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General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the Answer/Scoring Sheet are identical
- students and markers use only a pencil to complete the Answer/Scoring Sheets
- the totals of each of the four parts are written at the bottom
- each student’s final result is recorded, by booklet number, on the corresponding Answer/Scoring Sheet
- the Answer/Scoring Sheet is complete
- a photocopy has been made for school records

Once marking is completed, please forward the Answer/Scoring Sheets to Manitoba Education in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of short-answer questions, long-answer questions, and multiple-choice questions. Short-answer questions are worth 1 or 2 marks each, long-answer questions are worth 3 to 5 marks each, and multiple-choice questions are worth 1 mark each. An answer key for the multiple-choice questions can be found at the beginning of the section “Booklet 2 Questions.”

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student’s response must be complete and correct. Where alternative answering methods are possible, the Marking Guide attempts to address the most common solutions. For general guidelines regarding the scoring of students’ responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an Answer/Scoring Sheet is marked with “0” and/or “NR” only (e.g., student was present but did not attempt any questions) please document this on the Irregular Test Booklet Report.
**Assistance**

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Grade 12 Pre-Calculus Mathematics  
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Email: allison.potter@gov.mb.ca
**Communication Errors**

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e. committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

---

### COMMUNICATION ERRORS / ERREURS DE COMMUNICATION

Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).

Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).

- [ ] E1  
- [ ] E2  
- [ ] E3  
- [ ] E4  
- [ ] E5  
- [ ] E6  
- [ ] E7  
- [ ] E8  
- [ ] E9  
- [ ] E10

---

**Mark assigned to the student / Note accordée à l’élève**

<table>
<thead>
<tr>
<th>Booklet 1 / Cahier 1</th>
<th>+</th>
<th>Multiple Choice / Choix multiple</th>
<th>+</th>
<th>Booklet 2 / Cahier 2</th>
<th>-</th>
<th>Communication Errors / Erreurs de communication</th>
<th>=</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>+</td>
<td>7</td>
<td>+</td>
<td>40</td>
<td></td>
<td>1½</td>
<td></td>
<td>70½</td>
</tr>
</tbody>
</table>

Maximum deduction of 5 marks / déduction maximale de 5 points

90

---

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Scoring Guidelines
A central angle of a circle subtends an arc length of $5\pi$ cm.
Given the circle has a radius of 9 cm, find the measure of the central angle in degrees.

**Solution**

$s = \theta r$

$5\pi = \theta (9)$

$\theta = \frac{5\pi}{9}$

$\theta$ (in degrees) $= \frac{5\pi}{9} \cdot \frac{180^\circ}{\pi}$

$= 100^\circ$

½ mark for substitution into correct formula

½ mark for solving for $\theta$

1 mark for conversion to degrees

2 marks
Question 2  

Solve the equation \( \csc^2 \theta + 3 \csc \theta - 4 = 0 \) over the interval \([0, 2\pi]\). 
Express your answers as exact values or correct to 3 decimal places.

**Solution**

**Method 1**

\[
\csc^2 \theta + 3 \csc \theta - 4 = 0 \\
(\csc \theta - 1)(\csc \theta + 4) = 0
\]

\( \csc \theta = 1 \)  \( \csc \theta = -4 \)  

1 mark for solving for \( \csc \theta \)

\( \sin \theta = 1 \)  \( \sin \theta = -\frac{1}{4} \)

1 mark for reciprocal of \( \csc \theta \)

\( \theta = \frac{\pi}{2} \)  \( \theta = 0.252680 \)  

or

\( \theta = 1.570796 \)  \( \theta = 3.94273, 6.03050 \)

2 marks (1 mark for consistent solutions of each trigonometric equation)

\( \theta = \frac{\pi}{2}, 3.94, 6.031 \)

or

\( \theta = 1.571, 3.94, 6.031 \)
Question 2

Method 2–Graphing Calculator

\[ y = \left( \frac{1}{\sin \theta} \right)^2 + \frac{3}{\sin \theta} - 4 \]

Find all zeros from \([0, 2\pi]\).

\[ \theta = 1.571, 3.394, 6.031 \]

1 mark for equation

1 mark for justification

1 mark for restricted domain

1 mark for solutions

4 marks
**Question 3**

Jess invests $12,000 at a rate of 4.75% compounded monthly. How long will it take for Jess to triple her investment? Express your answer in years, correct to 3 decimal places.

