Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2026

Manitoba

Grade 12 Pre-Calculus Mathematics Achievement Test:
Marking Guide (January 2026)

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Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Contents

General Marking Instructions 1

Scoring Guidelines for Booklet 1 Questions 5

Scoring Guidelines for Booklet 2 Questions 57

Answer Key for Selected Response Questions 58

Appendices 118

Appendix A: Marking Guidelines 119

Appendix B: Irregularities in Provincial Tests 120

Irregular Test Booklet Report 121

Appendix C: Table of Questions by Unit and Learning Outcome 123

General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Early Childhood Learning in the envelope provided. (For more information see the administration manual.)

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If a *Scoring Sheet* is marked with "0" only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Early Childhood Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION

Shade in the circles below for a maximum total deduction of 5 marks (½ mark deduction per error).
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).

Example: Marks assigned to the student

Marks Awarded	Booklet 1 25	Selected Response 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**Maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

Question 1 T1

Liam went for a ride on a Ferris wheel with a radius of 5 metres. Determine the distance he travelled if the Ferris wheel stopped after rotating 1260°.

Solution

$\theta = 1260^\circ \bullet \frac{\pi}{180^\circ}$ 1 mark for conversion

$\theta = 7\pi$

or

$\theta = 21.991\,148...$

$s = \theta r$

$s = 7\pi(5)$ 1 mark for substitution

$s = 35\pi$ metres

or

$s = 109.956$ metres

2 marks

Exemplar 1

$s = \theta r$

$S = (1260)(5)$

$s = 6300$ metres

1 out of 2

+ 1 mark for substitution

Exemplar 2

$\theta = 1260 \cdot \frac{\pi}{180} = 21.99$

$s = \theta r$

$S = (21.99)(5)$

$s = 109.95$

2 out of 2

award full marks
E5 (units of measure omitted in final answer)
E6 (rounding error)

Question 2 P4

Determine and simplify the term that contains x^{12} in the binomial expansion of $\left(2x^4 - 3\right)^7$.

Solution

Method 1

$\left(x^4\right)^7, \left(x^4\right)^6, \left(x^4\right)^5, \ldots$

$x^{28}, x^{24}, x^{20}, \ldots$ — 1 mark for determining a pattern

$t_5 = {}_7C_4\left(2x^4\right)^3(-3)^4$

$= 22\,680x^{12}$ — 2 marks (1 mark for ${}_7C_4$; ½ mark for each consistent factor)

1 mark for simplification (½ mark for coefficient, ½ mark for exponent)

4 marks

Method 2

$x^{12} = \left(x^4\right)^{7-k}\left(x^0\right)^k$ — ½ mark for substitution

$12 = 28 - 4k$

$-16 = -4k$

$4 = k$ — ½ mark for solving for k

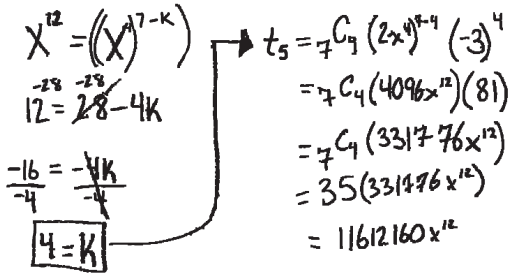
$t_5 = {}_7C_4\left(2x^4\right)^3(-3)^4$

$= 22\,680x^{12}$ — 2 marks (1 mark for ${}_7C_4$; ½ mark for each consistent factor)

1 mark for simplification (½ mark for coefficient, ½ mark for exponent)

4 marks

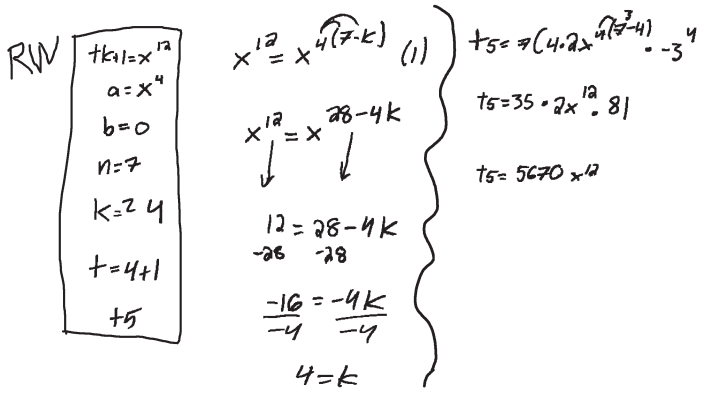
Exemplar 1



3½ out of 4

award full marks
– ½ mark for arithmetic error in line 6

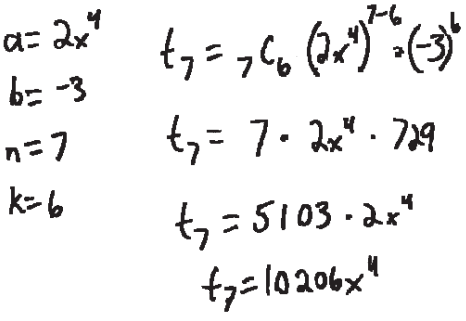
Exemplar 2



3½ out of 4

award full marks
– ½ mark for arithmetic error in line 7
E4 (missing brackets but still implied in line 6)

Exemplar 3



3 out of 4

+ 1 mark for ${}_7C_6$ (consistent with their value of k)
+ 1 mark for each consistent factor
+ 1 mark for simplification

Question 3 R10

The normal healing of wounds can be modeled by an exponential function. The area of the wound decreases according to the formula,

$$A = A_0 e^{-0.35t}$$

where: A is the area of the wound in cm^2, after t days,

A_0 is the initial area of the wound in cm^2, and

t is the time, in days, after healing begins.

If a wound has an initial area of $25\ \text{cm}^2$, determine:

a) the area of the wound after 3 days.

b) the amount of time it will take for the area of the wound to decrease to $4\ \text{cm}^2$.

Solution

a) $A = 25e^{-0.35(3)}$

$A = 8.748\ \text{cm}^2$

1 mark

b) $4 = 25e^{-0.35t}$

$0.16 = e^{-0.35t}$

$\ln 0.16 = \ln e^{-0.35t}$ 1 mark for applying logarithms

$\ln 0.16 = -0.35t \ln e$ ½ mark for power law

$\dfrac{\ln 0.16}{-0.35} = t$

$t = 5.235\,947...$

$t = 5.236\ \text{days}$ ½ mark for the value of t

2 marks

Exemplar 1

a)

$A = A_0 e^{-0.35t}$

$A = (25\ cm^2) e^{(-0.35(3))}$

$A = 625\, e^{(-1.05)}$

$A = 625(0.3499377)$

$A = 218.71$

$A = 14.7888\ cm$

$A = 14.789\ cm$

0 out of 1

b)

$A = A_0 e^{(-0.35t)}$

$4cm^2 = 25cm^2 e^{(-0.35t)}$

$\frac{16}{625} = \frac{625e^{(-0.35t)}}{625}$

$0.0256 = e^{(-0.35(t))}$

$\ln 0.0256 = (t)(-0.35) \ln e$

$\frac{-3.66516}{-0.35} = \frac{t(-0.35)}{-0.35}$

$t = 10.4718$ days

1½ out of 2

award full marks
– ½ mark for procedural error in line 3

Exemplar 2

a)

$A = 25e^{-0.35(3)}$

$= 52.852\ cm^2$

$\therefore$ the area of the wound is $52.852\ cm^2$

½ out of 1

award full marks
– ½ mark for arithmetic error

b)

$4cm^2 = 25cm^2\ e^{-0.35t}$

$\ln 4 = \ln 25\ (\ln e)^{-0.35t}$

$\frac{\ln 4}{\ln 25} = \frac{\ln 25}{\ln 25}(-0.35t)$

$\frac{\ln 4}{\ln 25} = -0.35t$

$t = -1.231$

1 out of 2

+ ½ mark for power law
+ ½ mark for consistent value of *t*
E1 (impossible solution not rejected in final answer)
E5 (units of measure omitted in final answer)

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Question 4 P3

During a hockey practice, 4 players are wearing a green jersey and 5 players are wearing a red jersey. Determine the number of ways three players can be selected if at least two of the players must be wearing a red jersey.

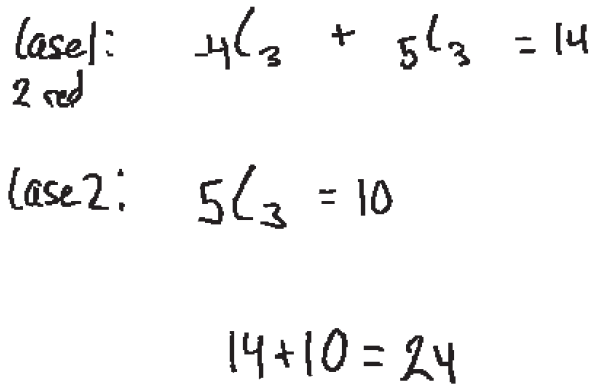
Solution

Case 1: ${}_5C_2 \bullet {}_4C_1 = 40$	1 mark for Case 1
Case 2: ${}_5C_3 \bullet {}_4C_0 = 10$	1 mark for Case 2
$40 + 10 = 50$ ways	1 mark for addition of cases

3 marks

Note:
${}_4C_0$ does not need to be shown.

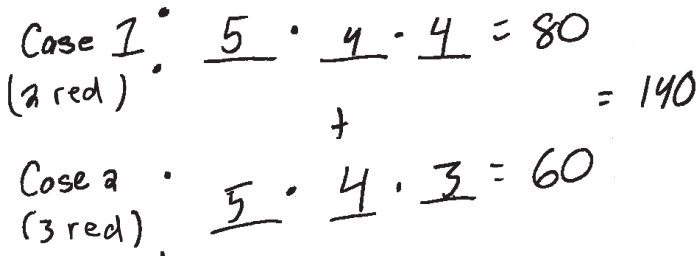
Exemplar 1



2 out of 3

+ 1 mark for Case 2
+ 1 mark for addition of cases

Exemplar 2



2 out of 3

award full marks
– 1 mark for concept error (using permutations instead of combinations)

Question 5 T5

Solve, algebraically, over the interval $[0,2\pi]$.

$$2\sin^2 x + 5\cos x + 1 = 0$$

Solution

$2(1-\cos^2 x) + 5\cos x + 1 = 0$ — 1 mark for substitution of an appropriate identity

$2 - 2\cos^2 x + 5\cos x + 1 = 0$

$-2\cos^2 x + 5\cos x + 3 = 0$

$2\cos^2 x - 5\cos x - 3 = 0$

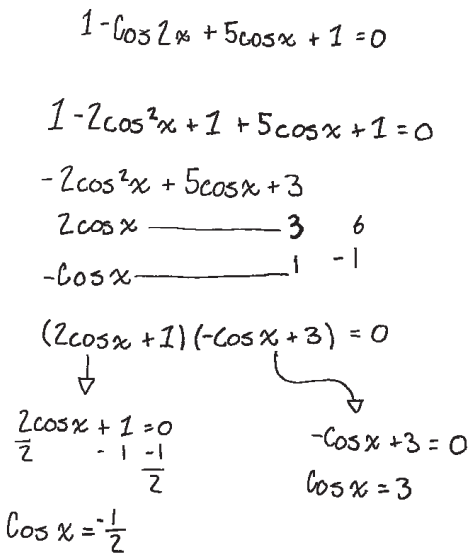
$(2\cos x + 1)(\cos x - 3) = 0$

$\cos x = -\frac{1}{2}$ $\quad \cos x = 3$ — 1 mark for solving for $\cos x$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad$ no solution — 2 marks for solving for x (½ mark for each value; 1 mark for indicating no solution)

4 marks

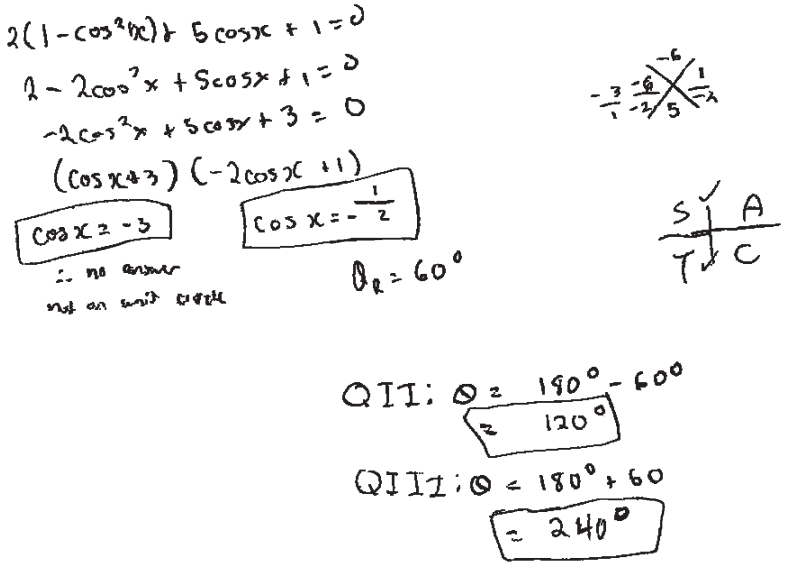
Exemplar 1



2 out of 4

+ 1 mark for substitution of an appropriate identity
+ 1 mark for solving for cos *x*
E2 (changing an equation into an expression in line 3)

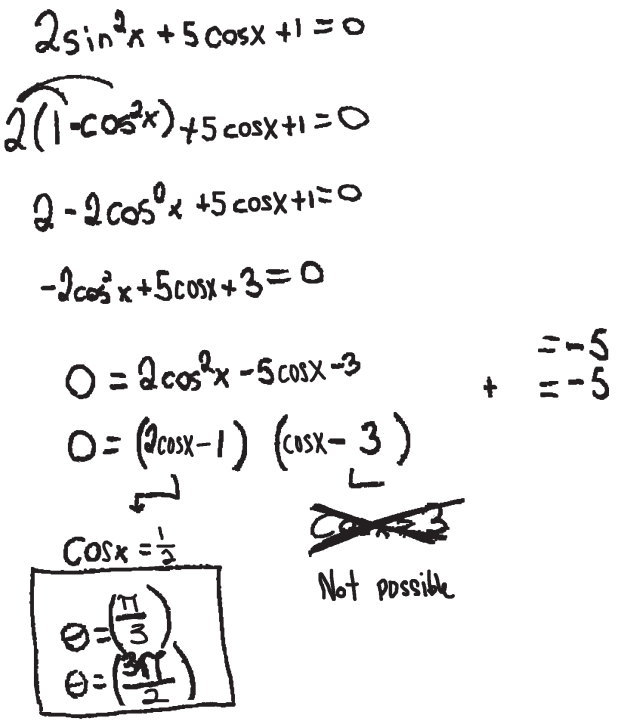
Exemplar 2



3 out of 4

award full marks
– ½ mark for procedural error in line 4 (incorrect factoring)
– ½ mark for arithmetic error in line 5
E2 (changing an equation into an expression in line 4)
E3 (variable introduced without being defined)
E5 (answer stated in degrees instead of radians)

Exemplar 3



3 out of 4

+ 1 mark for substitution of an appropriate identity
+ 1 mark for solving for cos *x*
+ ½ mark for consistent value of *x*
+ 1 mark for indicating no solution
– ½ mark for procedural error in line 6 (incorrect factoring)
E3 (variable introduced without being defined)

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Question 6 R3

The function, $y = f(x)$, has a domain of $[-8,4]$. The domain of the graph of $y = f(bx)$ is $[-4,2]$.

