Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre. Order online at www.manitobalrc.ca.

This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html. Websites are subject to change without notice.

Disponible en français.
While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.
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Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the Answer/Scoring Sheet are identical
- students and markers use only a pencil to complete the Answer/Scoring Sheets
- the totals of each of the four parts are written at the bottom
- each student’s final result is recorded, by booklet number, on the corresponding Answer/Scoring Sheet
- the Answer/Scoring Sheet is complete
- a photocopy has been made for school records

Once marking is completed, please forward the Answer/Scoring Sheets to Manitoba Education and Training in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section “Booklet 2 Questions.”

To receive full marks, a student’s response must be complete and correct. Where alternative answering methods are possible, the Marking Guide attempts to address the most common solutions. For general guidelines regarding the scoring of students’ responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an Answer/Scoring Sheet is marked with "0" only (e.g., student was present but did not attempt any questions), please document this on the Irregular Test Booklet Report.
**Assistance**

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Youyi Sun  
Assessment Consultant  
Grade 12 Pre-Calculus Mathematics  
Telephone: 204-945-7590  
Toll-Free: 1-800-282-8069, ext. 7590  
Email: youyi.sun@gov.mb.ca
Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Answer/Scoring Sheet that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the Answer/Scoring Sheet in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

<table>
<thead>
<tr>
<th>COMMUNICATION ERRORS / ERREURS DE COMMUNICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shade in the circles below for a maximum total deduction of 5 marks (½ mark deduction per error).</td>
</tr>
<tr>
<td>Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur)</td>
</tr>
<tr>
<td>E1 ● E2 ○ E3 ○ E4 ○ E5 ○</td>
</tr>
<tr>
<td>E6 ○ E7 ● E8 ● E9 ○ E10 ○</td>
</tr>
</tbody>
</table>

Example: Marks assigned to the student.

<table>
<thead>
<tr>
<th>Marks Awarded</th>
<th>Booklet 1</th>
<th>Selected Response</th>
<th>Booklet 2</th>
<th>Communication Errors (Deduct)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
<td>7</td>
<td>40</td>
<td>1½</td>
<td>70½</td>
</tr>
<tr>
<td>Total Marks</td>
<td>36</td>
<td>9</td>
<td>45</td>
<td>maximum deduction of 5 marks</td>
<td>90</td>
</tr>
</tbody>
</table>
Question 1

Determine the length of the radius, $r$, given an arc length of 20 metres and a central angle of $160^\circ$.

![Diagram](https://via.placeholder.com/150)

**Solution**

\[
\theta = (160) \left( \frac{\pi}{180} \right) = \frac{8\pi}{9}
\]

1 mark for conversion

\[ r = \frac{s}{\theta} \]

\[ r = \frac{20}{\left( \frac{8\pi}{9} \right)} \]

\[ r = \frac{180}{8\pi} \text{ m} \]

or

\[ r = 7.162 \text{ m} \]

2 marks
Exemplar 1

\[
160 \times \frac{\pi}{180} = \frac{8 \pi}{9} \\
20 = \frac{8 \pi}{9} \quad \text{r} \\
\frac{8 \pi}{9} = \frac{8 \pi}{9} \\
20 \times \frac{9}{8 \pi} = \text{r} \\
\boxed{70.685 = \text{r}}
\]

1½ out of 2
award full marks
– ½ mark for arithmetic error in line 5
E5 (units of measure omitted in final answer)

Exemplar 2

\[
S = \theta r \\
20m = 160 \text{r} \\
\frac{20m}{160} = 0.125 \text{m}
\]

1 out of 2
+ 1 mark for substitution
E1 (final answer not stated)
Exemplar 3

\[ S = \theta r \]

\[ 20 = 160 r \]

\[ 20 = \frac{2\pi}{3} r \]

\[ r = 9.5493 \text{ m} \]

1 out of 2

+ 1 mark for substitution
This page was intentionally left blank.
There are eight cars parked in a row. Determine the number of possible arrangements of these eight cars if Mrs. Jones must always park in the third spot and Mr. Rodriguez must always park in the last spot.

**Solution**

\[ 6 \cdot 5 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \]

720  

1 mark
Exemplar 1

\[ \begin{array}{ccccccc}
6 & 5 & 1 & 4 & 3 & a & 1 \\
= P_6 + P_1 + P_1 \\
= 720 + 1 + 1 \\
= 722
\end{array} \]

0 out of 1

Exemplar 2

\[ \begin{array}{ccccccc}
8 & 7 & 1 & 6 & 5 & 4 & 3 & 1 \\
= 20160 \text{ ways}
\end{array} \]

0 out of 1

Exemplar 3

\[ 6! \]

1 out of 1

award full marks
E1 (final answer not stated)
Bill wins $1 300 000 in a lottery and invests the entire amount at an annual interest rate of 2.5% compounded quarterly. He will withdraw $10 000 at the end of every three months.

Determine, algebraically, the total number of withdrawals, including the partial amount, that Bill can make until there is no money left. Express your answer as a whole number.

Use the formula:

\[ PV = \frac{R \left[ 1 - \left(1 + \frac{i}{4}\right)^{-n}\right]}{i} \]

where  
- \( PV \) = the present value deposited
- \( R \) = the amount of each withdrawal
- \( n \) = the number of equal withdrawals
- \( i \) = the annual interest rate (in decimal form)
- \( \frac{i}{4} \) = the number of compounding periods

**Solution**

\[ 1300000 = \frac{10000 \left[ 1 - \left(1 + \frac{0.025}{4}\right)^{-n}\right]}{0.025} \]

\[ 8125 = 10000 \left[ 1 - (1.00625)^{-n}\right] \]

\[ 0.8125 = 1 - (1.00625)^{-n} \]

\[ -0.1875 = -(1.00625)^{-n} \]

\[ \log(0.1875) = -n \log(1.00625) \]

\[ -\frac{\log(0.1875)}{\log(1.00625)} = n \]

\[ 268.672348 = n \]

Bill can make 269 withdrawals.

\[ \frac{1}{2} \text{ mark for substitution} \]

\[ \frac{1}{2} \text{ mark for simplification} \]

\[ \frac{1}{2} \text{ mark for applying logarithms} \]

\[ 1 \text{ mark for power law} \]

\[ \frac{1}{2} \text{ mark for solving for } n \]

**3 marks**
Exemplar 1

\[ P = \frac{10000 \times (1 - (1 + 0.00625)^{-n})}{0.00625} \]

\[ P = \frac{10000 \times (1 - (1.00625)^{-n})}{0.00625} \]

\[ 0.8125 = 1 - (1.00625)^{-n} \]

\[ -0.1875 = -(1.00625)^{-n} \]

\[ -0.1875 = \frac{-n \log 1.00625}{\log 1.00625} \]

\[ -0.1875 = n \]

Bill needs 69 equal withdrawals and then a partial withdrawal until there is no more money remaining.

