Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

June 2019

Manitoba

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This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Table of Contents

General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Training in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION				
Shade in the circles below for a maximum total deduction of 5 marks (½ mark deduction per error).				
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).				
E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

Example: Marks assigned to the student

	Booklet 1	Selected Response	Booklet 2	Communication Errors (Deduct)	Total
Marks Awarded	25	7	40	1½	70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

Question 1 P2

Avery has 4 adventure books, 5 mystery books, and 1 comic book.

Determine the number of ways he can arrange all of the books on his shelf if each type of book must be grouped together.

Solution

$$\underset{\text{types of books}}{\underline{3!}} \bullet \underset{\text{adventure books}}{\underline{4!}} \bullet \underset{\text{mystery books}}{\underline{5!}} \bullet \underset{\text{comic book}}{\underline{1!}} = 17280 \text{ ways}$$

1 mark for arrangement of types of books
1 mark for arrangement of adventure, mystery, and comic books

2 marks

Note:

- 1! does not need to be shown.

Exemplar 1

$4! \cdot 5! \cdot 1! = 2880$ ways

1 out of 2

+ 1 mark for arrangement of adventure, mystery, and comic books

Exemplar 2

$4!\, 5!\, 3!$

2 out of 2

award full marks
E1 (final answer not stated)

Exemplar 3

$3! = 6$

1 out of 2

+ 1 mark for arrangement of types of books

Solve the following equation, algebraically, over the interval $[0, 2\pi]$.

$$5\cos^2\theta - \cos\theta - \sin^2\theta = 0$$

Solution

$$5\cos^2\theta - \cos\theta - \left(1 - \cos^2\theta\right) = 0$$

1 mark for substitution of an appropriate identity

$$5\cos^2\theta - \cos\theta - 1 + \cos^2\theta = 0$$

$$6\cos^2\theta - \cos\theta - 1 = 0$$

$$\left(3\cos\theta + 1\right)\left(2\cos\theta - 1\right) = 0$$

$\cos\theta = -\dfrac{1}{3}$ $\qquad$ $\cos\theta = \dfrac{1}{2}$

1 mark for solving for $\cos\theta$ (½ mark for each branch)

$\theta_r = 1.230\ 959$

$\theta = 1.911,\ 4.373$ $\qquad$ $\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

2 marks for solving for θ (½ mark for each value of θ)

or

$\theta = 1.911,\ 4.373$ $\qquad$ $\theta = 1.047,\ 5.236$

4 marks

Exemplar 1

$$5\cos^2\theta - \cos\theta - \sin^2\theta = 0$$

$$5\cos^2\theta - \cos\theta = \sin^2\theta$$

$$5\cos^2\theta - \cos\theta = 1 - \cos^2\theta$$

$$-6\cos^2\theta + \cos\theta + 1 = 0$$

$$-(6\cos^2\theta - \cos\theta - 1) = 0$$

$$(2\cos\theta - 1)(3\cos\theta + 1)$$

$$2\cos\theta - 1 = 0 \qquad 3\cos\theta + 1 = 0$$

$$\cos\theta = 1/2 \qquad \cos\theta = -1/3$$

$$\frac{\pi}{6}, \frac{11\pi}{6}$$

---

2 out of 4

+ 1 mark for substitution of an appropriate identity
+ 1 mark for solving for $\cos\theta$
E2 (changing an equation to an expression in line 6)

Exemplar 2

$0 \le \theta \le 2\pi$

$5\cos^2\theta - \cos\theta - (1 - \cos^2\theta) = 0$

$5\cos^2\theta - \cos\theta - 1 + \cos^2\theta = 0$

$6\cos^2\theta - \cos\theta - 1 = 0$

$(3\cos\theta + 2)(\cos\theta - 1)$

$3\cos\theta + 2 = 0$

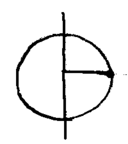
$\frac{3\cos\theta}{3} = \frac{-2}{3}$

$\cos\theta = -\frac{2}{3}$

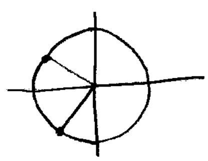
$\theta = 0.8411$

$\cos\theta - 1 = 0$

$\cos\theta = 1$



$\theta = 2\pi$



$\theta = \pi - 0.8411$

$\theta = 2.30$

$\theta = \pi + 0.8411$

$\theta = 3.983$

$\theta = 2.30;\ 3.983$

$\theta = 2.30;\ 3.983;\ 2\pi$

3 out of 4

+ 1 mark for substitution of an appropriate identity
+ 1 mark for solving for $\cos\theta$
+ 1 mark for solving for θ (left branch)
+ ½ mark for solving for θ (right branch)
– ½ mark for arithmetic error in line 4
E2 (changing an equation to an expression in line 4)
E6 (rounding error)

Exemplar 3

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta - 1 = -\sin^2\theta$

$5\cos^2\theta - \cos\theta + \cos^2\theta - 1$

$6\cos^2\theta - \cos\theta - 1$ $\quad -6 \to -3,\ 2$

$(6\cos^2\theta - 3\cos\theta)(+2\cos\theta - 1)$

$3\cos\theta(2\cos\theta - 1) + 1(2\cos\theta - 1)$

$(2\cos\theta - 1)(3\cos\theta + 1)$

$2\cos\theta = 1$ $\qquad 3\cos\theta = -1$

$\cos\theta = \frac{1}{2}$ $\qquad \cos\theta = -\frac{1}{3}$

no solutions

$\frac{\pi}{3}$ $\quad \frac{5\pi}{3}$

3 out of 4

+ 1 mark for substitution of an appropriate identity
+ 1 mark for solving for $\cos\theta$
+ 1 mark for solving for θ (left branch)

E1 (final answer not stated)
E2 (changing an equation to an expression in line 1)
E7 (notation error in line 3)

Given that $-6048x^2y^5$ is the sixth term in the expansion of $(3x - m)^7$, determine m.

Solution

$-6048x^2y^5 = {}_7C_5(3x)^2(-m)^5$ — 2 marks (1 mark for ${}_7C_5$; ½ mark for each consistent factor)

$-6048x^2y^5 = 21(9x^2)(-m^5)$

$-6048x^2y^5 = -189x^2m^5$

$32y^5 = m^5$ — ½ mark for simplification

$2y = m$ — ½ mark for m

3 marks

Exemplar 1

$$t_5 = {}_7C_5 \quad (3x)^2 \quad (y)^5$$

$$= 21 \cdot 9x^2 \cdot -32y$$

$$m = -32y$$

1½ out of 3

+ 1 mark for ${}_7C_5$

+ ½ mark for one consistent factor

Exemplar 2

$${}_7C_5 \, (3x)^2 \, (-m)^5$$

$$21 \, (9x^2) \, (-m^5)$$

$$189x^2 - m^5$$

$$\frac{-6048}{189} = -m^5$$

$$189x^2 - 32y^5$$

$$-6048x^2y^5$$

$$\sqrt[5]{32} = 2$$

$$\boxed{m = 2}$$

$${}_7C_5 \, (3x)^2 \, (-2)^5$$

$$21 \, (9x^2) \, (-32y^5)$$

$$= -6048x^2y^5$$

2 out of 3

+ 1 mark for ${}_7C_5$

+ 1 mark for both consistent factors

Exemplar 3

$$t_{k+1} = {}_nC_k(a)^{n-k}(b)^k$$

$$-6048x^2y^5 = {}_7C_5(3x)^{7-5}(m)^5$$

$$-6048x^2y^5 = 21(9x^2)(m)^5$$

$$\frac{-6048x^2y^5}{189x^2} = \frac{189x^2m^5}{189x^2}$$

$$\sqrt[5]{-32y^5} = \sqrt[5]{m^5}$$

$$\boxed{m = -2y}$$

2½ out of 3

+ 1 mark for ${}_7C_5$
+ ½ mark for one consistent factor
+ ½ mark for simplification
+ ½ mark for m

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A series of blood tests measures the concentration of a prescribed drug. This concentration decreases according to the formula $A = Pe^{rt}$ where:

A is the concentration at time t
P is the initial concentration
r is the rate of change
t is the time, in hours, after the first blood test

The initial concentration is 3.8900 units/mL. Three hours later, the concentration is 1.7505 units/mL.

a) Determine the rate of change, r, algebraically.

b) Determine the concentration of the prescribed drug four hours after the initial concentration of 3.8900 units/mL was measured. Express the answer correct to 4 decimal places.

Solution

a)

$$1.7505 = 3.8900e^{r(3)}$$ ½ mark for substitution

$$0.45 = e^{3r}$$

$$\ln 0.45 = 3r\ln e$$ ½ mark for applying logarithms
½ mark for power law

$$\frac{\ln 0.45}{3} = r$$ ½ mark for the value of r

or

$$-0.266\ 169... = r$$

2 marks

b)

$$A = 3.8900e^{-0.266\ 169(4)}$$

$$A = 1.341\ 424...$$

$$A = 1.3414 \text{ units/mL}$$ 1 mark for answer consistent with a)

1 mark

Exemplar 1

a)

$$1.7505 = 3.89^{r(3)}$$

$$\log 1.7505 = r3\log 3.89$$

$$r = \frac{\log 1.7505}{3\log 3.89}$$

$$\boxed{r = 0.137}$$

1 out of 2

award full marks
– 1 mark for concept error (omitting *e*)

b)

$$A = 3.89^{(0.137)(4)}$$

$$\boxed{A = 2.11 \text{ units/mL}}$$

1 out of 1

award full marks
E6 (rounding error)

Exemplar 2

a)

$$\frac{1.7505}{3.8900} = \frac{3.8900e^{r3}}{3.8900}$$

$$0.45 = e^{r3}$$

$$\ln 0.45 = r3 \ln e$$

$$\frac{\ln 0.45}{3} = \frac{r3}{3}$$

$$-0.266169232 = r$$

$$\boxed{-0.266 = r}$$

2 out of 2

b)

$$\frac{\ln 0.45}{4} = \frac{r4}{4}$$

$$-0.19962 6924 = r$$

$$\boxed{-0.200 \text{ mL/h} = r}$$

0 out of 1

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Ariane uses the formula $s = \theta r$ to determine the arc length of a circle that has a central angle of $20°$ and a radius of 15 cm.

Below is Ariane's work:

$$s = \theta r$$
$$s = (20)(15)$$
$$s = 300 \text{ cm}$$

Describe her error.

Solution

When using the formula $s = \theta r$, the angle must be in radians. Ariane did not convert the central angle from degrees to radians.

1 mark

Exemplar 1

The 15 radius must be changed to radians.

0 out of 1

Exemplar 2

$$S = \frac{20}{\pi} \cdot 15$$

$$= 6.36 \cdot 15$$

$$= 95.49 \text{ cm}$$

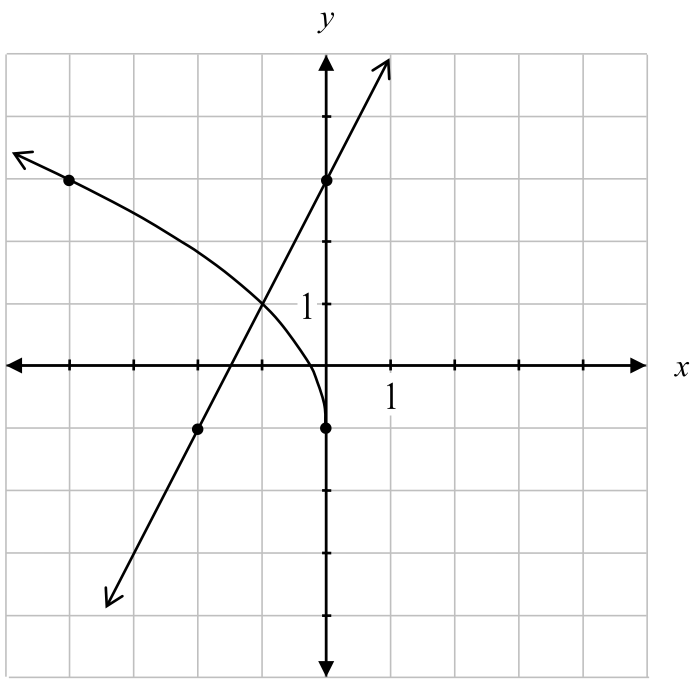
0 out of 1

Exemplar 3

She did not multiply by $\frac{\pi}{180°}$.

1 out of 1

Using the graphs below, state the solution of the equation $2x + 3 = 2\sqrt{-x} - 1$.