State the value of b.

Solution

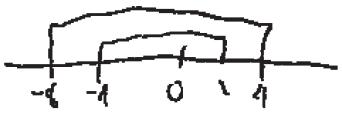
$b = 2$

1 mark

Exemplar 1

$f(x) = [-8, 4]$

$f(2x) = [-1, 2]$



$f(2x)$

1 out of 1

award full marks
E7 (notation error in lines 1 and 2)

Question 7 R14

Describe the difference between the graph of $f(x)=\dfrac{(x+3)(x-4)}{(x+3)}$ and the graph of $g(x)=x-4$.

Solution

The graph of $f(x)$ has a point of discontinuity at $x = -3$, where as the graph of $g(x)$ does not.

1 mark

Exemplar 1

One has a hole.

½ out of 1

award full marks
– ½ mark for lack of clarity in the description

Exemplar 2

$f(x) = \frac{(x+3)(x-4)}{(x+3)}$ $\quad g(x) = x - 4$

graph of $f(x)$ has a point of discontinuity

graph of $g(x)$ doesn't have point of discontinuity

1 out of 1

Question 8 T2

Determine if the point $\left(-\frac{3}{5}, \frac{2}{\sqrt{3}}\right)$ is on the unit circle. Justify your answer.

Solution

Method 1

$x^2 + y^2 = 1$

$$\text{Left-Hand Side} = \left(-\frac{3}{5}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2$$
$$= \frac{9}{25} + \frac{4}{3}$$
$$= \frac{27}{75} + \frac{100}{75}$$

$\frac{127}{75} \neq 1$

Therefore the point $\left(-\frac{3}{5}, \frac{2}{\sqrt{3}}\right)$ is not on the unit circle. 1 mark for justification

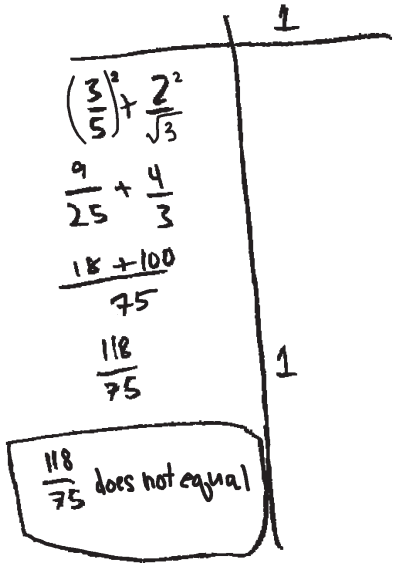
1 mark

Method 2

Since $\frac{2}{\sqrt{3}} > 1$, the point is not on the unit circle. 1 mark for justification

1 mark

Exemplar 1



½ out of 1

award full marks
– ½ mark for arithmetic error in line 3
E4(missing brackets but still implied in line 1)

Exemplar 2

$$\left(-\frac{3}{5}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 = 1$$

$$3 = \frac{9}{25} + \frac{4}{3} \times 25 = 1$$

$$\frac{27}{75} + \frac{100}{75} = 1$$

$$\frac{127}{75} \neq 1$$

No, the point is not on the unit circle as the two coordinates did not equal to 1 after the used equations

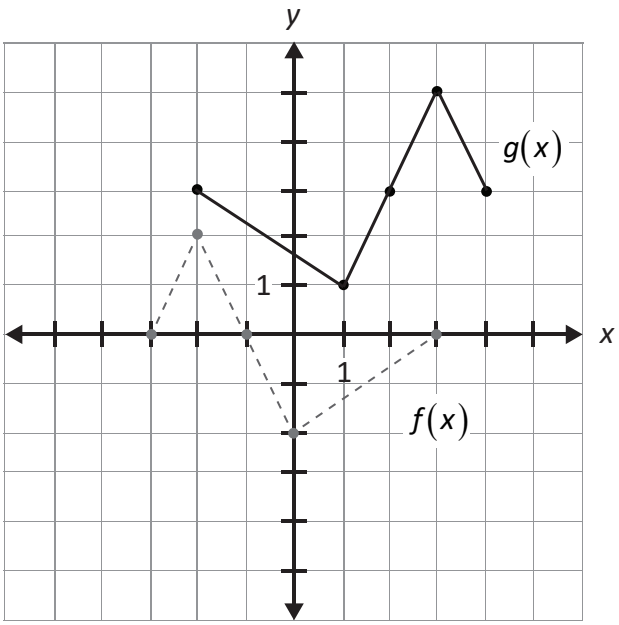
1 out of 1

award full marks
E7 (notation error in line 2)
E2 (equating the two sides when proving an identity)

Question 9 R2, R5

Given the graph of $f(x)$, sketch the graph of $g(x)=f\left(-(x-1)\right)+3$.

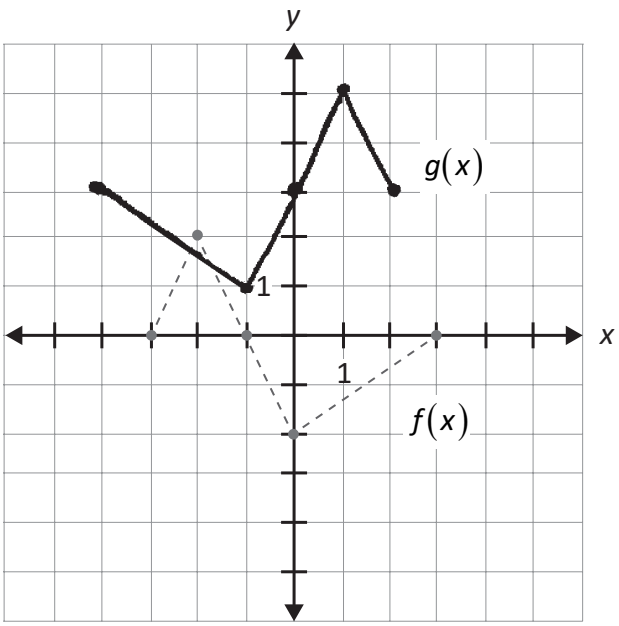
Solution



1 mark for horizontal reflection
1 mark for horizontal translation
1 mark for vertical translation

3 marks

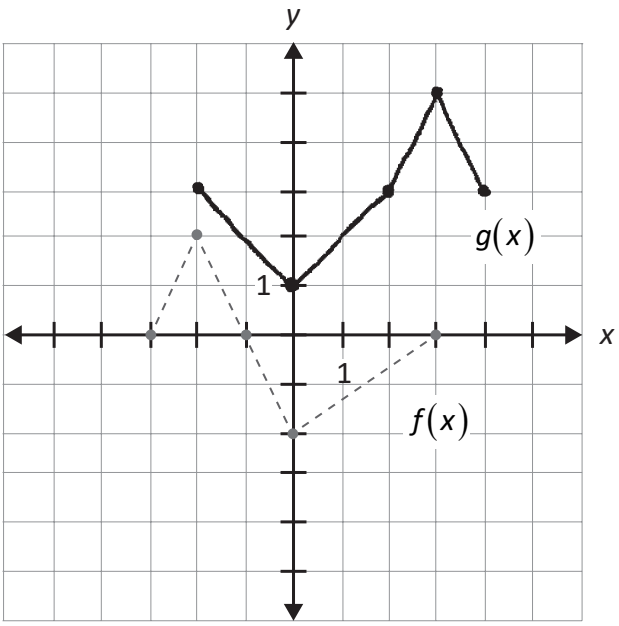
Exemplar 1



2 out of 3

+ 1 mark for horizontal reflection
+ 1 mark for vertical translation

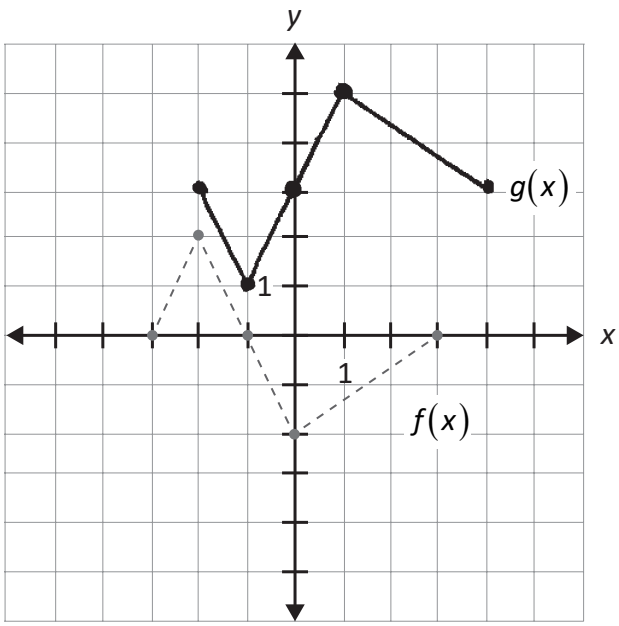
Exemplar 2



2½ out of 3

award full marks
– ½ mark for procedural error (one incorrect point)

Exemplar 3



2 out of 3

+ 1 mark for horizontal translation
+ 1 mark for vertical translation

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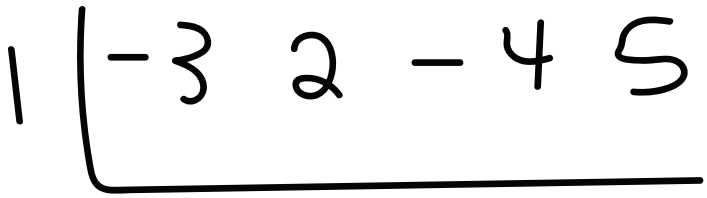
Question 10 R11

Bonnie was asked to determine the factors of $p(x) = -3x^4 + 2x^2 - 4x + 5$.

Describe the error she made in her set up of synthetic division.



Solution

Bonnie did not write the coefficient of 0 for the missing term, x^3.

1 mark

Exemplar 1

She forgot to include a value for x^3.

½ out of 1

award full marks
– ½ mark for lack of clarity in the description

Exemplar 2

Rose didn't add a "0" for the missing term

½ out of 1

award full marks
– ½ mark for lack of clarity in the description

Question 11 R1

Given the functions, $f(x)=\frac{1}{x-1}$ and $g(x)=x-3$,

a) state the equation of $f(g(x))$.

b) state the domain of $f(g(x))$.