2 out of 3

+ ½ mark for substitution
+ ½ mark for simplification
+ 1 mark for power law
+ ½ mark for solving for \( n \)
- ½ mark for arithmetic error in line 6
Exemplar 2

\[ 1300 000 = 10 000 \left( 1 - \left(1 + \frac{0.025}{4}\right)^{-\frac{n}{4}} \right) \]
\[ = 0.00625 \]

\[ 1300 000 = \left( \frac{10 000 \left(1 - (1.00625)^{-n}\right)}{0.00625} \right) \]
\[ = 0.00625 \]

\[ \frac{81.25}{1000} = \frac{10 000 \left(1 - (1.00625)^{-n}\right)}{10 000} \]
\[ -1 \cdot 0.8125 = \left(1 - (1.00625)^{-n}\right) - 1 \]
\[ -1 \cdot (1.8125) = - \left(1.00625^{-n}\right) \cdot -1 \]
\[ 1.8125 = (1.00625)^{-n} \]

\[ \log_{1.00625}(1.8125) = -n \]
\[ 95.45018201 = -n \]
\[ -95.450 = n \]

2½ out of 3

+ ½ mark for substitution
+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for solving for \( n \)
E1 (impossible solution not rejected in final answer)
Exemplar 3

\[
(0.0083) \frac{1300000}{10000} = 10000 \left[ \frac{1 - (1 + 0.0083)^{-n}}{0.0083} \right]
\]

\[
\frac{10790}{10000} = 10000 \left[ 1 - (1.0083)^{-n} \right]
\]

\[
1.079 = \frac{1 - (1.0083)^{-n}}{1}
\]

\[
1.079 = 1.0083^{-n}
\]

\[
\log 1.079 = -n \log 1.0083
\]

\[
\frac{\log 1.079}{\log 1.0083} = -n
\]

\[
-n = \frac{0.199}{-1}
\]

\[
n = -0.199
\]

2 out of 3

+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for solving for \( n \)
E1 (impossible solution not rejected in final answer)

:: He can make 10 withdrawals until there is no money remaining
Determine and simplify the 12th term in the binomial expansion of \( \left(x^3 - \frac{1}{2x^2}\right)^{12}\).

**Solution**

\[ t_{12} = \binom{12}{11} \left(x^3\right)^{11} \left(-\frac{1}{2x^2}\right)^{11} \]

2 marks (1 mark for \( \binom{12}{11} \); \( \frac{1}{2} \) mark for each consistent factor)

\[ t_{12} = (12)(x^3)^{11} \left(-\frac{1}{2048x^{22}}\right) \]

\[ t_{12} = -\frac{12}{2048x^{19}} \]

1 mark for simplification (\( \frac{1}{2} \) mark for coefficient; \( \frac{1}{2} \) mark for exponent)

or

\[ t_{12} = -\frac{3}{512x^{19}} \]

or

\[ t_{12} = -0.006x^{-19} \]

3 marks
Exemplar 1

\[ t_{k+1} = \binom{n}{k} (a)^{n-k} (b)^k \quad k \text{ is one less than solving for } \]

\[ t_{12} = 12 \binom{11}{1} (x^3)^1 \left( \frac{1}{2x} \right)^{11} \]

\[ = (12) (x^3) \left( \frac{1}{2048x^{22}} \right) \]

\[ = \frac{12x^3}{2048x^{22}} \]

\[ = \frac{3x^3}{52x^8} \]

2 out of 3

+ 1 mark for \( 12 \binom{11}{1} \)

+ 1 mark for consistent factors

Exemplar 2

\[ \binom{12}{2} \left( x^3 \right)^0 \left( \frac{-1}{2x^2} \right)^{12} = \left( 1 \right) \left( 1 \right) \left( \frac{1}{4096x^{24}} \right) \]

\[ = \frac{1}{4096x^{24}} \]

2 out of 3

+ 1 mark for consistent factors

+ 1 mark for simplification
Exemplar 3

\[
\begin{align*}
\frac{t}{12} &= 13 \times C_{12} \left( x^3 \right)^{13-12} \left( \frac{1}{2x^2} \right)^{12} \\
\frac{t}{12} &= 13 \times C_{12} \left( x^3 \right) \left( \frac{1}{4096x^{24}} \right) \\
\frac{t}{12} &= 12 \left( x^3 \right) \left( \frac{1}{4096x^{24}} \right) \\
\frac{t}{12} &= 12 \left( \frac{1}{4096x^{27}} \right) \\
\frac{t}{12} &= \frac{1}{4096x^{27}}
\end{align*}
\]

1 out of 3

+ 1 mark for consistent factors
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Question 5

Solve, algebraically.

\[ e^{2x-3} = 7^{x+1} \]

**Solution**

\[ \ln e^{2x-3} = \ln 7^{x+1} \]

\[ (2x - 3) \ln e = (x + 1) \ln 7 \]

\[ 2x - 3 = x \ln 7 + \ln 7 \]

\[ 2x - x \ln 7 = \ln 7 + 3 \]

\[ x(2 - \ln 7) = \ln 7 + 3 \]

\[ x = \frac{\ln 7 + 3}{2 - \ln 7} \]

\[ x = 91.438\,783 \]

\[ x = 91.439 \]

½ mark for collecting terms with \( x \)

½ mark for isolating \( x \)

½ mark for applying logarithms

1 mark for power law

½ mark for evaluating quotient of logarithms

3 marks
Exemplar 1

\[(2x-3)\log_e = (x+1)\log 7\]
\[2x\log_e - 3\log_e = x\log 7 + \log 7\]

1½ out of 3
+ ½ mark for applying logarithms
+ 1 mark for power law

Exemplar 2

\[\ln e^{2x-3} = \ln 7^{x+1}\]
\[2x-3\ln e = x+1\ln 7\]
\[2x-3 = x\ln 7 + \ln 7\]

1½ out of 3
+ ½ mark for applying logarithms
+ 1 mark for power law
E4 (missing brackets but still implied)

Exemplar 3

\[2x-3\ln e = x+1\ln 7\]
\[2x-x = \ln 7 + 3\]
\[x = \ln 7 + 3\]
\[x = 4.946\]

2½ out of 3
award full marks
- ½ mark for procedural error in line 1

Exemplar 4

\[2x-3\ln e = x+1\ln 7\]

1 out of 3
+ ½ mark for applying logarithms
+ 1 mark for power law
- ½ mark for procedural error
Sketch the angle $\frac{7\pi}{3}$ in standard position.

**Solution**

Note:
- If the directional arrow is not indicated, deduct an E1 error (final answer not stated).
Exemplar 1

0 out of 1

Exemplar 2

½ out of 1

+ ½ mark for correct number of revolutions
Exemplar 3

\[
\frac{7\pi}{3} - \frac{6\pi}{3} = \frac{\pi}{3}
\]

\[
\frac{\pi}{3} \text{ or } \frac{\pi}{3}
\]

\[\frac{1}{2} \text{ out of } 1\]

+ \frac{1}{2} \text{ mark for an appropriate angle in quadrant I}
This page was intentionally left blank.
Question 7

Determine, algebraically, all of the zeros of the polynomial function \( P(x) = x^4 - 5x^3 - 4x^2 + 20x. \)

**Solution**

\[
P(x) = x\left(x^3 - 5x^2 - 4x + 20\right)
\]

\[
P(2) = 2\left(2^3 - 5(2)^2 - 4(2) + 20\right) \quad \text{1 mark for identifying one possible value of } x
\]

\[
P(2) = 0
\]

\[
\therefore (x - 2) \text{ is a factor}
\]

\[
\begin{array}{c|cccc}
2 & 1 & -5 & -4 & 20 \\
\hline
 & 2 & -6 & -20 \\
 & 1 & -3 & -10 & 0 \\
\end{array}
\]

\[
P(x) = x(x - 2)(x^2 - 3x - 10)
\]

\[
0 = x(x - 2)(x - 5)(x + 2) \quad \frac{1}{2} \text{ mark for identifying all the factors}
\]

\[
x = 0, x = 2, x = 5, x = -2 \quad \frac{1}{2} \text{ mark for consistent zeros}
\]