Solution

$x = -1$

1 mark

Exemplar 1

$x = -1$
$y = 1$

0 out of 1

Exemplar 2

-1

1 out of 1

award full marks
E7 (notation error)

Question 7 R10

Solve, algebraically.

$$\log_2(x+3) = 5 - \log_2(x-1)$$

Solution

Method 1

$\log_2(x+3) + \log_2(x-1) = 5$	
$\log_2[(x+3)(x-1)] = 5$	1 mark for product law
$x^2 + 2x - 3 = 2^5$	1 mark for exponential form
$x^2 + 2x - 3 = 32$	
$x^2 + 2x - 35 = 0$	
$(x+7)(x-5) = 0$	
~~$x = -7$~~ $x = 5$	½ mark for solving for the permissible value of x ½ mark for showing the rejection of the extraneous root

3 marks

Method 2

$\log_2(x+3) + \log_2(x-1) = 5$	
$\log_2[(x+3)(x-1)] = \log_2 2^5$	1 mark for product law
$x^2 + 2x - 3 = 32$	1 mark for equating arguments
$x^2 + 2x - 35 = 0$	
$(x+7)(x-5) = 0$	
~~$x = -7$~~ $x = 5$	½ mark for the permissible value of x ½ mark for showing the rejection of the extraneous root

3 marks

Exemplar 1

$$\log_2 (x+3) + \log_2 (x-1) = 5$$

$$\log_2 (x+3)(x-1) = 5$$

$$\log_2 (x^2 - x + 3x - 3) = 5$$

$$\log_2 (x^2 + 2x - 3) = 5$$

$$2^5 = x^2 + 2x - 3$$

$$32 = x^2 + 2x - 3$$
$$-32 \qquad\qquad -32$$

$$= x^2 + 2x - 35$$

$$(x+5)(x-2)$$

$$x = -\not{5},\ 2$$

$$\boxed{x = 2}$$

2½ out of 3

award full marks
– ½ mark for arithmetic error in line 8
E2 (changing an equation to an expression in line 8)

Exemplar 2

$$\log_2(x+3)+\log_2(x-1) = 5$$

$$(x+3)(x-1) = 5$$

$$x^2+2x-3 = 5$$
$$\quad -5 \quad -5$$

$$x^2+2x-8=0$$

$$\underbrace{x^2+4x}_{x}\ \underbrace{-2x-8}_{-2} = 0$$

$$x(x+4)-2(x+4) = 0$$

$$(x-2)(x+4) = 0$$

$$x = 2, \cancel{-4}$$

$$\boxed{x=2}$$

---

2 out of 3

+ 1 mark for product law
+ ½ mark for the permissible value of *x*
+ ½ mark for showing the rejection of the extraneous root
E7 (notation error in line 5)

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Question 8 P3

Explain why the value of n must be greater than or equal to the value of r, when using ${}_nC_r$.

Solution

The number of objects to select, r, cannot be greater than the total number of objects, n.

1 mark

Exemplar 1

The equation expande to $\frac{n!}{(n-r)!r!}$

If n is less than r, you will have a negative factorial

You can't have a negative factorial.

1 out of 1

Exemplar 2

The "n" value must be greater or equal to "r", because in the formula,

you cannot have a zero in the denominator.

0 out of 1

Question 9 R2, R3, R5

Given that $y = |x|$, determine the equation of the resulting function, $g(x)$, after the following transformations:

- reflection in the x-axis
- vertical translation 5 units down
- horizontal stretch by a factor of 3

Solution

$g(x) = -\left|\frac{1}{3}x\right| - 5$

1 mark for vertical reflection
1 mark for vertical translation
1 mark for horizontal stretch

3 marks

Exemplar 1

$g(x) =$ $-\sqrt{(x-3)}\ -5$

1 out of 3

+ 1 mark for vertical reflection
+ 1 mark for vertical translation
– 1 mark for concept error (incorrect function)

Exemplar 2

$g(x) =$ $-\left(\frac{1}{3}(x)\right)-5$

2 out of 3

award full marks
– 1 mark for concept error (incorrect function)

Exemplar 3

$g(x) =$ $\left|-\frac{1}{3}x\right|-5$

2 out of 3

+ 1 mark for vertical translation
+ 1 mark for horizontal stretch

Question 10 R14

Explain why the graph of $y = \dfrac{x-1}{x^2+x-6}$ has a horizontal asymptote at $y = 0$.

Solution

As x approaches positive or negative infinity, y approaches zero. **1 mark**

or

The degree of the numerator is less than the degree of the denominator.

Exemplar 1

because the leading coefficient in the denominator is greater than the numerator, which means $y=0$.

0 out of 1

Exemplar 2

The graph has a horizontal asymptote at $y = 0$ because there isn't a value after the rational. Even when the bottom is factored, we have two vertical asymptotes, and no horizontal asymptotes.

0 out of 1

Exemplar 3

the m value, which is the exponent on x in the numerator which is 1 is less than n, which is the exponent on x in the numerator which is 2. $m<n$ then Horizontal Asymptote $y=0$.

½ out of 1

award full marks
– ½ mark for terminology error in explanation

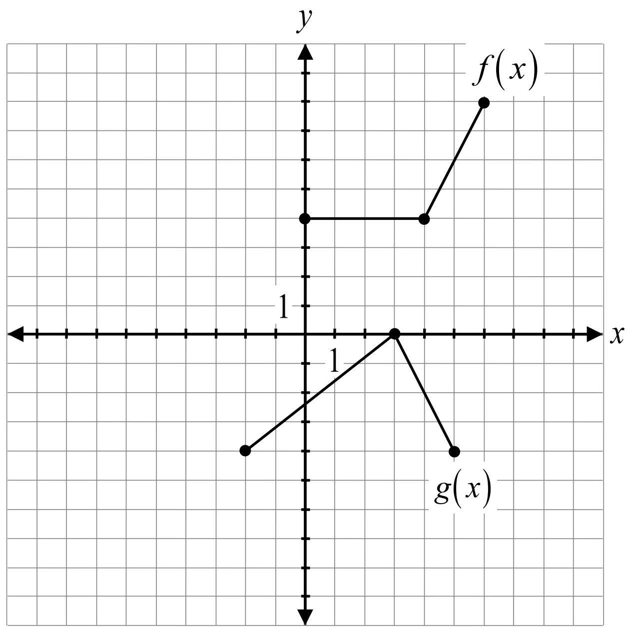
Exemplar 4

$$y = \frac{x-1}{(x+3)(x-2)}$$

The graph has a horizontal asymptote at $y=0$, because there is no vertical translation in the formula.

0 out of 1

Given the graphs of $f(x)$ and $g(x)$, evaluate $g(f(2))$.



Solution

$f(2) = 4$ ½ mark for the value of $f(2)$

$g(4) = -2$ ½ mark for consistent value of $g(f(2))$

1 mark

Exemplar 1

$g(f(2))$

$g(4)$

$g = -2$

1 out of 1

award full marks
E7 (notation error in line 3)

Exemplar 2

$g(4)$

2

½ out of 1

+ ½ mark for the value of $f(2)$

Question 12 T5

Kennedy was asked to solve the equation $\tan\theta = 1$ over all real numbers.

Below is Kennedy's solution:

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

Describe her error.

Solution

Kennedy did not include the general solution in her answer. **1 mark**

Exemplar 1

she only gave the solutions for the first and third quadrants since tan θ is positive in first and third quadrants.

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Exemplar 2

She only solved over the interval of $[0, 2\pi]$ and didn't account for all reals.

1 out of 1

Exemplar 3

because it's over all real numbers, one answer should be $\theta = \frac{\pi}{4} + \pi$.

0 out of 1

Question 13 P2, P3

Solve, algebraically.

$$ {}_{n}C_{3} = 3\left({}_{n}P_{2}\right) $$

Solution

$$ \frac{n!}{(n-3)!3!} = \frac{3n!}{(n-2)!} $$ 1 mark for substitution into equation (½ mark for each side)

$$ \frac{n!(n-2)!}{(n-3)!3!} = 3n! $$

$$ \frac{n!(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 3!(3n!) $$ 1 mark for factorial expansion

$$ n-2 = 3!(3) $$ ½ mark for simplification of factorials

$$ n-2 = 18 $$

$$ n = 20 $$ ½ mark for solving for n

3 marks

Exemplar 1

$$\frac{n!}{3!(n-3)!} = 3\left(\frac{n!}{(n-2)!}\right)$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\cancel{(n-3)!}} = 3\left(\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}\right)$$

$$\frac{n(n-1)(n-2)}{3!} = 3(n(n-1))$$

2½ out of 3

+ 1 mark for substitution into equation
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials

Exemplar 2

$$\frac{n!}{3!(n-3)!} = 3\left(\frac{n!}{(n-2)!}\right)$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\cancel{(n-3)!}} = 3\left(\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}\right)$$

$$\frac{n^2-n}{6} = 3(n^2-n)$$

$$n^2-n = (3n^2-3n)6$$

$$n^2-n = 18n^2-18n$$

$$0 = 18n^2-n^2-18n+n$$

$$\frac{0}{17} = \frac{17n^2-17n}{17}$$

$$0 = n^2-n$$

$$0 = (n+1)(n-1)$$

$$\cancel{n=-1} \qquad \boxed{n=1}$$

2 out of 3

award full marks
– ½ mark for procedural error in line 3
– ½ mark for arithmetic error in line 9
E1 (impossible solution not rejected in final answer)

Exemplar 3

$$\frac{n!}{(n-3)!\,3!} = 3\left(\frac{n!}{n-2!}\right)$$

$$\frac{n!}{(n-3)!\,3!} = \frac{3n!}{n-2!}$$

$$\frac{(n)(n-1)(n-2)\cancel{(n-3)}!}{\cancel{(n-3)!}\,6} = 3\,\frac{(n)(n-1)\cancel{(n-2)!}}{\cancel{n-2!}}$$

$$\frac{\cancel{(n)}\cancel{(n-1)}(n-2)}{\cancel{n}\cancel{(n-1)}} = \frac{3\cancel{(n)}\cancel{(n-1)}}{\cancel{(n)}\cancel{(n-1)}}$$

$$n-2=3$$

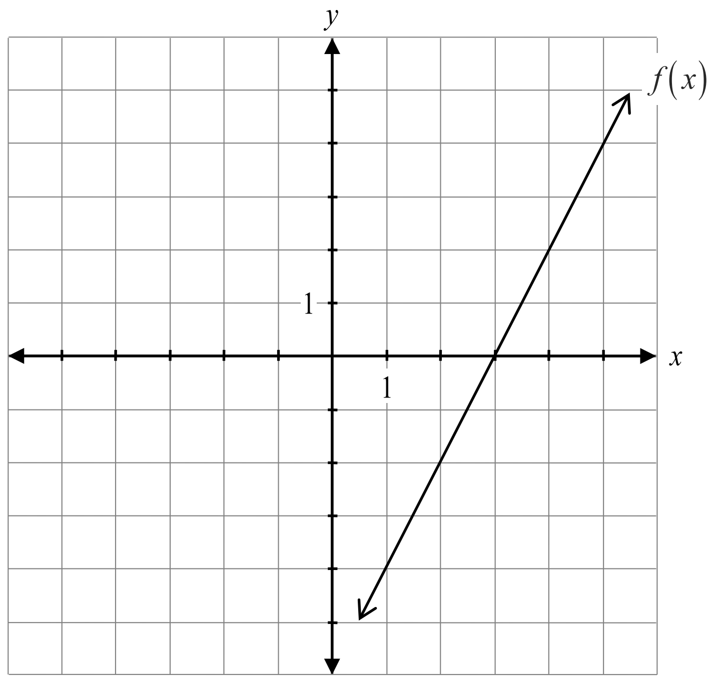
$$\boxed{n=5}$$

---

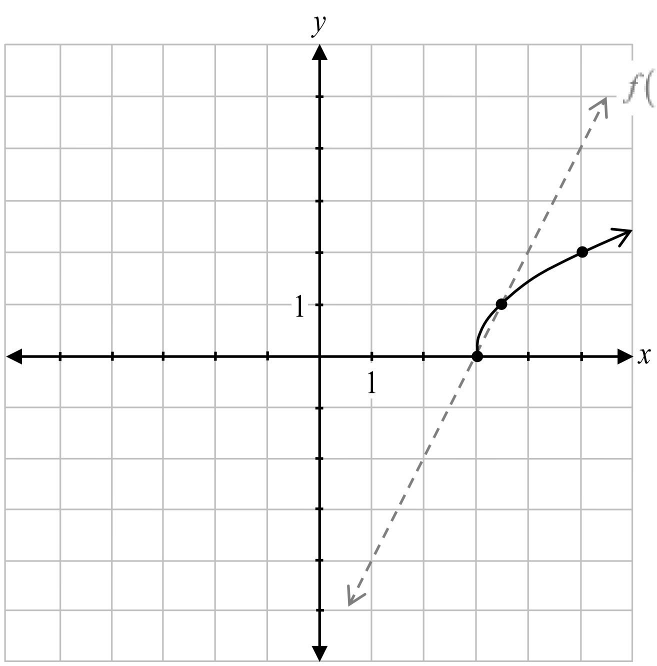
3 out of 3

award full marks
E4 (missing brackets but still implied in lines 1, 2, and 3)
E7 (transcription error in line 4)

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.



