Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2018

Manitoba

Manitoba Education and Training Cataloguing in Publication Data

Grade 12 pre-calculus mathematics achievement test.
Marking guide. January 2018

This resource is available in print and electronic formats.

ISBN: 978-0-7711-7663-0 (print)

ISBN: 978-0-7711-7664-7 (pdf)

1. Mathematics—Examinations, questions, etc.
2. Educational tests and measurements—Manitoba.
3. Mathematics—Study and teaching (Secondary)—Manitoba.
4. Pre-calculus—Study and teaching (Secondary)—Manitoba.
5. Mathematical ability—Testing.
I. Manitoba. Manitoba Education and Training.
510.76

Manitoba Education and Training
Winnipeg, Manitoba, Canada

After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre. Order online at www.manitobalrc.ca.

This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Training in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Training at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION

Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).

E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

Example: Marks assigned to the student.

Marks Awarded	Booklet 1 25	Selected Response 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

Question 1 P1

A group of 7 friends decide to go to a movie.

Determine how many ways the friends can sit in a row if two of the friends refuse to sit next to each other.

Solution

$7! - 6!\,2! = 3600$ ways

½ mark for $7!$
1 mark for product of $6!2!$
(½ mark for $6!$, ½ mark for $2!$)
½ mark for subtraction

2 marks

Exemplar 1

Case 1: two of the friends

$_7C_2 = 21$

Case 2: All together

$7! = 3440$

Total = 3440 − 21

Total = 3419 ways

½ out of 2

+ ½ mark for 7!
+ ½ mark for subtraction
– ½ mark for arithmetic error in line 4

Exemplar 2

$\frac{7!}{2!}$ ← Number of seats
← Friends that don't want to sit beside eachother.

=2520 different ways the Friends can sit together.

½ out of 2

+ ½ mark for 7!

Exemplar 3

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_7P_2 = \frac{7!}{(7-2)!}$$

$$= 42 \text{ ways.}$$

½ out of 2

+ ½ mark for 7!

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Question 2 R10

Gabrielle listens to her radio at a sound level of 80 dB. She attended a music concert that had a sound level of 115 dB. Determine how many times more intense the music concert was than the radio.

You may use the formula:

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

where β is the intensity level of sound, measured in dB

I is the intensity of sound

I_0 is the standard minimum intensity that a person can hear

Solution

Radio:

$80 = 10\log\left(\frac{I}{I_0}\right)$

$8 = \log\left(\frac{I}{I_0}\right)$

$10^8 = \frac{I}{I_0}$ ½ mark for exponential form

$10^8 I_0 = I$

Music concert:

$115 = 10\log\left(\frac{I}{I_0}\right)$

$11.5 = \log\left(\frac{I}{I_0}\right)$

$10^{11.5} = \frac{I}{I_0}$ ½ mark for exponential form

$10^{11.5} I_0 = I$

$$\frac{\text{intensity of music concert}}{\text{intensity of radio}} = \frac{10^{11.5} I_0}{10^8 I_0}$$

$$= 10^{3.5}$$

$$= 3162.27766$$

$$= 3162.278$$

1 mark for comparison

2 marks

Exemplar 1

radio

$\frac{80\ db}{10} = \frac{10 \log\left(\frac{I}{I_0}\right)}{10}$

$10^8 = \left(\frac{I}{I_0}\right)$

music concert

$\frac{115\ db}{10} = \frac{10 \log\left(\frac{I}{I_0}\right)}{10}$

$10^{11.5} = \left(\frac{I}{I_0}\right)$

$\left(\frac{I}{I_0}\right) = \frac{10^8}{10^{11.5}}$

1 out of 2

+ 1 mark for exponential form

Exemplar 2

$= \frac{\text{Concert}}{\text{Radio}}$

$= \frac{10^{115}}{10^{80}}$

$= 10^{115-80}$

$= 10^{35}$

$1 \times 10^{+35}$ times louder

1 ½ out of 2

award full marks
– ½ mark for procedural error

Exemplar 3

$B = 10 \log\left(\frac{115}{80}\right)$

$B = 10 \log 1.4375$

$B = 1.576 dB$

0 out of 2

Solve, algebraically.

$$2(7)^{x} = 3^{2x-3}$$

Solution

$\log\left(2\left(7^{x}\right)\right) = \log 3^{2x-3}$ ½ mark for applying logarithms

$\log 2 + x\log 7 = (2x-3)\log 3$ 1 mark for product law
1 mark for power law

$\log 2 + x\log 7 = 2x\log 3 - 3\log 3$

$\log 2 + 3\log 3 = 2x\log 3 - x\log 7$ ½ mark for collecting terms with x

$\log 2 + 3\log 3 = x(2\log 3 - \log 7)$

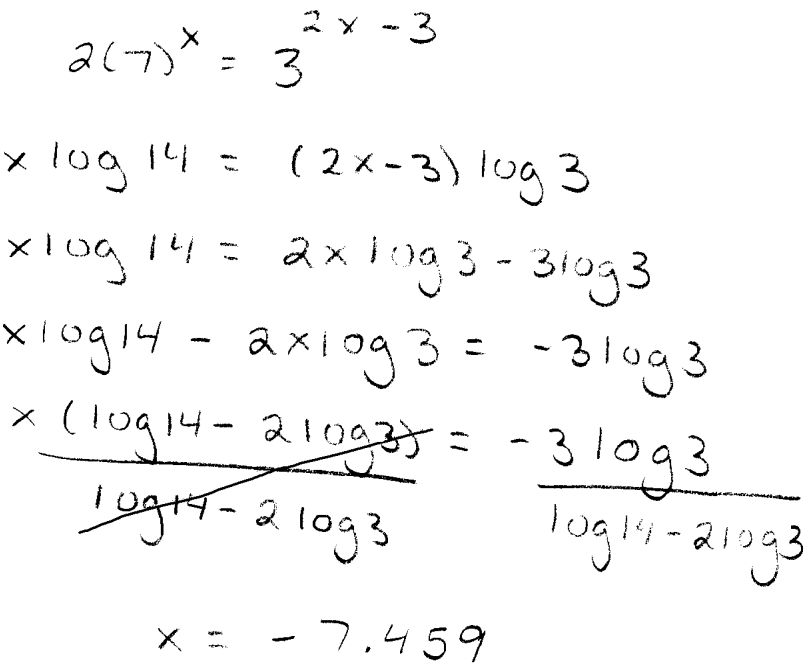
$\dfrac{\log 2 + 3\log 3}{2\log 3 - \log 7} = x$ ½ mark for isolating x

$15.872\,483 = x$

$15.872 = x$ ½ mark for evaluating quotient of logarithms

4 marks

Exemplar 1



3 out of 4

+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for collecting terms with *x*
+ ½ mark for isolating *x*
+ ½ mark for evaluating quotient of logarithms

Exemplar 2

$$\log(2(7^x)) = \log 3^{2x-3}$$

$$\log 2 + x\log 7 = 2x - 3\log 3$$

$$\log 2 + 3\log 3 = 2x - x\log 7$$

$$\log 2 + 3\log 3 = x(2 - \log 7)$$

$$x = \frac{\log 2 + 3\log 3}{(2 - \log 7)}$$

$$x = 1.500$$

3 ½ out of 4

award full marks
– ½ mark for procedural error in line 2

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Solve for θ, algebraically, over the interval $[0, 2\pi]$.

$$\csc^2\theta + 2\csc\theta - 8 = 0$$

Solution

$(\csc\theta + 4)(\csc\theta - 2) = 0$

$\csc\theta = -4 \qquad \csc\theta = 2$ — 1 mark for solving for $\csc\theta$

$\sin\theta = -\frac{1}{4} \qquad \sin\theta = \frac{1}{2}$ — 1 mark for reciprocal

$\theta_r = 0.252\ 680$

$\theta = 3.394 \qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ — 2 marks for solving for θ (½ mark for each value)

$\theta = 6.031$

4 marks

or

$\theta = 3.394 \qquad \theta = 0.524$

$\theta = 6.031 \qquad \theta = 2.618$

Exemplar 1

$(\csc\theta + 4)(\csc\theta - 2) = 0$

$\csc\theta = -4, \quad \csc\theta = -2$

$\sin\theta = -\frac{1}{4}, \quad \sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\theta R = 0.25268$

(QIII) $\theta = \pi + 0.25268$

$\theta = 3.3943$

(QIV) $\theta = 2\pi - 0.25268$

$\theta = 6.03051$

3½ out of 4

award full marks
– ½ mark for arithmetic error in line 2

Exemplar 2

$\csc^2\theta + 2\csc\theta - 8 = 0$

$(\csc\theta - 2)(\csc\theta + 4) = 0$

$\csc\theta = 2, \quad \csc\theta = 4$

$\sin\theta = \frac{1}{2}, \quad \sin\theta = \frac{1}{4}$

$\sin\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

1½ out of 4

+ 1 mark for solving for $\csc\theta$
+ 1 mark for reciprocal
+ 1 mark for solving for θ
– ½ mark for arithmetic error in line 3
– 1 mark for concept error in line 5

Exemplar 3

$$(\csc + 4)(\csc - 2) = 0$$

$$\csc = -4 \qquad \csc = 2$$

$$\frac{1}{\sin\theta} = -4 \qquad \frac{1}{\sin\theta} = 2$$

2 out of 4

+ 1 mark for solving for csc θ
+ 1 mark for reciprocal
E3 (variables omitted in an equation or identity)

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You have forgotten the code to unlock your cell phone. You know the code is made up of four numbers from 0 to 9.

Determine the number of possible codes, if repetition is allowed.

Solution

$\underline{10} \bullet \underline{10} \bullet \underline{10} \bullet \underline{10} = 10\ 000$ **1 mark**

Exemplar 1

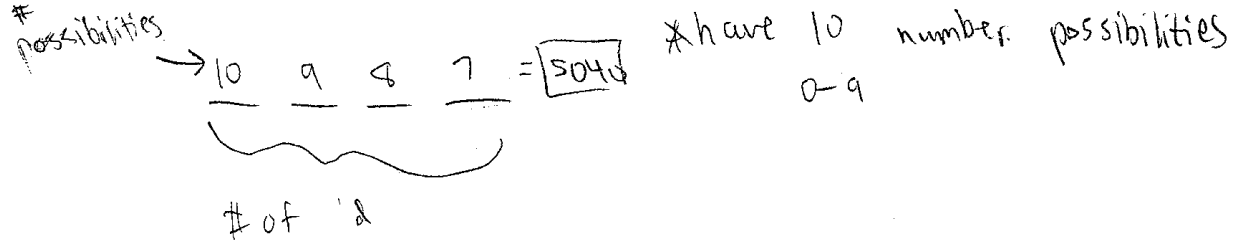
$= 9 \cdot 9 \cdot 9 \cdot 9$

$= 6561$

½ out of 1

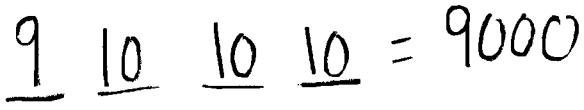
award full marks
– ½ mark for procedural error

Exemplar 2



0 out of 1

Exemplar 3



½ out of 1

award full marks
– ½ mark for procedural error

In the binomial expansion of $\left(\frac{7}{x^3} - 3x^7\right)^n$, the 5th term contains x^7.

Determine the value of *n*.

Solution

$$x^7 = \left(\frac{1}{x^3}\right)^{n-4}\left(x^7\right)^4$$

1 mark for $k = 4$
½ mark for substitution

$$x^7 = \left(x^{-3}\right)^{n-4}\left(x^7\right)^4$$

$$x^7 = x^{-3n+12+28}$$

$$7 = -3n + 40$$

$$-33 = -3n$$

$$11 = n$$

½ mark for solving for *n*

2 marks

$t_{k+1} = {}_nC_k\, a^{n-k} b^k$

$x^7 = {}_nC_4 \left(\frac{7}{x^3}\right)^{n-4} (-3x^7)^4$

$x^7 = \left(\frac{7}{x^3}\right)^{n-4} (-3x^7)^4$

$x^7 = \left(\frac{7}{x^3}\right)^{n-4} (81x^{28})$

$x^7 = (x^{-3})^{n-4} (81x^{28})$

$x^7 = x^{-3n+12} \times x^{28}$

$7 = -3n + 12\ (28)$

$7 = -84n + 336$

$84n = 329$

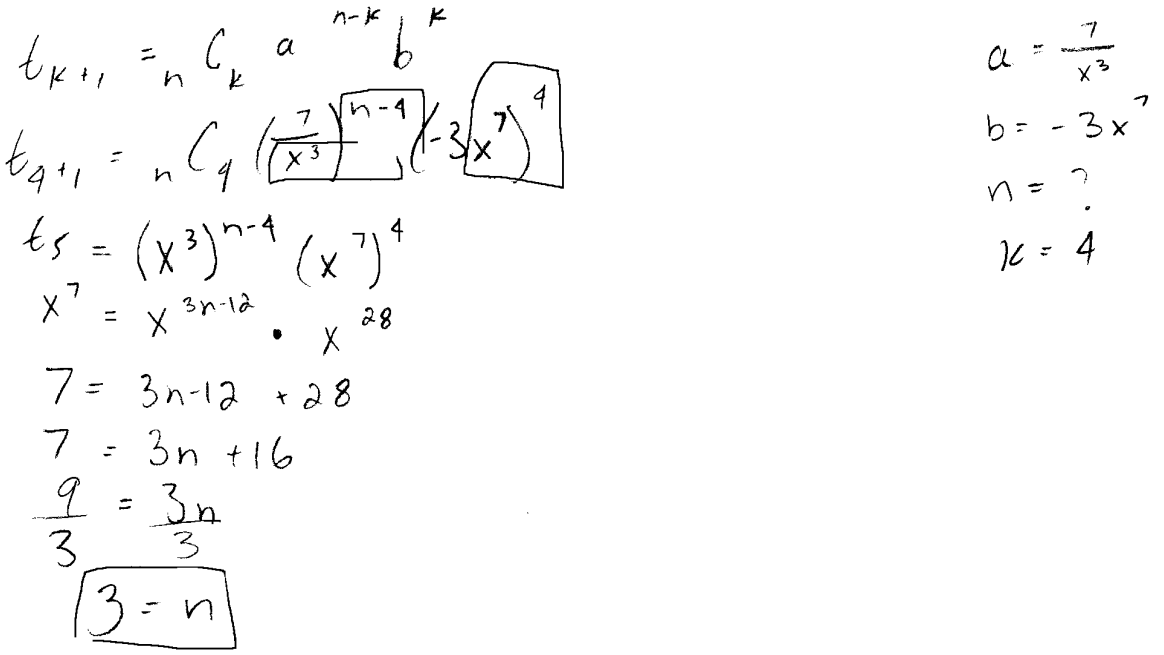
$n = \frac{329}{84}$

$n = \frac{47}{12}$

1 out of 2

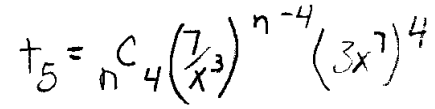
+ 1 mark for $k = 4$
+ ½ mark for substitution
– ½ mark for procedural error in line 2

Exemplar 2



1 out of 2
+ 1 mark for $k = 4$
+ ½ mark for substitution
+ ½ mark for solving for n
– ½ mark for procedural error in line 3
– ½ mark for arithmetic error in line 7

Exemplar 3



1 out of 2

+ 1 mark for $k = 4$

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Question 7 R6

Given the domain of $f(x)$ is $\{-6,1,3,4\}$ and the range of $f(x)$ is $\{-4,7,10,15\}$, state the domain of $f^{-1}(x)$.

