Grade 12 pre-calculus mathematics achievement test.
Marking guide. January 2018

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Manitoba Education and Training
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This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.
While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.
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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that
- the booklet number and the number on the Answer/Scoring Sheet are identical
- students and markers use only a pencil to complete the Answer/Scoring Sheets
- the totals of each of the four parts are written at the bottom
- each student’s final result is recorded, by booklet number, on the corresponding Answer/Scoring Sheet
- the Answer/Scoring Sheet is complete
- a photocopy has been made for school records

Once marking is completed, please forward the Answer/Scoring Sheets to Manitoba Education and Training in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section “Booklet 2 Questions.”

To receive full marks, a student’s response must be complete and correct. Where alternative answering methods are possible, the Marking Guide attempts to address the most common solutions. For general guidelines regarding the scoring of students’ responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an Answer/Scoring Sheet is marked with “0” and/or “NR” only (e.g., student was present but did not attempt any questions), please document this on the Irregular Test Booklet Report.
Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Youyi Sun
Assessment Consultant
Grade 12 Pre-Calculus Mathematics
Telephone: 204-945-7590
Toll-Free: 1-800-282-8069, ext. 7590
Email: youyi.sun@gov.mb.ca
Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Answer/Scoring Sheet that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the Answer/Scoring Sheet in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

<table>
<thead>
<tr>
<th>COMMUNICATION ERRORS / ERREURS DE COMMUNICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).</td>
</tr>
<tr>
<td>Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).</td>
</tr>
</tbody>
</table>

\[
\begin{array}{cccccc}
E1 & E2 & E3 & E4 & E5 \\
\bigcirc & \circ & \circ & \circ & \circ \\
E6 & E7 & E8 & E9 & E10 \\
\circ & \bigcirc & \bigcirc & \circ & \circ \\
\end{array}
\]

Example: Marks assigned to the student.

<table>
<thead>
<tr>
<th>Marks Awarded</th>
<th>Booklet 1</th>
<th>Selected Response</th>
<th>Booklet 2</th>
<th>Communication Errors (Deduct)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
<td>7</td>
<td>40</td>
<td>1½</td>
<td>70½</td>
</tr>
<tr>
<td>Total Marks</td>
<td>36</td>
<td>9</td>
<td>45</td>
<td>maximum deduction of 5 marks</td>
<td>90</td>
</tr>
</tbody>
</table>

Pre-Calculus Mathematics: Marking Guide (January 2018)
Scoring Guidelines for Booklet 1 Questions
A group of 7 friends decide to go to a movie.

Determine how many ways the friends can sit in a row if two of the friends refuse to sit next to each other.

**Solution**

\[ 7! - 6! \cdot 2! = 3600 \text{ ways} \]

\(
\begin{align*}
\text{½ mark for } 7! \\
\text{1 mark for product of } 6! \cdot 2! \\
\left( \frac{1}{2} \text{ mark for } 6!, \frac{1}{2} \text{ mark for } 2! \right) \\
\frac{1}{2} \text{ mark for subtraction}
\end{align*}
\)

2 marks
Exemplar 1

\[
\text{Case 1: two of the friends} \\
\binom{2}{2} = 21 \\
\text{Case 2: All together} \\
7! = 5,040 \\
\text{Total} = 5,040 - 21 \\
\text{Total} = 5,019 \text{ ways}
\]

½ out of 2
+ ½ mark for 7!
+ ½ mark for subtraction
− ½ mark for arithmetic error in line 4

Exemplar 2

\[
\frac{7!}{2!} \leftarrow \text{Number of seats} \\
2! \leftarrow \text{Friends that don't want to sit beside each other.} \\
= 2,520 \text{ different ways} \\
\text{the friends can sit together.}
\]

½ out of 2
+ ½ mark for 7!
Exemplar 3

\[ P_r = \frac{n!}{(n-r)!} \]

\[ P_2 = \frac{7!}{(7-2)!} \]

\[ = 42 \text{ ways} \]

\( \frac{1}{2} \text{ out of 2} \)

+ \( \frac{1}{2} \) mark for 7!
This page was intentionally left blank.
Gabrielle listens to her radio at a sound level of 80 dB. She attended a music concert that had a sound level of 115 dB. Determine how many times more intense the music concert was than the radio.

You may use the formula:

$$\beta = 10 \log \left( \frac{I}{I_0} \right)$$

where $\beta$ is the intensity level of sound, measured in dB

$I$ is the intensity of sound

$I_0$ is the standard minimum intensity that a person can hear

**Solution**

Radio:

$$80 = 10 \log \left( \frac{I}{I_0} \right)$$

$$8 = \log \left( \frac{I}{I_0} \right)$$

$$10^8 = \frac{I}{I_0} \quad \frac{1}{2} \text{ mark for exponential form}$$

$$10^8 I_0 = I$$

Music concert:

$$115 = 10 \log \left( \frac{I}{I_0} \right)$$

$$11.5 = \log \left( \frac{I}{I_0} \right)$$

$$10^{11.5} = \frac{I}{I_0} \quad \frac{1}{2} \text{ mark for exponential form}$$

$$10^{11.5} I_0 = I$$

\[
\frac{\text{intensity of music concert}}{\text{intensity of radio}} = \frac{10^{11.5} I_0}{10^8 I_0}
\]

$$= 10^{3.5}$$

$$= 3162.27766$$

$$= 3162.278 \quad 1 \text{ mark for comparison}$$

**2 marks**
Exemplar 1

\[
\begin{align*}
\text{radio} & \quad 80 \text{ dB} = 10 \log \left( \frac{I}{I_0} \right) \\
10^8 & = \left( \frac{I}{I_0} \right) \\
10^{11.5} & = \left( \frac{I}{I_0} \right) \\
\text{music concert} & \quad 115 \text{ dB} = 10 \log \left( \frac{I}{I_0} \right) \\
10^{11.5} & = \left( \frac{I}{I_0} \right)
\end{align*}
\]

1 out of 2
+ 1 mark for exponential form

Exemplar 2

\[
\begin{align*}
\frac{\text{Concert}}{\text{Radio}} & = \frac{10^{11.5}}{10^{80}} \\
& = 10^{11.5 - 80} \\
& = 10^{35} \\
& = 1 \times 10^{35} \text{ time louder}
\end{align*}
\]

1½ out of 2
award full marks
− ½ mark for procedural error

Exemplar 3

\[
\begin{align*}
B & = 10 \log \left( \frac{115}{80} \right) \\
B & = 10 \log 1.4375 \\
B & = 1.576 \text{ dB}
\end{align*}
\]

0 out of 2
Question 3

Solve, algebraically.

\[ 2(7)^x = 3^{2x-3} \]

**Solution**

\[
\log\left(2\left(7^x\right)\right) = \log 3^{2x-3}
\]

\[
\log 2 + x \log 7 = (2x - 3) \log 3
\]

\[
\log 2 + x \log 7 = 2x \log 3 - 3 \log 3
\]

\[
\log 2 + 3 \log 3 = 2x \log 3 - x \log 7
\]

\[
\log 2 + 3 \log 3 = x(2 \log 3 - \log 7)
\]

\[
\frac{\log 2 + 3 \log 3}{2 \log 3 - \log 7} = x
\]

\[
15.872 483 = x
\]

\[
15.872 = x
\]

½ mark for applying logarithms

1 mark for product law

1 mark for power law

½ mark for collecting terms with \(x\)

½ mark for isolating \(x\)

