Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2020

Manitoba

Grade 12 pre-calculus mathematics achievement test.
Marking guide. January 2020

This resource is available in print and electronic formats.

ISBN: 978-0-7711-6388-3 (print)

ISBN: 978-0-7711-6390-6 (pdf)

Manitoba Education
Winnipeg, Manitoba, Canada

After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre. Order online at www.manitobalrc.ca.

This resource will also be available on the Manitoba Education website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Table of Contents

General Marking Instructions 1

Scoring Guidelines for Booklet 1 Questions..5
Scoring Guidelines for Booklet 2 Questions...................................... 41
Answer Key for Selected Response Questions..................................... 42

Appendices 113
Appendix A: Marking Guidelines 115
Appendix B: Irregularities in Provincial Tests.....................................116
Irregular Test Booklet Report...117
Appendix C: Table of Questions by Unit and Learning Outcome 119

General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student’s final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section “Booklet 2 Questions.”

To receive full marks, a student’s response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students’ responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with “0” only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Youyi Sun
Assessment Consultant
Grade 12 Pre-Calculus Mathematics
Telephone: 204-945-7590
Toll-Free: 1-800-282-8069, ext. 7590
Email: youyi.sun@gov.mb.ca

Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION				
Shade in the circles below for a maximum total deduction of 5 marks (½ mark deduction per error).				
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).				
E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

Example: Marks assigned to the student

	Booklet 1	Selected Response	Booklet 2	Communication Erros (Deduct)	Total
Marks Awarded	25	7	40	1½	70½
Total Marks	**36**	**9**	**45**	**Maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

Determine and simplify the 8th term in the binomial expansion of $\left(x-\frac{2}{x^3}\right)^{10}$.

Solution

$$t_8 = {}_{10}C_7 (x)^3 \left(-\frac{2}{x^3}\right)^7$$ 2 marks (1 mark for ${}_{10}C_7$; ½ mark for each consistent factor)

$$= 120\left(x^3\right)\left(-\frac{128}{x^{21}}\right)$$

$$= -\frac{15\ 360}{x^{18}}$$ 1 mark for simplification (½ mark for coefficient; ½ mark for exponent)

3 marks

Exemplar 1

$t_8 = {}_{10}C_7 (x)^3 \left(-\frac{2}{x^3}\right)^7$

$= (120)(x^3)\left(-\frac{128}{x^{21}}\right)$

$= \frac{-15\,360}{x^{-18}}$

2 ½ out of 3

+ 2 marks (1 mark for ${}_{10}C_7$; ½ mark for each consistent factor)

+ ½ mark for simplification of coefficient

Exemplar 2

$t_8 = {}_9C_7 (x)^2 \left(-\frac{2}{x^3}\right)^7$ $\qquad t_{k+1} = {}_nC_k a^{n-k} b^k$

$t_8 = (36)(x^2)\left(\frac{-128}{x^{21}}\right)$

$t_8 = \frac{-4608\,\cancel{x^2}}{\cancel{x^{21}}\ 19}$

$t_8 = \frac{-4608}{x^{19}}$

2 out of 3

+ 1 mark for consistent factors
+ 1 mark for simplification

Question 2 R10

The temperature of hot chocolate is recorded as it cools down. The data shows that the temperature cools according to the equation:

$$T = 82(0.87)^t + 16$$

where T is the temperature of the hot chocolate, in degrees Celsius after t minutes and

t is the time, in minutes, after the hot chocolate is made.

In order to avoid burns, hot chocolate should not be served at a temperature higher than 71°C. Determine, algebraically, the amount of time it will take the hot chocolate to reach this temperature.

Solution

$$71 = 82(0.87)^t + 16$$

$$55 = 82(0.87)^t$$

$$\frac{55}{82} = 0.87^t$$

$$\log\left(\frac{55}{82}\right) = t\log(0.87)$$

½ mark for applying logarithms
1 mark for power law

$$t = \frac{\log\left(\frac{55}{82}\right)}{\log(0.87)}$$

$$t = 2.867\ 874$$

$$t = 2.868 \text{ minutes}$$

½ mark for evaluating quotient of logarithms

2 marks

Exemplar 1

$T = 82(0.87)^t + 16$

$71 = 82(0.87)^t + 16$

$-16 \qquad\qquad -16$

$55 = 82(0.87)^t$

$\log 55 = \log 82(0.87)^t$

$\log 55 = t \log 82(0.87)$

$\dfrac{\log 55}{\log 71.34} = \dfrac{t \log 71.34}{\log 71.34}$

$\boxed{.939 \text{ minutes} = t}$

1 out of 2

+ ½ mark for applying logarithms
+ ½ mark for evaluating quotient of logarithms

Exemplar 2

$71 = 82(0.87)^t + 16$

$55 = 82(0.87)^t$

$0.67 = 0.87^t$

$\dfrac{\log 0.67}{\log 0.87} = \dfrac{t \log 0.87}{\log 0.87}$

$t = 2.8757$

$t = 3 \text{ min}$

2 out of 2

award full marks
E6 (rounding too early)

Question 3 T5

Solve for θ, algebraically, over the interval $[0, 2\pi]$.

$$3\sin^2\theta + 6\sin\theta + 2 = 0$$

Solution

$$\sin\theta = \frac{-6 \pm \sqrt{6^2 - 4(3)(2)}}{2(3)}$$

½ mark for substitution into quadratic formula

$$\sin\theta = \frac{-6 \pm \sqrt{12}}{6}$$

½ mark for solving for $\sin\theta$

$\sin\theta = -0.422\ 649...$ $\quad$ $\sin\theta = -1.577\ 350...$

$\theta_r = 0.436\ 367...$ $\quad$ no solution

$\theta = 3.578, 5.847$

2 marks for solving for θ
(½ mark for each value; 1 mark for indicating no solution)

3 marks

Exemplar 1

$(\sin\theta+2)(3\sin\theta+1)=0$

$\sin\theta=-2$ | $\sin\theta=-\frac{1}{3}$

$\therefore$ No solution | $\theta_R=0.339836909$

$\theta=3.481, 5.943$

$\{\theta=3.481, 5.943\}$

2½ out of 3

+ ½ mark for solving for $\sin\theta$
+ 1 mark for indicating no solution
+ 1 mark (½ mark for each consistent value)

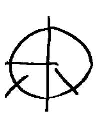
Exemplar 2

$\sin\theta=\frac{-6\pm\sqrt{6^2-4(3)(2)}}{2(3)}$

$\sin\theta=\frac{-6\pm\sqrt{36-24}}{6}$

$\sin\theta=\frac{-6\pm\sqrt{12}}{6}$

$\sin\theta=\frac{-6+\sqrt{12}}{6}$ $\quad$ $\sin\theta=\frac{-6-\sqrt{12}}{6}$



$\sin\theta=-0.42264973$ $\quad$ ~~$\sin\theta=-1.577350269$~~

$\theta_r=0.436367035$

$\theta=\pi+0.436367035$

$\boxed{\theta_1=2.705}$

$\theta=2\pi-0.436367035$

$\boxed{\theta_2=5.847}$

2½ out of 3

+ ½ mark for substitution into quadratic formula
+ ½ mark for solving for $\sin\theta$
+ ½ mark for consistent value of θ
+ 1 mark for indicating no solution

Question 4 P3

A committee of 4 people is to be selected from 6 high school students and 5 middle years students.

Determine the number of possible committees if the committee must have at least 3 high school students.

Solution

Case 1: ${}_6C_3 \bullet {}_5C_1 = 100$	1 mark for Case 1
Case 2: ${}_6C_4 \bullet {}_5C_0 = 15$	1 mark for Case 2
$100 + 15 = 115$ possible committees	1 mark for addition of cases

3 marks

Note:

- ${}_5C_0$ does not need to be shown.

Exemplar 1

case ①: $_6C_3 + {_5C_1} = 25$

case ②: $_6C_4 = 15$

$25 + 15 = 40$

40 possibilities?

2 out of 3

award full marks
– 1 mark for concept error (addition within Case 1)

Exemplar 2

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3} = 360$

+

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{5} = 600$

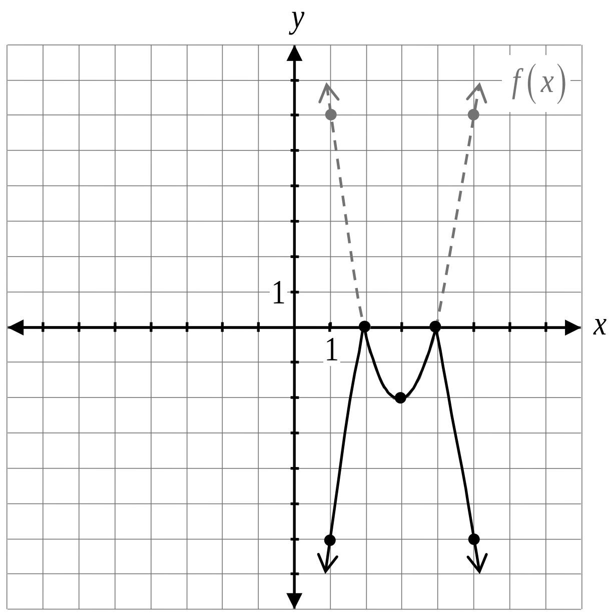
there are 960 possible committees

2 out of 3

award full marks
– 1 mark for concept error (using permutations)

Given the graph of $y = f(x)$, sketch the graph of $y = -|f(x)|$.

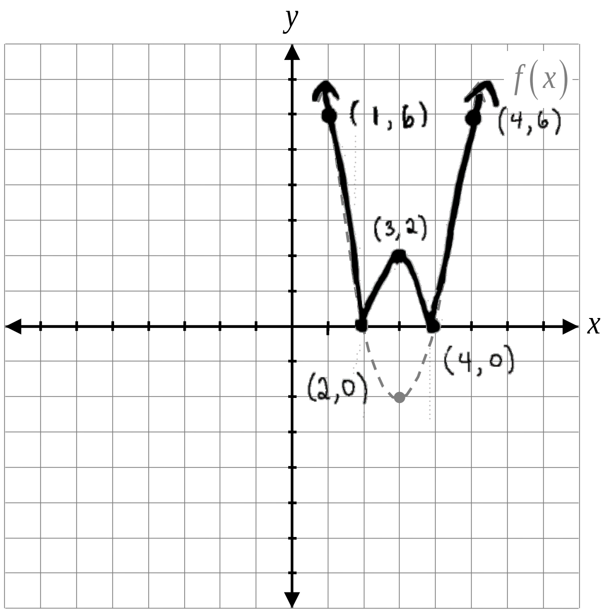
Solution



1 mark for absolute value
1 mark for vertical reflection

2 marks

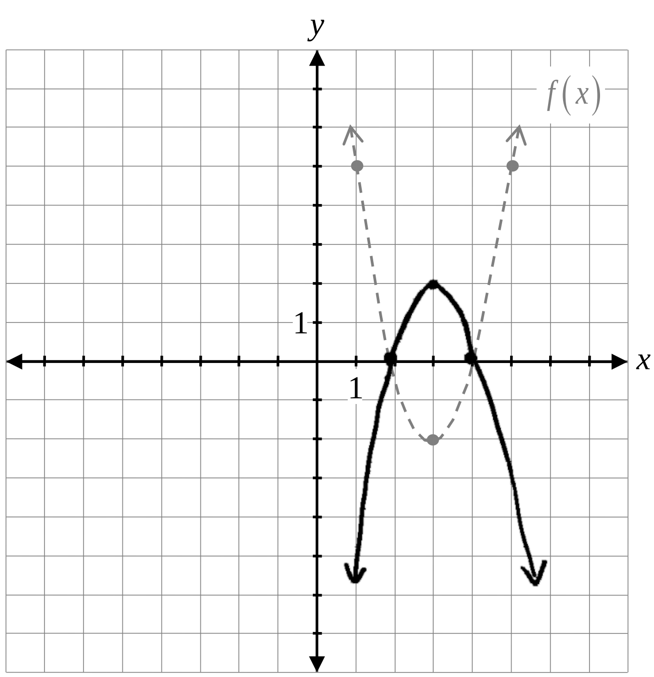
Exemplar 1



1 out of 2

+ 1 mark for absolute value
E9 (coordinate points labelled incorrectly)

Exemplar 2



1 out of 2

+ 1 mark for vertical reflection

Question 6 R2, R3

Describe the transformations used to obtain the graph of the function $y = 4f\left(\frac{x}{2}\right) - 3$ from the graph of $y = f(x)$.

Solution

The graph of $y = f(x)$ is vertically stretched by a factor of 4, horizontally stretched by a factor of 2, and translated down 3 units.

1 mark for vertical stretch
1 mark for horizontal stretch
1 mark for vertical translation

3 marks

Note:

- Deduct a maximum of 1 mark for concept error of incorrect order of vertical transformations.

Exemplar 1

- Vertical stretch by a factor of 4
- Horizonal compression by a factor of $\frac{1}{2}$
- shift down 3 units

3 out of 3

Exemplar 2

$y - k = aX(b(x-h))$

$k = -3$

$a = 4$

$b = \frac{1}{2}$

- graph will be moved down 3 units when $k = -3$.
- graph will be vertically stretched by a factor of 4
- graph will be horizontally compressed by a factor of 2.

1 out of 3

+ 1 mark for vertical stretch
+ 1 mark for vertical translation
– 1 mark for concept error of incorrect order of vertical transformations

Exemplar 3

You multiply all x-values by 2, and the y-values by 4. Then move all points 3 down.

3 out of 3

If $\log 4 = m$ and $\log 3 = n$, express $\log 48$ in terms of m and n.

Solution

$$\log 48 = \log\left(4^2 \bullet 3\right)$$
$$= 2\log 4 + \log 3$$
$$= 2m + n$$

1 mark for power law
1 mark for product law

2 marks

Exemplar 1

$\log 48 = \log 4^2 + \log 3$

$\log 48 = \log (16)(3)$

$\log 48 = \log 48$

$\therefore \boxed{m^2 + n.}$

1 out of 2

+ 1 mark for product law

Exemplar 2

$\log (48) = \log (4) + \log (3) + \log (4)$

2 out of 2

award full marks
E1 (final answer not stated)

Exemplar 3

$\log 48 = 2 \log 4 + \log 3$

$\log 48 = 2 \log m + \log n$

1½ out of 2

award full marks
– ½ mark for procedural error (incorrect substitution)

State the domain and range of the radical function, $f(x) = -3\sqrt{x-8} + 1$.