**Solution**

**Method 1**

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ 36,000 = 12,000 \left(1 + \frac{0.0475}{12}\right)^{12t} \]

\[ 3 = \left(1 + \frac{0.0475}{12}\right)^{12t} \]

\[ \ln 3 = \ln \left(1 + \frac{0.0475}{12}\right)^{12t} \]

\[ \ln 3 = 12t \ln \left(1 + \frac{0.0475}{12}\right) \]

\[ t = \frac{\ln 3}{12 \ln \left(1 + \frac{0.0475}{12}\right)} \]

\[ t = 23.174 \text{ years} \]

\[ t = 23.174 425 \]

½ mark for substitution

½ mark for applying logarithms

1 mark for power rule

½ mark for isolating \( t \)

½ mark for evaluating quotient of logarithms

3 marks
Question 3

Method 2–Graphing Calculator

\[ y = 3 \]

\[ y = \left( 1 + \frac{0.0475}{12} \right)^{12t} \]

1 mark for equations

or

Find the value of \( t \) at the point of intersection of these two functions.

\[ t = 23.174 \text{ years} \]

1 mark for justification

1 mark for solution

3 marks
Question 4

The 4th term in the binomial expansion of \( (qx^2 - \frac{3}{x})^{10} \) is \( 414720x^{11} \).

Determine the value of \( q \) algebraically.

**Solution**

\[
t_4 = \binom{10}{3} (qx^2)^7 \left(-\frac{3}{x}\right)^3
\]

\[
414720x^{11} = 120 \left(q^7x^{14}\right) \left(-\frac{27}{x^3}\right)
\]

\[
414720 = -3240q^7
\]

\[
q^7 = -128
\]

\[
q = -2
\]

2 marks (1 mark for \( \binom{10}{3} \), \( \frac{1}{2} \) mark for each consistent factor)

\( \frac{1}{2} \) mark for comparing coefficients

\( \frac{1}{2} \) mark for solving for \( q \)

3 marks
Question 5

Bella has 2 pairs of shoes, 3 pairs of pants, and 10 shirts. Carey has 4 pairs of shoes, 4 pairs of pants, and 4 shirts. An outfit is made up of one pair of shoes, one pair of pants, and one shirt. Who can make more outfits? Justify your answer.

Solution

Bella: \(2 \times 3 \times 10 = 60\) outfits

Carey: \(4 \times 4 \times 4 = 64\) outfits

\(\therefore\) Carey can make more outfits.

1 mark for justification

Question 6

In the binomial expansion of \((x - y)^{10}\), how many terms will be positive? Justify your answer.

Solution

Six terms will be positive.

The term will be positive when \(\text{“} -y \text{”} \) has an even exponent.

1 mark for six terms

1 mark for justification

2 marks
Question 7

Solve the following equation algebraically where $180^\circ \leq \theta \leq 360^\circ$.

$$2\sin^2 \theta + 5\cos \theta + 1 = 0$$

**Solution**

$$2\left(1 - \cos^2 \theta\right) + 5\cos \theta + 1 = 0$$

1 mark for identity

$$2 - 2\cos^2 \theta + 5\cos \theta + 1 = 0$$

$$2\cos^2 \theta - 5\cos \theta - 3 = 0$$

$$2\cos \theta + 1)(\cos \theta - 3) = 0$$

$\cos \theta = -\frac{1}{2}$

$\cos \theta = 3$

1 mark for solving for $\cos \theta$

$\theta_r = 60^\circ$

$\therefore$ no solution

1 mark for indicating no solution

$\theta = 240^\circ$

1 mark for solving for $\theta$

4 marks

---

**Note(s):**

- award a maximum of 3 marks if not solved algebraically
Solve the following equation algebraically:

\[ \log_3(x - 4) + \log_3(x - 2) = 1 \]

**Solution**

**Method 1**

\[ \log_3(x - 4) + \log_3(x - 2) = 1 \]

\[ \log_3(x - 4)(x - 2) = 1 \]

\[ 3^1 = (x - 4)(x - 2) \]

\[ 3 = x^2 - 6x + 8 \]

\[ 0 = x^2 - 6x + 5 \]

\[ 0 = (x - 5)(x - 1) \]

\[ x = 5 \]

½ mark for solving for \( x \) within a quadratic equation

½ mark for rejecting extraneous root

3 marks

**Method 2**

\[ \log_3(x - 4) + \log_3(x - 2) = 1 \]

\[ \log_3(x - 4)(x - 2) = 1 \]

\[ \log_3(x^2 - 6x + 8) = \log_3 3 \]

\[ x^2 - 6x + 8 = 3 \]

\[ x^2 - 6x + 5 = 0 \]

\[ (x - 1)(x - 5) = 0 \]

\[ x = 5 \]

½ mark for logarithmic form

½ mark for equating arguments

½ mark for solving for \( x \) within a quadratic equation

½ mark for rejecting extraneous roots

3 marks
Question 9

Given that \( f(x) = \{(1, 3), (2, 5), (3, 4), (4, 2)\} \), find \( f(f(3)) \).

