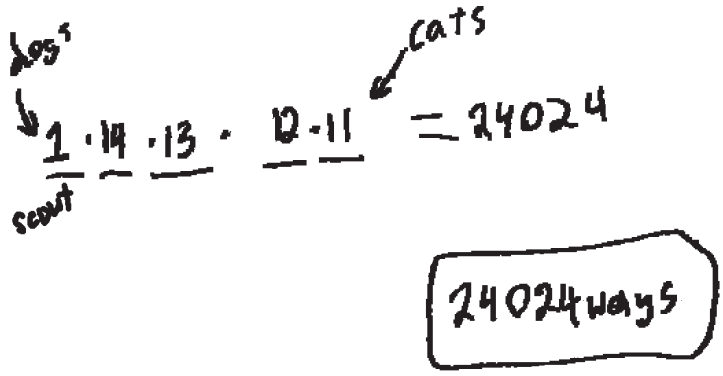
Exemplar 1



1½ out of 2

+ 1 mark for $_{14}C_2$
+ ½ mark for $_{12}C_2$

Exemplar 2



1 out of 2

award full marks
– 1 mark for concept error (using permutations instead of combinations)

Question 3 R10

Emily wants to save money to buy a car. She invests $180 per month at an annual interest rate of 4.5%, compounded monthly.

Determine, algebraically, the number of monthly investments she will need to make to obtain at least $15 000. Express the final answer as a whole number.

Use the formula: $FV = \dfrac{R\left[\left(1+i\right)^n - 1\right]}{i}$

where $FV =$ the future value

$R =$ the investment amount each period

$i = \left[\dfrac{\text{the annual interest rate (as a decimal)}}{\text{the number of compounding periods per year}}\right]$

$n =$ the number of investments

Solution

$$15\,000 = \frac{180\left[\left(1+\frac{0.045}{12}\right)^n - 1\right]}{\frac{0.045}{12}}$$

½ mark for substitution

$$15\,000 = \frac{180\left[\left(1+0.00375\right)^n - 1\right]}{0.00375}$$

$$0.3125 = 1.00375^n - 1$$

$$1.3125 = 1.00375^n$$

½ mark for simplification

$$\log\left(1.3125\right) = \log\left(1.00375\right)^n$$

½ mark for applying logarithms

$$\log\left(1.3125\right) = n\log\left(1.00375\right)$$

1 mark for power law

$$n = \frac{\log\left(1.3125\right)}{\log\left(1.00375\right)}$$

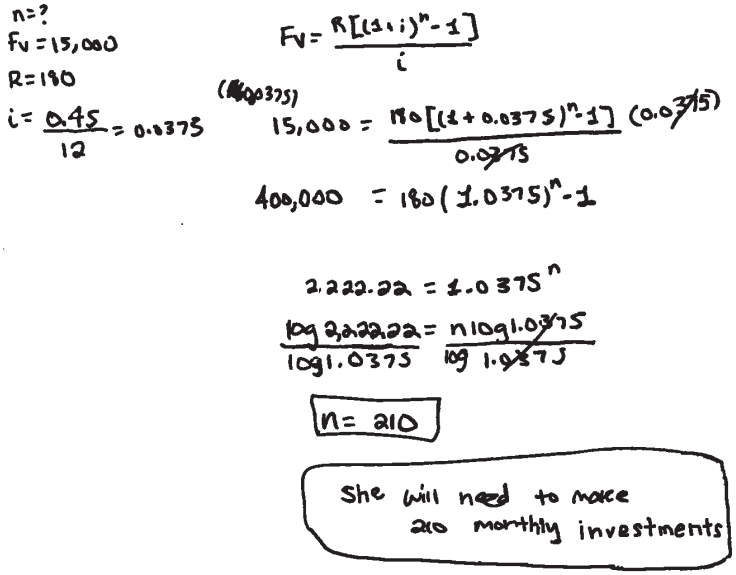
$$n = 72.651\,539...$$

½ mark for evaluating quotient of logarithms

$\therefore$ 73 monthly investments are needed

3 marks

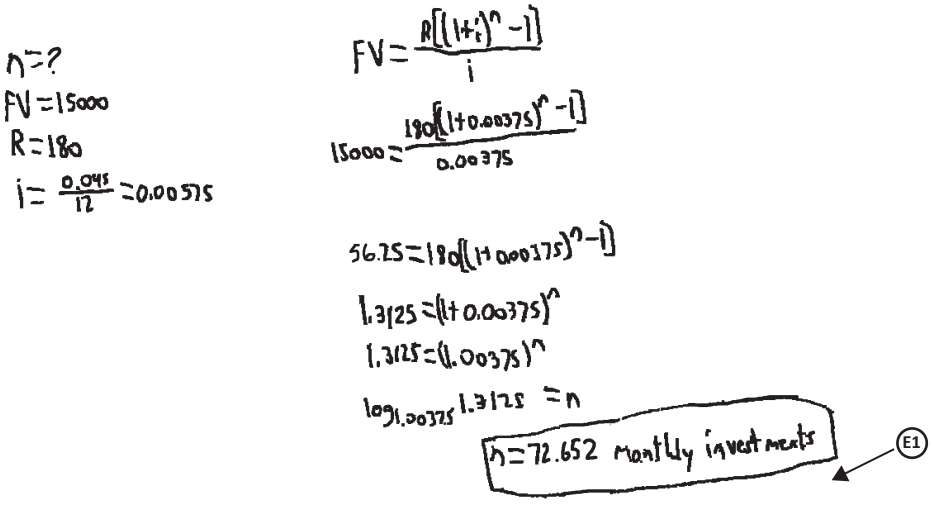
Exemplar 1



2 out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for evaluating quotient of logarithms

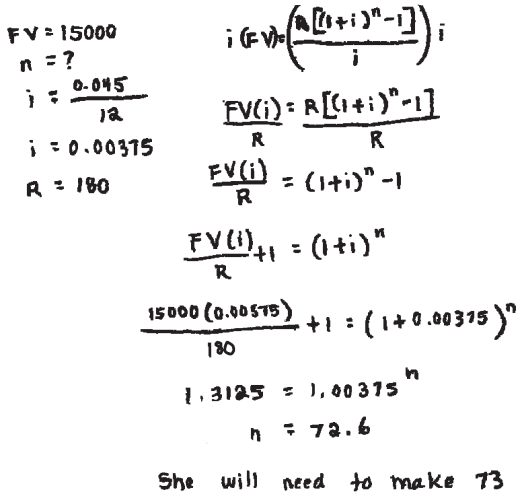
Exemplar 2



3 out of 3

award full marks
E1 (final answer not stated)

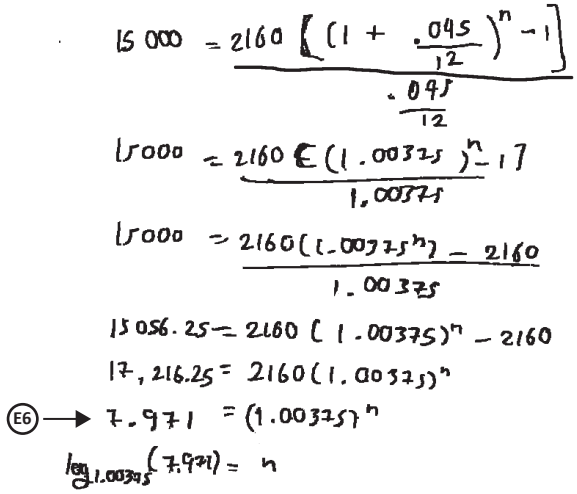
Exemplar 3



2 out of 3

+ ½ mark for substitution
+ ½ mark for simplification
+ 1 mark for guess and check solution

Exemplar 4



1½ out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law
E6 (rounding error)

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Question 4 P4

Determine and simplify the 4th term in the binomial expansion of $\left(3x-\frac{2}{x^2}\right)^6$.

Solution

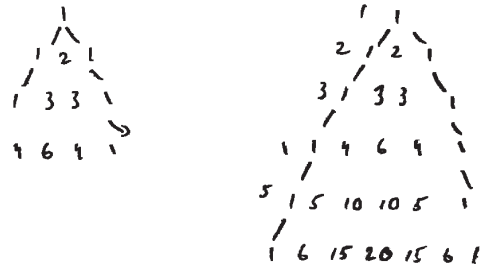
$t_4 = {}_6C_3\left(3x\right)^3\left(-\frac{2}{x^2}\right)^3$ 2 marks (1 mark for ${}_6C_3$; ½ mark for each consistent factor)

$= 20\left(27x^3\right)\left(\frac{-8}{x^6}\right)$

$= \frac{-4320}{x^3}$ 1 mark for simplification (½ mark for coefficient, ½ mark for exponent)

3 marks

Exemplar 1



$$1(3x)^6 + 6(3x)^5\left(-\frac{2}{x^2}\right) + 15(3x)^4\left(-\frac{2}{x^2}\right)^2$$
$$+20(3x)^3\left(-\frac{2}{x^2}\right)^3 + 15(3x)^2\left(-\frac{2}{x^2}\right)^4 + 6(3x)\left(-\frac{2}{x^2}\right)^5 \quad \leftarrow \text{E1}$$
$$+1\left(-\frac{2}{x^2}\right)^6$$

2 out of 3

+ 1 mark for ${}_6C_3$
+ 1 mark for consistent factors
E1 (final answer not stated)

Exemplar 2

$$t_{k+1} = {}_nC_k \cdot a^{n-k}\, b^k$$
$$t_4 = {}_7C_3 \cdot (3x)^{7-3} \left(\frac{-2}{x^2}\right)^3$$
$$= 35 \cdot 81x^4 \cdot \frac{-8}{x^6}$$
$$= \frac{-22680}{x^2}$$

2 out of 3

+ 1 mark for consistent factors
+ 1 mark for simplification

Question 5 T5

Solve, algebraically, over the interval $[0,2\pi]$.

$$4\cos^2 x - 3\cos x - 1 = 0$$

Solution

$4\cos^2 x - 3\cos x - 1 = 0$

$(4\cos x + 1)(\cos x - 1) = 0$

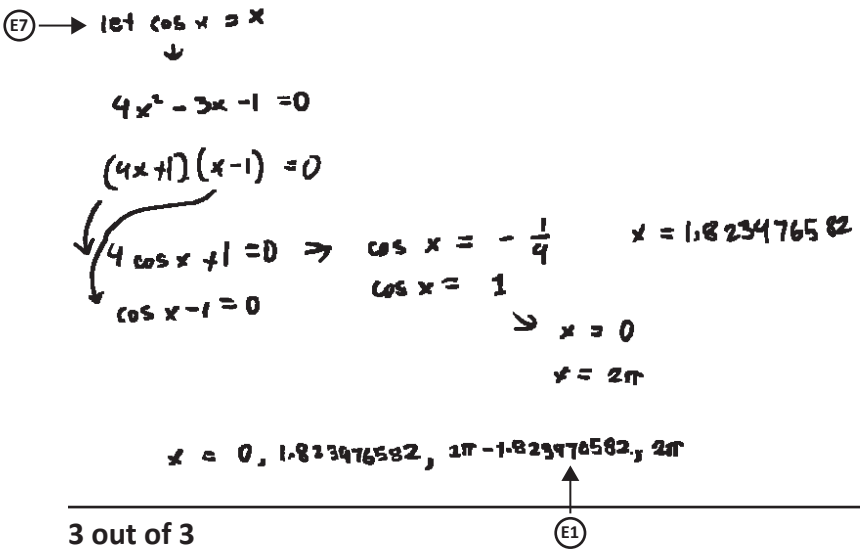
$\cos x = -\frac{1}{4}$ $\cos x = 1$ 1 mark for solving for $\cos x$ (½ mark for each branch)

$x_r = 1.318\ 116...$

$x = 1.823,\ 4.460$ $x = 0,\ 2\pi$ 2 marks for solving for x (½ mark for each value)

3 marks

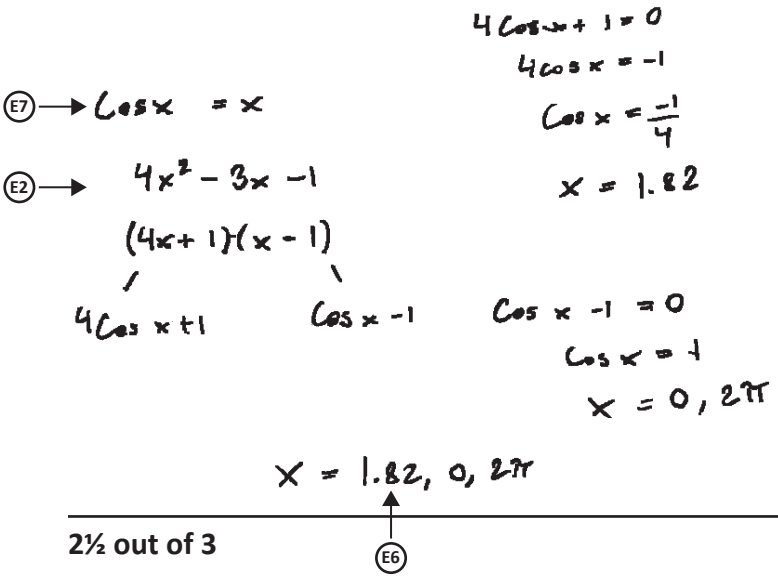
Exemplar 1



3 out of 3

award full marks
E7 (notation error in line 1)
E1 (final answer not stated)

Exemplar 2



2½ out of 3

+ 1 mark for solving for cos *x*
+ 1½ marks for solving for *x*
E7 (notation error in line 1)
E2 (changing an equation to an expression)
E6 (rounding error)

Question 6 R14

State an equation for a rational function, $g(x)$, whose graph has a vertical asymptote at $x = 7$.

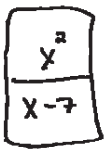
Solution

$$g(x) = \frac{1}{x-7}$$

1 mark

Note:
Other equations are possible.

Exemplar 1

$g(x) =$ 

1 out of 1

Exemplar 2

$g(x) =$ 1/(x-7)(x+1)

1 out of 1

Question 7 R13

State an equation of a radical function, $f(x)$, with a domain of $x \leq 0$ and a range of $y \geq 1$.

Solution

$f(x) = \underline{\sqrt{-x} + 1}$ 1 mark for a radical function with a domain of $x \leq 0$

1 mark for a radical function with a range of $y \geq 1$

2 marks

Note:
Other equations are possible.