Solution

Domain: $[8, \infty)$ 1 mark for domain

Range: $(-\infty, 1]$ 1 mark for range

2 marks

Exemplar 1

Domain: $8 \leq x \leq \infty$

Range: $1 \geq y \geq -\infty$

2 out of 2

award full marks
E7 (notation error)
E8 (bracket error made when stating domain and range; range written in incorrect order)

Exemplar 2

Domain: $x \geq 8$

Range: $x \leq 1$

1 out of 2

+ 1 mark for domain

Question 9 T4

Explain why the graph of $y = \tan x$ does not have an amplitude.

Solution

The range of $y = \tan x$ is all real numbers. **1 mark**

or

As the graph of $y = \tan x$ approaches the vertical asymptotes, the function value approaches positive and negative infinity.

Exemplar 1

Since the graph of tan doesn't have the amplitude because the way it's graphed it doesn't have the crest and trough to measure the amount of distance from starting therefore tanx won't have amp. None of the tan graphs have amplitude.

1 out of 1

Exemplar 2

y = tanx doesn't have an amplitude because tan graphs aren't sinosodial and therefore have no definitive amplitude.

1 out of 1

Exemplar 3

- Because y = tanx 's range is infinite ((-∞, ∞))

- basically it has an infinite amplitude already.

½ out of 1

award full marks
– ½ mark for terminology error in explanation

Prove the following identity for all permissible values of θ.

$$\frac{\sec\theta - \sin^2\theta\sec\theta}{\tan\theta\sin\theta} = \csc^2\theta - 1$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\dfrac{\dfrac{1}{\cos\theta} - \dfrac{\sin^2\theta}{\cos\theta}}{\left(\dfrac{\sin\theta}{\cos\theta}\right)(\sin\theta)}$	$\csc^2\theta - 1$
$\dfrac{\dfrac{1-\sin^2\theta}{\cancel{\cos\theta}}}{\dfrac{\sin^2\theta}{\cancel{\cos\theta}}}$	
$\dfrac{\cos^2\theta}{\sin^2\theta}$	
$\cot^2\theta$	
$\csc^2\theta - 1$	

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Method 2

Left-Hand Side	Right-Hand Side
$\dfrac{\sec\theta\left(1-\sin^2\theta\right)}{\left(\dfrac{\sin\theta}{\cos\theta}\right)(\sin\theta)}$	$\csc^2\theta-1$
$\dfrac{\left(\dfrac{1}{\cos\theta}\right)\left(\cos^2\theta\right)}{\dfrac{\sin^2\theta}{\cos\theta}}$	
$\dfrac{\dfrac{\cos^2\theta}{\cancel{\cos\theta}}}{\dfrac{\sin^2\theta}{\cancel{\cos\theta}}}$	
$\cot^2\theta$	
$\csc^2\theta-1$	

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$= \dfrac{\frac{1}{\cos\theta} - \sin^2\theta\left(\frac{1}{\cos\theta}\right)}{\frac{\sin\theta}{\cos\theta}\left(\sin\theta\right)}$	$= \dfrac{1}{\sin^2\theta} - 1$
$= \dfrac{\frac{1-\sin^2\theta}{\cos\theta}}{\frac{\sin^2\theta}{\cos\theta}}$	
$= \dfrac{\frac{\cos^2\theta}{\cos\theta}}{\frac{1-\cos^2\theta}{\cos\theta}}$	
$= \cos\theta\left(\dfrac{\cos\theta}{1-\cos^2\theta}\right)$	

$$= \frac{\cos^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

2 out of 3

+ 1 mark for correct substitution of appropriate identities
+ 1 mark for algebraic strategies

Exemplar 2

Left-Hand Side	Right-Hand Side
$\dfrac{\sec\theta - \sin^2\theta\sec\theta}{\tan\theta\sin\theta}$	$\csc^2\theta - 1$
$= \dfrac{\left(\frac{1}{\cos\theta}\right) - \sin^2\theta\left(\frac{1}{\cos\theta}\right)}{\left(\frac{\sin\theta}{\cos\theta}\right)\sin\theta}$	$= \cot^2\theta$
$= \dfrac{\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}}{\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\sin\theta\cos\theta}{\cos\theta}\right)}$	$= \left(\dfrac{\cos^2\theta}{\sin^2\theta}\right)$
$= \dfrac{\cos^2\theta}{\cos\theta}\left(\dfrac{\cos\theta}{\sin^2\theta\cos\theta}\right)$	
$= \dfrac{\cos^3\theta}{\sin^2\theta\cos^2\theta}$	
$= \dfrac{\cos^2\theta}{\sin^2\theta}$	LHS = RHS

2 out of 3

+ 1 mark for correct substitution of appropriate identities
+ 1 mark for logical process to prove the identity

Exemplar 3

Left-Hand Side	Right-Hand Side
$\frac{\sec(1-\sin^2\theta)}{\tan\theta \sin}$	$\cot^2\theta$
$\frac{\sec(1-\sin^2\theta)}{\frac{\sin}{\cos} \quad \frac{\sin}{1}}$	
$\frac{\frac{1}{\cos}(1-\sin^2\theta)}{\frac{\sin^2}{\cos}}$	

$$\frac{1-\sin^2\theta}{\sin^2}$$

$$\frac{\cos^2\theta}{\sin^2\theta} \qquad LHS = RHS$$

$$\cot^2\theta$$

3 out of 3

award full marks
E3 (variable omitted in an identity)

This page was intentionally left blank.

Question 11 R11

Given that $x = 3$ is one of the zeros of $p(x) = x^3 - 7x - 6$, express $p(x)$ in completely factored form.

Solution

3	1	0	–7	–6
		3	9	6
	1	3	2	0

1 mark for synthetic division (or equivalent strategy)

$p(x) = (x-3)\left(x^2 + 3x + 2\right)$

½ mark for factor of $(x-3)$

$p(x) = \underline{(x-3)(x+2)(x+1)}$

½ mark for other consistent factors

2 marks

Exemplar 1

-3	1	0	-7	-6
+		-3	+9	-6
x	1	-3	2	0

$= (x-3)(x^2-3x+2)$

$= (x-3)(x-2)(x-1)$

$p(x) =$ $(x-3)(x-2)(x-1)$

1 out of 2

+ ½ mark for factor of $(x-3)$

+ ½ mark for other consistent factors

Exemplar 2

-3	1	0	-7	-6
-	↓	-3	-9	-6
	1	3	2	0

$p(x) =$ $(x^2+3x+2)(x-3)$

1½ out of 2

+ 1 mark for synthetic division

+ ½ mark for factor of $(x-3)$

Exemplar 3

3	1	-7	-6
		3	-12
	1	-4	-18

$p(x) =$ $(x-3)(x^2-4x-18)$

1 out of 2

+ 1 mark for synthetic division

+ ½ mark for factor of $(x-3)$

– ½ mark for procedural error in line 1 (missing coefficient of zero)

Solve, algebraically.

$${}_{n+1}P_2 = 6$$

Solution

$$\frac{(n+1)!}{(n+1-2)!} = 6$$
½ mark for substitution

$$\frac{(n+1)!}{(n-1)!} = 6$$

$$\frac{(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}} = 6$$
1 mark for factorial expansion
½ mark for simplification of factorials

$$n^2 + n = 6$$

$$n^2 + n - 6 = 0$$

$$(n+3)(n-2) = 0$$

$$\xcancel{n = -3} \qquad n = 2$$
½ mark for the permissible value of *n*
½ mark for showing the rejection of the extraneous root

3 marks

Exemplar 1

$$\frac{n+1!}{((n+1)-2)!} = 6$$

$$\frac{n+1!}{n-1!} = 6$$

$$\frac{(n+1)(n)\cancel{(n-1)!}}{\cancel{n-1!}} = 6$$

$$(n+1)(n) = 6$$

$$n = 2$$

2½ out of 3

+ ½ mark for substitution
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for the permissible value of *n*

E4 (missing brackets but still implied in lines 1 to 3)

Exemplar 2

$$\frac{n!}{(n-2)!} = 6$$

$$\frac{n(n-1)\cancel{(n-2)}}{\cancel{(n-2)}}$$

$$n(n-1) = 6 \quad -6 \quad -6$$

$$n^2 - n - 6 = 0$$

$$(n-3)(n+2) = 0$$

$n = 3$ ~~$n = -2$~~

2½ out of 3

+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for permissible value of *n*
+ ½ mark for showing the rejection of the extraneous root

E2 (changing an equation to an expression in line 2)

E7 (notation error in line 2)

Exemplar 3

$$\frac{(n+1)!}{(n+1-2)!} = 6$$

$$\frac{(n+1)!}{(n-1)!} = 6$$

$$\frac{(n+1)(n)(n-1!)}{(n-1)!} = 6$$

$$(n+1)n = 6$$

$$n^2+1=6$$

$$n^2 = 5$$

$$n = \sqrt{5}$$

2 out of 3

+ ½ mark for substitution
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
E7 (notation error in line 3)

This page was intentionally left blank.

Question 13 R8

Express $\log(3x-1) - \log x + \log 9$ as a single logarithm.

Solution

$$\log\left(\frac{9(3x-1)}{x}\right)$$

1 mark for product law
1 mark for quotient law

2 marks

Exemplar 1

$$\log(3x-1) - \log x + \log 9$$

$$= \log\left(\frac{3\cancel{x}-1}{\cancel{x}}\right) + \log 9$$

$$= \log 2 + \log 9$$

$$= \log(2 \cdot 9)$$

$$= \log 18$$

1½ out of 2

award full marks
– ½ mark for procedural error in line 2

Exemplar 2

$$\log\left(\frac{3x-1 \cdot 9}{x}\right)$$

2 out of 2

award full marks
E7 (notation error)

Exemplar 3

$$\log \frac{(3x-1)}{(x)(9)}$$

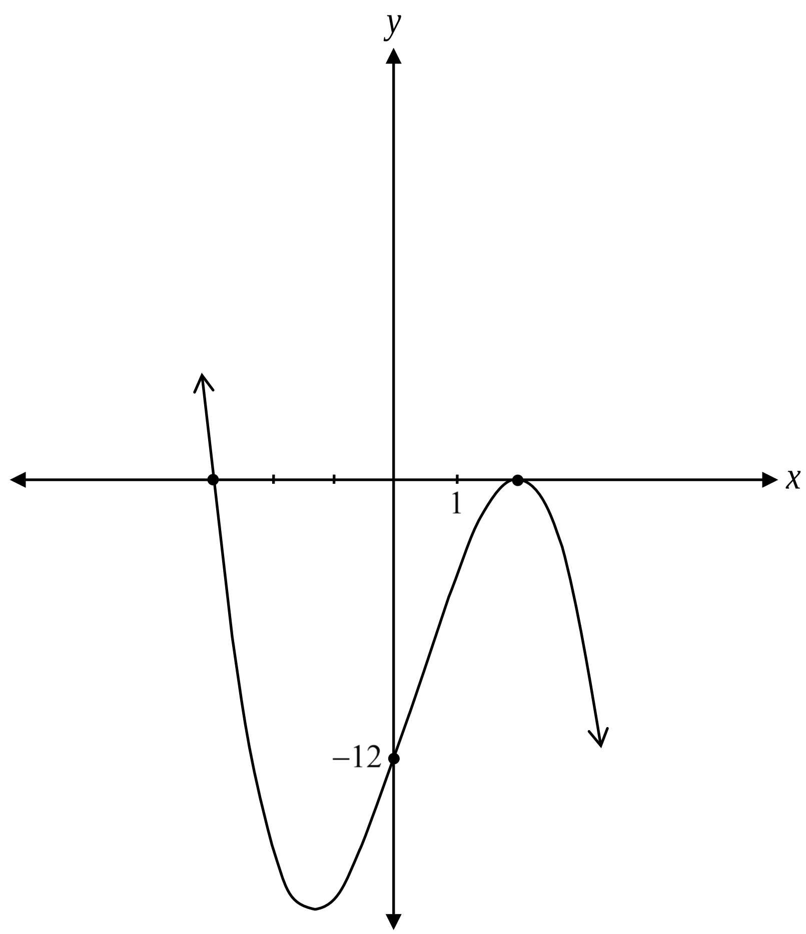
$$= \log \frac{(3x-1)}{(9x)}$$

1 out of 2

+ 1 mark for quotient law

Sketch the graph of $P(x) = -(x-2)^2(x+3)$.