**3 marks**
Exemplar 1

\[ \begin{array}{c|cccc}
-2 & 1 & -5 & -4 & 20 \\
\hline
 & 1 & 14 & -20 \\
 & 1 & -7 & 10 & 0 \\
\end{array} \]

\[ x^3 - 7x^2 + 10x = x \left( x^2 - 7x + 10 \right) = x(x-2)(x-5) \]

Zero's: \( x, (x+2), (x-2), (x-5) \)

2 out of 3

+ 1 mark for identifying one possible value of \( x \)
+ 1 mark for synthetic division
Exemplar 2

\[ \frac{1}{2} \]

\[ \begin{array}{c|cccc}
2 & 1 & -5 & -4 & 0 \\
 & & 2 & -3 & -20 \\
\hline
1 & -3 & -10 & 0
\end{array} \]

\[ \text{Zeros: } 2, 5, -2 \]

\[ (x-2)(x^2 - 3x - 10) \]

\[ (x-2)(x-5)(x+2) \]

2 out of 3

+ 1 mark for identifying one possible value of \( x \)
+ 1 mark for synthetic division
+ \( \frac{1}{2} \) mark for consistent zeros
- \( \frac{1}{2} \) mark for procedural error in line 3
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Question 8

Justify why four of the terms in the binomial expansion of \((-x + y)^6\) are positive.

**Solution**

\((-x)^6(y)^0, (-x)^5(y)^1, (-x)^4(y)^2,\ldots\)

\(x^6, -x^5y, x^4y^2,\ldots\)

There are 7 terms. The first term is positive and the signs alternate.

1 mark for justification
Exemplar 1

because every even \( k \) value will give you a positive term and there is 4 even numbers between (0-6) inclusive.

1 out of 1

Exemplar 2

In every even term the negative \( k \) would become positive due to the exponent. In the last term the negative \( k \) would be to the power of 0, removing the negative again.

½ out of 1

award full marks
– ½ mark for terminology error
Determine the equation of $g(x)$ in terms of $f(x)$.

**Solution**

$$g(x) = -f(x - 3)$$

1 mark for vertical reflection
1 mark for horizontal translation

2 marks
Exemplar 1

\[ g(x) = -\sqrt{x - 3} \]

1 out of 2
award full marks
– 1 mark for concept error (not including \( f(x) \))

Exemplar 2

\[ g(x) = -\sqrt{f(x - 3)} - 3 \]

1 out of 2
award full marks
– 1 mark for concept error (reflection over \( y = 3 \))
Question 10

Prove the identity for all permissible values of $x$.

$$\frac{\sin x + \tan x}{\cot x + \csc x} = \frac{\sin^2 x}{\cos x}$$

Solution

Method 1

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sin x + \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{1}{\sin x}}$</td>
<td>$\frac{\sin^2 x}{\cos x}$</td>
</tr>
<tr>
<td>$\frac{\sin x \cos x + \sin x}{\cos x} \cdot \frac{\cos x + 1}{\sin x}$</td>
<td>$\frac{\sin x}{(\cos x + 1)}$</td>
</tr>
<tr>
<td>$\frac{\sin x}{\cos x}$</td>
<td>$\frac{\sin^2 x}{\cos x}$</td>
</tr>
</tbody>
</table>

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks
Question 10

Solution

Method 2

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
</table>
| \[
\sin x + \tan x \\
\frac{1}{\tan x} + \frac{1}{\sin x}
\] | \[
\sin^2 x \\
\cos x
\] |
| \[
\frac{\sin x + \tan x}{\sin x + \tan x} \\
\sin x \tan x
\] | \[
\sin x \cdot \frac{\sin x}{\cos x}
\] |
| \[
\sin x \cdot \frac{\sin x}{\cos x}
\] | \[
\sin^2 x \\
\cos x
\] |

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks
Exemplar 1

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{(\cos \sin x + \sin \cos \sin x)}{\cos \sin x + \sin x} )</td>
<td>( \frac{\sin^2 x}{\cos x} )</td>
</tr>
<tr>
<td>( \frac{\cos x \sin x}{\cos x} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{\cos x}{\sin x} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{\sin x}{\cos x} )</td>
<td></td>
</tr>
<tr>
<td>( \sin x \cdot \sin x \cdot \sin x \cdot \cos x )</td>
<td></td>
</tr>
<tr>
<td>( = \frac{\sin^2 x}{\cos x} )</td>
<td>√</td>
</tr>
</tbody>
</table>

1 out of 3

+ 1 mark for correct substitution of identities
E3 (variable omitted in an identity)
<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\sin x + \tan x}{\cos x} + \frac{1}{\sin x} )</td>
<td>( \frac{\sin^2 x}{\cos x} )</td>
</tr>
</tbody>
</table>

0 out of 3
### Exemplar 3

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\sin x + \tan x}{\cot x + \csc x} )</td>
<td>( \frac{\sin^2 x}{\cos x} )</td>
</tr>
<tr>
<td>( \frac{(\cos x) \sin x + \sin x}{\cos x} )</td>
<td>( \frac{\cos x \sin x + \sin x}{\cos x} )</td>
</tr>
<tr>
<td>( \frac{\cos x \sin x + \sin x}{\cos x} )</td>
<td>( \frac{1 + \cos x}{\sin x} )</td>
</tr>
</tbody>
</table>

**2 out of 3**

- + 1 mark for correct substitution of identities
- + 1 mark for algebraic strategies
Given that the point \((-2,1)\) is on the graph of \(y = f(x)\), describe how the coordinates of the corresponding point on the graph of \(y = f(4x)\) are different.

**Solution**

The \(x\)-value is divided by 4.

1 mark
Exemplar 1

\[ y = 1 \text{ will stay the same because there are no changes for } y. \]

\[ x \text{ will be compressed by } 4 \text{ so } x = -2 \text{ will be } x = -\frac{1}{2}. \]

1 out of 1

Exemplar 2

\[ (-2, 1) \rightarrow \left[ \left( \frac{-1}{2}, 1 \right) \right] \]

0 out of 1

Exemplar 3

The x value is different.

\(\frac{1}{2}\) out of 1

award full marks
– \(\frac{1}{2}\) mark for lack of clarity in explanation
Question 12

Using the laws of logarithms, completely expand the expression:

$$\log \left( \frac{x^2 \sqrt{y}}{w-1} \right)$$

**Solution**

$$2 \log x + \frac{1}{2} \log y - \log (w-1)$$

1 mark for product law
1 mark for power law (½ mark for each)
1 mark for quotient law

3 marks
Exemplar 1

\[
\log x^2 + \log y^{0.5} - \frac{\log w}{\log 1} = \log x^2 + 0.5 \log y - \frac{\log w}{\log 1}
\]

1½ out of 3
+ 1 mark for product law
+ ½ mark for power law

Exemplar 2

\[
\frac{\log x^2 + \log \sqrt{y}}{\log (w-1)}
\]

2 out of 3
+ 1 mark for product law
+ 1 mark for power law

Exemplar 3

\[
\log x^2 + \log \sqrt{y} - (\log (w-1))
\]

3 out of 3
award full marks
E7 (notation error in lines 1 to 3)
Question 13

Given \( \sec \theta = -\frac{5}{4} \) and \( \tan \theta > 0 \), state the quadrant in which \( \theta \) terminates.

Justify your answer.

**Solution**

Since \( \sec \theta \) is negative in quadrants II and III, and \( \tan \theta \) is positive in quadrants I and III, \( \theta \) terminates in quadrant III.