½ mark for evaluating quotient of logarithms

4 marks
Exemplar 1

\[ 2(7)^x = 3 \]

\[ x \log 14 = (2x-3) \log 3 \]

\[ x \log 14 = 2 \log 3 - 3 \log 3 \]

\[ x \log 14 - 2 \log 3 = -3 \log 3 \]

\[ x \frac{\log 14 - 2 \log 3}{\log 14 - 2 \log 3} = -3 \log 3 \]

\[ x = \frac{-3 \log 3}{\log 14 - 2 \log 3} \]

\[ x = -7.459 \]

3 out of 4

+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for collecting terms with \( x \)
+ ½ mark for isolating \( x \)
+ ½ mark for evaluating quotient of logarithms
Exemplar 2

\[
\log(2(x^7)) = \log 3^{2x^2 - 3}
\]
\[
\log 2 + 7\log x = 2x^2 - 3 \log 3
\]
\[
\log 2 + 3\log 3 = 2x - x \log 7
\]
\[
\log 2 + 3\log 3 = x(2 - \log 7)
\]
\[
x = \frac{\log 2 + 3\log 3}{2 - \log 7}
\]
\[
x = 1.500
\]

3½ out of 4

Award full marks

- ½ mark for procedural error in line 2
This page was intentionally left blank.
Question 4

Solve for $\theta$, algebraically, over the interval $[0, 2\pi]$.

\[ \csc^2 \theta + 2 \csc \theta - 8 = 0 \]

**Solution**

\[ (\csc \theta + 4)(\csc \theta - 2) = 0 \]

$csc \theta = -4 \quad csc \theta = 2$

$\sin \theta = -\frac{1}{4} \quad \sin \theta = \frac{1}{2}$

$\theta_r = 0.252680$

$\theta = 3.394 \quad \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = 6.031$

\[ \theta = 3.394 \quad \theta = 0.524 \]

$\theta = 6.031 \quad \theta = 2.618$

1 mark for solving for $csc \theta$

1 mark for reciprocal

2 marks for solving for $\theta$ (½ mark for each value)

4 marks
Exemplar 1

\[(\csc \theta + 4)(\csc \theta - 2) = 0\]
\[
\csc \theta = -4, \quad \csc \theta = -2
\]
\[
\sin \theta = -\frac{1}{4}, \quad \sin \theta = -\frac{1}{2}
\]
\[
\theta = 0, 25268
\]

\(Q11\) \(\theta = \pi + 0.25268\)
\[
\theta = 3.3943
\]

\(Q14\) \(\theta = 2\pi - 0.25268\)
\[
\theta = 6.03051
\]

3½ out of 4
award full marks
– ½ mark for arithmetic error in line 2

Exemplar 2

\[\csc^2 \theta + 2\csc \theta - 8 = 0\]
\[
(\csc \theta - 2)(\csc \theta + 4) = 0
\]
\[
\csc \theta = 2, \quad \csc \theta = 4
\]
\[
\sin \theta = \frac{1}{2}, \quad \sin \theta = \frac{1}{4}
\]
\[
\sin \theta = \frac{\pi}{6}, \quad \frac{5\pi}{6}
\]

1½ out of 4
+ 1 mark for solving for \(\csc \theta\)
+ 1 mark for reciprocal
+ 1 mark for solving for \(\theta\)
– ½ mark for arithmetic error in line 3
– 1 mark for concept error in line 5
Exemplar 3

\[(\csc \theta + 4)(\csc \theta - 2) = 0\]

\[\csc \theta = -4 \quad \csc \theta = 2\]

\[\frac{1}{\sin \theta} = -4 \quad \frac{1}{\sin \theta} = 2\]

2 out of 4
+ 1 mark for solving for \(\csc \theta\)
+ 1 mark for reciprocal
E3 (variables omitted in an equation or identity)
This page was intentionally left blank.
Question 5

You have forgotten the code to unlock your cell phone. You know the code is made up of four numbers from 0 to 9.

Determine the number of possible codes, if repetition is allowed.

Solution

\[10 \times 10 \times 10 \times 10 = 10000\]  

1 mark
Exemplar 1

\[
\begin{array}{c}
\frac{9}{9} = 0.9999 \\
= 6561
\end{array}
\]

\(\frac{1}{2}\) out of 1
award full marks
– \(\frac{1}{2}\) mark for procedural error

Exemplar 2

\[\frac{10}{9} \div 7 = \frac{50}{49} \times 10 \text{ number possibilities} \]

0 out of 1

Exemplar 3

\[\frac{9}{10} \times 10 = 9000\]

\(\frac{1}{2}\) out of 1
award full marks
– \(\frac{1}{2}\) mark for procedural error
In the binomial expansion of \( \left( \frac{7}{x^3} - 3x^7 \right)^n \), the 5\textsuperscript{th} term contains \( x^7 \).

Determine the value of \( n \).

**Solution**

\[
x^7 = \left( \frac{1}{x^3} \right)^{n-4} \left( x^7 \right)^4
\]

1 mark for \( k = 4 \)  
\( \frac{1}{2} \) mark for substitution

\[
x^7 = \left( x^{-3} \right)^{n-4} \left( x^7 \right)^4
\]

\[
x^7 = x^{-3n+12+28}
\]

\[
7 = -3n + 40
\]

\[
-33 = -3n
\]

\[
11 = n \quad \frac{1}{2} \text{ mark for solving for } n
\]

**2 marks**
Exemplar 1

\[ + k_{11} = \binom{n}{k} a^{n-k} b^k \]

\[ x^3 = n C_4 \left( \frac{3}{x^3} \right)^{n-4} (-3x^3)^4 \]

\[ x^3 = \left( \frac{3}{x^3} \right)^{n-4} (-3x^3)^4 \]

\[ x^3 = \left( \frac{3}{x^3} \right)^{n-4} (81x^{28}) \]

\[ x = \frac{-3n+12}{28} \]

\[ 7 = -3n + 12 \text{ (28)} \]

\[ 7 = -84n + 336 \]

\[ 84n = 329 \]

\[ n = \frac{329}{84} \]

\[ n = \frac{47}{12} \]

1 out of 2

+ 1 mark for \( k = 4 \)

+ \( \frac{1}{2} \) mark for substitution

- \( \frac{1}{2} \) mark for procedural error in line 2
**Exemplar 2**

\[
\begin{align*}
\binom{n-k}{1} &= \binom{n}{k} \cdot a^{n-k} \cdot b^k \\
\binom{n-1}{4} &= \binom{7}{n-1} \cdot (\frac{7}{x^3})^4 \\
\binom{7}{4} &= (x^3)^{n-1} \cdot (x^7)^4 \\
x^7 &= x^{3n-12} \cdot x^{28} \\
7 &= 3n-12 + 28 \\
7 &= 3n + 16 \\
\frac{9}{3} &= \frac{3n}{3} \\
3 &= n
\end{align*}
\]

1 out of 2
+ 1 mark for \( k = 4 \)
+ \( \frac{1}{2} \) mark for substitution
+ \( \frac{1}{2} \) mark for solving for \( n \)
- \( \frac{1}{2} \) mark for procedural error in line 3
- \( \frac{1}{2} \) mark for arithmetic error in line 7

**Exemplar 3**

\[
\begin{align*}
\hat{\binom{7}{4}} = \binom{n}{4} \cdot \left(\frac{7}{x^3}\right)^{n-4} \cdot (3x^7)^{4}
\end{align*}
\]

1 out of 2
+ 1 mark for \( k = 4 \)
This page was intentionally left blank.
Given the domain of $f(x)$ is $\{-6, 1, 3, 4\}$ and the range of $f(x)$ is $\{-4, 7, 10, 15\}$, state the domain of $f^{-1}(x)$.