Solution

a) $f(g(x))=$ $\frac{1}{x-4}$

1 mark

b) Domain: $\{x|x\neq 4, x\in\mathbb{R}\}$

or

Domain: $(-\infty,4)\cup(4,\infty)$

Exemplar 1

a)

$f(g(x))=$ $\frac{1}{x-4}$

1 out of 1

b)

Domain: $x \neq 4, x \neq 1$

0 out of 1

Exemplar 2

a)

$f(g(x))=$ $\frac{1}{(x-4)}$

1 out of 1

b)

Domain: $\{x \in \mathbb{R} \mid x \neq 43$

1 out of 1

award full marks
E7 (notation error)

Exemplar 3

a)

$f(x-3) = \frac{1}{(x-3)-1}$

$= \frac{1}{-x+3}$

$f(g(x))$= _____

½ out of 1

award full marks
– ½ mark for procedural error in line 2

b)

Domain: $\{x \mid x \in \mathbb{R}, x \neq 3\}$

1 out of 1

answer consistent with (a)

Exemplar 4

a)

$f(g(x))$= $\frac{1}{(x-3)-1}$

1 out of 1

b)

Domain: $[-\infty, \infty)$

0 out of 1

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Question 12 T6

Prove the identity for all permissible values of x.

$$\tan 2x = \frac{2}{\cot x - \tan x}$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\tan 2x$	$\dfrac{2}{\dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}}$
	$\dfrac{2}{\dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x}}$
	$\dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x}$
	$\dfrac{\sin 2x}{\cos 2x}$
	$\tan 2x$

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Method 2

Left-Hand Side	Right-Hand Side
$\dfrac{2\tan x}{1-\tan^2 x}$	$\dfrac{2}{\dfrac{\cos x}{\sin x}-\dfrac{\sin x}{\cos x}}$
$\dfrac{\dfrac{2\sin x}{\cos x}}{1-\dfrac{\sin^2 x}{\cos^2 x}}$	$\dfrac{2}{\dfrac{\cos^2 x-\sin^2 x}{\sin x\cos x}}$
$\dfrac{\dfrac{2\sin x}{\cos x}}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}}$	$\dfrac{2\sin x\cos x}{\cos^2 x-\sin^2 x}$
$\dfrac{2\sin x\cos x}{\cos^2 x-\sin^2 x}$	

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Question 12 T6

Method 3

Left-Hand Side	Right-Hand Side
$\frac{2\tan x}{1-\tan^2 x}$	$\frac{2}{\frac{1}{\tan x}-\tan x}$
	$\frac{2}{\frac{1-\tan^2 x}{\tan x}}$
	$\frac{2\tan x}{1-\tan^2 x}$

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$\tan 2x$	$\frac{2}{\cot x - \tan x}$ $2 \div \frac{\cos x \times \cos x}{\sin x \times \cos x} - \frac{\sin x \times \sin x}{\cos x \times \sin x}$ $2 \div \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$ $2 \times \frac{\sin x \cos x}{\cos^2 x - \sin^2 x} \frac{(\cos^2 x - \sin^2 x)}{(\cos^2 x - \sin^2 x)}$

2 out of 3

+ 1 mark for correct substitution of appropriate identities
+ 1 mark for algebraic strategies
E4 (missing brackets but still implied in line 2)

Exemplar 2

Left-Hand Side	Right-Hand Side
$\frac{2\tan x}{1-\tan^2 x}$ $\frac{2\cdot\frac{\sin x}{\cos x}}{(\cos^2 x)1-\frac{\sin x^2}{\cos y^2}}$ $\frac{2\cdot\frac{\sin x}{\cos x}}{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}$ $\frac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}$ $\frac{2\sin x}{\frac{\cos^2 x-\sin^2 x}{\cos x}}$ $2\sin x\cdot\frac{\cos x}{\cos 2x-\sin^2 x}$	$\frac{2}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}(\sin x)}$ $\frac{2}{\frac{\cos^2 x-\sin^2 x}{\cos x\sin x}}$ $2\cdot\frac{\cos x\sin x}{\cos^2 x-\sin^2 x}$ $\frac{2\cos x\sin x}{\cos^2 x-\sin^2 x}$

3 out of 3

award full marks

E4 ($\sin x^2$ written instead of $\sin^2 x$)
E7 (notation error in line 2)

Exemplar 3

Left-Hand Side	Right-Hand Side
$\tan 2x$ $\frac{2\tan x}{1-\tan^2 x}$ $\frac{2\frac{\sin x}{\cos x}}{1-\left(\frac{\sin x}{\cos x}\right)^2}$ $2\frac{\sin x}{\cos x} \cdot \left\|-\left(\frac{\cos x}{\sin x}\right)^2\right.$ $2\left(\frac{\sin x \cos x^2}{\cos x \sin x^2}\right.$	$\frac{2}{\cot - \tan x}$

1 out of 3

+ 1 mark for correct substitution of appropriate identities
E3 (variable omitted in an identity in line 1)

Question 13 R1

Given $f(x) = -x^2 + 2$ and $g(x) = x^2 - 1$,

a) determine the equation $h(x) = g(x) - f(x)$.

b) state the range of $h(x)$.

Solution

a) $h(x) = g(x) - f(x)$

$h(x) = \left(x^2 - 1\right) - \left(-x^2 + 2\right)$

$h(x) = x^2 - 1 + x^2 - 2$

$h(x) = \underline{2x^2 - 3}$

1 mark for subtraction of $g(x) - f(x)$

1 mark

b) Range: $\underline{\{y | y \geq -3, y \in \mathbb{R}\}}$

or

Range: $\underline{[-3, \infty)}$

1 mark for range consistent with $h(x)$

1 mark

Exemplar 1

a)

$h(x) = x^2 - 1 - (-x^2 + 2)$

$x^2 - 1 + x^2 - 2$

$h(x) = 2x^2 - 3$

---

1 out of 1

award full marks
E2 (changing an equation to an expression in line 2)

b)

y int
$x = 0$
$2(0)^2 - 3$

Range: $(-3, \infty)$

½ out of 1

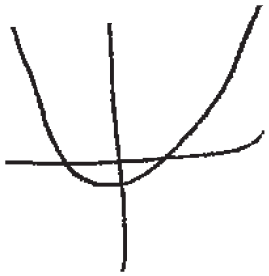
award full marks
– ½ mark for procedural error (bracket error made when stating range)

Exemplar 2

a)

$h(x) = x^2 - 1 - (-x^2 + 2)$

$h(x) = x^2 - 1 + x^2 - 2$



$h(x) =$ $2x^2 - 1$

½ out of 1

award full marks
– ½ mark for arithmetic error in line 3

b)

Range: $[-1, \infty)$

1 out of 1

answer consistent with a)

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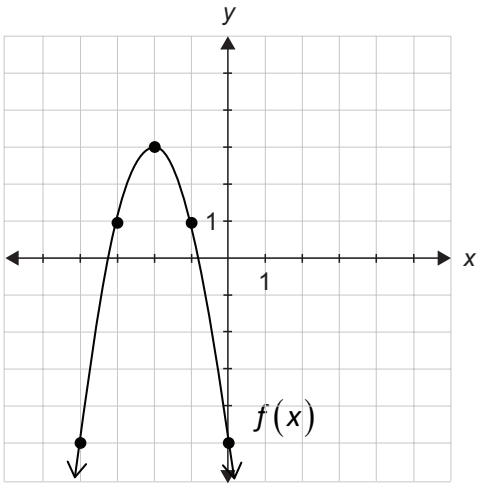
Question 14 R5, R6

Given the graph of $f(x)$,

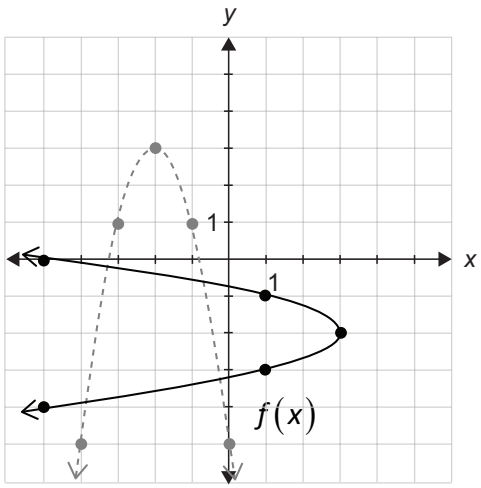


a) sketch the graph after a reflection over the line $y = x$.

b) state a restriction on the domain of $f(x)$ in order for its inverse to be a function.

Solution

a)



1 mark

b) $\{x \mid x \geq -2, x \in \mathbb{R}\}$

or

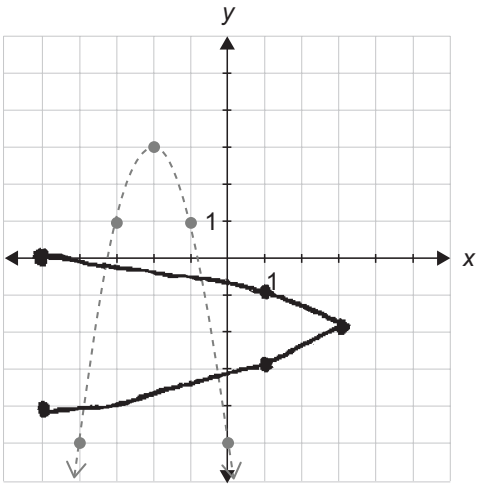
$\{x \mid x \leq -2, x \in \mathbb{R}\}$

1 mark

Note:
Other answers are possible for b).

Exemplar 1

a)



1 out of 1

award full marks
E9 (arrowheads omitted)

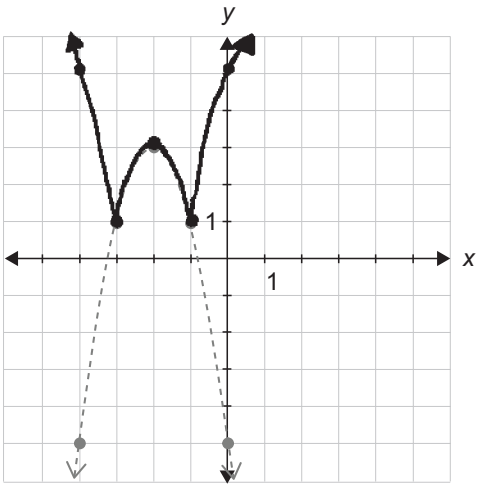
b)

$(-\infty, -2]$

1 out of 1

Exemplar 2

a)



0 out of 1

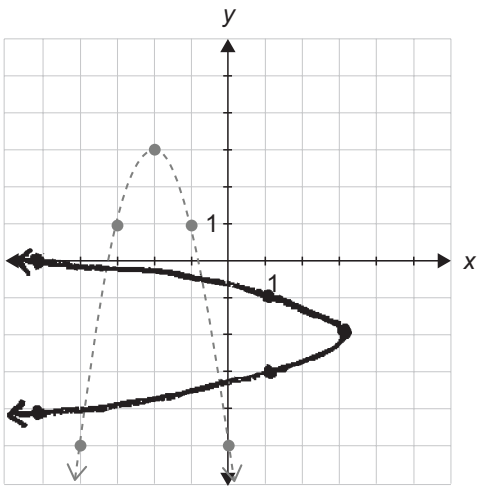
b)

$\{x \in \mathbb{R} \mid x \geq 13$

1 out of 1

Exemplar 3

a)



1 out of 1

b)

$[-2, \infty]$

½ out of 1

award full marks
– ½ mark for procedural error (bracket error made when stating domain)

Question 15 R4, R5

Describe a transformation of a function that would not affect its domain.

Solution

1 mark

vertical stretch by a factor of a

or

vertical reflection

or

vertical translation k units up/down

Exemplar 1

absolute value $y = |f(x)|$

1 out of 1

Exemplar 2

$f(x) = 3(x) + 2$

0 out of 1

Exemplar 3

$f(x) + 1$

↳ change y to 1
(a vertical shift of 1 up)

↳ Only affects the range

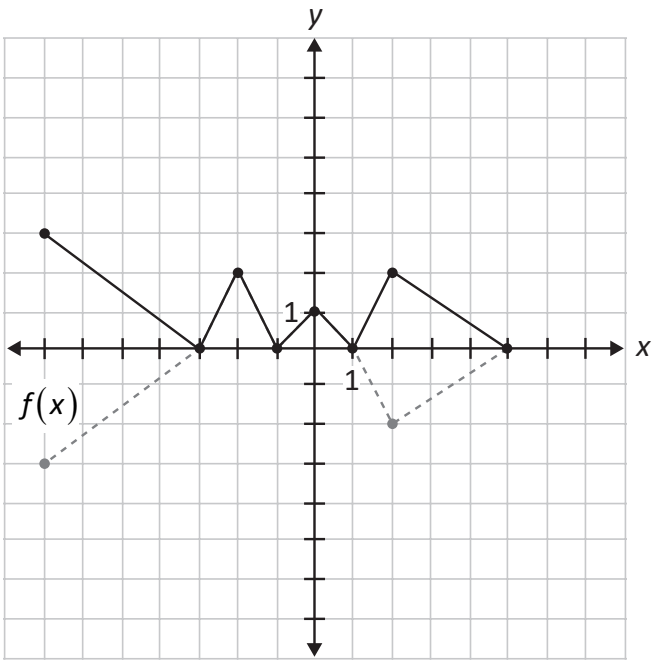
½ out of 1

award full marks
– ½ mark for error in terminology in the description

Question 16 R1

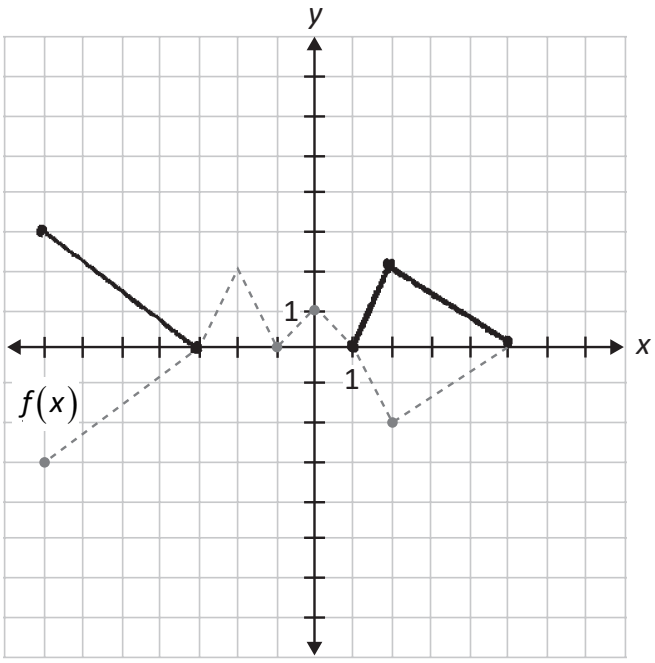
Given the graph of $f(x)$, sketch the graph of the function $y = |f(x)|$.

Solution



1 mark

Exemplar 1



0 out of 1

Question 17 R11

Determine, algebraically, all of the zeros of the polynomial function, $p(x)=3x^3+x^2-20x+12$.