Solution

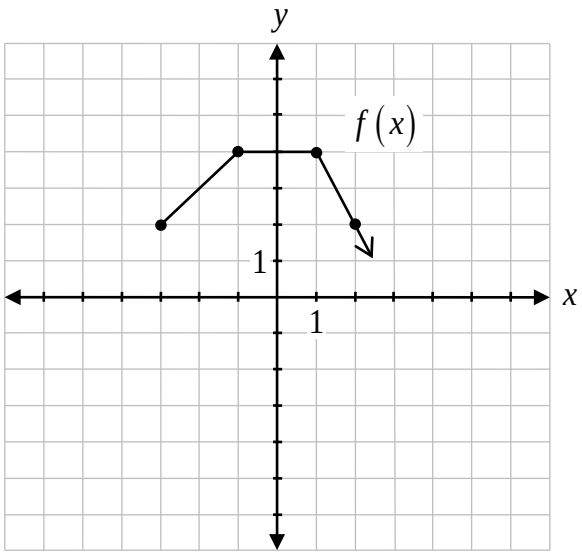
$\{-4,7,10,15\}$ **1 mark**

Exemplar 1

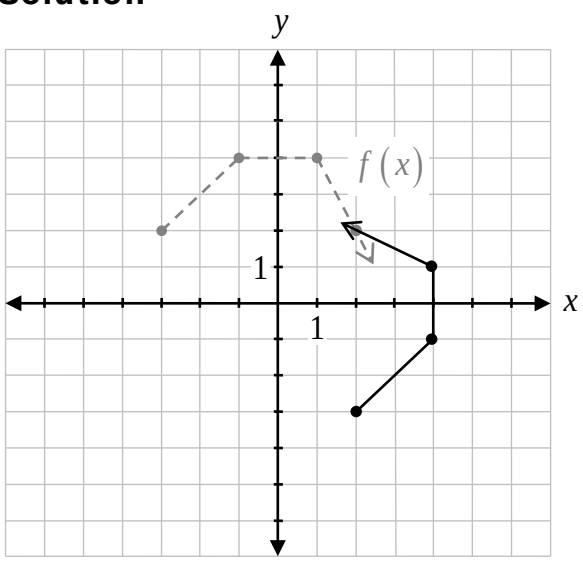
D: $-4 \leq x \leq 15$

0 out of 1

Given the graph of $y = f(x)$, sketch the graph of its inverse.

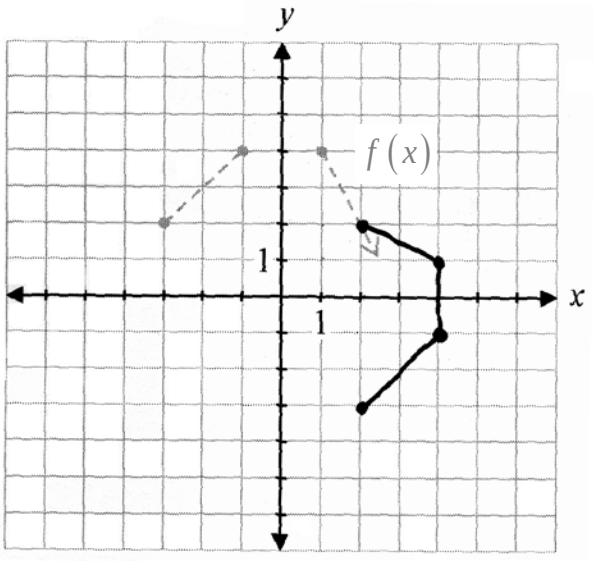


Solution



1 mark

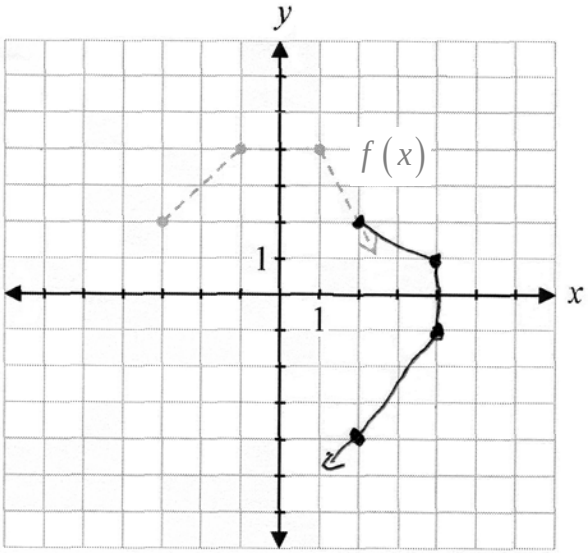
Exemplar 1



1 out of 1

award full marks
E9 (endpoints or arrowheads omitted or incorrect)

Exemplar 2



½ out of 1

award full marks
– ½ mark for procedural error (one incorrect point)
E9 (endpoints or arrowheads omitted or incorrect)

Question 9 T6

Prove the following identity for all permissible values of θ.

$$\frac{1+\cos\theta}{1-\sin^2\theta} = \sec\theta + \tan^2\theta + 1$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\frac{1+\cos\theta}{1-\sin^2\theta}$	$\sec\theta + \tan^2\theta + 1$
$\frac{1+\cos\theta}{\cos^2\theta}$	
$\frac{1}{\cos^2\theta} + \frac{1}{\cos\theta}$	
$\sec^2\theta + \sec\theta$	
$\tan^2\theta + 1 + \sec\theta$	

1 mark for algebraic strategies

1 mark for logical process to prove the identity

1 mark for correct substitution of appropriate identities

3 marks

Solution

Method 2

Left-Hand Side	Right-Hand Side	
$\frac{1+\cos\theta}{\cos^2\theta}$	$\frac{1}{\cos\theta}+\sec^2\theta$	
	$\frac{1}{\cos\theta}+\frac{1}{\cos^2\theta}$	1 mark for algebraic strategies
	$\frac{\cos\theta+1}{\cos^2\theta}$	1 mark for logical process to prove the identity
		1 mark for correct substitution of appropriate identities
		3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
	$\frac{1}{\cos\theta} + \frac{\sin^2\theta}{\cos^2\theta} + 1$ $\frac{1}{\cos\theta} + \frac{\sin^2\theta}{1-\sin^2\theta} + 1$ $\frac{1}{\cos\theta} + \frac{1-\cos^2\theta}{1-\sin^2\theta} + 1$ $\frac{1+\cos\theta}{1-\sin^2\theta} = MG.$

1 out of 3

+ 1 mark for correct substitution of appropriate identities

Exemplar 2

Left-Hand Side	Right-Hand Side
$\frac{1+\cos\theta}{\cos^2\theta}$	$\frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos^2\theta} + 1$ $\frac{\cos\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} + 1$ $\frac{\cos\theta + \sin^2\theta}{\cos^2\theta} + 1$ $\frac{\cos\theta + \sin^2\theta}{1-\sin^2\theta} + 1$

2 out of 3

+ 1 mark for correct substitution of appropriate identities
+ 1 mark for algebraic strategies

Exemplar 3

Left-Hand Side	Right-Hand Side
$\frac{1+\cos\theta}{1-\sin^2\theta}$ $\frac{1+\cos\theta}{\cos^2\theta}$ $\frac{1}{\cos\theta}$	$\frac{1}{\cos\theta} + \sec^2\theta - 1 + 1$ $\frac{1}{\cos\theta} + \frac{1}{\tan^2\theta + 1}$ $\frac{1}{\cos\theta} + \frac{1}{\quad}$

0 out of 3

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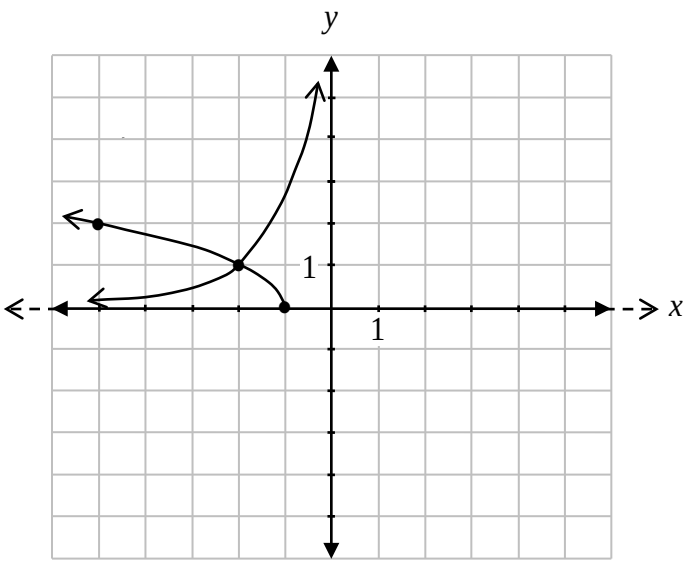
Thomas used graphs to solve the equation $e^{x+2} = \sqrt{-(x+1)}$.



He incorrectly states the solution as $(-2, 1)$.

Describe how Thomas should have stated the solution.

Solution

He stated his solution as a coordinate point; his solution should have only been the value of *x*.

1 mark

Exemplar 1

(-2,1) is not the solution, (-2,1) is where the graphs intersect.

0 out of 1

Exemplar 2

He can't include y.

1 out of 1

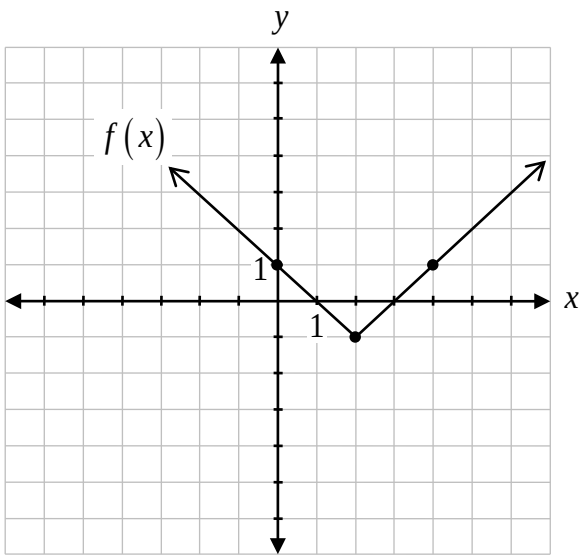
Exemplar 3

he must state $x = -2$

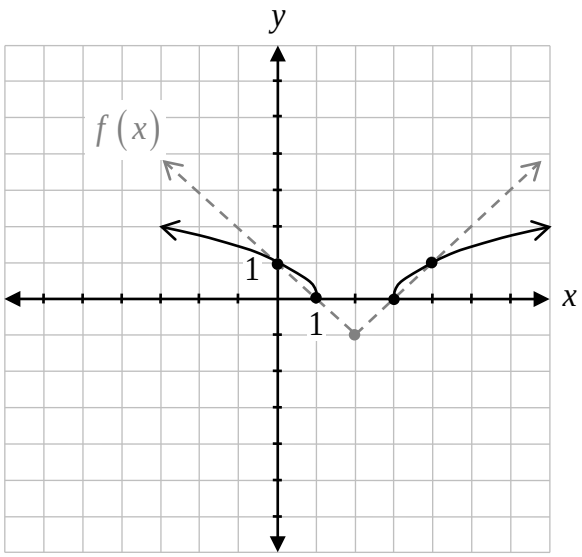
$y = 1$

0 out of 1

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.



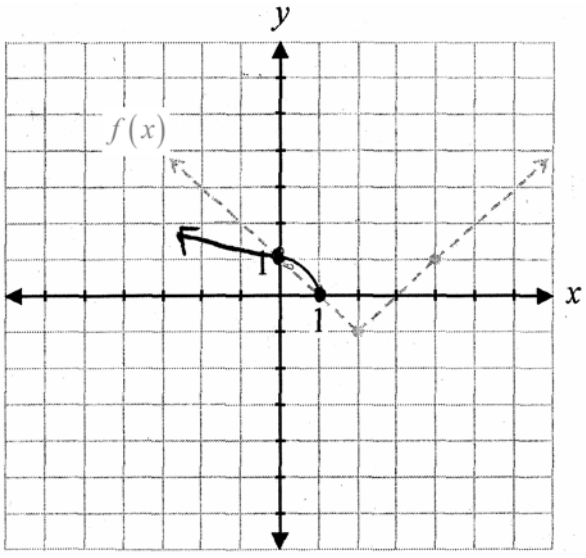
Solution



1 mark for restricting domain
½ mark for shape between both pairs of invariant points
½ mark for shape above both pairs of invariant points

2 marks

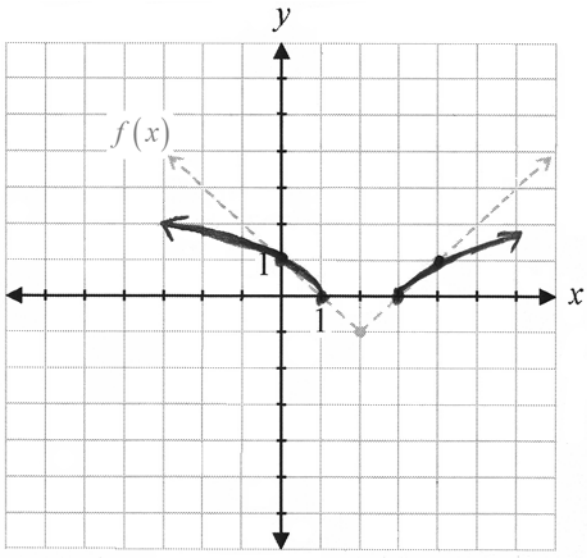
Exemplar 1



1 out of 2

award full marks
– 1 mark for concept error (omitting right branch)

Exemplar 2



1½ out of 2

+ 1 mark for restricting domain
+ ½ mark for shape above both pairs of invariant points

When a polynomial, $P(x)$, is divided by $(x-2)$ the resulting equation is $\frac{P(x)}{x-2} = x^2 - x + 1 + \frac{3}{x-2}$.

a) Explain why $x-2$ is not a factor of $P(x)$.

b) Determine the equation for the polynomial function $P(x)$.

