Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

January 2016

Manitoba

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This resource will also be available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/assess/archives/index.html>.

Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.

Table of Contents

General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Advanced Learning in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Advanced Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION

Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).

E1 ○	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ○	**E8** ○	**E9** ○	**E10** ○

Example: Marks assigned to the student.

Marks Awarded	Booklet 1 25	Selected Response 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines

Booklet 1 Questions

Question 1 T1

A pizza with a diameter of 15 inches is cut into equal slices, each with a central angle of 36°. Determine the length of the crust on the outer edge of one slice of pizza.

Solution

$$\frac{36^\circ}{180^\circ} = \frac{\theta}{\pi}$$

$$\theta = \frac{\pi}{5}$$

1 mark for conversion

$$s = \theta r$$

$$s = \left(\frac{\pi}{5}\right)\left(\frac{15}{2}\right)$$

1 mark for substitution

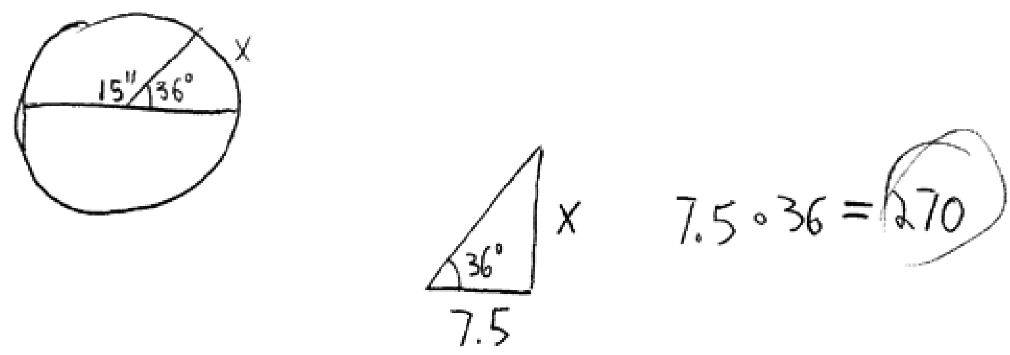
$$s = \frac{3\pi}{2} \text{ inches}$$

or

$$s = 4.712 \text{ inches}$$

2 marks

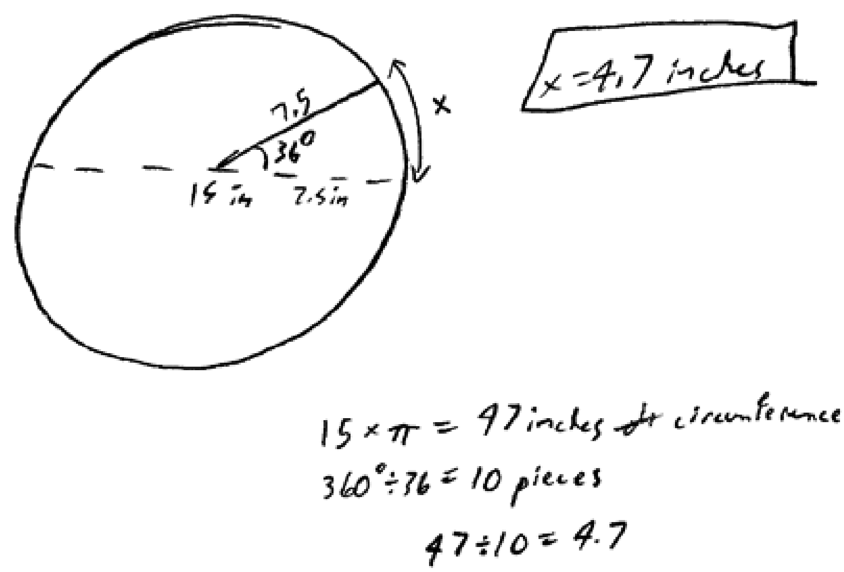
Exemplar 1



1 out of 2

+ 1 mark for substitution
E5 (missing units of measure)

Exemplar 2



2 out of 2

award full marks
E6 (rounding too early)

Question 2 P3

There are 9 girls and 7 boys in a math class from which a committee of 5 is to be chosen.

a) How many different committees of 5 can be formed if one of the boys, William, must be on the committee?

b) How many different committees of 5 can be formed if there must be 2 girls and 3 boys on the committee?

Solution

a) ${}_{1}C_{1} \bullet {}_{15}C_{4} = 1365$

1 mark

b) ${}_{9}C_{2} \bullet {}_{7}C_{3} = 1260$

½ mark for ${}_{9}C_{2}$
½ mark for ${}_{7}C_{3}$
1 mark for the product of combinations

2 marks

Note(s):

- ${}_{1}C_{1}$ does not need to be shown

Exemplar 1

a)

$$\underset{\text{William}}{\underline{1}} \cdot \underline{15} \cdot \underline{14} \cdot \underline{13} \cdot \underline{12} = 32760 \text{ ways}$$

32760 committees can be formed.

0 out of 1

award full marks
– 1 mark for concept error (using permutations instead of combinations)

b)

$$\underset{\text{girl}}{\underline{9}} \cdot \underset{\text{girl}}{\underline{8}} \cdot \underset{\text{boy}}{\underline{7}} \cdot \underset{\text{boy}}{\underline{6}} \cdot \underset{\text{boy}}{\underline{5}} = 15120 \text{ committees can be formed}$$

2 out of 2

award full marks
(consistent with concept error in a)

Exemplar 2

a)

$_{1}P_{1} = 1$

$_{15}P_{4} = 32760$

32760

0 out of 1

award full marks
– 1 mark for concept error (using permutations instead of combinations)

b)

$_{9}C_{2} = 36$

$_{7}C_{3} = 35$

$\boxed{1260}$

2 out of 2

Exemplar 3

a)

${}_{15}C_4$

1 out of 1

award full marks
E1 (final answer not stated)

b)

${}_{9}C_2 \quad {}_{7}C_3$

$36 \cdot 35$

1260

2 out of 2

Question 3 T5

Solve the following equation over the interval $[0, 2\pi]$:

$$\sin^2\theta + 6\sin\theta - 2 = 0$$

Solution

$$\sin\theta = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-2)}}{2(1)}$$

$$\sin\theta = \frac{-6 \pm \sqrt{36+8}}{2}$$

$$\sin\theta = \frac{-6 \pm \sqrt{44}}{2}$$

$\sin\theta = 0.316\ 624\ldots$ $\quad$ $\sin\theta = -6.316\ 624\ldots$ $\quad$ 1 mark for solving for $\sin\theta$

$\theta_r = 0.322\ 169$

$\theta = 0.322$ $\quad$ no solution $\quad$ 2 marks for solving for θ (½ mark for each value, 1 mark for indicating no solution)

$\theta = 2.819$

3 marks

Exemplar 1

$$x^2 + 6x - 2 = 0$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 + 8}}{2}$$

$$= \frac{-6 \pm \sqrt{44}}{2}$$

$$= \frac{-6 + \sqrt{44}}{2} \qquad = \frac{-6 - \sqrt{44}}{2}$$

$$= 0.316 \qquad = -6.316$$

1 out of 3

+ 1 mark for solving for $\sin\theta$
E3 (variable introduced without being defined)
E7 (notation error in line 5)

Exemplar 2

$$\sin\theta = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{44}}{2}$$

$$= \frac{-6 \pm 2\sqrt{11}}{2}$$

$$= -3 \pm \sqrt{11}$$

$$\theta = \sin^{-1}(-3 \pm \sqrt{11})$$

$$\theta = \sin^{-1}(-3 + \sqrt{11})$$

QI: $\theta_r = 0,32217$

QII: $\pi - \theta_r = 2,819$

QIII: $\pi + \theta_r = 3,464$

QIV: $2\pi - \theta_r = 5,961$

$\theta = 0,322;\ 2,819;\ 3,464;\ 5,961$

1 out of 3

+ 1 mark for solving for $\sin\theta$
+ 1 mark for two correct values of θ
– 1 mark for concept error (solutions in quadrants 3 and 4)