**Solution**

$\{-4, 7, 10, 15\}$

1 mark
Exemplar 1

$D: -4 \leq x \leq 5$

0 out of 1
Given the graph of \( y = f(x) \), sketch the graph of its inverse.

**Solution**
Exemplar 1

award full marks
E9 (endpoints or arrowheads omitted or incorrect)

Exemplar 2

½ out of 1
award full marks
– ½ mark for procedural error (one incorrect point)
E9 (endpoints or arrowheads omitted or incorrect)
Question 9

Prove the following identity for all permissible values of $\theta$.

$$\frac{1 + \cos \theta}{1 - \sin^2 \theta} = \sec \theta + \tan^2 \theta + 1$$

**Solution**

**Method 1**

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1 + \cos \theta}{1 - \sin^2 \theta}$</td>
<td>$\sec \theta + \tan^2 \theta + 1$</td>
</tr>
<tr>
<td>$\frac{1 + \cos \theta}{\cos^2 \theta}$</td>
<td>$\frac{1}{\cos^2 \theta} + \frac{1}{\cos \theta}$</td>
</tr>
<tr>
<td>$\sec^2 \theta + \sec \theta$</td>
<td>$\tan^2 \theta + 1 + \sec \theta$</td>
</tr>
</tbody>
</table>

1 mark for algebraic strategies

1 mark for logical process to prove the identity

1 mark for correct substitution of appropriate identities

3 marks
### Solution

#### Method 2

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1 + \cos \theta}{\cos^2 \theta} )</td>
<td>( \frac{1}{\cos \theta} + \sec^2 \theta )</td>
<td>1 mark for algebraic strategies</td>
</tr>
<tr>
<td>( \frac{1}{\cos \theta} + \frac{1}{\cos^2 \theta} )</td>
<td>( \cos \theta + 1 )</td>
<td>1 mark for logical process to prove the identity</td>
</tr>
<tr>
<td>( \frac{1}{\cos^2 \theta} )</td>
<td>1 mark for correct substitution of appropriate identities</td>
<td></td>
</tr>
</tbody>
</table>

3 marks
### Exemplar 1

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{\cos \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} + 1 )</td>
<td>( \frac{1}{\cos \theta} + \frac{\sin^2 \theta}{1 - \sin^2 \theta} + 1 )</td>
</tr>
<tr>
<td>( \frac{1}{\cos \theta} + \frac{\ln \cos \theta}{1 - \sin^2 \theta} + 1 )</td>
<td>( \frac{1 + \cos \theta}{1 - \sin^2 \theta} = MG )</td>
</tr>
</tbody>
</table>

**1 out of 3**

+ 1 mark for correct substitution of appropriate identities
Exemplar 2

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
</table>
| \[
\frac{1 + \cos \theta}{\cos^2 \theta}
\] | \[
\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} + 1
\] |
|               | \[
\frac{\cos \theta}{\cos^2 \theta} + \frac{\sin \theta}{\cos \theta} + 1
\] |
|               | \[
\frac{\cos \theta + \sin \theta}{\cos \theta} + 1
\] |
|               | \[
\frac{\cos \theta + \sin \theta}{1 - \sin^2 \theta} + 1
\] |

2 out of 3
+ 1 mark for correct substitution of appropriate identities
+ 1 mark for algebraic strategies
<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1 + \cos \theta}{1 - \sin^2 \theta} )</td>
<td>( \frac{1}{\cos \theta} + \sec^2 \theta - 1 + 1 )</td>
</tr>
<tr>
<td>( \frac{1 + \cos \theta}{\cos^2 \theta} )</td>
<td>( \frac{1}{\cos \theta} + \frac{1}{\tan^2 \theta + 1} )</td>
</tr>
<tr>
<td>( \frac{1}{\cos \theta} )</td>
<td>( \frac{1}{\cos \theta} + \frac{1}{\tan^2 \theta + 1} )</td>
</tr>
</tbody>
</table>

0 out of 3
This page was intentionally left blank.
Thomas used graphs to solve the equation \( e^{x+2} = \sqrt[3]{x+1} \).

He incorrectly states the solution as \((-2, 1)\).

Describe how Thomas should have stated the solution.

**Solution**

He stated his solution as a coordinate point; his solution should have only been the value of \( x \).
Exemplar 1

\((-2, 1)\) is not the solution, \((-2, 1)\) is where the graphs intersect.

0 out of 1

Exemplar 2

He can’t include \(y\).

1 out of 1

Exemplar 3

He must state \(x = -2\)

\(y = 1\)

0 out of 1
Question 11

Given the graph of \( y = f(x) \), sketch the graph of \( y = \sqrt{f(x)} \).

Solution

1 mark for restricting domain
½ mark for shape between both pairs of invariant points
½ mark for shape above both pairs of invariant points

2 marks
Exemplar 1

1 out of 2
award full marks
– 1 mark for concept error (omitting right branch)

Exemplar 2

1½ out of 2
+ 1 mark for restricting domain
+ ½ mark for shape above both pairs of invariant points
Question 12

When a polynomial, $P(x)$, is divided by $(x - 2)$ the resulting equation is

\[ \frac{P(x)}{x - 2} = x^2 - x + 1 + \frac{3}{x - 2}. \]

a) Explain why $x - 2$ is not a factor of $P(x)$.

b) Determine the equation for the polynomial function $P(x)$.

**Solution**

a) There is a remainder when $P(x)$ is divided by $x - 2$.  

b) $P(x) = (x - 2)(x^2 - x + 1) + 3$  

or

\[ P(x) = x^3 - 3x^2 + 3x + 1 \]
Exemplar 1

a) If it was a factor there would not be the term $x - 2$ as the denominator.

1 out of 1

b) 

\[
\frac{(x^2 - x + 1 + \frac{3}{x+3})}{(x+2)}
\]

\[
x^3 - x^2 + x + 2x^2 - 3x + 2 + 3
\]

\[
P(x) = \frac{x^2 + x^2 - x + 5}{x+3}
\]

1 out of 1

award full marks
E7 (transcription error)
Exemplar 2

a) 

"to be a factor, the remainder must be $\emptyset$."

1 out of 1

b) 

\[ P(x) = (x-2)\left(x^2-x+1 + \frac{3}{x-2}\right) \]

1 out of 1
Exemplar 3

a)

Because there is a remainder of \( \frac{3}{x-2} \)

½ out of 1

Award full marks

- ½ mark for terminology error (including \( x - 2 \) as part of the remainder)

b)

\[
P(x) = (x^2 - x + 1)(x - 2)
= x^3 - 2x^2 - x^2 + 2x + x - 2
= x^3 - 3x^2 + 3x - 2
\]

\[
P(x) = x^3 - 3x^2 + 3x - 2
\]

0 out of 1
Question 13

Determine the equation for $g(x)$ in terms of $f(x)$.

![Graphs of $f(x)$ and $g(x)$]

**Solution**

$$g(x) = -f(x-1) + 3$$

1 mark for vertical reflection
1 mark for horizontal translation
1 mark for vertical translation

3 marks
Exemplar 1

\[ g(x) = \frac{-1}{x-1} - 3 \]

1 out of 3
+ 1 mark for vertical reflection
+ 1 mark for horizontal translation
− 1 mark for concept error (omitting \( f \))

Exemplar 2

\[ g(x) = \frac{-9}{x-1} - 2 \]

1½ out of 3
+ 1 mark for vertical reflection
+ 1 mark for horizontal translation
− ½ mark for procedural error (\( g \) instead of \( f \))

Exemplar 3

\[ g(x) = f(-x+1) + 3 \]

2 out of 3
+ 1 mark for horizontal translation
+ 1 mark for vertical translation
Explain why the binomial expansion of \((2x + y)^9\) does not have a middle term.