Solution

a) There is a remainder when $P(x)$ is divided by $x-2$. **1 mark**

b) $P(x) = \underline{(x-2)(x^2 - x + 1) + 3}$ **1 mark**

or

$P(x) = \underline{x^3 - 3x^2 + 3x + 1}$

Exemplar 1

a)

If it was a factor there would not be the term $x - 2$ as the denominator.

1 out of 1

b)

$$\left(x^2 - x + 1 + \frac{3}{x+2}\right)(x+2)$$

$$x^3 - x^2 + x + 2x^2 - 2x + 2 + 3$$

$$P(x) = x^3 + x^2 - x + 5$$

1 out of 1

award full marks
E7 (transcription error)

Exemplar 2

a)

to be a factor, the remainder must be Ø.

1 out of 1

b)

$P(x) =$ $(x-2)\left(x^2-x+1+\frac{3}{x-2}\right)$

1 out of 1

Exemplar 3

a)

Because there is a remainder of $\frac{3}{x-2}$

½ out of 1

award full marks
– ½ mark for terminology error (including $x - 2$ as part of the remainder)

b)

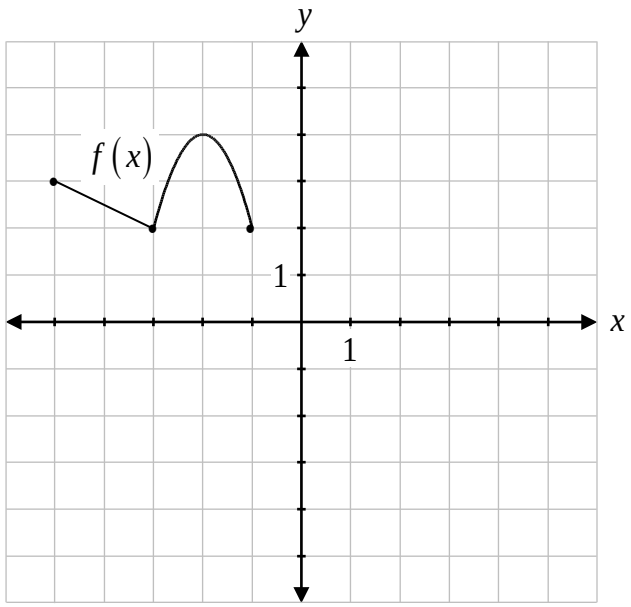
$$P(x) = (x^2 - x + 1)(x - 2)$$
$$= x^3 - 2x^2 - x^2 + 2x + x - 2$$
$$= x^3 - 3x^2 + 3x - 2$$

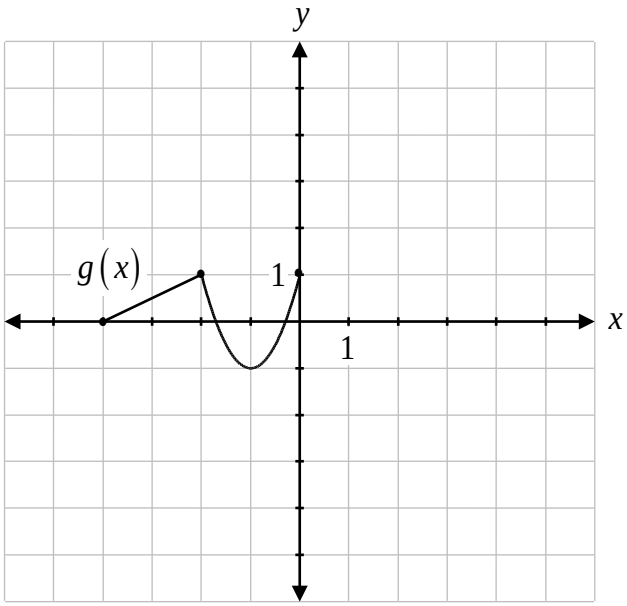
$$P(x) = x^3 - 3x^2 + 3x - 2$$

0 out of 1

Question 13 R2, R5

Determine the equation for $g(x)$ in terms of $f(x)$.





Solution

$g(x) = -f(x-1)+3$

1 mark for vertical reflection
1 mark for horizontal translation
1 mark for vertical translation

3 marks

Exemplar 1

$g(x) =$ $-1(x-1)-3$

f(x)

X	Y
-5	3
-3	2
-1	2

+1 x

g(x)

x	Y
-4	0
-2	1
0	1

h = +1

ref y's - a

$a(x-h)+k$

1 out of 3

+ 1 mark for vertical reflection
+ 1 mark for horizontal translation
– 1 mark for concept error (omitting *f*)

Exemplar 2

$g(x) =$ $-g(x-1)-2$

1½ out of 3

+ 1 mark for vertical reflection
+ 1 mark for horizontal translation
– ½ mark for procedural error (*g* instead of *f*)

Exemplar 3

$g(x) =$ $f(-x+1)+3$

2 out of 3

+ 1 mark for horizontal translation
+ 1 mark for vertical translation

Explain why the binomial expansion of $(2x + y)^9$ does not have a middle term.

Solution

The expansion contains $n + 1$ terms. Since *n* equals 9, there are 10 terms, which would not allow for a middle term.

1 mark

Exemplar 1

because that 9 would be a 10 and since 10 is an even number there is no middle term.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

n = 9

k = 8

if there are 8 terms there is no middle term, 1 1 1 1 | 1 1 1 1.

0 out of 1

award full marks
– 1 mark for concept error (incorrect number of terms)

Exemplar 3

doesnt have a middle term because they're an even amount

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 15 R8

Using the laws of logarithms, completely expand the expression $\log\left(\frac{5\sqrt{a}}{b^3}\right)$.

Solution

$\log 5 + \frac{1}{2}\log a - 3\log b$

1 mark for product law
1 mark for power law (½ mark for each)
1 mark for quotient law

3 marks

Exemplar 1

$$\log\left(\frac{5\sqrt{a}}{b^3}\right)$$

$$\log_a M - \log_a N = \log_a\left(\frac{M}{N}\right)$$

$$\log 5\, a^{\frac{1}{2}} - \log b^3 =$$

$$\frac{1}{2}\log 5 - 3\log b =$$

1½ out of 3

+ ½ mark for power law
+ 1 mark for quotient law
E7 (transcription error in line 4)

Exemplar 2

$$\log 5\sqrt{a} - \log b^3$$

$$\boxed{\log 5\sqrt{a} - 3\log b}$$

1½ out of 3

+ ½ mark for power law
+ 1 mark for quotient law

Exemplar 3

$$= \log 5\sqrt{a} - \log b^3$$
$$= \log 5a^{\frac{1}{2}} - 3\log b$$
$$= \frac{1}{2}\log 5a - 3\log b$$
$$= \frac{1}{2}\log 5 + \log a - 3\log b$$

2 out of 3

+ 1 mark for product law
+ ½ mark for power law
+ 1 mark for quotient law
– ½ mark for procedural error in line 4

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Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
16	D	T1
17	B	R12
18	A	R7
19	C	T1
20	D	P2
21	B	R14
22	B	R3
23	C	T6
24	A	R9

Question 16 T1

Identify $10°$ in radians.

a) $\frac{1800}{\pi}$

b) $\frac{\pi}{1800}$

c) $\frac{18}{\pi}$

d) $\frac{\pi}{18}$

Question 17 R12

The polynomial function, $P(x) = a(x-1)^2(x+4)^2$, has a *y*-intercept of -8.

Identify the value of *a*.

a) -2

b) $-\frac{1}{2}$

c) $\frac{1}{2}$

d) 2

Question 18 R7

Identify the value of $\log_4\left(\frac{1}{16}\right)$.

a) -2

b) $-\frac{1}{2}$

c) $\frac{1}{2}$

d) 2

Question 19 T1

Given the angle $\frac{25\pi}{7}$, identify the coterminal angle on the interval $[-2\pi, 0]$.

a) $\frac{18\pi}{7}$

b) $\frac{11\pi}{7}$

c) $-\frac{3\pi}{7}$

d) $-\frac{10\pi}{7}$

Question 20 P2

Identify which expression cannot be evaluated.

a) ${}_7P_0$

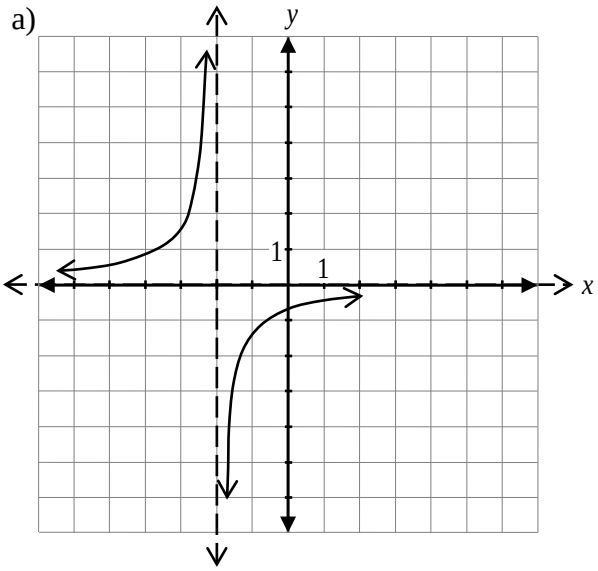
b) ${}_7P_6$

c) ${}_7P_7$

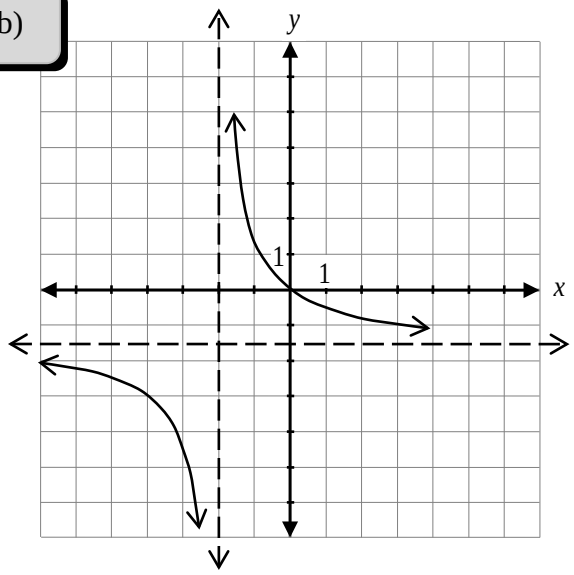
d) ${}_7P_8$

Identify the graph of $f(x) = \dfrac{-3x}{2x+4}$.

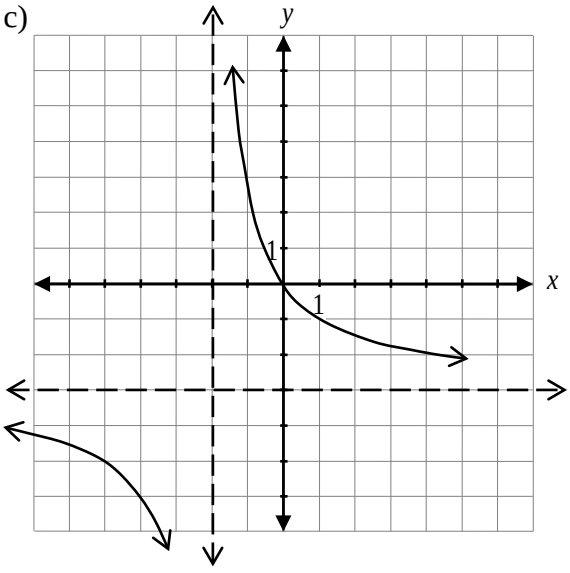
a)



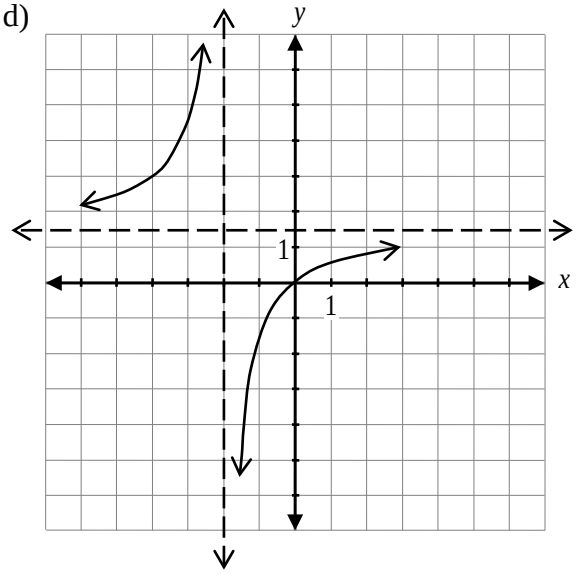
b)



c)



d)



Question 22 R3

Given a point $(-2, 0)$ on the graph of $y = f(x)$, identify the coordinates of the corresponding point on the graph of $y = 4f\left(\frac{1}{2}x\right)$.

a) $(-8, 0)$

b) $(-4, 0)$

c) $(-2, 0)$

d) $(-1, 0)$

Question 23 T6

Identify the non-permissible value of θ for the expression $\frac{\cos\theta}{1+\sin\theta}$.

a) $\frac{\pi}{2}$

b) π

c) $\frac{3\pi}{2}$

d) 2π

Question 24 R9

Identify the function with an asymptote at $x = -3$.

a) $y = \log(x+3)$

b) $y = \log x + 3$

c) $y = \log(x-3)$

d) $y = \log x - 3$

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Evaluate the following expression.

$$\tan\left(\frac{2\pi}{3}\right)\csc\left(\frac{-2\pi}{3}\right)+\cos\left(3\pi\right)$$

Solution

$\left(-\sqrt{3}\right)\left(-\frac{2}{\sqrt{3}}\right)+\left(-1\right)$

$2-1$

1

1 mark for $\tan\left(\frac{2\pi}{3}\right)$ (½ mark for quadrant, ½ mark for value)

1 mark for $\csc\left(-\frac{2\pi}{3}\right)$ (½ mark for quadrant, ½ mark for value)

1 mark for $\cos\left(3\pi\right)$

3 marks

Exemplar 1

$= \left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)\left(\frac{1}{\frac{\sqrt{3}}{2}}\right) + (1)$

$= \left(\frac{\sqrt{3}}{2} \cdot \frac{2}{1}\right)\left(\frac{1}{1} \cdot \frac{2}{\sqrt{3}}\right)$

$= (\sqrt{3})\left(\frac{2}{\sqrt{3}}\right) + 1$

$= \frac{2\sqrt{3}}{\sqrt{3}} + 1$

$= 2 + 1$

$= 3$

1 out of 3

+ ½ mark for value of $\tan\left(\frac{2\pi}{3}\right)$

+ ½ mark for value of $\csc\left(-\frac{2\pi}{3}\right)$

E7 (transcription error in line 2)