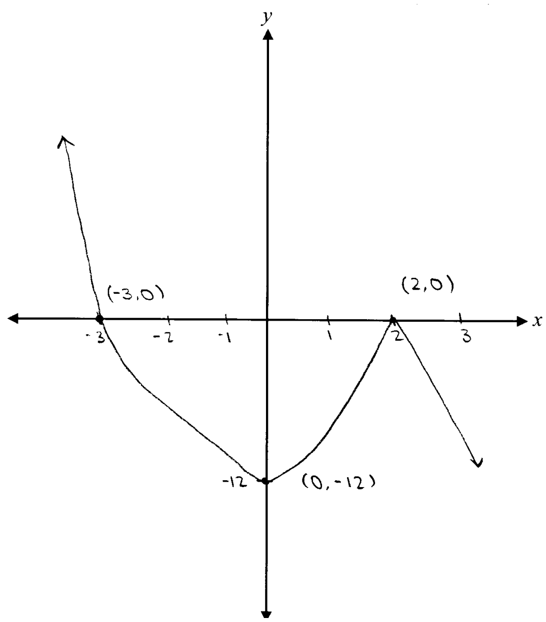
Solution



1 mark for *x*-intercepts
½ mark for *y*-intercept
1 mark for multiplicity of 2 at $x = 2$
½ mark for end behaviour

3 marks

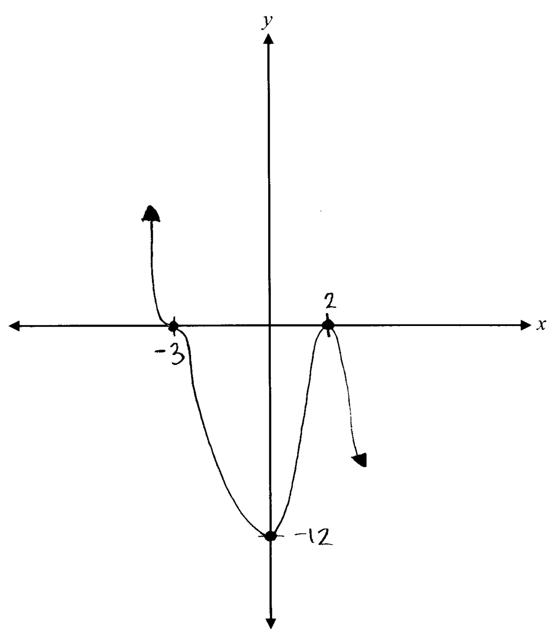
Exemplar 1



2 ½ out of 3

award full marks
– ½ mark for incorrect shape of graph at $x = 2$

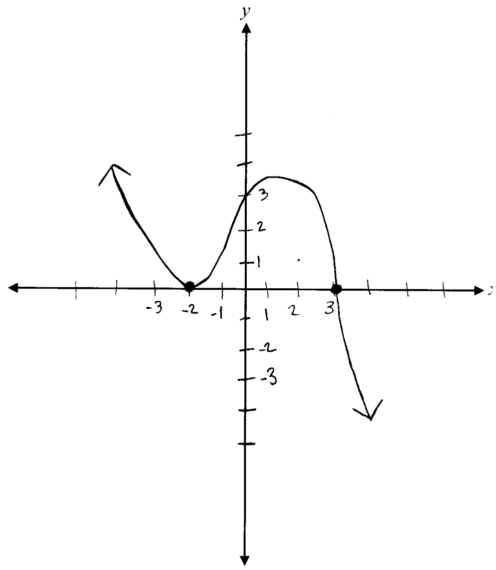
Exemplar 2



2 out of 3

award full marks
– 1 mark for concept error (multiplicity of 3 at $x = -3$)

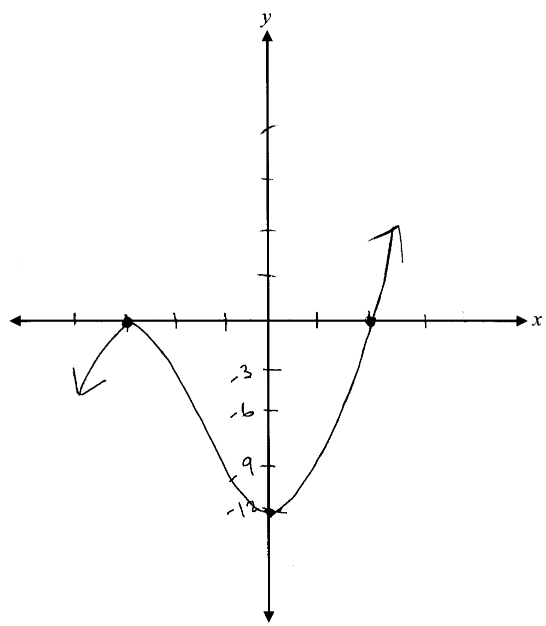
Exemplar 3



1½ out of 3

+ 1 mark for multiplicity of 2 at $x = -2$
+ ½ mark for end behaviour

Exemplar 4



1½ out of 3

+ 1 mark for *x*-intercepts
+ ½ mark for *y*-intercept
E9 (scale values on *x*-axis not indicated)

Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
15	B	R1
16	D	R6
17	A	R14
18	C	R7
19	A	R11
20	B	P2
21	B	T3
22	C	T1

Question 15 R1

Given $f(x) = -2x - 5$ and $g(x) = x + 7$, identify the equation of $h(x) = f(g(x))$.

a) $h(x) = -2x^2 - 35$

b) $h(x) = -2x - 19$

c) $h(x) = -2x^2 - 19x - 35$

d) $h(x) = -2x + 2$

Question 16 R6

The function $y = f(x)$ has a domain of $[2, 5]$ and a range of $[-6, 3]$. Identify the range of the function $y = f^{-1}(x)$.

a) $[-6, 3]$

b) $[-5, 2]$

c) $[-3, 6]$

d) $[2, 5]$

Question 17 R14

Identify the function that has a graph with a point of discontinuity (hole) at $x = -3$.

a) $y = \dfrac{x+3}{x^2-9}$

b) $y = \dfrac{x-3}{x^2-9}$

c) $y = \dfrac{x^2-9}{x-3}$

d) $y = \dfrac{x^2+9}{x+3}$

Question 18 R7

Identify the value of $\ln e$.

a) 0

b) $\log e$

c) 1

d) e

Question 19 R11

When the polynomial function, $p(x)$, is divided by $(x-4)$ the remainder is 17. Identify which statement is true.

a) $p(4)=17$

b) $p(-4)=17$

c) $p(4)=0$

d) $p(-4)=0$

Question 20 P2

Identify the expression equivalent to $\frac{(n-6)!}{(n-4)!}$.

a) $(n-4)(n-5)$

b) $\frac{1}{(n-4)(n-5)}$

c) $(n-6)(n-5)$

d) $\frac{1}{(n-6)(n-5)}$

Question 21 T3

Identify the quadrant in which θ terminates if $\sec\theta = -\frac{4}{3}$ and $\sin\theta > 0$.

a) I

b) II

c) III

d) IV

Question 22 T1

Identify the expression that represents all angles that are coterminal with $\frac{\pi}{3}$.

a) $\frac{\pi}{3} + \pi k,\ k \in \mathbb{Z}$

b) $\frac{\pi}{3} + \pi k,\ k \in \mathbb{R}$

c) $\frac{\pi}{3} + 2\pi k,\ k \in \mathbb{Z}$

d) $\frac{\pi}{3} + 2\pi k,\ k \in \mathbb{R}$

Question 23 R13

Given $f(x)=\frac{1}{2}x-3,$ state the coordinates of an invariant (unchanged) point when sketching the graph of $y=\sqrt{f(x)}.$

Solution

$$\frac{1}{2}x-3=0$$

$$\frac{1}{2}x=3$$

$$x=6$$

$$(6,0)$$

1 mark

or

$$\frac{1}{2}x-3=1$$

$$\frac{1}{2}x=4$$

$$x=8$$

$$(8,1)$$

Exemplar 1

$f(x) = \frac{1}{2}x - 3$

$f(x) = 2\left(\frac{1}{2}x - 3\right)$

$= x - 6$

$(6, 0)$

$y = \sqrt{f(x)}$

$(0, 0)$ or $(1, 1)$

Coordinates: $(6, 0)$, $(0, 0)$, $(1, 1)$

0 out of 1

award full marks

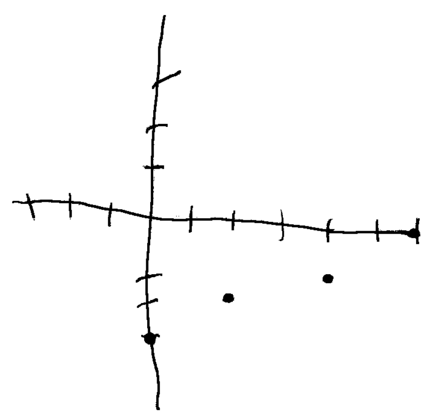
– 1 mark for concept error (not relating $f(x)$ to its radical)

Exemplar 2

$f(x) = \frac{1}{2}x - 3 \qquad y = \sqrt{f(x)}$

$y = \sqrt{\frac{1}{2}x - 3}$

$x = 6$



½ out of 1

award full marks

– ½ mark for procedural error in line 3

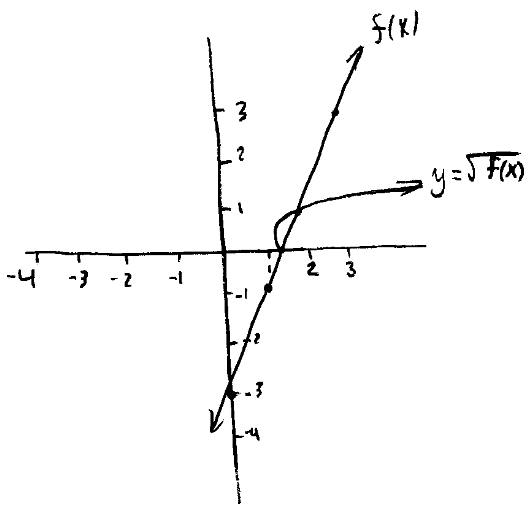
Exemplar 3

8, 1

1 out of 1

award full marks
E7 (notation error)

Exemplar 4



∴ (2, 1) because when we take the square root of 1 it's 1, where as y = 1 x is equal to 2.

½ out of 1

award full mark
– ½ mark for procedural error (incorrect slope of $f(x)$)

This page was intentionally left blank.

Question 24 R10

Solve, algebraically.

$$125^{3x+4} = \left(\frac{1}{5}\right)^{x}$$

Solution

$\left(5^{3}\right)^{3x+4} = \left(5^{-1}\right)^{x}$ 1 mark for changing to a common base (½ mark for each side)

$5^{9x+12} = 5^{-x}$ ½ mark for exponent law

$9x + 12 = -x$ ½ mark for equating exponents

$10x = -12$

2 marks

$x = -\frac{12}{10}$

or

$x = -\frac{6}{5}$

Exemplar 1

$$125^{3x+4} = 125^{-5x}$$

$$3x+4 = -5x$$

$$8x + 4 = 0$$

$$\frac{8x}{8} = \frac{-4}{8}$$

$$\boxed{x = -\frac{1}{2}}$$

1 out of 2

+ ½ mark for exponent law
+ ½ mark for equating exponents

Exemplar 2

$$\log 125^{3x+4} = \log\left(\frac{1}{5}\right)^x$$

$$(3x+4)\log 125 = x\log\left(\frac{1}{5}\right)$$

$$3x\log 125 + 4\log 125 = x\log\left(\frac{1}{5}\right)$$

$$3x\log 125 + 4\log 125 - x\log\left(\frac{1}{5}\right) = 0$$

1½ out of 2

+ 1 mark for equivalent strategy
+ ½ mark for power law
E1 (final answer not stated)

Exemplar 3

$$(5^3)^{3x+4} = (5^{-1})^x$$

$$5^{9x+12} = 5^{-x}$$

$$9x+12 = -x$$

$$8x+12 = 0$$

$$\frac{8x}{8} = \frac{-12}{8} \qquad \frac{12}{8} = \frac{3}{2} = 1\tfrac{1}{2}$$

$$\boxed{x = -1\tfrac{1}{2}}$$

1 ½ out of 2

award full marks
– ½ mark for arithmetic error in line 4

Exemplar 4

$$125^{3x+4} = \left(\frac{1}{5}\right)^x$$

$$3x+4 = x$$

$$\frac{4}{-2} = \frac{-2x}{-2}$$

$$x = -2$$

0 out of 2

This page was intentionally left blank.

Question 25 T6

Given $\sin\theta = -\frac{\sqrt{2}}{2}$ and $\cos\theta = -\frac{\sqrt{2}}{2}$, justify that the value of $\tan(2\theta)$ is undefined.

Solution

Method 1

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2(1)}{1-1}$$ 1 mark for value of $\tan\theta$

$$= \frac{2}{0}$$

$\therefore$ undefined 1 mark for justification

2 marks

Method 2

$$\tan\theta = 1$$

$$\theta = \frac{5\pi}{4}$$ 1 mark for value of θ

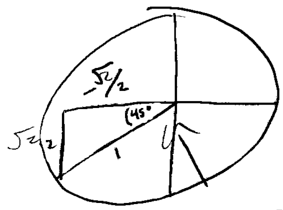
$$\tan(2\theta) = \tan\left(2\left(\frac{5\pi}{4}\right)\right)$$

$$= \tan\frac{5\pi}{2}$$

$\therefore$ undefined 1 mark for justification

2 marks

Exemplar 1

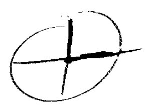


When $\sin\theta = -\frac{\sqrt{2}}{2}$ and $\cos\theta = -\frac{\sqrt{2}}{2}$, $\tan\theta = 45°$ or $225°$.

Either way, when doubled, θ becomes 90°.

$\theta = 225°$

$2\theta = 450°$



$\sin 90° = 1$

$\cos 90° = 0$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{0}$

$=$ undefined

$\tan(2 \cdot 225) = \frac{1}{0}$ which is undefined

1 ½ out of 2

award full marks
– ½ mark for procedural error in line 1

Exemplar 2

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{-\sqrt{2}}{2} \div \frac{-\sqrt{2}}{2}$

$\tan\theta = \frac{-\sqrt{2}}{2} \times \frac{2}{-\sqrt{2}}$

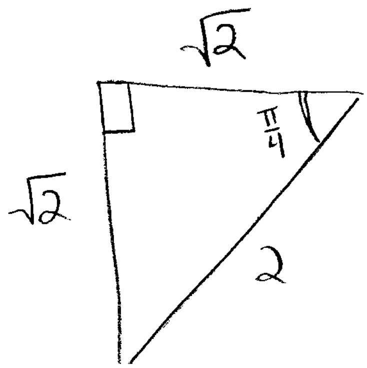
$\tan\theta = 1$

$\tan 2\theta = 2$ out of parameter and can't be used therefore undefined.

1 out of 2

+ 1 mark for value of $\tan\theta$

Exemplar 3



$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\tan\frac{\pi}{4}}{1-\tan^2\frac{\pi}{4}}$$

$$= \frac{2\left(\frac{\sqrt{2}}{\sqrt{2}}\right)}{1-(1\cdot 1)}$$

$$= \frac{2}{0}$$

↳ undefined

1½ out of 2

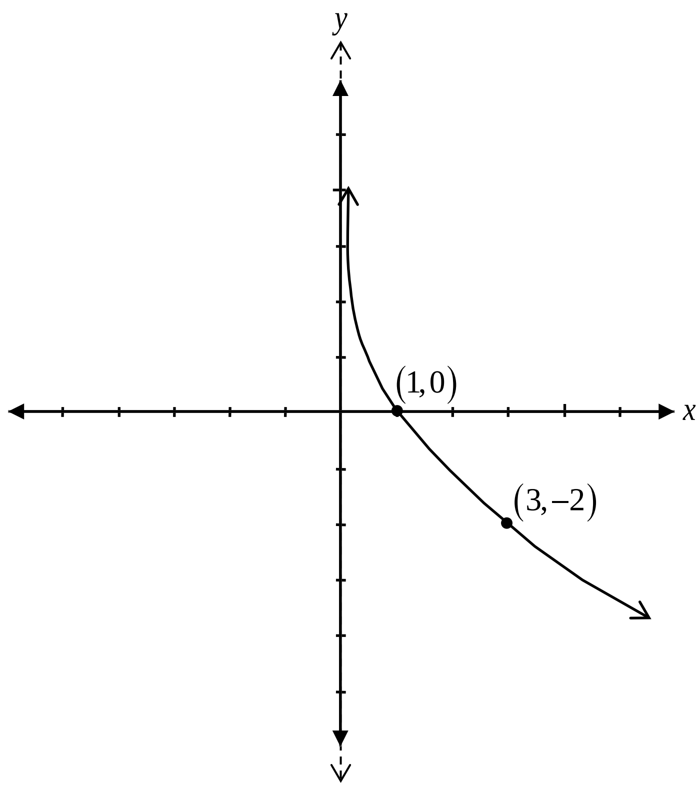
award full marks
– ½ mark for procedural error in line 2 (incorrect value of θ)

This page was intentionally left blank.