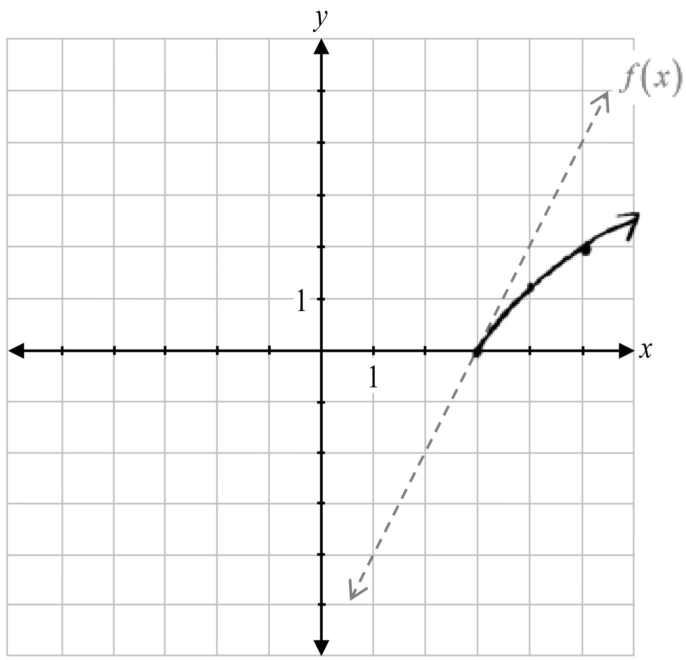
Solution



1 mark for restricting domain
½ mark for shape between invariant points
½ mark for shape to the right of invariant points

2 marks

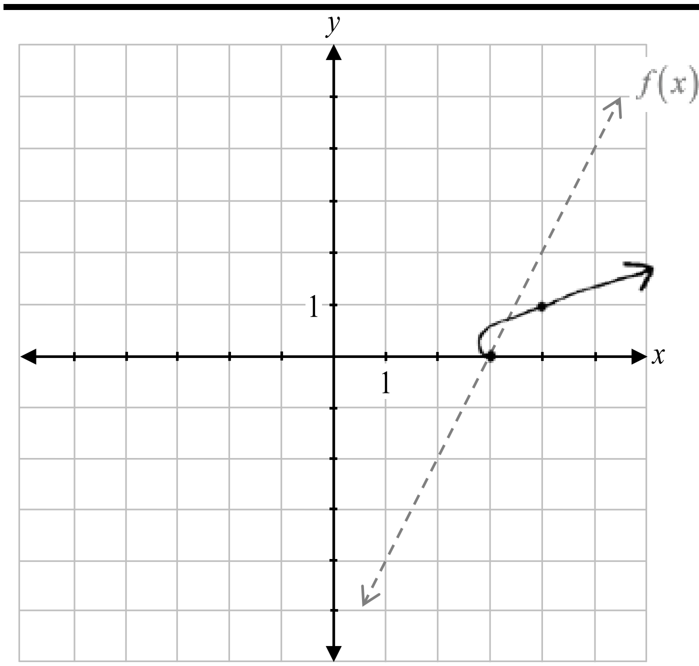
Exemplar 1



1½ out of 2

+ 1 mark for restricting domain
+ ½ mark for shape to the right of the invariant points

Exemplar 2



1 out of 2

+ 1 mark for restricting domain

Question 15 T6

Prove the identity for all permissible values of θ.

$$\frac{\sec\theta - \tan\theta\sin\theta}{\tan\theta\sin\theta} = \csc^2\theta - 1$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\dfrac{\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta} \bullet \sin\theta}{\dfrac{\sin\theta}{\cos\theta} \bullet \sin\theta}$	$\csc^2\theta - 1$
$\dfrac{\dfrac{1-\sin^2\theta}{\cancel{\cos\theta}}}{\dfrac{\sin^2\theta}{\cancel{\cos\theta}}}$	
$\dfrac{1-\sin^2\theta}{\sin^2\theta}$	
$\dfrac{1}{\sin^2\theta} - \dfrac{\sin^2\theta}{\sin^2\theta}$	
$\csc^2\theta - 1$	

1 mark for substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Method 2

Left-Hand Side	Right-Hand Side
$\dfrac{\sec\theta}{\tan\theta\sin\theta}-1$	$\csc^2\theta-1$
$\dfrac{\dfrac{1}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}\bullet\sin\theta}-1$	
$\dfrac{1}{\cos\theta}\bullet\dfrac{\cos\theta}{\sin^2\theta}-1$	
$\dfrac{1}{\sin^2\theta}-1$	
$\csc^2\theta-1$	

1 mark for substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$\dfrac{\left(\dfrac{1}{\cos\theta} - \left(\dfrac{\cos\theta}{\sin\theta}\right)\sin\theta\right)}{\left(\left(\dfrac{\cos\theta}{\sin\theta}\right)\sin\theta\right)}$	$1+\cot\theta - 1$
$\dfrac{\left(\dfrac{1}{\cos\theta} - \cos\theta\right)}{\cos\theta}$	$1+\dfrac{\sin\theta^2}{\cos\theta^2}-1$
$\dfrac{\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right)}{\cos\theta}$	$\dfrac{\sin\theta^2}{\cos\theta^2}$
$\dfrac{\dfrac{\sin\theta^2}{\cos\theta}}{\cos\theta}$	
$\dfrac{\sin\theta^2}{\cos^2\theta}$	

2 out of 3

+ 1 mark for algebraic strategies
+ 1 mark for logical process to prove the identity

E4 (“$\sin x^2$” written instead of “$\sin^2 x$”)

Exemplar 2

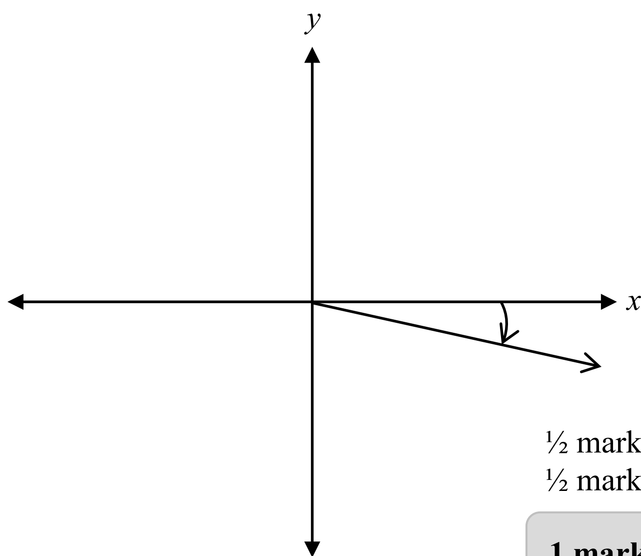
Left-Hand Side	Right-Hand Side
$\dfrac{\sec\theta - \tan\theta\sin\theta}{\tan\theta\sin\theta}$	$\csc^2\theta - 1$
$\dfrac{\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta}{\dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta}$	$\dfrac{1}{\sin^2\theta} - 1$
$\dfrac{\dfrac{1-\sin\theta}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}}$	
$\dfrac{1-\sin\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$	
$\dfrac{1-\sin\theta}{\sin\theta}$	

1 out of 3

+ 1 mark for substitution of appropriate identities

Sketch the angle of $-\frac{\pi}{12}$ radians in standard position.

Solution



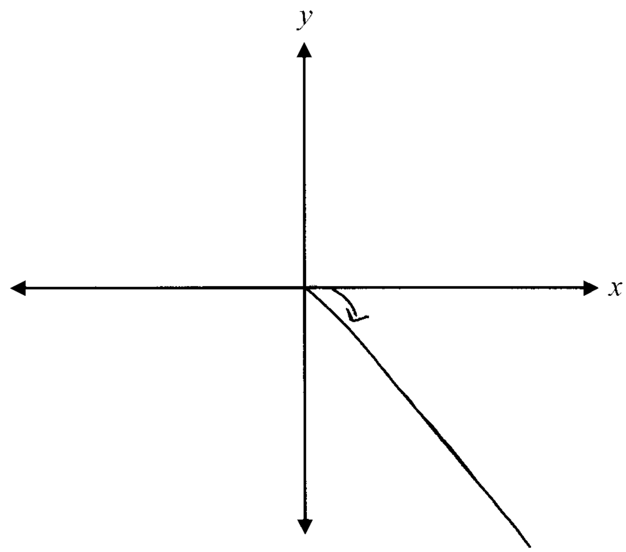
½ mark for an appropriate angle in quadrant IV
½ mark for correct direction

1 mark

Note:

- If the directional arrow is not indicated, deduct an E1 error (final answer not stated)

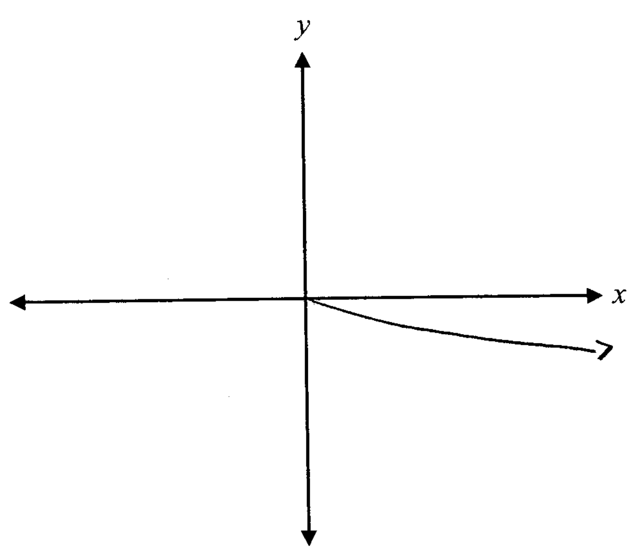
Exemplar 1



½ out of 1

+ ½ mark for correct direction

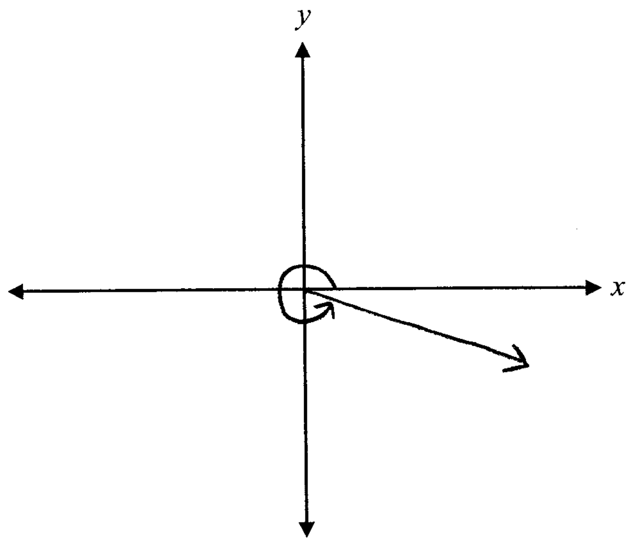
Exemplar 2



½ out of 1

+ ½ mark for an appropriate angle in quadrant IV

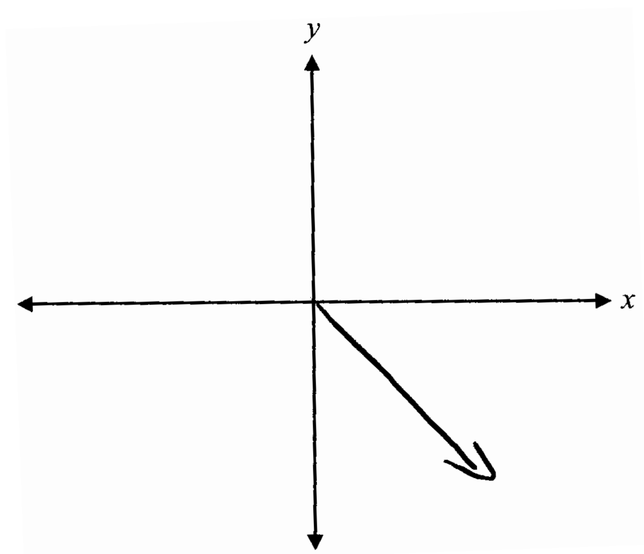
Exemplar 3



½ out of 1

+ ½ mark for an appropriate angle in quadrant IV

Exemplar 4



0 out of 1

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Question 17 R1

Given that $h(x) = 2x^2 - 7x - 15$, determine possible equations of the functions $f(x)$ and $g(x)$ if $h(x) = f(x) \bullet g(x)$.

Solution

$f(x) = \underline{2x + 3}$

$g(x) = \underline{x - 5}$

1 mark for two correct factors of $h(x)$

1 mark

Note:

- Other answers are possible.

Exemplar 1

$2x^2 - 10x + 3x - 15$
$2x(x-5) + 3(x-5)$
$(2x-3)(x-5)$

$f(x) =$ $2x-3$

$g(x) =$ $x-5$

1 out of 1

award full marks
E7 (transcription error in line 3)

Exemplar 2

$f(x) =$ $2x+3$

$g(x) =$ $x-10$

0 out of 1

Exemplar 3

$f(x) =$ $(-2x-3)$

$g(x) =$ $(-x+5)$

1 out of 1

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Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
18	C	R3
19	B	R8
20	B	T1
21	D	R6
22	A	R12
23	C	P4
24	C	T6
25	B	R2
26	B	R7

Question 18 R3

The range of $y = f(x)$ is $-6 \le y \le 12$. The range of the transformed function $y = af(x)$ is $-2 \le y \le 4$. Identify the value of a.

a) -3

b) $-\frac{1}{3}$

c) $\frac{1}{3}$

d) 3

Question 19 R8

Identify the expression which is equivalent to $3\log y - \frac{1}{2}\log x + \log z$.

a) $\log\left(\frac{y^3}{\sqrt{xz}}\right)$

b) $\log\left(\frac{y^3 z}{\sqrt{x}}\right)$

c) $\log\left(\frac{y^3}{x^2 z}\right)$

d) $\log\left(\frac{y^3 z}{x^2}\right)$

Question 20 T1

Identify the measure of the angle $-\frac{2\pi}{9}$ in degrees.

a) $-400°$

b) $-40°$

c) $40°$

d) $320°$

Question 21 R6

If $y = f(x)$ has a domain of $[2, 5]$ and a range of $[6, 10]$, identify the domain of $y = f^{-1}(x)$.

a) $\left[\frac{1}{2}, \frac{1}{5}\right]$

b) $[-5, -2]$

c) $[-10, -6]$

d) $[6, 10]$

Question 22 R12

Identify which of the following is a polynomial function.

a) $p(x) = -\frac{1}{2}(x+2)^3(x-3)$

b) $p(x) = 2x^{\frac{1}{2}} + x - 3$

c) $p(x) = 3x^{-4} + x^2 - 6$

d) $p(x) = 2^x + 3$

Question 23 P4

Identify the total number of terms in the expansion of $(x - y)^9$.