Exemplar 2

$\left(\frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}\right) \cdot \left(-\frac{2}{\sqrt{3}}\right)$

$\left(-\sqrt{3}\right) \cdot \left(-\frac{2}{\sqrt{3}}\right) + (-1)$

$\frac{-\sqrt{3}}{1} \cdot -\frac{2}{\sqrt{3}} - 1$

$\frac{-2\sqrt{3}}{\sqrt{3}} - 1$

$-2 - 1$

$= -3$

2½ out of 3

award full marks
– ½ mark for arithmetic error in line 4
E7 (transcription error in line 1)

Exemplar 3

$\frac{\sin}{\cos}\left(\frac{2\pi}{3}\right)\frac{1}{\sin}\left(-\frac{2\pi}{3}\right)+\cos(3\pi)$

$\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\cdot\frac{1}{\frac{-\sqrt{3}}{2}}+0$

$\frac{\sqrt{3}}{1}\cdot\frac{-2}{\sqrt{3}}$

$\frac{-2}{1}$

1½ out of 3

+ 1 mark for $\tan\left(\frac{2\pi}{3}\right)$

+ 1 mark for $\csc\left(-\frac{2\pi}{3}\right)$

– ½ mark for arithmetic error in line 3
E7 (notation error in line 1)

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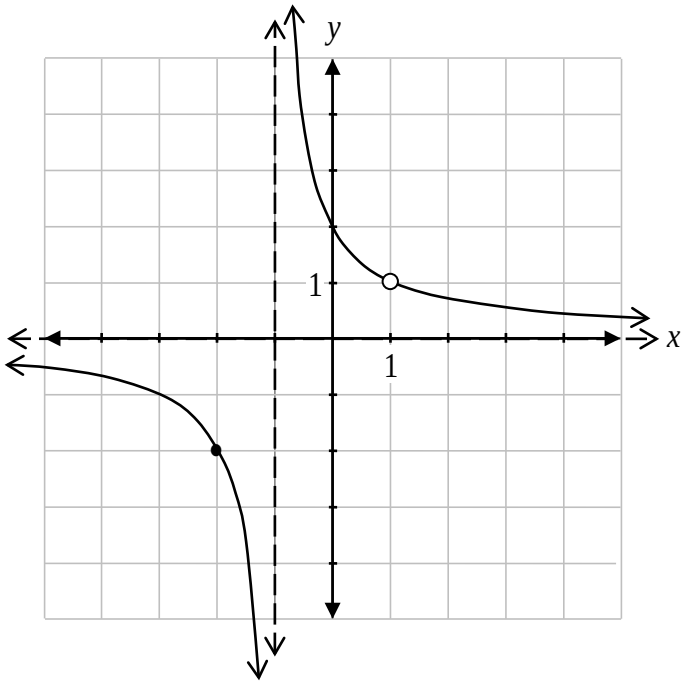
State the range of the graph below.



Solution

Range: $\{y \in \mathbb{R},\ y \neq 0 \text{ and } y \neq 1\}$ 1 mark (½ mark for $y \neq 0$, ½ mark for $y \neq 1$)

1 mark

Exemplar 1

Range: $(-\infty, 0] \cup [0, 1) \cup (1, \infty)$

1 out of 1

award full marks
E8 (bracket error made when stating domain or range)

Exemplar 2

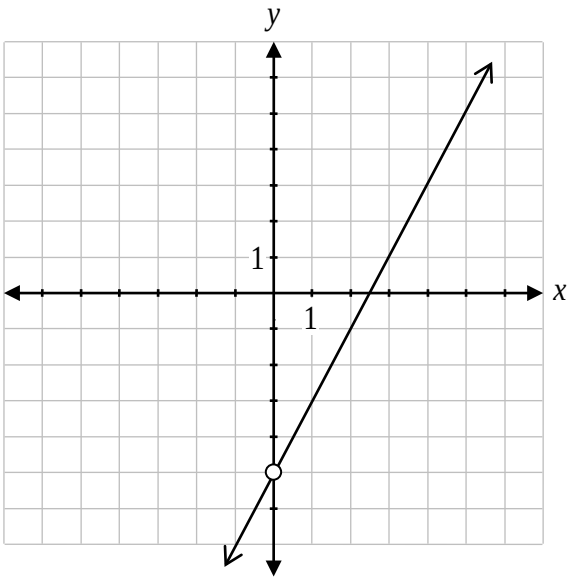
Range: $(\infty, 0) \cup (0, -\infty)$

½ out of 1

+ ½ mark for $y \neq 0$
E8 (domain or range written in incorrect order)

Sketch the graph of the function $f(x) = \dfrac{2x^2 - 5x}{x}$.

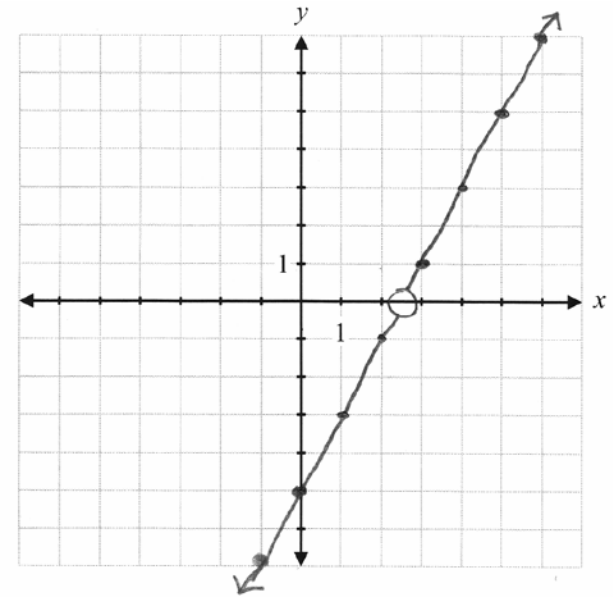
Solution



1 mark for point of discontinuity (hole) at $(0, -5)$
(½ mark for $x = 0$, ½ mark for $y = -5$)
1 mark for shape of a linear function

2 marks

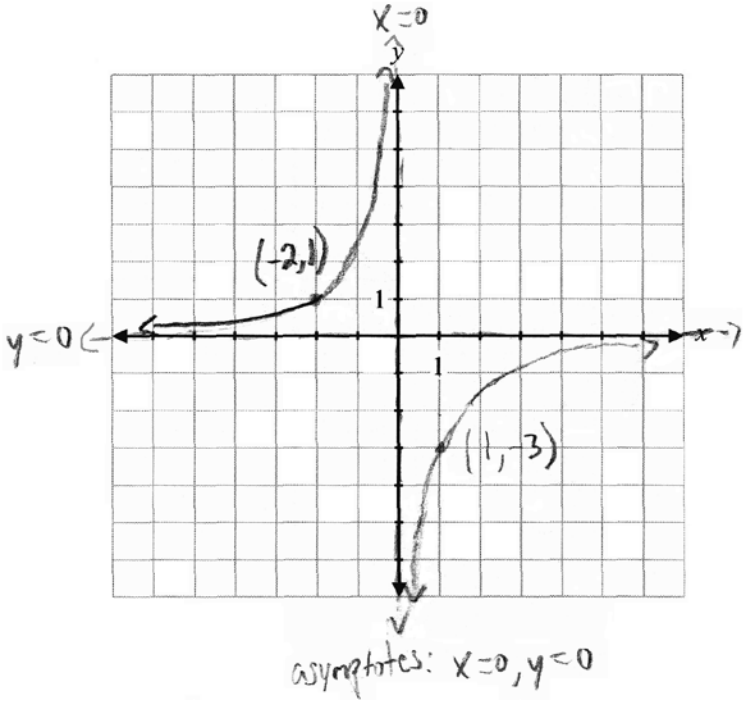
Exemplar 1



1 out of 2

+ 1 mark for shape of a linear function

Exemplar 2



0 out of 2

Question 28 R12

State a possible value of *n* if the polynomial function $P(x) = (x-1)^2 (x+2)^n$ has a range of $[0, \infty)$.

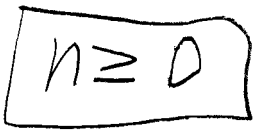
Solution

$n = 2$ **1 mark**

Note(s):

- Accept any even positive value of *n*, including zero.

Exemplar 1



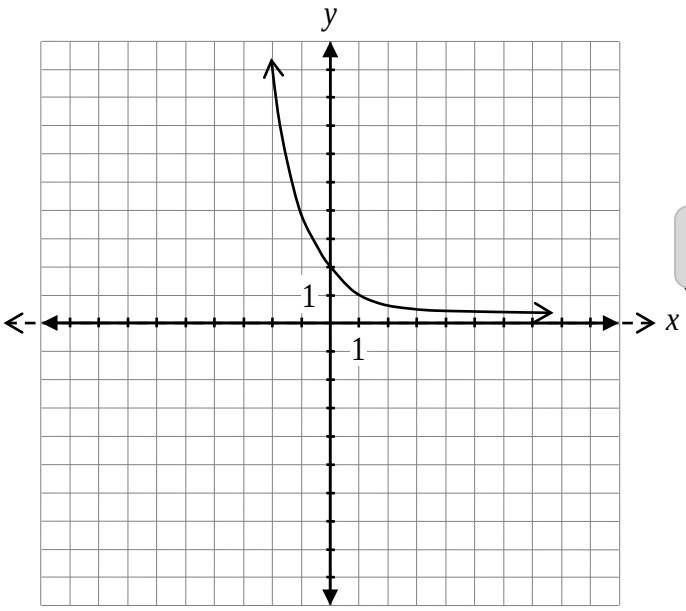
0 out of 1

Exemplar 2

1 out of 1

Sketch the graph of $y = \left(\frac{1}{2}\right)^{x-1}$.

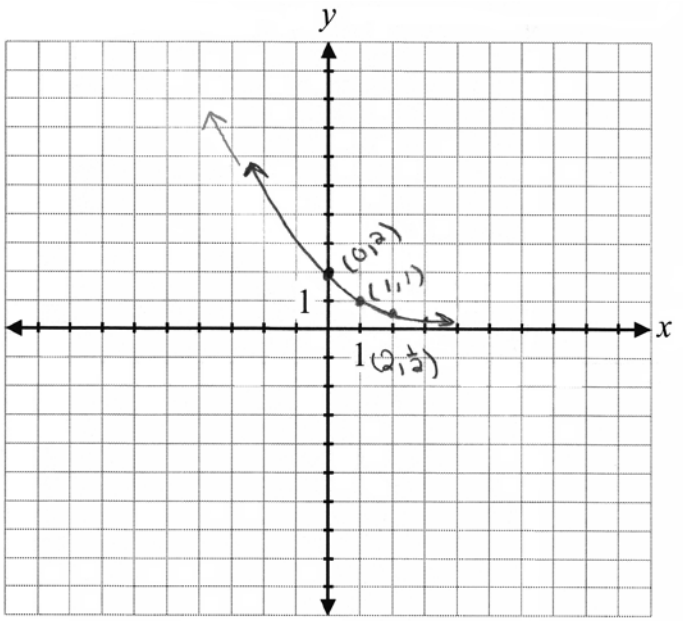
Solution



1 mark for decreasing exponential function
1 mark for horizontal translation

2 marks

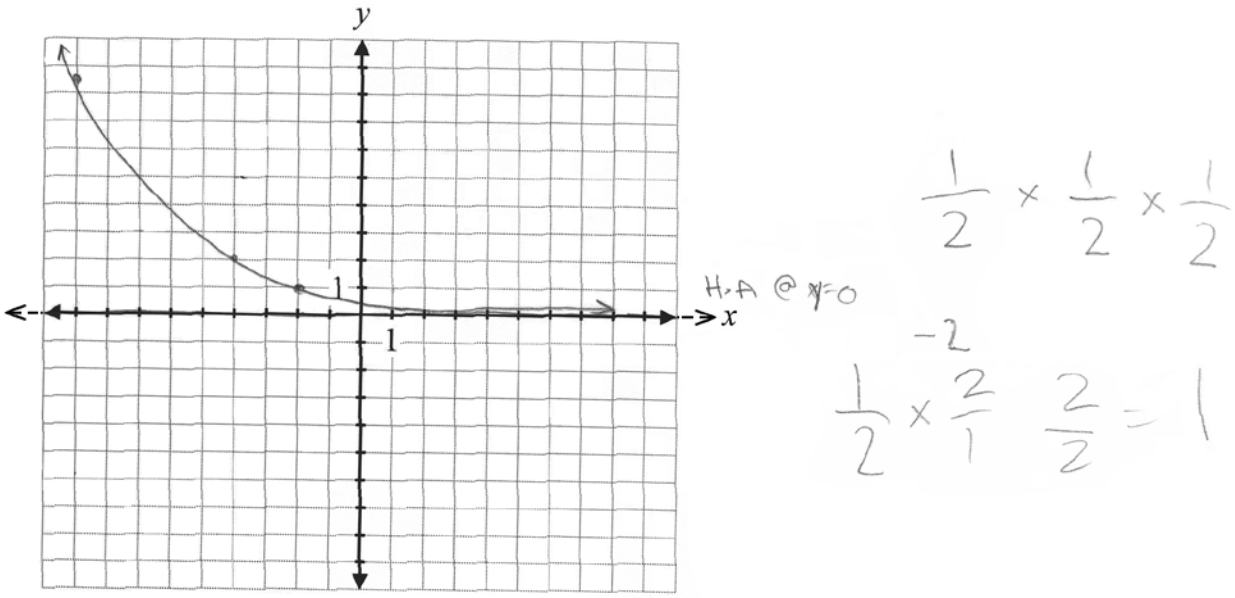
Exemplar 1



2 out of 2

award full marks
E10 (asymptotes omitted but still implied)

Exemplar 2



1 out of 2

+ 1 mark for decreasing exponential function

Solve.

$$\log_x 27 = 3$$

Solution

$x^3 = 27$ 1 mark for exponential form

$x = 3$

1 mark

Exemplar 1

$x^3 = 27$

$3^3 = 27$ $\qquad 3 \cdot 3 = 9 \times 3$

1 out of 1

award full marks
E1 (final answer not stated)

Exemplar 2

$\sqrt[3]{x^3} = \sqrt[3]{27}$

$\boxed{x = 9}$

½ out of 1

+ 1 mark for exponential form
– ½ mark for arithmetic error

Exemplar 3

$3^x = 27$

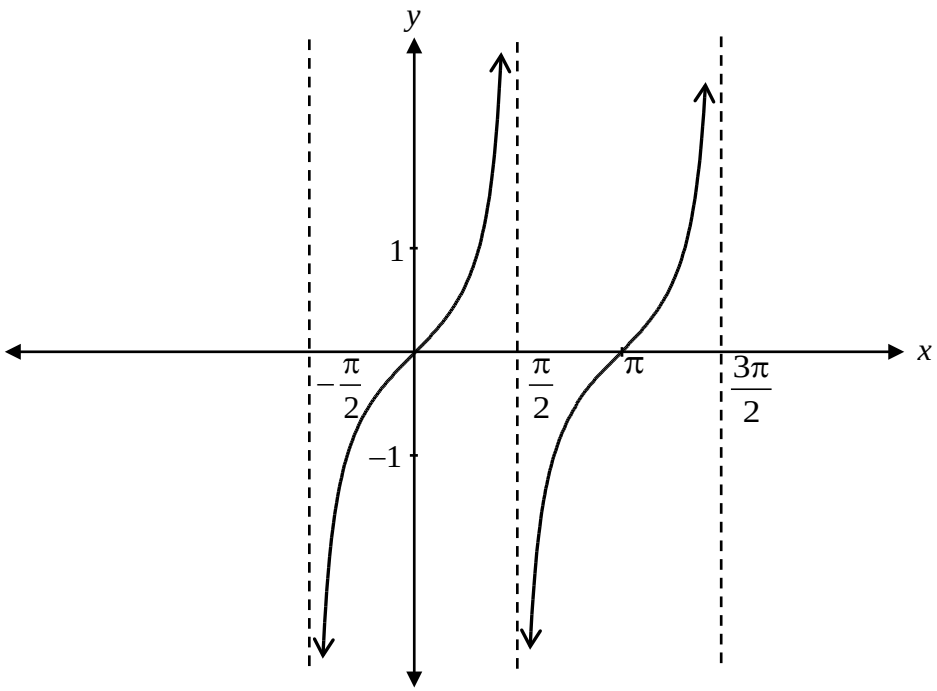
$3^x = 3^3$

$\boxed{x = 3}$

0 out of 1

Sketch at least two periods of the graph $y = \tan x$.