Question 26 R9

Sketch the graph of $y = -2\log_3 x$.

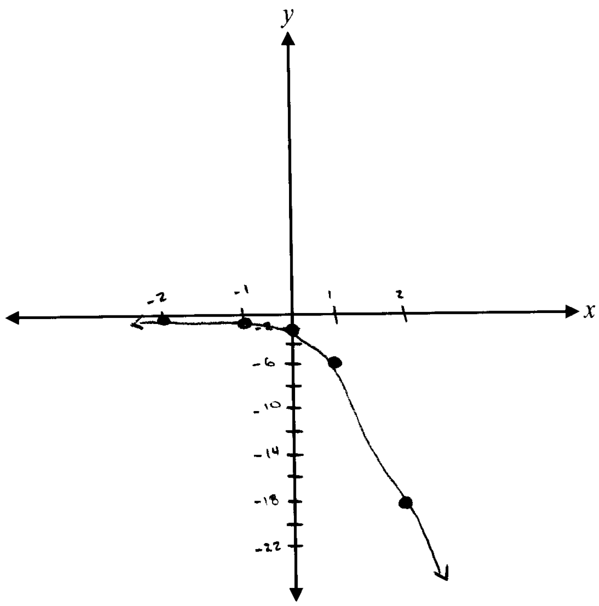
Solution



1 mark for asymptotic behaviour approaching $x = 0$
1 mark for vertical reflection
1 mark for vertical stretch

3 marks

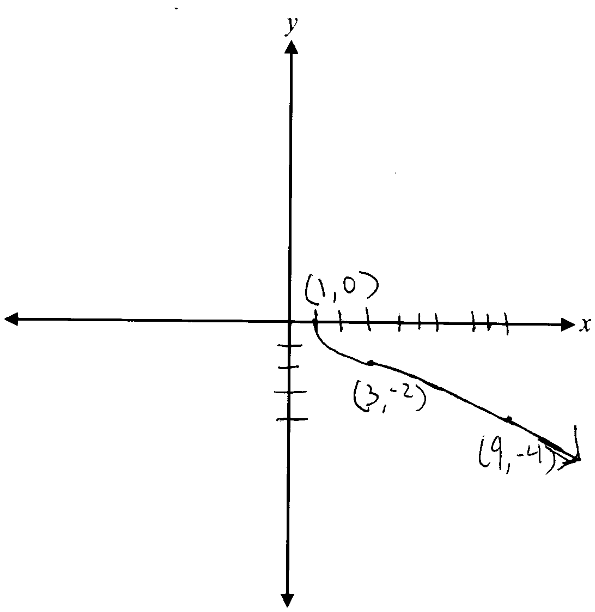
Exemplar 1



2 out of 3

award full marks
– 1 mark for concept error (exponential graph)
E10 (asymptote omitted but still implied)

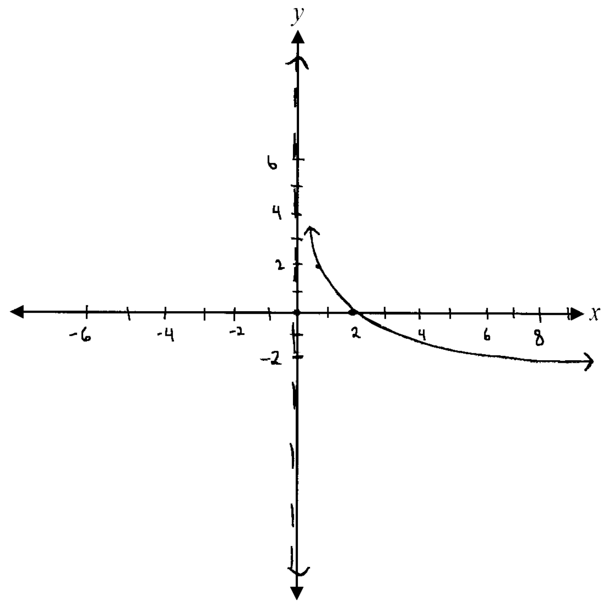
Exemplar 2



2 out of 3

+ 1 mark for vertical reflection
+ 1 mark for vertical stretch

Exemplar 3



2 out of 3

+ 1 mark for asymptotic behaviour approaching $x = 0$
+ 1 mark for vertical reflection

This page was intentionally left blank.

Evaluate.

$$\cos\left(\pi \bullet \sin\left(-\frac{\pi}{6}\right)\right)$$

Solution

$\cos\left(\pi\left(-\frac{1}{2}\right)\right)$ — 1 mark for $\sin\left(-\frac{\pi}{6}\right)$ (½ mark for quadrant; ½ mark for value)

$\cos\left(-\frac{\pi}{2}\right)$

0 — 1 mark for consistent value

2 marks

Exemplar 1

$\sin(-30^{\circ})$

$\cos\left(-\frac{1}{2}\right)$

$= 210^{\circ}$

1 out of 2

+ 1 mark for $\sin\left(-\frac{\pi}{6}\right)$

Exemplar 2

$\cos\left(\frac{1}{2}\pi\right)$

$\cos\left(\frac{\pi}{2}\right) = 0$

1 ½ out of 2

+ ½ mark for the value of $\sin\left(-\frac{\pi}{6}\right)$

+ 1 mark for consistent value

Exemplar 3

$$\cos\left(\pi \sin\left(-\frac{\pi}{6}+\frac{6\pi}{6}\right)\right)$$

$$\cos\left(\pi \sin\left(\frac{5\pi}{6}\right)\right)$$

$$\cos\left(\pi\left(\frac{1}{2}\right)\right)$$

$$\cos\frac{\pi}{2}$$

$$0$$

1 ½ out of 2

award full marks
– ½ mark for procedural error in line 1

Exemplar 4

$$= \cos\left(\pi \sin\left(-\frac{\pi}{6}\right)\right)$$

$$= \cos\left(0\left(-\frac{\pi}{6}\right)\right)$$

$$= \cos(0)$$

$$= \frac{3\pi}{2} \text{ or } \frac{\pi}{2}$$

0 out of 2

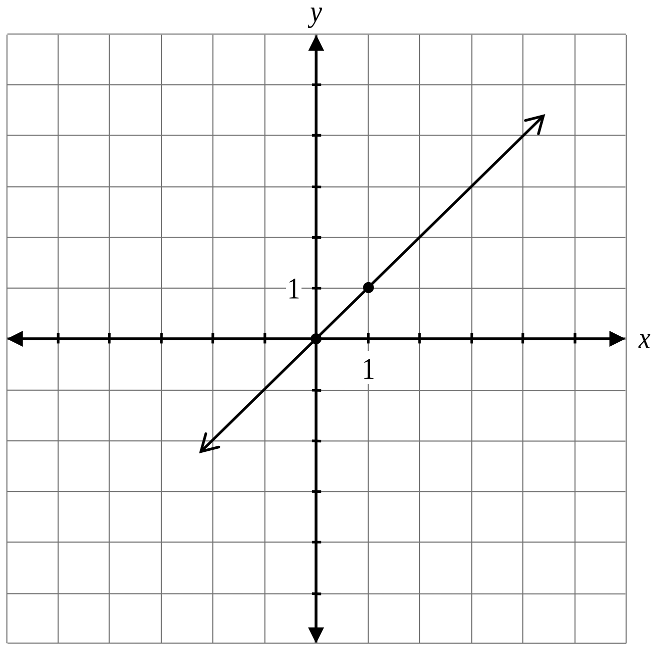
This page was intentionally left blank.

a) Given $f(x) = -x + 2$, sketch the graph of $h(x) = f(f(x))$.

b) Explain why the domain of $h(x) = f(f(x))$ does not have any restrictions.

Solution

a)



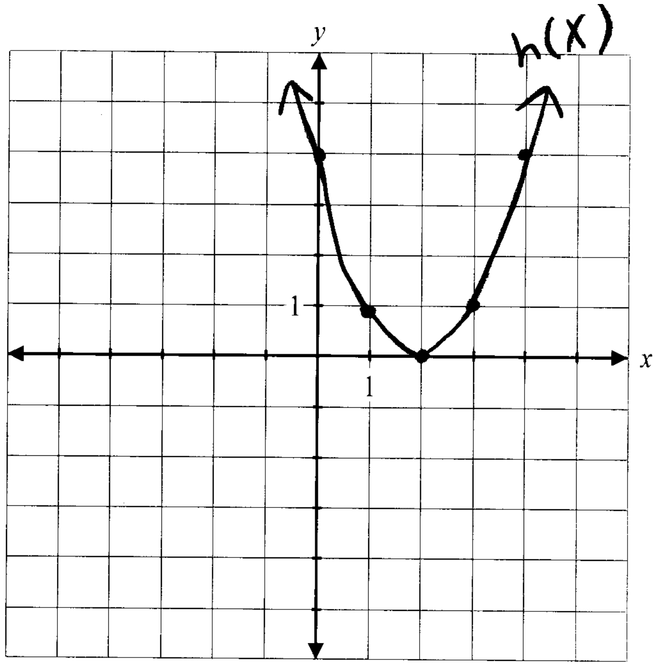
1 mark

b) Since the domain of $y = f(x)$ does not have any restrictions, then the domain of the composition does not have any restrictions.

1 mark

Exemplar 1

a)



0 out of 1

b)

Because it's a quadratic graph and has no non-permissibles or square roots etc... it's simply f(x) times itself (two simple binomials)

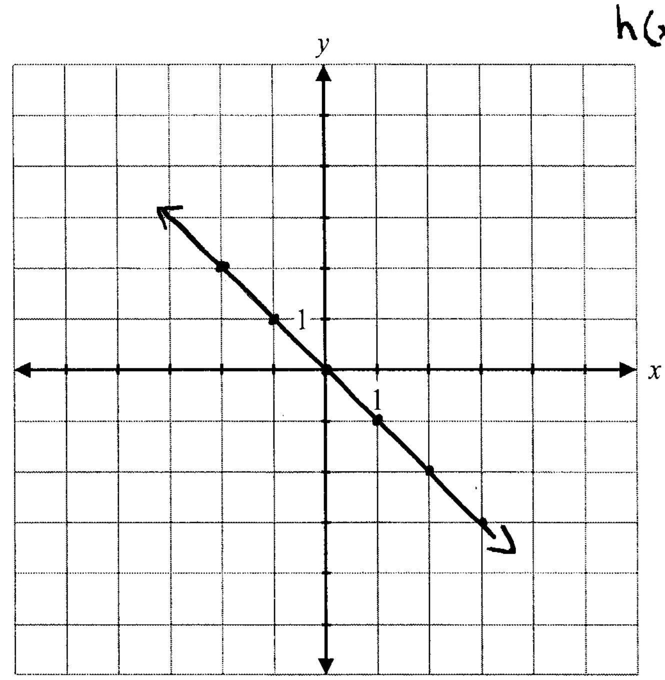
1 out of 1

Exemplar 2

a)

$h(x) = -(-x+2)+2$

$h(x) = (x-2)+2$



½ out of 1

award full marks
– ½ mark for procedural error (incorrect slope)

b)

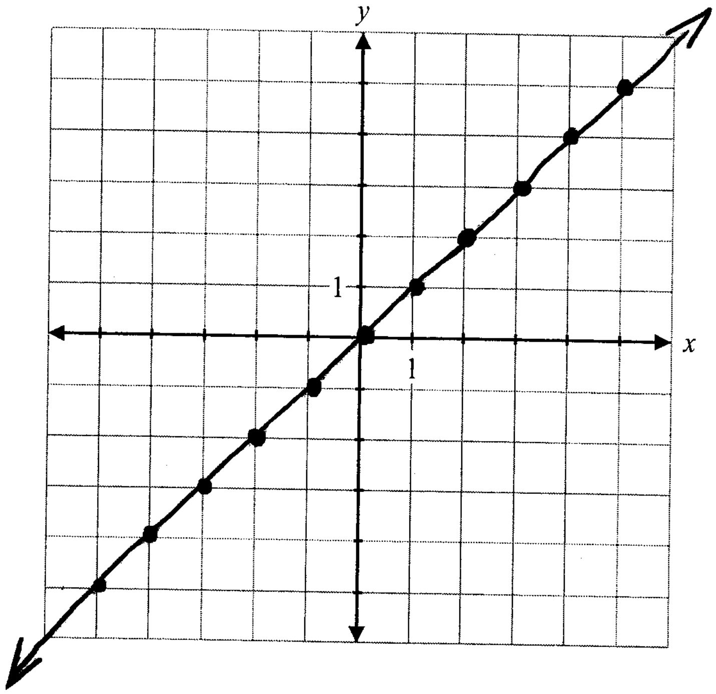
Because it is linear.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3

a)



1 out of 1

b) There are no holes and it continuously goes on, it does not say it stops.

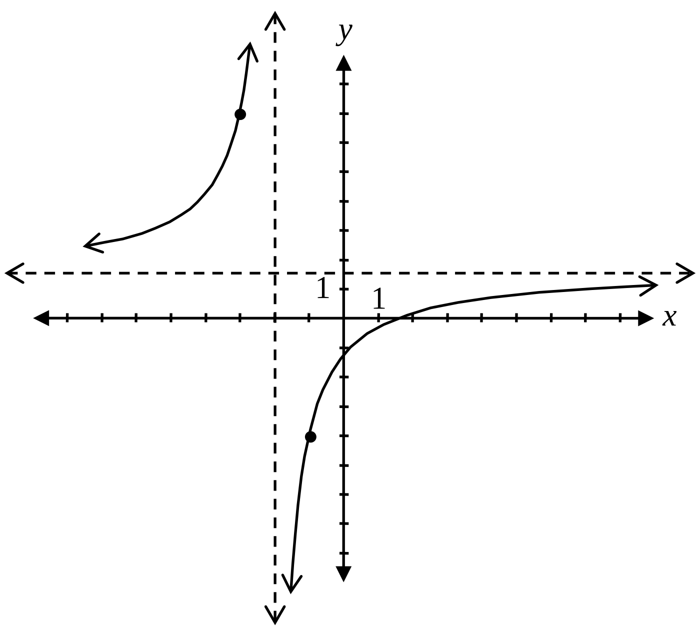
½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

a) Sketch the graph of $f(x) = \frac{3x-5}{2x+4}$.

b) State the range of $f(x)$.