a) 8

b) 9

c) 10

d) 11

Question 24 T6

Identify the exact value of $2\cos^2(15°) - 1$.

a) 1

b) $\frac{1}{2}$

c) $\frac{\sqrt{3}}{2}$

d) $\sqrt{3}$

Question 25 R2

The zeros of the function $y = f(x)$ are $x = -2$ and $x = 3$. Identify the zeros of the function $g(x) = 2f(x-4)$.

a) $x = -6$ and $x = -1$

b) $x = 2$ and $x = 7$

c) $x = -4$ and $x = 6$

d) $x = 0$ and $x = 10$

Question 26 R7

Identify the value of $\log_4\left(\frac{1}{4}\right)$.

a) -16

b) -1

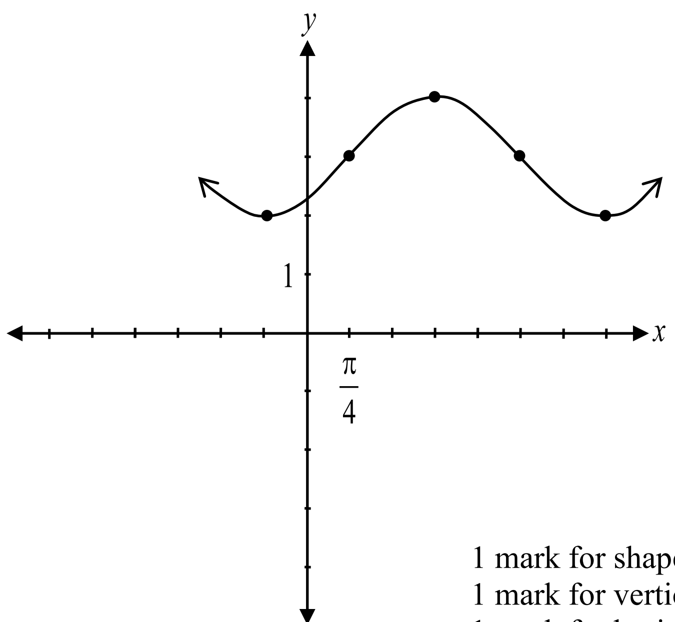
c) 1

d) 16

Question 27 T4

Sketch the graph of at least one period of the function $y = -\cos\left(x + \frac{\pi}{4}\right) + 3$.

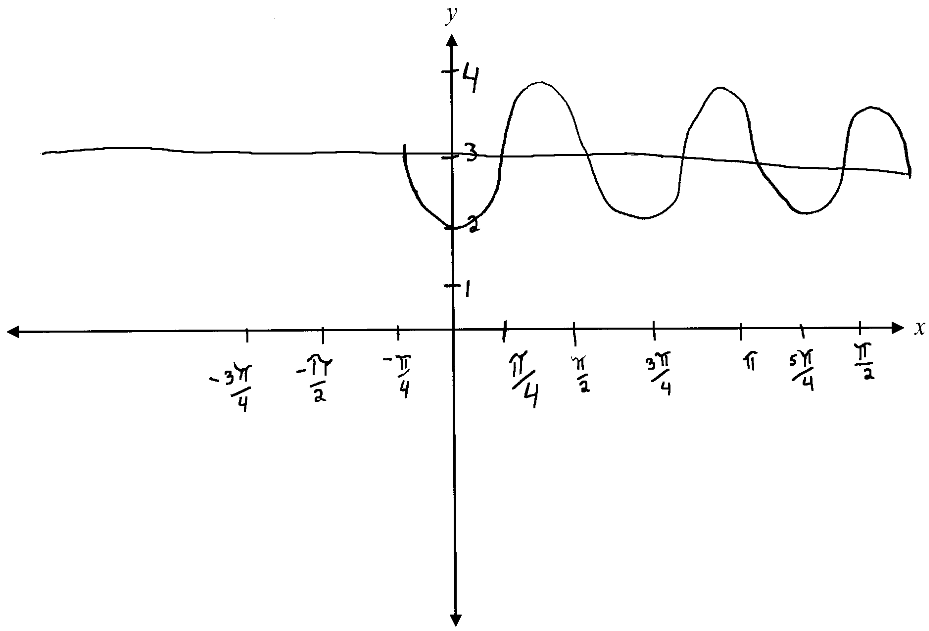
Solution



1 mark for shape of a sinusoidal function with correct period
1 mark for vertical reflection
1 mark for horizontal translation
1 mark for vertical translation

4 marks

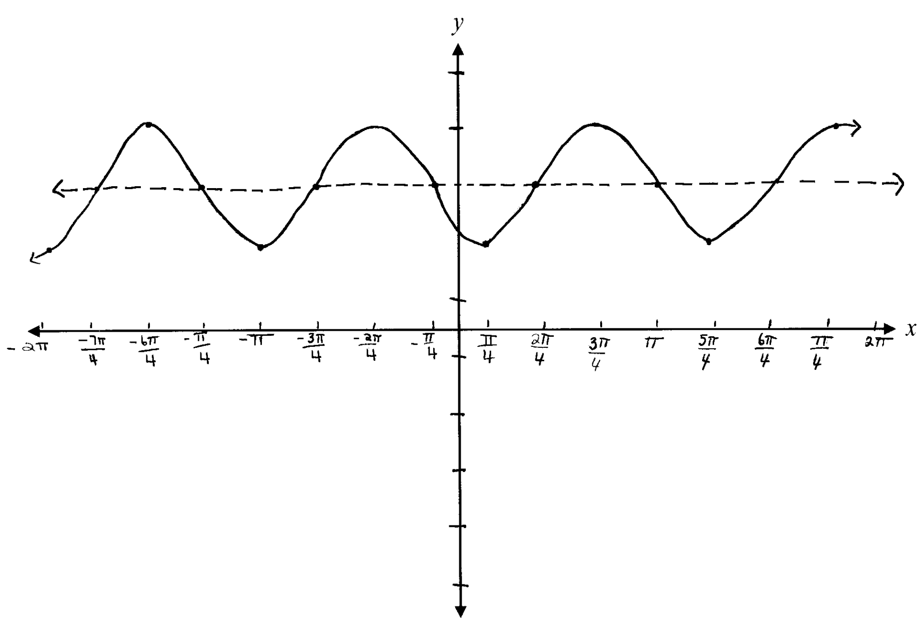
Exemplar 1



2 out of 4

+ 1 mark for vertical reflection
+ 1 mark for vertical translation

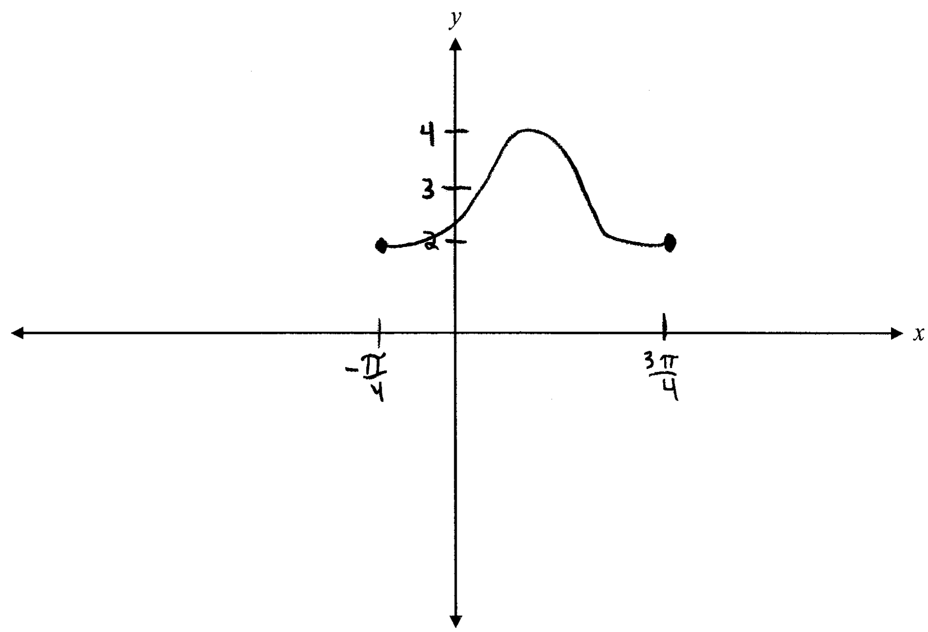
Exemplar 2



2 out of 4

+ 1 mark for vertical reflection
+ 1 mark for vertical translation
E9 (scale values on y-axis not included)

Exemplar 3



3 out of 4

+ 1 mark for vertical reflection
+ 1 mark for horizontal translation
+ 1 mark for vertical translation

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Justify that $(x-5)$ is not a possible factor of the function $P(x) = x^3 - 3x^2 - 4x + 12$.

Solution

When $x = 5$ is substituted into $P(x)$, $P(5)$ does not equal 0.

1 mark

Exemplar 1

$P(5) = (5)^3 - 3(5)^2 - 4(5) + 12$

$= 125 - 75 - 20 + 12$

$P(5) = 42$

$(x-5)$ is not a factor, because the constant is not equal zero.

½ out of 1

award full marks
– ½ mark for terminology error

Exemplar 2

5	1	−3	−4	12
	↓	6	10	−30
	1	2	−6	−18

The remainder does not equal zero

½ out of 1

award full marks
– ½ mark for arithmetic error in line 3

Exemplar 3

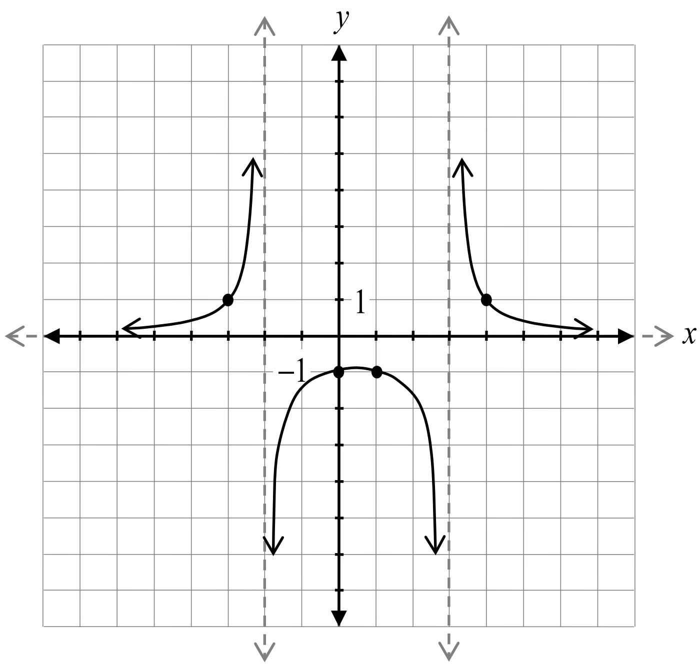
5	1	−3	−4	12
	↓	5	10	30
	1	2	6	42

has a remainder of 42

1 out of 1

Sketch the graph of $f(x) = \dfrac{6}{(x+2)(x-3)}$ and state the y-intercept.

Solution

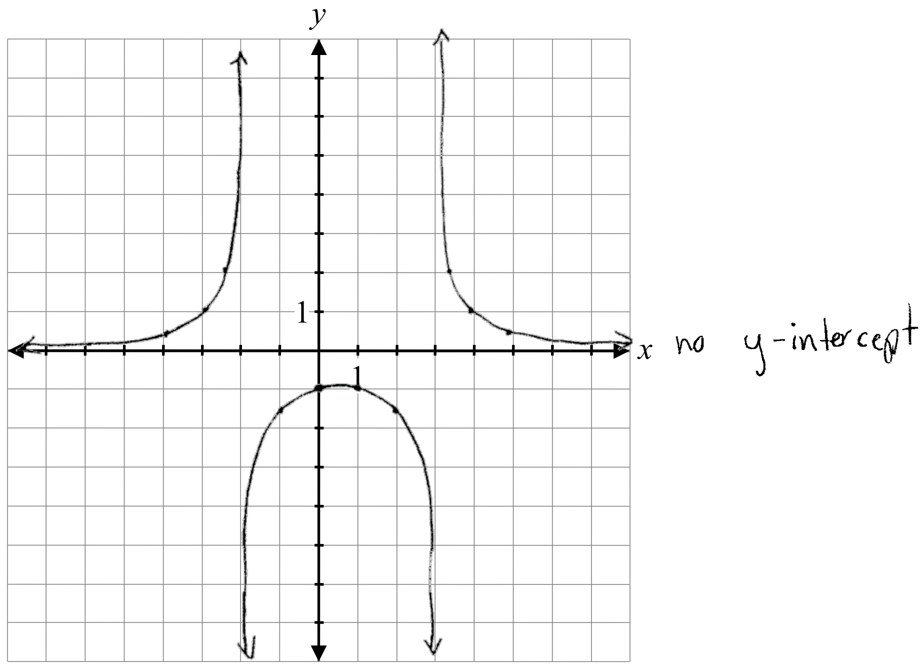


y-intercept: -1

1 mark for vertical asymptotic behaviour
(½ mark for behaviour approaching $x = -2$; ½ mark for behaviour approaching $x = 3$)
1 mark for horizontal asymptotic behaviour approaching $y = 0$
1½ marks for shape (½ mark for shape in each section)
½ mark for y-intercept