Solution

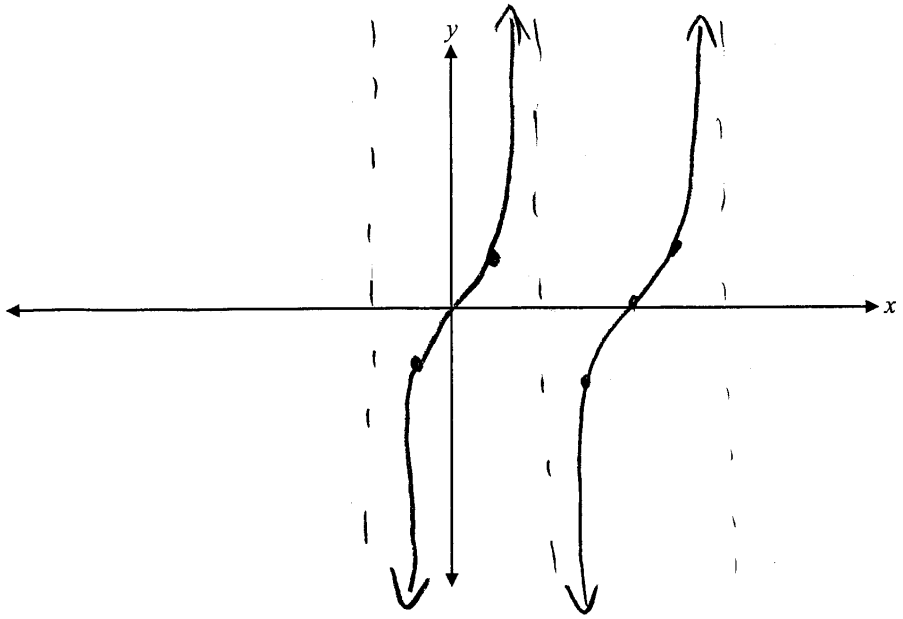


1 mark for increasing trigonometric function

1 mark for asymptotic behaviour approaching $x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

2 marks

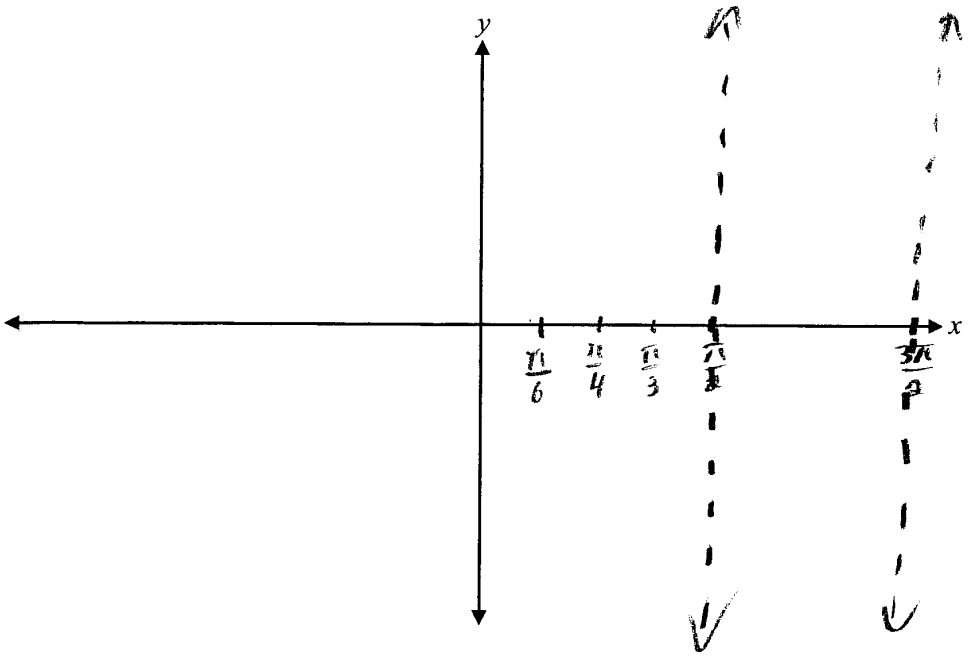
Exemplar 1



1 out of 2

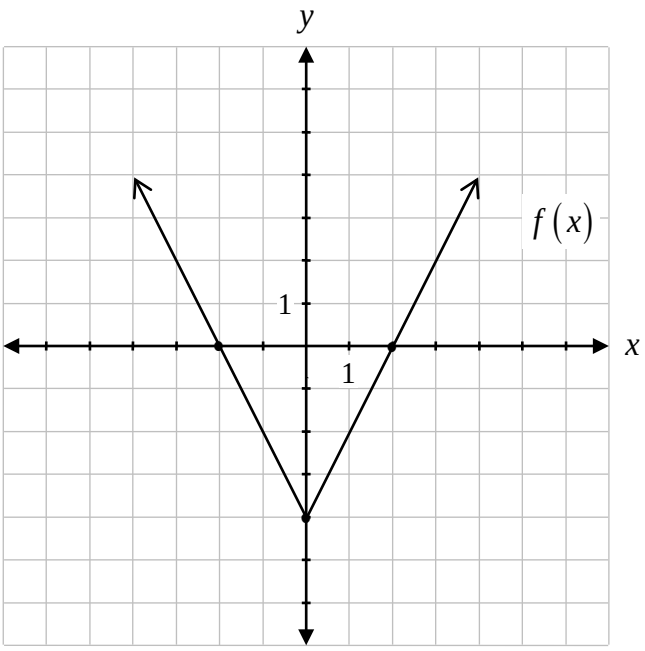
+ 1 mark for increasing trigonometric function
E9 (scale values on axes not indicated)

Exemplar 2



0 out of 2

Given the graph of $f(x)$, state the domain of $\frac{1}{f(x)}$.



Solution

Domain: $\{x \in \mathbb{R},\ x \neq \pm 2\}$ 1 mark (½ mark for $x \neq 2$, ½ mark for $x \neq -2$)

1 mark

Exemplar 1

Domain: $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$

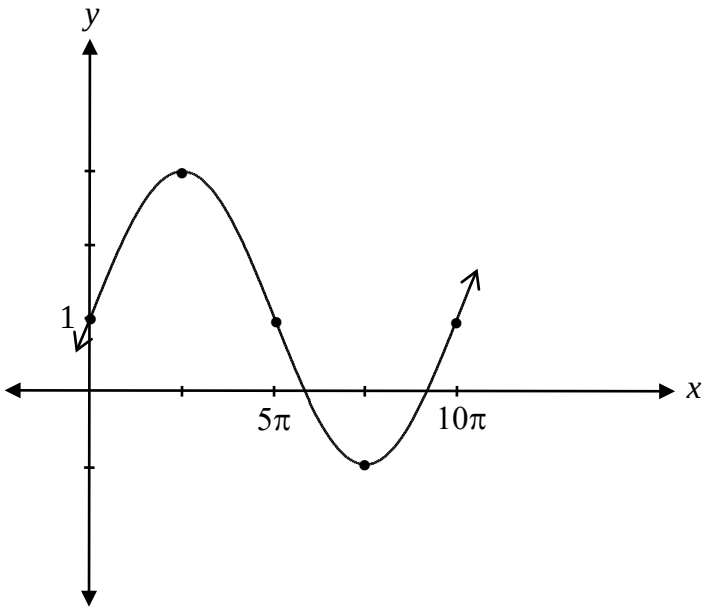
1 out of 1

Exemplar 2

Domain: $x \in \mathbb{R}, x \geq \pm 2$

0 out of 1

Determine the values of A, B, and D of the sinusoidal function in the form $y = \mathrm{A}\sin\left(\mathrm{B}x\right) + \mathrm{D}$.



Solution

A = 2 1 mark for A

B = $\frac{1}{5}$ 1 mark for B

D = 1 1 mark for D

3 marks

Exemplar 1

A = 2

$\frac{2\pi}{10\pi} = \frac{\pi}{5}$

B = $\frac{\pi}{5}$

D = 1

2½ out of 3

award full marks
– ½ mark for arithmetic error while calculating value of B

Exemplar 2

A = 1

B = $\frac{1}{5}$

D = 2

1 out of 3

+ 1 mark for B

Determine if the point $\left(-\frac{\sqrt{7}}{5}, \frac{2}{5}\right)$ is on the unit circle.

Justify your answer.

Solution

$$x^2 + y^2 = 1$$

$$\text{Left-hand side} = \left(-\frac{\sqrt{7}}{5}\right)^2 + \left(\frac{2}{5}\right)^2$$

$$= \frac{7}{25} + \frac{4}{25}$$

$$= \frac{11}{25}$$

$$\frac{11}{25} \neq 1$$

$\therefore$ not on the unit circle

1 mark for justification

1 mark

Exemplar 1

$a^2 + b^2$

$\left(-\frac{\sqrt{7}}{5}\right)^2 + \left(\frac{2}{5}\right)^2$

$\frac{-7}{25} + \frac{4}{25}$

$= \frac{-3}{25} \neq 0$

No it is not on the unit circle

0 out of 1

Exemplar 2

No.

0 out of 1

Exemplar 3

$\left(-\frac{\sqrt{7}}{5}\right)^2 + \left(\frac{2}{5}\right)^2$

$-\frac{7}{25} + \frac{4}{25} = 1$

$-\frac{3}{25} \neq 1$

no

½ out of 1

award full marks
– ½ mark for arithmetic error in line 2
E7 (notation error in line 2)

Solve, algebraically.

$$\frac{{}_{n}C_{5}}{{}_{n}C_{4}} = 6$$

Solution

$$\frac{\frac{n!}{(n-5)!5!}}{\frac{n!}{(n-4)!4!}} = 6$$ ½ mark for substitution into equation

$$\frac{n!(n-4)!4!}{n!(n-5)!5!} = 6$$

$$\frac{\cancel{n!}\,(n-4)\,\cancel{(n-5)!}\,\cancel{4!}}{\cancel{n!}\,\cancel{(n-5)!}\,5 \bullet \cancel{4!}} = 6$$

1 mark for factorial expansion
(½ mark for numerical factors; ½ mark for factors with variables)
1 mark for simplification of factorials
(½ mark for numerical factors; ½ mark for factors with variables)

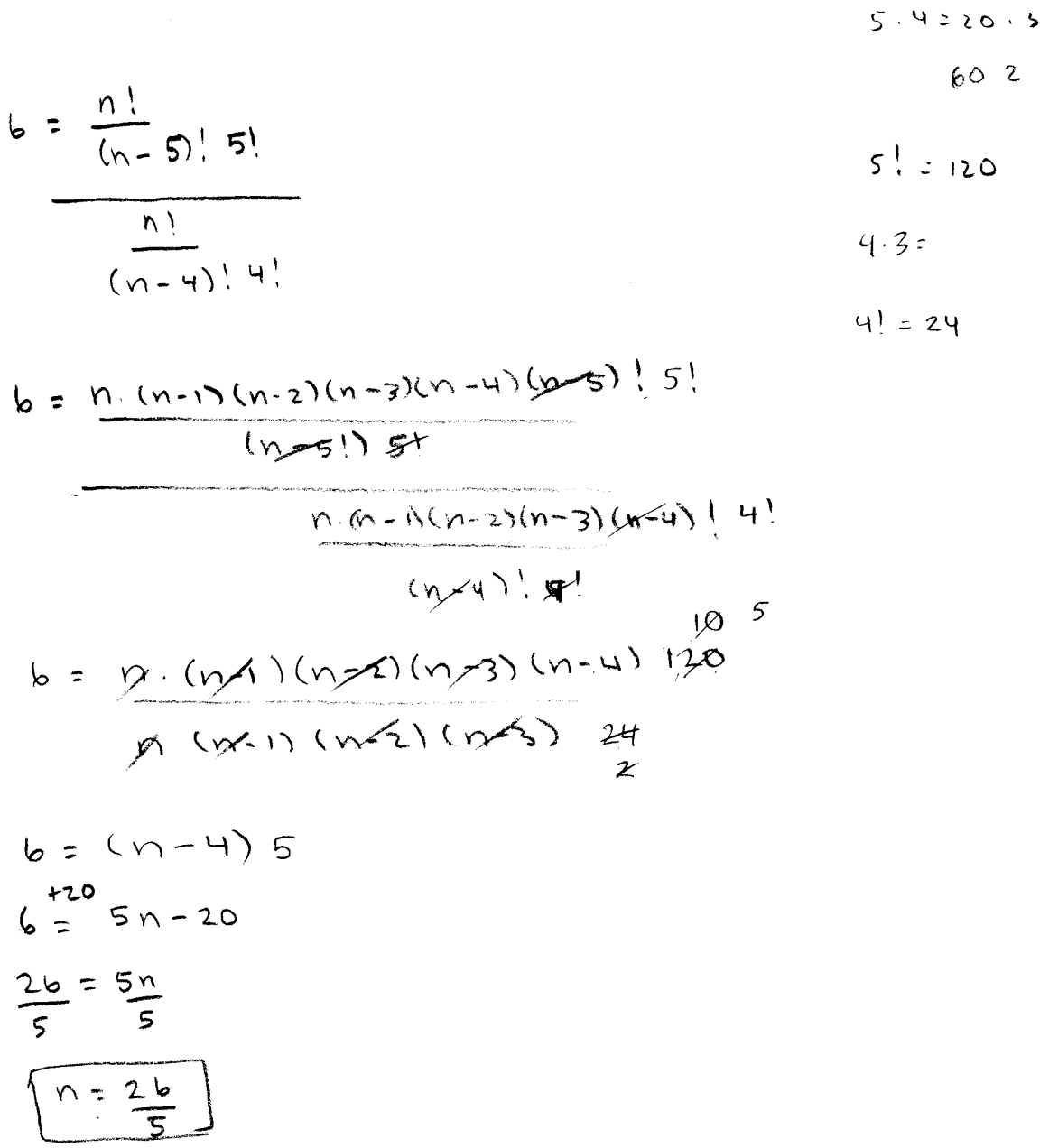
$$\frac{n-4}{5} = 6$$

$$n - 4 = 30$$

$n = 34$ ½ mark for solving for n

3 marks

Exemplar 1



2½ out of 3

award full marks
– ½ mark for procedural error in line 2
E7 (notation error in line 2)
E1 (impossible solution not rejected in final answer)