Solution



1 mark for asymptotic behaviour approaching $x = -2$

1 mark for asymptotic behaviour approaching $y = \frac{3}{2}$

½ mark for graph right of $x = -2$

½ mark for graph left of $x = -2$

3 marks

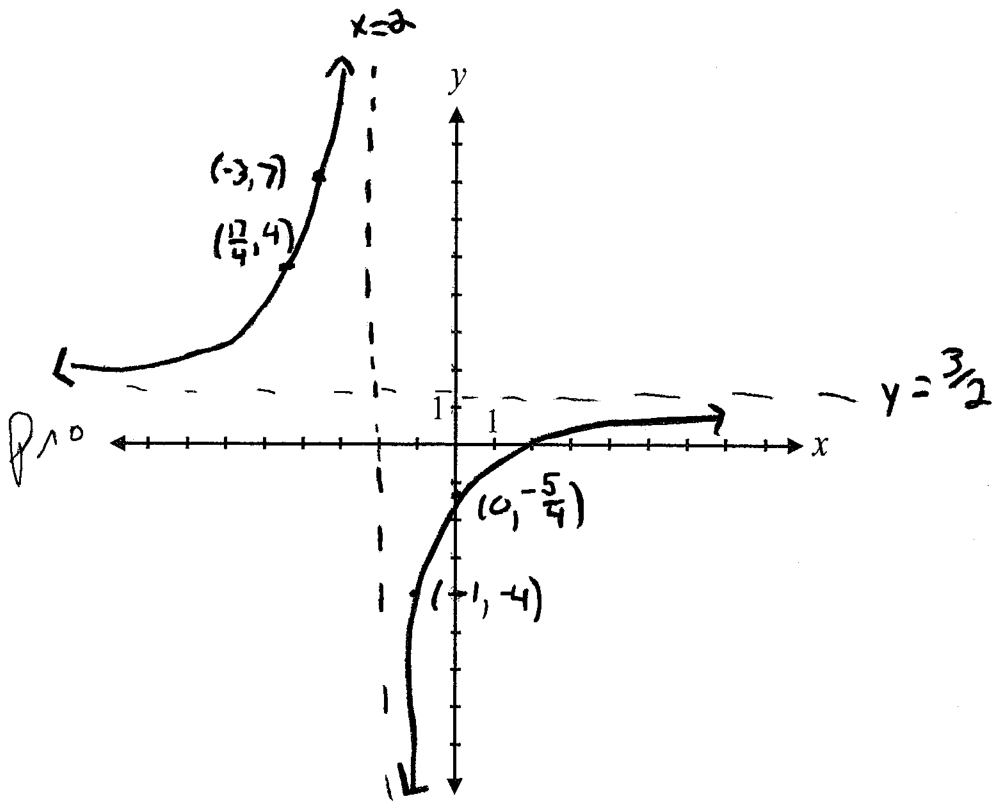
b) Range: $\left\{ y \in \mathbb{R} \,\middle|\, y \neq \frac{3}{2} \right\}$

1 mark for range consistent with graph

1 mark

Exemplar 1

a)



3 out of 3

award full marks
E9 (coordinate point labelled incorrectly)

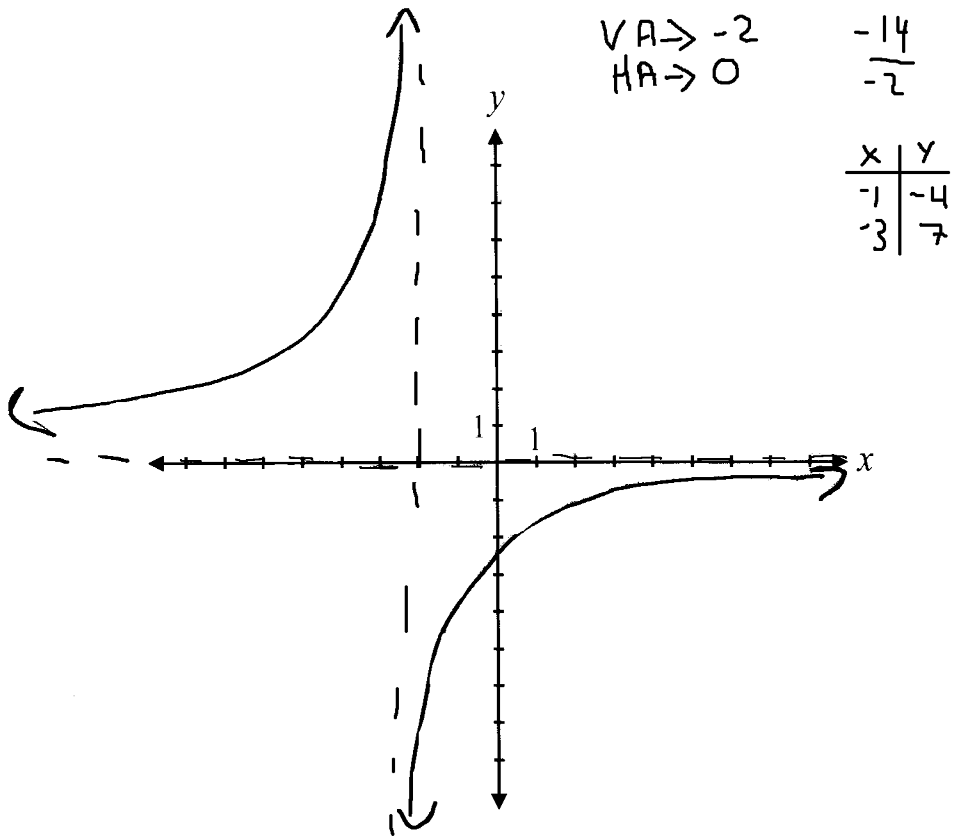
b)

Range: $(-\infty, \infty)$

0 out of 1

Exemplar 2

a)



2 out of 3

+ 1 mark for asymptotic behaviour approaching $x = -2$
+ ½ mark for graph right of $x = -2$
+ ½ mark for graph left of $x = -2$

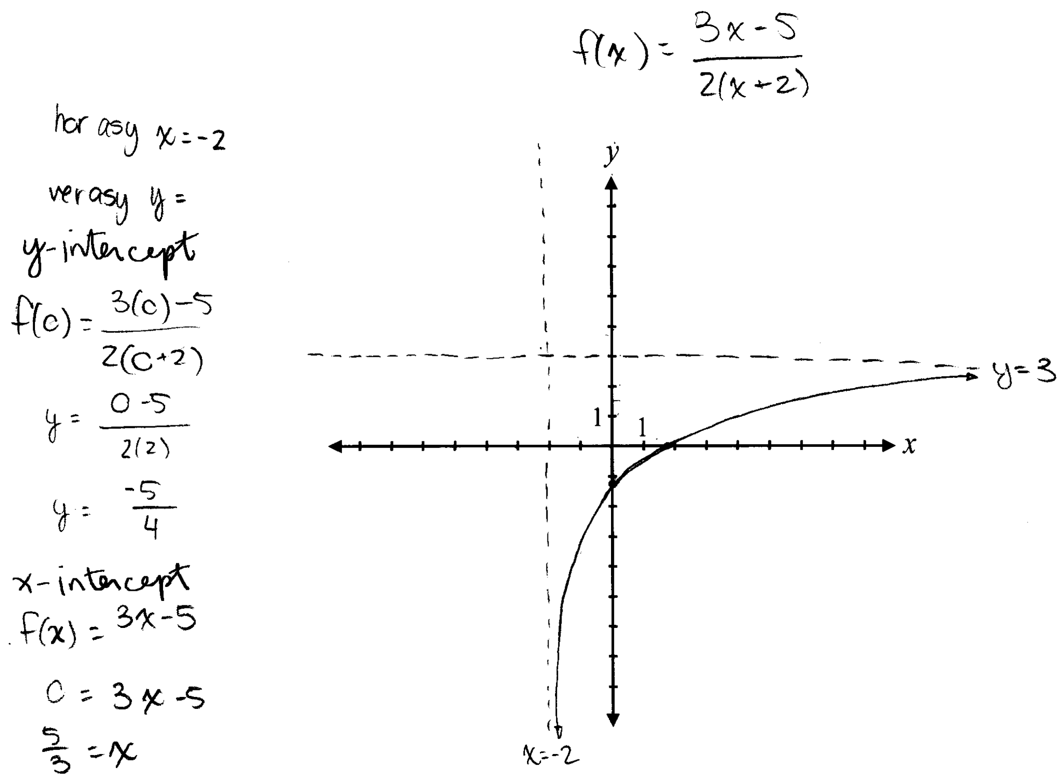
b)

Range: $y \neq 0$

1 out of 1

Exemplar 3

a)



1 ½ out of 3

+ 1 mark for asymptotic behaviour approaching $x = -2$
+ ½ mark for graph right of $x = -2$

b)

Range: $\{y \in \mathbb{R} \mid y \neq 3\}$

0 out of 1

Question 30 R12

Determine, algebraically, the equation of $p(x)$ that satisfies all of the following conditions:

- $p(x)$ is a polynomial function of degree 4
- $p(x)$ has a zero at 3 with a multiplicity of 2
- $p(x)$ has zeroes at -1 and -2
- $p(x)$ passes through the point $(2, 24)$.

Solution

	½ mark for factors of $p(x)$
$p(x) = a(x+1)(x+2)(x-3)^2$	½ mark for multiplicity of 2 at $x = 3$
$24 = a(2+1)(2+2)(2-3)^2$	½ mark for substitution of $p(2) = 24$
$24 = 12a$	
$a = 2$	½ mark for consistent value of *a*
$p(x) = \underline{2(x+1)(x+2)(x-3)^2}$	**2 marks**

Exemplar 1

$p(x) = (x+2)(x+1)(x-3)^2$

1 out of 2

+ ½ mark for factors of $p(x)$
+ ½ mark for multiplicity of 2 at $x = 3$

Exemplar 2

$p(x) = a(x+1)(x+2)(x-3)^2$

$24 = a(2+1)(2+2)(2-3)^2$

$24 = a \cdot 2 \cdot 4 \cdot 1$

$24 = 8a$

$a = 3$

$p(x) = 3(x+1)(x+2)(x-3)^2$

1½ out of 2

+ ½ mark for factors of $p(x)$
+ ½ mark for multiplicity of 2 at $x = 3$
+ ½ mark for substitution of $p(2) = 24$

Question 31 T5

Verify, by substitution, that the equation $\dfrac{\cos\theta + \cot\theta}{\cot\theta} = 1 + \sin\theta$ is true for $\theta = \dfrac{2\pi}{3}$.

Solution

Left-Hand Side	Right-Hand Side	
$\dfrac{\cos\left(\frac{2\pi}{3}\right) + \cot\left(\frac{2\pi}{3}\right)}{\cot\left(\frac{2\pi}{3}\right)}$	$1 + \sin\left(\dfrac{2\pi}{3}\right)$	½ mark for substitution
$\dfrac{-\frac{1}{2} - \frac{1}{\sqrt{3}}}{-\frac{1}{\sqrt{3}}}$	$1 + \dfrac{\sqrt{3}}{2}$	1½ marks for exact values (½ mark for each)
$\dfrac{\frac{-\sqrt{3} - 2}{2\sqrt{3}}}{-\frac{1}{\sqrt{3}}}$	$\dfrac{2}{2} + \dfrac{\sqrt{3}}{2}$	
$\dfrac{-\sqrt{3} - 2}{-2}$	$\dfrac{2 + \sqrt{3}}{2}$	
$\dfrac{-\left(\sqrt{3} + 2\right)}{-2}$	$\dfrac{\sqrt{3} + 2}{2}$	1 mark for simplification (½ mark for LHS; ½ mark for RHS)
$\dfrac{\sqrt{3} + 2}{2}$		

3 marks

Note:

- Deduct a maximum of 1 mark if student proves identity without substitution of $\theta = \dfrac{2\pi}{3}$.

Exemplar 1

Left-Hand Side	Right-Hand Side
$= \dfrac{\cos\frac{2\pi}{3} + \cot\frac{2\pi}{3}}{\cot\frac{2\pi}{3}}$	$= 1 + \sin\frac{2\pi}{3}$
$= \dfrac{\frac{1}{2} + \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}}$	$= 1 + \left(\frac{\sqrt{3}}{2}\right)$
$= \dfrac{\frac{\sqrt{3}}{2\sqrt{3}} + \frac{2}{2\sqrt{3}}}{\frac{1}{\sqrt{3}}}$	$= \frac{2}{2} + \frac{\sqrt{3}}{2}$
$= \dfrac{\frac{2+\sqrt{3}}{2\sqrt{3}}}{\frac{1}{\sqrt{3}}}$	$= \dfrac{2+\sqrt{3}}{2}$
$= \dfrac{2+\sqrt{3}}{2}$	

✓

2 out of 3

+ ½ mark for substitution

+ ½ mark for exact value of $\sin\frac{2\pi}{3}$

+ 1 mark for simplification

Exemplar 2

Left-Hand Side	Right-Hand Side
$\dfrac{\cos\left(\frac{2\pi}{3}\right) + \dfrac{\cos\left(\frac{2\pi}{3}\right)}{\sin\left(\frac{2\pi}{3}\right)}}{\dfrac{\cos\left(\frac{2\pi}{3}\right)}{\sin\left(\frac{2\pi}{3}\right)}}$	$1 + \sin\left(\frac{2\pi}{3}\right)$
$\dfrac{\left(-\frac{1}{2}\right) + \dfrac{\left(-\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}}{\dfrac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}}$	$1 + \dfrac{\sqrt{3}}{2}$
$\dfrac{-\frac{1}{2} + \left(-\frac{1}{2}\right)\left(\frac{2}{\sqrt{3}}\right)}{-\frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)}$	$\dfrac{2}{2} + \dfrac{\sqrt{3}}{2}$
$\dfrac{-\frac{1}{2} - \frac{1}{\sqrt{3}}}{-\frac{1}{\sqrt{3}}}$	$\dfrac{2 + \sqrt{3}}{2}$

2½ out of 3

+ ½ mark for substitution
+ 1½ mark for exact values
+ ½ mark for simplification of RHS

This page was intentionally left blank.