**Solution**

\[
\begin{align*}
   f(f(3)) &= f(4) \\
   &= 2
\end{align*}
\]

\( \frac{1}{2} \) mark for \( f(3) = 4 \)

\( \frac{1}{2} \) mark for \( f(4) = 2 \)

1 mark
Given the graphs of \( f(x) \) and \( g(x) \) below,

\[ y = f(x) - g(x). \]

**Solution**

\[
\begin{array}{c|c|c|c}
 x & f(x) & g(x) & f(x) - g(x) \\
-4 & -2 & 3 & -5 \\
-2 & 0 & 1 & -1 \\
-1 & 1 & 0 & 1 \\
0 & 2 & 1 & 1 \\
2 & 4 & 3 & 1 \\
\end{array}
\]

1 mark for subtraction of \( f(x) - g(x) \)
1 mark for restricted domain

**2 marks**
Question 11

Given the graph of \( y = f(x) \), describe the transformations to obtain the graph of the function \( y = f(2x - 6) \).

Solution

Method 1

Factor out the 2.

\[ y = f(2(x-3)) \]

Horizontally compress by a factor of 2. Then shift 3 units to the right.

Method 2

\[ y = f(2x - 6) \]

Shift 6 units to the right. Then horizontally compress by a factor of 2.

Question 12

Given \( f(x) = \{(-3, 4), (2, 7), (8, 6)\} \), state the domain of the resulting function after \( f(x) \) is reflected through the line \( y = x \).

Solution

Domain: \( \{4, 6, 7\} \)

1 mark for correct domain

Note(s):

- award \( \frac{1}{2} \) mark for stating the inverse of the function: \( f^{-1}(x) = \{(4, -3), (7, 2), (8, 6)\} \)
Question 13  

Determine the value of \( y \) in the following equation:

\[
\log_x 27 - \log_x 3 = 2\log_x y
\]

**Solution**

\[
\log_x 27 - \log_x 3 = 2\log_x y
\]

\[
\log_x \frac{27}{3} = 2\log_x y \\
\log_x 9 = \log_x y^2
\]

\[
9 = y^2 \\
y = \pm 3
\]

\[
y = 3
\]

1 mark for quotient rule
1 mark for power rule
½ mark for positive value of \( y \)
½ mark for negative value of \( y \) and rejecting extraneous root

3 marks

---

Question 14  

Angle \( \theta \), measuring \( \frac{5\pi}{4} \), is drawn in standard position as shown below.

Determine the measures of all angles in the interval \([ -4\pi, 2\pi ]\) that are coterminal with \( \theta \).

**Solution**

\[
\theta = -\frac{3\pi}{4}
\]

½ mark

\[
\theta = -\frac{11\pi}{4}
\]

½ mark

1 mark
Prove the identity below for all permissible values of $x$:

\[
\frac{\sin^2 x}{\sec x + 1} = \cos x - \cos^2 x
\]

**Solution**

**Method 1**

\[
LHS = \frac{1 - \cos^2 x}{\cos x + 1}
\]

\[
= \frac{1 - \cos^2 x}{1 + \cos x}
\]

\[
= \left(1 - \cos^2 x\right) \left(\frac{\cos x}{1 + \cos x}\right)
\]

\[
= (1 - \cos x)(1 + \cos x) \left(\frac{\cos x}{1 + \cos x}\right)
\]

\[
= (1 - \cos x)(\cos x)
\]

\[
= \cos x - \cos^2 x
\]

\[
= RHS
\]