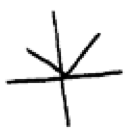
Exemplar 3

$$\sin\theta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\sin\theta = \frac{-6 \pm \sqrt{6^2-4(1)(-2)}}{2(1)}$$

$$\sin\theta = \frac{-6 + \sqrt{44}}{2}$$

$$\sin\theta = 0.31662479$$



$$\theta_r = 18.459^\circ$$

$$\theta_1 = 18.459^\circ$$

$$\theta_2 = 161.541^\circ$$

2 out of 3

+ 1 mark for solving for $\sin\theta$
+ 1 mark for two correct values of θ
E5 (answer stated in degrees instead of radians)

Question 4 R10

Solve:

$$6(5)^{3x+2} = 9^{2-x}$$

Solution

$\log\left[6(5)^{3x+2}\right] = \log 9^{2-x}$	½ mark for applying logarithms
$\log 6 + \log 5^{3x+2} = \log 9^{2-x}$	1 mark for product law
$\log 6 + (3x+2)\log 5 = (2-x)\log 9$	1 mark for power law
$\log 6 + 3x\log 5 + 2\log 5 = 2\log 9 - x\log 9$	
$3x\log 5 + x\log 9 = 2\log 9 - 2\log 5 - \log 6$	½ mark for collecting terms with x
$x(3\log 5 + \log 9) = 2\log 9 - 2\log 5 - \log 6$	
$x = \dfrac{2\log 9 - 2\log 5 - \log 6}{3\log 5 + \log 9}$	½ mark for solving for x
$x = -0.087\ 707$	½ mark for evaluating quotient of logarithms
$= -0.088$	

4 marks

Exemplar 1

$$6\left(5^{3x+2}\right) = 9^{2-x}$$

$$\log 6(5)^{3x+2} = \log 9^{2-x}$$

$$(3x+2)\log 30 = 2-x\log 9$$

$$3x\log 30 + 2\log 30 = 2\log 9 - x\log 9$$

$$3x\log 30 + x\log 9 = 2\log 9 - 2\log 30$$

$$\frac{x\ (3\log 30 + \log 9)}{3\log 30 + \log 9} = \frac{2\log 9 - 2\log 30}{3\log 30 + \log 9}$$

$$x = -0.194$$

3 out of 4

+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for collecting terms with x
+ ½ mark for solving for x
+ ½ mark for evaluating quotient of logarithms
E4 (missing brackets but still implied in line 3)

Exemplar 2

$$6(3x+2)\log 5 = (2-x)\log 9$$

$$(18x+12)\log 5 = 2\log 9 - x\log 9$$

$$18x\log 5 + 12\log 5 = 2\log 9 - x\log 9$$

$$18x\log 5 + x\log 9 = 2\log 9 - 12\log 5$$

$$x(18\log 5 + \log 9) = 2\log 9 - 12\log 5$$

$$x = \frac{2\log 9 - 12\log 5}{18\log 5 + \log 9}$$

$$x = -.4786$$

3 out of 4

+ ½ mark for applying logarithms
+ 1 mark for power law
+ ½ mark for collecting terms with *x*
+ ½ mark for solving for *x*
+ ½ mark for evaluating quotient of logarithms

Exemplar 3

$$6(3x+2)(5) = (2-x)9$$
$$6(15x+10) = 18-9x$$
$$90x+60 = 18-9x.$$
$$+9x$$
$$99x+60 = 18.$$
$$-60 \quad -60$$
$$\frac{99x}{99} = \frac{-42}{99}$$
$$x = -.4242.$$

1 out of 4

+ 1 mark for power law
+ ½ mark for collecting terms with x
+ ½ mark for solving for x
– 1 mark for concept error (using power law without logarithms)

Exemplar 4

$$3x+2 = 2-x$$
$$4x = 0$$
$$x = 0$$

0 out of 4

Solve $(2\sin\theta - 1)(\sin\theta + 1) = 0$ where $\theta \in \mathbb{R}$.

Solution

$2\sin\theta - 1 = 0$ $\quad\quad$ $\sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ $\quad\quad$ $\sin\theta = -1$ $\quad\quad$ 1 mark for solving for $\sin\theta$

$\theta_r = \frac{\pi}{6}$ $\quad\quad$ $\theta_r = \frac{3\pi}{2}$

$\theta = \frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$ $\quad\quad$ $\theta = \frac{3\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

$\theta = \frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

2 marks for solving for θ (1 mark for each branch)
1 mark for general solution

or $\quad\quad$ **or**

$\theta_r = 30^\circ$ $\quad\quad$ $\theta_r = 270^\circ$

$\theta = 30^\circ + 360^\circ k,\ k \in \mathbb{Z}$ $\quad\quad$ $\theta = 270^\circ + 360^\circ k,\ k \in \mathbb{Z}$

$\theta = 150^\circ + 360^\circ k,\ k \in \mathbb{Z}$

4 marks

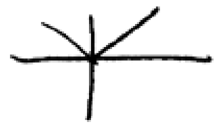
Exemplar 1

$$2\sin\theta - 1 = 0$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$



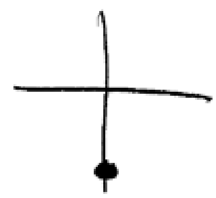
$$\frac{\pi}{6} + 2k\pi, k \in Z$$

$$\sin\theta + 1 = 0$$

$$\sin\theta = -1$$

$$\theta = \sin^{-1}(-1)$$

$$\theta = \frac{3\pi}{2}$$



$$\frac{3\pi}{2} + 2k\pi, k \in Z$$

3½ out of 4

+ 1 mark for solving for $\sin\theta$
+ 2 marks for solving for θ
+ 1 mark for general solution

− ½ mark for procedural error (missing $\theta = \frac{5\pi}{6} + 2k\pi$)

E7 (notation error in lines 4 and 10)

Exemplar 2

$\sin\theta = \frac{1}{2}$ | $\sin\theta = 1$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ | $\theta = \frac{\pi}{2}$

2½ out of 4

+ 1 mark for solving for $\sin\theta$
+ 2 marks for consistent values of θ
– ½ mark for arithmetic error in line 1

Exemplar 3

$\sin\theta = \frac{1}{2}$ $\quad \sin\theta = -1$

$\sin\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad \sin\theta = \frac{3\pi}{2}$

$$\therefore \sin\theta = \begin{cases} \frac{\pi}{6} + 2k\pi / k \in R \\ \frac{5\pi}{6} + 2k\pi / k \in R \\ \frac{3\pi}{2} + 2k\pi / k \in R \end{cases}$$

2½ out of 4

award full marks
– 1 mark for concept error in lines 2 and 3
– ½ mark for procedural error when stating general solution $(k \in \mathbb{R})$

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Question 6 R12

The roots of the polynomial equation $3(x-2)(x+1)^2 = 0$ are $x = 2$ and $x = -1$.

Explain what these roots represent on the graph of $p(x) = 3(x-2)(x+1)^2$.

Solution

They are the x-intercepts of the graph of $p(x)$.