**Solution**

The expansion contains \(n + 1\) terms. Since \(n\) equals 9, there are 10 terms, which would not allow for a middle term.

1 mark
Exemplar 1

Because that $9$ would be a $10$ and since $10$ is an even number, there is no middle term.

½ out of 1
award full marks
— ½ mark for lack of clarity in explanation

Exemplar 2

$n = 9$

$k = 8$

If there are $8$ terms, there is no middle term, $1 \underline{1111} 1$.

0 out of 1
award full marks
— 1 mark for concept error (incorrect number of terms)

Exemplar 3

doesn’t have a middle term because they’re an even amount

½ out of 1
award full marks
— ½ mark for lack of clarity in explanation
Using the laws of logarithms, completely expand the expression $\log \left( \frac{5\sqrt{a}}{b^3} \right)$.

**Solution**

$\log 5 + \log \left( \frac{1}{2} a \right) - 3 \log b$

1 mark for product law
1 mark for power law (½ mark for each)
1 mark for quotient law

**3 marks**
Exemplar 1

\[
\log \left( \frac{5\sqrt[3]{a}}{b^3} \right)
\]

\[
\log_a M - \log_a N = \log_a \left( \frac{M}{N} \right)
\]

\[
\log 5 a^{\frac{1}{2}} - \log b^3 = \frac{1}{2} \log 5 - 3 \log b
\]

1½ out of 3
+ ½ mark for power law
+ 1 mark for quotient law
E7 (transcription error in line 4)

Exemplar 2

\[
\log 5\sqrt[3]{a} - \log b^3
\]

\[
\log 5\sqrt[3]{a} - 3 \log b
\]

1½ out of 3
+ ½ mark for power law
+ 1 mark for quotient law

Exemplar 3

\[
= \log 5\sqrt[3]{a} - \log b^3
\]

\[
= \log 5 a^{\frac{1}{3}} - 3 \log b
\]

\[
= \frac{1}{2} \log 5a - 3 \log b
\]

\[
= \frac{1}{2} \log 5 + \log a - 3 \log b
\]

2 out of 3
+ 1 mark for product law
+ ½ mark for power law
+ 1 mark for quotient law
− ½ mark for procedural error in line 4
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## Answer Key for Selected Response Questions

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Question 16

Identify $10^\circ$ in radians.

a) $\frac{1800}{\pi}$

b) $\frac{\pi}{1800}$

c) $\frac{18}{\pi}$

d) $\frac{\pi}{18}$

Question 17

The polynomial function, $P(x) = a(x-1)^2(x+4)^2$, has a y-intercept of $-8$.

Identify the value of $a$.

a) $-2$

b) $-\frac{1}{2}$

c) $\frac{1}{2}$

d) 2
Question 18
Identify the value of \( \log_4 \left( \frac{1}{16} \right) \).

a) \(-2\)
b) \(-\frac{1}{2}\)
c) \(\frac{1}{2}\)
d) 2

Question 19
Given the angle \( \frac{25\pi}{7} \), identify the coterminal angle on the interval \([-2\pi, 0]\).

a) \(\frac{18\pi}{7}\)
b) \(\frac{11\pi}{7}\)
c) \(-\frac{3\pi}{7}\)
d) \(-\frac{10\pi}{7}\)

Question 20
Identify which expression cannot be evaluated.

a) \(\gamma P_0\)
b) \(\gamma P_6\)
c) \(\gamma P_7\)
d) \(\gamma P_8\)
Identify the graph of \( f(x) = \frac{-3x}{2x + 4} \).
Question 22

Given a point \((-2, 0)\) on the graph of \(y = f(x)\), identify the coordinates of the corresponding point on the graph of \(y = 4f\left(\frac{1}{2}x\right)\).

a) \((-8, 0)\)

b) \((-4, 0)\)

c) \((-2, 0)\)

d) \((-1, 0)\)

Question 23

Identify the non-permissible value of \(\theta\) for the expression \(\frac{\cos\theta}{1 + \sin\theta}\).

a) \(\frac{\pi}{2}\)

b) \(\pi\)

c) \(\frac{3\pi}{2}\)

d) \(2\pi\)

Question 24

Identify the function with an asymptote at \(x = -3\).

a) \(y = \log(x + 3)\)

b) \(y = \log x + 3\)

c) \(y = \log(x - 3)\)

d) \(y = \log x - 3\)
This page was intentionally left blank.
Evaluate the following expression.

\[ \tan \left( \frac{2\pi}{3} \right) \csc \left( -\frac{2\pi}{3} \right) + \cos (3\pi) \]

**Solution**

\[ (-\sqrt{3}) \left( -\frac{2}{\sqrt{3}} \right) + (-1) \]

\[ \frac{2 - 1}{1} \]

1 mark for \( \tan \left( \frac{2\pi}{3} \right) \) (½ mark for quadrant, ½ mark for value)

1 mark for \( \csc \left( -\frac{2\pi}{3} \right) \) (½ mark for quadrant, ½ mark for value)

1 mark for \( \cos (3\pi) \)

3 marks
Exemplar 1

\[
= \left( \frac{1}{\sqrt{3}} \right) \left( \frac{\sqrt{3}}{2} \right) + 1
\]

\[
= \left( \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{1} \right) \left( \frac{\sqrt{3}}{2} \right) + 1
\]

\[
= \left( \frac{\sqrt{3}}{3} \right) \left( \frac{2}{\sqrt{3}} \right) + 1
\]

\[
= \frac{2}{\sqrt{3}} + 1
\]

\[
= 2 + 1
\]

\[
= 3
\]

1 out of 3

+ \frac{1}{2} \text{ mark for value of } \tan \left( \frac{2\pi}{3} \right)

+ \frac{1}{2} \text{ mark for value of } \csc \left( -\frac{2\pi}{3} \right)

E7 (transcription error in line 2)

Exemplar 2

\[
\left( \frac{\sqrt{3}}{2} \right) \left( \frac{\sqrt{3}}{2} \right)
\]

\[
\left( -\frac{\sqrt{3}}{2} \right) \left( -\frac{\sqrt{3}}{2} \right) + (-1)
\]

\[
\left( -\frac{\sqrt{3}}{2} \right) \left( -\frac{\sqrt{3}}{2} \right) - 1
\]

\[
-\frac{2}{\sqrt{3}} - 1
\]

\[
-2\sqrt{3} - 1
\]

\[
= -3
\]

2\frac{1}{2} out of 3

award full marks

- \frac{1}{2} \text{ mark for arithmetic error in line 4}

E7 (transcription error in line 1)
Exemplar 3

\[
\frac{\sin\left(\frac{2\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)} + \frac{1}{\sin\left(-\frac{2\pi}{3}\right)} + \cos\left(\frac{2\pi}{3}\right)
\]

\[
\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + 0
\]

\[
\frac{\sqrt{3} \cdot -2}{1}
\]

\[
\frac{-2}{1}
\]

1½ out of 3

+ 1 mark for \(\tan\left(\frac{2\pi}{3}\right)\)

+ 1 mark for \(\csc\left(-\frac{2\pi}{3}\right)\)

− ½ mark for arithmetic error in line 3

E7 (notation error in line 1)
This page was intentionally left blank.
State the range of the graph below.