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks
Question 15

Method 2

\[ \text{LHS} = \frac{\sin^2 x}{\sec x + 1} \cdot \frac{(\sec x - 1)}{(\sec x - 1)} \]

\[ \begin{align*}
= & \frac{\sin^2 x}{\sec^2 x - 1} \\
= & \frac{\sin^2 x (\sec x - 1)}{\tan^2 x} \\
= & \frac{\sin^2 x (\sec x - 1)}{\frac{\sin^2 x}{\cos^2 x}} \\
= & \cos^2 x (\sec x - 1) \\
= & \cos^2 x \left( \frac{1}{\cos x} - 1 \right) \\
= & \cos x - \cos^2 x \\
= & \text{RHS}
\]

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks
Question 16

Solve algebraically:

\[ \binom{n}{2} = 4n + 5 \]

Solution

\[ \binom{n}{2} = 4n + 5 \]

\[ \frac{n!}{(n-2)!2!} = 4n + 5 \quad \frac{1}{2} \text{ mark for factorial notation} \]

\[ \frac{n(n-1)(n-2)!}{(n-2)!2!} = 4n + 5 \quad \frac{1}{2} \text{ mark for factorial expansion} \]

\[ n(n-1) = 2!(4n + 5) \quad \frac{1}{2} \text{ mark for simplification of factorial} \]

\[ n^2 - n = 8n + 10 \]

\[ n^2 - 9n - 10 = 0 \quad \frac{1}{2} \text{ mark for simplification} \]

\[ (n - 10)(n + 1) = 0 \]

\[ n = 10 \quad n = -1 \quad \frac{1}{2} \text{ mark for both values of } n \]

\[ n = 10 \quad \frac{1}{2} \text{ mark for rejecting extraneous root} \]

3 marks
Booklet 2 Questions
## Answer Key for Multiple-Choice Questions

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<th>Answer</th>
<th>Learning Outcome</th>
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<td>C</td>
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</tr>
<tr>
<td>24</td>
<td>C</td>
<td>R12</td>
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</tbody>
</table>
Question 17

How many different arrangements are possible when arranging all of the letters of the word SEPTEMBER?

a) \(9!\)  
b) \(6!3!\)  
c) \(\frac{9!}{3!}\)  
d) \(\frac{6!}{3!}\)

Question 18

Which one of the following angles terminates in Quadrant III?

a) 3 radians  
b) \(\frac{7\pi}{5}\) radians  
c) \(-210^\circ\)  
d) \(500^\circ\)

Question 19

There are 13 terms in the expansion of \((3x - y)^{2n}\). Determine the value of \(n\).

a) 6  
b) 6.5  
c) 7  
d) 26
Question 20

Which of the following is true about the periods of the three functions below?

\[ f(\theta) = 2\sin\left(\theta - \frac{\pi}{2}\right) \quad g(\theta) = \sin 3\theta + 6 \quad k(\theta) = 3\sin \theta + 6 \]

a) The graphs of \( f(\theta) \) and \( g(\theta) \) have the same period.

b) The graphs of \( g(\theta) \) and \( k(\theta) \) have the same period.

c) All of the graphs have the same period.

d) None of the graphs have the same period.

Question 21

Which of the following represents the general solution to the equation \( \tan \theta = -1 \)?

a) \( \theta = \frac{\pi}{4} + 2k\pi, k \in \mathbb{I} \)

b) \( \theta = \frac{\pi}{4} + k\pi, k \in \mathbb{I} \)

c) \( \theta = \frac{3\pi}{4} + 2k\pi, k \in \mathbb{I} \)

d) \( \theta = \frac{3\pi}{4} + k\pi, k \in \mathbb{I} \)

Question 22

If \((3, -2)\) is a point on the graph of \( y = f(x) \), what point must be on the graph of \( y = 2f(x + 1) \)?

a) \((4, -1)\)

b) \((4, -4)\)

c) \((2, 1)\)

\[ \text{d) } (2, -4) \]
Question 23

Which equation is represented by the graph sketched below?

a) \[ y = \left( \frac{1}{2} \right)^{-x} \]

b) \[ y = \left( \frac{1}{2} \right)^{x} \]

c) \[ y = 2^x \]

d) \[ y = -2^x \]

Question 24

What is the degree of the polynomial represented below?

a) 2

b) 3

c) 4

d) 5
Question 25

Given the graph of \( y = 2 \cos \pi x + 1 \) below, determine another equation that will produce the same graph.