1 mark

Exemplar 1

The roots represent where the graph touches 0.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

(2, 0) and (−1, 0)

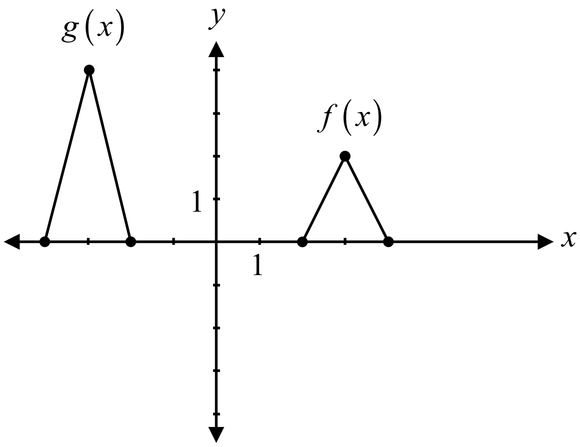
0 out of 1

Exemplar 3

They are the zeroes of the graph.

1 out of 1

Determine an equation for $g(x)$ as a transformation of $f(x)$.



Solution

$g(x) = \underline{2f(x+6)}$

1 mark for vertical stretch
1 mark for horizontal translation

2 marks

or

$g(x) = \underline{2f(-x)}$

1 mark for vertical stretch
1 mark for horizontal reflection

2 marks

Exemplar 1

$g(x) = 2(x+6)$

1 out of 2

award full marks
– 1 mark for concept error (not stating in terms of $f(x)$).

Exemplar 2

$g(x) = 2g(x+6)$

1½ out of 2

award full marks
– ½ mark for procedural error (stating g instead of f).

Exemplar 3

$g(x) = 2(-f(x))$

1 out of 2

+ 1 mark for vertical stretch

Question 8 R11

A student must determine the factors of $5x^4 - 2x^3 + 4x - 1$. He used 5, -2, 4, and -1 as the coefficients of the polynomial when using synthetic division.

Explain the student's error.

Solution

The student did not write the coefficient of 0 for the x^2 term.

1 mark

Exemplar 1

they didn't put in an x^2 term

1 out of 1

Exemplar 2

They need a placeholder.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3

They forgot a zero

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 4

The equation $5x^4 - 2x^3 + 4x - 1$ doesn't have an x^2
So the student forgot to put a 1 in it's place
when using synthetic division.

0 out of 1

Question 9 R2, R5

Describe the transformations of $y = f(x)$ when asked to sketch the graph of $y = -f(x-4)$.

Solution

$f(x)$ is reflected over the x-axis and translated 4 units to the right.

1 mark for vertical reflection
1 mark for horizontal translation

2 marks

Exemplar 1

the graph would shift 4 units right and the graph would flip over the x axis

2 out of 2

Exemplar 2

- the graph reflects over the y-axis
- the graph shifts right 4 units

1 out of 2

+ 1 mark for horizontal translation

Question 10 T6

Prove the identity below for all permissible values of θ:

$$\sin\theta + \frac{\cos\theta}{\tan\theta} = \frac{1}{\cos\theta\tan\theta}$$

Solution

Method 1

Left-Hand Side	Right-Hand Side	
$\sin\theta + \dfrac{\cos\theta}{\tan\theta}$	$\dfrac{1}{\cos\theta\tan\theta}$	
$\sin\theta + \dfrac{\cos\theta}{\dfrac{\sin\theta}{\cos\theta}}$	$\dfrac{1}{\cos\theta\dfrac{\sin\theta}{\cos\theta}}$	1 mark for correct substitution of identities
$\sin\theta + \dfrac{\cos^2\theta}{\sin\theta}$	$\dfrac{1}{\sin\theta}$	1 mark for algebraic strategies
$\dfrac{\sin^2\theta}{\sin\theta} + \dfrac{\cos^2\theta}{\sin\theta}$		1 mark for logical process to prove an identity
$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta}$		**3 marks**
$\dfrac{1}{\sin\theta}$		

Method 2

Left-Hand Side	Right-Hand Side	
$\sin\theta + \dfrac{\cos\theta}{\tan\theta}$	$\dfrac{1}{\cos\theta\tan\theta}$	
	$\dfrac{1}{\cos\theta\dfrac{\sin\theta}{\cos\theta}}$	1 mark for correct substitution of identities
	$\dfrac{1}{\sin\theta}$	
	$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta}$	
	$\dfrac{\sin^2\theta}{\sin\theta} + \dfrac{\cos^2\theta}{\sin\theta}$	
	$\sin\theta + \cos\theta\dfrac{\cos\theta}{\sin\theta}$	1 mark for algebraic strategies
	$\sin\theta + \cos\theta\cot\theta$	
	$\sin\theta + \dfrac{\cos\theta}{\tan\theta}$	1 mark for logical process to prove an identity

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$LHS = \sin\theta + \frac{\cos\theta}{\tan\theta}$	$\frac{1}{\cos\theta\tan\theta} = RHS$
$= \frac{\sin\theta\tan\theta + \cos\theta}{\tan\theta}$	$\frac{1}{\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)}$
$= \frac{\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) + \cos\theta}{\tan\theta}$	$\frac{1}{\frac{\sin\theta\cos\theta}{\cos\theta}}$
$= \frac{\frac{\sin^2\theta}{\cos\theta} + \cos\theta}{\tan\theta}$	$\frac{\cos\theta}{\sin\theta\cos\theta}$
$= \frac{\frac{1-\cos^2\theta}{\cos\theta} + \cos\theta}{\tan\theta}$	
$= \frac{1-\cos\theta + \cos\theta}{\tan\theta}$	

1 out of 3

+ 1 mark for correct substitution of identities

Exemplar 2

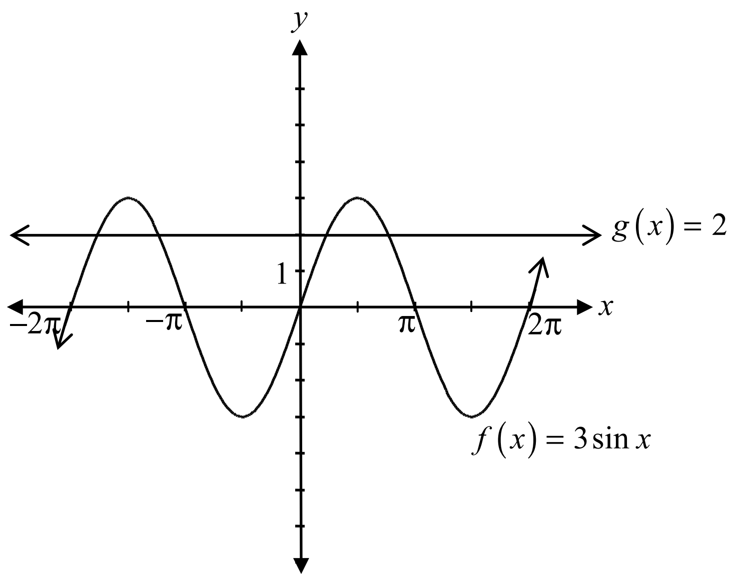
Left-Hand Side	Right-Hand Side
$\frac{\sin\theta}{1} + \frac{\cos\theta}{\tan\theta}$	$\boxed{\frac{1}{\cos\theta\tan\theta}}$
$\frac{\tan\theta\sin\theta + \cos\theta}{\tan\theta}$	
$\frac{\frac{\sin\theta}{\cos\theta}\sin\theta + \cos\theta}{\tan\theta}$	
$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta}{1}$	
$\frac{\frac{\sin^2\theta + \cos^2\theta}{\cos\theta}}{\tan\theta}$	
$\frac{1}{\cos\theta} \cdot \frac{1}{\tan\theta}$	
$\boxed{\frac{1}{\cos\theta\tan\theta}}$	

3 out of 3

award full marks
E7 (transcription error in line 4)

Question 11 T5

Describe how to use the graphs of $f(x) = 3\sin x$ and $g(x) = 2$ to solve the equation $3\sin x = 2$.