1 mark for justification

1 mark
Exemplar 1

\[
\cos \theta = \frac{4}{5} \quad \tan \theta = 70
\]

\[
\text{(Quadrant III)}
\]

½ out of 1

award full marks
– ½ mark for lack of clarity in justification

Exemplar 2

For \( \tan \theta \) to be greater than 0 it must be \( Q1 \) or \( Q3 \).

For \( \sec \theta \) to be negative it must be \( Q3 \) or \( Q2 \).

∴ \( \theta \) terminates in \( Q2 \).

1 out of 1

award full marks
E7 (transcription error)

Exemplar 3

III

0 out of 1
State an equation of a rational function that has a vertical asymptote at \( x = -8 \) and a horizontal asymptote at \( y = 9 \).

Solution

\[
y = \frac{9x}{x + 8}
\]

1 mark for vertical asymptote
1 mark for horizontal asymptote

Note:
- Other equations are possible.
Exemplar 1

\[ f(x) = \frac{1}{(x+8)} + q \]

2 out of 2

Exemplar 2

\[ g(x) = \frac{x}{x+8} \]

2 out of 2

award full marks
E2 (changing an equation to an expression)

Exemplar 3

\[ y = \frac{(x+8)(x-q)}{(x+8)} \]

0 out of 2

Exemplar 4

\[ y = \frac{9x^2 + 2}{(x^2-64)} \]

2 out of 2
Given the point \((5, 1)\), state the coordinates of the corresponding point after a reflection across the line \(y = x\).

**Solution**

\((1, 5)\)  

1 mark
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Question 16

Simplify.

\[
\frac{(n-13)!}{(n-12)!}
\]

**Solution**

\[
\frac{(n-13)!}{(n-12)(n-13)!} = \frac{1}{n-12}
\]

1 mark for factorial expansion

1 mark
Exemplar 1

\[
\frac{(n-13)(n-14)}{(n-12)(n-13)(n-14)} = \frac{1}{n-12}
\]

1 out of 1
award full marks
E7 (notation error)

Exemplar 2

\[
\frac{(n-13)!}{(n-12)(n-13)!} \cdot \frac{1}{n-12}
\]

\[\frac{(n-13)!}{(n-12)(n-13)!} \]

½ out of 1
award full marks
– ½ mark for arithmetic error

Exemplar 3

\[
\frac{(n-13)!}{(n-12)(n-13)!} \cdot \frac{1}{(n-12)!}
\]

1 out of 1
E1 (final answer not stated)

Exemplar 4

\[
\frac{(n-13)!}{(n-12)(n-13)!} \cdot \frac{1}{(n-12)!}
\]

½ out of 1
award full marks
– ½ mark for procedural error
Question 17

Given the graphs of \( y = f(x) \) and \( y = g(x) \),

\[
\begin{align*}
\text{Solution} \\
\text{a) determine the value of } f\left( g(2) \right). \\
g(2) &= 0 \quad \text{½ mark for the value of } g(2) \\
f(0) &= 4 \quad \text{½ mark for the value of } f\left( g(2) \right) \text{ consistent with } g(2) \\
& \quad \boxed{1 \text{ mark}} \\
\text{b) determine the value of } (g - f)(-3). \\
g(-3) - f(-3) &= \frac{1}{3} \quad \text{½ mark for the values of } g(-3) \text{ and } f(-3) \\
& \quad \text{½ mark for value of } (g - f)(-3) \text{ consistent with } g(-3) \text{ and } f(-3) \\
& \quad \boxed{1 \text{ mark}}
\end{align*}
\]
Exemplar 1

a) 
\[
g(2) = 1 \\
f(1) = 6
\]

½ out of 1
+ ½ mark for the value of \(f(g(2))\) consistent with \(g(2)\)

b) 
\[
f(-3) = -2 \\
g(-3) = 1
\]

½ out of 1
+ ½ mark for the values of \(g(-3)\) and \(f(-3)\)
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# Answer Key for Selected Response Questions

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Question 18

Identify the remainder when \( P(x) = 3x^3 - x^2 + 1 \) is divided by \( (x - 2) \).

a) \(-27\)
b) \(-19\)
c) 11
d) 21

d) 21

Question 19

Identify the logarithmic form of \( 2^x = \frac{1}{4} \).

a) \( \log_2 x = \frac{1}{4} \)
b) \( \log_2 \frac{1}{4} = x \)
c) \( \log_2 \left( \frac{1}{4} \right) = x \)
d) \( \log_2 \left( \frac{1}{4} \right) = 2 \)

d) \( \log_2 \left( \frac{1}{4} \right) = 2 \)

Question 20

Leah’s Pizzeria offers 9 different pizza toppings. Identify the expression that represents the number of different types of pizzas, with 3 different toppings, that can be made.

a) \( \binom{9}{3} \)
b) \( 9P_3 \)
c) \( \frac{9!}{3!} \)
d) \( 9!3! \)
Question 21

Given \((5, -4)\) is a point on the graph of \(y = f(x)\), identify the corresponding point on the graph of \(y = \frac{1}{f(x)}\).

a) \(\left(\frac{1}{5}, -4\right)\)

b) \(\left(5, -\frac{1}{4}\right)\)

c) \(\left(\frac{1}{5}, -\frac{1}{4}\right)\)

d) \((-4, 5)\)

Question 22

Identify the non-permissible value of \(x\) for \(1 + \sec x\) over \([0, \pi]\).

a) 0

b) \(\frac{\pi}{4}\)

c) \(\frac{\pi}{2}\)

d) \(\pi\)

Question 23

Indicate the combination that represents the circled term in the given row of Pascal’s triangle.

\[
\begin{array}{cccccc}
1 & 4 & 6 & 4 & 1 \\
\end{array}
\]

a) \(\binom{4}{3}\)

b) \(\binom{4}{4}\)

c) \(\binom{5}{3}\)

d) \(\binom{5}{4}\)
Question 24

Identify the x-intercept on the graph of \( f(x) = \sqrt{2(x + 5)} \).

a) \(-5\)
b) 0
c) \(\sqrt{10}\)
d) 5

Question 25

Identify the coterminal angle of \( \frac{\pi}{5} \) over the interval \(-\pi \leq \theta \leq 4\pi\).

a) \(\frac{9\pi}{5}\)
b) \(\frac{\pi}{5}\)
c) \(\frac{3\pi}{5}\)
d) \(\frac{11\pi}{5}\)

Question 26

Given \( f(x) = \{(2,6),(3,2),(3,4),(6,5)\}, \) identify the value of \( f(f(2)) \).

a) 3
b) 4
c) 5

Pre-Calculus Mathematics: Marking Guide (January 2019)
The graph of \( f(x) = (x - 1)^2 \) is translated 2 units to the left and 3 units up. Identify the equation of the transformed graph, \( g(x) \).

a) \( g(x) = (x + 1)^2 + 3 \)

b) \( g(x) = (x - 3)^2 + 3 \)

c) \( g(x) = (x + 2)^2 + 3 \)

d) \( g(x) = (x - 2)^2 + 3 \)
This page was intentionally left blank.
Given $\csc \theta = -\frac{8}{5}$, determine the exact value of $\cos 2\theta$.