![Graph of \( y = 2 \cos \pi x + 1 \)](image)

**Solution**

Some sample equations are:

\[
y = 2 \cos \pi (x - 2) + 1 \\
y = -2 \cos \pi (x - 1) + 1 \\
y = -2 \cos \pi (x + 1) + 1 \\
y = 2 \sin \pi \left( x + \frac{1}{2} \right) + 1 \\
y = 2 \sin \pi \left( x - \frac{3}{2} \right) + 1
\]

1 mark for correct equation

1 mark for correct equation

Other answers are possible.
Question 26

Given \( f(x) = 3 \) and \( g(x) = x + 2 \), determine the domain and range of \( h(x) = \frac{f(x)}{g(x)} \).

**Solution**

**Domain:** \( \{x \mid x \in \mathbb{R}, x \neq -2\} \)

**Range:** \( \{y \mid y \in \mathbb{R}, y \neq 0\} \)

1 mark for domain

1 mark for range

2 marks

---

Question 27

Explain how to find the exact value of \( \sec\left(\frac{19\pi}{6}\right) \).

**Solution**

Find the exact value of \( \cos\left(\frac{19\pi}{6}\right) \).

1 mark for \( \cos\left(\frac{19\pi}{6}\right) \)

Then take the reciprocal of the value of \( \cos\left(\frac{19\pi}{6}\right) \).

1 mark for reciprocal

2 marks
Given \( f(x) = 4 - x \), verify that \( f^{-1}(x) = f(x) \).

**Solution**

**Method 1**

\( y = 4 - x \)

To find \( f^{-1}(x) \), switch \( x \) and \( y \) values.

\[
x = 4 - y
\]

\[-y = x - 4
\]

\[y = 4 - x
\]

\[f^{-1}(x) = 4 - x \quad \text{1 mark for verifying } f^{-1}(x) = f(x)
\]

**Method 2**

When \( y = 4 - x \) is reflected over the line \( y = x \) it produces the same graph.

**Method 3**

Assume \( f^{-1}(x) = 4 - x \).

\[
f \left( f^{-1}(x) \right) = 4 - (4 - x)
\]

\[= x
\]

\[\therefore f(x) \text{ and } f^{-1}(x) \text{ are inverses of one another.} \]
Sketch the graph of:

\[ f(x) = (2 - x)(x + 3)(x + 1)^2 \]

Label the \( x \)-intercepts and \( y \)-intercept.

**Solution**

\( x \)-intercepts: \(-3, -1, \) and 2

\( y \)-intercept: 6

1 mark for \( x \)-intercepts

\( \frac{1}{2} \) mark for \( y \)-intercept

1 mark for multiplicity of 2 at \( x = -1 \) only

\( \frac{1}{2} \) mark for end behaviour

3 marks
Question 30

Which expression has a larger value?

\[ \log_3 36 \quad \text{or} \quad \log_3 80 \]

Justify your answer.

**Solution**

**Method 1**

\[
\log_2 36 \quad 2^5 = 32 \quad \Rightarrow \quad 2^6 = 64 \\
\log_3 80 \quad 3^3 = 27 \quad \Rightarrow \quad 3^4 = 81
\]

\[ \therefore \log_2 36 \text{ is the larger value} \]

**Method 2**

\[
\log_2 32 = 5 \quad \therefore \log_2 36 \text{ is a little more than 5} \\
\log_3 81 = 4 \quad \therefore \log_3 80 \text{ is a little less than 4}
\]

\[ \therefore \log_2 36 \text{ is the larger value} \]
The graph below represents the equation \( y = ax^3 + 6x^2 + 5x - 10. \)

What must be true about the value of \( a \)? Explain your reasoning.

**Solution**

\( a \) is any negative number.  
Explanations with reference to end behaviour. \[ \frac{1}{2} \text{ mark} \]

or

\( a \) cannot be zero. 
The graph is of a cubic function, not a quadratic function. \[ \frac{1}{2} \text{ mark} \]

\[ \frac{1}{2} \text{ mark for explanation} \]

\[ 1 \text{ mark} \]
The terminal arm of an angle $\theta$, in standard position, intersects the unit circle in Quadrant IV at a point $P\left(\frac{\sqrt{5}}{4}, y\right)$. Determine the value of $\sin \theta$.