Question 32 R14

Jyugo was asked to state the equation of the vertical asymptote(s) on the graph of $f(x) = \dfrac{3x-15}{x^2-5x}$.

His solution:

$$f(x) = \frac{3(x-5)}{x(x-5)}$$

$$x = 0 \qquad x = 5$$

Explain why his solution is incorrect.

Solution

His solution at $x = 5$ is the x-value of a point of discontinuity (hole), not a vertical asymptote.

1 mark

Exemplar 1

You can cancel out (x-5) which will give you a point of discontinuity, which he missed.

1 out of 1

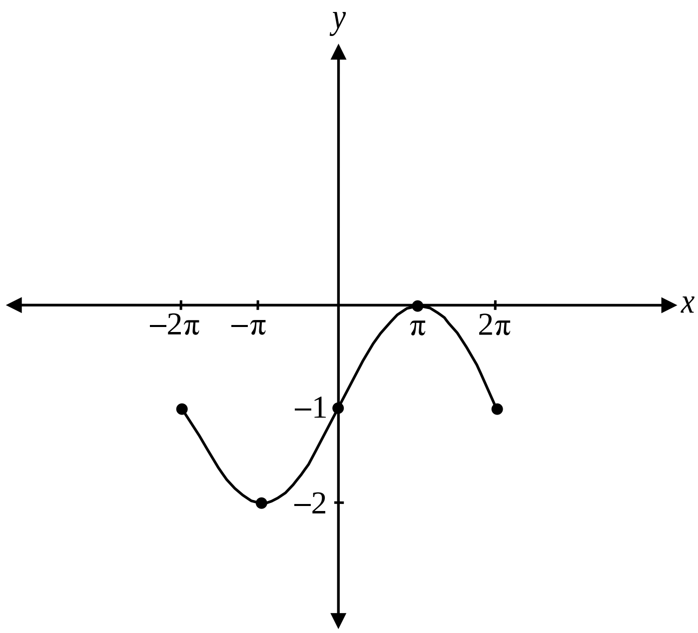
Exemplar 2

He couldve canceled out the x-5 which will give him a diff. vertical asymptote.

0 out of 1

Sketch the graph of $y = \sin\left(\frac{1}{2}x\right) - 1$ over the interval $[-2\pi, 2\pi]$.

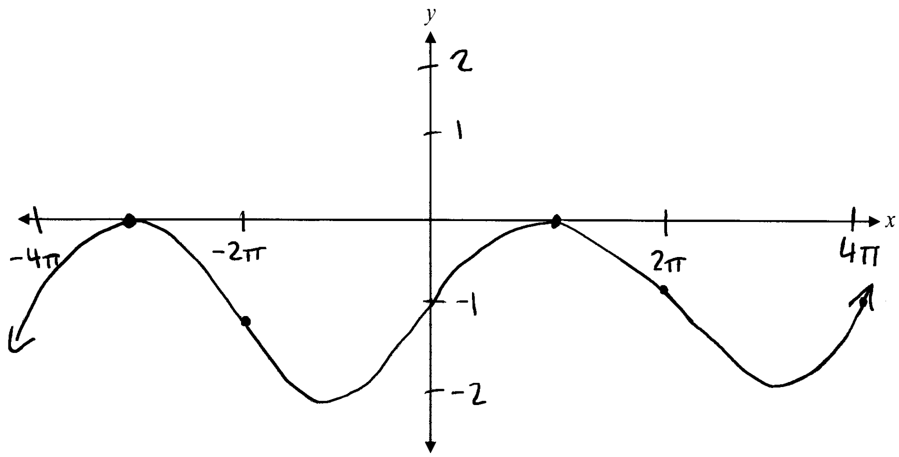
Solution



1 mark for shape of $y = \sin x$
1 mark for period
1 mark for vertical translation

3 marks

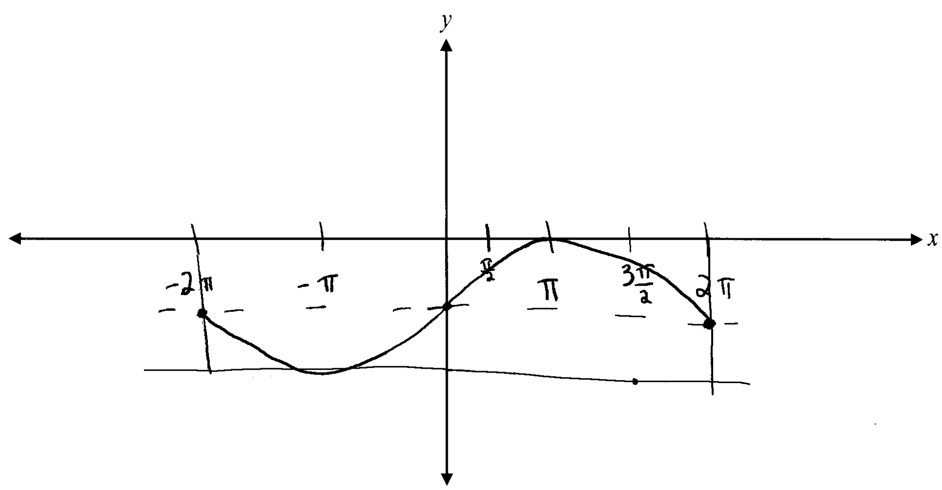
Exemplar 1



3 out of 3

award full marks
E8 (answer outside the given domain)

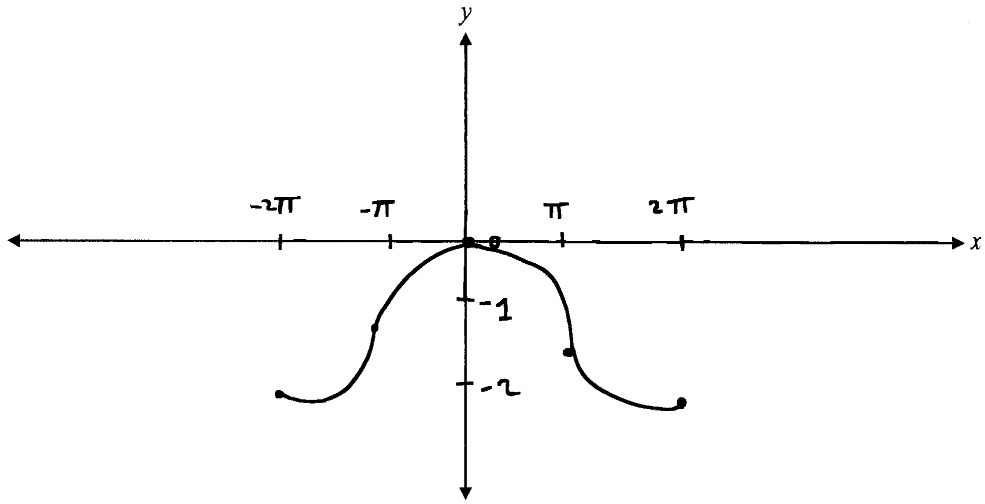
Exemplar 2



3 out of 3

award full marks
E9 (scale values on *y*-axis not indicated)

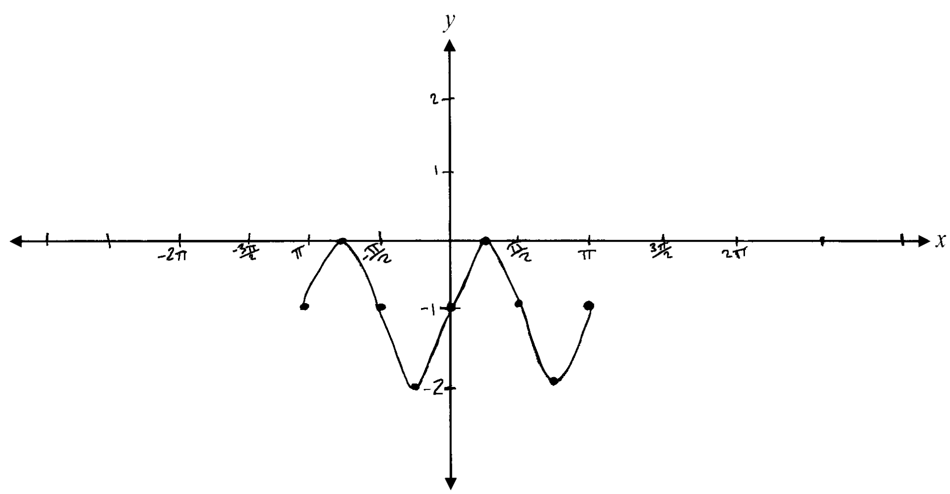
Exemplar 3



2 out of 3

+ 1 mark for period
+ 1 mark for vertical translation

Exemplar 4



1½ out of 3

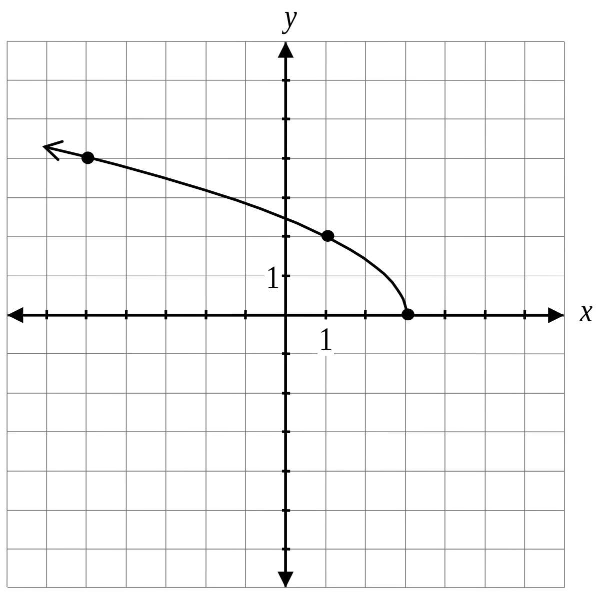
+ 1 mark for shape of $y = \sin x$
+ 1 mark for vertical translation
– ½ mark for procedural error (incomplete domain of graph)

This page was intentionally left blank.

Question 34 R13

Sketch the graph of $y = \sqrt{-2x + 6}$.

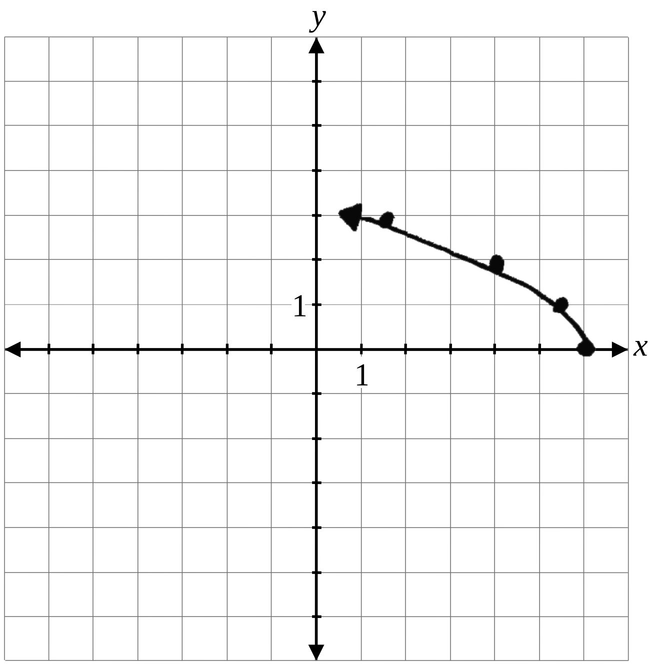
Solution



1 mark for shape of a radical function
1 mark for horizontal reflection
1 mark for horizontal compression
1 mark for horizontal translation

4 marks

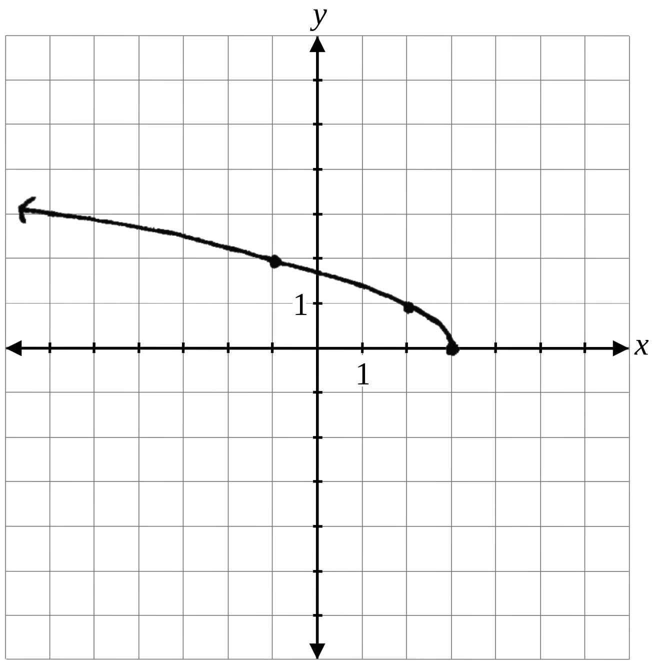
Exemplar 1



3 out of 4

+ 1 mark for shape of a radical function
+ 1 mark for horizontal reflection
+ 1 mark for horizontal compression

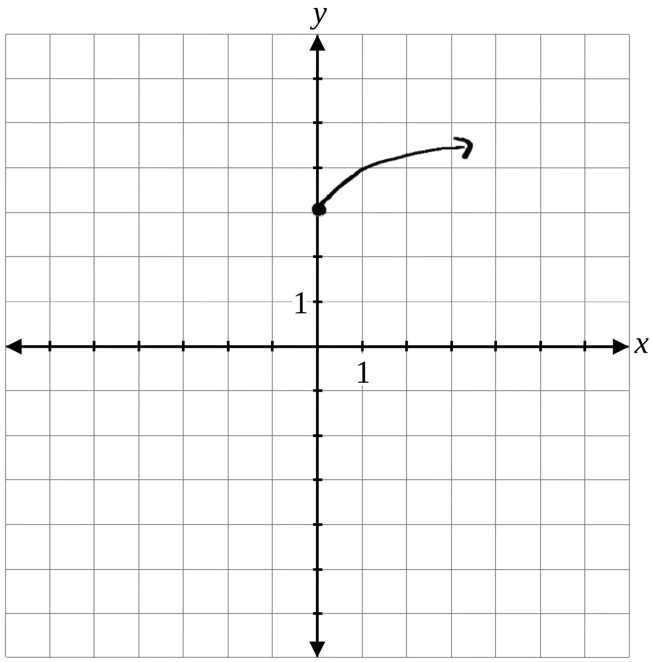
Exemplar 2



3 out of 4

+ 1 mark for shape of radical function
+ 1 mark for horizontal reflection
+ 1 mark for horizontal translation

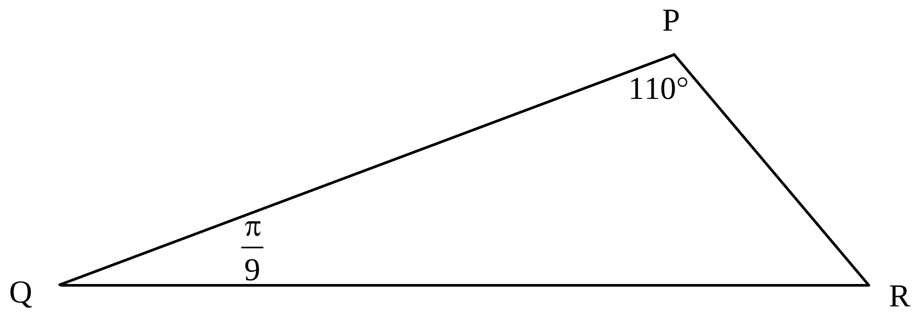
Exemplar 3



1 out of 4

+ 1 mark for shape of a radical function

Determine the measure of angle R, in radians.