Solution

The solution will be the *x*-values where the two graphs intersect.

1 mark

Exemplar 1

you would look at where the
y value of graph 3sinx is when the x value
is 2

0 out of 1

Exemplar 2

set F(x) = g(x) and solve algebraically

0 out of 1

Question 12 P1

A hockey arena has 5 doors.

Determine the number of ways that you can enter through one door but exit through a different door.

Solution

$\underline{\quad 5 \quad} \cdot \underline{\quad 4 \quad}$ = 20 ways

1 mark

Exemplar 1

$5! = \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = \boxed{120}$

0 out of 1

Question 13 R11

Given that $(x+3)$ is one of the factors, express $2x^3 + 7x^2 + 2x - 3$ as a product of factors.

Solution

−3	2	7	2	−3
	↓	−6	−3	3
	2	1	−1	0

½ mark for $x = -3$
1 mark for synthetic division (or for any other equivalent strategy)

$(x+3)(2x^2 + x - 1)$

or

$(x+3)(2x-1)(x+1)$

½ mark for consistent product of factors

2 marks

Exemplar 1

-3	2	7	$^{-}2$	-3
		-6	$^{-}3$	3
	2	1	-1	0

$$\boxed{2x^2 + x - 1}$$

1½ out of 2

+ ½ mark for $x = -3$
+ 1 mark for synthetic division

Exemplar 2

$$
\begin{array}{r}
2x^2 + x - 1 \\
(x+3)\overline{)\,2x^3 + 7x^2 + 2x - 3} \\
-\underline{2x^3 + 6x^2} \\
x^2 + 2x \\
-\underline{x^2 + 3x} \\
-x - 3 \\
-(-x - 3)
\end{array}
$$

$$(x+3)(2x^2 + x - 1)$$

$$\boxed{(x+3)(2x-1)(x+1)}$$

2 out of 2

award full marks
E4 (missing brackets but still implied in lines 3 and 5)

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Booklet 2 Questions

Answer Key for Selected Response Questions

Question	Answer	Learning Outcome
14	C	R12
15	C	R3
16	B	T1
17	B	T6
18	D	P4
19	B	R12
20	C	T6
21	A	R14

Question 14 R12

Identify the maximum number of x-intercepts for a polynomial function of degree 3.

a) 1

b) 2

c) 3

d) 4

Question 15 R3

The graph of $y = f(x)$ contains the point (a, b). The graph of $g(x)$ is a transformation of the graph of $f(x)$ and contains the point $(3a, b)$.

Identify the function that represents $g(x)$.

a) $g(x) = f(3x)$

b) $g(x) = 3f(x)$

c) $g(x) = f\left(\frac{x}{3}\right)$

d) $g(x) = \frac{1}{3}f(x)$

Question 16 T1

The angle 2.95 radians, in standard position, terminates in quadrant:

a) I

b) II

c) III

d) IV

Question 17 T6

Evaluate:

$$2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$$

a) $\frac{1}{2}$

b) $\frac{\sqrt{2}}{2}$

c) 1

d) $\sqrt{2}$

Question 18 P4

Identify which of the following represents the 5th term in the expansion of $\left(4x^2 - 2y^3\right)^{15}$.

a) ${}_{15}C_5\left(4x^2\right)^{10}\left(-2y^3\right)^5$

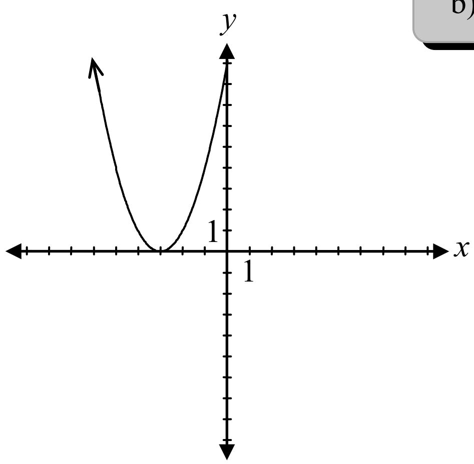
b) ${}_{15}C_5\left(4x^2\right)^{11}\left(-2y^3\right)^4$

c) ${}_{15}C_4\left(4x^2\right)^{10}\left(-2y^3\right)^5$

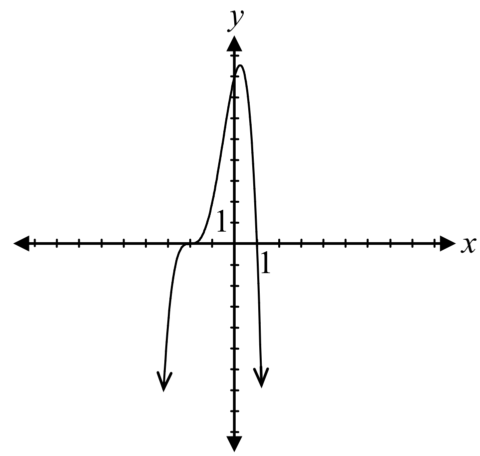
d) ${}_{15}C_4\left(4x^2\right)^{11}\left(-2y^3\right)^4$

Identify which of the following graphs of polynomial functions has a zero with a multiplicity of 3.

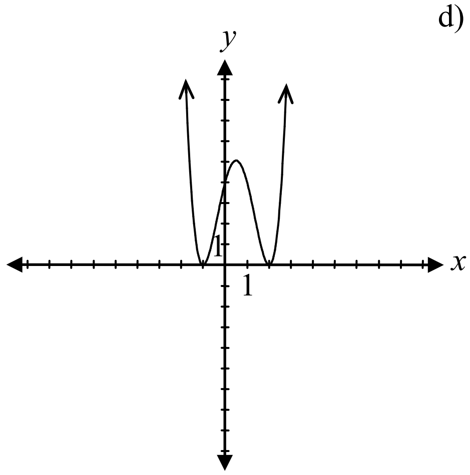
a)



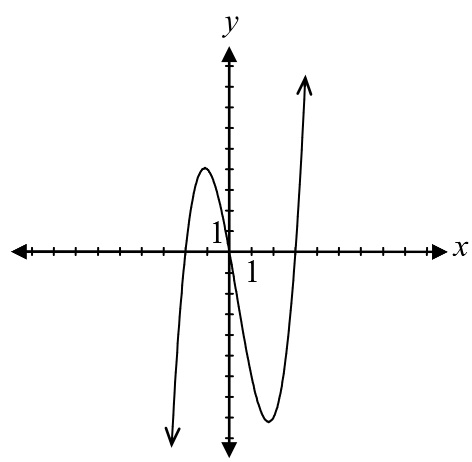
b)



c)



d)



Question 20 T6

A non-permissible value of x for the function $f(x) = \frac{1}{\cos x + 1}$ is:

a) -1

b) 0

c) π

d) $\frac{3\pi}{2}$

Question 21 R14

Identify which of the following statements is true for the rational function $f(x) = \frac{4(x-1)(x-2)}{(x-1)(x+3)}$.

a) The equation of the horizontal asymptote is $y = 4$.

b) The equation of the vertical asymptote is $x = 1$.

c) The y-intercept is 0.

d) There is a point of discontinuity (hole) when $x = 2$.