**Solution**

**Method 1**

The point $P(\theta)$ on the unit circle has coordinates $(\cos \theta, \sin \theta)$.

\[
\cos^2 \theta + \sin^2 \theta = 1
\]

\[
\left(\frac{\sqrt{5}}{4}\right)^2 + \sin^2 \theta = 1
\]

\[
\sin^2 \theta = 1 - \frac{5}{16}
\]

\[
\sqrt{\sin^2 \theta} = \frac{\sqrt{11}}{4}
\]

\[
\sin \theta = \pm \frac{\sqrt{11}}{4}
\]

\[
\sin \theta = -\frac{\sqrt{11}}{4}
\]

**Method 2**

\[
\left(\sqrt{5}\right)^2 + y^2 = 4^2
\]

\[
5 + y^2 = 16
\]

\[
y^2 = 11
\]

\[
y = \pm \sqrt{11}
\]

\[
\sin \theta = -\frac{\sqrt{11}}{4}
\]

2 marks
Question 33

Given the sinusoidal function \( f(x) \) below, sketch the graph of \( g(x) = |f(x)| - 1 \).

\[ f(x) \]

\[ \begin{array}{c}
\text{Solution} \\
\end{array} \]

\[ g(x) \]

1 mark for absolute value
1 mark for vertical shift

2 marks
Question 34  

The graph of a rational function, \( f(x) \), has a point of discontinuity when \( x = 2 \) and an asymptote when \( x = 4 \). Write a possible equation for \( f(x) \).

**Solution**

A possible equation is:

\[
f(x) = \frac{x - 2}{(x - 2)(x - 4)}
\]

1 mark for \( \frac{x - 2}{x - 2} \) (point of discontinuity when \( x = 2 \))

1 mark for \( x - 4 \) in denominator (asymptote when \( x = 4 \))

2 marks

Question 35  

Given that \( (x - 1) \) is one of the factors, express \( x^3 - 57x + 56 \) as a product of factors.

**Solution**

\[
\begin{array}{cccc|c}
1 & 1 & 0 & -57 & 56 \\
\hline
1 & 1 & 1 & -56 & 0 \\
1 & 1 & -56 & 0 & \\
\end{array}
\]

\( (x - 1)(x^2 + x - 56) \)

or

\( (x - 1)(x + 8)(x - 7) \)

\( \frac{1}{2} \) mark for \( x = 1 \)

1 mark for synthetic division (or for any other equivalent strategy)

\( \frac{1}{2} \) mark for consistent factors

2 marks
Question 36

Give an example using values for $A$ and $B$, in degrees or radians, to verify that $\cos(A + B) = \cos A + \cos B$ is not an identity.

**Solution**

**Method 1**

Let $A = 45^\circ$ and $B = 90^\circ$.

<table>
<thead>
<tr>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos(45^\circ + 90^\circ)$</td>
<td>$\cos 45^\circ + \cos 90^\circ$</td>
</tr>
<tr>
<td>$\cos(135^\circ)$</td>
<td>$\cos 45^\circ + \cos 90^\circ$</td>
</tr>
<tr>
<td>$-\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{2}}{2} + 0$</td>
</tr>
<tr>
<td>$-\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
</tr>
</tbody>
</table>

LHS $\neq$ RHS $\therefore \cos(A + B) = \cos A + \cos B$ is not an identity.

1 mark for simplification of $\cos(A + B)$
1 mark for simplification of $\cos A + \cos B$

2 marks
Method 2

\[
\cos(A + B) = \cos A \cos B - \sin A \sin B
\]

Let \(A = 60^\circ\) and \(B = 30^\circ\).

\[
\begin{aligned}
\cos\left(60^\circ + 30^\circ\right) &= \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ \\
&= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) \\
&= \frac{\sqrt{3} - \sqrt{3}}{4} \\
&= 0 \\
\end{aligned}
\]

1 mark for simplification of \(\cos(A + B)\)

\[
\cos A + \cos B = \cos 60^\circ + \cos 30^\circ \\
= \frac{1}{2} + \frac{\sqrt{3}}{2} \\
= \frac{1 + \sqrt{3}}{2}
\]

1 mark for simplification of \(\cos A + \cos B\)

These two solutions are not equal \(\therefore \cos(A + B) = \cos A + \cos B\) is not an identity.
Question 37

Sketch the graph of \( y = \sqrt{x + 1} - 2 \) and verify that the value of the \( x \)-intercept is the same as the solution to the equation \( \sqrt{x + 1} - 2 = 0 \).