Solution

$p(x)=3x^3+x^2-20x+12$

$p(2)=3(2)^3+(2)^2-20(2)+12$

$p(2)=0$ 1 mark for identifying one zero of $p(x)$

$\therefore (x-2)$ is a factor

2	3	1	−20	12
		6	14	−12
	3	7	−6	0

1 mark for synthetic division (or equivalent strategy)

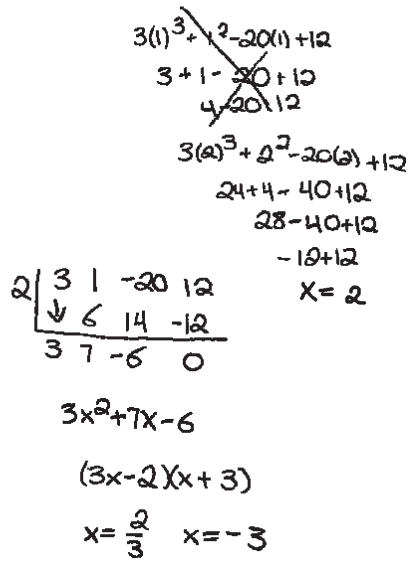
$p(x)=(x-2)\left(3x^2+7x-6\right)$

$0=(x-2)(3x-2)(x+3)$ ½ mark for consistent factors

$x=2,\ \ x=\frac{2}{3},\ \ x=-3$ ½ mark for consistent zeros

3 marks

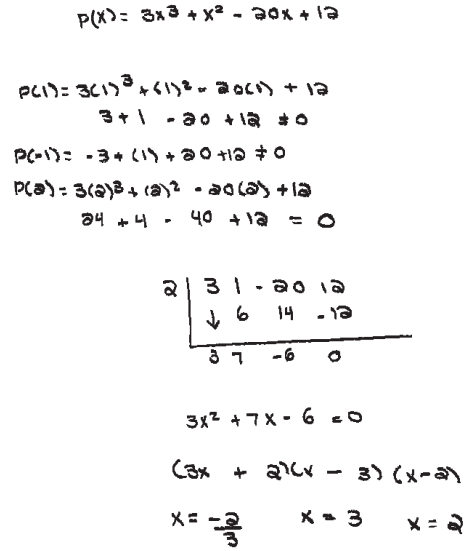
Exemplar 1



2½ out of 3

award full marks
– ½ mark for procedural error in line 8

Exemplar 2



2½ out of 3

+ 1 mark for identifying one zero of $p(x)$
+ 1 mark for synthetic division
+ ½ mark for consistent zeros
E2 (changing an equation to an expression in line 9)

Exemplar 3

$3x^3+x^2-20x+12$ $(x-2)$ is a factor

$$
\begin{array}{r|l}
x-2 & 3x^3+x^2-20x+12 \\
 & -(3x^3-6x^2) \\
\hline
 & 7x^2-20x \\
 & -(7x^2-14x) \\
\hline
 & -6x+12 \\
 & -(-6x+12) \\
\hline
 & 0
\end{array}
\qquad 3x^2+7x-6
$$

$3x^2+9x-2x-6$

$3x(x+3)-2(x+3)$

$(3x-2)(x+3)$

2½ out of 3

+ 1 mark for identifying one zero of $p(x)$
+ 1 mark for equivalent strategy
+ ½ mark for consistent factors

Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Reponse Questions

Question	Answer	Learning Outcome
18	C	T2
19	D	R1
20	C	R3
21	A	R12
22	B	T5
23	B	R13
24	D	P3
25	A	R11

Question 18 T2

Given $\theta = 240°$, identify the coordinates of the point, $P(\theta)$, on the unit circle.

a. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

b. $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

c. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

d. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Question 19 R1

Given the functions, $f(x) = -3x + 5$ and $g(x) = x^2 + x - 1$, identify the value of $g(f(2))$.

a. −10

b. −5

c. −3

d. −1

Question 20 R3

Given $f(x)=\sqrt{x}+5$, identify the equation of the transformed graph that has the same *y*-intercept.

a. $y=f(x-3)$

b. $y=-3f(x)$

c. $y=f(-3x)$

d. $y=f(x)-3$

Question 21 R12

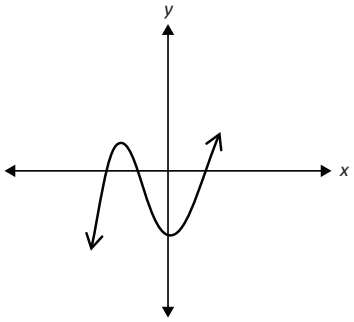
Given the graph of $p(x)=ax^3+bx^2+cx+d$, identify the statement that is true.

a. $a>0,\ d<0$

b. $a>0,\ d>0$

c. $a<0,\ d<0$

d. $a<0,\ d>0$



Question 22 T5

Identify the general solution of the equation, $\csc\theta = -1$.

a. $\theta=\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$

b. $\theta=\frac{3\pi}{2}+2k\pi,\ k\in\mathbb{Z}$

c. $\theta=\frac{3\pi}{2}+k\pi,\ k\in\mathbb{Z}$

d. $\theta=\pi+2k\pi,\ k\in\mathbb{Z}$

Question 23 R13

Identify the equation of a radical function with a domain of $[-6,\infty)$ and a range of $(-\infty,3]$.

a. $y=-\sqrt{x+6}-3$

b. $y=-\sqrt{x+6}+3$

c. $y=\sqrt{-(x+3)}-6$

d. $y=\sqrt{x-6}+3$

Question 24 P3

Identify the value of *n* in the equation, ${}_nC_3 = {}_nC_7$.

a. 3

b. 4

c. 7

d. 10

Question 25 R11

Identify the remainder when $p(x)=x^5-1$ is divided by $(x+1)$.

a. −2

b. −1

c. 0

d. 4

Question 26 R13

Match each function with its corresponding graph.

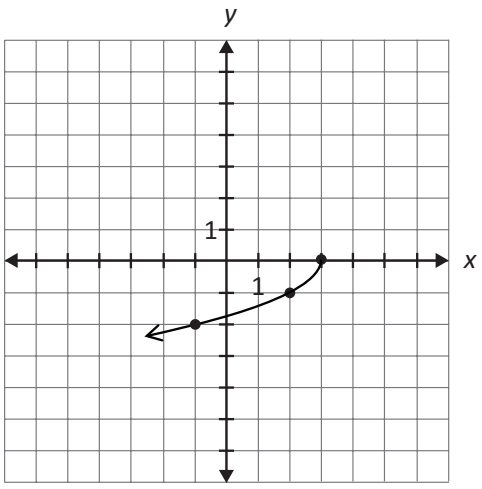
Solution

	Write the corresponding letter in this column.
$y = \sqrt{x} - 3$	B
$y = \sqrt{-x} + 3$	D
$y = -\sqrt{x+3}$	C
$y = -\sqrt{-(x-3)}$	A

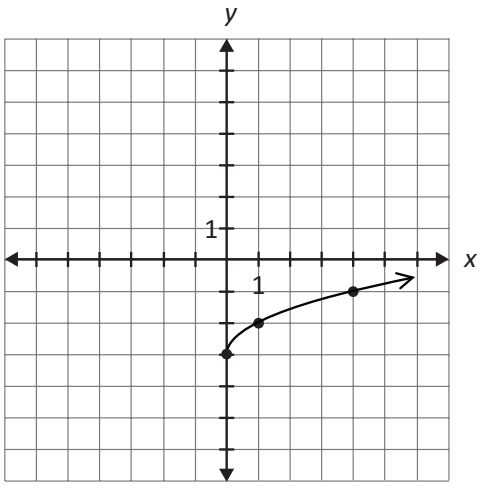
½ mark for each correct answer

2 marks

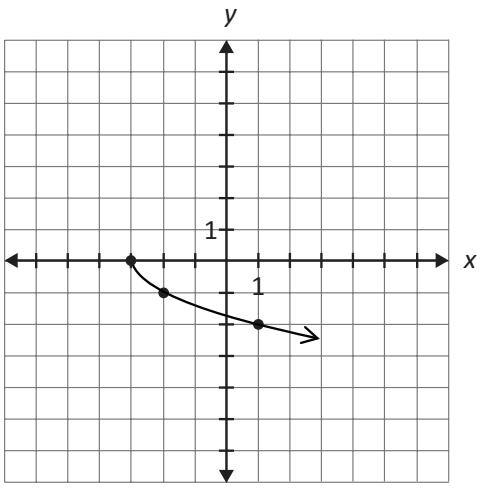
A)



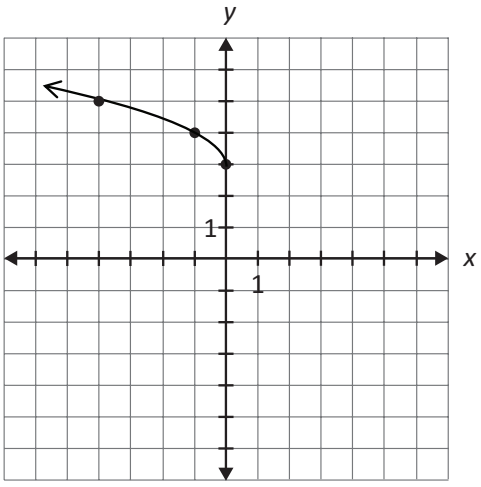
B)



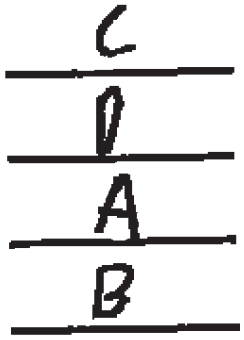
C)



D)



Exemplar 1

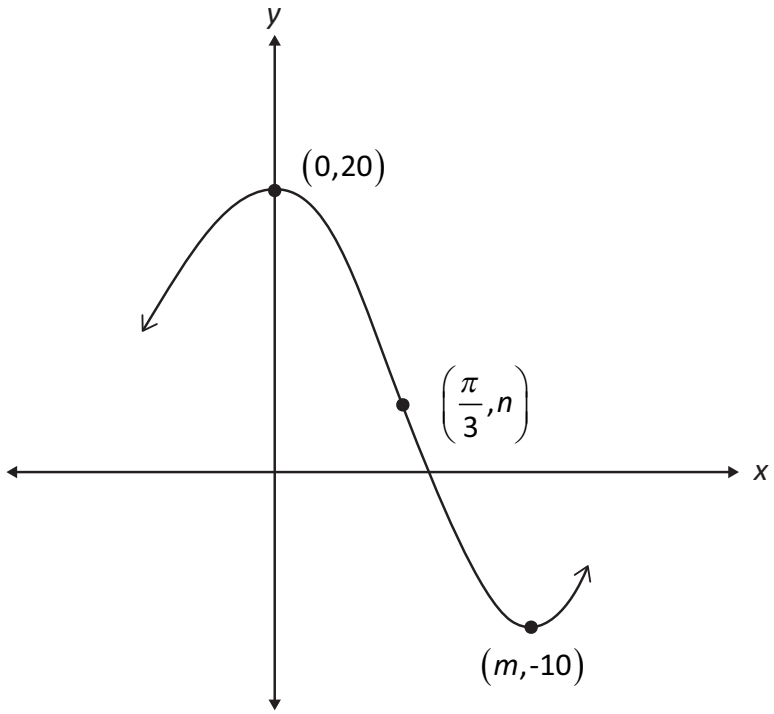


½ out of 2

+ ½ mark for D

Question 27 T4

The amplitude, A, of the sinusoidal function can be determined using the equation $A = 20 - n$. State the values of m and n.



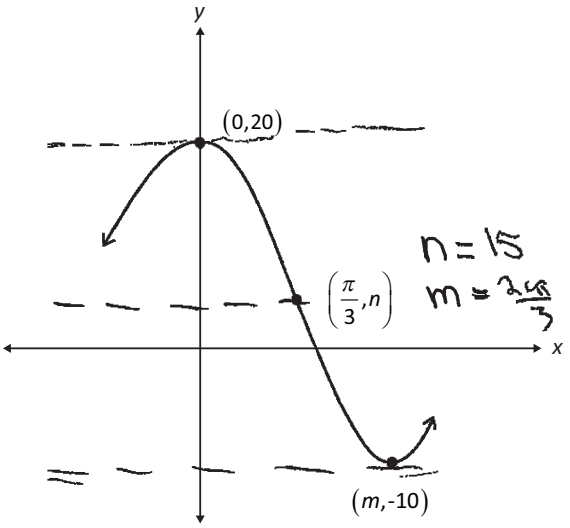
Solution

$m = \frac{2\pi}{3}$ 1 mark for value of m

$n = 5$ 1 mark for value of n

2 marks

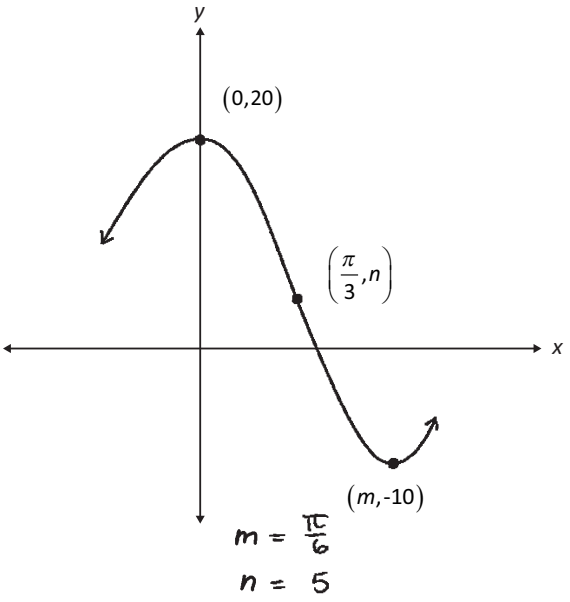
Exemplar 1



1 out of 2

+ 1 mark for value of *m*

Exemplar 2



1 out of 2

+ 1 mark for value of *n*

Question 28 P2

Solve, algebraically.

$${}_{n-1}P_2 = 20$$

Solution

$$\frac{(n-1)!}{(n-1-2)!} = 20$$

½ mark for substitution

$$\frac{(n-1)!}{(n-3)!} = 20$$

$$\frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 20$$

1 mark for factorial expansion
½ mark for simplification of factorials

$$n^2 - 3n + 2 = 20$$

$$n^2 - 3n - 18 = 0$$

$$(n-6)(n+3) = 0$$

$$n = 6 \quad \cancel{n = -3}$$

½ mark for the permissible value of n
½ mark for showing the rejection of the extraneous root