**Solution**

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$= 1 - 2 \left( -\frac{5}{8} \right)^2$$

$$= 1 - 2 \cdot \frac{25}{64}$$

$$= 1 - \frac{50}{64}$$

$$= \frac{14}{64}$$

or

$$= \frac{7}{32}$$

1 mark for the value of $\sin \theta$

1 mark for substitution into correct identity

2 marks
Exemplar 1

\[
\cos 2 \theta = 1 - 2 \sin^2 \theta \\
= 1 - 2 \left( -\frac{5}{8} \right)^2 \\
= 1 - 2 \left( \frac{25}{64} \right) \\
= 1 - \frac{50}{64} \\
\]

\[
\cos 2 \theta = \frac{14}{64}
\]

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 5
E1 (impossible solution not rejected in final answer)

Exemplar 2

\[
\cos 2 \theta = 1 - \sin^2 \theta \\
= 1 - \left( -\frac{5}{8} \right)^2 \\
= 1 - \frac{25}{64} \\
= \frac{64}{64} - \frac{25}{64} \\
\]

\[
\cos 2 \theta = \frac{39}{64}
\]

1 out of 2

+ 1 mark for the value of \( \sin \theta \)
Exemplar 3

\[
\cos 2\theta = 1 - 2 \left( \frac{5}{8} \right)^2 \\
= 1 - 2 \left( \frac{25}{64} \right) \\
= \frac{64}{64} - \frac{50}{64} \\
= \frac{14}{64}
\]

1½ out of 2

award full marks
- ½ mark for procedural error in line 1
E2 (changing an equation to an expression)

Exemplar 4

\[
\cos \theta = -\frac{5}{8} \\
\cos 2\theta = 2 \cos^2 \theta - 1 \\
= 2 \left( -\frac{5}{8} \right)^2 - 1 \\
= 2 \left( \frac{25}{64} \right) - 1 \\
= \frac{50}{64} - \frac{64}{64} \\
= \frac{-14}{64}
\]

1 out of 2

+ 1 mark for substitution into correct identity
This page was intentionally left blank.
Determine the period of the sinusoidal function, $f(x) = -6 \cos \left( \frac{\pi}{6} (x + 1) \right) + 5$.

**Solution**

Period = $\frac{2\pi}{\left( \frac{\pi}{6} \right)}$

= 12  

1 mark
Exemplar 1

\[
\text{Period} = \frac{\pi}{\frac{\pi}{6}}
\]

\[= 6 \]

0 out of 1

Exemplar 2

\[
\text{Period} = \frac{2\pi}{\frac{\pi}{6}}
\]

1 out of 1

award full marks
E1 (final answer not stated)
Determine, algebraically, the equation of \( P(x) \), given the graph of the polynomial function \( P(x) \).

**Solution**

\[
P(x) = a(x - 3)^2(x - 1)(x + 2)
\]

\[
-36 = a(-3)^2(-1)(2)
-36 = 18a
\]

\[
a = 2
\]

\[
P(x) = 2(x - 3)^2(x - 1)(x + 2)
\]

½ mark for factors of \( P(x) \)

½ mark for multiplicity of 2 at \( x = 3 \)

½ mark for substitution of \( P(0) = -36 \)

½ mark for correct value of \( a \)

2 marks
Exemplar 1

\[(x+2)(x-1)(x-3)(x-3)
\]
\[\left( x^2 + 2 \right) \left( x^2 - 6x + 9 \right) \]
\[x^4 - 6x^3 + 9x^2 + x^3 - 6x^2 + 9x - 2x^2 + 12x - 18 \]
\[2 \left( x^4 - 5x^3 + x^2 + 21x - 18 \right) \]

\[ P(x) = \frac{2x^4 + 10x^3 + 12x^2 - 36}{2} \]

2 out of 2

Exemplar 2

\[ P(x) = \frac{(x+2)(x-1)(x-3)^2 - 36}{(x-3)^2 - 36} \]

1 out of 2

+ ½ mark for factors of \( P(x) \)
+ ½ mark for multiplicity of 2 at \( x = 3 \)
Solve $2 \sin^2 \theta - 7 \sin \theta - 4 = 0$ where $\theta \in \mathbb{R}$.

**Solution**

$$(2 \sin \theta + 1)(\sin \theta - 4) = 0$$

$\sin \theta = -\frac{1}{2}$  \hspace{1cm} $\sin \theta = 4$  \hspace{1cm} 1 mark for solving for $\sin \theta$ (½ mark for each branch)

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$  \hspace{1cm} No solution  \hspace{1cm} 2 marks for solving for $\theta$ (1 mark for each branch)

$\theta = \frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}$

$\theta = \frac{11\pi}{6} + 2k\pi, k \in \mathbb{Z}$  \hspace{1cm} 1 mark for general solution

**4 marks**
Exemplar 1

\[ 2x^2 - 2x - 4 \geq 0 \]
\[ S \sin \theta = 4 \]
\[ S \frac{\sin \theta}{2} = -\frac{1}{2} \]
\[ \theta = \frac{7\pi}{6} \]
\[ \frac{7\pi}{6} + 2\pi k, \ k \in \mathbb{R} \]
\[ \frac{5\pi}{6} + 2\pi k, \ k \in \mathbb{R} \]

2 out of 4

+ 1 mark for solving for \( \sin \theta \)
+ \( \frac{1}{2} \) mark for solving for \( \theta \)
+ 1 mark for general solution

- \( \frac{1}{2} \) mark for procedural error in lines 6 and 7 (\( k \in \mathbb{R} \))
E3 (variable introduced without being defined)
E2 (changing an equation to an expression)
Exemplar 2

\[(2\sin\theta+4)(\sin\theta-1)=0\]

\[
\begin{align*}
2\sin\theta+4 & = 0 \\
\sin\theta & = -\frac{4}{2} = -2
\end{align*}
\]

no solution

\[
\begin{align*}
\sin\theta-1 & = 0 \\
\sin\theta & = 1
\end{align*}
\]

\[
\begin{align*}
\theta & = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z} \\
\frac{3\pi}{2} + 2\pi k, k \in \mathbb{Z}
\end{align*}
\]

2½ out of 4

award full marks
– ½ mark for arithmetic error in line 1
– 1 mark for concept error in line 5

Exemplar 3

\[(2\sin\theta+1)(\sin\theta-4)=0\]

\[
\begin{align*}
\sin\theta & = -\frac{1}{2} \\
\sin\theta & = 4
\end{align*}
\]

\[
\begin{align*}
0 & = 30^\circ \\
0 & = 210^\circ \\
0 & = 330^\circ
\end{align*}
\]

3 out of 4

+ 1 mark for solving for \(\sin\theta\)
+ 2 marks for solving for \(\theta\)
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Question 32

Justify that the shapes of the graphs of $f(x) = (x + 1)^2 (x - 1)$ and $g(x) = (x + 1)^2 (x - 1)^3$ are different as they approach the $x$-intercept at $x = 1$.

**Solution**

At $x = 1$, both graphs pass through the $x$-axis, however, the graph of $g(x)$ flattens as it passes through the $x$-axis.

1 mark
Exemplar 1

The $g(x)$ graph will have a $3$ multiplicity that will look like this.

As the $f(x)$ graph will go straight through.

1 out of 1

Exemplar 2

The graph will have a different type of curve going through the coordinate of $(1, 0)$. 

0 out of 1

Exemplar 3

The graph of $f(x)$ would pass through the graph at $x = 1$, the graph $g(x)$ would "slide" along the $x$-axis before passing through.

$\frac{1}{2}$ out of 1

award full marks
– $\frac{1}{2}$ mark for lack of clarity in justification

Exemplar 4

The graph of $g(x)$ will flatten as it crosses the zero at $x = 1$, while $f(x)$ will not flatten.