Exemplar 1

$f(x)=$ $-\sqrt{-x}+1$

1 out of 2

+ 1 mark for a radical function with a domain of $x \leq 0$

Exemplar 2

$f(x)=$ $\sqrt{x}+1$

1 out of 2

+ 1 mark for a radical function with a range of $y \geq 1$

Question 8 T6

Prove the identity for all permissible values of x.

$$\frac{\cos x + \sin^2 x \sec x}{\sin x} = \sec x \csc x$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\frac{\cos x + \sin^2 x \sec x}{\sin x}$	$\sec x \csc x$
$\frac{\cos x + \frac{\sin^2 x}{\cos x}}{\sin x}$	
$\frac{\frac{\cos^2 x + \sin^2 x}{\cos x}}{\sin x}$	
$\frac{\frac{1}{\cos x}}{\sin x}$	
$\frac{1}{\cos x} \cdot \frac{1}{\sin x}$	
$\sec x \csc x$	

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Method 2

Left-Hand Side	Right-Hand Side
$\dfrac{\cos x + \sin^2 x \sec x}{\sin x}$	$\sec x \csc x$
$\dfrac{\cos x}{\sin x} + \dfrac{\sin^2 x \ \sec x}{\cancel{\sin x}}$	
$\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$	
$\dfrac{\cos^2 x + \sin^2 x}{\sin x \cos x}$	
$\dfrac{1}{\sin x \cos x}$	
$\sec x \csc x$	

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Method 3

Left-Hand Side	Right-Hand Side
$\dfrac{\cos x + \sin^2 x \sec x}{\sin x}$	$\sec x \csc x$
$\dfrac{\cos x + \dfrac{1-\cos^2 x}{\cos x}}{\sin x}$	
$\dfrac{\dfrac{\cos^2 x + 1 - \cos^2 x}{\cos x}}{\sin x}$	
$\dfrac{1}{\cos x \sin x}$	
$\sec x \csc x$	

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$\frac{\cos x + \sin^2 x \sec x}{\sin x}$ $\frac{\cos x + \sin^2 x \frac{1}{\cos x}}{\sin x}$ $\frac{\cos x + \frac{\sin^2 x}{\cos x}}{\sin x}$ $\frac{\cos x + \frac{1-\cos^2 x}{\cos x}}{\sin x}$ $\frac{\frac{\cos^2 x}{\cos x} + \frac{1-\cos^2 x}{\cos x}}{\sin x}$ $\frac{\frac{1-\sin^2 x}{\cos x} + \frac{1-\cos^2 x}{\cos x}}{\sin x}$ $\frac{\frac{1-\sin^2 x + 1-\cos^2 x}{\cos x}}{\sin x}$	$\sec x \csc x$ $\frac{1}{\cos x} \frac{1}{\sin x}$

2 out of 3

+ 1 mark for correct substitution of appropriate identities
+ 1 mark for algebraic strategies

Exemplar 2

Left-Hand Side	Right-Hand Side
$\frac{\cos x + \sin^2 x \sec x}{\sin x}$ $\frac{\cos x + (\sin^2 x)(\frac{1}{\cos x})}{\sin x}$ $\frac{\frac{\cos x}{1} + \frac{\sin^2 x}{\cos x}}{\sin x}$ $\frac{\frac{\cos x}{\cos x} + \frac{\sin^2 x}{\cos x}}{\sin x}$ $\frac{\frac{\cos x + \sin^2 x}{\cos x}}{\sin x}$ $\frac{\cos x + \sin^2 x}{\cos x} \cdot \frac{1}{\sin x}$ $\frac{\cos x (1 - \cos^2 x)}{\cos x} \cdot \frac{1}{\sin x}$	$\sec x \csc x$ $(\frac{1}{\cos x})(\frac{1}{\sin x})$ $\frac{1}{\cos x \sin x}$

1 out of 3

+ 1 mark for correct substitution of appropriate identities

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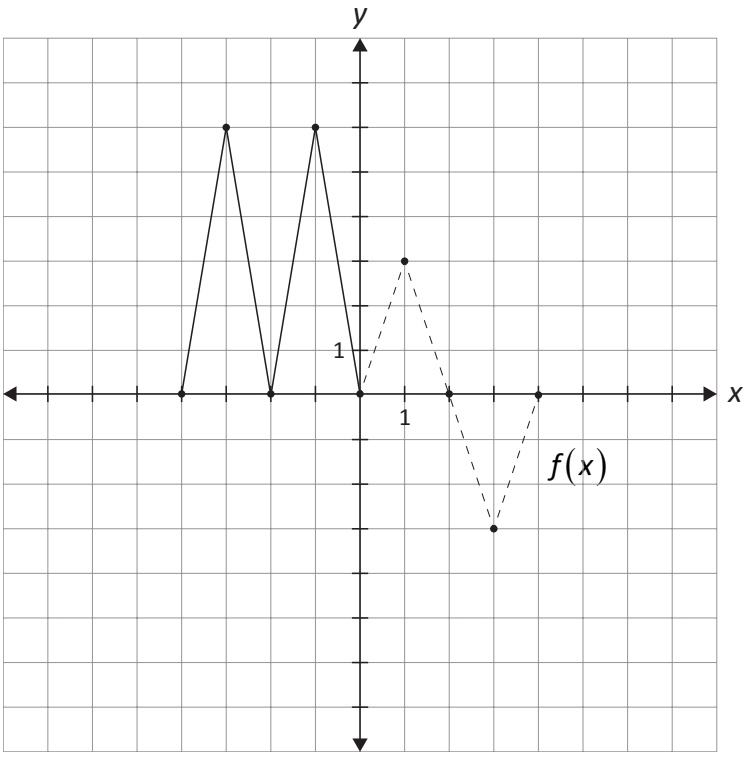
Question 9 R1, R3, R5

Given the graph of, $y = f(x)$, sketch the graph of $y = 2\left|f(-x)\right|$.

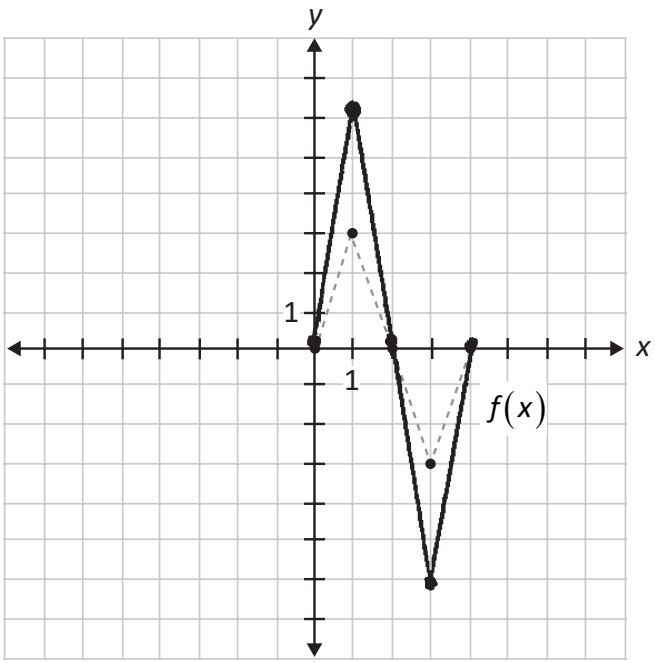
Solution



1 mark for horizontal reflection
1 mark for absolute value
1 mark for vertical stretch

3 marks

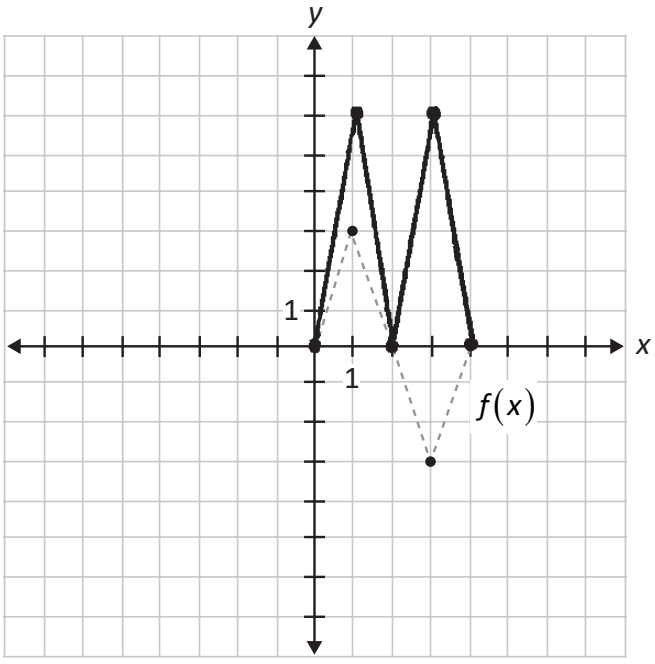
Exemplar 1



1 out of 3

+ 1 mark for vertical stretch

Exemplar 2



2 out of 3

+ 1 mark for absolute value
+ 1 mark for vertical stretch

Question 10 R10

Solve, algebraically.

$$\log(x-1)+\log(x+2)=1$$

Solution

$\log\left[(x-1)(x+2)\right]=1$ 1 mark for product law

$(x-1)(x+2)=10^1$ 1 mark for exponential form

$x^2+x-2=10$

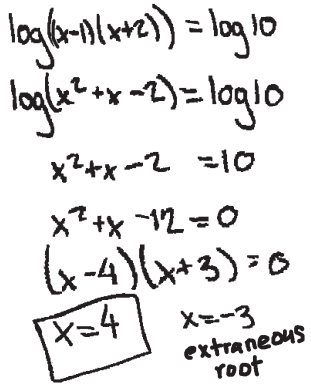
$x^2+x-12=0$

$(x+4)(x-3)=0$

~~$x=-4$~~ $x=3$ ½ mark for the permissible value of x
½ mark for showing the rejection of the extraneous root

3 marks

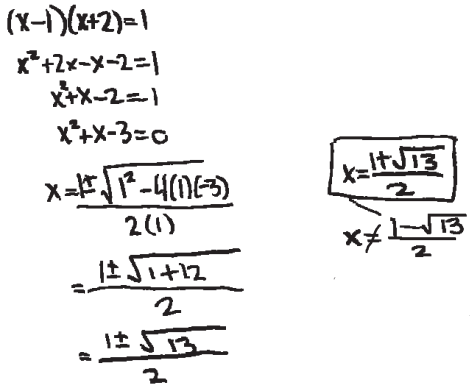
Exemplar 1



2½ out of 3

award full marks
– ½ mark for procedural error (incorrect factoring)

Exemplar 2



1 out of 3

+ 1 mark for product law
+ ½ mark for the permissible value of *x*
+ ½ mark for showing the rejection of the extraneous root
– ½ mark for procedural error (not including logarithm in line 1)
– ½ mark for procedural error (incorrect substituion in quadratic formula)

Exemplar 3

$$\log \frac{x-1}{x+2} = 1$$

$$\frac{x-1}{x+2} = 10^1$$

$$(x-1) = 10(x+2)$$

$$x - 1 = 10x + 20$$

$$-21 = 9x$$

$$-7 = 3x$$

$$x = -\frac{7}{3}$$

1½ out of 3

+ 1 mark for exponential form
+ ½ mark for showing the rejection of the extraneous root

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Question 11 P4

Justify that there are only two negative terms in the expansion of $(-2x+5y)^4$.

Solution

$t_1 = {}_4C_0(-2x)^4(5y)^0 \rightarrow +$

$t_2 = {}_4C_1(-2x)^3(5y)^1 \rightarrow -$

$t_3 = {}_4C_2(-2x)^2(5y)^2 \rightarrow +$

$t_4 = {}_4C_3(-2x)^1(5y)^3 \rightarrow -$

$t_5 = {}_4C_4(-2x)^0(5y)^4 \rightarrow +$

$\therefore$ The term is negative only when $(-2x)$ has an odd exponent, which occurs twice.

1 mark

Exemplar 1

It's only negative when the exponent is odd which only is twice compared to the three times it's even.

½ out of 1

award full marks
– ½ mark for lack of clarity in justification

Exemplar 2

$t_{0+1} = {}_4C_0 (-2x)^{4-0} (5y)^0$

$t_1 = 16x^4 \rightarrow (+)$

$t_2 \rightarrow (-)$

$t_3 \rightarrow (+)$

$t_4 \rightarrow (-)$

2 negative terms

½ out of 1

award full marks
– ½ mark for incomplete justification

Exemplar 3

t_1 : contains $(-2x)^4 \rightarrow$ pos

t_2 : $\downarrow (-2x)^3 \rightarrow$ neg

t_3 : pos

t_4 : neg

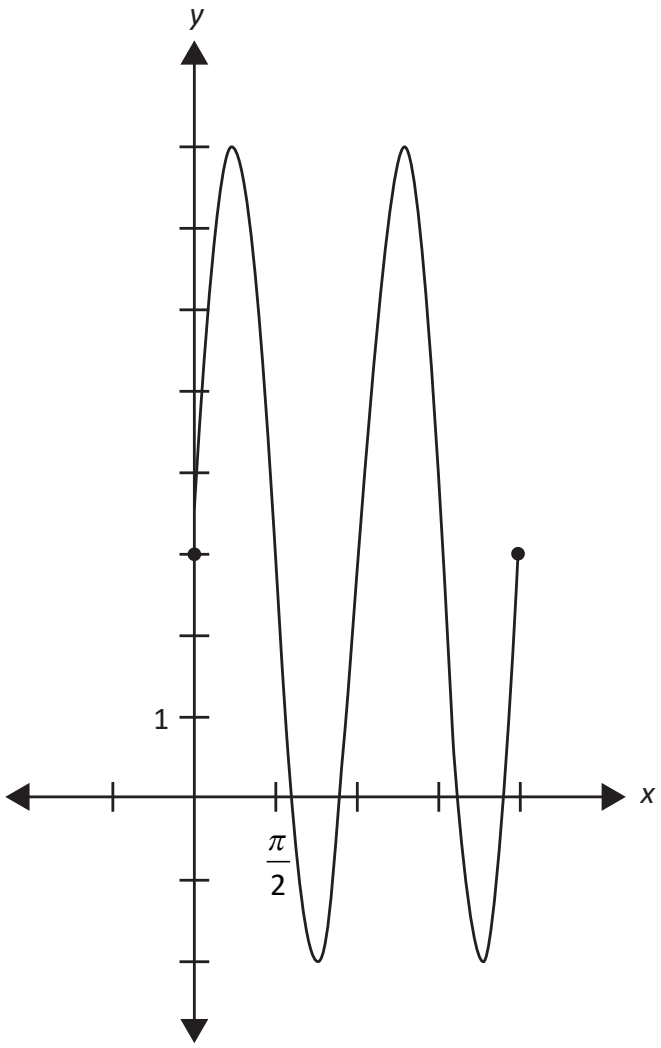
t_5 : pos

∴ all 2 negative terms

1 out of 1

Question 12 T5

The graph of $y = 5\sin(2x) + 3$ below can be used to solve the equation $0 = 5\sin(2x) + 3$.
State how many solutions there are to the equation $0 = 5\sin(2x) + 3$ over the interval $[0, 2\pi]$.