3 marks

Exemplar 1

$$\frac{(n-1)!}{(n-3)!2!} = 20$$

$$\frac{(n-1)(n-2)(n-3)!}{(n-3)!\,2!} = 20$$

$$\frac{(n-1)(n-2)}{2} = 20$$

$$n^2 - 3n + 2 = 40$$

$$n^2 - 3n + -38 = 0$$

1½ out of 3

+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials

Exemplar 2

$$\frac{(n-1)!}{(n-1-2)!} = 20$$

$$\frac{(n-1)!}{(n-3)!} = 20$$

$$\frac{(n-1)(n-2)(n-3)!}{(n-3)!} = 20$$

$$(n-1)(n-2) = 20$$

$$6-1 \cdot 6-2 = 20$$

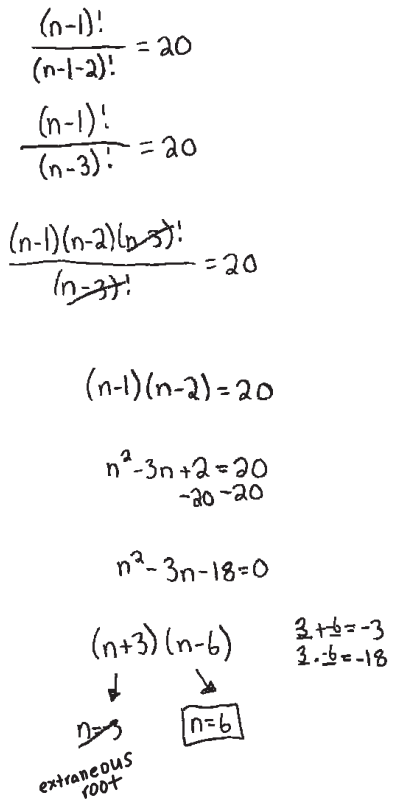
$$5 \cdot 4 = 20$$

2½ out of 3

+ ½ mark for substitution
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for the permissible value of *n*

E4 (missing brackets but still implied in line 5)

Exemplar 3



3 out of 3

award full marks
E2 (changing an equation into an expression in line 7)

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Question 29 T6

Determine the exact value of $\sin\left(\frac{23\pi}{12}\right)$.

Solution

$$\sin\left(\frac{23\pi}{12}\right) = \sin\left(\frac{21\pi}{12} + \frac{2\pi}{12}\right)$$

$$\sin\left(\frac{7\pi}{4} + \frac{\pi}{6}\right) = \sin\left(\frac{7\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{7\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

1 mark for substitution into correct identity

$$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

2 marks for exact values (½ mark for each)

$$= \frac{-\sqrt{6} + \sqrt{2}}{4}$$

3 marks

Note:
Other combinations are possible.

Exemplar 1

$$\sin\left(\frac{23\pi}{12}\right) = \sin\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$$

$$\sin\left(\frac{5\pi}{3} + \frac{\pi}{4}\right) = \left(\sin\frac{5\pi}{3}\right)\left(\cos\frac{\pi}{4}\right) + \left(\cos\frac{5\pi}{3}\right)\left(\sin\frac{\pi}{4}\right)$$

$$\sin\left(\frac{23\pi}{12}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\sin\left(\frac{23\pi}{12}\right) = \left(\frac{\sqrt{6}}{4}\right) + \left(\frac{\sqrt{2}}{4}\right)$$

$$\boxed{\sin\left(\frac{23\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}}$$

2½ out of 3

+ 1 mark for substitution into correct identity
+ 1½ marks for exact values

Exemplar 2

$$\sin\left(\frac{9\pi}{12} + \frac{14\pi}{12}\right)$$

$$\sin\left(\frac{3\pi}{4} + \frac{7\pi}{6}\right)$$

$$\sin\left(\frac{23\pi}{12}\right) = \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{7\pi}{6}\right) + \cos\left(\frac{3\pi}{4}\right) + \sin\left(\frac{7\pi}{6}\right)$$

$$\sin\left(\frac{23\pi}{12}\right) = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) + \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{-1}{2}\right)$$

$$\sin\left(\frac{23\pi}{12}\right) = \frac{-\sqrt{6}}{4} + \frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{23\pi}{12}\right) = \frac{-\sqrt{6}}{4} + \frac{2\sqrt{2}}{4}$$

$$\boxed{\sin\left(\frac{23\pi}{12}\right) = \frac{-\sqrt{6}+2\sqrt{2}}{4}}$$

2½ out of 3

award full marks
– ½ mark for arithmetic error in line 5
E7 (notation error in line 3)

Exemplar 3

$$\sin\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$$

$$\sin\left(\frac{5\pi}{3}\right)\left(\cos\frac{\pi}{4}\right) - \cos\frac{5\pi}{3}\left(\sin\frac{\pi}{4}\right)$$

$$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \quad \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

2 out of 3

+ 2 marks for exact values

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Question 30 R10

Cameron was asked to solve the equation, $\log_3(x-4)=2$.

Cameron's solution:

$$x-4=2^3$$
$$x-4=8$$
$$x=12$$

Describe Cameron's error.

Solution

Cameron mixed up the exponent and the base when changing to exponential form.

1 mark

Exemplar 1

$x - 4 = 3^2$

3 is the base
in the log

½ out of 1

award full marks
– ½ mark for lack of clarity in the description

Exemplar 2

$3^2 = x - 4$

$9 = x - 4$
$+4 \quad +4$

$13 = x$

Cameron should have done 3^2 instead of 2^3. You do base to the argument.

½ out of 1

award full marks
– ½ mark for terminology error in the description

Question 31 T6

Given the identity $\sin^2\theta = \frac{\tan\theta\sin\theta}{\sec\theta}$, state the non-permissible value of θ over the interval $[\pi, 2\pi]$.

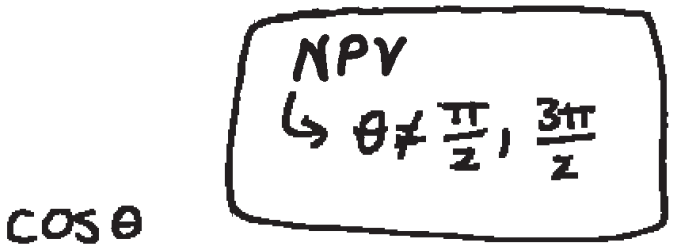
Solution

$\cos\theta = 0$

$\theta = \frac{3\pi}{2}$

1 mark

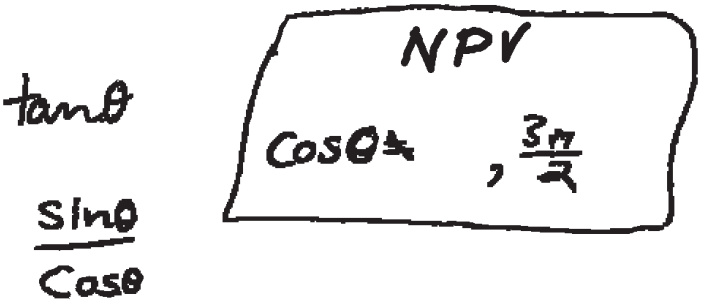
Exemplar 1



1 out of 1

award full marks
E8 (answer outside the given domain)

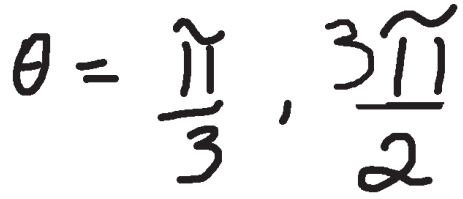
Exemplar 2



0 out of 1

award full marks
− 1 mark for concept error (stating $\cos\theta$ as final answer)

Exemplar 3

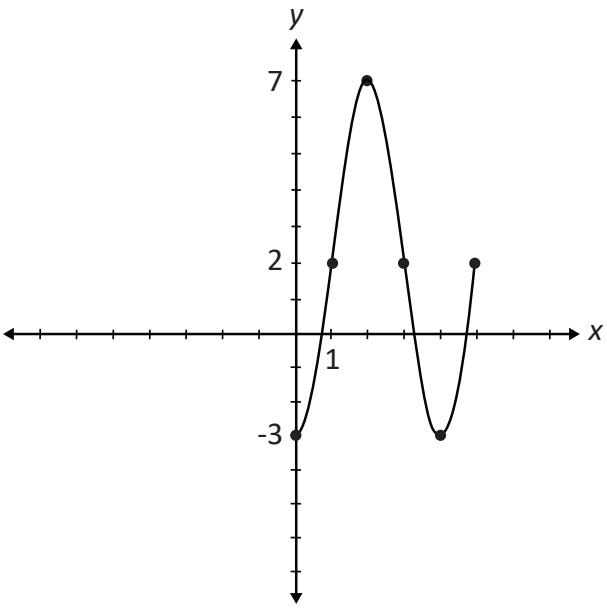


0 out of 1

Question 32 T4

Sketch the graph of $y = -5\cos\left(\frac{\pi}{2}x\right) + 2$ over the domain $[0,5]$.

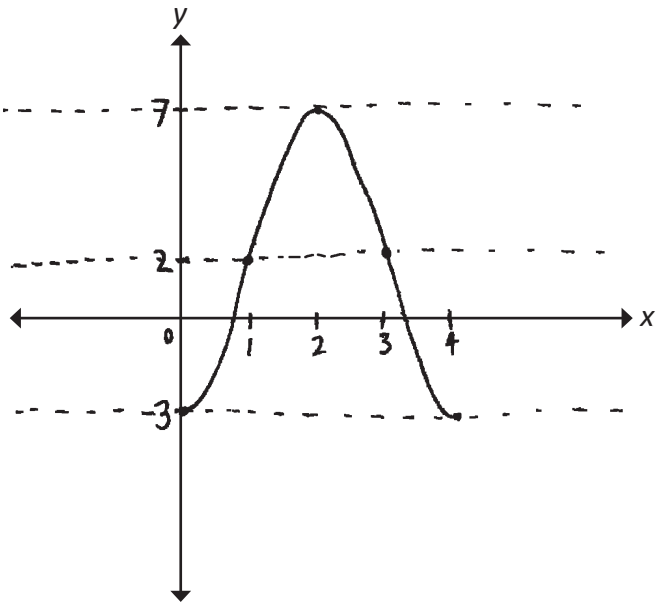
Solution



1 mark for shape of a sinusoidal function with correct amplitude
1 mark for vertical reflection
1 mark for vertical translation
1 mark for period

4 marks

Exemplar 1



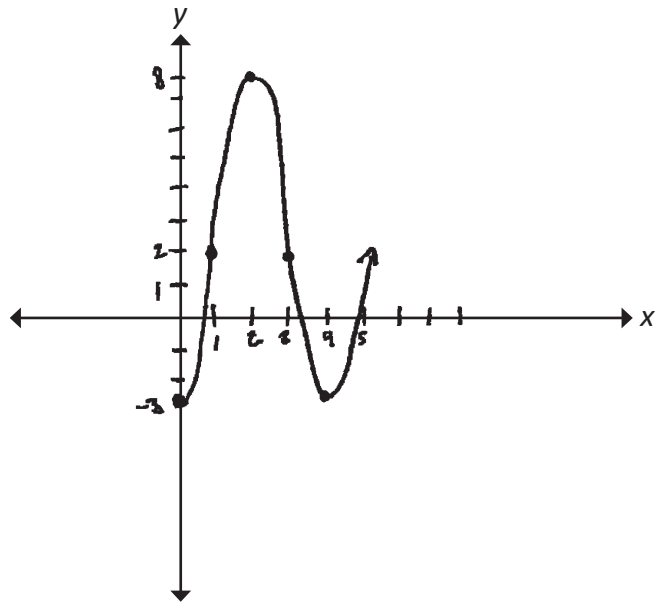
$\frac{2\pi}{\frac{\pi}{2}} = 4$

0	1	2	3	4

3½ out of 4

award full marks
– ½ mark for procedural error (incomplete domain of graph)

Exemplar 2



0,1 -> 0,-5 -> 0,-3

$\frac{\pi}{2}$,0 -> 1,0 -> 0,2

π,-1 -> 2,5 -> 2,8

$\frac{3\pi}{2}$,0

2π,1

a -5

b $\frac{\pi}{2}$

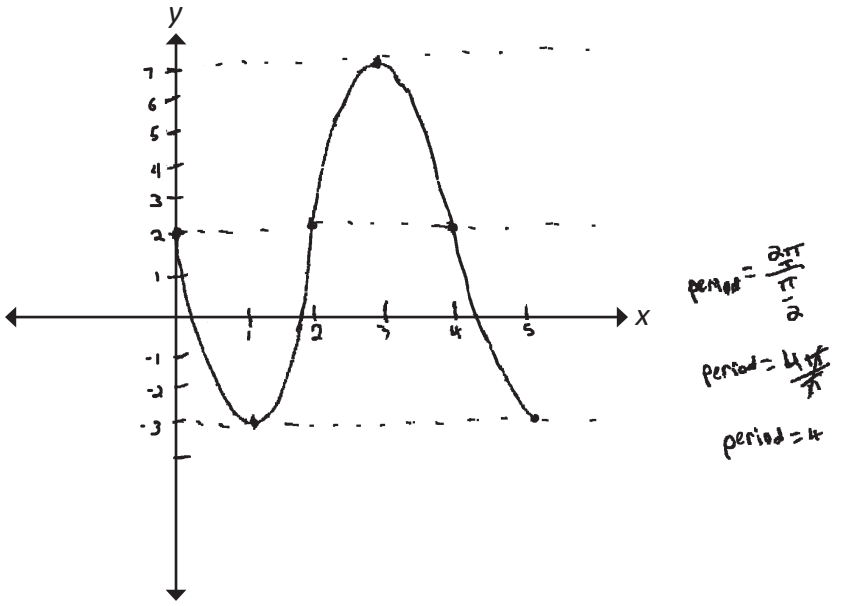
c 0 $2\pi \div \frac{\pi}{2} = 1$

d 2

3½ out of 4

award full marks
– ½ mark for arithmetic error (incorrect calculation of maximum)
E8 (answer outside the given domain)

Exemplar 3



3 out of 4

award full marks

− 1 mark for concept error (sketching $y = \sin x$ instead of $y = \cos x$)

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Question 33 R12

Describe the behaviour of the graph of the polynomial function, $p(x)=(x-3)^2(x-1)$, as it approaches the x-intercept at $x=3$.