**Solution**

Range: \( \{ y \in \mathbb{R}, y \neq 0 \text{ and } y \neq 1 \} \)  \( 1 \) mark (½ mark for \( y \neq 0 \), ½ mark for \( y \neq 1 \))
Exemplar 1

Range: $(-\infty, 0] \cup [0, 1) \cup (1, \infty)$

1 out of 1

Award full marks
E8 (bracket error made when stating domain or range)

Exemplar 2

Range: $(\infty, 0) \cup (0, -\infty)$

½ out of 1

+ ½ mark for $y \neq 0$
E8 (domain or range written in incorrect order)
Sketch the graph of the function \( f(x) = \frac{2x^2 - 5x}{x} \).

**Solution**

1 mark for point of discontinuity (hole) at \((0, -5)\)

\((\frac{1}{2} \text{ mark for } x = 0, \; \frac{1}{2} \text{ mark for } y = -5)\)

1 mark for shape of a linear function
Exemplar 1

1 out of 2
+ 1 mark for shape of a linear function

Exemplar 2

0 out of 2
State a possible value of $n$ if the polynomial function $P(x) = (x - 1)^2(x + 2)^n$ has a range of $[0, \infty)$.

**Solution**

$n = 2$  

1 mark

---

**Note(s):**

- Accept any even positive value of $n$, including zero.
Exemplar 1

\[ n \geq 0 \]

0 out of 1

Exemplar 2

\[ n = 0 \]

1 out of 1
Question 29

Sketch the graph of $y = \left(\frac{1}{2}\right)^{x-1}$.

**Solution**

1 mark for decreasing exponential function
1 mark for horizontal translation

2 marks
Exemplar 1

2 out of 2
award full marks
E10 (asymptotes omitted but still implied)

Exemplar 2

1 out of 2
+ 1 mark for decreasing exponential function
Solve.

\[ \log_x 27 = 3 \]

**Solution**

\[ x^3 = 27 \]
\[ x = 3 \]

1 mark for exponential form

1 mark
Exemplar 1

\[ x^3 = 27 \]
\[ 3^3 = 27 \quad 3 \cdot 3 = 9 \times 3 \]

1 out of 1

award full marks
E1 (final answer not stated)

Exemplar 2

\[ \sqrt[3]{x^3} = \sqrt[3]{27} \]
\[ x = 9 \]

½ out of 1

+ 1 mark for exponential form
– ½ mark for arithmetic error

Exemplar 3

\[ 3^x = 27 \]
\[ 3^x = 3^3 \]
\[ \sqrt[3]{x} = 3 \]

0 out of 1
Question 31

Sketch at least two periods of the graph \( y = \tan x \).

**Solution**

1 mark for increasing trigonometric function

1 mark for asymptotic behaviour approaching \( x = \frac{\pi}{2} + k\pi, \ k \in \mathbb{Z} \)

2 marks
Exemplar 1

1 out of 2
+
1 mark for increasing trigonometric function
E9 (scale values on axes not indicated)

Exemplar 2

0 out of 2
Question 32

Given the graph of \( f(x) \), state the domain of \( \frac{1}{f(x)} \).

Solution

Domain: \( \{ x \in \mathbb{R}, x \neq \pm 2 \} \) 1 mark (½ mark for \( x \neq 2 \), ½ mark for \( x \neq -2 \))

1 mark
Exemplar 1

Domain: \((-\infty, -2) \cup (-2, 2) \cup (2, \infty)\)

1 out of 1

Exemplar 2

Domain: \(x \in \mathbb{R}, x \geq -2\)

0 out of 1
Question 33

Determine the values of $A$, $B$, and $D$ of the sinusoidal function in the form $y = A \sin (Bx) + D$.

Solution

$A = \boxed{2}$ \hspace{1cm} 1 mark for $A$

$B = \boxed{\frac{1}{5}}$ \hspace{1cm} 1 mark for $B$

$D = \boxed{1}$ \hspace{1cm} 1 mark for $D$

$\boxed{3}$ marks
Exemplar 1

\[ A = \frac{2}{2} \quad \frac{\theta \pi}{100} = \frac{\pi}{5} \]

\[ B = \frac{\pi}{5} \]

\[ D = 1 \]

2½ out of 3

award full marks
- ½ mark for arithmetic error while calculating value of B

Exemplar 2

\[ A = \frac{1}{2} \]

\[ B = \frac{1}{5} \]

\[ D = 2 \]

1 out of 3

+ 1 mark for B
Question 34

Determine if the point \( \left( -\frac{\sqrt{7}}{5}, \frac{2}{5} \right) \) is on the unit circle.

Justify your answer.

**Solution**

\[ x^2 + y^2 = 1 \]

Left-hand side = \( \left( -\frac{\sqrt{7}}{5} \right)^2 + \left( \frac{2}{5} \right)^2 \)

\[ = \frac{7}{25} + \frac{4}{25} \]

\[ = \frac{11}{25} \neq 1 \]

\[ \therefore \text{not on the unit circle} \]

1 mark for justification
Exemplar 1

\[
\begin{align*}
\left(-\frac{\sqrt{7}}{5}\right)^2 + \left(\frac{2}{5}\right)^2 &= \frac{7}{25} + \frac{4}{25} \\
&= \frac{11}{25} \\
&= \frac{-3}{25} \neq 0 \\
\text{No, it is not on the unit circle}
\end{align*}
\]

0 out of 1

Exemplar 2

No.

0 out of 1

Exemplar 3

\[
\begin{align*}
\left(-\frac{\sqrt{7}}{5}\right)^2 + \left(\frac{2}{5}\right)^2 &= \frac{7}{25} + \frac{4}{25} \\
&= \frac{11}{25} \\
&= \frac{-3}{25} \neq 1 \\
\text{No}
\end{align*}
\]

½ out of 1

award full marks
– ½ mark for arithmetic error in line 2
E7 (notation error in line 2)
Question 35

Solve, algebraically.

\[ \frac{n \choose 5}{n \choose 4} = 6 \]

**Solution**

\[
\frac{n!}{(n - 5)!5!} = 6 \quad \frac{1}{2} \text{ mark for substitution into equation}
\]

\[
\frac{n!(n - 4)!4!}{n!(n - 5)!5!} = 6 \quad 1 \text{ mark for factorial expansion}
\]

\[
\frac{n!(n - 4)(n - 5)!4!}{n!(n - 5)!5! \cdot 4!} = 6 \quad (\frac{1}{2} \text{ mark for numerical factors; } \frac{1}{2} \text{ mark for factors with variables})
\]

\[
\frac{n - 4}{5} = 6 \quad 1 \text{ mark for simplification of factorials}
\]

\[
\frac{n - 4}{5} = 6 \quad (\frac{1}{2} \text{ mark for numerical factors; } \frac{1}{2} \text{ mark for factors with variables})
\]

\[
n - 4 = 30
\]

\[
n = 34 \quad \frac{1}{2} \text{ mark for solving for } n
\]

3 marks
Exemplar 1

\[ b = \frac{n!}{(n-5)! \cdot 5!} \]

\[ \frac{n!}{(n-4)! \cdot 4!} \]

\[ b = \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-4) \cdot 5!}{(n-5)! \cdot 5!} \]

\[ \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-4) \cdot 4!}{(n-4)! \cdot 4!} \]

\[ \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot 24}{24} \]

\[ b = (n-5) \]

\[ b = 5n - 20 \]

\[ \frac{2b}{5} = \frac{5n}{5} \]

\[ n = \frac{2b}{5} \]

\[ 5 \cdot 4 = 20 \]

\[ 60 \]

\[ 5! = 120 \]

\[ 4 \cdot 3 = \]

\[ 4! = 24 \]