1 out of 1
Question 33

Determine the exact value of \( \cot \theta \) if \( \cos \theta = -\frac{4}{7} \) and \( \sin \theta \) is positive.

**Solution**

\[ x^2 + y^2 = r^2 \]

\[ (-4)^2 + y^2 = (7)^2 \]

\[ 16 + y^2 = 49 \]

\[ y^2 = 49 - 16 \]

\[ y = \pm \sqrt{33} \]

\[ \cot \theta = \frac{-4}{\sqrt{33}} \]

\[ \text{or} \]

\[ \cot \theta = \frac{-4\sqrt{33}}{33} \]

Note:
- Accept any of the following values for \( y \): \( y = \pm \sqrt{33}, y = \sqrt{33}, \text{ or } y = -\sqrt{33} \).
Exemplar 1

\[7^2 - 4^2 = b^2\]
\[41 - 16 = b^2\]
\[\sqrt{35} = b\]
\[\cot \theta = \frac{4}{\sqrt{35}}\]

1 out of 2
+ ½ mark for substitution
+ ½ mark for consistent value of \(\cot \theta\)

Exemplar 2

\[4^2 + y^2 = 7^2\]
\[16 + y^2 = 49\]
\[y^2 = 33\]
\[y = \sqrt{33}\]

\[\tan \theta = \frac{\sqrt{33}}{7}\]
\[\cot \theta = \frac{7}{\sqrt{33}}\]

1 out of 2
+ ½ mark for substitution
+ ½ mark for solving for \(y\)
Exemplar 3

\[ \cot \theta = \frac{\cos \theta}{\sin \theta} \]

\[ = \frac{-4}{7} \cdot \frac{1}{\sqrt{33}} \]

\[ = \frac{-4}{7} \times \frac{1}{\sqrt{33}} \]

**1 out of 2**

award full marks

– 1 mark for concept error in line 2
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Question 34

Sketch the graph of \( f(x) = -\log_2(x) + 2 \).

**Solution**

1 mark for asymptotic behaviour approaching \( x = 0 \)
1 mark for vertical reflection
1 mark for vertical translation

**3 marks**
Exemplar 1

3 out of 3

award full marks
E10 (asymptote omitted but still implied)
E9 (arrowhead omitted)

Exemplar 2

2½ out of 3

award full marks
– ½ mark for procedural error (x-intercept omitted)
Exemplar 3

2 out of 3

award full marks
– ½ mark for procedural error (x-intercept omitted)
– ½ mark for incorrect shape
E10 (asymptote omitted but still implied)

Exemplar 4

2 out of 3

award full marks
– 1 mark for concept error (including a horizontal asymptote)
This page was intentionally left blank.
State the range of \( f(x) = \sqrt{x + 4} \).

**Solution**

Range: \([0, \infty)\) \hspace{1cm} 1 mark

or

Range: \( \{ y \in \mathbb{R} \mid y \geq 0 \} \)
Exemplar 1

Range: $\left( 0, \infty \right)$

1 out of 1
award full marks
E8 (bracket error made when stating domain)

Exemplar 2

Range: $\left[ 0, \infty \right)$

1 out of 1
award full marks
E8 (range written in incorrect order)

Exemplar 3

Range: $y \geq 0$

1 out of 1
Sophie correctly solved the logarithmic equation, \( \log_7(x - 1) = \log_7(2x - 2) \).

\[
\begin{align*}
\frac{x - 1}{2x - 2} &= 1 \\
-x + 2 &= 2x - x \\
-x &= -2x + x \\
-x &= -2
\end{align*}
\]

Explain why \( x = 1 \) is an extraneous root.

**Solution**

The root, \( x = 1 \), is an extraneous root because the argument of a logarithm cannot be zero.

1 mark
Exemplar 1

If you substitute the answer you get back into the equation and it doesn't work, it is extraneous.

0 out of 1

Exemplar 2

You can't have a negative argument in a logarithmic equation so if you solve & get a value that's a negative, it is an extraneous root.

0 out of 1

Exemplar 3

You can't take the log of zero.

1 out of 1

Exemplar 4

When you plug $x=1$ into the equation, you get 0.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation
Sketch the graph of \( f(x) = \sqrt{4x} - 1 \).

**Solution**

1 mark for shape of a radical function
1 mark for horizontal compression
1 mark for vertical translation

3 marks
Exemplar 1

2 out of 3
+ 1 mark for shape of a radical function
+ 1 mark for vertical translation

Exemplar 2

2 out of 3
+ 1 mark for shape of a radical function
+ 1 mark for horizontal compression
Solve, algebraically.

\[ _n \binom{C}{2} = 2n + 7 \]

**Solution**

\[ \frac{n!}{(n-2)!2!} = 2n + 7 \]

\[ \frac{n(n-1)(n-2)!}{(n-2)!} = 2!(2n + 7) \]

\[ n(n-1) = 2(2n + 7) \]

\[ n^2 - n = 4n + 14 \]

\[ n^2 - 5n - 14 = 0 \]

\[ (n + 2)(n - 7) = 0 \]

\[ n = 7 \]

½ mark for substitution into equation

½ mark for factorial expansion

½ mark for simplification of factorial

½ mark for simplification

½ mark for the permissible value of \( n \)

½ mark for showing the rejection of the extraneous root

3 marks
Exemplar 1

\[
\frac{n!}{(n-2)!} = 2n + 7
\]

\[
\frac{n(n-1)(n-2)!}{(n-3)!} = 2n + 7
\]

\[n(n-1) = 2n + 7\]

\[n^2 - n = 2n + 7\]

\[-2n - 7 = -2n - 7\]

\[n^2 - 3n - 7 = 0\]

1½ out of 3

+ ½ mark for factorial expansion
+ ½ mark for simplification of factorial
+ ½ mark for simplification
Exemplar 2

\[ \frac{n!}{n-2!\cdot 2!} = 2n+7 \]

\[ \frac{(n)(n-1)(n-2)!}{n-2!\cdot 2!} = 2n+7 \]

\[ \frac{n^2-n}{2!} = 2n+7 \]

\[ n^2 - n = 4n + 14 \]

\[ n^2 - 5n - 14 = 0 \]

\[ (n - 7)(n + 2) = 0 \]

\[ n = 7 \]

3 out of 3

award full marks
E4 (missing brackets but still implied in lines 1 and 2)
Exemplar 3

\[ \frac{n!}{2!(n-2)!} = 2n + 7 \]
\[ \frac{n(n-1)}{2} = 2n + 7 \]
\[ n^2 - n = 4n + 14 \]
\[ n^2 - 5n - 14 = 0 \]
\[ (n-7)(n+2) = 0 \]
\[ n = 7 \]

2½ out of 3
+ ½ mark for substitution into equation
+ ½ mark for factorial expansion
+ ½ mark for simplification of factorial
+ ½ mark for simplification
+ ½ mark for the permissible value of \( n \)
E7 (notation error in line 1)
Given \( f(x) = x^2 - 1 \) and \( g(x) = x - 3 \), explain why the domain of \( h(x) = \frac{f(x)}{g(x)} \) has a restriction when \( x = 3 \).

**Solution**

When \( x = 3 \), the denominator is equal to zero and it is not possible to divide by zero.

1 mark
Exemplar 1

\[ \frac{x^2 - 1}{x - 3} \]

* It has a restriction of 3 because that's where the graph does not exist because \( x=3 \) is an asymptote.