Solution

\[
\begin{align*}
\sqrt{x + 1} &= 2 \\
(\sqrt{x + 1})^2 &= (2)^2 \\
x + 1 &= 4 \\
x &= 3
\end{align*}
\]

\[
\begin{align*}
\sqrt{x + 1} - 2 &= 0 \\
\sqrt{x + 1} &= 2 \\
(\sqrt{x + 1})^2 &= (2)^2 \\
x + 1 &= 4 \\
x &= 3
\end{align*}
\]

1 mark for general shape
½ mark for horizontal shift
½ mark for vertical shift

1 mark for verification

3 marks
Mohamed is asked to sketch the graph of \( y = \tan x \).
His graph is shown below.

Explain why his graph is incorrect.

**Solution**

The graph of \( y = \tan x \) should have zeros at \( k\pi, k \in \mathbb{I} \).

or

The graph of \( y = \tan x \) should have asymptotes at \( (2k + 1)\frac{\pi}{2} \) or \( \frac{\pi}{2} + k\pi, k \in \mathbb{I} \).

or

Mohamed sketched the incorrect graph. He sketched the graph of \( y = \tan \left(x - \frac{\pi}{2}\right) \).
On the interval $0 \leq \theta < 2\pi$, identify the non-permissible values of $\theta$ for the trigonometric identity:

$$\tan \theta = \frac{1}{\cot \theta}$$

**Solution**

$$\frac{\sin \theta}{\cos \theta} = \frac{1}{\cos \theta} \frac{\cos \theta}{\sin \theta}$$

$\therefore$ the above identity is non-permissible when $\cos \theta = 0$ or $\sin \theta = 0.$

1 mark for identifying non-permissible values

(½ mark for $\cos \theta = 0$, ½ mark for $\sin \theta = 0$)

$\cos \theta \neq 0 \quad \sin \theta \neq 0$

$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2} \quad \theta \neq 0, \pi$

1 mark for solving for $\theta$ (½ mark for each solution set)

$\theta \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

2 marks
Question 40

a) Sketch the graph of \( y = \ln(x) \).

b) Sketch the graph of \( y = -\ln(x - 2) \).

**Solutions**

a)

\[
(1, 0) \quad (e, 1)
\]

½ mark for increasing logarithmic function
½ mark for \( x \)-intercept at \((1, 0)\)
½ mark for consistent point on logarithmic function
½ mark for vertical asymptotic behaviour

2 marks

b)

\[
(3, 0) \quad (e + 2, -1)
\]

1 mark for reflection in \( x \)-axis
1 mark for horizontal shift

2 marks
Given \( f(x) = \sqrt{x - 2} \) and \( g(x) = 3x \), write the equation for \( h(x) = f(g(x)) \).

What are the restrictions on the domain of \( h(x) \)?

Explain your reasoning.

**Solution**

\[
h(x) = \sqrt{3x - 2}
\]

1 mark for \( h(x) = f(g(x)) \)

\[
3x - 2 \geq 0
\]
\[
3x \geq 2
\]
\[
x \geq \frac{2}{3}
\]

\( x \geq \frac{2}{3} \)

½ mark for identifying restriction

Since we cannot find a square root of a negative number, there is a restriction on the domain, \( x \geq \frac{2}{3} \).

½ mark for explanation

2 marks
Question 42

Sketch the graph of \( y = 10\cos\left(\frac{\pi}{2}(x - 2)\right) \) over the interval \([0, 6]\).

**Solution**

\[
\text{period} = \frac{2\pi}{\frac{\pi}{2}} = 4
\]

1 mark for amplitude
1 mark for period
1 mark for horizontal shift

Note(s):
- deduct \( \frac{1}{2} \) mark if the interval \([0, 6]\) is not completely sketched
Question 43

Sketch the graph of the function \( f(x) = \frac{x^2}{x^2 - x} \).

Solution

\[
f(x) = \frac{x^2}{x(x - 1)}
= \frac{x}{x - 1} \quad \text{with a point of discontinuity where } x = 0
\]

Point of discontinuity: \( f(0) = \frac{0}{0 - 1} = 0 \)

\[\therefore \text{there is a point of discontinuity at } (0, 0).\]

Divide:

\[
\frac{x - 1}{x - 1} \cdot \frac{1}{x + 0}
= \frac{x - 1 + 1}{x - 1}
\]

\[\therefore f(x) = \frac{1}{x - 1} + 1\]

\[\therefore \text{horizontal asymptote at } y = 1\]
\[\therefore \text{vertical asymptote at } x = 1\]

1 mark for vertical asymptote at \( x = 1 \)
1 mark for horizontal asymptote at \( y = 1 \)
1 mark for point of discontinuity at \((0, 0)\) or a point of discontinuity consistent with graph
\(\frac{1}{2}\) mark for graph left of vertical asymptote
\(\frac{1}{2}\) mark for graph right of vertical asymptote

4 marks
Question 44

Is \((x - 3)\) a factor of \(x^4 - x^3 - 3x^2 + x - 1\)? Justify your answer.