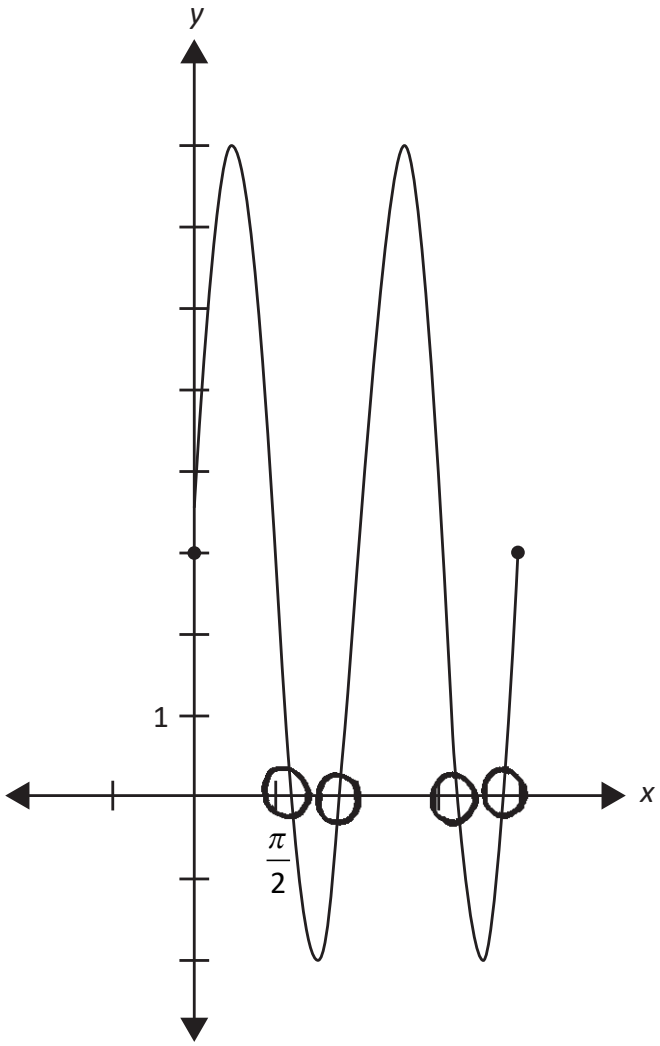


Solution

There are four solutions.

1 mark

Exemplar 1



1 out of 1

award full marks
E1 (final answer not stated)

Question 13 R1

Given the functions, $f(x) = x^2 - 1$ and $g(x) = x + 1$,

a) state the equation of $h(x) = \dfrac{f(x)}{g(x)}$.

b) sketch the graph of $h(x)$.

Solution

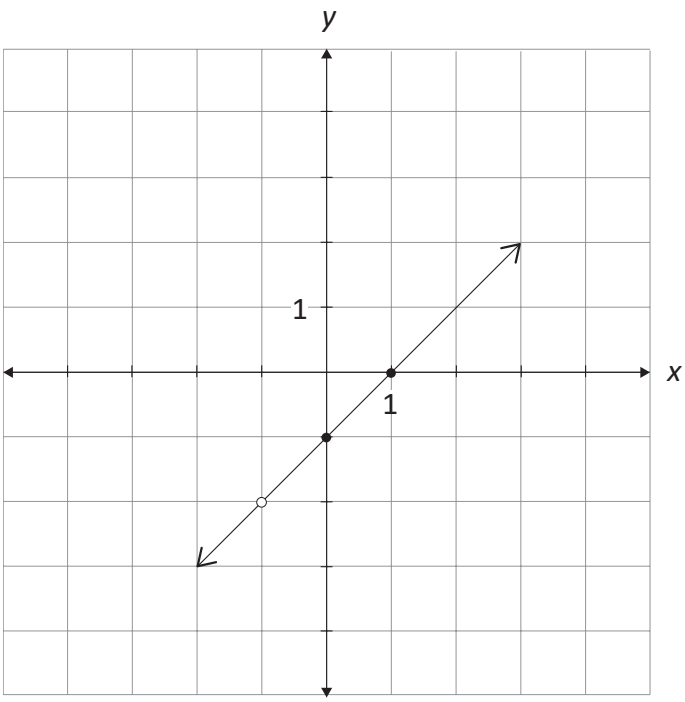
a) $h(x) = \dfrac{x^2 - 1}{x + 1}$

$= \dfrac{(x+1)(x-1)}{(x+1)}$

$= x - 1, x \neq -1$

1 mark

b)



1 mark for shape of graph consistent with a)

1 mark for point of discontinuity (hole) at $x = -1$

2 marks

Note:

Deduct a maximum of 1 mark for the concept error of not restricting domain.
Deduct ½ mark for procedural error (not stating domain of simplified function in part a) if graph shows the correct domain.

Exemplar 1

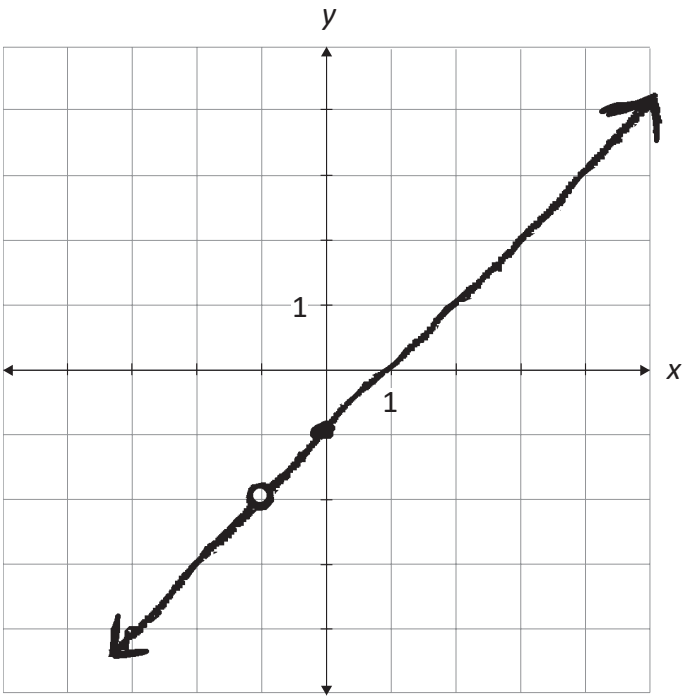
a)

$h(x)=$ $\frac{(x-1)(x+1)}{x+1} = \boxed{x-1}$

½ out of 1

award full marks
– ½ mark for procedural error (not stating restricted domain)

b)



2 out of 2

Exemplar 2

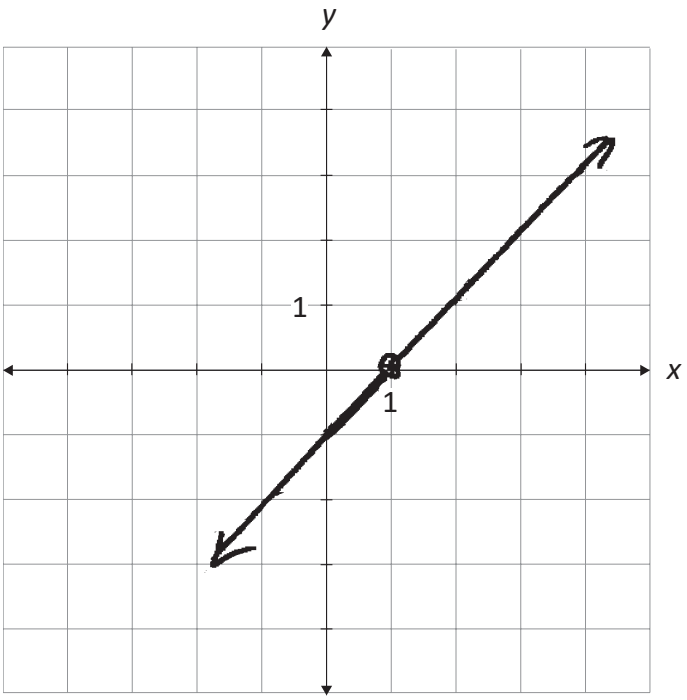
a)

$h(x)=$ $x-1,\ x\neq 1$

½ out of 1

award full marks
– ½ mark for procedural error (incorrect restricted domain)

b)



2 out of 2

graph consistent with a)

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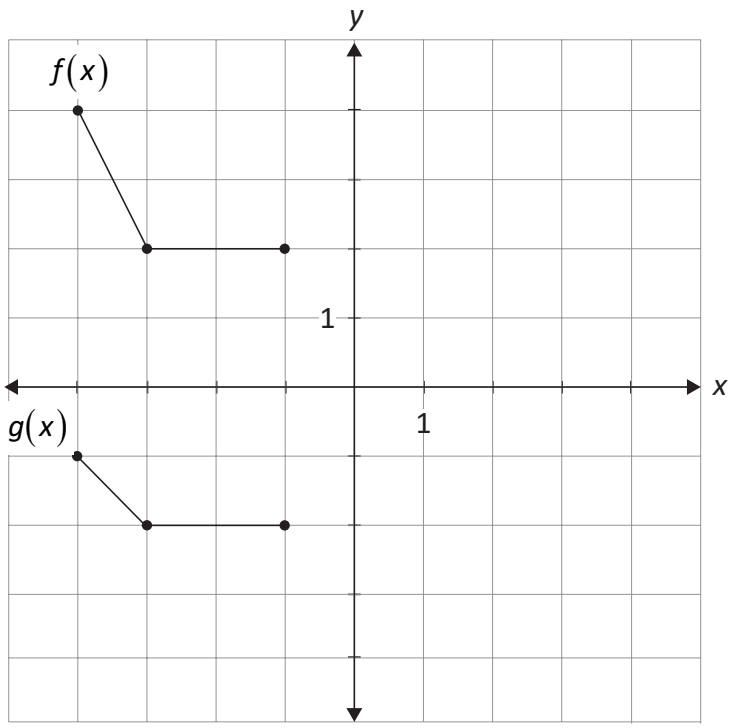
Question 14 R2, R3

State an equation for $g(x)$, in terms of $f(x)$.



Solution

$g(x)=\frac{1}{2}f(x)-3$

1 mark for vertical compression
1 mark for vertical translation

2 marks

Exemplar 1

$g(x) =$ $\frac{1}{2}x - 3$

1 out of 2

award full marks
– 1 mark for concept error (omitting *f*)

Exemplar 2

$g(x) =$ $2g(x) - 3$

½ out of 2

+ 1 mark for vertical translation
– ½ mark for procedural error (stating *g* instead of *f*)

Exemplar 3

$g(x) =$ $2g(x) + 6$

1 out of 2

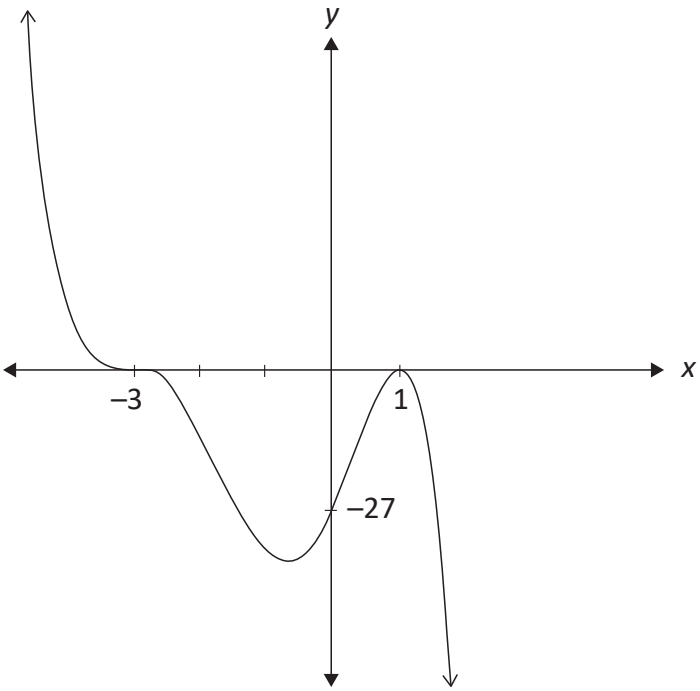
award full marks
– 1 mark for concept error (transforming $g(x)$ instead of $f(x)$)

Question 15 R12

Sketch the graph of a polynomial function, $p(x)$, with the following characteristics:

- degree 5
- leading coefficient of –1
- a zero at –3, with a multiplicity of 3
- a zero at 1, with a multiplicity of 2

Solution



1 mark for x-intercept at –3 with a multiplicity of 3

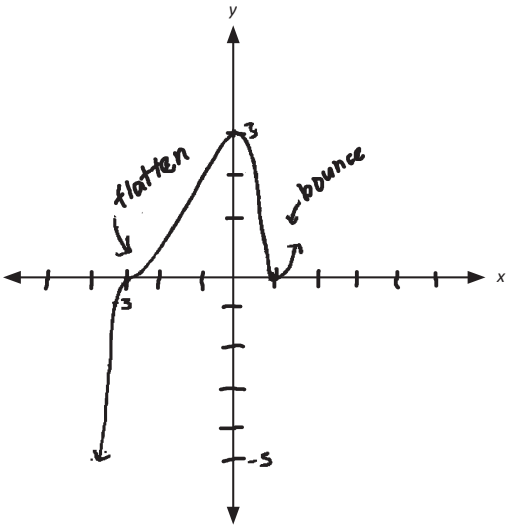
1 mark for x-intercept at 1 with a multiplicity of 2

½ mark for end behaviour

½ mark for y-intercept

3 marks

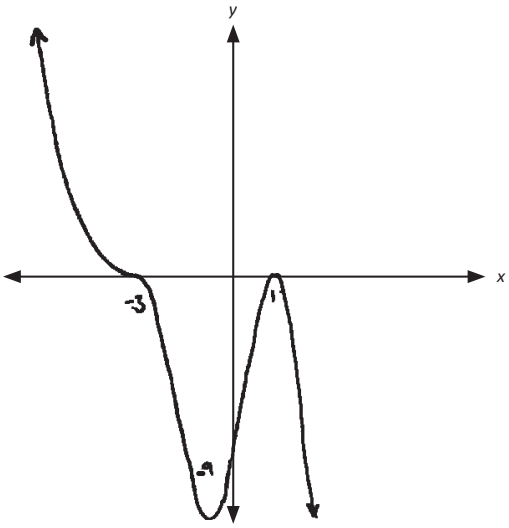
Exemplar 1



2 out of 3

+ 1 mark for *x*-intercept at –3 with a multiplicity of 3
+ 1 mark for *x*-intercept at 1 with a multiplicity of 2

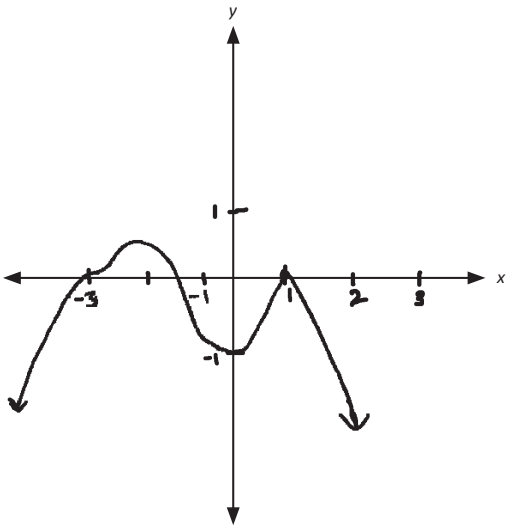
Exemplar 2



2½ out of 3

+ 1 mark for *x*-intercept at –3 with a multiplicity of 3
+ 1 mark for *x*-intercept at 1 with a multiplicity of 2
+ ½ mark for end behaviour

Exemplar 3



1 out of 3

+ 1 mark for *x*-intercept at –3 with a multiplicity of 3
+ 1 mark for *x*-intercept at 1 with a multiplicity of 2
– 1 mark for concept error (introducing another *x*-intercept in their graph)

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Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Reponse Questions

Question	Answer	Learning Outcome
16	A	T6
17	C	P2
18	C	R10
19	D	R1
20	B	R14
21	D	T4
22	A	R11
23	A	T1
24	B	R8

Question 16 T6

Identify the expression that is equivalent to $\tan\theta\left(\csc^2\theta - 1\right)$.

a. $\cot\theta$

b. $\tan\theta$

c. $\sec\theta$

d. $\sin\theta$

Question 17 P2

Identify the expression that represents the number of ways five books can be arranged on a shelf if two of them must be together.

a. 5!2!

b. $5! - 2!$

c. 4!2!

d. 3!2!