4 marks

Exemplar 1

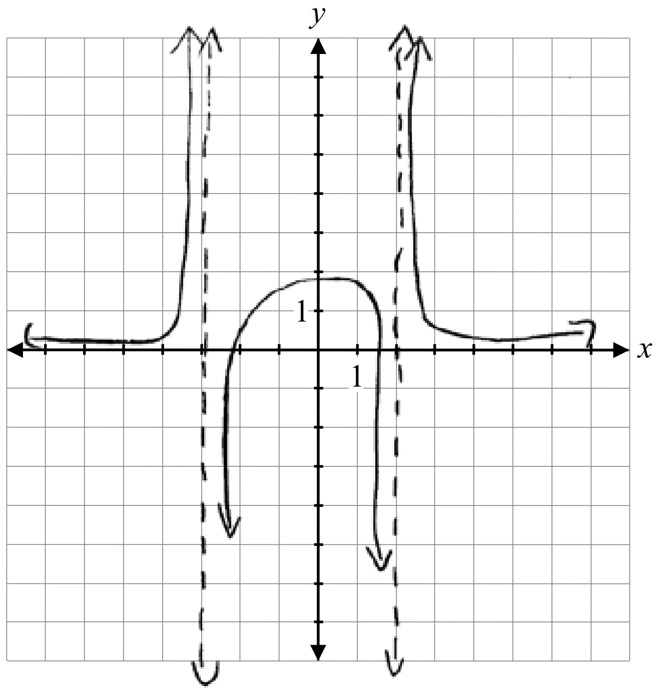


y-intercept: none

3½ out of 4

+ 1 mark for vertical asymptotic behaviour
+ 1 mark for horizontal asymptotic behaviour approaching $y = 0$
+ 1½ marks for shape
E10 (asymptote omitted but still implied)

Exemplar 2

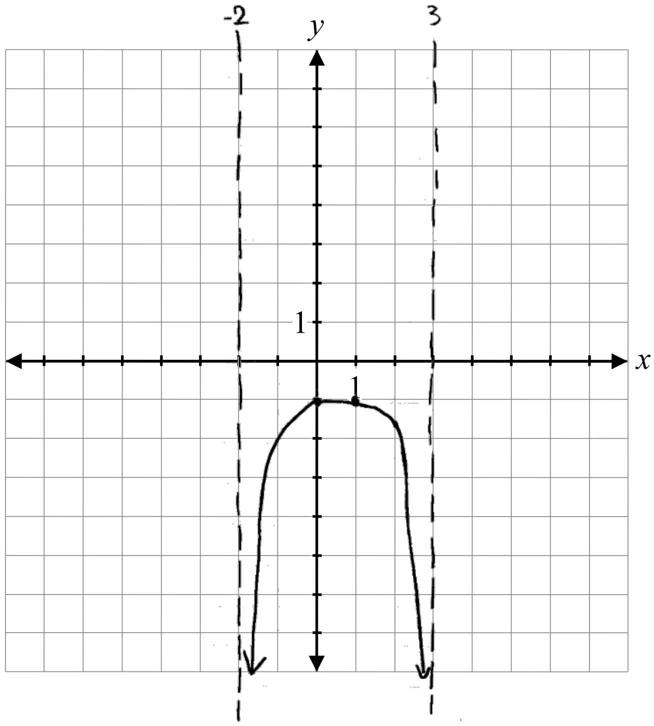


y-intercept: 6, 2, −3

1½ out of 4

+ 1 mark for horizontal asymptotic behaviour approaching $y = 0$
+ ½ mark for shape left of the vertical asymptote
+ ½ mark for shape right of the vertical asymptote
– ½ mark for procedural error (not including a minimum of one point in each section)
E10 (asymptote omitted but still implied)

Exemplar 3



y-intercept: -6

1½ out of 4

+ 1 mark for vertical asymptotic behaviour
+ ½ mark for shape between vertical asymptotes

Question 30 P2

Determine how many 3-digit odd numbers less than 300 are possible using the digits 1, 2, 3, 4, 5, 6 if repetition is not allowed.

Solution

case 1 : $\underset{1}{\underline{1}} \bullet \underline{4} \bullet \underset{\text{odd}}{\underline{\quad 2 \quad}} = 8$ — ½ mark for case 1

case 2 : $\underset{2}{\underline{1}} \bullet \underline{4} \bullet \underset{\text{odd}}{\underline{\quad 3 \quad}} = 12$ — ½ mark for case 2

$8 + 12 = 20$ numbers — 1 mark for addition of cases

2 marks

Exemplar 1

$$\underset{1 \text{ or } 2}{\underline{2}} \quad \underline{4} \quad \underset{1,3,5}{\underline{3}} = 24$$

1 out of 2

award full marks
– 1 mark for concept error

Exemplar 2

$$\underline{6} \cdot \underline{5} \cdot \underline{4} = 120$$

0 out of 2

Exemplar 3

$$\underset{1}{\underline{1}} \cdot \underline{5} \cdot \underline{4} = 20$$

$$+$$

$$\underset{2}{\underline{1}} \cdot \underline{5} \cdot \underline{4} = 20$$

$$40$$

1 out of 2

+ 1 mark for addition of cases

Exemplar 4

$$\underline{1} \cdot \underline{4} \cdot \underline{2} = 8$$

$$\bullet$$

$$\underline{1} \quad \underline{4} \quad \underline{3} = 12$$

$$96$$

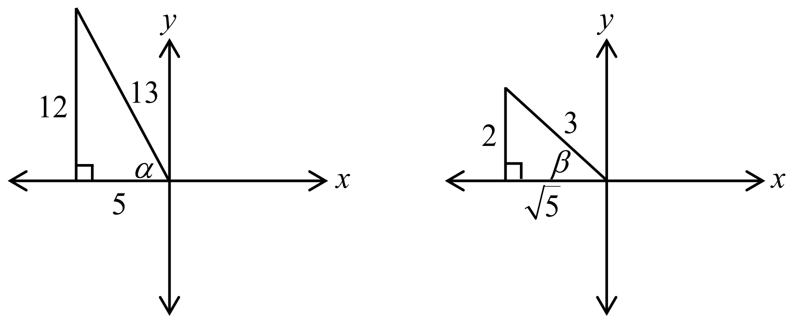
1 out of 2

+ ½ mark for case 1
+ ½ mark for case 2

Question 31 T6

Given that $\cos\alpha = -\frac{5}{13}$ and $\sin\beta = \frac{2}{3}$, where α and β terminate in the same quadrant, determine the exact value of $\cos(\alpha - \beta)$.

Solution



$$x^2 + y^2 = r^2 \qquad x^2 + y^2 = r^2$$

$$25 + y^2 = 169 \qquad x^2 + 4 = 9$$

$$y^2 = 144 \qquad x^2 = 5$$

$$y = \pm 12 \qquad x = \pm\sqrt{5}$$

½ mark for value of x
½ mark for value of y

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= \left(-\frac{5}{13}\right)\left(-\frac{\sqrt{5}}{3}\right) + \left(\frac{12}{13}\right)\left(\frac{2}{3}\right)$$

$$= \frac{5\sqrt{5}}{39} + \frac{24}{39}$$

$$= \frac{5\sqrt{5} + 24}{39}$$

½ mark for $\cos\beta$
½ mark for $\sin\alpha$
1 mark for substitution into correct identity

3 marks

Notes:

- Accept any of the following values for x: $x = \pm\sqrt{5}$, $x = \sqrt{5}$, or $x = -\sqrt{5}$.
- Accept any of the following values for y: $y = \pm 12$ or $y = 12$.

Exemplar 1

$\cos a = \frac{-5}{13}$ $\sin B = \frac{2}{3}$

$\cos(a-b) = \cos a \cos B + \sin a \sin B$

$= \left(-\frac{5}{13}\right)\left(\frac{\sqrt{5}}{3}\right) + \left(\frac{-12}{13}\right)\left(\frac{2}{3}\right)$

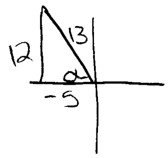
$= -\frac{5\sqrt{5}}{39} + \frac{-24}{39}$

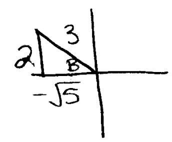
$= \frac{-5\sqrt{5}-24}{39}$

2 out of 3

+ ½ mark for value of *x*
+ ½ mark for value of *y*
+ 1 mark for substitution into correct identity

Exemplar 2





$r^2 - x^2 = y^2$

$13^2 - (-5^2) = y^2$

$169 - 25 = y^2$

$\sqrt{144} = \sqrt{y^2}$

$169 - 25 = 144$

$12 = y$

$r^2 - y^2 = x^2$

$3^2 - 2^2 = x^2$

$9 - 4 = x^2$

$\sqrt{5} = \sqrt{x^2}$

$\pm\sqrt{5} = x$ (reject +)

$\cos\left(\frac{-5}{13}\right)\cos\left(\frac{-\sqrt{5}}{3}\right) + \sin\left(\frac{12}{13}\right)\sin\left(\frac{2}{3}\right)$

$\left(\frac{-5}{13}\right)\left(\frac{-\sqrt{5}}{3}\right) + \left(\frac{12}{13}\right)\left(\frac{2}{3}\right)$

$\frac{-5-\sqrt{5}}{39} + \frac{24}{39}$ $\boxed{\frac{-\sqrt{5}+19}{39}}$

---

2 out of 3

award full marks

– ½ mark for procedural error in line 6

– ½ mark for arithmetic error in line 8

Exemplar 3

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$
$$= \left(\frac{-5}{13}\right)\cos b + \sin a\left(\frac{2}{3}\right)$$
$$= \left(\frac{-5}{13}\right)\cos b + \left(\frac{12}{13}\right)\left(\frac{2}{3}\right)$$
$$= \left(\frac{-5}{13}\right)\left(\frac{\sqrt{5}}{3}\right) + \left(\frac{12}{13}\right)\left(\frac{2}{3}\right)$$
$$= \frac{-5\sqrt{5}}{39} + \frac{24}{39}$$
$$= \boxed{\frac{-5\sqrt{5}+24}{39}}$$

$$\sin^2 a + \left(\frac{-5}{13}\right)^2 = 1$$
$$\sin^2 a + \frac{25}{169} = 1$$
$$\sin a^2 = \frac{144}{169}$$
$$\sin a = \frac{12}{13}$$
$$\sin a = \frac{12}{13}$$

$$\left(\frac{2}{3}\right)^2 + \cos b^2 = 1$$
$$\frac{4}{9} + \cos b^2 = 1$$
$$\cos b^2 = \frac{5}{9}$$
$$\cos b = \frac{\sqrt{5}}{3}$$

2½ out of 3

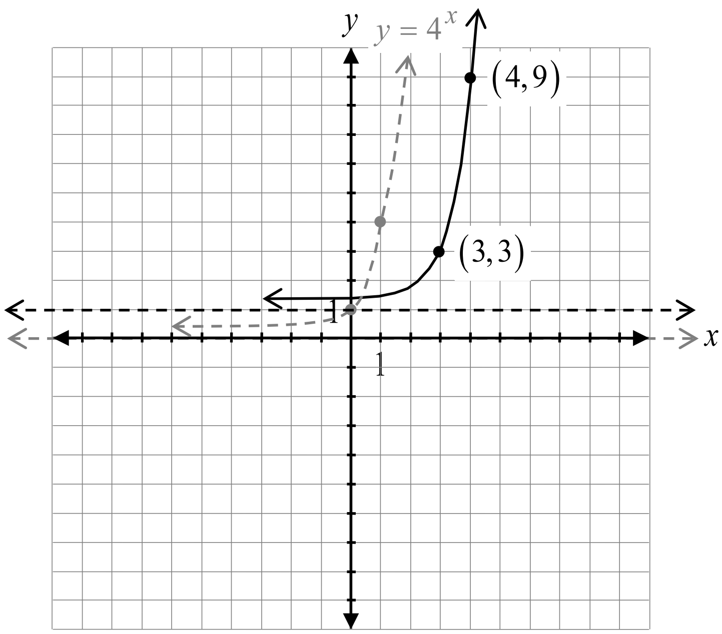
+ ½ mark for value of x
+ ½ mark for value of y
+ ½ mark for $\sin\alpha$
+ 1 mark for substitution into correct identity

E4 ("$\sin\alpha^2$" written instead of "$\sin^2\alpha$")

Question 32 R9

Given the graph of $y = 4^x$, sketch the graph of $y = 2(4)^{x-3} + 1$.