2½ out of 3

award full marks
- ½ mark for procedural error in line 2
E7 (notation error in line 2)
E1 (impossible solution not rejected in final answer)
Exemplar 2

\[ \frac{n!}{(n-5)!5!} = 6 \]

\[ \frac{n!}{(n-4)!4!} \]

\[ \frac{n(n-1)(n-2)(n-3)(n+5)!}{(n-5)!} \times \frac{5!}{n(n-1)(n-2)(n-3)} \]

\[ \frac{5!}{4!} \]

1\frac{1}{2} out of 3

+ ½ mark for substitution into equation
+ ½ mark for factorial expansion (factors with variables)
+ ½ mark for simplification of factorials (factors with variables)
E2 (changing an equation into an expression in lines 2 and 3)
E7 (notation error in line 2)
Exemplar 3

\[
\left( \frac{\frac{n!}{(n-5)!}}{\frac{n!}{(n-4)!}} \right) = 6 \\
\left( \frac{(n)(n-1)(n-2)(n-3)(n-4)(n-5)!}{(n-5)!} \right) = 6 \\
\left( \frac{(n)(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} \right) = 6 \\
\left( \frac{(n)(n-1)(n-2)(n-3)(n-4)}{(n)(n-1)(n-2)(n-3)} \right) = 6 \\
\]

\[
n - 4 = 6 \\
+ 4 + 4 \\
n = 10
\]

2½ out of 3

+ 1 mark for factorial expansion
+ 1 mark for simplification of factorials
+ ½ mark for solving for \( n \)
Given \( \sin \alpha = \frac{4}{5}, \) where \( \alpha \) is in quadrant II, determine the exact value of \( \sin 2\alpha. \)

**Solution**

\[
\begin{align*}
\cos \alpha &= -\frac{3}{5} \\
\sin 2\alpha &= 2 \sin \alpha \cos \alpha \\
&= 2 \left( \frac{4}{5} \right) \left( -\frac{3}{5} \right) \\
&= -\frac{24}{25}
\end{align*}
\]

Note(s):
- Accept any of the following values for \( x \): \( x = \pm 3; x = -3; \) or \( x = 3. \)
Exemplar 1

\[ 8 \sin 2\alpha = 2(\sin \alpha \cos \alpha) \]
\[ = 2 \left( \frac{4}{5} \cdot \frac{\sqrt{41}}{5} \right) \]
\[ = 2 \left( \frac{4 \sqrt{41}}{10} \right) \]
\[ = \frac{4 \sqrt{41}}{5} \]

½ out of 2
+ 1 mark for substitution into correct identity
- ½ mark for arithmetic error in line 3
E1 (impossible solution not rejected in final answer)

Exemplar 2

\[ 2 \sin \alpha \cos \alpha \]
\[ = 2 \left( \frac{4}{\sqrt{5}} \right) \left( \frac{3}{\sqrt{5}} \right) \]
\[ = \frac{24}{25} \]

1½ out of 2
+ ½ mark for value of \( x \)
+ 1 mark for substitution into correct identity
Given the functions $f(x) = x + 1$ and $g(x) = \sqrt{x}$,

a) determine the equation of $g\left(f(x)\right)$.

b) sketch the graph of $g\left(f(x)\right)$.

**Solution**

a) $g\left(f(x)\right) = \sqrt{x + 1}$

b)

1 mark for domain of $g\left(f(x)\right)$
1 mark for shape consistent with $g\left(f(x)\right)$
Exemplar 1

a) 
\[ g(f(x)) = \sqrt[3]{x^2 - 1} \]

1 out of 1

---

b) 

1 out of 2

+ 1 mark for shape consistent with \( g(f(x)) \)
Exemplar 2

a) $g(f(x)) = \sqrt{x + 1}$

1 out of 1

b)

1½ out of 2

award full marks
– ½ mark for procedural error (incorrect second point)
Exemplar 3

a)

\[ g(f(x)) = \sqrt{x + 1} \]

**1 out of 1**

b)

**1 out of 2**

award full marks
– 1 mark for concept error of asymptote
Exemplar 4

a)

\[ g(f(x)) = \sqrt{x} + 1 \]

0 out of 1

b)

2 out of 2

award full marks (consistent with answer in a)
This page was intentionally left blank.
Steve is asked to determine an equation with a larger period than the period of the graph of $y = \cos(2x)$.

Justify why Steve’s answer of $y = \cos(6x)$ is incorrect.

**Solution**

Steve’s equation needs to have a value of $|b|$ less than 2. 

or

Steve’s graph would have a period of $\frac{2\pi}{6} = \frac{\pi}{3}$, which is smaller than $\frac{2\pi}{2} = \pi$, the period of the given graph.
Exemplar 1

Because when determining Period, we turn it into a fraction. So $\frac{1}{2} > \frac{1}{6}$

0 out of 1

Exemplar 2

because making the number in front of x will make the graph bigger. It needs to be a fraction for the graph to expand.

0 out of 1

Exemplar 3

because B values that are $>1$ make the period smaller.

0 out of 1
Given the graphs of \( f(x) \) and \( g(x) \),

a) determine the value of \( (f \cdot g)(-1) \).

b) determine the value of \( g(f(0)) \).

**Solution**

a) \( (f \cdot g)(-1) = (-1)(4) = -4 \)  
   1 mark for value of \( (f \cdot g)(-1) \)

b) \( f(0) = 1 \)  
   \( g(f(0)) = 2 \)  
   \( \frac{1}{2} \) mark for \( f(0) \)  
   \( \frac{1}{2} \) mark for \( g(f(0)) \) consistent with \( f(0) \) value  
   1 mark
Exemplar 1

a)  
\[ f(-1) = -1 \]
\[ g(-1) = 4 \]
\[ (f \cdot g)(-1) = 4 \]

½ out of 1
award full marks
– ½ mark for arithmetic error

b)  
\[ g(f(8)) = 1 \]
\[ g(1) = 2 \]

1 out of 1
award full marks
E7 (notation error)

Exemplar 2

a)  
\[ (-1 \cdot 4) (-1) \]
\[ (-4) (-1) \]

0 out of 1
award full marks
– 1 mark for concept error

b)  
\[ 3 \cdot 1 = 3 \]

0 out of 1
Sketch the graph of \( P(x) = -(x-1)^3 (x-3)(x+1) \).

**Solution**

1 mark for \( x \)-intercepts  
½ mark for \( y \)-intercept  
1 mark for multiplicity (degree 3 at \( x = 1 \))  
½ mark for end behaviour

3 marks
Exemplar 1

2 out of 3
+ 1 mark for x-intercepts
+ 1 mark for multiplicity (degree 3 at $x = 1$)
E9 (scale values on axes not indicated)
Exemplar 2

2½ out of 3

award full marks
– ½ mark for procedural error (multiplicity of degree 3 at $x = -1$ instead of $x = 1$)
Exemplar 3

1½ out of 3

+ 1 mark for x-intercepts
+ ½ mark for end behaviour
1 out of 3

+ 1 mark for multiplicity (consistent with incorrect x-intercepts)
+ $\frac{1}{2}$ mark for end behaviour
− $\frac{1}{2}$ mark for incorrect shape
This page was intentionally left blank.
The point \((-\sqrt{3}, 1)\) is on the terminal arm of an angle \(\theta\), in standard position.

a) Determine \(\tan \theta\).

b) Determine a possible value of \(\theta\), in radians.