Question 18 R10

Identify the value of *x* in the equation, $\log_8 x = -\frac{1}{3}$.

a. −2

b. $-\frac{1}{2}$

c. $\frac{1}{2}$

d. 2

Question 19 R1

Given the functions, $f(x)=\frac{1}{x}$ and $g(x)=\sqrt{x-1}$, identify the domain of $f(g(x))$.

a. $(-\infty,\infty)$

b. $(-\infty,1)\cup(1,\infty)$

c. $[1,\infty)$

d. $(1,\infty)$

Question 20 R14

Given $f(x)=\frac{3(x+2)}{5(x+2)(x-2)}$, identify the equation of the horizontal asymptote.

a. $y=-2$

b. $y=0$

c. $y=\frac{3}{5}$

d. $y=2$

Question 21 T4

Identify the range of the sinusoidal function, $f(x)=2\sin x+3$.

a. $\{y \mid y\in\mathbb{R}\}$

b. $\{y \mid 2\le y\le 4, y\in\mathbb{R}\}$

c. $\{y \mid -2\le y\le 2, y\in\mathbb{R}\}$

d. $\{y \mid 1\le y\le 5, y\in\mathbb{R}\}$

Question 22 R11

Given that $(x-3)$ is a factor of the polynomial $P(x)$, identify which of the following is true.

a. $P(3)=0$

b. $P(0)=3$

c. $P(0)=-3$

d. $P(-3)=0$

Question 23 T1

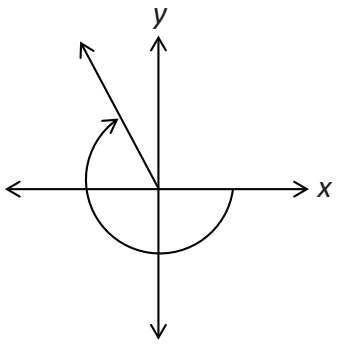
Identify the value that best represents the angle in standard position.

a. −4.4 radians

b. −2 radians

c. 1.3 radians

d. 1.9 radians



Question 24 R8

Identify the expression that is equivalent to $\log_a(x+2)+3\log_a(x)$.

a. $\log_a(3(x+2))$

b. $\log_a\left(x^3(x+2)\right)$

c. $\log_a\left(\frac{(x+2)}{x^3}\right)$

d. $\log_a x+\log_a 2+\log_a x^3$

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Question 25 T3

Evaluate.

$$\tan^3\left(\frac{3\pi}{4}\right)+\csc\left(\frac{-4\pi}{3}\right)\cos\left(\frac{13\pi}{6}\right)$$

Solution

$\left(-1\right)^3+\left(\frac{2}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$-1+1$

0

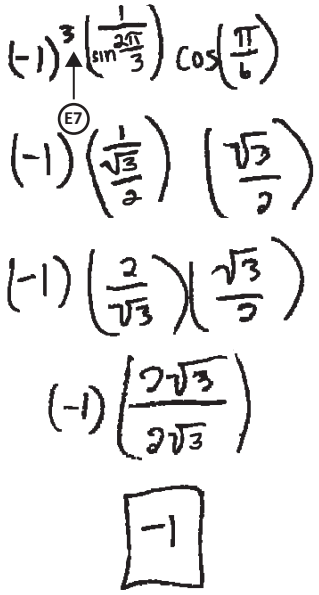
1 mark for $\tan\left(\frac{3\pi}{4}\right)$ (½ mark for value, ½ mark for quadrant)

1 mark for $\csc\left(\frac{-4\pi}{3}\right)$ (½ mark for value, ½ mark for quadrant)

1 mark for $\cos\left(\frac{13\pi}{6}\right)$ (½ mark for value, ½ mark for quadrant)

3 marks

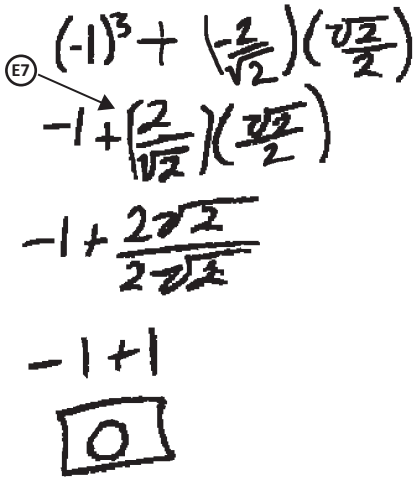
Exemplar 1



3 out of 3

award full marks
E7 (transcription error in line 1)

Exemplar 2



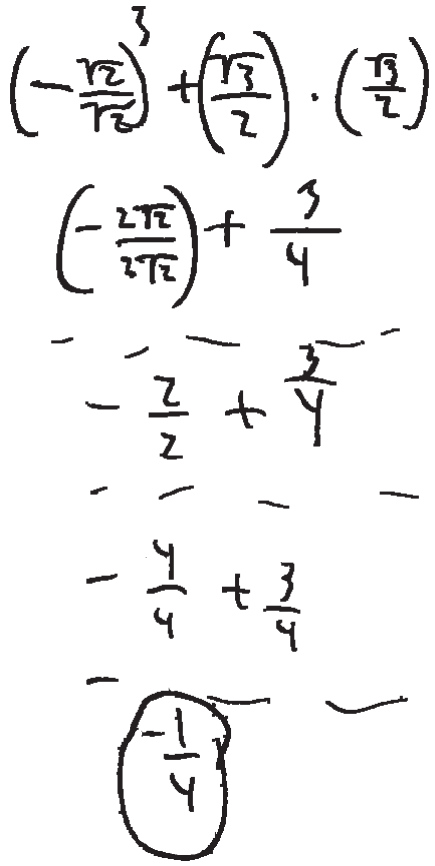
1½ out of 3

+ 1 mark for $\tan\left(\frac{3\pi}{4}\right)$

+ ½ mark for the quadrant of $\cos\left(\frac{13\pi}{6}\right)$

E7 (transcription error in line 2)

Exemplar 3



2½ out of 3

+ 1 mark for $\tan\left(\frac{3\pi}{4}\right)$

+ ½ mark for quadrant of $\csc\left(\frac{-4\pi}{3}\right)$

+ 1 mark for $\cos\left(\frac{13\pi}{6}\right)$

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Question 26 R8

If $a = \log 4$ and $b = \log 3$, express $\log 36$ in terms of a and b.

Solution

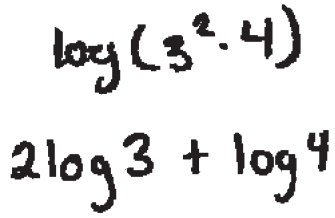
$$\begin{aligned}\log 36 &= \log 4 + \log 3^2 \\ &= \log 4 + 2\log 3 \\ &= a + 2b\end{aligned}$$

1 mark for product law

1 mark for power law

2 marks

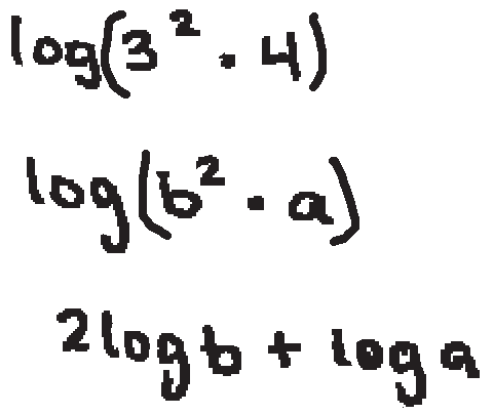
Exemplar 1



1½ out of 2

award full marks
– ½ mark for procedural error (not stating answer in terms of *a* and *b*)

Exemplar 2



1 out of 2

award full marks
– 1 mark for concept error (substituting $a = 4$ and $b = 3$)

Question 27 T6

Justify that $\tan 2x$ has a non-permissible value at $x = \frac{\pi}{4}$.

Solution

Method 1

$\tan 2x$

Let $x = \frac{\pi}{4}$,

$\tan\left(2 \bullet \frac{\pi}{4}\right)$ 1 mark for substitution

$\tan\frac{\pi}{2}$

$\frac{1}{0}$ which is undefined 1 mark for justification

$\therefore x \neq \frac{\pi}{4}$

2 marks

Method 2

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ 1 mark for correct substitution of appropriate identity

$1-\tan^2 x \neq 0$

$\tan^2 x \neq 1$

$\tan x \neq \pm 1$

$\therefore x \neq \frac{\pi}{4}$ 1 mark for justification

2 marks

Method 3

$\tan 2x = \dfrac{\sin 2x}{\cos 2x}$

$\cos 2x \neq 0$

Let $x = \dfrac{\pi}{4}$,

$\cos\left(2 \bullet \dfrac{\pi}{4}\right)$ 1 mark for substitution

$\cos\left(\dfrac{\pi}{2}\right) = 0$ 1 mark for justification

$\therefore x \neq \dfrac{\pi}{4}$

2 marks

Exemplar 1

$\tan 2x = \frac{2\tan}{1-\tan^2}$ ← E3

tan 2x has a NPV at π/4, because

$\tan(\frac{\pi}{4}) = \frac{\sqrt{2}}{\sqrt{2}} = 1$

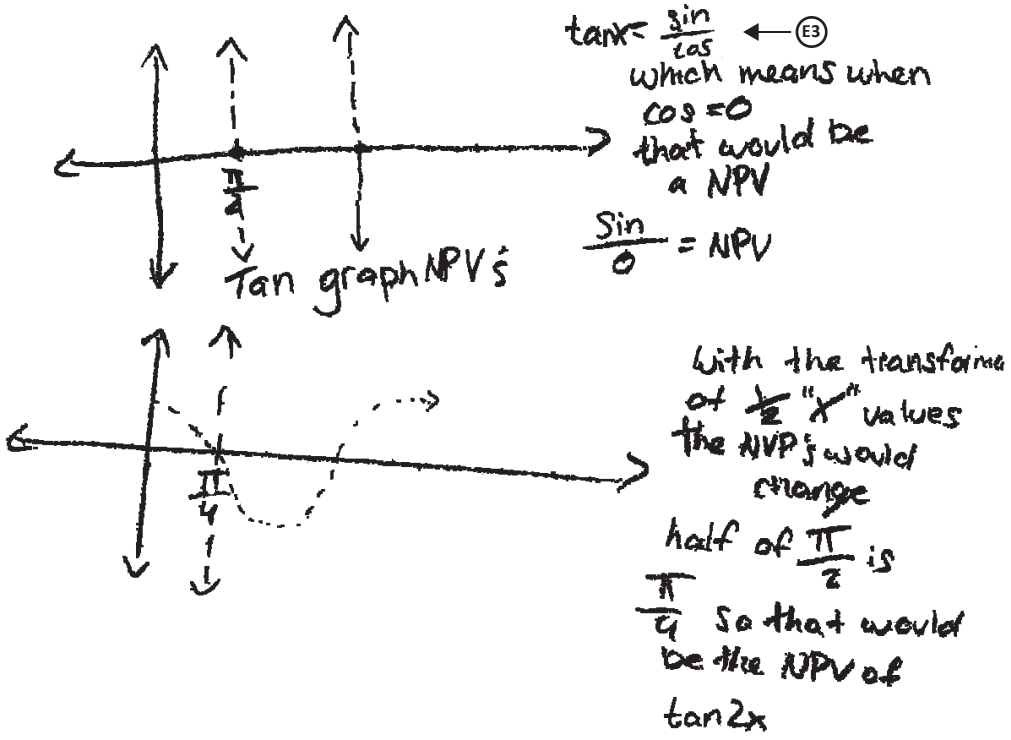
$1^2 = 1$ still and $1-1=0$ and

zero can't be in the donimnator

2 out of 2

award full marks
E3 (variable omitted in an identity in line 1)

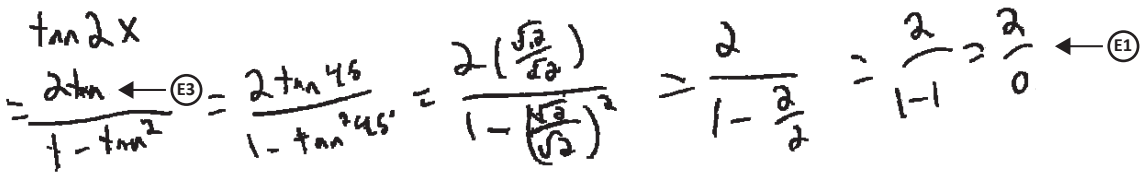
Exemplar 2



2 out of 2

award full marks
E3 (variable omitted in an identity)

Exemplar 3



2 out of 2

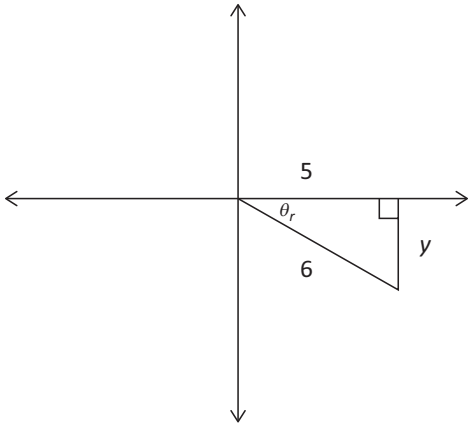
award full marks
E3 (variable omitted in an identity in line 2)
E1 (final answer not stated)

Question 28 T3

Determine the exact value of $\tan\theta$, if $\sec\theta = \frac{6}{5}$ and θ terminates in quadrant IV.

Solution

Method 1



$$\cos\theta = \frac{5}{6}$$

$$5^2 + y^2 = 6^2$$

$$y^2 = 36 - 25$$

$$y^2 = 11$$

$$y = \pm\sqrt{11}$$ 1 mark for value of y

$$\tan\theta = \frac{-\sqrt{11}}{5}$$ 1 mark for $\tan\theta$ (½ mark for quadrant, ½ mark for value)

2 marks

Note:
Accept any of the following values for y: $y = \pm\sqrt{11}$, $y = \sqrt{11}$, $y = -\sqrt{11}$.