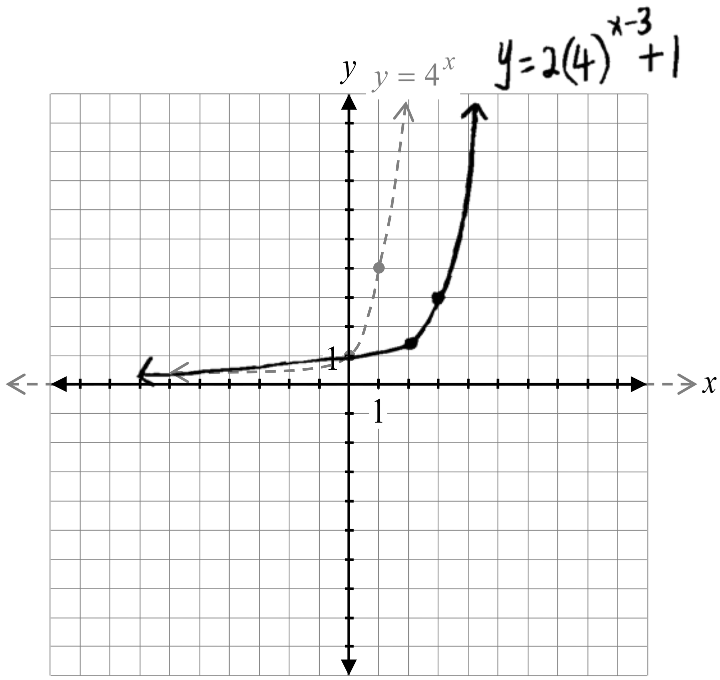
Solution



1 mark for vertical stretch
1 mark for horizontal translation
1 mark for vertical translation

3 marks

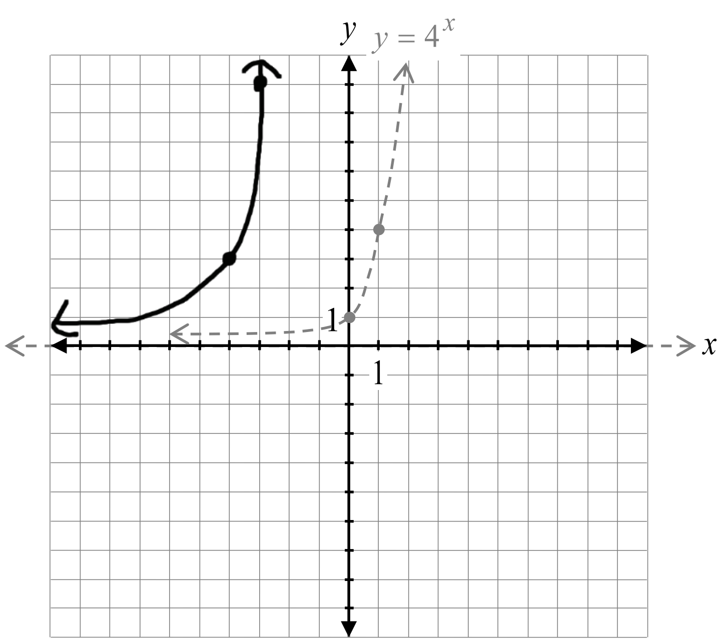
Exemplar 1



2 out of 3

+ 1 mark for vertical stretch
+ 1 mark for horizontal translation

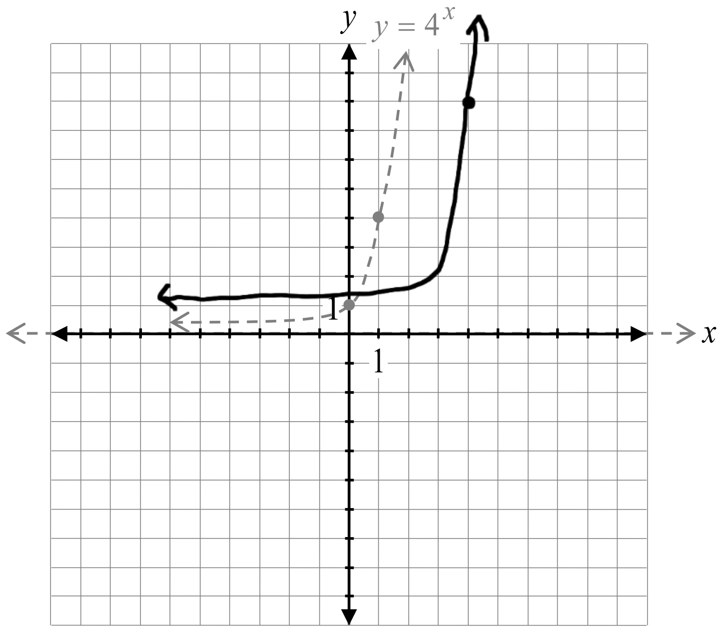
Exemplar 2



1 out of 3

+ 1 mark for vertical stretch

Exemplar 3



2 out of 3

+ 1 mark for vertical stretch
+ 1 mark for horizontal translation

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Question 33 T1

Determine the coterminal angle of $\frac{\pi}{5}$ over the interval $[-2\pi, 0]$.

Solution

$-\frac{9\pi}{5}$

1 mark

Exemplar 1

$$\frac{11\pi}{5}$$

1 out of 1

award full marks
E8 (answer outside the given domain)

Question 34 R9

State the domain of the graph of $y = \log(x - 4) - 8$.

Solution

$x > 4$

or

$(4, \infty)$

1 mark

Exemplar 1

$x \geq 4$

0 out of 1

award full marks
– 1 mark for concept error for including the asymptote in the solution

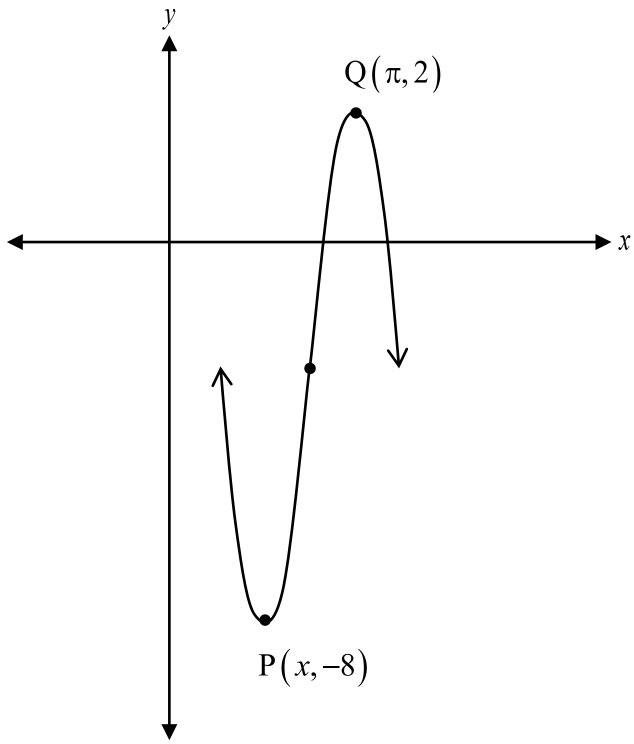
Exemplar 2

$x \neq 4$

0 out of 1

Question 35 T4

Given the graph of $y = 5\sin\left[2\left(x + \frac{\pi}{4}\right)\right] - 3$, determine the exact value of the *x*-coordinate in the point P.



Solution

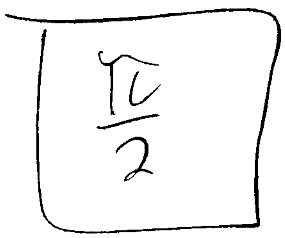
$x = \frac{\pi}{2}$ **1 mark**

Exemplar 1

1 out of 1

award full marks
E7 (notation error)

Exemplar 2



1 out of 1

Verify that the following equation is true for $x = \frac{5\pi}{6}$.

$$\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$$

Solution

Left-Hand Side	Right-Hand Side	
$\dfrac{\cos\frac{5\pi}{6}}{1 - \sin\frac{5\pi}{6}}$	$\dfrac{1 + \sin\frac{5\pi}{6}}{\cos\frac{5\pi}{6}}$	
$\dfrac{-\frac{\sqrt{3}}{2}}{1 - \frac{1}{2}}$	$\dfrac{1 + \frac{1}{2}}{-\frac{\sqrt{3}}{2}}$	1 mark for exact values (½ mark for $\cos\frac{5\pi}{6}$; ½ mark for $\sin\frac{5\pi}{6}$)
$\dfrac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$	$\dfrac{\frac{3}{2}}{-\frac{\sqrt{3}}{2}}$	
$-\dfrac{\sqrt{3}}{1}$	$-\dfrac{3}{\sqrt{3}} \bullet \dfrac{\sqrt{3}}{\sqrt{3}}$	1 mark for simplification (½ mark for LHS; ½ mark for RHS)
$-\sqrt{3}$	$-\sqrt{3}$	**2 marks**

Exemplar 1

Left-Hand Side	Right-Hand Side
$\dfrac{\cos\frac{5\pi}{6}}{1-\sin\frac{5\pi}{6}}$	$\dfrac{1+\sin\frac{5}{6}\pi}{\cos\frac{5}{6}\pi}$
$\dfrac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}}$	$\dfrac{1-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$
$\dfrac{\frac{\sqrt{3}}{2}}{+\frac{2+1}{2}}$	$\dfrac{+\frac{1}{2}}{\frac{\sqrt{3}}{2}}$
$\dfrac{\sqrt{3}}{2}\cdot\dfrac{2}{3}$	$\dfrac{1}{2}\cdot\dfrac{2}{\sqrt{3}}$
$\dfrac{\sqrt{3}}{3}$	$\dfrac{1}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}$
$\dfrac{\sqrt{3}}{3}$	$\dfrac{\sqrt{3}}{3}$

LHS = RHS

1 out of 2

+ 1 mark for simplification

Exemplar 2

Left-Hand Side	Right-Hand Side
$\dfrac{\frac{\sqrt{3}}{2}}{1-\left(\frac{1}{2}\right)}$	$\dfrac{1+\left(\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}$
$\dfrac{\frac{\sqrt{3}}{2}}{\frac{2}{2}-\frac{1}{2}}$	$\dfrac{\frac{2}{2}+\frac{1}{2}}{\frac{\sqrt{3}}{2}}$
$\dfrac{\sqrt{3}}{2}\left(\dfrac{2}{1}\right)$	$\dfrac{3}{2}\left(\dfrac{2}{\sqrt{3}}\right)$
$\dfrac{2\sqrt{3}}{2}$	$\dfrac{6}{2\sqrt{3}}$
$\sqrt{3}$	$\dfrac{3}{\sqrt{3}}$

LHS = RHS

1½ out of 2

+ ½ mark for $\sin\frac{5\pi}{6}$

+ 1 mark for simplification

Exemplar 3

Left-Hand Side	Right-Hand Side
$\dfrac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}}$	$\dfrac{1+\frac{1}{2}}{\frac{-\sqrt{3}}{2}}$
$\dfrac{-\sqrt{3}}{\cancel{2}} \cdot \dfrac{\cancel{2}}{1}$	$\dfrac{3}{2} \cdot \dfrac{2}{-\sqrt{3}}$
$-\sqrt{3}$	$\dfrac{3}{-\sqrt{3}}$

2 out of 2

award full marks
E1 (final answer not stated)

Question 37 R11

Given that $(x+1)$ is one of the factors of $P(x) = x^3 - x^2 + kx - 8$, determine the value of k.

Solution

Method 1

$x = -1$	½ mark for $x = -1$
$0 = (-1)^3 - (-1)^2 + k(-1) - 8$	1 mark for remainder theorem
$0 = -1 - 1 - k - 8$	
$k = -10$	½ mark for solving for k

Method 2

$x = -1$ ½ mark for $x = -1$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & k & -8 \\ & & -1 & 2 & -k-2 \\ \hline & 1 & -2 & k+2 & -k-10 \end{array}$$

1 mark for synthetic division

$-k - 10 = 0$

$k = -10$ ½ mark for solving for k

2 marks

Exemplar 1

$P(-1) = (-1)^3 - (-1)^2 + k(-1) - 8$

$= -1 + 1 + k(-1) - 8$

$= -8 + k(-1)$

$8 = k(-1)$

$\boxed{-8 = k}$

1 out of 2

award full marks
– ½ mark for procedural error in line 2 (did not show the equation equal to zero before solving)
– ½ mark for arithmetic error in line 2

Exemplar 2

-1 1 -8 8

1	1	-1	k	-8
	↓	1	0	8
	1	0	8	0

$-1^3 + 1^2 + -8(-1) - 8 = 0$

-1

$K = -8$

1 out of 2

+ 1 mark for synthetic division

Question 38 R13

Given the function $f(x) = \sqrt{x}$, describe how to use transformations to determine the domain of the function $g(x) = f(x+2)+1$.

Solution

The graph of $g(x)$ is a horizontal translation 2 units to the left of the graph of $f(x)$, which changes the domain from $x \geq 0$ to $x \geq -2$.

1 mark

Exemplar 1

- move the points to the left two units.
- move the points up one unit.

* keep in mind that "x" must be greater than or equal to −2, so domain will be $[-2, \infty)$.

1 out of 1

Exemplar 2

shift 2 units left

shift 1unit up

0 out of 1

Exemplar 3

$\sqrt{x-2}$

$x+2 \geq 0$

$x \geq -2$

$D: [-2, \infty)$

0 out of 1

Exemplar 4

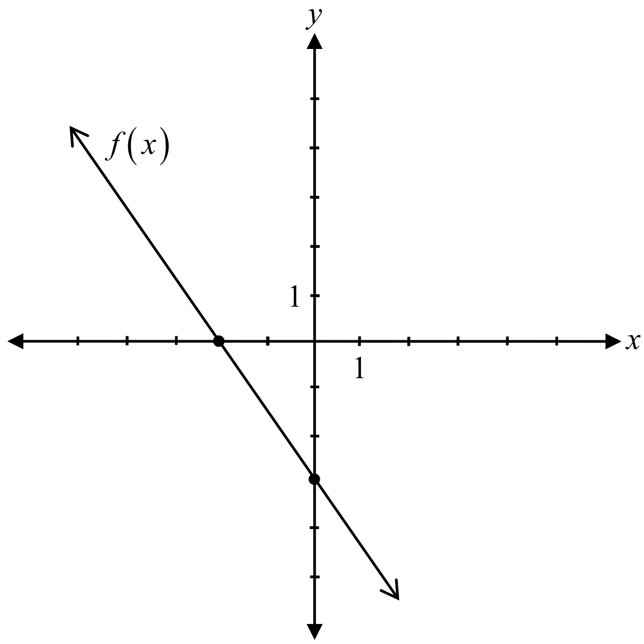
take the domain of $f(x)=\sqrt{x}$ and subtract 2 from it, that will be the domain of $g(x)$.