Exemplar 2

$$\frac{\frac{n!}{(n-5)!5!}}{\frac{n!}{(n-4)!4!}} = 6$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)\ldots}{(n-5)!\,5!}$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)\ldots}{(n-4)!\,4!}$$

$$\frac{\frac{n(n-1)(n-2)(n-3)(n-4)}{5!}}{\frac{n(n-1)(n-2)(n-3)}{4!}}$$

1½ out of 3

+ ½ mark for substitution into equation
+ ½ mark for factorial expansion (factors with variables)
+ ½ mark for simplification of factorials (factors with variables)

E2 (changing an equation into an expression in lines 2 and 3)
E7 (notation error in line 2)

Exemplar 3

$$\frac{\left(\frac{n!}{(n-5)!}\right)}{\left(\frac{n!}{(n-4)!}\right)} = 6$$

$$\frac{\left(\frac{(n)(n-1)(n-2)(n-3)(n-4)(n-5)!}{(n-5)!}\right) = 6}{\left(\frac{(n)(n-1)(n-2)(n-3)(n-4)!}{(n-4)!}\right)}$$

$$\frac{(n)(n-1)(n-2)(n-3)(n-4)}{(n)(n-1)(n-2)(n-3)} = 6$$

$$n - 4 = 6$$
$$+4 \quad +4$$

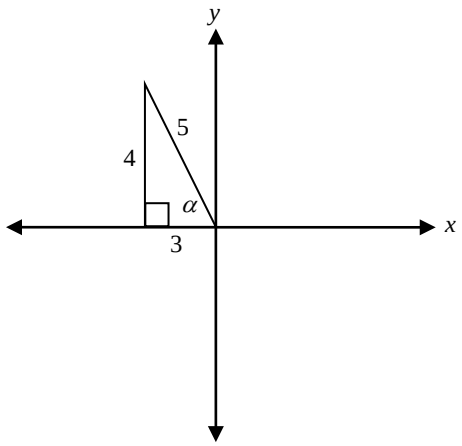
$$n = 10$$

2 ½ out of 3

+ 1 mark for factorial expansion
+ 1 mark for simplification of factorials
+ ½ mark for solving for *n*

Given $\sin\alpha = \frac{4}{5}$, where α is in quadrant II, determine the exact value of $\sin 2\alpha$.

Solution



$$x^2 + y^2 = r^2$$
$$x^2 + 16 = 25$$
$$x^2 = 9$$
$$x = \pm 3$$
$$x = -3$$

$\cos\alpha = -\frac{3}{5}$

½ mark for value of x
½ mark for $\cos\alpha$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$$
$$= -\frac{24}{25}$$

1 mark for substitution into correct identity

2 marks

Note(s):

- Accept any of the following values for x : $x = \pm 3; x = -3;$ or $x = 3.$

Exemplar 1

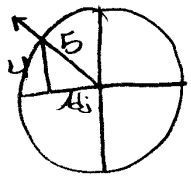
$\sin 2\alpha = 2(\sin\alpha \cos\alpha)$

$= 2\left(\frac{4}{5} \cdot \frac{\sqrt{41}}{5}\right)$

$= 2\left(\frac{4\sqrt{41}}{105}\right)$

$= \frac{4\sqrt{41}}{5}$

$\sin\alpha$



$\cos\alpha = \frac{Adj}{hyp}$

$Adj^2 + Opp^2 = hyp^2$

$Adj = \sqrt{5^2 + 4^2}$

$Adj = \sqrt{25+16}$

$Adj = \sqrt{41}$

$\cos\alpha = \frac{\sqrt{41}}{5}$

½ out of 2

+ 1 mark for substitution into correct identity
– ½ mark for arithmetic error in line 3
E1 (impossible solution not rejected in final answer)

Exemplar 2

$2\sin\alpha\cos\alpha$

$= 2(4/5)(3/5)$

$= \frac{24}{25}$

1½ out of 2

+ ½ mark for value of x
+ 1 mark for substitution into correct identity

Given the functions $f(x) = x + 1$ and $g(x) = \sqrt{x}$,

a) determine the equation of $g(f(x))$.

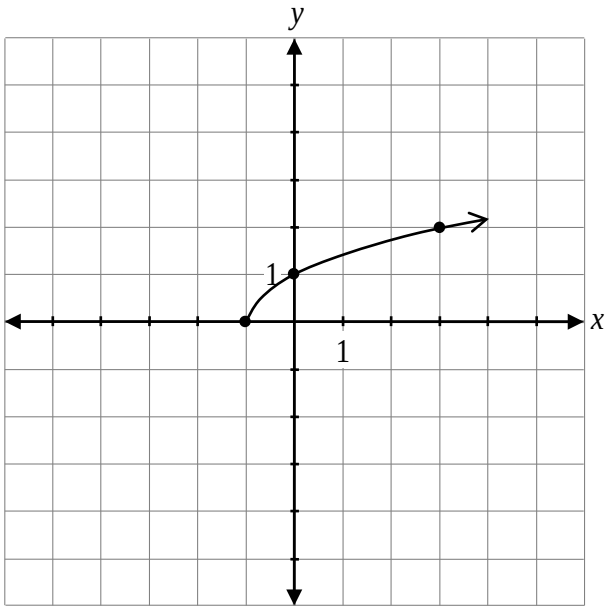
b) sketch the graph of $g(f(x))$.

Solution

a) $g(f(x)) = \sqrt{x+1}$

1 mark

b)



1 mark for domain of $g(f(x))$

1 mark for shape consistent with $g(f(x))$

2 marks

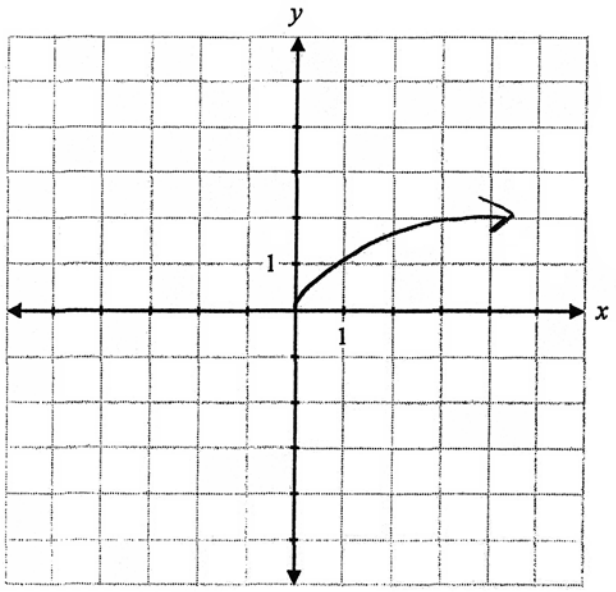
Exemplar 1

a)

$g(f(x)) =$ $\sqrt{x+1}$

1 out of 1

b)



1 out of 2

+ 1 mark for shape consistent with $g\left(f(x)\right)$

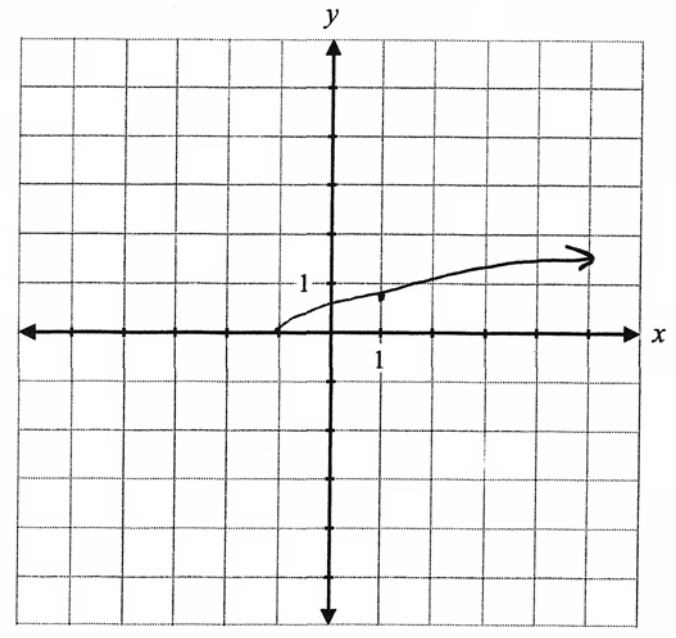
Exemplar 2

a)

$g(f(x)) =$ $\sqrt{x+1}$

1 out of 1

b)



1½ out of 2

award full marks
– ½ mark for procedural error (incorrect second point)

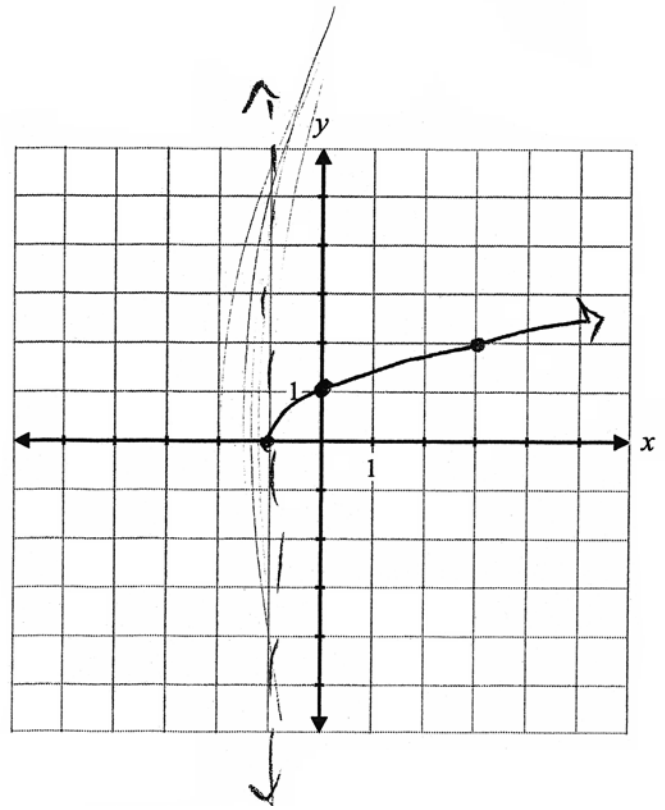
Exemplar 3

a)

$g(f(x)) =$ $\sqrt{x+1}$

1 out of 1

b)



1 out of 2

award full marks
– 1 mark for concept error of asymptote

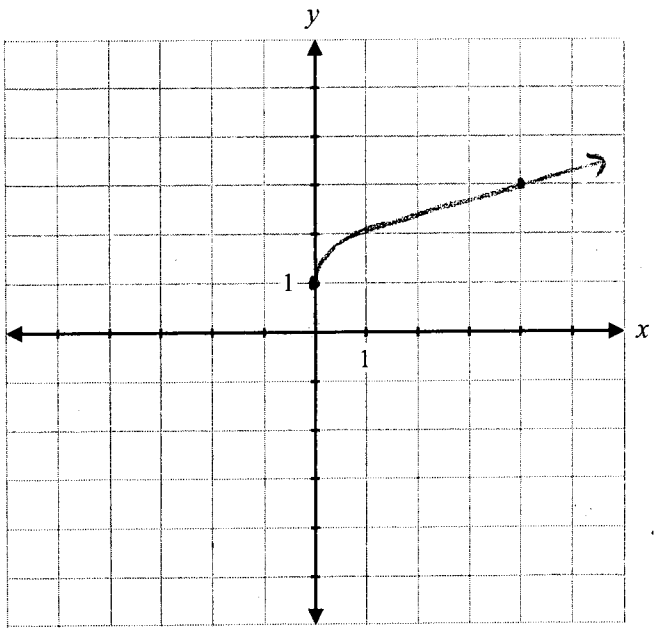
Exemplar 4

a)

$g(f(x)) =$ $\sqrt{x} + 1$

0 out of 1

b)



2 out of 2

award full marks (consistent with answer in a))

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Question 38 T4

Steve is asked to determine an equation with a larger period than the period of the graph of $y = \cos(2x)$.

Justify why Steve's answer of $y = \cos(6x)$ is incorrect.

Solution

Steve's equation needs to have a value of $|b|$ less than 2. **1 mark**

or

Steve's graph would have a period of $\frac{2\pi}{6} = \frac{\pi}{3}$, which is smaller than $\frac{2\pi}{2} = \pi$, the period of the given graph.