Question 28 T3

Method 2

$\tan^2\theta = \sec^2\theta - 1$

$\tan^2\theta = \left(\frac{6}{5}\right)^2 - 1$

1 mark for substitution into appropriate identity

$\tan^2\theta = \frac{36}{25} - 1$

$\tan^2\theta = \frac{11}{25}$

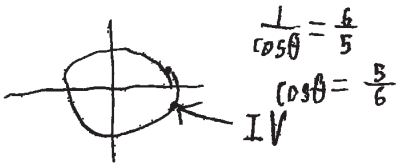
$\tan\theta = \frac{\pm\sqrt{11}}{5}$

$\therefore \tan\theta = \frac{-\sqrt{11}}{5}$ in quadrant IV

1 mark for $\tan\theta$ (½ mark for quadrant, ½ mark for value)

2 marks

Exemplar 1



$\frac{1}{\cos\theta} = \frac{6}{5}$

$\cos\theta = \frac{5}{6}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{5}{6}\right)^2 = 1$

$\sin^2\theta + \left(\frac{25}{36}\right) = 1$

$\sin^2\theta = 1 - \frac{25}{36}$

$\sin^2\theta = \frac{11}{36}$

$\sin\theta = -\sqrt{\frac{11}{36}}$

$\tan\theta = \frac{-\sqrt{\frac{11}{36}}}{\frac{5}{6}} = \frac{-6\sqrt{\frac{11}{36}}}{5}$ ← E1

2 out of 2

award full marks
E1 (answer given as a complex fraction)

Exemplar 2

$\cos\theta = \frac{5}{6}$

$\sin\theta = \sqrt{6^2 - 5^2}$

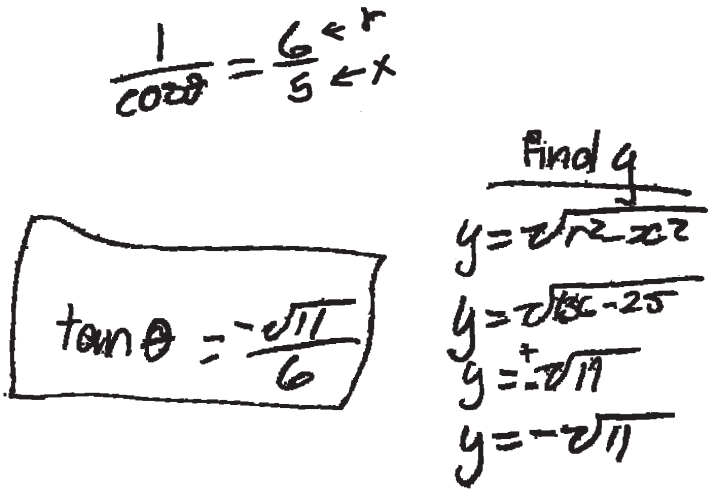
$\sin\theta = \frac{3}{6}$

$\tan\theta = \frac{5}{3}$

½ out of 2

+ 1 mark for value of *y*
– ½ mark for arithmetic error in line 3

Exemplar 3



1½ out of 2

+ 1 mark for value of *y*
+ ½ mark for quadrant of $\tan\theta$

Question 29 R14

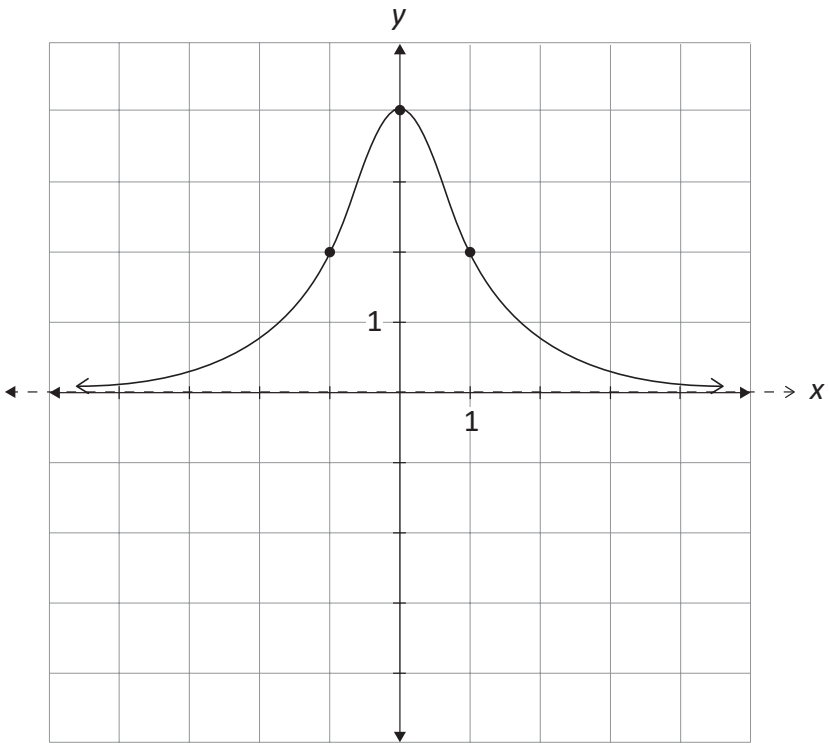
Given the function, $g(x)=\dfrac{4}{x^2+1}$,

a) sketch the graph of $g(x)$.

b) state the range of the graph of $g(x)$.

Solution

a)



1 mark for asymptotic behaviour approaching $y = 0$

1 mark for shape
(½ mark for graph left of $x = 0$;
½ mark for graph right of $x = 0$)

2 marks

b) Range: $\underline{\{y \mid 0 < y \leq 4,\ y \in \mathbb{R}\}}$

or

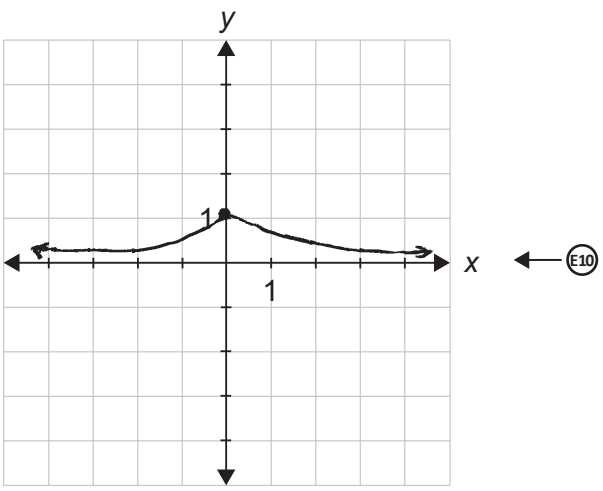
Range: $\underline{(0,4]}$

1 mark for range consistent with a)

1 mark

Exemplar 1

a)



1 out of 2

+ 1 mark for asymptotic behaviour approaching $y = 0$
E10 (asymptote omitted but still implied)

b)

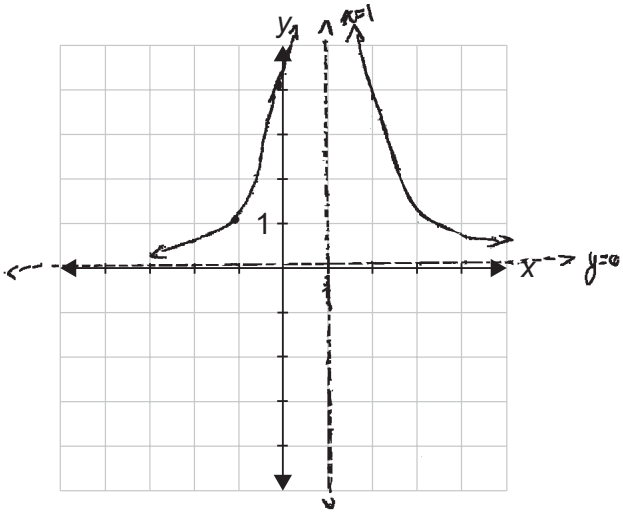
Range: $R: \{0 \leq y \leq 1\}$

½ out of 1

award full marks
– ½ mark for procedural error (including 0 in the range)

Exemplar 2

a)



1 out of 2

+ 1 mark for asymptotic behaviour approaching $y = 0$

b)

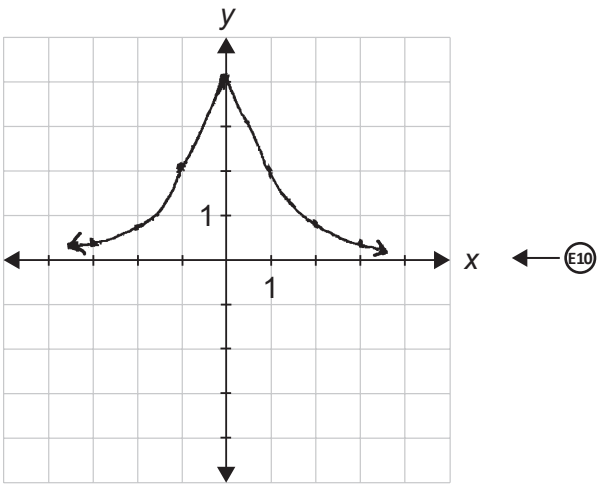
Range: $R: \{y \mid y > 0, y \in R\}$

1 out of 1

range consistent with a)

Exemplar 3

a)



1½ out of 2

award full marks
– ½ mark for procedural error (shape of graph)
E10 (asymptote omitted by still implied)

b)

Range: $\{y \mid y \neq 0, y \in \mathbb{R}\}$

0 out of 1

Question 30 R6

Given $y = x^2 - 4$,

a) determine the equation of the inverse.

b) state a restriction on the domain of $y = x^2 - 4$, in order for its inverse to be a function.

Solution

a) $y = x^2 - 4$

$x = y^2 - 4$ 1 mark for switching *x* and *y*

$y^2 = x + 4$

$y = \underline{\pm\sqrt{x+4}}$ 1 mark for solving for *y*

2 marks

b) Domain: $\underline{\{x | x \geq 0, x \in \mathbb{R}\}}$

or

Domain: $\underline{[0, \infty)}$

1 mark

Note:
Other solutions are possible for b).

Exemplar 1

a)

$y =$ $(\sqrt{x} + 2)$

$x = y^2 - 4$

$\sqrt{x+4} = y$

1 out of 2

+ 1 mark for switching *x* and *y*

b)

$x \leq 0$

1 out of 1

Exemplar 2

a)

$y =$ $\sqrt{x+4}$

$x = y^2 - 4$

$\sqrt{x+4} = \sqrt{y^2}$

1½ out of 2

award full marks
– ½ mark for procedural error (missing ±)

b)

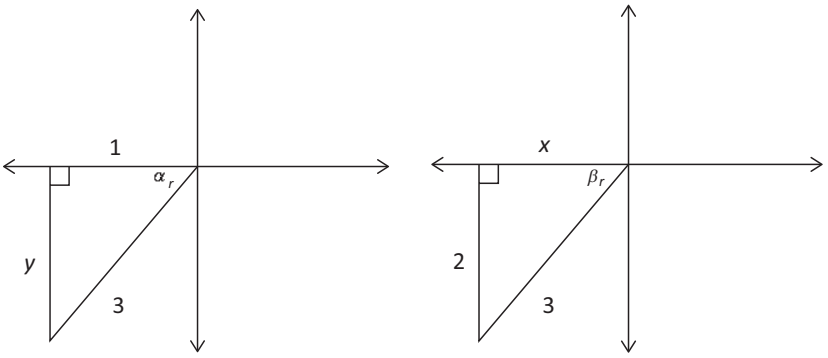
remove "y" negative values since is a $\sqrt{x}$

0 out of 1

Question 31 T6

Given $\cos\alpha = -\frac{1}{3}$ and $\sin\beta = -\frac{2}{3}$ where α and β terminate in the same quadrant, determine the exact value of $\sin(\alpha - \beta)$.

Solution



$x^2 + y^2 = r^2$ $\qquad$ $x^2 + y^2 = r^2$

$1 + y^2 = 9$ $\qquad$ $x^2 + 4 = 9$

$y^2 = 8$ $\qquad$ $x^2 = 5$

$y = \pm\sqrt{8}$ $\qquad$ $x = \pm\sqrt{5}$

$y = -\sqrt{8}$ $\qquad$ $x = -\sqrt{5}$

½ mark for value of x
½ mark for value of y

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$= \left(\frac{-\sqrt{8}}{3}\right)\left(\frac{-\sqrt{5}}{3}\right) - \left(\frac{-1}{3}\right)\left(\frac{-2}{3}\right)$$

$$= \frac{\sqrt{40}}{9} - \frac{2}{9}$$

$$= \frac{\sqrt{40} - 2}{9}$$

or

$$= \frac{2\sqrt{10} - 2}{9}$$

½ mark for consistent value of $\sin\alpha$

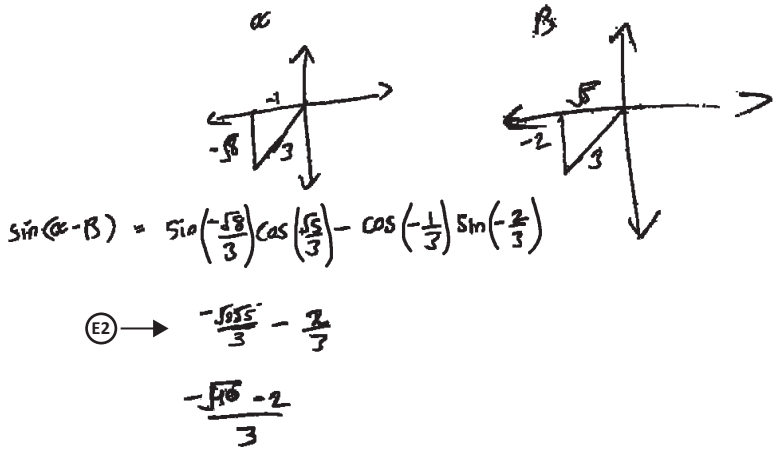
½ mark for consistent value of $\cos\beta$

1 mark for substitution into correct identity

3 marks

Note:
Accept any of the following values for x: $x = \pm\sqrt{5}$, $x = -\sqrt{5}$, $x = \sqrt{5}$.
Accept any of the following values for y: $y = \pm\sqrt{8}$, $y = -\sqrt{8}$, $y = \sqrt{8}$.