0 out of 1

Exemplar 2

because the domain can only be where both graphs exist.

0 out of 1

Exemplar 3

When \( x=3 \) \( h(x) \neq \frac{9}{0} \) and because we can't divide by 0, the point \( a^+ \) \( x=3 \) becomes a point of discontinuity

0 out of 1

Exemplar 4

\[ h(x) = \frac{3^2 - 1}{3 - 3} \]

\( h(x) \) is undefined when \( x=3 \)

0 out of 1

Exemplar 5

\[ h(x) = \frac{x^2 - 1}{x - 3} \]

NPV is \( x \neq 3 \) because it's on the denominator

0 out of 1
Question 40

Evaluate.

\[
\frac{\cot \left( \frac{11\pi}{6} \right) \sin \left( -\frac{4\pi}{3} \right)}{\cos \left( \frac{2\pi}{3} \right)}
\]

**Solution**

\[
\left( -\sqrt{3} \right) \left( \frac{\sqrt{3}}{2} \right) \left( -\frac{1}{2} \right) \left( -\frac{2}{1} \right)
\]

1 mark for \( \cot \left( \frac{11\pi}{6} \right) \) (½ mark for quadrant; ½ mark for value)

1 mark for \( \sin \left( -\frac{4\pi}{3} \right) \) (½ mark for quadrant; ½ mark for value)

1 mark for \( \cos \left( \frac{2\pi}{3} \right) \) (½ mark for quadrant; ½ mark for value)

3 marks
Exemplar 1

\[
\left( \frac{1}{\sqrt{3}} \right) \left( \frac{1}{2} \right) \]

\[
- \frac{\sqrt{3}}{2}
\]

\[
\frac{1}{2 \sqrt{3}} \times \frac{a}{\sqrt{3}} = \frac{1}{2(3)}
\]

\[
= \frac{2}{6}
\]

\[
= \frac{1}{3}
\]

1 out of 3

+ ½ mark for quadrant of \( \sin \left( -\frac{4\pi}{3} \right) \)

+ ½ mark for quadrant of \( \cos \left( \frac{2\pi}{3} \right) \)

E7 (transcription error in line 2)
Exemplar 2

\[
\left( \frac{\sqrt{3}}{1} \right) \left( -\frac{\sqrt{3}}{2} \right) \left( \frac{1}{2} \right) \left( \frac{2}{-1} \right) \left[ -3 \right]
\]

1½ out of 3

+ ½ mark for value of \( \cot \left( \frac{11\pi}{6} \right) \)

+ ½ mark for value of \( \sin \left( -\frac{4\pi}{3} \right) \)

+ 1 mark for value of \( \cos \left( \frac{2\pi}{3} \right) \)

- ½ mark for arithmetic error in line 2
Exemplar 3

\[
\frac{\cos \left( \frac{11\pi}{6} \right) \sin \left( -\frac{4\pi}{3} \right)}{\cos \left( \frac{2\pi}{3} \right)} \\
\left( \frac{\sqrt{3}}{a} \right) \left( -\frac{2}{1} \right) \left( \frac{\sqrt{3}}{2} \right) \left( -\frac{2}{1} \right) \\
\left( \frac{3}{4} \right) \left( -\frac{4}{1} \right) \\
-\frac{12}{4} \\
-3
\]

2½ out of 3

award full marks
– ½ mark for arithmetic error in line 4
E7 (notation error in line 1)
Question 41

Solve, algebraically.

\[ \log_2 \left( \log_3 x \right) = 2 \]

**Solution**

\[ 2^2 = \log_3 x \quad \text{½ mark for exponential form} \]

\[ 4 = \log_3 x \]

\[ x = 3^4 \quad \text{½ mark for exponential form} \]

\[ x = 81 \quad \text{1 mark} \]
Exemplar 1

\[ 2^2 = x \]

\[ 4 = x \]

0 out of 1

Exemplar 2

\[ \log_2 2^2 = \log_3 x \]

\[ 1 = \log_3 x \]

\[ 3^1 = x \]

\[ 3 = x \]

\( \frac{1}{2} \) out of 1

+ \( \frac{1}{2} \) mark for exponential form
Sketch the graph of the function \( y = 5 \sin \left( \frac{\pi}{4} x \right) + 1 \) over the domain \([-4, 8]\).

**Solution**

- 1 mark for shape of \( y = \sin x \)
- 1 mark for amplitude
- 1 mark for period
- 1 mark for vertical translation

4 marks
Exemplar 1

2 out of 4

+ 1 mark for amplitude
+ 1 mark for vertical translation
Exemplar 2

3 out of 4

+ 1 mark for amplitude
+ 1 mark for period
+ 1 mark for vertical translation
Exemplar 3

3 out of 4

+ 1 mark for shape of \( y = \sin x \)
+ 1 mark for amplitude
+ 1 mark for vertical translation
Exemplar 4

2 out of 4

+ 1 mark for amplitude
+ 1 mark for vertical translation
This page was intentionally left blank.
Given the graph of \( y = f(x) \), sketch the graph of \( y = |f(-x)| \).

**Solution**

1 mark for horizontal reflection
1 mark for absolute value

**2 marks**
Exemplar 1

1 out of 2
+ 1 mark for horizontal reflection

Exemplar 2

1 out of 2
+ 1 mark for absolute value
E9 (endpoints or arrowheads omitted or incorrect)
Exemplar 3

0 out of 2
This page was intentionally left blank.
Savannah used the graph of \( y = f(x) \) to sketch the graph of \( y = \sqrt{f(x)} \). Her solution is given below. Describe her error.

**Solution**

Savannah did not restrict the domain at \( x = 2 \).
Exemplar 1

her arrow is the wrong direction, you can't have it going on forever that way if $f(x)$ ends at 2, 4.

0 out of 1

Exemplar 2

She made the line continuous, but it ends at $(2, 4)$, so she should have just ended it there.

½ out of 1

award full marks
– ½ mark for lack of clarity in description
Determine the exact value of $\sin \left( \frac{13\pi}{12} \right)$.

**Solution**

\[
\sin \left( \frac{3\pi}{4} + \frac{\pi}{3} \right) = \sin \left( \frac{3\pi}{4} \right) \cos \left( \frac{\pi}{3} \right) + \cos \left( \frac{3\pi}{4} \right) \sin \left( \frac{\pi}{3} \right)
\]

\[
= \left( \frac{\sqrt{2}}{2} \right) \left( \frac{1}{2} \right) + \left( -\frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right)
\]

\[
= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}
\]

\[
= \frac{\sqrt{2} - \sqrt{6}}{4}
\]

1 mark for substitution into correct identity

2 marks (½ mark for each exact value)

3 marks

**Note:**

- Other combinations are possible.
Exemplar 1

\[ \frac{13 \pi}{12} \times \frac{15}{40} = (13)(15) = 195^\circ \]

\[ \sin \left( \frac{3\pi}{4} + \frac{5\pi}{6} \right) = \left( \sin \frac{3\pi}{4} \right) \left( \cos \frac{5\pi}{6} \right) + \left( \cos \frac{3\pi}{4} \right) \left( \sin \frac{5\pi}{6} \right) \]

\[ = \left( \frac{\sqrt{2}}{2} \right) \left( -\frac{\sqrt{3}}{2} \right) + \left( \frac{\sqrt{2}}{2} \right) \left( \frac{1}{2} \right) \]

\[ = -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{2} \]

\[ = -\sqrt{6} \times \frac{\sqrt{2}}{2} \]

\[ \sin \left( \frac{13\pi}{12} \right) = \frac{2\sqrt{2} - \sqrt{6}}{4} \]