½ out of 1

award full marks
– ½ mark for lack of clarity in description

Question 39 R1

Given the graph of $y = f(x)$, state the equation of the vertical asymptote of $y = \frac{1}{f(x)}$.



Solution

$x = -2$ **1 mark**

Exemplar 1

vertical asymptote

$y = -2$

0 out of 1

Exemplar 2

$x \neq -2$

1 out of 1

award full marks
E7 (notation error)

Exemplar 3

$VA = -2$

½ out of 1

award full marks
– ½ mark for procedural error (omitting variable)

Question 40 R10

Solve, algebraically.

$$16^{x} = 64^{2x-1}$$

Solution

$4^{2x} = 4^{3(2x-1)}$	1 mark for changing to a common base (½ mark for each)
$4^{2x} = 4^{6x-3}$	½ mark for exponent law
$2x = 6x - 3$	½ mark for equating exponents
$3 = 4x$	**2 marks**
$\frac{3}{4} = x$	

Exemplar 1

$$\log 16^{x} = \log 64^{2x-1}$$

$$x\log 16 = 2x\log 64 - \log 64$$

$$x\log 16 - 2x\log 64 = -\log 64$$

$$\frac{x(\log 16 - 2\log 64)}{\log 16 - 2\log 64} = \frac{-\log 64}{\log 16 - 2\log 64}$$

$$x = \frac{-\log 64}{\log 16 - 2\log 64}$$

2 out of 2

award full marks
E1 (final answer not stated)

Exemplar 2

$$16^{x} = 16^{4(2x-1)}$$

$$x = 4(2x-1)$$

$$x = 8x - 4$$

$$x - 8x = -4$$

$$\frac{-7x}{-7} = \frac{-4}{-7}$$

$$x = \frac{4}{7}$$

1 out of 2

+ ½ mark for exponent law
+ ½ mark for equating exponents

Exemplar 3

$$\left(4^2\right)^x = \left(4^4\right)^{2x-1}$$

$$4^{2x} = 4^{8x-4}$$

$$2x = 8x - 4$$
$$-2x \quad -2x + 4$$
$$+4$$

$$\frac{4}{6} = \frac{6x}{6}$$

$$\boxed{x = \frac{2}{3}}$$

1½ out of 2

+ ½ mark for changing to a common base
+ ½ mark for exponent law
+ ½ mark for equating exponents

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Question 41 R11

Given that one of the factors of $P(x) = x^3 + 2x^2 - 5x - 6$ is $(x+3)$, express $P(x)$ in completely factored form.

Solution

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ & \downarrow & -3 & 3 & 6 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

½ mark for $x = -3$

1 mark for synthetic division (or any equivalent strategy)

$P(x) = (x+3)\left(x^2 - x - 2\right)$

$P(x) = \underline{(x+3)(x-2)(x+1)}$

½ mark for consistent product of factors

2 marks

Exemplar 1

-3	1	2	-5	-6
	↓	-3	3	6
	1	-1	-2	0

$x^2 - x - 2$

$P(x) =$ $(x^2 - x - 2)(x + 3)$

1½ out of 2

+ ½ mark for $x = -3$
+ 1 mark for synthetic division

Exemplar 2

-3	1	2	-5	-6
		-3	3	6
	1	-1	-2	0

$x^2 - 1x - 2$

$(x - 2)(x + 1)$

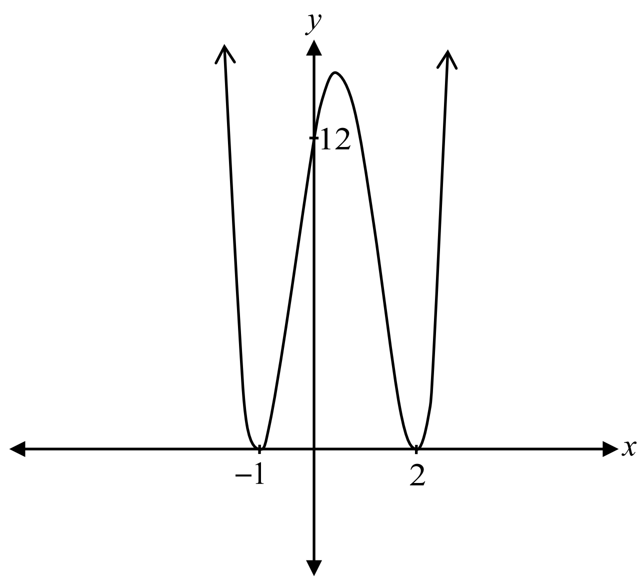
$P(x) =$ $(x - 2)(x + 1)$

1½ out of 2

+ ½ mark for $x = -3$
+ 1 mark for synthetic division

Sketch the graph of $p(x) = 3(x+1)^2(x-2)^2$.

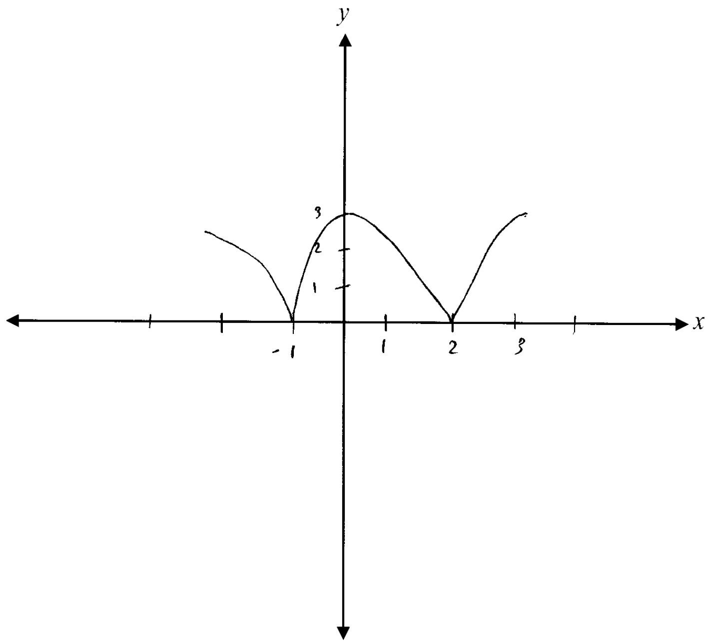
Solution



1 mark for x-intercepts
½ mark for y-intercept
1 mark for multiplicity of 2 at $x = -1$ and at $x = 2$
(½ mark for each)
½ mark for end behaviour

3 marks

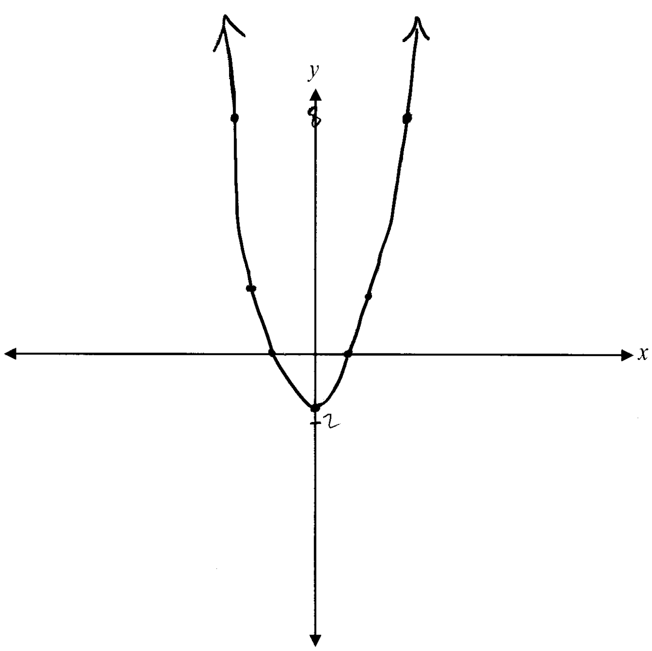
Exemplar 1



1½ out of 3

+ 1 mark for x-intercepts
+ 1 mark for multiplicity of 2 at $x = -1$ and at $x = 2$
– ½ mark for incorrect shape of graph at x-intercepts

Exemplar 2



½ out of 3

+ ½ mark for end behaviour

Question 43 R1

Given that $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$, determine $f(g(x))$ and state its domain.

Solution

$f(g(x)) = (\sqrt{x})^2 - 4$

$f(g(x)) = \underline{x - 4, x \geq 0}$

1 mark for composite function
1 mark for domain consistent with composite function

2 marks

Exemplar 1

$f(g(x)) = \sqrt{x}^2 - 4$

$f(g(x)) =$ $x - 4$

1 out of 2

+ 1 mark for composite function

Exemplar 2

$f(g(x)) =$ $\sqrt{x^2 - 4}$, $x \geq 2$, $x \leq -2$

1 out of 2

+ 1 mark for domain consistent with the composite function

Exemplar 3

$f(g(x)) =$ $(\sqrt{x})^2 - 4$

1 out of 2

+ 1 mark for composite function

Exemplar 4

$f(x) = \sqrt{x}^2 - 4$

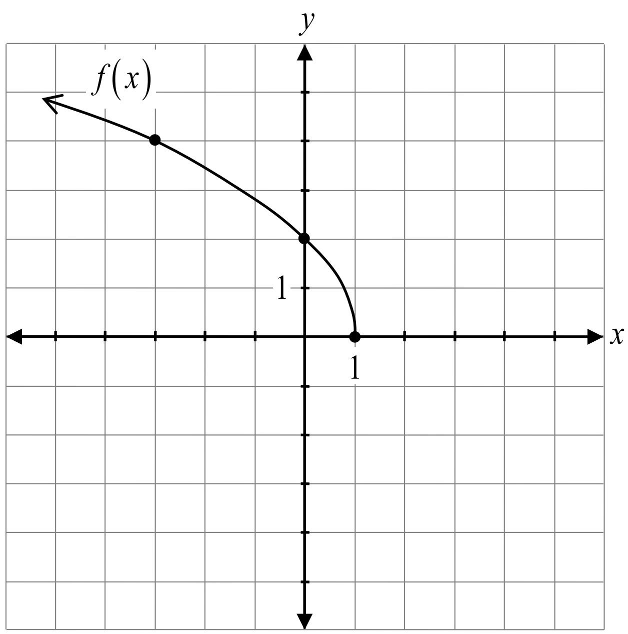
$\downarrow$

$f(g(x)) =$ $x - 4$, $x > 0$

2 out of 2

award full marks
E8 (error made when stating domain)

Determine a possible equation of the function $f(x)$.



Solution

$f(x) = \underline{2\sqrt{-(x-1)}}$

1 mark for vertical stretch
1 mark for horizontal reflection
1 mark for horizontal translation

3 marks

or

$f(x) = \underline{\sqrt{-4(x-1)}}$

1 mark for horizontal compression
1 mark for horizontal reflection
1 mark for horizontal translation

3 marks

Exemplar 1

$f(x) = 2(-x-1)$

1 out of 3

+ 1 mark for vertical stretch
+ 1 mark for horizontal reflection
– 1 mark for concept error (incorrect function)

Exemplar 2

$f(x) = 2f\sqrt{-(x-1)}$

2½ out of 3

award full marks
– ½ mark for procedural error (including *f*)

Exemplar 3

$f(x) = -2f(x-1)$

1 out of 3

+ 1 mark for vertical stretch
+ 1 mark for horizontal translation
– 1 mark for concept error (using *f* instead of radical)

Question 45 R9

Explain why the graph of $y = \log_2 x$ does not have a y-intercept.

Solution

The domain of the graph is $x > 0$. **1 mark**

or

There is a vertical asymptote at $x = 0$.

Exemplar 1

This graph does not have a y-intercept because there is a vertical asymptote.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

At the y-intercept, $x = 0$; logarithms cannot have arguments of zero or they would be undefined. Therefore, no y-intercept.

1 out of 1

Exemplar 3

It doesn't have a y-intercept b/c it never crosses the y-axis.

0 out of 1

Question 46 T3

Evaluate.

$$\sin^2\left(-\frac{\pi}{3}\right)+\cos\left(\frac{17\pi}{6}\right)\sec\left(\frac{\pi}{6}\right)$$

Solution

$\left(-\frac{\sqrt{3}}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{3}}\right)$	1 mark for $\sin\left(-\frac{\pi}{3}\right)$ (½ mark for quadrant; ½ mark for value)
$\frac{3}{4}-1$	1 mark for $\cos\left(\frac{17\pi}{6}\right)$ (½ mark for quadrant; ½ mark for value)
$-\frac{1}{4}$	1 mark for $\sec\left(\frac{\pi}{6}\right)$ (½ mark for quadrant; ½ mark for value)

3 marks

Exemplar 1

$$\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

$$-\sqrt{3}$$

1 out of 3

+ 1 mark for $\sin\left(-\frac{\pi}{3}\right)$
+ ½ mark for quadrant of $\cos\left(\frac{17\pi}{6}\right)$
+ ½ mark for quadrant of $\sec\left(\frac{\pi}{6}\right)$
– ½ mark for procedural error in line 1
– ½ mark for arithmetic error in line 2

Exemplar 2

$$\sin^2\left(-\frac{\sqrt{3}}{2}\right) + \cos\left(-\frac{\sqrt{3}}{2}\right)\sec\left(\frac{2}{\sqrt{3}}\right)$$

2 out of 3

award full marks
– 1 mark for concept error

Exemplar 3

$$\left(-\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2}\left(\frac{2}{1}\right)$$

$$\frac{-3}{4} + 1 \quad = \frac{1}{4}$$

1 out of 3

+ 1 mark for $\sin\left(-\frac{\pi}{3}\right)$

+ ½ mark for quadrant of $\sec\left(\frac{\pi}{6}\right)$

– ½ mark for arithmetic error in line 2

Exemplar 4

$$\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{3}}\right)$$

$$1 - 1$$

$$0$$

2 out of 3

+ ½ mark for quadrant of $\sin\left(-\frac{\pi}{3}\right)$

+ 1 mark for $\cos\left(\frac{17\pi}{6}\right)$

+ 1 mark for $\sec\left(\frac{\pi}{6}\right)$

– ½ mark for arithmetic error in line 2

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Question 47 R14

Determine the coordinates of the point of discontinuity (hole) on the graph of $y = \dfrac{x^2 - 3x}{x}$.