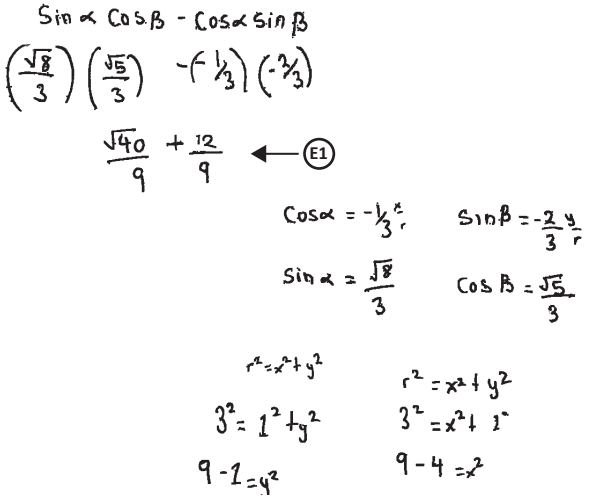
Exemplar 1



1½ out of 3

+ ½ mark for value of *x*
+ ½ mark for value of *y*
+ ½ mark for consistent value of $\sin\alpha$
+ 1 mark for substitution into correct identity
– ½ mark for procedural error in line 1 (including sin and cos when substituting values with correction in next line)
– ½ mark for arithmetic error in line 2
E2 (changing an equation to an expression)

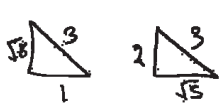
Exemplar 2



1½ out of 3

+ ½ mark for value of *x*
+ ½ mark for value of *y*
+ 1 mark for substitution into correct identity
– ½ mark for arithmetic error in line 3
E1 (final answer not stated)

Exemplar 3



$\sin\left(\frac{1}{3}-\frac{2}{3}\right)=\left(\frac{\sqrt{8}}{3}\right)\left(\frac{\sqrt{5}}{3}\right)-\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$ ← E7

E2 → $\frac{(\sqrt{8})(\sqrt{5})}{9}-\frac{2}{9}$

E1 → $\frac{(\sqrt{8})(\sqrt{5})-2}{9}$

1½ out of 3

+ ½ mark for value of *x*
+ ½ mark for value of *y*
+ 1 mark for substitution into correct identity
– ½ mark for procedural error in line 1 (substituting values for α and β)
E7 (transcription error in line 1)
E2 (changing an equation to an expression)
E1 (final answer not stated)

Exemplar 4

α

$x^2+y^2=r^2$

$1+y^2=9$

$y^2=8$

$y=\sqrt{8}$

β

$x^2+4=9$

$x^2=5$

$x=\sqrt{5}$

$\sin(\alpha-\beta)=\sin\left(\frac{-\sqrt{8}}{3}\right)\cos\left(\frac{-\sqrt{5}}{3}\right)-\sin\left(\frac{-1}{2}\right)\cos\left(\frac{-2}{3}\right)$

2 out of 3

award full marks
– 1 mark for concept error (including sin and cos when substituting values, without subsequent correction)

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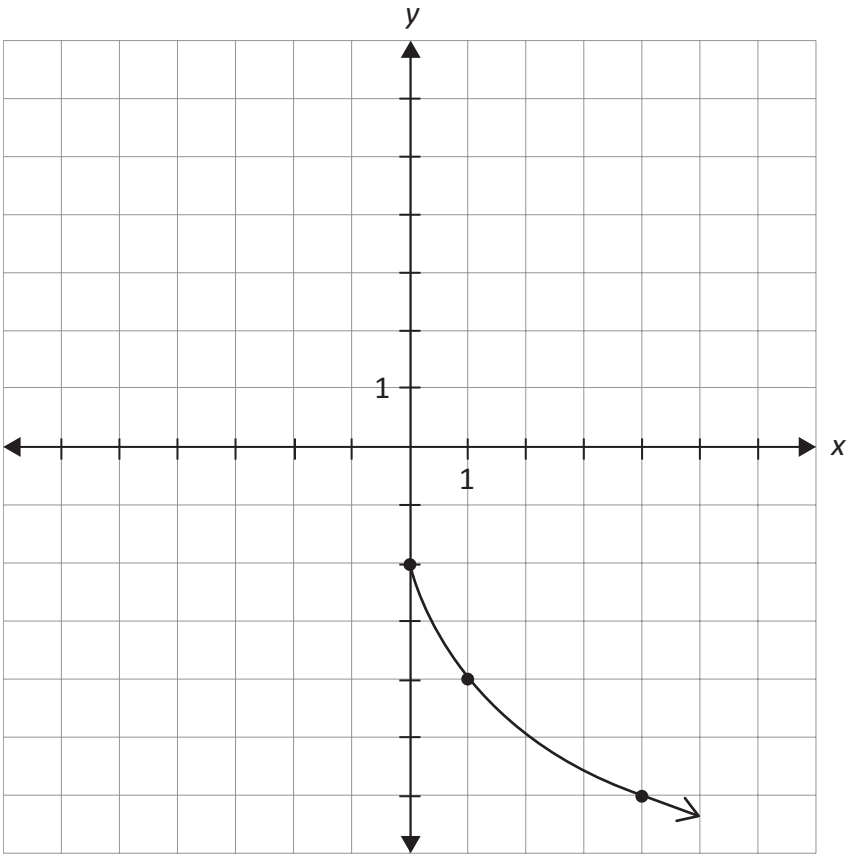
Question 32 R13

Sketch the graph of $y + 2 = -\sqrt{4x}$.

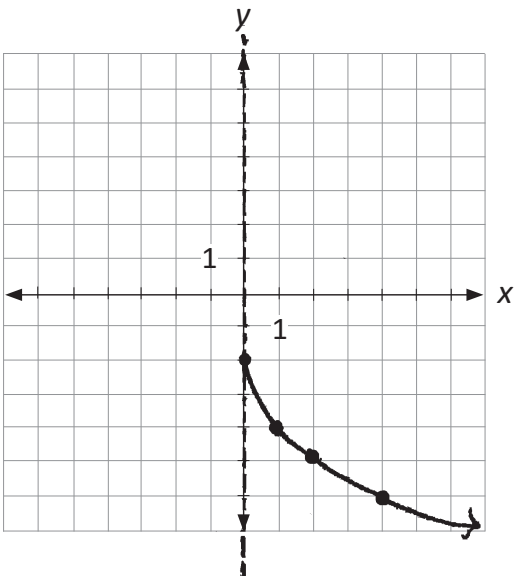
Solution



1 mark for shape of a radical function
1 mark for vertical reflection
1 mark for horizontal compression
1 mark for vertical translation

4 marks

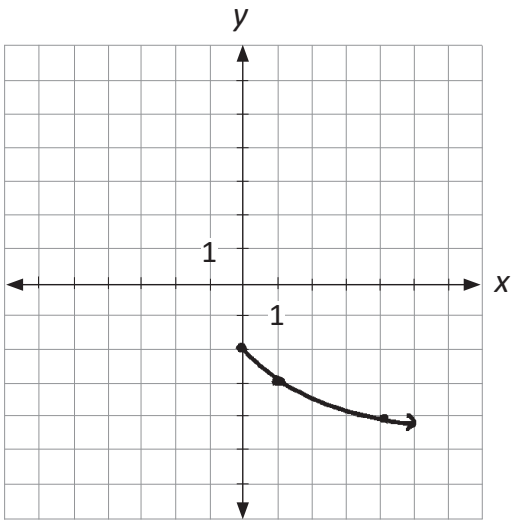
Exemplar 1



3 out of 4

+ 1 mark for vertical reflection
+ 1 mark for horizontal compression
+ 1 mark for vertical translation

Exemplar 2



3 out of 4

+ 1 mark for shape of a radical function
+ 1 mark for vertical reflection
+ 1 mark for vertical translation

Question 33 R11

Express $p(x)=x^4-x^3-6x^2+4x+8$ in completely factored form, given that $(x+1)$ is a factor of $p(x)$.

Solution

-1	1	-1	-6	4	8
		-1	2	4	-8
	1	-2	-4	8	0

½ mark for synthetic division (or equivalent strategy)

$p(x)=(x+1)\left(x^3-2x^2-4x+8\right)$

$p(2)=(2+1)\left(2^3-2(2)^2-4(2)+8\right)$

$p(2)=0$

1 mark for identifying a second zero of $p(x)$

$\therefore (x-2)$ is a factor

2	1	-2	-4	8
		2	0	-8
	1	0	-4	0

½ mark for synthetic division (or equivalent strategy)

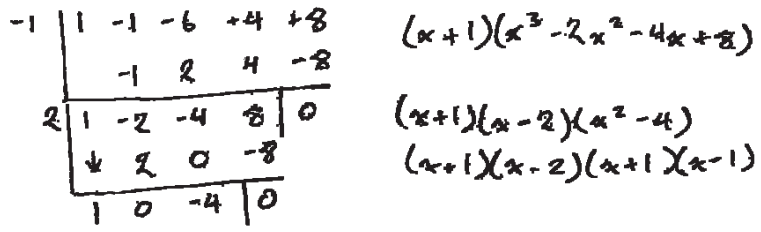
$p(x)=(x+1)(x-2)\left(x^2-4\right)$

$p(x)=(x+1)(x-2)^2(x+2)$

1 mark for consistent product of factors

3 marks

Exemplar 1

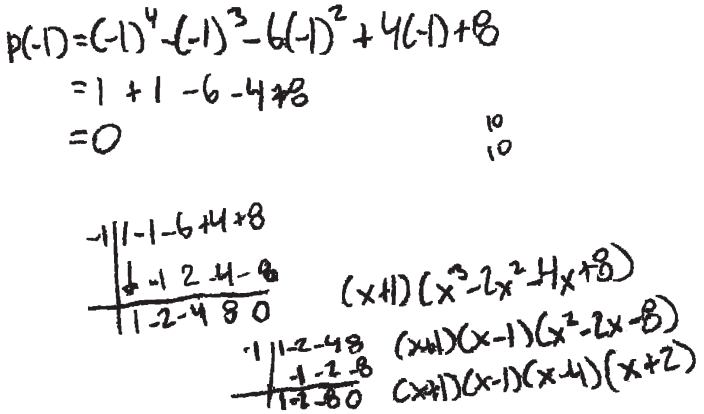


$p(x) =$ $(x+1)^2(x-2)(x-1)$

2½ out of 3

award full marks
– ½ mark for procedural error (incorrect factoring)

Exemplar 2

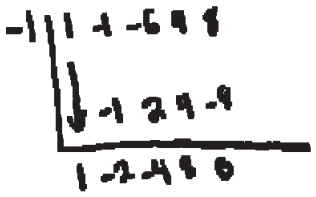


$p(x) =$ $(x+1)(x-1)(x-4)(x+2)$

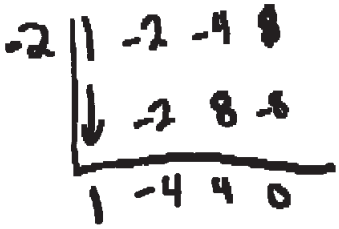
1 out of 3

+ ½ mark for synthetic division
+ 1 mark for consistent product of factors
– ½ mark for procedural error (one incorrect factor)

Exemplar 3



$(x+1)(x^3-2x^2-4x+8)$



$p(x) = (x+1)(x+2)(x^2-4x+4)$

2½ out of 3

award full marks
– ½ mark for procedural error (not completely factored)

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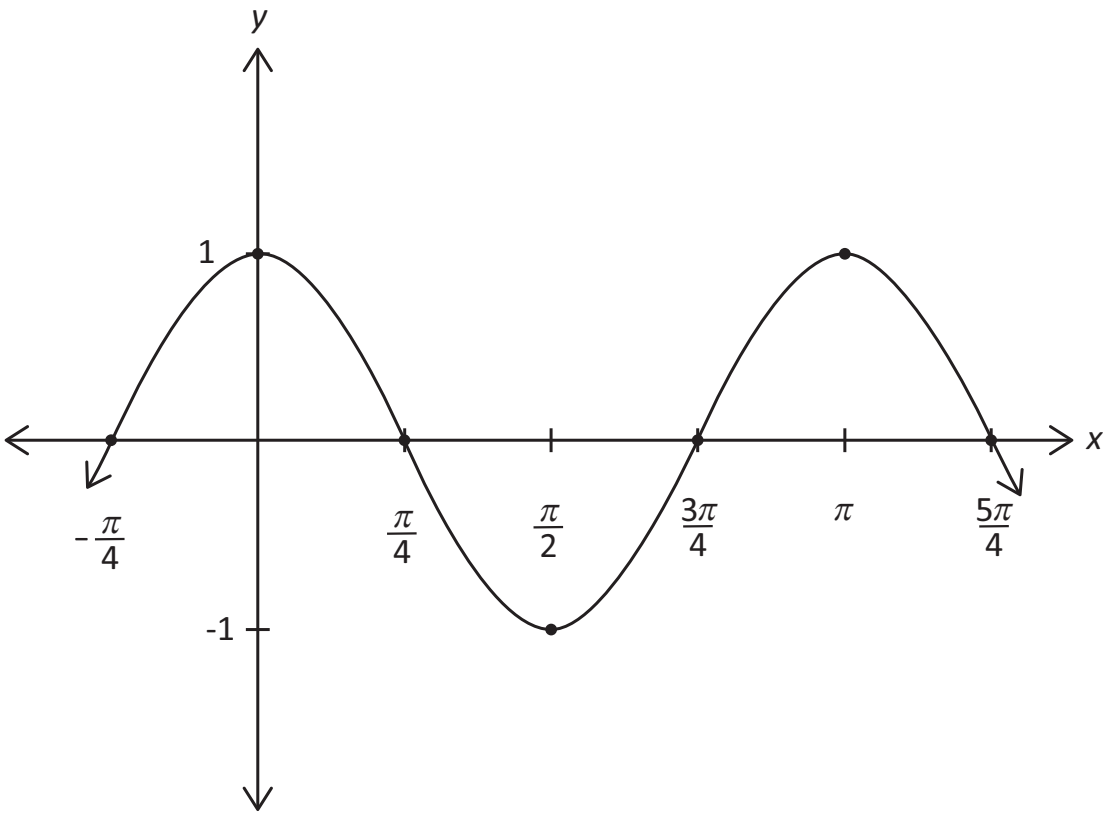
Question 34 T4

Sketch at least one period of the graph of $y = -\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$.