Solution

As it approaches $x=3$, the graph touches but does not cross the x-axis.

1 mark

Exemplar 1

It bounces off the x-axis
at $x = 3$

1 out of 1

Exemplar 2

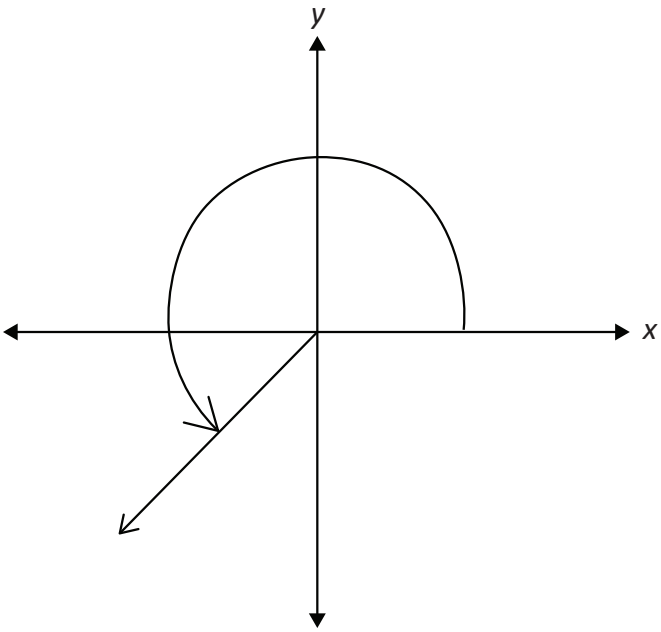
There's a multiplicity of 2 at $x = 3$

0 out of 1

Question 34 T1

Sketch the angle of 4 radians in standard position.

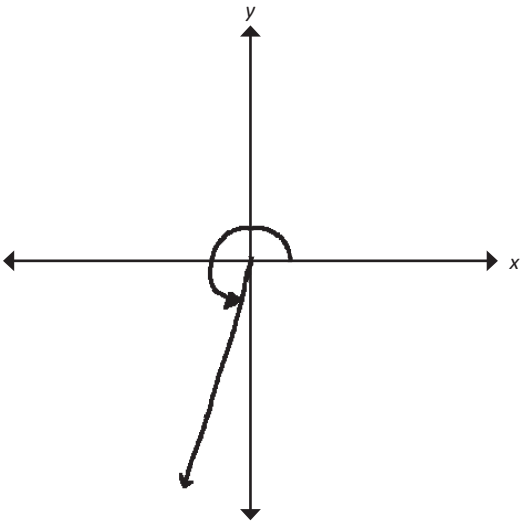
Solution



½ mark for an appropriate angle in quadrant III
½ mark for the correct direction of an angle in standard position

1 mark

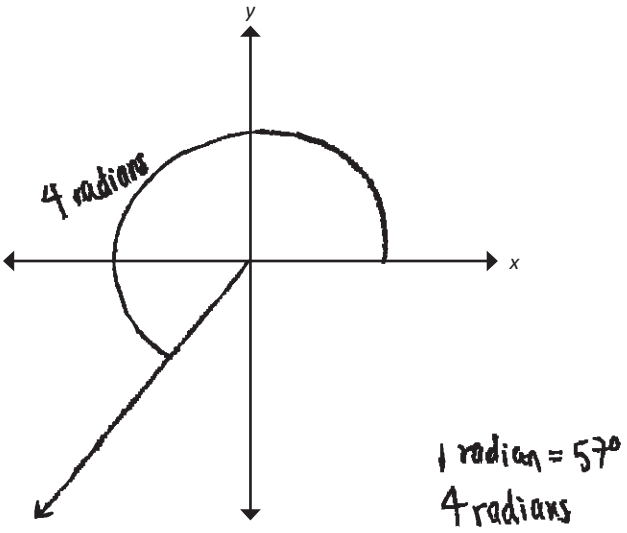
Exemplar 1



½ out of 1

+ ½ mark for the correct direction of an angle in standard position

Exemplar 2



½ out of 1

+ ½ mark for an appropriate angle in quadrant III

Question 35 R10

Solve, algebraically.

$$\log_5 12 + 2\log_5 x = \log_5 48$$

Solution

Method 1

$\log_5 12 + 2\log_5 x = \log_5 48$

$\log_5 12 + \log_5 x^2 = \log_5 48$ 1 mark for power law

$\log_5 12x^2 = \log_5 48$ 1 mark for product law

$12x^2 = 48$ 1 mark for equating arguments

$x^2 = 4$

$x = 2$ ~~$x = -2$~~ ½ mark for permissible value of x
½ mark for showing the rejection of the extraneous root

4 marks

Method 2

$\log_5 12 + 2\log_5 x = \log_5 48$

$\log_5 12 + \log_5 x^2 - \log_5 48 = 0$ 1 mark for power law

$\log_5\left(\frac{12x^2}{48}\right) = 0$ ½ mark for product law
½ mark for quotient law

$\frac{12x^2}{48} = 5^0$ 1 mark for exponential form

$x^2 = 4$

$x = 2$ ~~$x = -2$~~ ½ mark for permissible value of x
½ mark for showing the rejection of the extraneous root

4 marks

Exemplar 1

$\log_5 12 + \log_5 x^2 = \log_5 48$

$12 + x^2 = 48$

$\sqrt{x^2} = \sqrt{36}$

$x = 6$

1½ out of 4

+ 1 mark for power law
+ ½ mark for permissible value of *x*

Exemplar 2

$\log_5 12 + 2\log_5 x = \log_5 48$

$\log_5 (12 \times 2)x = \log_5 48$

$\frac{\log_5 (24)x}{\log_5} = \frac{\log_5 48}{\log_5}$

$\frac{24}{24} x = \frac{48}{24}$

$x = 2$

1½ out of 4

+ 1 mark for product law
+ 1 mark for equating arguments
+ ½ mark for permissible value of *x*
− 1 mark for concept error (dividing by log)

Exemplar 3

$2\log_5 x = \log_5 48 - \log_5 12$

$2\log_5 x = \log_5\left(\frac{48}{12}\right)$

$\log_5 x^2 = \log_5 4$

$x^2 = 4$

$x = 2$

3½ out of 4

+ 1 mark for power law
+ 1 mark for product law
+ 1 mark for equating arguments
+ ½ mark for permissible value of *x*

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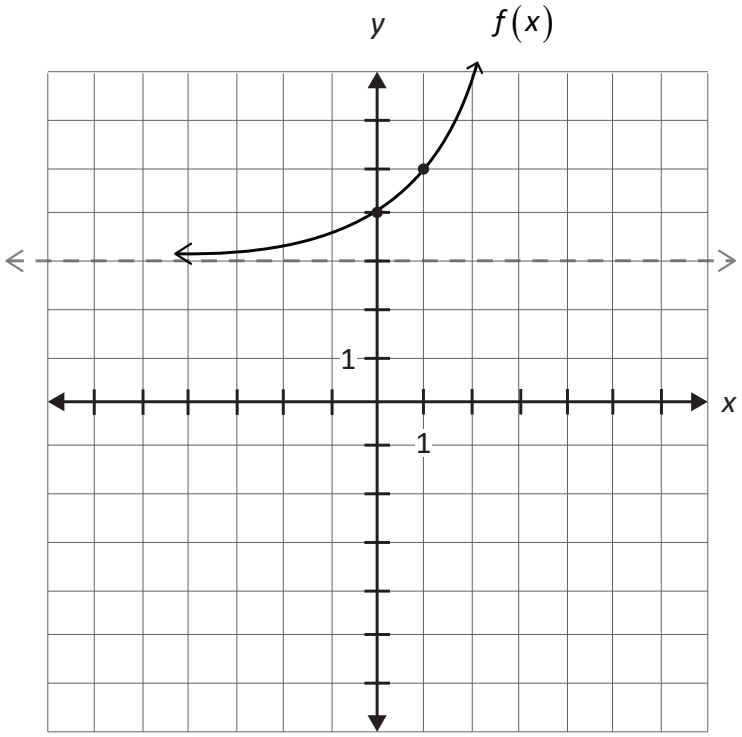
Question 36 R9

State the equation of the exponential function represented by the graph of $f(x)$.



Solution

$f(x)=2^x+3$

1 mark for exponential function with a base of 2
1 mark for vertical translation

2 marks

Exemplar 1

$f(x)=$ $y=2^x+3$

2 out of 2

Exemplar 2

$f(x)=$ $\frac{1}{2}\sqrt{x}+3$

1 out of 2

+ 1 mark for vertical translation

Question 37 T3

Evaluate.

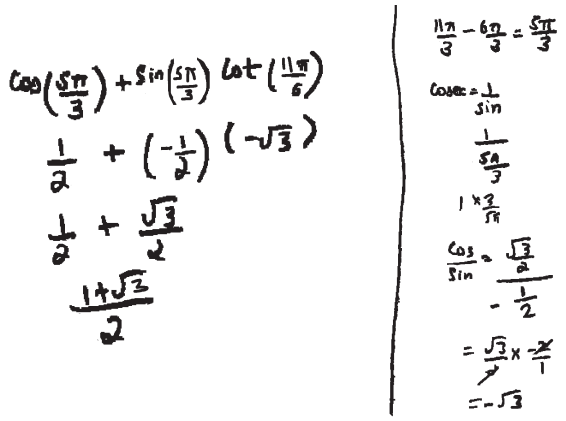
$$\cos\left(\frac{11\pi}{3}\right)+\csc\left(-\frac{\pi}{3}\right)\cot\left(\frac{11\pi}{6}\right)$$

Solution

$\left(\frac{1}{2}\right)+\left(-\frac{2}{\sqrt{3}}\right)\left(-\frac{\sqrt{3}}{1}\right)$	1 mark for $\cos\left(\frac{11\pi}{3}\right)$ (½ mark for value; ½ mark for quadrant)
$\frac{1}{2}+2$	1 mark for $\csc\left(-\frac{\pi}{3}\right)$ (½ mark for value; ½ mark for quadrant)
$\frac{5}{2}$	1 mark for $\cot\left(\frac{11\pi}{6}\right)$ (½ mark for value; ½ mark for quadrant)

3 marks

Exemplar 1



2 out of 3

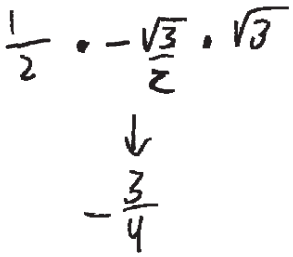
+ 1 mark for $\cos\left(\frac{11\pi}{3}\right)$

+ ½ mark for quadrant of $\csc\left(-\frac{\pi}{3}\right)$

+ 1 mark for $\cot\left(\frac{11\pi}{6}\right)$

− ½ mark for procedural error in line 1

Exemplar 2



2 out of 3

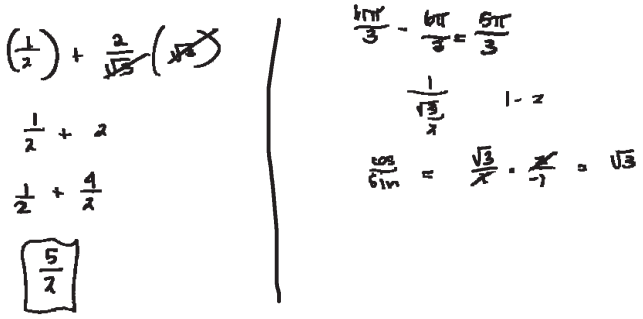
+ 1 mark for $\cos\left(\frac{11\pi}{3}\right)$

+ ½ mark for quadrant of $\csc\left(-\frac{\pi}{3}\right)$

+ ½ mark for the value of $\cot\left(\frac{11\pi}{6}\right)$

E7 (transcription error in line 1)

Exemplar 3



2 out of 3

+ 1 mark for $\cos\left(\frac{11\pi}{3}\right)$

+ ½ mark for the value of $\csc\left(-\frac{\pi}{3}\right)$

+ ½ mark for the value of $\cot\left(\frac{11\pi}{6}\right)$

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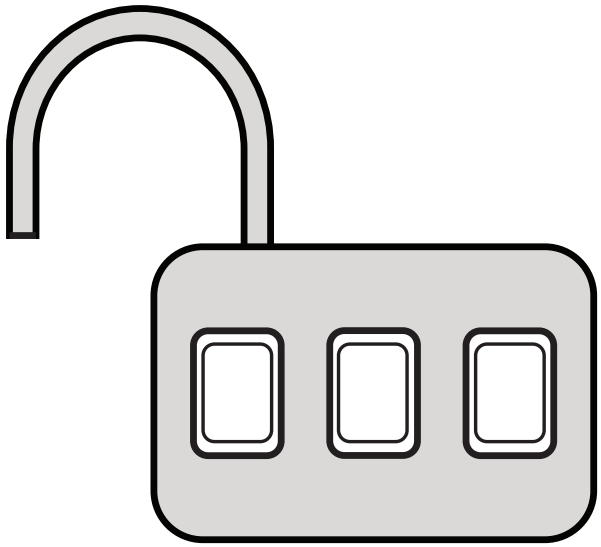
Question 38 P1

Min Li has a 3-digit code for his lock. Determine the number of possible codes for his lock, if repetition is allowed.