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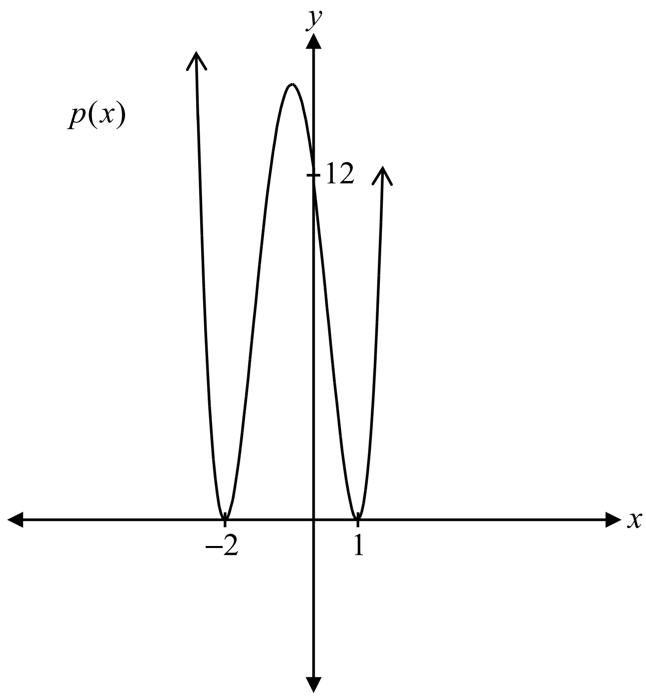
Determine the equation of the polynomial function, $p(x)$, represented by the graph.



Solution

$p(x) = \underline{3(x+2)^2(x-1)^2}$

1 mark for factors

1 mark for multiplicity of 2 (½ mark for each)

1 mark for correct value of *a* (consistent with factors and multiplicity)

3 marks

Exemplar 1

$p(x) = (x+2)(x-1)^2$

1½ out of 3

+ 1 mark for factors
+ ½ mark for multiplicity of 2

Exemplar 2

$a(0-2)^2(0+1)^2 = 12$

$a(-2)^2(1)^2 = 12$

$4a = 12$

$a = 3$

$p(x) = 3(x-2)^2(x+1)^2$

2 out of 3

+ 1 mark for multiplicity of 2
+ 1 mark for correct value of a (consistent with factors and multiplicity)

Exemplar 3

$p(x) = (x+2)^2(x-1)^2 + 12$

2 out of 3

+ 1 mark for factors
+ 1 mark for multiplicity of 2

Question 23 R7

Evaluate:

$$\log_4 2$$

Solution

$\frac{1}{2}$ **1 mark**

Exemplar 1

$4^{x} = 2$

$4^{1/2} = 2$

1 out of 1

award full marks
E3 (variable introduced without being defined)

Exemplar 2

$= 2$

0 out of 1

Exemplar 3

$\log_4 2 = x$

$4^{x} = 2$

$2^{2x} = 2$

$2x = 1$

$x = 1/2$

1 out of 1

Question 24 T3

Evaluate:

$$\left(\cos\frac{11\pi}{3}\right)\left(\csc\frac{11\pi}{6}\right)$$

Solution

$$\left(\frac{1}{2}\right)(-2)$$

1 mark for $\cos\frac{11\pi}{3}$ (½ mark for the quadrant, ½ mark for the value)

1 mark for $\csc\frac{11\pi}{6}$ (½ mark for the quadrant, ½ mark for the value)

$$-1$$

2 marks

Exemplar 1

$$\left(\cos\frac{11\pi}{3}\right)\left(\csc\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{2\sqrt{3}}{2\sqrt{3}}$$

$$\left(\cos\frac{11\pi}{3}\right)\left(\csc\frac{11\pi}{6}\right) = 1$$

½ out of 2

+ ½ mark for correct quadrant for $\cos\frac{11\pi}{3}$

Exemplar 2

$$-\left(\frac{1}{2}\right)\cdot(-2) = 1$$

1½ out of 2

+ ½ mark for the value of $\cos\frac{11\pi}{3}$

+ 1 mark for $\csc\frac{11\pi}{6}$

Exemplar 3

$$\left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{3}}\right) \quad \frac{2}{2\sqrt{3}} \quad \boxed{\frac{1}{\sqrt{3}}}$$

1 out of 2

+ 1 mark for $\cos\frac{11\pi}{3}$

Question 25 R7

Estimate the value of $\log_2 5$.

Justify your answer.

Solution

$$\left.\begin{array}{l}\log_2 4 = 2 \\ \log_2 8 = 3\end{array}\right\}$$ ½ mark for justification

$\therefore \log_2 5 \approx 2.3$ ½ mark for estimated value in the interval $[2.1, 2.4]$

1 mark

Exemplar 1

$\log_2 5 = x$

$2^x = 5$

$2^2 = 4$

4 is close to 5
therefore my estimation
for the value of
$\log_2 5$ is 2

$\log_2 5 \approx 2$

½ out of 1

+ ½ mark for justification

Exemplar 2

$\log_2 5 = y$

$2^y = 5$

The reason it's a decimal is
because no whole number gives 5
once we put 2 with an exponent,
therefore must be a decimal.

0 out of 1

Question 26 T2

If θ terminates in quadrant III and $\cos\theta = -\frac{6}{7}$, determine the exact value of $\tan\theta$.

Solution

$$\cos\theta = \frac{x}{r}$$

$$x^2 + y^2 = r^2$$

$$y^2 = (7)^2 - (-6)^2$$ ½ mark for substitution of $x = -6$ and $r = 7$

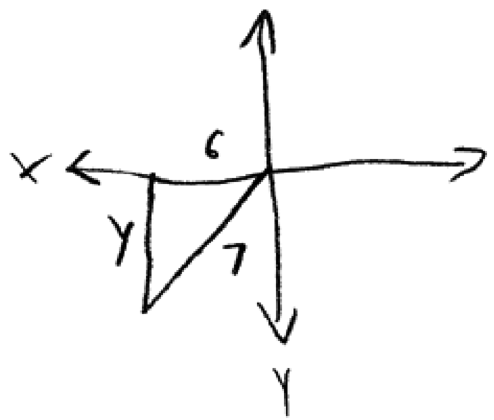
$$y^2 = 13$$

$$y = \pm\sqrt{13}$$ ½ mark for solving for y

$$\tan\theta = \frac{\sqrt{13}}{6}$$ 1 mark for the value of $\tan\theta$ (½ mark for the quadrant, ½ mark for the value)

2 marks

Exemplar 1



$$6^2 + y^2 = 7^2$$
$$36 + y^2 = 49$$
$$\sqrt{y^2} = \sqrt{85}$$
$$y = \sqrt{85}$$
$$\tan = \frac{\sqrt{85}}{-6}$$

1 out of 2

+ ½ mark for substitution of $x = -6$ and $r = 7$
+ ½ mark for solving for y
+ ½ mark for the value of $\tan\theta$
− ½ mark for arithmetic error in line 3
E3 (variable omitted in an equation in line 5)

Question 27 R1

Given $f(x) = x^2 + x - 4$ and $g(x) = \sqrt{x+5}$, Taz was asked to find $f(g(x))$.

Taz's solution:

$$f(g(x)) = \left(\sqrt{x+5}\right)^2 + x - 4$$
$$= x + 5 + x - 4$$
$$= 2x + 1, \ x \geq -5$$

Describe the error in Taz's solution.

Solution

Taz must substitute $g(x)$ in both terms containing x in $f(x)$ and then simplify.

1 mark

Exemplar 1

He should have multiplied $f(x)$ and $g(x)$.

0 out of 1

Exemplar 2

Taz made a mistake because he didn't substitute $g(x)$ correctly.