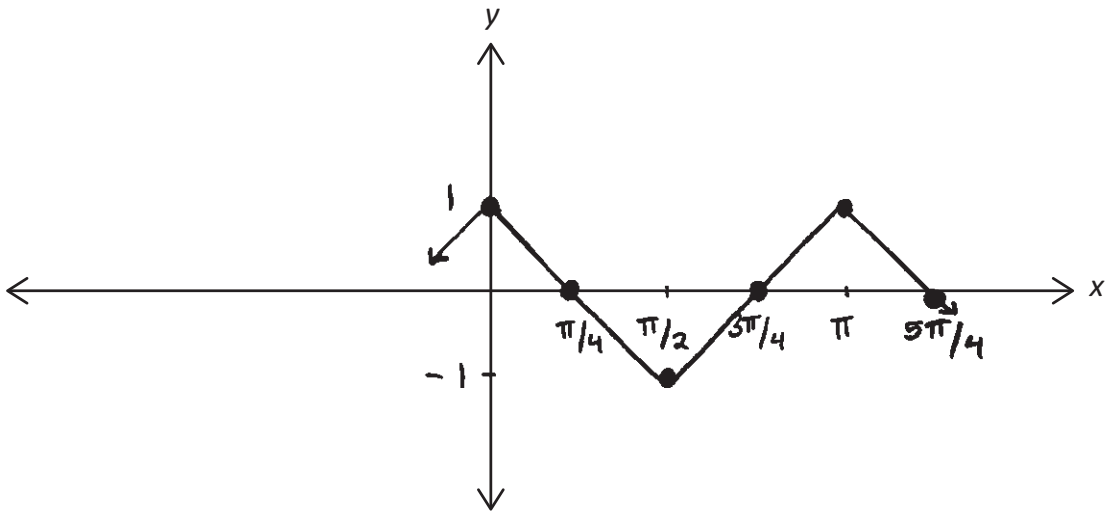
Solution



1 mark for shape of a sinusoidal function with correct amplitude
1 mark for vertical reflection
1 mark for horizontal translation
1 mark for period

4 marks

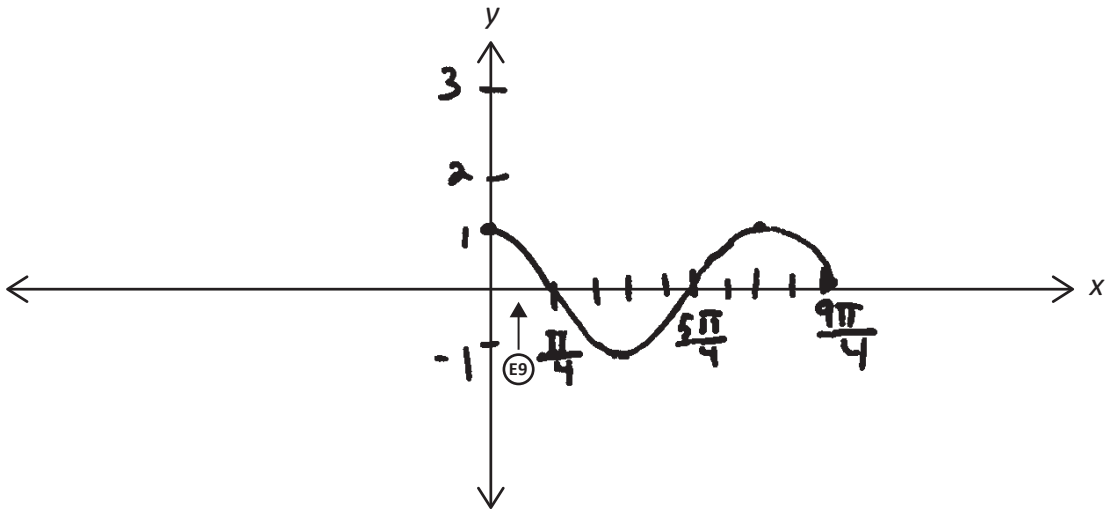
Exemplar 1



3 out of 4

+ 1 mark for vertical reflection
+ 1 mark for horizontal translation
+ 1 mark for period

Exemplar 2



3 out of 4

+ 1 mark for shape of a sinusoidal function with correct amplitude
+ 1 mark for vertical reflection
+ 1 mark for horizontal translation
E9 (scale values on axes incorrectly spaced)

Question 35 R9

Given $g(x)=\log_2(x)-3$,

a) determine the *x*-intercept of the graph.

b) sketch the graph of $g(x)$.

Solution

a) $0=\log_2(x)-3$

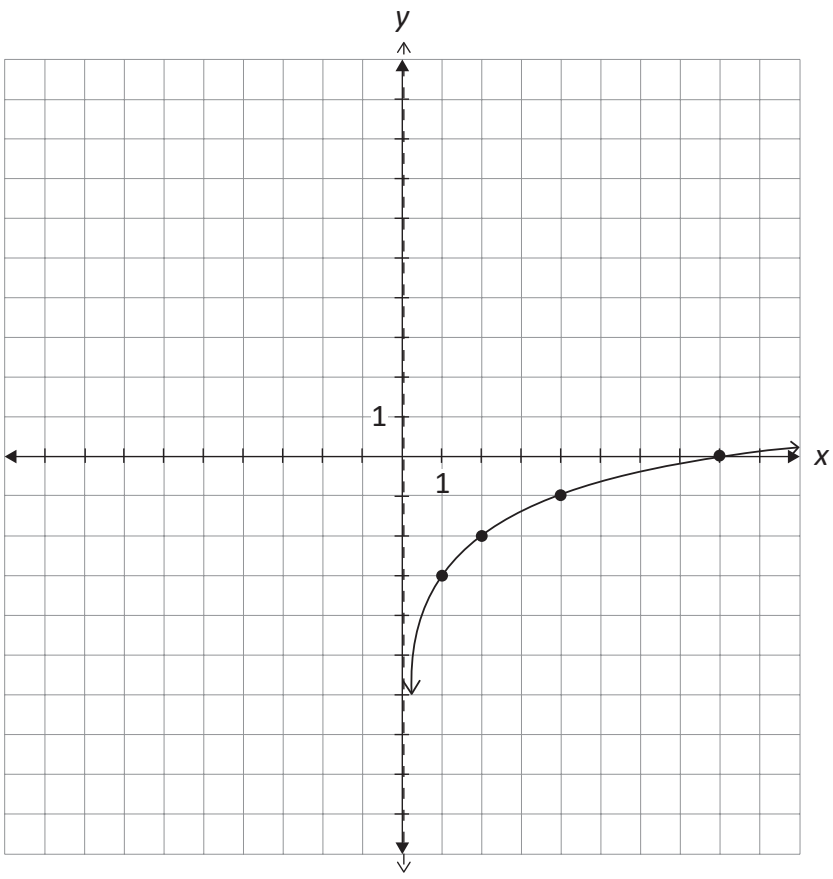
$3=\log_2(x)$

$2^3=x$

$x=8$

1 mark

b)



1 mark for shape of increasing logarithmic function with base 2

1 mark for vertical translation

2 marks

Exemplar 1

a)

$0 = \log_2 x - 3$

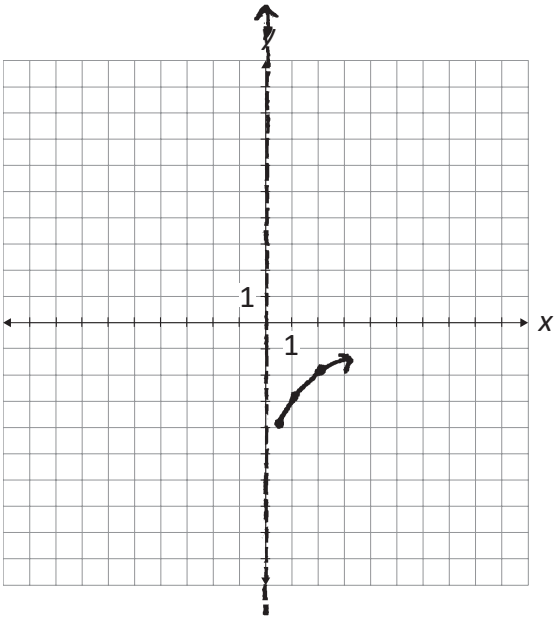
$3 = \log_2 x$

$2^3 = x$ ← E1

1 out of 1

award full marks
E1 (final answer not stated)

b)



$y = 2^x$

x	y
-1	1/2
0	1
1	2

$y = \log_2 x$

x	y
1/2	-1
1	0
2	1

1 out of 2

+ 1 mark for vertical translation

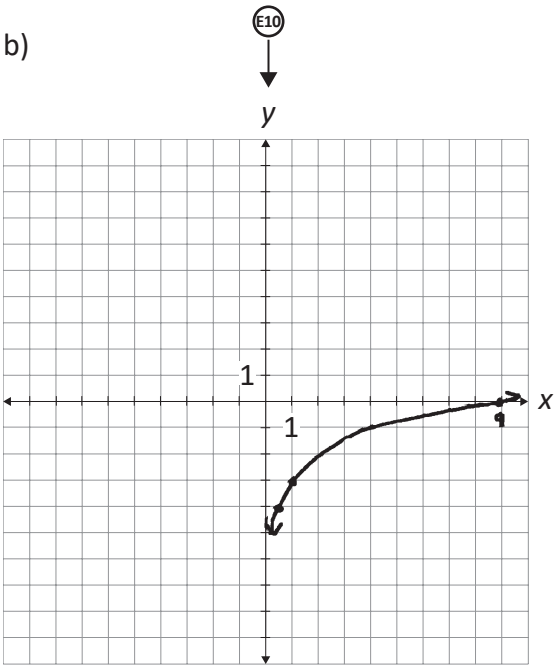
Exemplar 2

a)

x-int at 9

0 out of 1

b)



2 out of 2

award full marks
graph consistent with a)
E10 (asymptote omitted but still implied)

Exemplar 3

a)

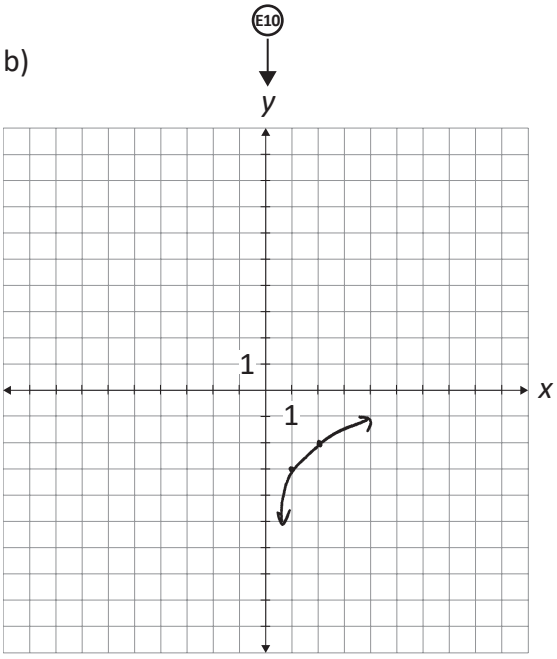
$0 = \log_2 x - 3$

$3 = \log_2 x$

$x = 8$

1 out of 1

b)



1½ out of 2

award full marks
– ½ mark for procedural error (not including *x*-intercept)
E10 (asymptote omitted but still implied)

Question 36 T6

Modesola incorrectly evaluated the expression, $\cos^2\left(30^\circ\right)-\sin^2\left(30^\circ\right)$.

Modesola's answer:

$\cos^2(30^\circ) - \sin^2(30^\circ)$

$= 2\cos(30^\circ)$

$= 2\left(\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3}$

Describe their error.

Solution

Modesola incorrectly applied the double angle identity. They should have written $\cos 2\left(30^\circ\right)$ instead of $2\cos\left(30^\circ\right)$.

1 mark

Exemplar 1

$1 - \sin^2(30°) - \sin^2(30°)$

$= 1 - 2\sin^2(30°)$

$= 1 - 2\left(\frac{1}{2}^2\right)$

$= 1 - 2\left(\frac{1}{4}\right)$

$= \frac{2}{2} - \frac{1}{2}$

$= \frac{1}{2}$

0 out of 1

Exemplar 2

$\cos 2(30°)$

$= \cos(60°)$

$= \frac{1}{2}$

Modesola multiplied cos30° by 2 when they should have only multiplied 30° by 2.

1 out of 1

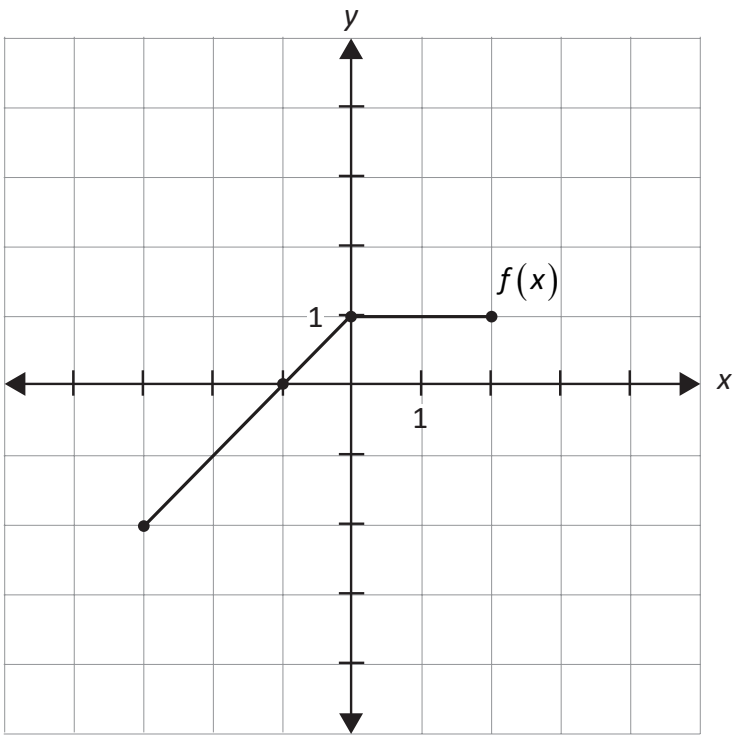
Exemplar 3

They should've found the value of cos, & sin of 30° first and then squared them substracted the two from eachother.

1 out of 1

Question 37 R13

Given the graph of $y = f(x)$, state the range of $y = \sqrt{f(x)}$.



Solution

$\{y | 0 \le y \le 1, y \in \mathbb{R}\}$

or

$[0,1]$

1 mark

Exemplar 1

½ out of 1

award full marks
– ½ mark for procedural error (not including 0 in the range)

Exemplar 2

0 out of 1

Question 38 T2

Determine a value of x that satisfies $\sin^2(2x-1)+\cos^2(x+5)=1$.