Exemplar 1

Because when determining Period, we turn it into a fraction. So $\frac{1}{2} > \frac{1}{6}$

0 out of 1

Exemplar 2

because making the number in front of x will make the graph bigger. It needs to be a fraction for the graph to expand

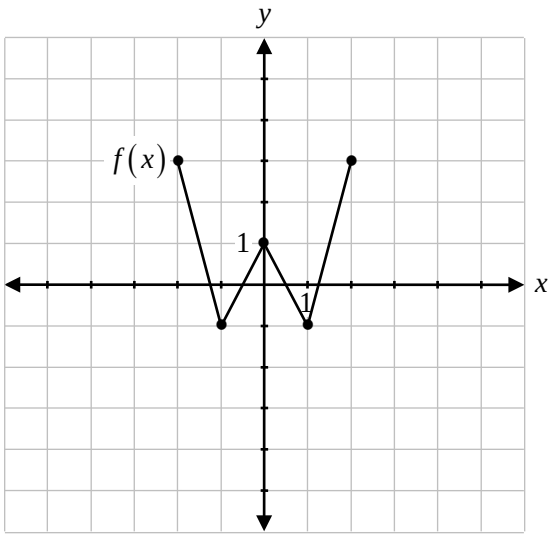
0 out of 1

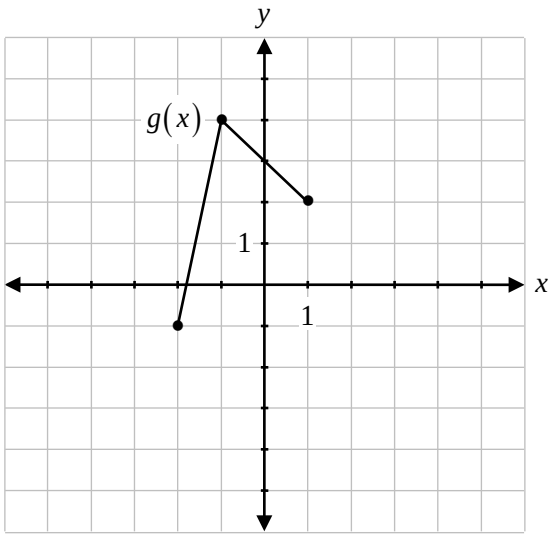
Exemplar 3

because B values ~~higher~~ that are >1 make the period smaller.

0 out of 1

Given the graphs of $f(x)$ and $g(x)$,





a) determine the value of $(f \bullet g)(-1)$.

b) determine the value of $g(f(0))$.

Solution

a) $(f \bullet g)(-1) = (-1)(4)$ 1 mark for value of $(f \bullet g)(-1)$

$= -4$

1 mark

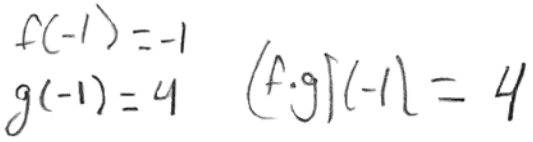
b) $f(0) = 1$ ½ mark for $f(0)$

$g(f(0)) = 2$ ½ mark for $g(f(0))$ consistent with $f(0)$ value

1 mark

Exemplar 1

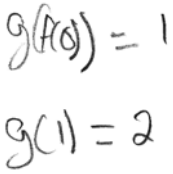
a)



½ out of 1

award full marks
– ½ mark for arithmetic error

b)

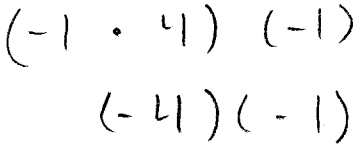


1 out of 1

award full marks
E7 (notation error)

Exemplar 2

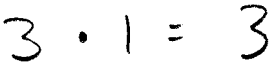
a)



0 out of 1

award full marks
– 1 mark for concept error

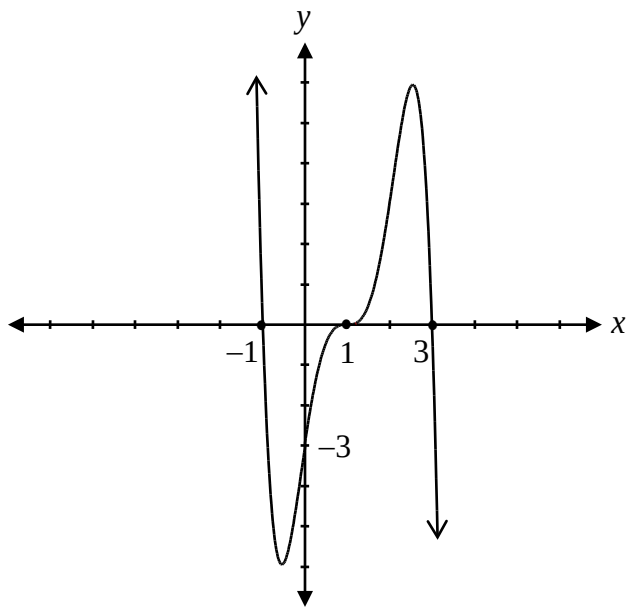
b)



0 out of 1

Sketch the graph of $P(x) = -(x-1)^3(x-3)(x+1)$.

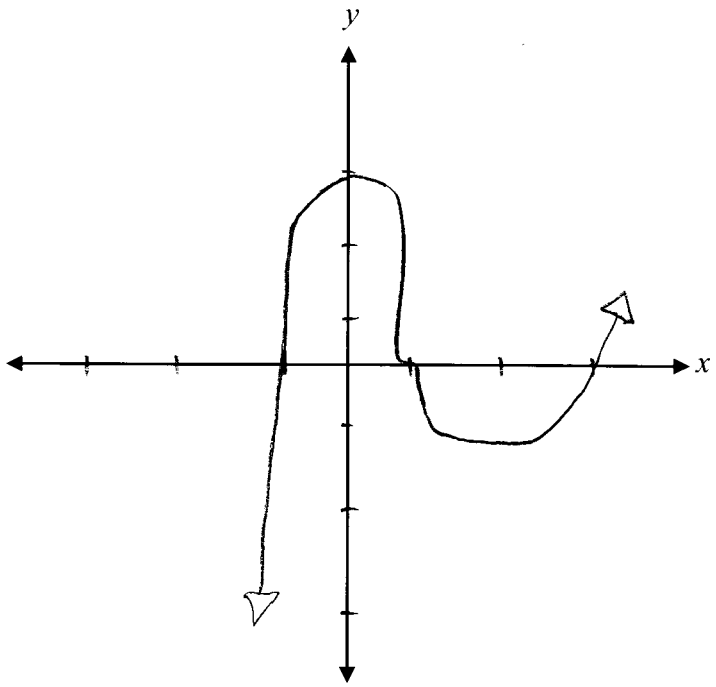
Solution



1 mark for x-intercepts
½ mark for y-intercept
1 mark for multiplicity (degree 3 at $x = 1$)
½ mark for end behaviour

3 marks

Exemplar 1



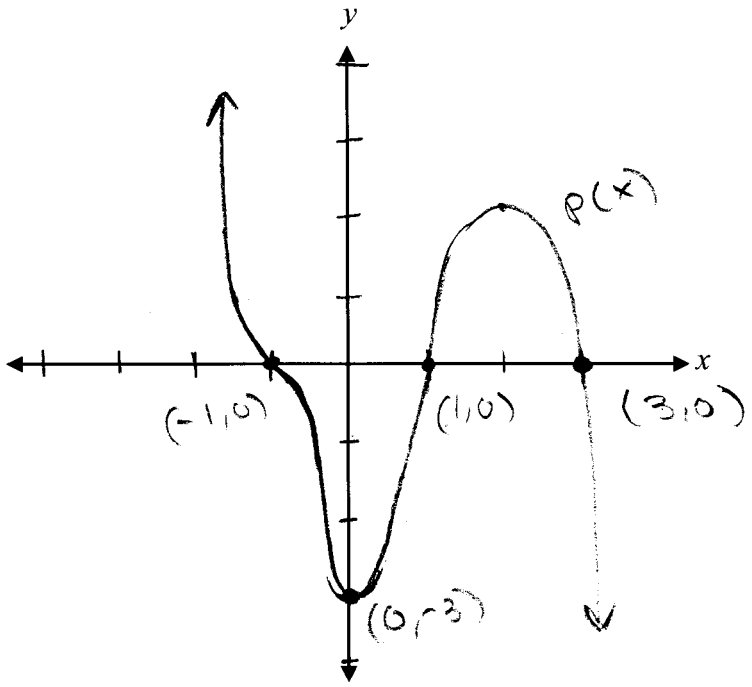
2 out of 3

+ 1 mark for x-intercepts

+ 1 mark for multiplicity (degree 3 at $x = 1$)

E9 (scale values on axes not indicated)

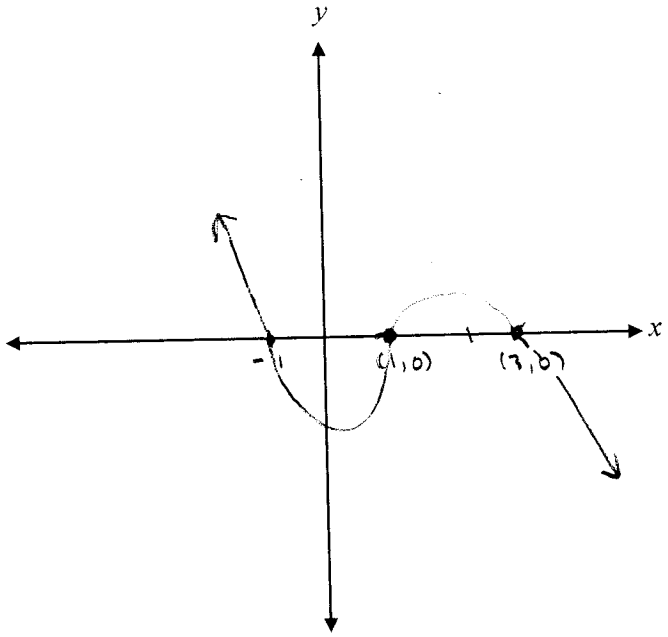
Exemplar 2



2½ out of 3

award full marks
– ½ mark for procedural error (multiplicity of degree 3 at $x = -1$ instead of $x = 1$)

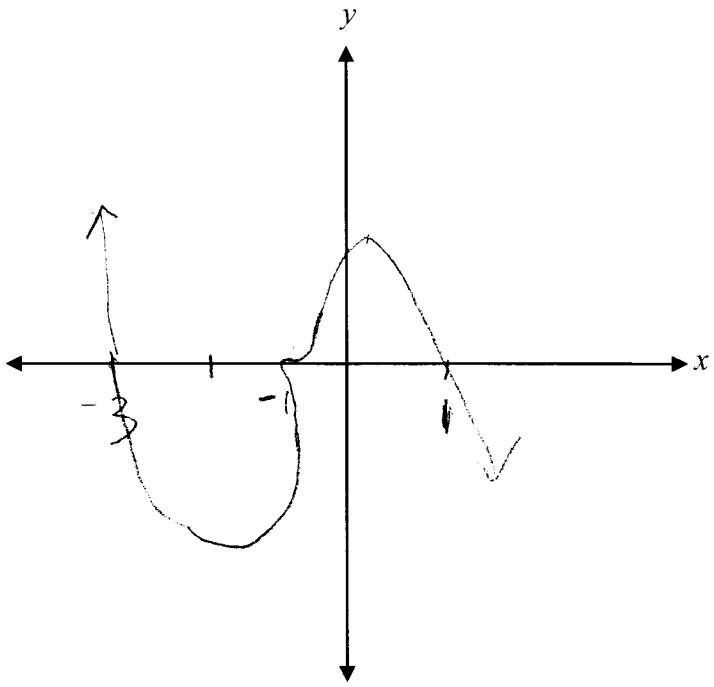
Exemplar 3



1½ out of 3

+ 1 mark for *x*-intercepts
+ ½ mark for end behaviour

Exemplar 4



1 out of 3

+ 1 mark for multiplicity (consistent with incorrect x-intercepts)
+ ½ mark for end behaviour
– ½ mark for incorrect shape

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Question 41 T2, T3

The point $\left(-\sqrt{3},1\right)$ is on the terminal arm of an angle $\theta,$ in standard position.

a) Determine $\tan\theta.$

b) Determine a possible value of $\theta,$ in radians.

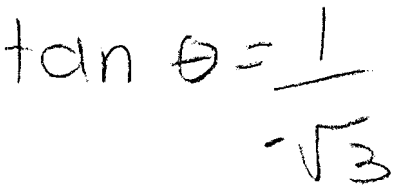
Solution

a) $\tan\theta = -\frac{1}{\sqrt{3}}$ **1 mark**

b) $\theta = \frac{5\pi}{6}$ **1 mark**

Exemplar 1

a)



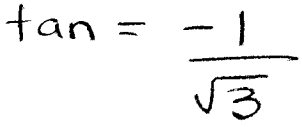
1 out of 1

b)

0 out of 1

Exemplar 2

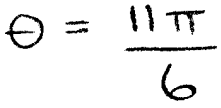
a)



1 out of 1

award full marks
E3 (variable omitted in an equation or identity)

b)



0 out of 1

Exemplar 3

a)

$\tan\theta = -1/\sqrt{3}$

1 out of 1

b)

$\theta = 150^{\circ}$

1 out of 1

award full marks
E5 (answer stated in degrees instead of radians)

Exemplar 4

a)

$\tan\theta = -\sqrt{3}$

0 out of 1

b)

$\theta = \frac{2\pi}{3}$

1 out of 1

award full marks (consistent with answer in a))

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Describe the transformation used to obtain the graph of $y = \log_5 x$ given the graph of $y = 5^x$.

Solution

The graph of $y = \log_5 x$ is obtained by reflecting the graph of $y = 5^x$ over the line $y = x$.

or

1 mark

The graph of $y = \log_5 x$ is the inverse of $y = 5^x$.

Exemplar 1

One has a vertical asymptote and the other has a horizontal asymptote.

0 out of 1

Exemplar 2

a logarithm is ~~the~~ inverse of exponential equation
∴ They are simply the opposite of each other.

½ out of 1

award full marks
– ½ mark for terminology error

Exemplar 3

The graph $y = \log_5 x$ is found by switching the x and y values.

1 out of 1

Solve $\sin\theta = -\frac{\sqrt{3}}{2}$, where $\theta \in \mathbb{R}$.

Solution

$\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$ 1 mark for values of θ (½ mark for each value)

$\theta = \left\{\begin{matrix}\frac{4\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \\ \frac{5\pi}{3} + 2k\pi,\ k \in \mathbb{Z}\end{matrix}\right\}$ 1 mark for general solution

2 marks

or

$\theta = 240°, 300°$

$\theta = \left\{\begin{matrix}240° + 360°k,\ k \in \mathbb{Z} \\ 300° + 360°k,\ k \in \mathbb{Z}\end{matrix}\right\}$

Exemplar 1

$\sin\theta$ (-) in Q III, Q IV

$\theta = \frac{4\pi}{3} + \pi k, \; k \in z$

$\theta = \frac{5\pi}{3} + \pi k, \; k \in z$

1 out of 2

+ 1 mark for values of θ

Exemplar 2

$240°, 300°$

$\theta = 360° + 240°k$

$\theta = 360° + 300°k$ $\quad k \in Z$

1 out of 2

+ 1 mark for values of θ
E7 (notation error in line 1)

Exemplar 3

$\sin\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

$\theta = \frac{4\pi}{3} + 2k\pi \quad k \in \mathbb{Z}$

$\theta = \frac{5\pi}{3} + 2k\pi \quad k \in \mathbb{Z}$

1 ½ out of 2

award full marks
– ½ mark for procedural error in line 1

Exemplar 4

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{4\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi \Big\} k \in R$

½ out of 2

award full marks
– 1 mark for concept error in line 2
– ½ mark for procedural error in line 2 ($k \in \mathbb{R}$ instead of $k \in \mathbb{Z}$)

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Given that the point (a,b) is on the graph of $f(x)$, describe how you would determine the corresponding point on the graph of $y = \sqrt{f(x)}$.