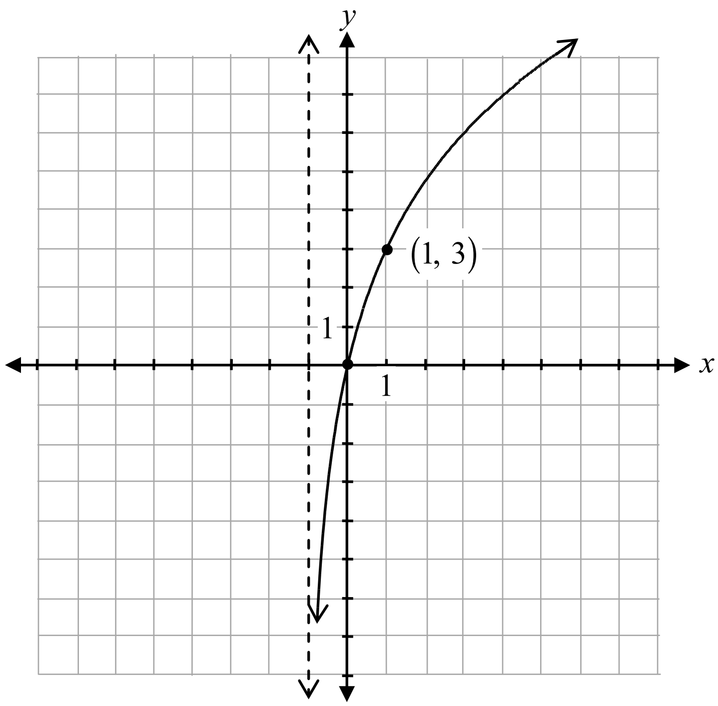
½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 28 R9

Sketch the graph of the function $f(x) = 3\log_2(x+1)$.

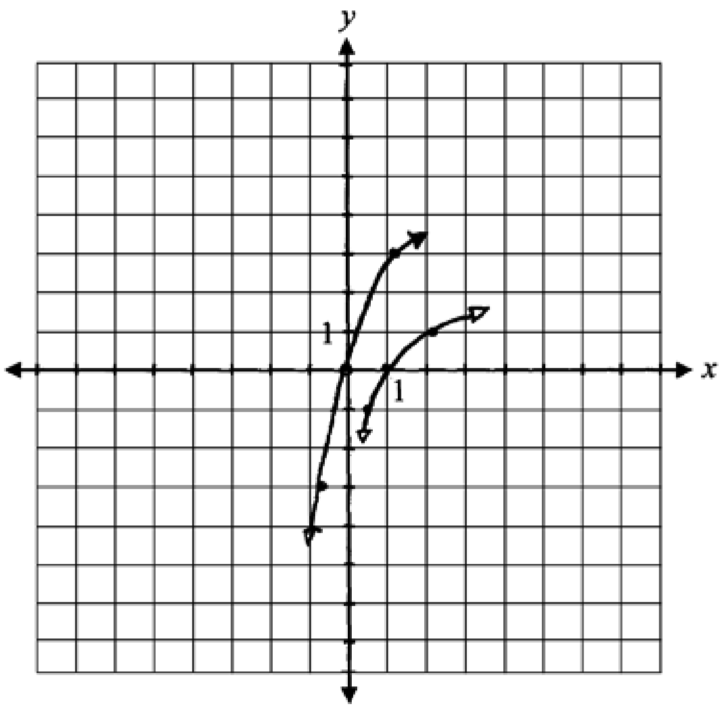
Solution



1 mark for increasing logarithmic function
1 mark for vertical stretch
1 mark for asymptotic behaviour at $x = -1$

3 marks

Exemplar 1

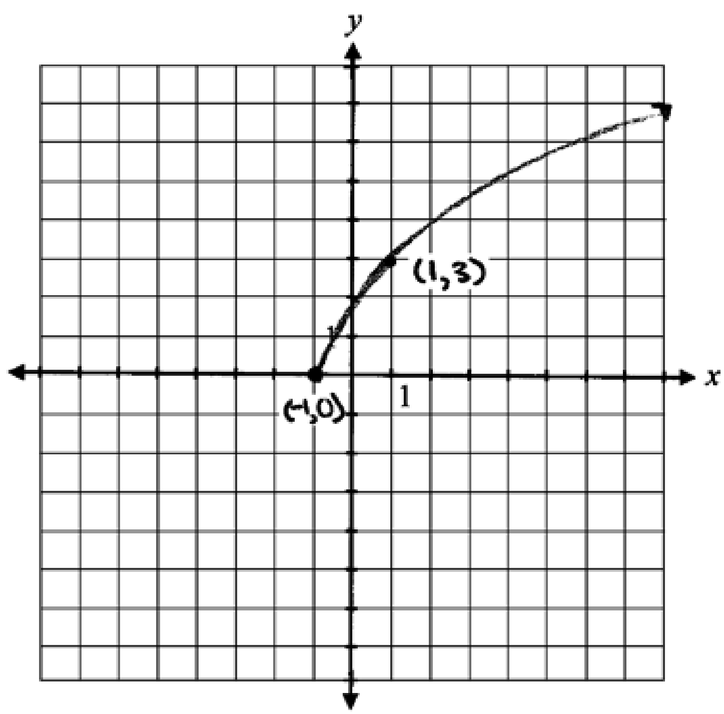


x	y
1/2	-1
1	0
2	1

2 out of 3

+ 1 mark for increasing logarithmic function
+ 1 mark for vertical stretch
E1 (final answer not stated)

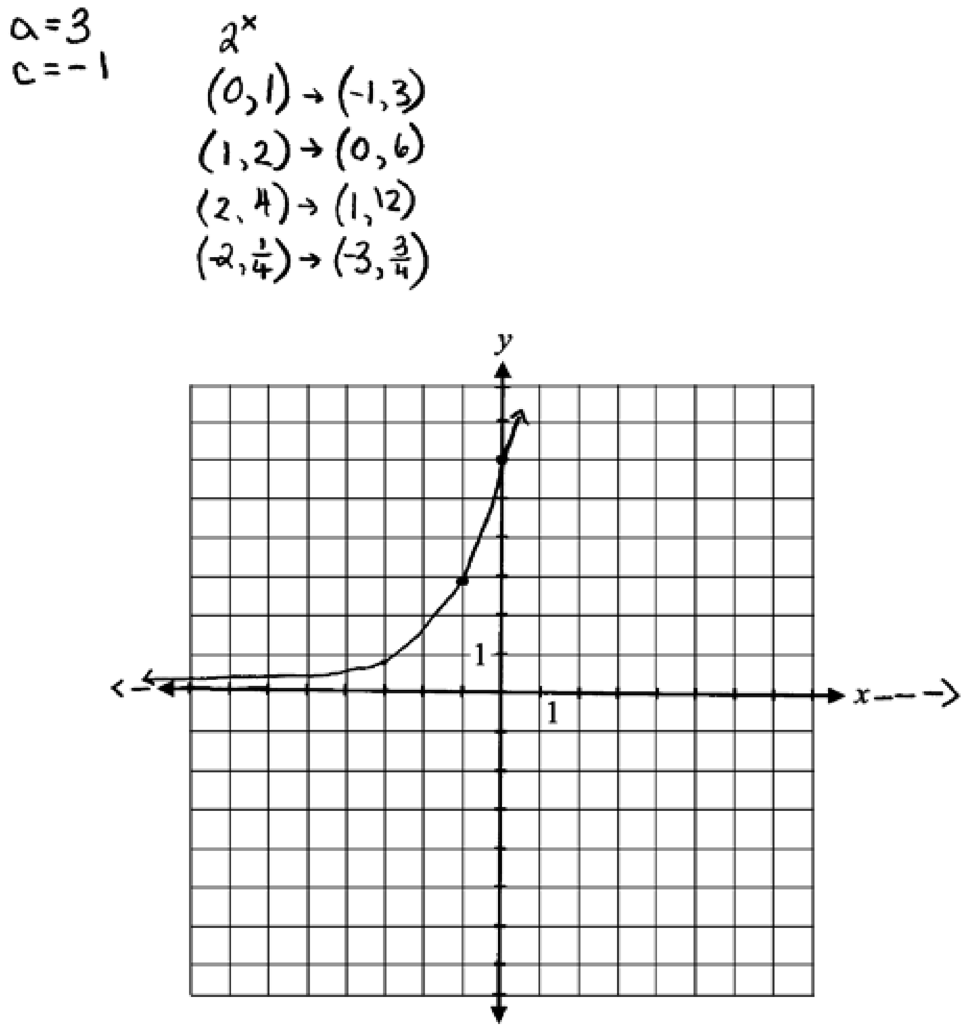
Exemplar 2



1 out of 3

+ 1 mark for vertical stretch

Exemplar 3



2 out of 3

award full marks

– 1 mark for concept error (a graph of an exponential function was sketched instead of a logarithmic function)

Question 29 R14

Write an equation of a rational function that would not have any vertical asymptotes.