Solution

$(110)\left(\frac{\pi}{180}\right) = \frac{11\pi}{18}$ 1 mark for conversion

$\angle R = \pi - \left(\frac{\pi}{9} + \frac{11\pi}{18}\right)$

$\angle R = \frac{18\pi}{18} - \left(\frac{2\pi}{18} + \frac{11\pi}{18}\right)$

$\angle R = \frac{5\pi}{18}$ 1 mark for angle R

2 marks

Exemplar 1

$\frac{\pi}{9} \cdot \frac{180}{\pi} = 20°$

$180° - (110° + 20°)$

$180° - (130°)$

Angle R $= 50°$

2 out of 2

award full marks
E5 (answer stated in degrees instead of radians)

Exemplar 2

$\frac{\pi}{6} \times \frac{180}{\pi}$

$\frac{180}{6} = 30°$

$180° - 140° = 40°$

$\frac{40°}{1} \times \frac{\pi}{180}$

$\frac{40\pi}{180} =$

$\frac{4\pi}{18}$

$\frac{2\pi}{9}$

$R = \frac{2\pi}{9}$

2 out of 2

award full marks
E7 (transcription error in line 1)

Question 36 R5

Given $f(x) = \sqrt{x-4}$, state the equation of the resulting function, $g(x)$, after a reflection over the *x*-axis.

Solution

$g(x) = \underline{-\sqrt{x-4}}$ 1 mark for vertical reflection

1 mark

Exemplar 1

$g(x) = \sqrt{(-x-4)}$

0 out of 1

Exemplar 2

$g(x) = -f(x)$

1 out of 1

Question 37 T1, T6

a) Determine the coterminal angle of $\frac{29\pi}{12}$ over the interval $[0, 2\pi]$.

b) Determine the exact value of $\sin\left(\frac{29\pi}{12}\right)$.

Solution

a) $\frac{5\pi}{12}$

1 mark

b)

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

1 mark for substitution into correct identity

$$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

2 marks (½ mark for each exact value)

3 marks

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$

or

$$= \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

Note:

- Other combinations are possible.

Exemplar 1

a)

there is no coterminal angle with $2\frac{9}{12}\pi$ in this domain.

0 out of 1

b)

$$\sin\left(\frac{3\pi}{4}+\frac{5\pi}{6}\right)=\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{5\pi}{6}\right)+\cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{5\pi}{6}\right)$$

$$=\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$=\left(\frac{-\sqrt{6}}{4}\right)+\left(-\frac{\sqrt{2}}{4}\right)$$

$$=\frac{-\sqrt{6}-\sqrt{2}}{4}$$

2 ½ out of 3

award full marks

– ½ mark for procedural error (incorrect combination)

Exemplar 2

a)

$$\frac{5\pi}{12}$$

1 out of 1

b)

$$\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$$

$$=\sin\left(\frac{1}{2}\right)\cos\left(\frac{\sqrt{2}}{2}\right)+\cos\left(\frac{\sqrt{3}}{2}\right)\sin\left(\frac{\sqrt{2}}{2}\right)$$

2 out of 3

award full marks

– 1 mark for concept error

Exemplar 3

a)

$$\frac{29\pi}{12} - \frac{2\pi}{1}\left(\frac{12}{12}\right)$$

$$= \frac{29\pi}{12} - \frac{24\pi}{12}$$

$$= \boxed{\frac{5\pi}{12}}$$

1 out of 1

b)

$$\frac{9\pi}{12} = \frac{3\pi}{4}$$

$$\frac{20\pi}{12} = \frac{10\pi}{6} = \frac{5\pi}{3}$$

$$\sin\left(\frac{5\pi}{3}\right) + \sin\left(\frac{3\pi}{4}\right)$$

$$-\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} = \boxed{\frac{-\sqrt{3}+\sqrt{2}}{2}}$$

1 out of 3

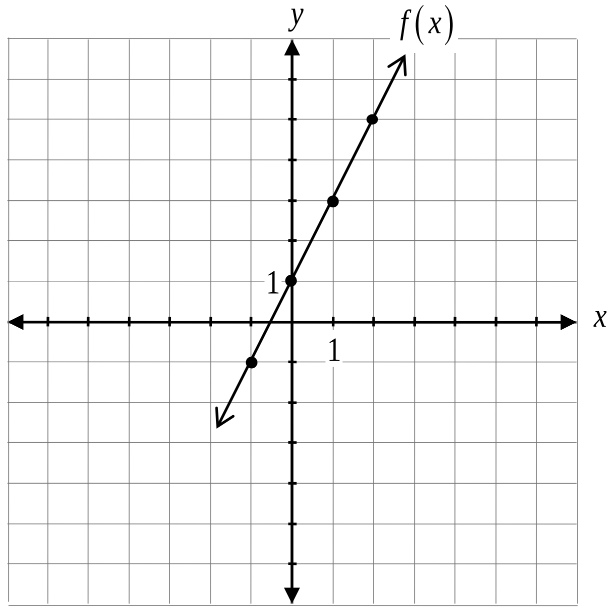
+ ½ mark for exact value of $\sin\frac{5\pi}{3}$

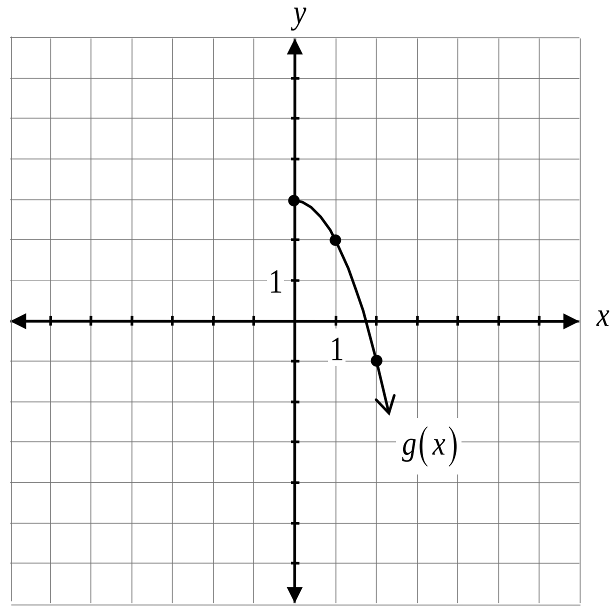
+ ½ mark for exact value of $\sin\frac{3\pi}{4}$

This page was intentionally left blank.

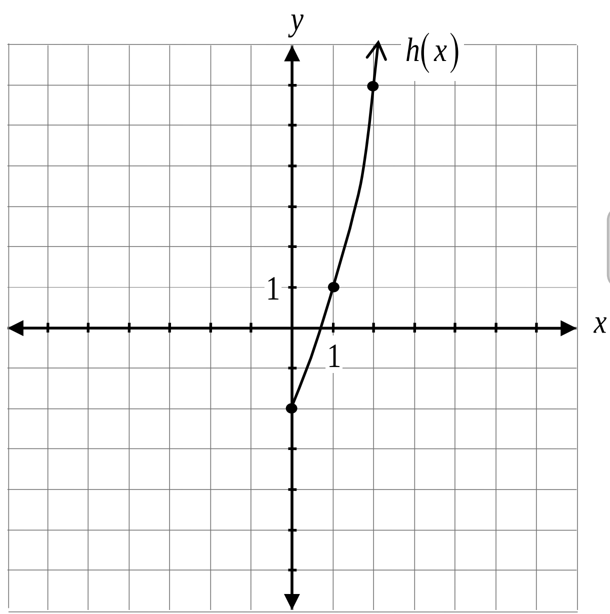
Question 38 R1

Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $h(x) = f(x) - g(x)$.





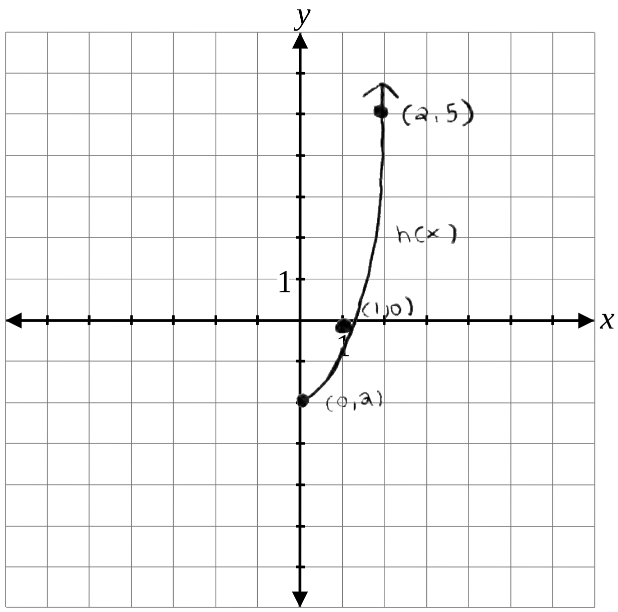
Solution



1 mark for operation of subtraction
1 mark for restricted domain

2 marks

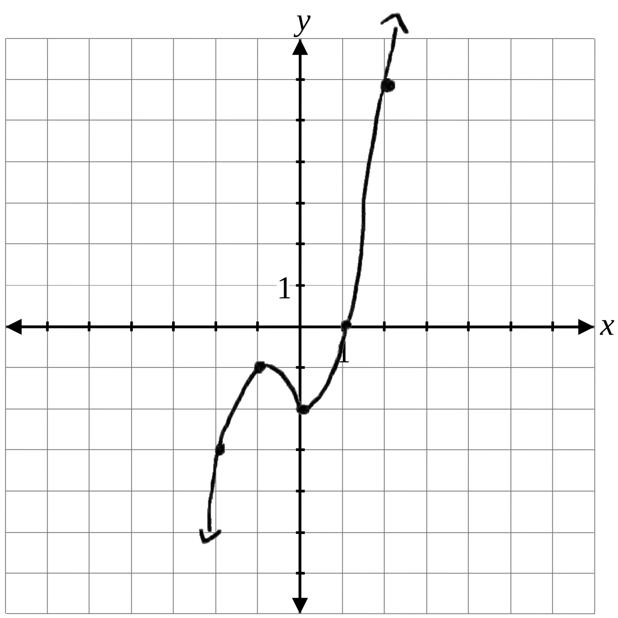
Exemplar 1



1 out of 2

+ 1 mark for restricted domain

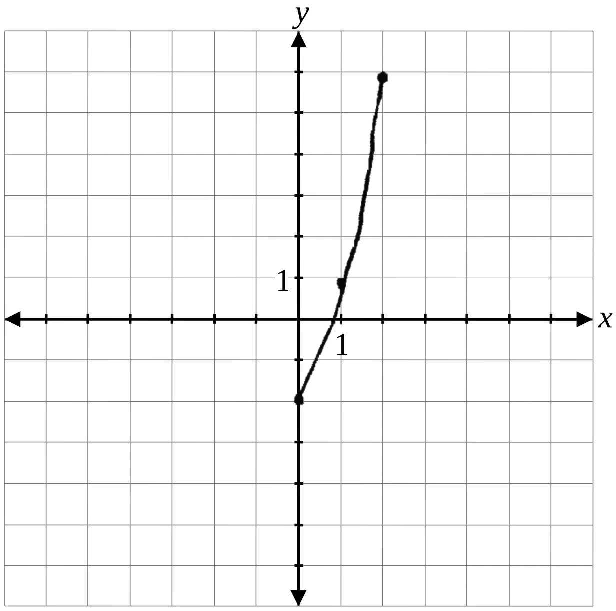
Exemplar 2



½ out of 2

+ 1 mark for operation of subtraction
– ½ mark for procedural error (1 incorrect point)

Exemplar 3



2 out of 2

award full marks
E9 (arrowhead omitted)

This page was intentionally left blank.

Question 39 T4

Determine an equation of a sinusoidal function that has the following characteristics:

- an amplitude of 3
- a period of 6
- a minimum value of -5

Solution

$$y = A\sin(Bx) + D$$

$$y = 3\sin\left(\frac{\pi}{3}x\right) - 2$$

or

$$y = A\cos(Bx) + D$$

$$y = 3\cos\left(\frac{\pi}{3}x\right) - 2$$

1 mark for value of A
1 mark for value of B
1 mark for value of D

3 marks

Note:

- Other equations are possible.