Solution

The value of *a* stays the same, square root the value of *b*. **1 mark**

Exemplar 1

$(a, \sqrt{b})$

0 out of 1

Exemplar 2

You would determine $y = \sqrt{f(x)}$ from f(x) by square rooting it.

0 out of 1

Evaluate.

$$\cos\left(\frac{\pi}{20}\right)\cos\left(\frac{\pi}{5}\right) - \sin\left(\frac{\pi}{20}\right)\sin\left(\frac{\pi}{5}\right)$$

Solution

$\cos\left(\frac{\pi}{20} + \frac{\pi}{5}\right)$ ½ mark for substitution of an appropriate identity

$\cos\left(\frac{\pi}{20} + \frac{4\pi}{20}\right)$

$\cos\left(\frac{5\pi}{20}\right)$

$\cos\left(\frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2}$ ½ mark for exact value

1 mark

Exemplar 1

$\cos(\alpha + \beta)$

$\cos\left(\frac{\pi}{20} + \frac{\pi}{5}\right)$

$\cos\left(\frac{\pi}{20} + \frac{4\pi}{20}\right)$

$\cos\left(\frac{5\pi}{20}\right)$

$\boxed{\cos\frac{\pi}{5}}$

½ out of 1

+ ½ mark for substitution of an appropriate identity

Exemplar 2

$\cos\left(\frac{\pi}{20} + \frac{\pi}{5}\right)$

$\cos\left(\frac{\pi}{20} + \frac{4\pi}{20}\right)$

$\cos\left(\frac{5\pi}{20}\right)$

$\cos\left(\frac{\pi}{4}\right)$

½ out of 1

+ ½ mark for substitution of an appropriate identity

Exemplar 3

$= \cos\left(\frac{1\pi}{20} + \frac{\pi \times 4}{5 \times 4}\right)$

$= \cos\left(\frac{4\pi}{20}\right)$

$= \cos\left(\frac{1}{4}\pi\right)$

$= \frac{\sqrt{2}}{2}$

½ out of 1

award full marks
– ½ mark for arithmetic errors in lines 2 and 3

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Question 46 R2, R5

Describe the transformations used to obtain the graph of the function $y = f(-x + 6) - 8$ from the graph of $y = f(x)$.

Solution

Reflect the graph of $y = f(x)$ over the *y*-axis and then translate 6 units right and 8 units down.

1 mark for horizontal reflection
1 mark for horizontal translation
1 mark for vertical translation

3 marks

Note(s):

- Deduct 1 mark if correct transformations are given in the wrong order.

Exemplar 1

$y = f(-(x-6)) - 8$

- shift 8 units down
- shift 6 units right
- reflection in the y-axis

2 out of 3

award full marks
– 1 mark for correct transformations given in the wrong order

Exemplar 2

- It moves 6 units to the left
- It moves 8 units down

- There's a horizontal reflection

3 out of 3

Exemplar 3

Reflection over the y-axis
shift 8 units down
shift 6 units right

3 out of 3

State the equations of all the asymptotes of the function, $y = \dfrac{1}{3x+1}$.

Solution

$y = 0$ 1 mark for horizontal asymptote

$x = -\dfrac{1}{3}$ 1 mark for vertical asymptote

2 marks

Exemplar 1

$x = 0$

$y = -\frac{1}{3}$

1 out of 2

award full marks
– 1 mark for concept error

Exemplar 2

horizontal asymptote @ $y \neq 0$
vertical asymptote @ $x \neq \frac{1}{3}$

1 out of 2

+ 1 mark for horizontal asymptote
E7 (notation error)

Exemplar 3

vertical asymptote = $-1/3$

horizontal asymptote = 0

1 ½ out of 2

award full marks
– ½ mark for procedural error

Determine the zeros of the polynomial function $P(x) = 2x^3 + 5x^2 - 4x - 3.$

Solution

$P(1) = 2(1)^3 + 5(1)^2 - 4(1) - 3$ 1 mark for identifying one possible value of *x*

$P(1) = 0$

$(x-1)$ is a factor

1	2	5	-4	-3
	$\downarrow$	2	7	3
	2	7	3	0

1 mark for synthetic division (or for any equivalent strategy)

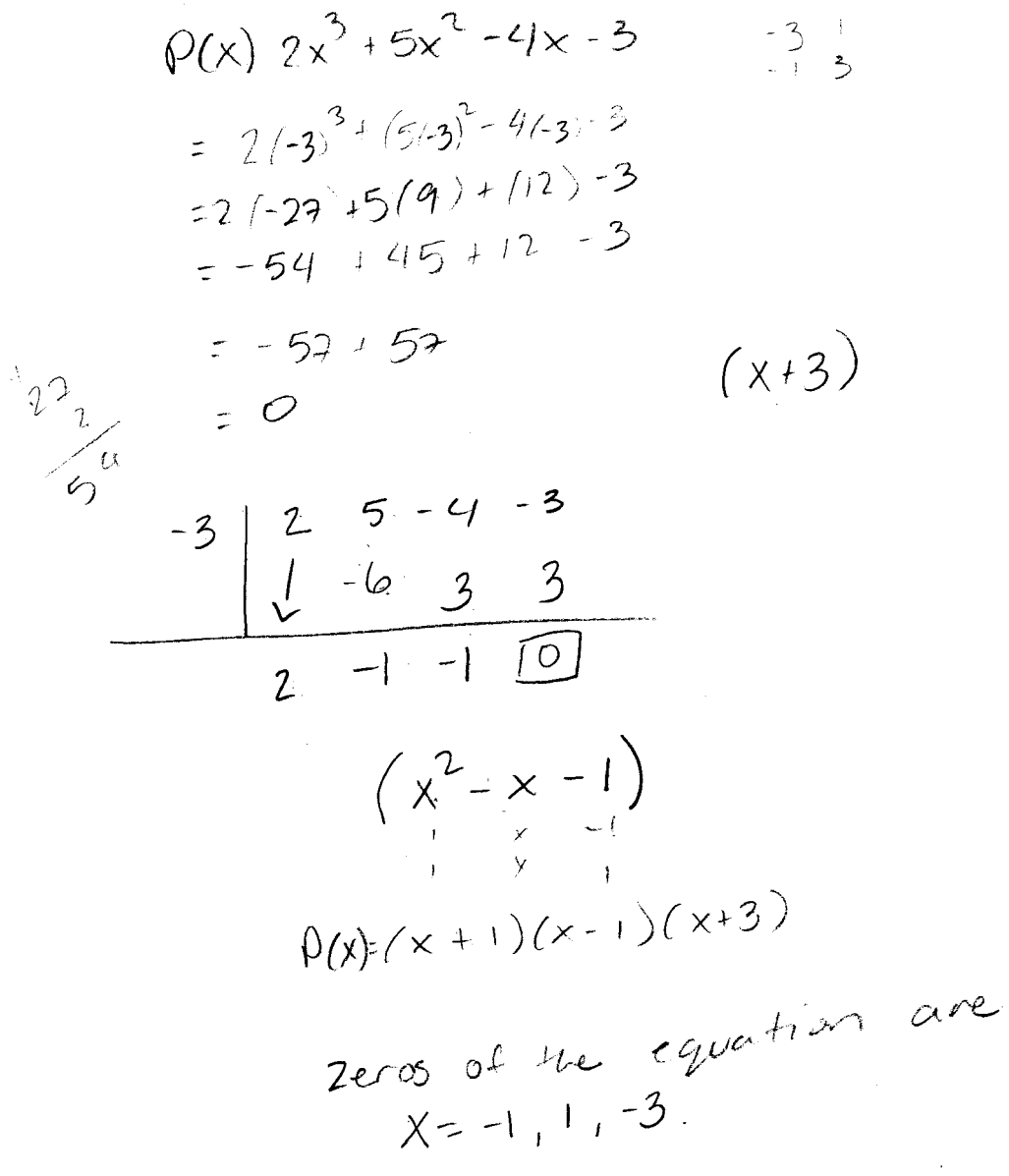
$P(x) = (x-1)(2x^2 + 7x + 3)$ ½ mark for consistent factors

$0 = (x-1)(2x+1)(x+3)$

$x = 1 \quad x = -\frac{1}{2} \quad x = -3$ ½ mark for all zeros

3 marks

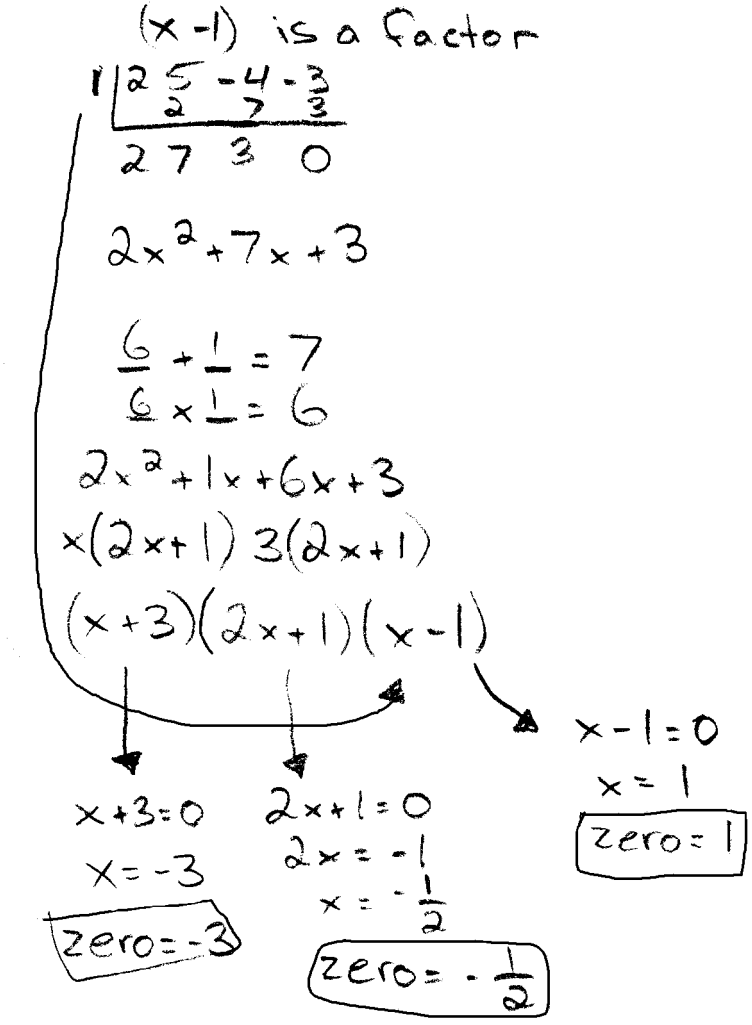
Exemplar 1



2½ out of 3

+ 1 mark for identifying one possible value of *x*
+ 1 mark for synthetic division
+ ½ mark for consistent zeros
E7 (notation error in line 2)
E7 (transcription error in line 8)

Exemplar 2



3 out of 3

award full marks
E2 (changing an equation to an expression in line 6)
E7 (notation error in lines 5 and 7)

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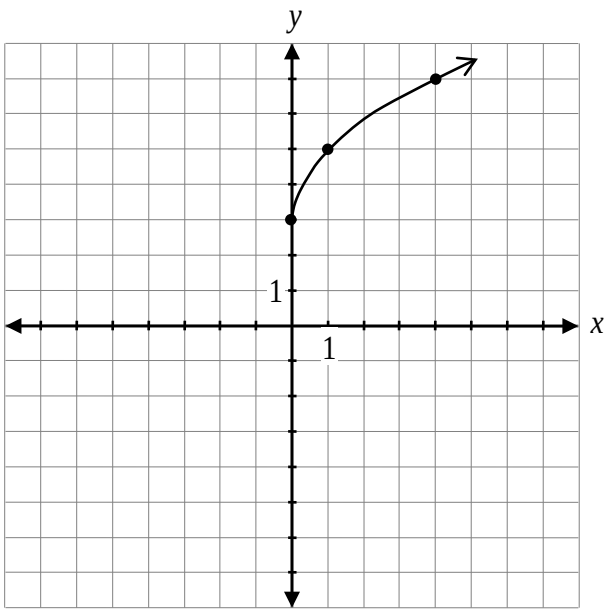
Determine the equation of the radical function represented by the graph.



Solution

$y = 2\sqrt{x} + 3$ — 1 mark for vertical stretch
1 mark for vertical translation

or

$y = \sqrt{4x} + 3$ — 1 mark for horizontal stretch
1 mark for vertical translation

2 marks

Exemplar 1

$y = \sqrt{2x} + 3$

1 out of 2

+ 1 mark for vertical translation

Exemplar 2

$y = 2\sqrt{x+3}$

1 out of 2

+ 1 mark for vertical stretch

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Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in steps leading to final answer
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: _____

Date marked: _____

Booklet No.: _____

Problem(s) noted: _____

_____

_____

_____

_____

_____

Question(s) affected: _____

_____

_____

Action taken or rationale for assigning marks: _____

_____

_____

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_____

Follow-up: _____

_____

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_____

Decision: _____

_____

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_____

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_____

Marker's Signature: _____

Principal's Signature: _____

For Department Use Only—After Marking Complete

Consultant: _____

Date: _____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
7	R6	1
8	R6	1
13	R2, R5	3
22	R3	1
32	R1	1
37 a)	R1	1
37 b)	R1	2
39 a)	R1	1
39 b)	R1	1
42	R6	1
46	R2, R5	3
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
16	T1	1
19	T1	1
25	T3	3
31	T4	2
33	T4	3
34	T2	1
38	T4	1
41 a)	T2	1
41 b)	T3	1
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
1	P1	2
5	P1	1
6	P4	2
14	P4	1
20	P2	1
35	P3	3
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
12 a)	R11	1
12 b)	R11	1
17	R12	1
28	R12	1
40	R12	3
48	R11	3

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
4	T5	4
9	T6	3
23	T6	1
36	T6	2
43	T5	2
45	T6	1
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
2	R10	2
3	R10	4
10	R9	½
15	R8	3
18	R7	1
24	R9	1
29	R9	2
30	R10	1
42	R9	1
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
10	R13	½
11	R13	2
21	R14	1
26	R14	1
27	R14	2
44	R13	1
47	R14	2
49	R13	2