Solution

$(0, -3)$ 1 mark for point of discontinuity (hole) at $x = 0$

1 mark

Note:

- Deduct ½ mark for procedural error of incorrect y-value.

Exemplar 1

$$y = \frac{x(x-3)}{x}$$ hole@ $x=0$, $y=0$

hole at $(0,0)$

½ out of 1

award full marks
– ½ mark for procedural error (incorrect y-value)

Exemplar 2

$$y = \frac{x^2 - 3x}{x} = \frac{x(x-3)}{x}$$

hole at $x = 3$, $y = 0$

0 out of 1

Exemplar 3

O

$y = x - 3$
$y = 0 - 3$
$y = -3$

$0, -3$

1 out of 1

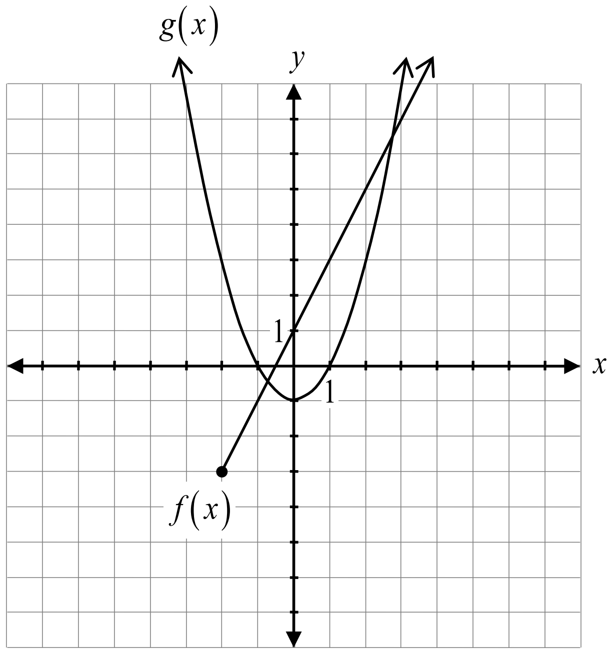
award full marks
E7 (notation error)

Exemplar 4

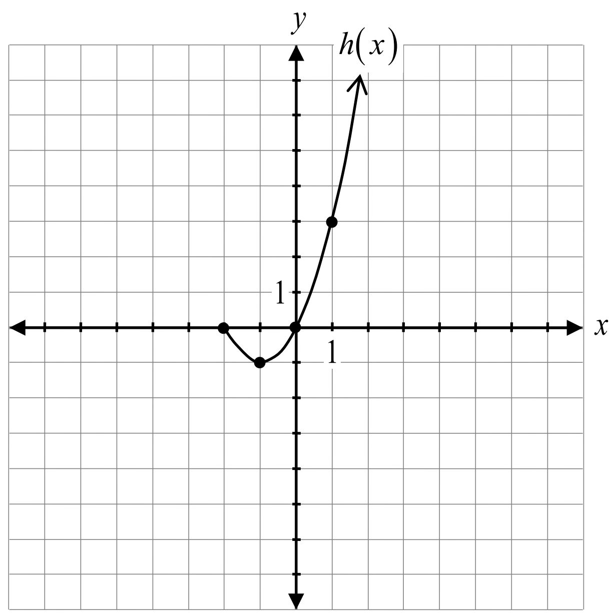
$x = 0 \qquad y = -3$

1 out of 1

Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $h(x) = f(x) + g(x)$.



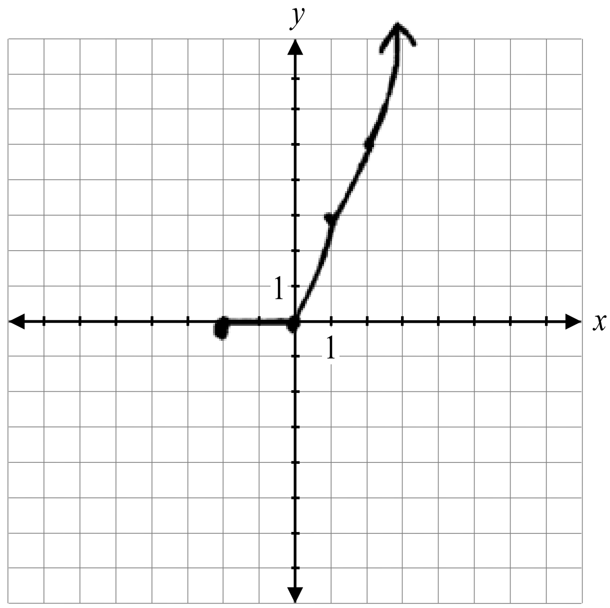
Solution



1 mark for operation of addition
1 mark for restricted domain

2 marks

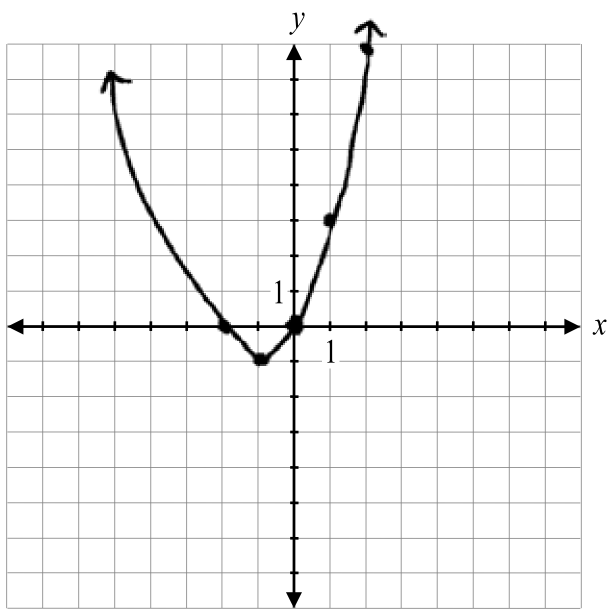
Exemplar 1



1 out of 2

+ 1 mark for restricted domain

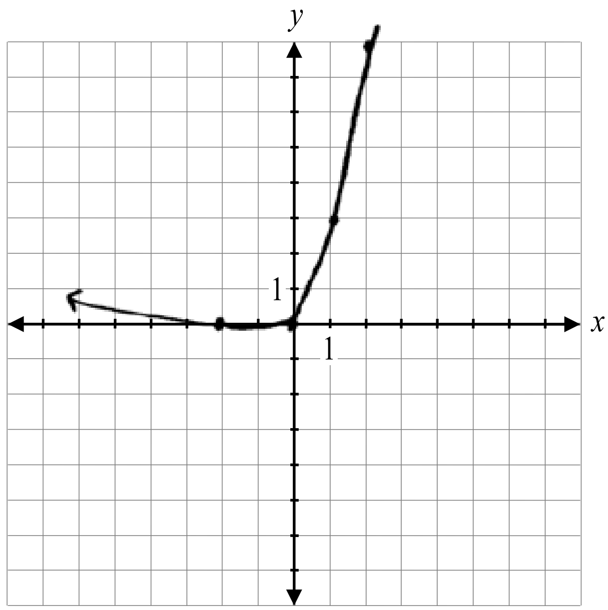
Exemplar 2



1 out of 2

+ 1 mark for operation of addition

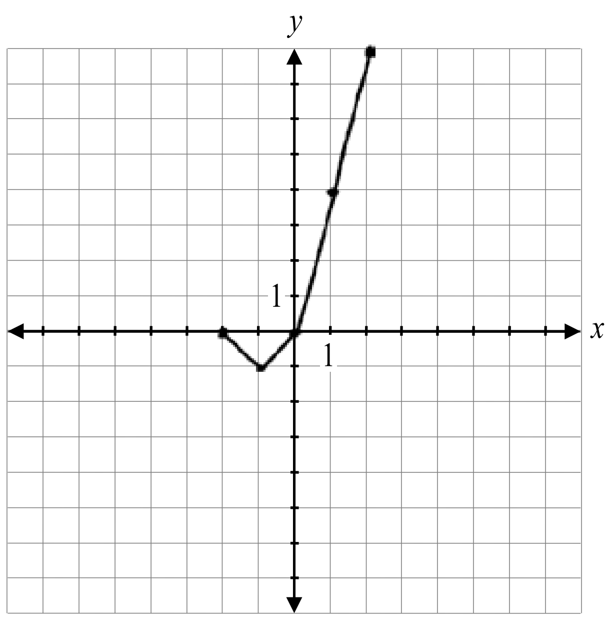
Exemplar 3



½ out of 2

+ 1 mark for operation of addition
– ½ mark for procedural error (one incorrect point)

Exemplar 4



1½ out of 2

award full marks
– ½ mark for incorrect shape
E9 (arrowhead omitted)

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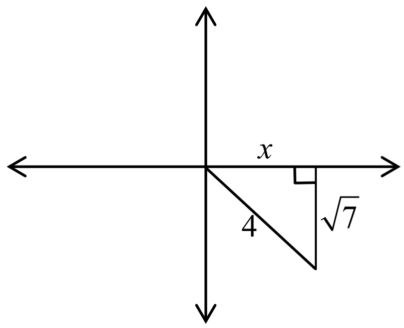
Question 49 T2

Given that $\csc\theta = -\frac{4}{\sqrt{7}}$ and $\cos\theta > 0$, determine the exact value of $\tan\theta$.

Solution



$\sin\theta = -\frac{\sqrt{7}}{4}$

$x^2 + \left(\sqrt{7}\right)^2 = (4)^2$ ½ mark for substitution

$x^2 = 16 - 7$

$x^2 = 9$

$x = \pm 3$ ½ mark for solving for x

$\tan\theta = -\frac{\sqrt{7}}{3}$ 1 mark for $\tan\theta$ (½ mark for quadrant; ½ mark for value)

2 marks

Note:

- Accept any of the following values for x : $x = \pm 3,\ x = 3,$ or $x = -3.$

Exemplar 1

$\sin\theta = \frac{-\sqrt{7}}{4}$

S	A
T	O

$x^2 = 4^2 - (-\sqrt{7})^2$

$= 16 - 7$

$= \sqrt{11}$

$= +\sqrt{11}$

$\tan\theta = \frac{+\sqrt{11}}{-\sqrt{7}}$

1 out of 2

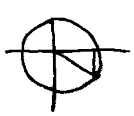
+ ½ mark for substitution
+ ½ mark for the quadrant of $\tan\theta$

Exemplar 2

$\csc\theta = -\frac{4}{\sqrt{7}}$

S	A
T	C•

$\sin\theta = -\frac{\sqrt{7}}{4}$



$r^2 = y^2 + x^2$

$4^2 = \sqrt{7}^2 + x^2$

$16 + 7 = x^2$

$\sqrt{23} = \sqrt{x^2}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = -\frac{\sqrt{7}}{\sqrt{21}}$

1½ out of 2

+ ½ mark for substitution
+ 1 mark for $\tan\theta$

E7 (transcription error in line 8)

Exemplar 3

$\tan\theta =$

$\csc\theta = \frac{-4}{\sqrt{7}} \quad \sin = \frac{\sqrt{7}}{4}$

$\sin \quad y/r$

$\cos$

$\tan \quad y/x$

$y = \sqrt{7} \quad r = 4 \quad x = ?$

$x^2 + y^2 \overset{-y^2}{=} r^2 - y^2$

$x^2 = 4^2 - \sqrt{7}^2$

$x^2 = 16 - 7$

$\sqrt{x^2} = \sqrt{9}$

$x = 3$

$\tan = \sqrt{7}/3$

1½ out of 2

+ ½ mark for substitution
+ ½ mark for solving for *x*
+ ½ mark for the value of $\tan\theta$

E3 (variable omitted in an equation or identity)

Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in step leading to final answer
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: ______________________________

Date marked: ______________________________

Booklet No.: ______________________________

Problem(s) noted: ______________________________

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Question(s) affected: ______________________________

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Action taken or rationale for assigning marks: ______________________________

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Follow-up: ____

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Decision: ____

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Marker’s Signature: ____

Principal’s Signature: ____

For Department Use Only—After Marking Complete

Consultant: ____

Date: ____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
9	R2, R3, R5	3
11	R1	1
17	R1	1
18	R3	1
21	R6	1
25	R2	1
39	R1	1
43	R1	2
48	R1	2
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
5	T1	1
16	T1	1
20	T1	1
27	T4	4
33	T1	1
35	T4	1
46	T3	3
49	T2	2
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
1	P2	2
3	P4	3
8	P3	1
13	P2, P3	3
23	P4	1
30	P2	2
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
22	R12	1
28	R11	1
37	R11	2
41	R11	2
42	R12	3

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
2	T5, T6	4
12	T5	1
15	T6	3
24	T6	1
31	T6	3
36	T6	2

Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
4a)	R10	2
4b)	R10	1
7	R10	3
19	R8	1
26	R7	1
32	R9	3
34	R9	1
40	R10	2
45	R9	1

Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
6	R13	1
10	R14	1
14	R13	2
29	R14	4
38	R13	1
44	R13	3
47	R14	1