Exemplar 1

$$3\sin\left(\frac{\pi}{3}x\right) - 5$$

$a = 3$

$b =$

$c =$

$d = -5$

1½ out of 3

+ 1 mark for value of A
+ 1 mark for value of B
– ½ mark for procedural error (not written as an equation)

Exemplar 2

$$y = 3\sin\left(\frac{1}{3}x\right) - 5$$

1 out of 3

+ 1 mark for value of A

Exemplar 3

$$y = 3\left(\sin\left(\frac{\pi}{3}\right)\right) - 2$$

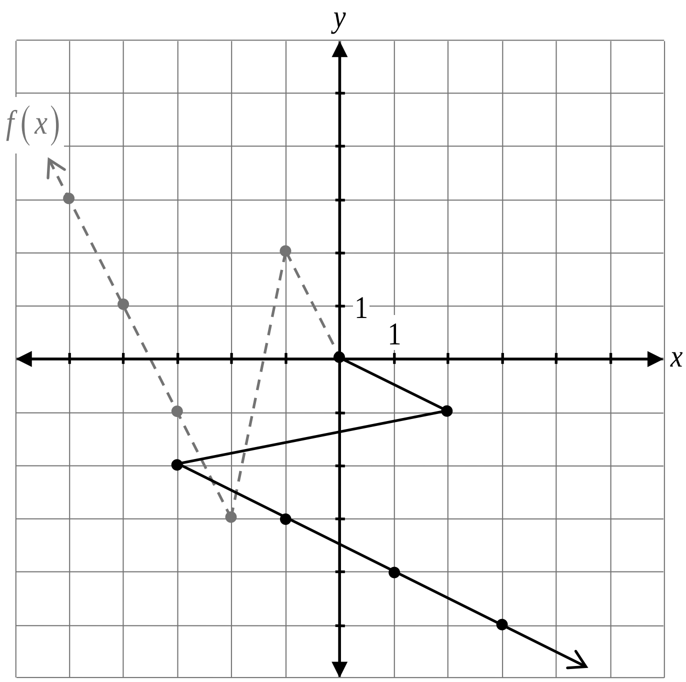
2½ out of 3

award full marks
– ½ mark for procedural error (omitted "*x*")

Question 40 R5

Given the graph of $y = f(x)$, sketch the graph after a reflection over the line $y = x$.

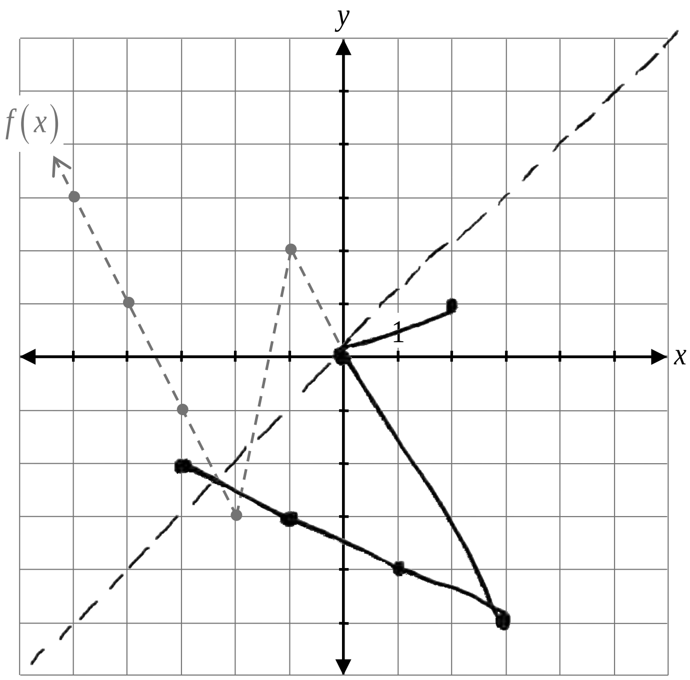
Solution



1 mark for reflection over the line $y = x$

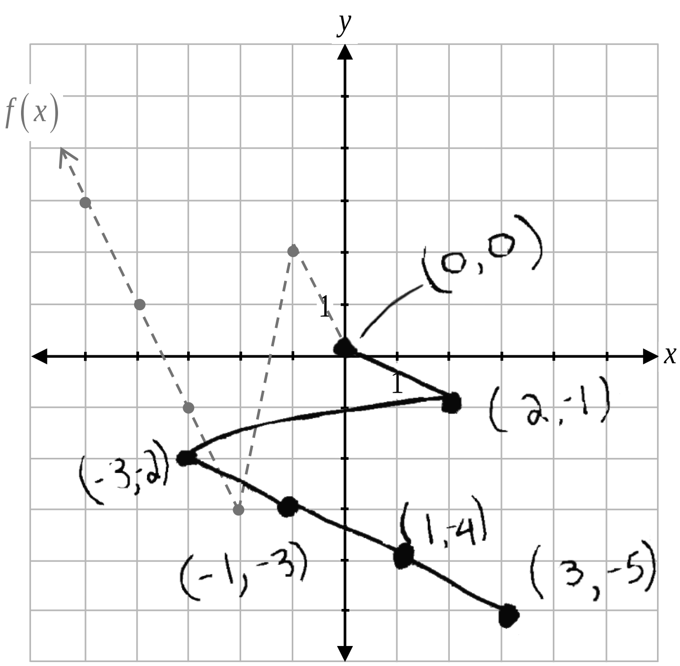
1 mark

Exemplar 1



0 out of 1

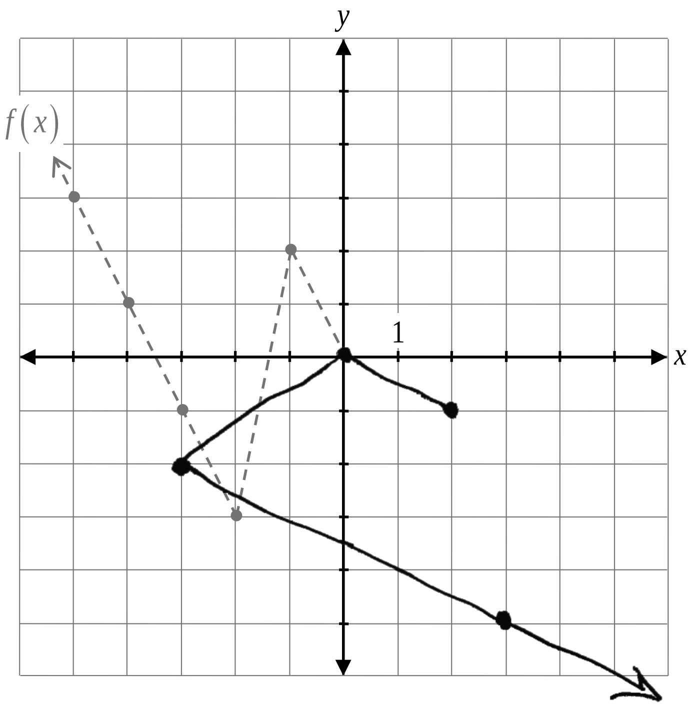
Exemplar 2



1 out of 1

award full marks
E9 (arrowhead omitted)

Exemplar 3



½ out of 1

award full marks
– ½ mark for procedural error (connected points in incorrect order)

This page was intentionally left blank.

a) Determine the remainder when $3x^3 + 5x^2 - 13x - 3$ is divided by $(x + 3)$.

b) Is $(x + 3)$ a factor of $3x^3 + 5x^2 - 13x - 3$? Explain your reasoning.

Solution

a) $3(-3)^3 + 5(-3)^2 - 13(-3) - 3$

$3(-27) + 5(9) - 13(-3) - 3$

0

1 mark for remainder theorem

or

-3	3	5	-13	-3
		-9	12	3
	3	-4	-1	0

1 mark for synthetic division
(or any equivalent strategy)

The remainder is 0.

1 mark

b) Yes, because the remainder is 0.

1 mark for explanation

1 mark

Exemplar 1

a)

−3	3	5	−13	3
		−9	−12	75
	3	4	−25	78

Remainder = 78

½ out of 1

award full marks
– ½ mark for arithmetic error in line 3
E7 (transcription error in line 1)

b)

No, because there is a remainder of 78. If the remainder was zero, $(x+3)$ would be a factor.

1 out of 1

Exemplar 2

a)

−3	3	5	−13	−3
		−9	12	3
	3	−4	−1	0

R = 0

1 out of 1

b)

The remainder = 0

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 42 R9

State the range, the y-intercept, and the equation of the asymptote of the exponential function, $f(x) = 3^{x-1} + 2$.

Solution

Range: $(2, \infty)$ — 1 mark for range

y-intercept: $\frac{7}{3}$ — 1 mark for y-intercept

Equation of the asymptote: $y = 2$ — 1 mark for equation of the asymptote

3 marks

Exemplar 1

Range: $[2, \infty)$

y-intercept: $7/3$

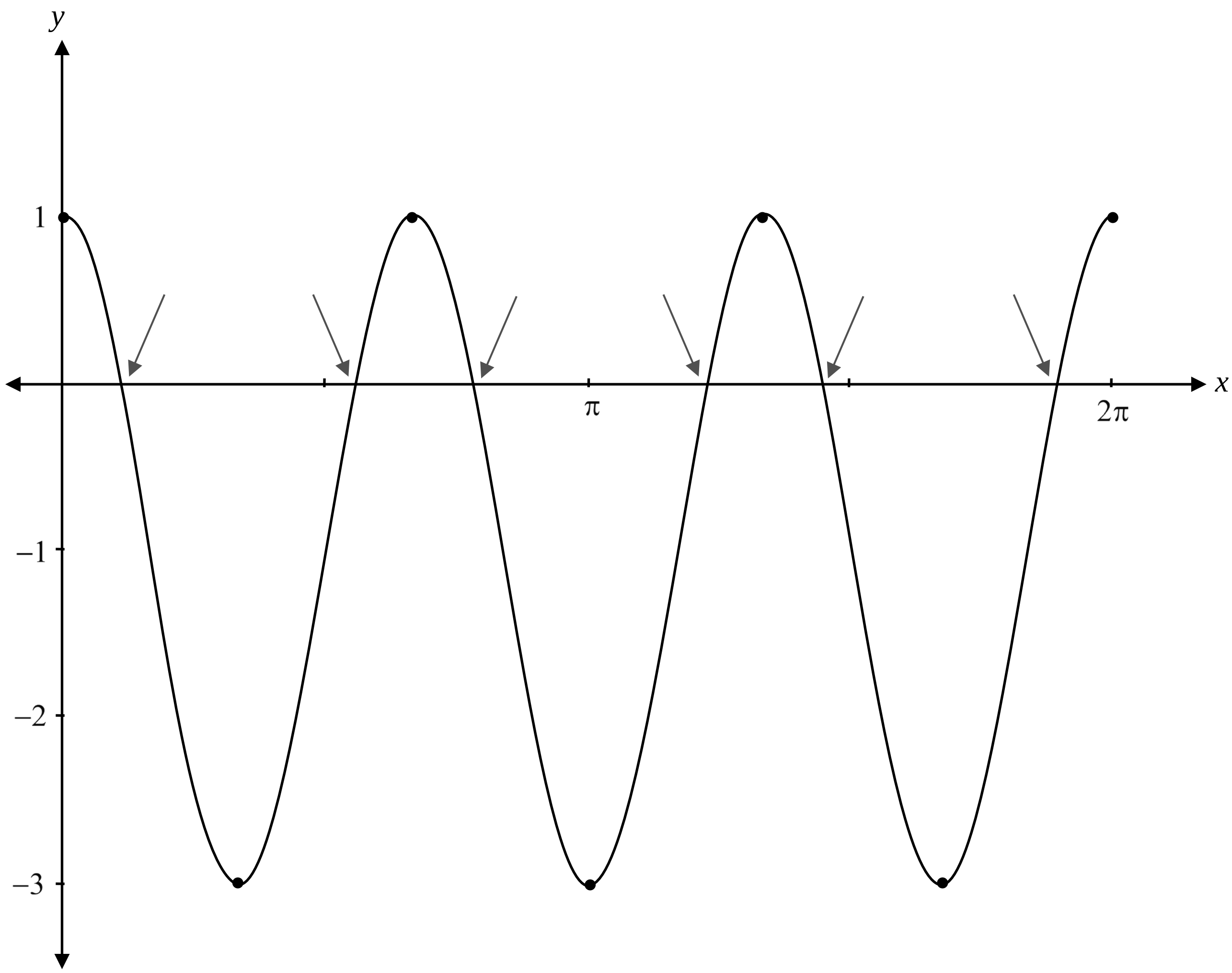
Equation of the asymptote: $x = 2$

2 out of 3

+ 1 mark for range
+ 1 mark for *y*-intercept
E8 (bracket error made when stating range)

Question 43 T5

The graph of $y = 2\cos(3x) - 1$ below can be used to solve the equation $0 = 2\cos(3x) - 1$ over the interval $[0, 2\pi]$. Indicate on the graph where to find at least one solution to the equation $0 = 2\cos(3x) - 1$.



Solution

See arrows on graph above. 1 mark for indicating a solution on graph

1 mark

Note:

- Students do not need to show all six arrows.

Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply:

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in step leading to final answer
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: ____

Date marked: ____

Booklet No.: ____

Problem(s) noted: ____

____

____

____

____

____

Question(s) affected: ____

____

____

Action taken or rationale for assigning marks: __________________________

____

____

____

____

____

____

____

____

Follow-up: ____

____

____

____

____

____

Decision: ____

____

____

____

____

____

____

Marker's Signature: ____

Principal's Signature: ____

For Department Use Only—After Marking Complete

Consultant: ____

Date: ____

Appendix C

Table of Questions by Unit and Learning Outcome

Question	Learning Outcome	Mark
Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
5	R1, R5	2
6	R2, R3	3
15	R1	1
16	R6	1
28a)	R1	1
28b)	R1	1
36	R5	1
38	R1	2
40	R5	1
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
9	T4	1
21	T3	1
22	T1	1
27	T3	2
33	T4	3
35	T1	2
37a)	T1	1
39	T4	3
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
1	P4	3
4	P3	3
12	P2	3
20	P2	1
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
11	R11	2
14	R12	3
19	R11	1
30	R12	2
41a)	R11	1
41b)	R11	1

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
3	T5	3
10	T6	3
25	T6	2
31	T5	3
37b)	T6	3
43	T5	1

Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
2	R10	2
7	R8	2
13	R8	2
18	R7	1
24	R10	2
26	R9	3
42	R9	3

Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
8	R13	2
17	R14	1
23	R13	1
29a)	R14	3
29b)	R14	1
32	R14	1
34	R13	4