**Solution**

a) \[ \tan \theta = -\frac{1}{\sqrt{3}} \]  
   1 mark

b) \[ \theta = \frac{5\pi}{6} \]  
   1 mark
Exemplar 1

a) \[ \tan \theta = \frac{1}{\sqrt{3}} \]

1 out of 1

b) \[ \Theta = \frac{5\pi}{6}, \frac{5\pi}{3} \]

0 out of 1

Exemplar 2

a) \[ \tan = \frac{-1}{\sqrt{3}} \]

1 out of 1

award full marks
E3 (variable omitted in an equation or identity)

b) \[ \Theta = \frac{11\pi}{6} \]

0 out of 1
Exemplar 3

a) \[ \tan \theta = -\sqrt{3} \]

1 out of 1

b) \[ \theta = 150^\circ \]

1 out of 1

award full marks
E5 (answer stated in degrees instead of radians)

Exemplar 4

a) \[ \tan \theta = -\sqrt{3} \]

0 out of 1

b) \[ \theta = \frac{2\pi}{3} \]

1 out of 1

award full marks (consistent with answer in a)
This page was intentionally left blank.
Describe the transformation used to obtain the graph of $y = \log_5 x$ given the graph of $y = 5^x$.

**Solution**

The graph of $y = \log_5 x$ is obtained by reflecting the graph of $y = 5^x$ over the line $y = x$.

or

The graph of $y = \log_5 x$ is the inverse of $y = 5^x$. 

1 mark
Exemplar 1

One has a vertical asymptote and the other has a horizontal asymptote.

0 out of 1

Exemplar 2

A logarithm is the inverse of exponential equation. They are simply the opposite of each other.

½ out of 1

Award full marks
- ½ mark for terminology error

Exemplar 3

The graph \( y = \log_5 x \) is found by switching the \( x \) and \( y \) values.

1 out of 1
Solve $\sin \theta = -\frac{\sqrt{3}}{2}$, where $\theta \in \mathbb{R}$.

**Solution**

$\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$  \hspace{1cm} 1 mark for values of $\theta$ (½ mark for each value)

$\theta = \left\{ \frac{4\pi}{3} + 2k\pi, \ k \in \mathbb{Z} \right\}$  \hspace{1cm} 1 mark for general solution

or

$\theta = 240^\circ, 300^\circ$

$\theta = \left\{ 240^\circ + 360^\circ k, \ k \in \mathbb{Z} \right\}$

$\left\{ 300^\circ + 360^\circ k, \ k \in \mathbb{Z} \right\}$
Exemplar 1

\[
\sin \theta < 0 \text{ in } Q \text{ III, IV}
\]

\[
\theta = \frac{4\pi}{3} + \pi k, \ k \in \mathbb{Z}
\]

\[
\theta = \frac{5\pi}{3} + \pi k, \ k \in \mathbb{Z}
\]

1 out of 2
+ 1 mark for values of $\theta$

Exemplar 2

\[
240^\circ, 300^\circ
\]

\[
\theta = 360^\circ + 240^\circ k
\]

\[
\theta = 360^\circ + 300^\circ k, \ k \in \mathbb{Z}
\]

1 out of 2
+ 1 mark for values of $\theta$
E7 (notation error in line 1)
Exemplar 3

\[
\sin \theta = \frac{4\pi}{3}, \frac{5\pi}{3}
\]

\[
\theta = \frac{4\pi}{3} + 2k\pi \quad k \in \mathbb{Z}
\]

\[
\theta = \frac{5\pi}{3} + 2k\pi \quad k \in \mathbb{Z}
\]

1½ out of 2

award full marks
– ½ mark for procedural error in line 1

Exemplar 4

\[
\sin \theta = \frac{-\sqrt{3}}{2}
\]

\[
\sin \theta = \left\{ \frac{4\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi \right\} \quad k \in \mathbb{R}
\]

½ out of 2

award full marks
– 1 mark for concept error in line 2
– ½ mark for procedural error in line 2 (\( k \in \mathbb{R} \) instead of \( k \in \mathbb{Z} \))
This page was intentionally left blank.
Given that the point \((a, b)\) is on the graph of \(f(x)\), describe how you would determine the corresponding point on the graph of \(y = \sqrt{f(x)}\).

**Solution**

The value of \(a\) stays the same, square root the value of \(b\).  

1 mark
Exemplar 1

\((a, \sqrt[5]{b})\)

0 out of 1

Exemplar 2

You would determine \(y = \sqrt[5]{f(x)}\) from \(f(x)\) by square rooting it.

0 out of 1
Question 45

Evaluate.

\[ \cos\left(\frac{\pi}{20}\right) \cos\left(\frac{\pi}{5}\right) - \sin\left(\frac{\pi}{20}\right) \sin\left(\frac{\pi}{5}\right) \]

**Solution**

\[ \cos\left(\frac{\pi}{20} + \frac{\pi}{5}\right) \]  \( \frac{1}{2} \) mark for substitution of an appropriate identity

\[ \cos\left(\frac{\pi}{20} + \frac{4\pi}{20}\right) \]

\[ \cos\left(\frac{5\pi}{20}\right) \]

\[ \cos\left(\frac{\pi}{4}\right) \]

\[ \frac{\sqrt{2}}{2} \]

\( \frac{1}{2} \) mark for exact value

1 mark
Exemplar 1

\[
\cos(a+b) \\
\cos\left(\frac{\pi}{20} + \frac{\pi}{3}\right) \\
\cos\left(\frac{\pi}{20} + \frac{4\pi}{20}\right) \\
\cos\left(\frac{5\pi}{20}\right) \\
\cos\left(\frac{\pi}{3}\right)
\]

\(\frac{1}{2}\) out of 1
+ ½ mark for substitution of an appropriate identity

Exemplar 2

\[
\cos\left(\frac{\pi}{20} + \frac{\pi}{3}\right) \\
\cos\left(\frac{\pi}{20} + \frac{4\pi}{20}\right) \\
\cos\left(\frac{5\pi}{20}\right) \\
\cos\left(\frac{\pi}{4}\right)
\]

\(\frac{1}{2}\) out of 1
+ ½ mark for substitution of an appropriate identity
Exemplar 3

\[
\begin{align*}
&= \cos \left( \frac{\pi}{20} + \frac{\pi x y}{5 x y} \right) \\
&= \cos \left( \frac{\pi}{20} \right) \\
&= \cos \left( \frac{\pi}{4} \right) \\
&= \frac{\sqrt{2}}{2}
\end{align*}
\]

½ out of 1

award full marks
– ½ mark for arithmetic errors in lines 2 and 3
Describe the transformations used to obtain the graph of the function \( y = f(-x + 6) - 8 \) from the graph of \( y = f(x) \).

**Solution**

Reflect the graph of \( y = f(x) \) over the \( y \)-axis and then translate 6 units right and 8 units down.

- 1 mark for horizontal reflection
- 1 mark for horizontal translation
- 1 mark for vertical translation

**3 marks**

---

**Note(s):**
- Deduct 1 mark if correct transformations are given in the wrong order.
Exemplar 1

\[ y = f(\overline{-(x-u)}) - 8 \]

- Shift 8 units down
- Shift 6 units right
- Reflection in the y-axis

2 out of 3

award full marks
- 1 mark for correct transformations given in the wrong order

Exemplar 2

- It moves 6 units to the left
- It moves 8 units down
- There's a horizontal reflection

3 out of 3

Exemplar 3

Reflection over the y-axis
- Shift 8 units down
- Shift 6 units right

3 out of 3
State the equations of all the asymptotes of the function, \( y = \frac{1}{3x + 1} \).