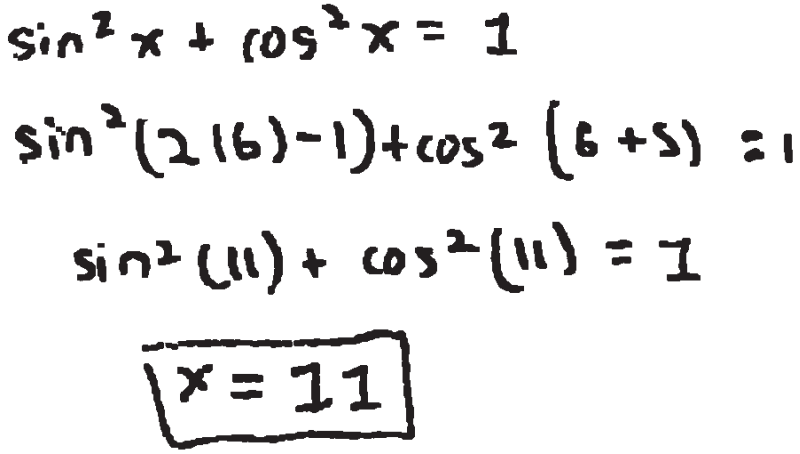
Solution

$2x-1=x+5$

$x=6$

1 mark

Exemplar 1



½ out of 1

award full marks
– ½ mark for procedural error (incorrectly identifying the value of *x*)

Question 39 R10

Solve, algebraically.

$$9^{2x+3} = \left(\frac{1}{3}\right)^{x+1}$$

Solution

$\left(3^2\right)^{2x+3} = \left(3^{-1}\right)^{x+1}$ 1 mark for changing to a common base

$3^{4x+6} = 3^{-x-1}$

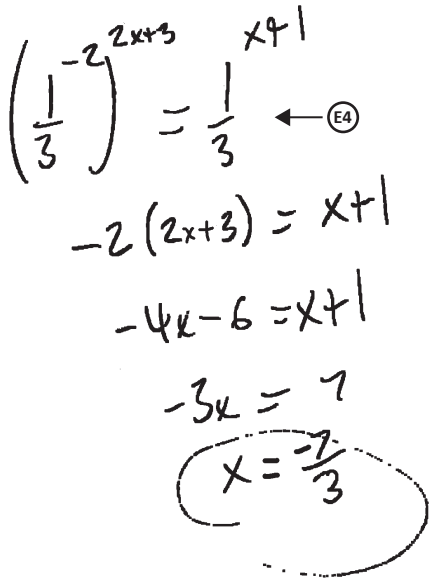
$4x + 6 = -x - 1$ 1 mark for equating exponents with a common base

$5x = -7$

$x = -\frac{7}{5}$

2 marks

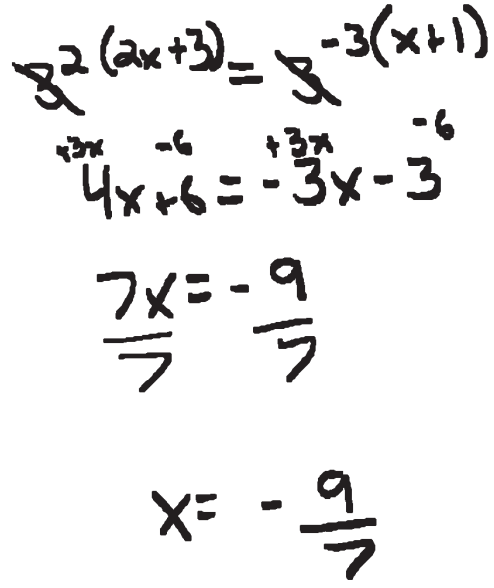
Exemplar 1



1½ out of 2

award full marks
– ½ mark for arithmetic error in line 4
E4 (missing brackets but still implied in line 1)

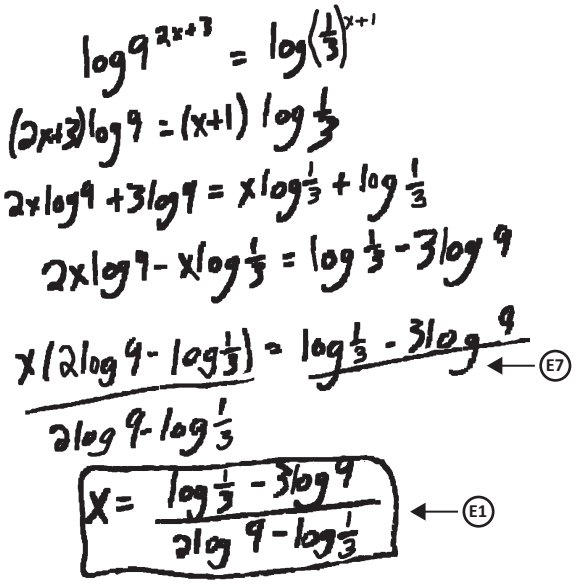
Exemplar 2



1 out of 2

award full marks
– ½ mark for procedural error in line 1 (crossing out bases)
– ½ mark for procedural error in line 1 (incorrect exponent on right hand side)

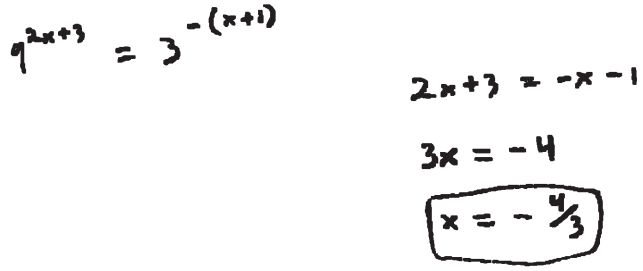
Exemplar 3



2 out of 2

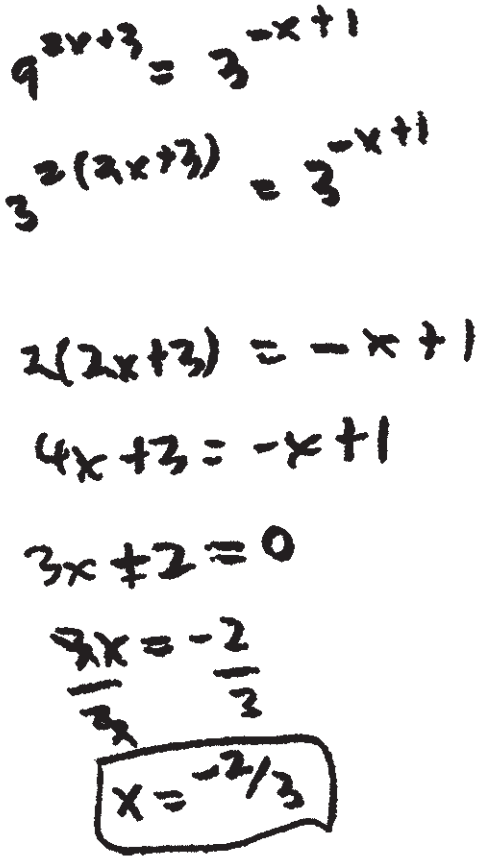
award full marks
E7 (notation error in line 5)
E1 (final answer not stated)

Exemplar 4



0 out of 2

Exemplar 5



1 out of 2

award full marks
– ½ mark for procedural errors lines 1 and 4 (error in distributive property)
– ½ mark for arithmetic error in line 5

Question 40 P2

Determine the total number of arrangements of the letters in the word GUIDES if the vowels and consonants must alternate.

Solution

Method 1

$\underline{6} \bullet \underline{3} \bullet \underline{2} \bullet \underline{2} \bullet \underline{1} \bullet \underline{1} = 72$

1 mark for beginning with any letter

1 mark for alternating consonants and vowels

2 marks

Method 2

$case\ 1: \ \underset{v}{\underline{3}} \bullet \underset{c}{\underline{3}} \bullet \underset{v}{\underline{2}} \bullet \underset{c}{\underline{2}} \bullet \underset{v}{\underline{1}} \bullet \underset{c}{\underline{1}} = 36$

1 mark for alternating consonants and vowels

$case\ 2: \ \underset{c}{\underline{3}} \bullet \underset{v}{\underline{3}} \bullet \underset{c}{\underline{2}} \bullet \underset{v}{\underline{2}} \bullet \underset{c}{\underline{1}} \bullet \underset{v}{\underline{1}} = 36$

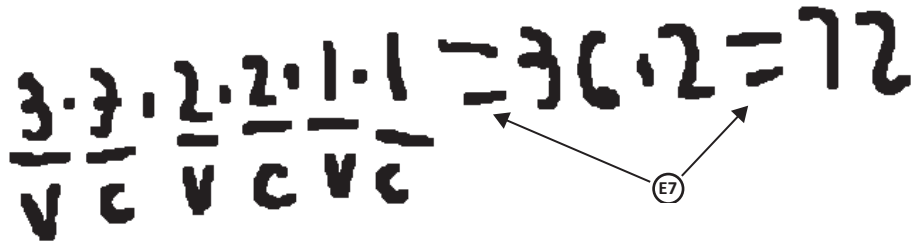
½ mark for second case

$36 + 36 = 72$

½ mark for addition of cases

2 marks

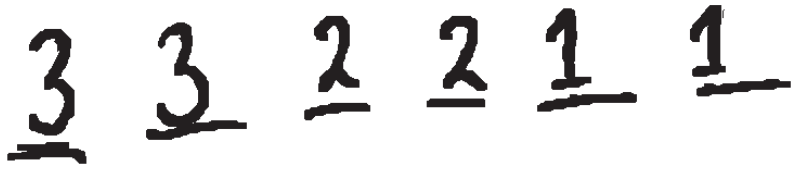
Exemplar 1



2 out of 2

award full marks
E7 (notation error)

Exemplar 2



½ out of 2

+ 1 mark for alternating consonants and vowels
– ½ mark for procedural error (not demonstrating an operation)

Question 41 P2

Solve, algebraically.

$$\frac{5!(n-5)!}{3!(n-6)!}=80$$

Solution

$$\frac{5 \bullet 4 \bullet \cancel{3!}\ (n-5)\cancel{(n-6)!}}{\cancel{3!}\cancel{(n-6)!}}=80$$

1 mark for factorial expansion (½ mark for each)
½ mark for simplification of factorials

$$20(n-5)=80$$

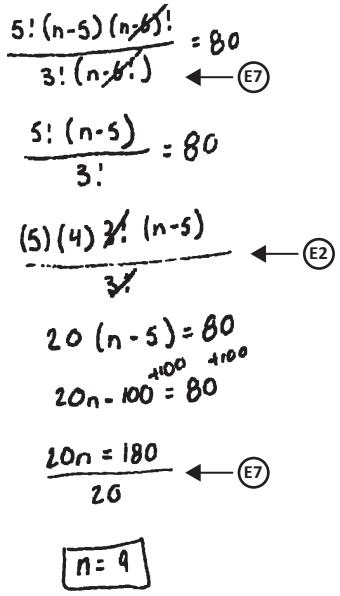
$$n-5=4$$

$$n=9$$

½ mark for solving for n

2 marks

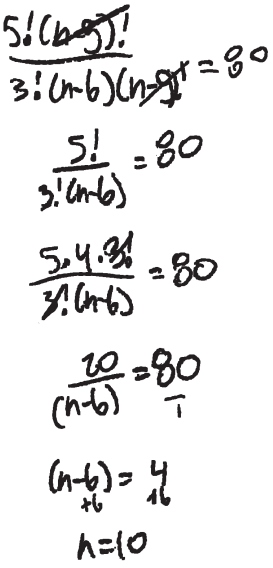
Exemplar 1



2 out of 2

award full marks
E7 (notation error in lines 1 and 6)
E2 (changing an equation to an expression in line 3)

Exemplar 2



1 out of 2

+ ½ mark for factorial expansion
+ ½ mark for simplification of factorials

Exemplar 3

$$\frac{5!(n-5)(n-6)!}{3!(n-6)!} = 80$$

$$\frac{5!(n-5)}{3\cdot 4\cdot 5!} = 80$$

$$\frac{(n-5)}{12} = 80$$

$$(n-5) = 960$$
$$+5 \quad +5$$

$$n = 965$$

1½ out of 2

+ ½ mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for solving for *n*

Exemplar 4

$$5!(n-5)! = 80(3!(n-6)!)$$

$$\frac{120}{120}(n-5)! = \frac{480}{120}(n-6)!$$

$$\frac{(n-5)!}{(n-6)!} = 4$$

½ out of 2

+ ½ mark for factorial expansion

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Question 42 R7

Justify that 2.1 is a better estimate than 2.6 for the value of $\log_5 29$.

Solution

$5^2 = 25$ $5^3 = 125$

$\log_5 25 = 2$ $\log_5 125 = 3$

29 is closer to 25 than 125

$\therefore \log_5 29$ is closer to 2 than 3

1 mark

Exemplar 1

$5^2 = 25$

$5^3 = 125$

½ out of 1

award full marks
– ½ mark for incomplete justification

Exemplar 2

Since 24 is a lot closer to 25 than 125 and logs scale up exponentially 2.1 is a better estimate

½ out of 1

award full marks
– ½ mark for lack of clarity in justification

Question 43 R11

When $p(x) = x^3 + kx + 6$ is divided by $(x+2)$, the remainder is 4. Determine the value of k.

Solution

Method 1

$4 = (-2)^3 + k(-2) + 6$ ½ mark for $x = -2$

1 mark for remainder theorem

$4 = -8 - 2k + 6$

$4 = -2 - 2k$

$6 = -2k$

$k = -3$ ½ mark for solving for k

2 marks

Method 2

$x = -2$ ½ mark for $x = -2$

-2	1	0	k	6
		-2	4	$-2k-8$
	1	-2	$k+4$	$-2k-2$

1 mark for synthetic division (or equivalent strategy)

$-2k - 2 = 4$

$-2k = 6$

$k = -3$ ½ mark for solving for k

2 marks

Exemplar 1

$(-2)^3 - 2k + 6 = 4$

$8 - 2k + 6 = 4$

$-2k = -10$

$k = 5$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 2

Exemplar 2

$$\begin{array}{r|lll} -2 & 1 & k & +6 \\ & \downarrow & -2 & (-2) \\ \hline & 1 & 1 & 4 \end{array}$$

$k - 2 = 1$

$k = 3$

1½ out of 2

award full marks
– ½ mark for procedural error in line 1 (missing 0 placeholder)

Question 44 R2, R3, R5

Describe the transformations used to obtain the graph of the function $y = -f\left(7x + 14\right)$ from the graph of $y = f\left(x\right)$.

Solution

• reflection over the *x*-axis	1 mark for vertical reflection
• horizontal stretch by a factor of $\frac{1}{7}$	1 mark for horizontal stretch
• horizontal translation 2 units to the left	1 mark for horizontal translation

3 marks

Note:

Deduct a maximum of 1 mark for concept error of incorrect order of horizontal transformations.

Exemplar 1

reflect over x-axis
horizontal shift to the left by 2
7 times narrower

2 out of 3

award full marks
– 1 mark for concept error (incorrect order of horizontal transformations)

Exemplar 2

- have a vertical stretch of 7
- move to the left 2 places.

1 out of 3

+ 1 mark for horizontal translation

Appendices

Appendix A

Marking Guidelines

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply:

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- error in or incomplete justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet.*

E1 **final answer**	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in step leading to final answer
E2 **equation/expression**	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 **variables**	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 **brackets**	▪ "sin x^2" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 **units**	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 **rounding**	▪ rounding error ▪ rounding too early
E7 **notation/transcription**	▪ notation error ▪ transcription error
E8 **domain/range**	▪ answer outside the given domain ▪ domain or range written in incorrect order
E9 **graphing**	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated or incorrectly spaced ▪ coordinate points labelled incorrectly
E10 **asymptotes**	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

Irregularities in Provincial Tests

A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an Irregular Test Booklet Report should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how they assisted a student during test administration
- student offering that they received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report.*

Except in the case of cheating or plagiarism where the result is a provinical test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares and *Irregular Test Booklet Report* documenting the situation, the people contacted and the follow-up. The orginal copy of this report is to be returned by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: _____

Date marked: _____

Booklet No.: _____

_____

Problem(s) noted: _____

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Question(s) affected: _____

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Action taken or rationale for assigning marks: _____

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Follow-up: _____

Decision: _____

Marker's Signature: _____

Principal's Signature: _____

For Department Use Only—After Marking Complete

Consultant: _____

Date: _____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
9	R1, R3, R5	3
13a)	R1	1
13b)	R1	2
14	R2, R3	2
19	R1	1
30a)	R6	2
30b)	R6	1
44	R2, R3, R5	3
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
21	T4	1
23	T1	1
25	T3	3
28	T3	2
34	T4	4
38	T2	1
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
2	P3	2
4	P4	3
11	P4	1
17	P2	1
40	P2	2
41	P2	2
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
15	R12	3
22	R11	1
33	R11	3
43	R11	2

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
5	T5	3
8	T6	3
12	T5	1
16	T6	1
27	T6	2
31	T6	3
36	T6	1
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
3	R10	3
10	R10	3
18	R10	1
24	R8	1
26	R8	2
35a)	R9	1
35b)	R9	2
39	R10	2
42	R7	1
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
6	R14	1
7	R13	2
20	R14	1
29a)	R14	2
29b)	R14	1
32	R13	4
37	R13	1