Solution

$\underline{10} \bullet \underline{10} \bullet \underline{10} = 1000$

1 mark

Exemplar 1

$\frac{10}{1^{st}\text{ digit}} \cdot \frac{10}{2^{nd}\text{ digit}} \cdot \frac{10}{3^{rd}\text{ digit}} = 30$ possible combinations

½ out of 1

award full marks
– ½ mark for procedural error (addition instead of multiplication)

Exemplar 2

9 · 9 · 9

81 · 9

81
9
729 ways

½ out of 1

award full marks
– ½ mark for procedural error (9 digits instead of 10)

Exemplar 3

10 · 9 · 8

or

case 1: $\frac{9}{10}$ 8 7

case 2: $\frac{8}{7}$ 7 6

case 3: $\frac{7}{8}$ 6 5

case 4: $\frac{6}{7}$ 5 4

case 5: $\frac{5}{6}$ 4 3

case 6: $\frac{4}{5}$ 3 2

0 out of 1

Question 39 R10

Solve, algebraically.

$$\left(\frac{1}{6}\right)^{3x+2} = 6^{2x}$$

Solution

$\left(6^{-1}\right)^{3x+2} = 6^{2x}$ 1 mark for changing to a common base

$6^{-3x-2} = 6^{2x}$

$-3x - 2 = 2x$ 1 mark for equating exponents

$-5x = 2$

$x = -\frac{2}{5}$

2 marks

Exemplar 1

$6^{6(3x+2)} = 6^{2x}$

$18x + 12 = 2x$

$\frac{12}{-16} = \frac{-16x}{-16}$

$-\frac{3}{4} = x$

1 out of 2

+ 1 mark for equating exponents

Exemplar 2

$\left(\frac{1}{6}\right)^{3x+2} \left(\frac{6}{1}\right)^{2x}$

$3x + 2 = 2x$

$-2x \quad -2x$

$x = -2$

0 out of 2

Exemplar 3

$\log(1/6)^{3x+2} = \log(6)^{2x}$

$(3x+2)\log 1/6 = 2x \log 6$

$3x \log 1/6 + 2\log 1/6 = 2x \log 6$

$-2\log 1/6 \quad -2x\log 6$

$3x \log 1/6 - 2x \log 6 = -2\log 1/6$

$x(3\log 1/6 - 2\log 6) = -2\log 1/6$

$\frac{x(\log 1/6^3 - \log 6^2)}{\log 1/6^3 - \log 6^2} = \frac{-2\log 1/6}{\log 1/6^3 - \log 6^2}$

$x = \frac{-2\log 1/6}{\log 1/6^3 - \log 6^2}$

2 out of 2

award full marks
E1 (final answer not stated)
E7 (notation error in lines 3, 6 and 7)

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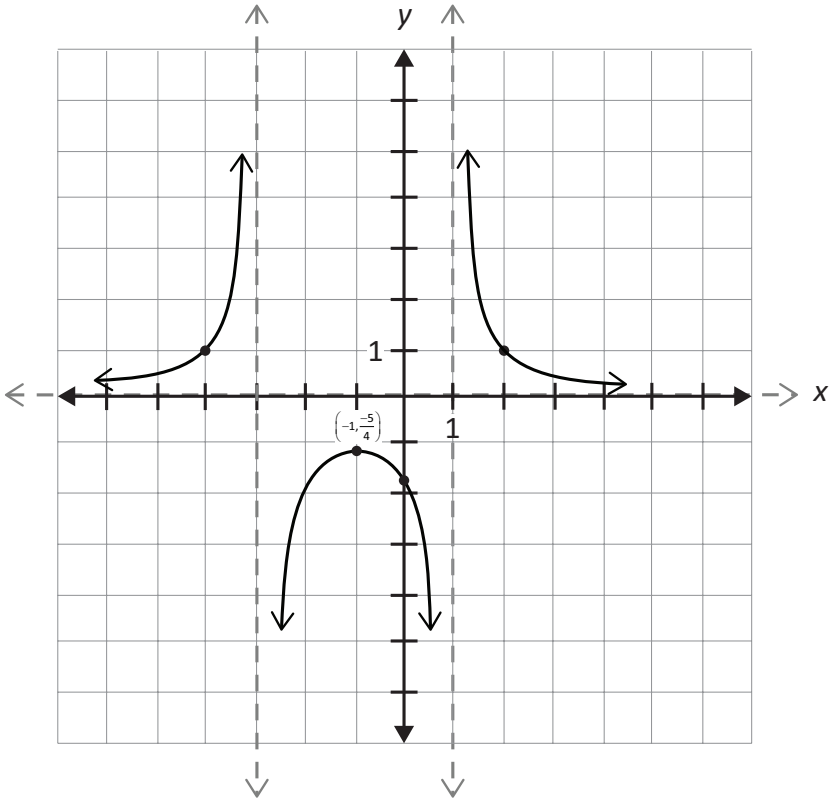
Question 40 R14

Sketch the graph of $f(x)=\dfrac{5}{(x-1)(x+3)}$ and state the *y*-intercept.

Solution

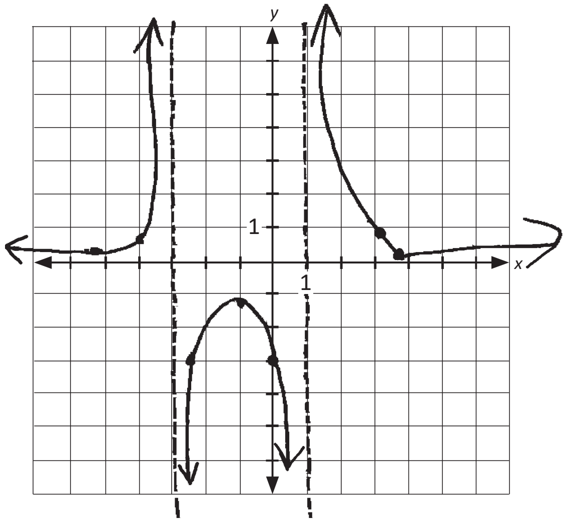


y-intercept: $\left(0,-\dfrac{5}{3}\right)$

1 mark for vertical asymptotic behaviour
(½ mark for behaviour approaching $x = -3$; ½ mark for behaviour approaching $x = 1$)
1 mark for horizontal asymptotic behaviour approaching $y = 0$
1½ marks for shape (½ mark for shape of each branch)
½ mark for *y*-intercept

4 marks

Exemplar 1



$y =$

$x = 2 \quad \frac{5}{5} = 1$

$x = 3 \quad \frac{5}{12} = 2\frac{5}{12}$

$x = -0 \quad = -3$

$x > -1 \quad -2, 2 > \frac{5}{-4}$

$x = -4 \quad \frac{5}{(-5)(-1)} = \frac{5}{8} = 1$

$x = -5 \quad = \frac{5}{(-6)(-2)} \frac{5}{12}$

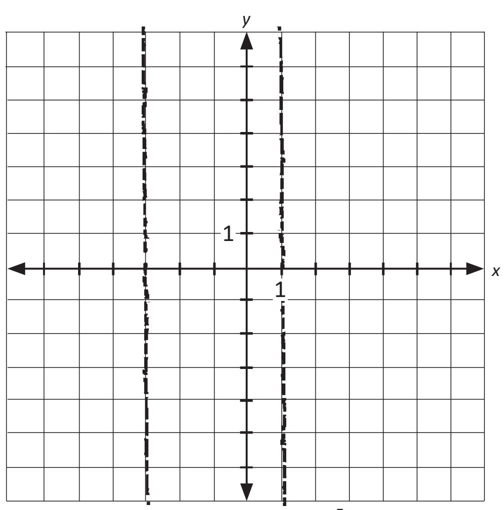
$\frac{5}{(0-1)(0+3)} = \frac{5}{-3}$

y–intercept: $\left(0, \frac{5}{-3}\right)$

2½ out of 4

+ 1 mark for vertical asymptotic behaviour
+ 1 mark for horizontal asymptotic behaviour approaching *y* = 0
+ ½ mark for *y*-intercept
E10 (asymptote omitted but still implied)
E10 (graph crosses or curls away from asymptotes)

Exemplar 2



y-int : $(0-1)(0+3)$

$= \frac{5}{(-1)(3)}$

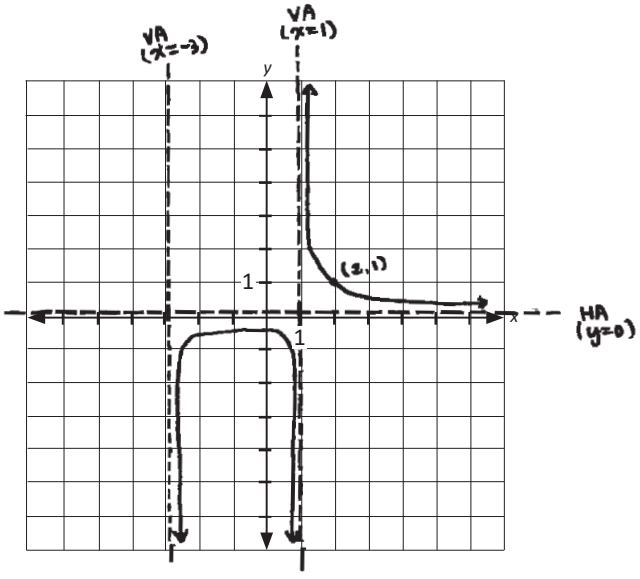
$= \frac{5}{-3}$

y–intercept: $-\frac{5}{3}$

½ out of 4

+ ½ mark for *y*-intercept

Exemplar 3



$y = \frac{5}{(-1-1)(-1+3)}$

$= \frac{5}{-2(2)}$

KP: $y = \frac{5}{(0-1)(0+3)}$

$= \frac{5}{-1(3)}$

$= -\frac{5}{3}$

$y = \frac{5}{(-4-1)(-4+3)}$

5

$y = \frac{5}{(2-1)(2+5)} \to 1(5)$

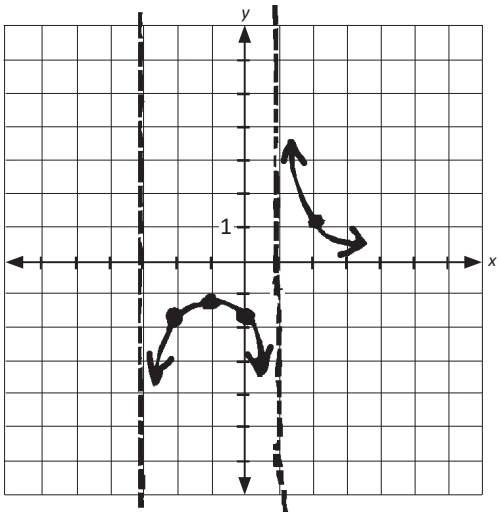
$= \frac{5}{5} = 1$

y–intercept: $(0, -\frac{5}{3})$

2½ out of 4

+ 1 mark for vertical asymptotic behaviour
+ 1 mark for horizontal asymptotic behaviour approaching $y = 0$
+ ½ mark for shape (to the right of $x = 1$)
+ ½ mark for y-intercept
− ½ mark for procedural error (missing branch)

Exemplar 4



y–intercept: $(0, -\frac{5}{3})$

3 out of 4

+ 1 mark for vertical asymptotic behaviour
+ 1 mark for horizontal asymptotic behaviour approaching $y = 0$
+ 1 mark for shape
+ ½ mark for y-intercept
− ½ mark for procedural error (missing branch)
E10 (asymptote omitted but still implied)

Question 41 R7, R10

Given $\log_2 x = 5$, determine the value of $\log_4 (2x)$.

Solution

$x = 2^5$

$x = 32$

1 mark for solving for x

$\log_4 (2(32))$

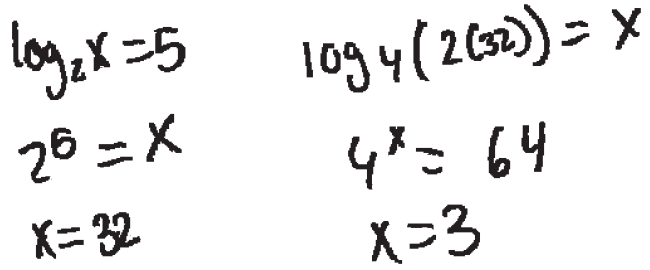
$\log_4 (64)$

3

1 mark for evaluating the logarithm

2 marks

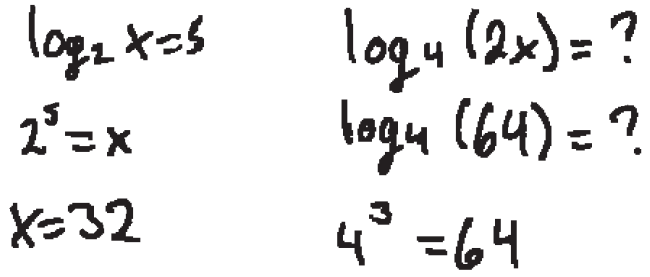
Exemplar 1



2 out of 2

award full marks
E7 (notation error in line 4)

Exemplar 2



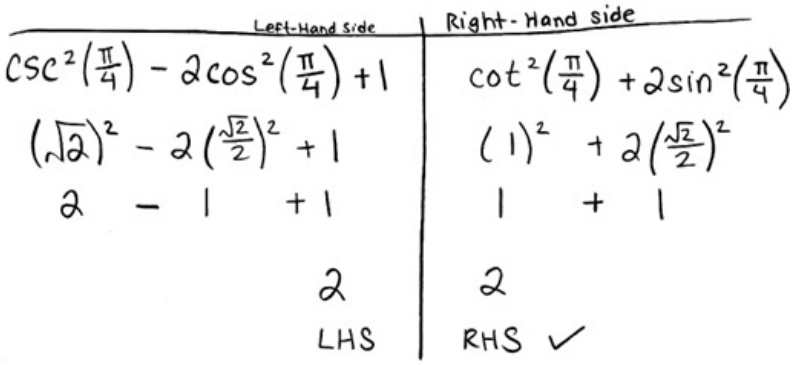
2 out of 2

award full marks
E1 (final answer not stated)

Question 42 T6

Henry was asked to prove the identity, $\csc^2\theta - 2\cos^2\theta + 1 = \cot^2\theta + 2\sin^2\theta$, for all permissible values of θ.