Solution

Method 1

\[ x = 3 \]
\[ \therefore (3)^4 - (3)^3 - 3(3)^2 + (3) - 1 = 81 - 27 - 27 + 3 - 1 = 29 \]

The remainder does not equal zero, therefore \((x - 3)\) is not a factor.

Method 2

\[ \begin{array}{c|cccc}
3 & 1 & -1 & -3 & 1 & -1 \\
\hline & & 3 & 6 & 9 & 30 \\
\end{array} \]

\[ \begin{array}{c|cccc}
& 1 & 2 & 3 & 10 & 29 \\
\end{array} \]

The remainder does not equal zero, therefore \((x - 3)\) is not a factor.
Given $f(x) = x - 1$ and $g(x) = x^2$, write the equation of $y = f(g(x))$ and sketch the graph.

**Solution**

$f(g(x)) = x^2 - 1$  

or

$y = x^2 - 1$

1 mark for composition

1 mark for consistent graph

2 marks
MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the Answer/Scoring Sheet.

<table>
<thead>
<tr>
<th>E1</th>
<th>answer given as a complex fraction</th>
<th>final answer not stated</th>
<th>answer stated in degrees instead of radians or vice versa</th>
</tr>
</thead>
<tbody>
<tr>
<td>E2</td>
<td>changing an equation to an expression or vice versa</td>
<td>equating the two sides when proving an identity</td>
<td></td>
</tr>
<tr>
<td>E3</td>
<td>variable omitted in an equation or identity</td>
<td>variables introduced without being defined</td>
<td></td>
</tr>
<tr>
<td>E4</td>
<td>“$\sin x^2$” written instead of “$\sin^2 x$”</td>
<td>missing brackets but still implied</td>
<td></td>
</tr>
<tr>
<td>E5</td>
<td>missing units of measure</td>
<td>incorrect units of measure</td>
<td></td>
</tr>
<tr>
<td>E6</td>
<td>rounding error</td>
<td>rounding too early</td>
<td></td>
</tr>
<tr>
<td>E7</td>
<td>transcription error</td>
<td>notation error</td>
<td></td>
</tr>
<tr>
<td>E8</td>
<td>answer given outside the domain</td>
<td>bracket error made when stating domain or range</td>
<td></td>
</tr>
<tr>
<td>E9</td>
<td>incorrect or missing endpoints or arrowheads</td>
<td>scale values on axes not indicated</td>
<td></td>
</tr>
<tr>
<td>E10</td>
<td>asymptotes drawn as solid lines</td>
<td>graph crosses or curls away from asymptotes</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

IRREGULARITIES IN PROVINCIAL TESTS
A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an Irregular Test Booklet Report should be completed and sent to the Department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the Department is made aware that follow-up has taken place by completing an Irregular Test Booklet Report.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an Irregular Test Booklet Report documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Department along with the test materials.
## Table of Questions by Unit and Learning Outcome

### Unit A: Transformations of Functions

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<th>Learning Outcome</th>
<th>Mark</th>
</tr>
</thead>
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<td>9</td>
<td>R1</td>
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</tr>
<tr>
<td>10</td>
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</tr>
<tr>
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<td>12</td>
<td>R5</td>
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<td>33</td>
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<tr>
<td>40 b)</td>
<td>R2, R5</td>
<td>2</td>
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<tr>
<td>41</td>
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<td>2</td>
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<td>45</td>
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### Unit B: Trigonometric Functions

<table>
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<th>Learning Outcome</th>
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<td>14</td>
<td>T1</td>
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</tr>
<tr>
<td>18</td>
<td>T1</td>
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<tr>
<td>20</td>
<td>T4</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>T4</td>
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<td>T3</td>
<td>2</td>
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### Unit C: Binomial Theorem

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<td>31</td>
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<tr>
<td>35</td>
<td>R11</td>
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<td>44</td>
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</table>
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<tbody>
<tr>
<td>2</td>
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</tr>
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<th>Learning Outcome</th>
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### Unit G: Radicals and Rationals

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<td>R1, R13</td>
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<tr>
<td>43</td>
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