**Solution**

\( y = 0 \)  
1 mark for horizontal asymptote

\( x = -\frac{1}{3} \)  
1 mark for vertical asymptote

2 marks
Exemplar 1

\[ x = 0 \]
\[ y = -\frac{1}{3} \]

1 out of 2
award full marks
- 1 mark for concept error

Exemplar 2

horizontal asymptote @ \( y \neq 0 \)
vertical asymptote @ \( x \neq \frac{1}{3} \)

1 out of 2
+ 1 mark for horizontal asymptote
E7 (notation error)

Exemplar 3

vertical asymptote = \(-\frac{1}{3}\)
horizontal asymptote = 0

1½ out of 2
award full marks
- \(\frac{1}{2}\) mark for procedural error
Determine the zeros of the polynomial function \( P(x) = 2x^3 + 5x^2 - 4x - 3. \)

**Solution**

\[ P(1) = 2(1)^3 + 5(1)^2 - 4(1) - 3 \]

1 mark for identifying one possible value of \( x \)

\[ P(1) = 0 \]

\((x - 1)\) is a factor

\[
\begin{array}{c|cccc}
1 & 2 & 5 & -4 & -3 \\
& & 2 & 7 & 3 \\
\hline
& 2 & 7 & 3 & 0
\end{array}
\]

1 mark for synthetic division (or for any equivalent strategy)

\[ P(x) = (x - 1)(2x^2 + 7x + 3) \]

\[ 0 = (x - 1)(2x + 1)(x + 3) \]

\[ x = 1 \quad x = -\frac{1}{2} \quad x = -3 \]

½ mark for consistent factors

½ mark for all zeros

3 marks
Exemplar 1

\[ p(x) = 2x^3 + 5x^2 - 4x - 3 \]
\[ = 2(-3)^3 + (5+3)^2 - 4(-3) - 3 \]
\[ = 2(-27) + 8^2 + 12 - 3 \]
\[ = -54 + 64 + 12 - 3 \]
\[ = -54 + 5 + 12 - 3 \]
\[ = -52 + 5 \]
\[ = 0 \]

\[ \frac{2}{x+3} \]
\[ \begin{array}{c|cccc}
-3 & 2 & 5 & -4 & -3 \\
\hline
1 & -6 & 3 & 3 \\
\hline
2 & -1 & -1 & 0 \\
\end{array} \]

\[ (x^2 - x - 1) \]
\[ \frac{\begin{array}{c}
(x^2 + 1)(x - 1)(x + 3) \\
\end{array}}{p(x)} \]

Zeros of the equation are
\[ x = -1, 1, -3. \]

2½ out of 3
+ 1 mark for identifying one possible value of \( x \)
+ 1 mark for synthetic division
+ ½ mark for consistent zeros
E7 (notation error in line 2)
E7 (transcription error in line 8)
Exemplar 2

\[ (x-1) \text{ is a factor } \]
\[
\begin{array}{c}
| 2 & -4 & 3 \\
2 & 7 & 3 & 0 \\
2x^2 + 7x + 3 \\
6 + 1 = 7 \\
6x + 1 = 6 \\
2x^2 + 1x + 6x + 3 \\
x(2x+1) = (2x+1) \\
(x+3)(2x+1)(x-1) \\
\end{array}
\]

\[ \begin{align*}
x-1 &= 0 \\
x &= 1 \\
\text{zero} &= 1 \\
x + 3 &= 0 \\
x &= -3 \\
\text{zero} &= -3 \\
2x + 1 &= 0 \\
x &= -1 \\
\text{zero} &= \frac{-1}{2}
\end{align*} \]

3 out of 3

Award full marks
E2 (changing an equation to an expression in line 6)
E7 (notation error in lines 5 and 7)
This page was intentionally left blank.
Determine the equation of the radical function represented by the graph.

**Solution**

\[ y = 2\sqrt{x} + 3 \]

1 mark for vertical stretch
1 mark for vertical translation

or

\[ y = \sqrt{4x} + 3 \]

1 mark for horizontal stretch
1 mark for vertical translation

2 marks
Exemplar 1

\[ y = \sqrt{2x} + 3 \]

1 out of 2
+ 1 mark for vertical translation

Exemplar 2

\[ y = 2\sqrt{x+3} \]

1 out of 2
+ 1 mark for vertical stretch
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Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.
- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the Answer/Scoring Sheet.

<table>
<thead>
<tr>
<th>Error Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>final answer</td>
</tr>
<tr>
<td></td>
<td>- answer given as a complex fraction</td>
</tr>
<tr>
<td></td>
<td>- final answer not stated</td>
</tr>
<tr>
<td></td>
<td>- impossible solution(s) not rejected in final answer and/or in steps leading to final answer</td>
</tr>
<tr>
<td>E2</td>
<td>equation/expression</td>
</tr>
<tr>
<td></td>
<td>- changing an equation to an expression or vice versa</td>
</tr>
<tr>
<td></td>
<td>- equating the two sides when proving an identity</td>
</tr>
<tr>
<td>E3</td>
<td>variables</td>
</tr>
<tr>
<td></td>
<td>- variable omitted in an equation or identity</td>
</tr>
<tr>
<td></td>
<td>- variables introduced without being defined</td>
</tr>
<tr>
<td>E4</td>
<td>brackets</td>
</tr>
<tr>
<td></td>
<td>- “( \sin x^2 )” written instead of “( \sin^2 x )”</td>
</tr>
<tr>
<td></td>
<td>- missing brackets but still implied</td>
</tr>
<tr>
<td>E5</td>
<td>units</td>
</tr>
<tr>
<td></td>
<td>- units of measure omitted in final answer</td>
</tr>
<tr>
<td></td>
<td>- incorrect units of measure</td>
</tr>
<tr>
<td></td>
<td>- answer stated in degrees instead of radians or vice versa</td>
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<tr>
<td>E6</td>
<td>rounding</td>
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<tr>
<td></td>
<td>- rounding error</td>
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<td></td>
<td>- rounding too early</td>
</tr>
<tr>
<td>E7</td>
<td>notation/transcription</td>
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<tr>
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<td>- notation error</td>
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<tr>
<td></td>
<td>- transcription error</td>
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<td>- bracket error made when stating domain or range</td>
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<td>- domain or range written in incorrect order</td>
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<td>E9</td>
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<td>- endpoints or arrowheads omitted or incorrect</td>
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<td>- scale values on axes not indicated</td>
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<td>- coordinate points labelled incorrectly</td>
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<td>- asymptotes drawn as solid lines</td>
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<td>- asymptotes omitted but still implied</td>
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<tr>
<td></td>
<td>- graph crosses or curls away from asymptotes</td>
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</tbody>
</table>
IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.
Irregular Test Booklet Report

Test: ________________________________

Date marked: ________________________

Booklet No.: ________________________

Problem(s) noted: ____________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

Question(s) affected: ____________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

Action taken or rationale for assigning marks: ________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________
Follow-up: ...........................................................................................................

...........................................................................................................

...........................................................................................................

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Decision: ...........................................................................................................

...........................................................................................................

...........................................................................................................

...........................................................................................................

...........................................................................................................

Marker’s Signature: ...........................................................................................................

Principal’s Signature: ...........................................................................................................

For Department Use Only—After Marking Complete

Consultant: ...........................................................................................................

Date: ...........................................................................................................
# Table of Questions by Unit and Learning Outcome

## Unit A: Transformations of Functions

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## Unit B: Trigonometric Functions

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## Unit C: Binomial Theorem

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## Unit D: Polynomial Functions

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### Unit E: Trigonometric Equations and Identities

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### Unit F: Exponents and Logarithms

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### Unit G: Radicals and Rationals

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