His work:



Explain why his proof is insufficient.

Solution

Henry proved the identity for $\theta = \frac{\pi}{4}$, but not for all permissible values of θ.

1 mark

Exemplar 1

Henry must prove it for all values of θ, not just $\frac{\pi}{4}$

1 out of 1

Exemplar 2

because he plugged in a value for θ

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3

May not work if other angles are substituted for θ.

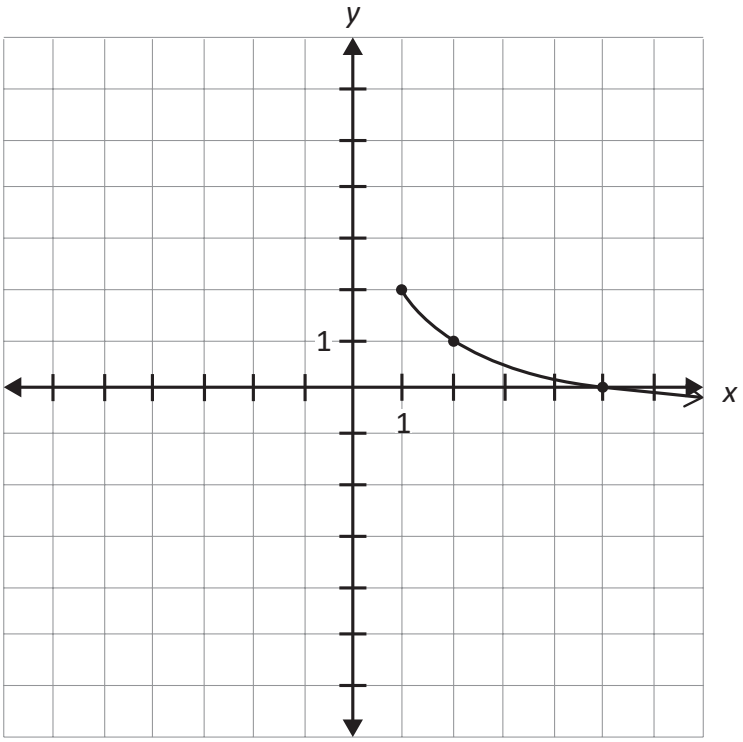
½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 43 R13

Sketch the graph of $y-2=-\sqrt{x-1}$.

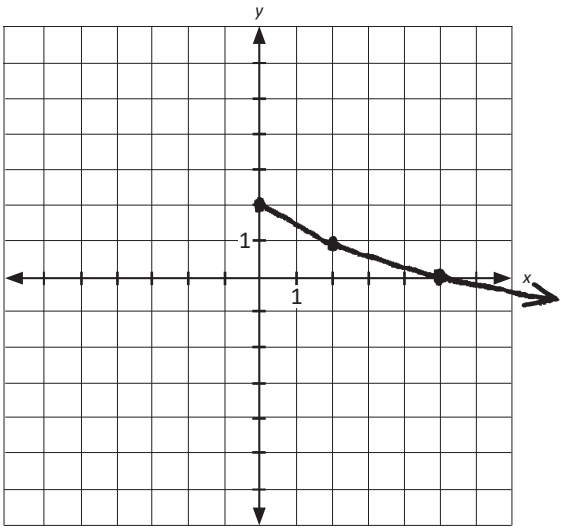
Solution



1 mark for shape of a radical function
1 mark for vertical reflection
1 mark for horizontal translation
1 mark for vertical translation

4 marks

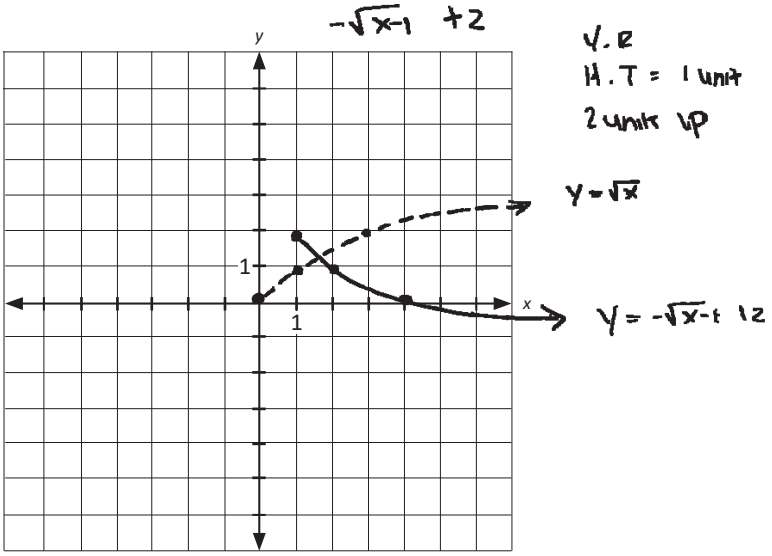
Exemplar 1



3½ out of 4

award full marks
– ½ mark for procedural error (one incorrect point)

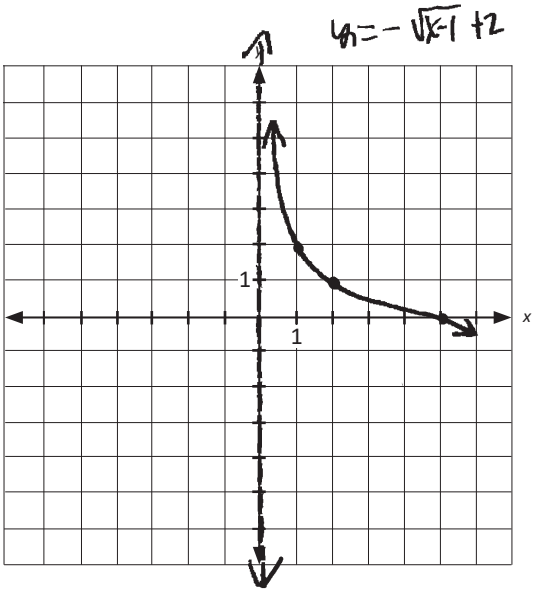
Exemplar 2



3½ out of 4

award full marks
– ½ mark for procedural error (one incorrect point)

Exemplar 3



$y=-\sqrt{x-1}+2$

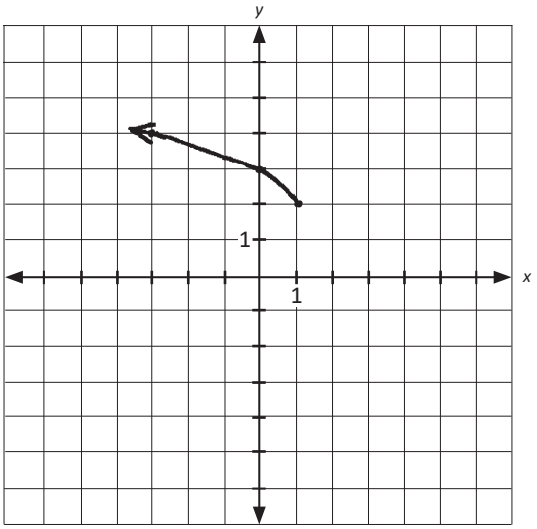
$y=-1 \quad x+1$
$+2$

$(0,0) \rightarrow (1,2)$
$(1,1) \rightarrow (2,1)$
$(4,2) \rightarrow (5,0)$

3 out of 4

+ 1 mark for vertical reflection
+ 1 mark for horizontal translation
+ 1 mark for vertical translation

Exemplar 4



3 out of 4

+ 1 mark for shape of a radical function
+ 1 mark for horizontal translation
+ 1 mark for vertical translation

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Question 44 R7, R8

Evaluate.

$$\log_3 54 - \log_3 6$$

Solution

$\log_3\left(\frac{54}{6}\right)$ 1 mark for quotient law

$\log_3 9$

2 1 mark for evaluating the logarithm

2 marks

Exemplar 1

$\log_3 \left(\frac{54}{6}\right)$

$\log_3 8$

1 out of 2

+ 1 mark for quotient law

Exemplar 2

$\log_3 \left(\frac{54}{6}\right)$

$\log_3 9 = x$

$3^x = 9$

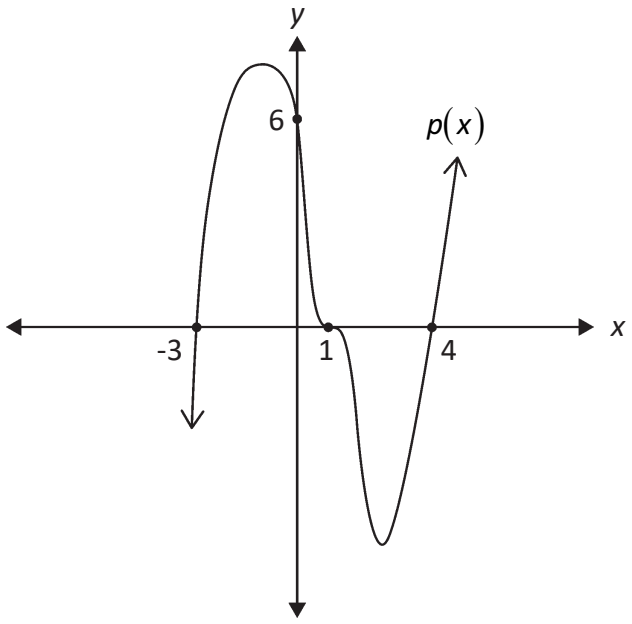
$x = 3$

1½ out of 2

award full marks
− ½ mark for arithmetic error in line 4

Question 45 R12

Determine, algebraically, the equation of the graph of the polynomial function, $p(x)$.



Solution

$p(x) = a(x-1)^3(x+3)(x-4)$

½ mark for factors of $p(x)$
½ mark for multiplicity of 3 at $x = 1$

$p(0) = a(0-1)^3(0+3)(0-4)$

$6 = a(-1)^3(3)(-4)$

$6 = 12a$

$\frac{1}{2} = a$

1 mark for consistent value of a

$p(x) = \frac{1}{2}(x-1)^3(x+3)(x-4)$

2 marks

Exemplar 1

$p(x)=$ $(x+3)(x-1)(x-4)$

½ out of 2

+ ½ mark for factors of $p(x)$

Exemplar 2

$p(x)=$ $\frac{1}{3}(x+3)(x-1)^3(x-4)+6$

1 out of 2

award full marks
− 1 mark for concept error (including a vertical shift)

Exemplar 3

$p(x)=$ $-(x-1)^3(x-4)(x+3)$

1 out of 2

+ ½ mark for factors of $p(x)$
+ ½ mark for multiplicity of 3 at $x = 1$

Exemplar 4

$p(x) = (x-1)^3(x+3)(x-4)$

$p(x) = a(x-1)^3(x+3)(x-4)$

$6 = a(-1)^3(3)(-4)$

$6 = a(12)$

$a = 2$

$p(x) = 2(x-1)^3(x+3)(x-4)$

1½ out of 2

award full marks
− ½ mark for arithmetic error in line 5
E7 (notation error in line 1)

Appendices

Appendix A

Marking Guidelines

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply:

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- error in or incomplete justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet.*

E1 **final answer**	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in step leading to final answer
E2 **equation/expression**	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 **variables**	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 **brackets**	▪ "sin x^2" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 **units**	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 **rounding**	▪ rounding error ▪ rounding too early
E7 **notation/transcription**	▪ notation error ▪ transcription error
E8 **domain/range**	▪ answer outside the given domain ▪ domain or range written in incorrect order
E9 **graphing**	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated or incorrectly spaced ▪ coordinate points labelled incorrectly
E10 **asymptotes**	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

Irregularities in Provincial Tests

A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report.*

Except in the case of cheating or plagiarism where the result is a provinical test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares and *Irregular Test Booklet Report* documenting the situation, the people contacted and the follow-up. The orginial copy of this report is to be returned by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: _____

Date marked: _____

Booklet No.: _____

_____

Problem(s) noted: _____

_____

_____

_____

_____

_____

Question(s) affected: _____

_____

_____

Action taken or rationale for assigning marks: _____

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Follow-up: _____

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Decision: _____

_____

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_____

Marker's Signature: _____

Principal's Signature: _____

For Department Use Only—After Marking Complete

Consultant: _____

Date: _____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
6	R3	1
9	R2, R5	3
11a)	R1	1
11b)	R1	1
13a)	R1	1
13b)	R1	1
14a)	R5	1
14b)	R6	1
15	R4, R5	1
16	R1	1
19	R1	1
20	R3	1
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
8	T2	1
18	T2	1
27	T4	2
32	T4	4
34	T1	1
37	T3	3
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
2	P4	4
4	P3	3
24	P3	1
28	P2	3
38	P1	1

Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
10	R11	1
17	R11	3
21	R12	1
25	R11	1
33	R12	1
45	R12	2
Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
5	T5	4
12	T6	3
22	T5	1
29	T6	3
31	T6	1
42	T6	1
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
3a)	R10	1
3b)	R10	2
30	R10	1
35	R10	4
36	R9	2
39	R10	2
41	R7, R10	2
44	R7, R8	2
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
7	R14	1
23	R13	1
26	R13	2
40	R14	4
43	R13	4