1½ out of 3

+ 1 mark for substitution into correct identity

+ ½ mark for exact value of \( \sin \frac{3\pi}{4} \)

+ ½ mark for exact value of \( \cos \frac{5\pi}{6} \)

+ ½ mark for exact value of \( \sin \frac{5\pi}{6} \)

− ½ mark for procedural error of incorrect combination

− ½ mark for arithmetic error in line 4
Exemplar 2

\[
\sin \left( \frac{9\pi}{12} + \frac{4\pi}{12} \right) = \\
\sin \left( \frac{2\pi}{4} + \frac{\pi}{3} \right) = \\
\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \\
\frac{2\sqrt{2}}{2} 
\]

0 out of 3

Exemplar 3

\[
\sin \left( \frac{3\pi}{4} + \frac{\pi}{3} \right) = \sin \frac{3\pi}{4} \cos \frac{\pi}{3} + \cos \frac{3\pi}{4} \sin \frac{\pi}{3} \\
\quad = \sin \frac{\sqrt{2}}{2} \cos \frac{1}{2} + \cos \frac{-\sqrt{2}}{2} \sin \frac{\sqrt{3}}{2} \\
\quad = \frac{\sqrt{2} - \sqrt{6}}{4} 
\]

2½ out of 3

award full marks

– ½ mark for procedural error in line 2
This page was intentionally left blank.
State the equation of the horizontal asymptote of \( f(x) = \frac{3x}{x - 1} \).

**Solution**

\[ y = 3 \]  

1 mark
Exemplar 1

\[ y = 0 \]

0 out of 1

Exemplar 2

\[ x = 3 \]

0 out of 1

Exemplar 3

\[ H.A. = 3 \]

\( \frac{1}{2} \) out of 1

award full marks
– \( \frac{1}{2} \) mark for procedural error
Sketch the graph of \( f(x) = \frac{5x - 10}{x^2 + x - 6} \).

**Solution**

\[
f(x) = \frac{5x - 10}{x^2 + x - 6} = \frac{5(x - 2)}{(x - 2)(x + 3)} = \frac{5}{x + 3}, x \neq 2
\]

\[
\therefore \text{there is a point of discontinuity (hole) at } (2,1)
\]

corresponding vertical asymptote at \( x = -3 \)

corresponding horizontal asymptote at \( y = 0 \)

\[
\text{1 mark for asymptotic behaviour approaching } x = -3
\]

\[
\text{1 mark for asymptotic behaviour approaching } y = 0
\]

\[
\text{1 mark for a point of discontinuity (hole) at } (2,1)
\]

\[
\frac{1}{2} \text{ mark for graph left of } x = -3
\]

\[
\frac{1}{2} \text{ mark for graph right of } x = -3
\]

4 marks
Exemplar 1

2½ out of 4

+ 1 mark for asymptotic behaviour approaching $x = -3$
+ 1 mark for asymptotic behaviour approaching $y = 0$
+ ½ mark for graph left of $x = -3$
E10 (asymptote omitted but still implied)

Exemplar 2

3 out of 4

+ 1 mark for asymptotic behaviour approaching $x = -3$
+ 1 mark for asymptotic behaviour approaching $y = 0$
+ ½ mark for graph left of $x = -3$
+ ½ mark for graph right of $x = -3$
Exemplar 3

3½ out of 4
award full marks
- ½ mark for procedural error (incorrect y-value for point of discontinuity (hole))
E10 (graph curls away from asymptote)
This page was intentionally left blank.
Determine, algebraically, the inverse of $f(x) = 3x + 4$.

**Solution**

Let $f(x) = y$

$y = 3x + 4$

$x = 3y + 4$

$x - 4 = 3y$

$x - 4 \div 3 = y$  

$\frac{x - 4}{3} = y$  

1 mark for switching $x$ and $y$-values

$\frac{1}{2}$ mark for solving for $y$

$f^{-1}(x) = \frac{x - 4}{3}$  

$\frac{1}{2}$ mark for writing equation of $f^{-1}(x)$

2 marks
Exemplar 1

\[ x = 3y + 4 \]

\[ \frac{x - 4}{3} = \frac{3y}{3} \]

\[ f(x)^{-1} = \frac{x - 4}{3} \]

2 out of 2

award full marks
E7 (notation error in line 3)
Sketch the graph of $P(x) = -(x+1)(x-2)(x+3)$.

**Solution**

1 mark for $x$-intercepts
½ mark for $y$-intercept
½ mark for end behaviour

2 marks
Exemplar 1

1 out of 2
+ 1 mark for $x$-intercepts

Exemplar 2

1½ out of 2
+ 1 mark for $x$-intercepts
+ ½ mark for end behaviour
Exemplar 3

1 out of 2

+ 1 mark for $x$-intercepts
+ $\frac{1}{2}$ mark for $y$-intercept
− $\frac{1}{2}$ mark for procedural error (one incorrect $x$-intercept)
## MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allowed for shape)

### Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the Answer/Scoring Sheet.

| E1 final answer | answer given as a complex fraction  
|                 | final answer not stated  
|                 | impossible solution(s) not rejected in final answer and/or in steps leading to final answer |
| E2 equation/expression | changing an equation to an expression or vice versa  
|                     | equating the two sides when proving an identity |
| E3 variables | variable omitted in an equation or identity  
|             | variables introduced without being defined |
| E4 brackets | “\( \sin x^2 \)” written instead of “\( \sin^2 x \)”  
|            | missing brackets but still implied |
| E5 units | units of measure omitted in final answer  
|           | incorrect units of measure  
|           | answer stated in degrees instead of radians or vice versa |
| E6 rounding | rounding error  
|             | rounding too early |
| E7 notation/transcription | notation error  
|                     | transcription error |
| E8 domain/range | answer outside the given domain  
|                   | bracket error made when stating domain or range  
|                   | domain or range written in incorrect order |
| E9 graphing | endpoints or arrowheads omitted or incorrect  
|            | scale values on axes not indicated  
|            | coordinate points labelled incorrectly |
| E10 asymptotes | asymptotes drawn as solid lines  
|               | asymptotes omitted but still implied  
|               | graph crosses or curls away from asymptotes |
Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an Irregular Test Booklet Report should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an Irregular Test Booklet Report.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an Irregular Test Booklet Report documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.
Irregular Test Booklet Report

Test: ________________________________________________

Date marked: _______________________________________

Booklet No.: _______________________________________

Problem(s) noted: __________________________________

__________________________________________________________________

__________________________________________________________________

__________________________________________________________________

__________________________________________________________________

Question(s) affected: ___________________________________________________________________________

__________________________________________________________________

__________________________________________________________________

Action taken or rationale for assigning marks: _____________________________________________________

__________________________________________________________________

__________________________________________________________________

__________________________________________________________________

__________________________________________________________________

__________________________________________________________________
Follow-up: _________________________________________________________
________________________________________________________
__________________________________________________________________
__________________________________________________________________
__________________________________________________________________
__________________________________________________________________
__________________________________________________________________

Decision: _________________________________________________________
________________________________________________________
__________________________________________________________________
__________________________________________________________________
__________________________________________________________________
__________________________________________________________________
__________________________________________________________________

Marker’s Signature: _____________________________________________

Principal’s Signature: ___________________________________________

For Department Use Only—After Marking Complete

Consultant: ______________________________________________________

Date: __________________________________________________________
### Table of Questions by Unit and Learning Outcome

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