Solution

Various equations, such as the following, are possible:

$$y = \frac{(x-2)(x+1)}{(x-2)}$$

or

$$y = \frac{4}{x^2+4}$$

1 mark

Exemplar 1

$$f(x) = \frac{1(x+2)}{(x+2)}$$

1 out of 1

Exemplar 2

$$f(x) = \frac{(x+4)(x+1)}{(x+4)(x+1)}$$

they would cancel out and become holes of discontinuity instead of asymptotes.

1 out of 1

Question 30 T6

Determine the exact value of $\tan 75^\circ$.

Solution

$$\tan 75^\circ = \tan\left(30^\circ + 45^\circ\right)$$

1 mark for combination

$$= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ}$$

$$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}(1)}$$

1 mark for exact values (½ mark for each)

2 marks

$$= \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{1+\sqrt{3}}{\sqrt{3}-1}$$

or

$$= \frac{\sqrt{3}+3}{3-\sqrt{3}}$$

Note(s):

- Other combinations are possible.

Exemplar 1

$$\tan 75° = \tan 45° + \tan 30°$$
$$= (1) + \left(\frac{1}{\sqrt{3}}\right)$$
$$= \frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}$$
$$= \frac{\sqrt{4}}{\sqrt{3}}$$

½ out of 2

+ 1 mark for exact values
– ½ mark for arithmetic error in line 4

Exemplar 2

$$\tan(30° + 45°) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(30° + 45°) = \frac{\sqrt{3} + 1}{1 - \sqrt{3}(1)}$$
$$= \boxed{\frac{\sqrt{3}+1}{1-\sqrt{3}}}$$

1½ out of 2

+ 1 mark for combination
+ ½ mark for exact value of $\tan 45^{o}$

Exemplar 3

$$\tan(120^\circ - 45^\circ) = \frac{\tan 120^\circ - \tan 45^\circ}{1 + \tan 120^\circ (\tan 45^\circ)}$$

$$= \frac{-\sqrt{3} - 1}{1 + (-\sqrt{3})(1)}$$

$$= \frac{-\sqrt{3} - 1}{1 - \sqrt{3}}$$

2 out of 2

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Sketch the graph of the following function:

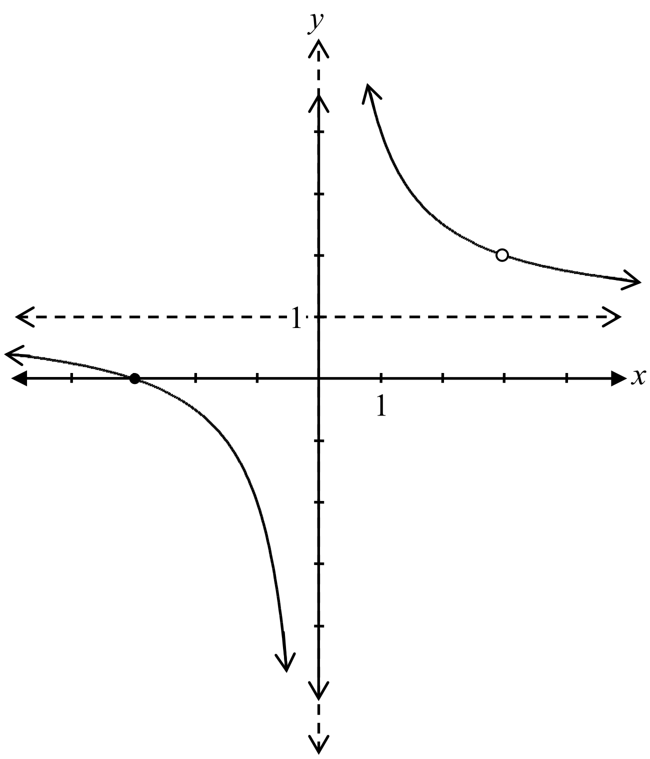
$$f(x) = \frac{(x+3)(x-3)}{x(x-3)}$$

Solution

$$f(x) = \frac{(x+3)\cancel{(x-3)}}{x\cancel{(x-3)}}$$

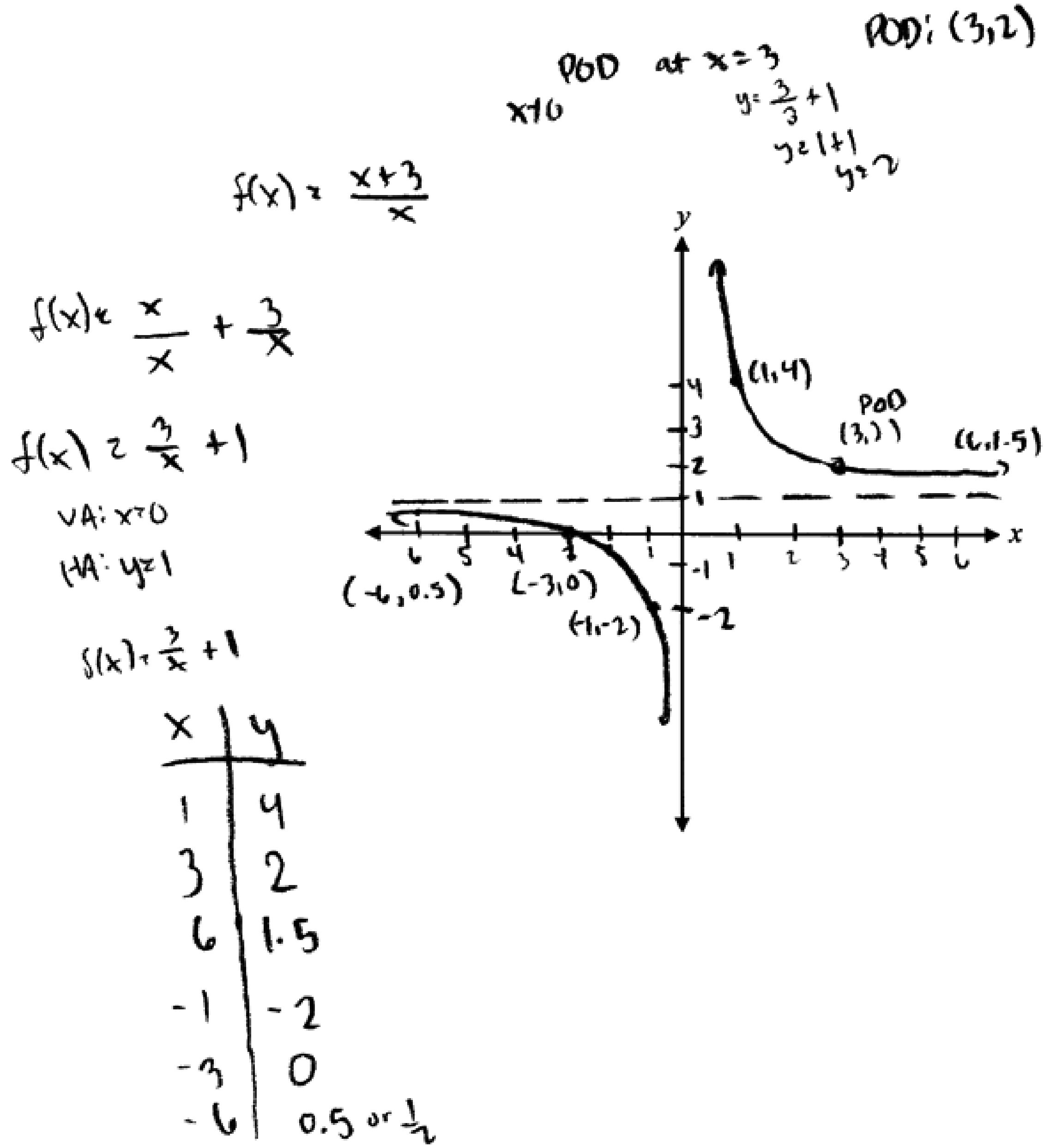
$$= \frac{x+3}{x}, \; x \neq 3$$

$\therefore$ there is a point of discontinuity (hole) at $(3,\ 2)$



1 mark for asymptotic behaviour at $y = 1$
1 mark for asymptotic behaviour at $x = 0$
1 mark for point of discontinuity (hole) at $(3,\ 2)$
(½ mark for $x = 3$, ½ mark for $y = 2$)
½ mark for graph left of vertical asymptote
½ mark for graph right of vertical asymptote

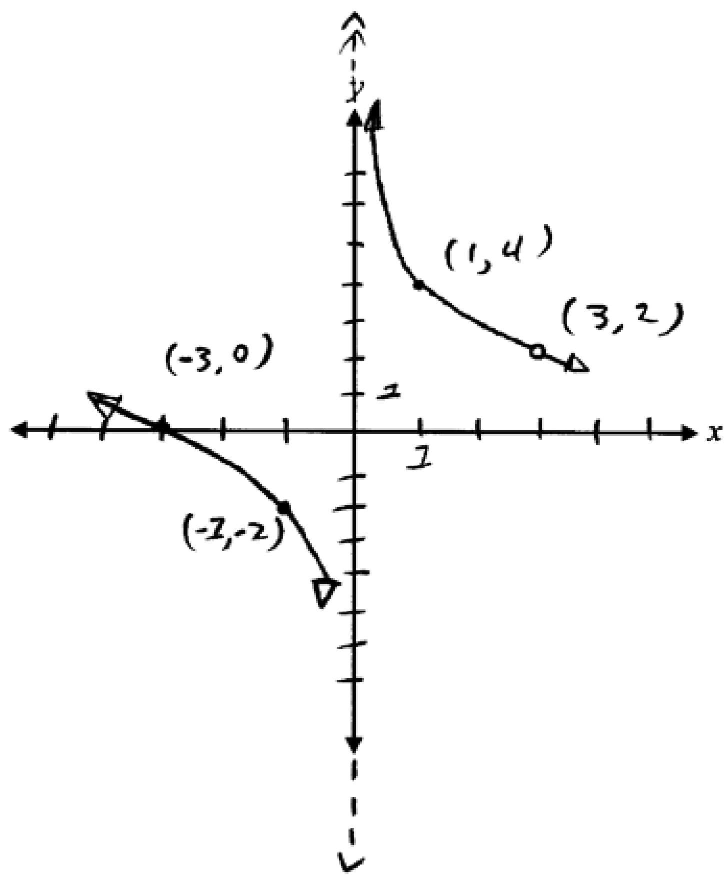
4 marks



4 out of 4

award full marks
E10 (asymptote missing but still implied at $x = 0$)

Exemplar 2



3 out of 4

+ 1 mark for asymptotic behaviour at $x = 0$
+ 1 mark for point of discontinuity (hole) at (3, 2)
+ ½ mark for graph left of vertical asymptote
+ ½ mark for graph right of vertical asymptote

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Question 32 P4

In the binomial expansion of $\left(\frac{1}{x^3} - 2x^2\right)^9$, determine which term contains x^3.

Solution

Method 1

$x^3 = \left(x^{-3}\right)^{9-k} \left(x^2\right)^k$ ½ mark for substitution

$x^3 = x^{-27+3k+2k}$

$3 = -27 + 3k + 2k$

$30 = 5k$

$6 = k$ ½ mark for solving for k

$\therefore$ term 7 would contain x^3

1 mark for identifying the 7th term
(or consistent term with the value of k)

2 marks

Method 2

$\left(\frac{1}{x^3}\right)^9, \left(\frac{1}{x^3}\right)^8 \left(x^2\right), \left(\frac{1}{x^3}\right)^7 \left(x^2\right)^2$

$x^{-27}, x^{-22}, x^{-17}$

1 mark for determining the pattern

$\therefore$ term 7 would contain x^3

1 mark for identifying the 7th term
(or consistent term with the pattern)

2 marks

Exemplar 1

$$t_{k+1} = {}_9C_k \left(\frac{1}{x^3}\right)^{9-k} \left(-2x^2\right)^k \qquad k=6$$

$$\left(\frac{1}{x^3}\right)^{9-6} \left(-2x^2\right)^6$$

$$\left(\frac{1}{x^3}\right)^3 \left(-2x^{12}\right)$$

$$\left(\frac{1}{x^9}\right)\left(-2x^{12}\right) = \frac{-2x^{12}}{x^9}$$

$$= -2x^3$$

½ out of 2

Method 1
+ ½ mark for substitution
+ ½ mark for solving for k
− ½ mark for arithmetic error in line 3
E2 (changing an equation to an expression)

Exemplar 2

the 7th term
will contain x^3

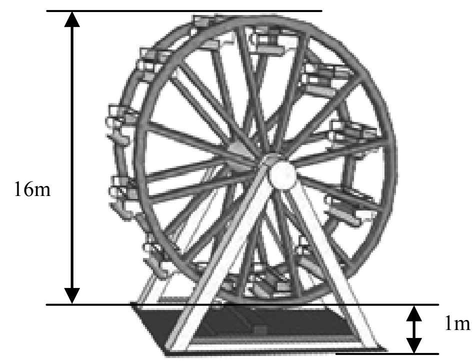
1 out of 2

+ 1 mark for identifying the 7th term

Question 33 T4

José and Dana get on a Ferris wheel, which is 1 metre off the ground. The diameter of the Ferris wheel is 16 metres. Their ride lasts for 4 minutes, in which time the Ferris wheel makes one revolution.

Determine the values of A, B, C, and D, if the sinusoidal function that models the situation is $h(t) = A\cos[B(t - C)] + D$, where h is the height at which José and Dana are located on the Ferris wheel, from the ground, in metres, and t is the time, in minutes.



A = ________________

B = ________________

C = ________________

D = ________________

Solution

$A =$ 8	**or**	$A =$ -8	1 mark for A
$B = \frac{\pi}{2}$		$B = \frac{\pi}{2}$	1 mark for B
$C =$ 2		$C =$ 0	1 mark for C
$D =$ 9		$D =$ 9	1 mark for D

4 marks

Note(s)

- Other answers are possible.

Exemplar 1

A = 8

B = $\frac{\pi}{2} = 8\pi$

C = 2

D = 9

3½ out of 4

award full marks
– ½ mark for arithmetic error in calculating the value of B

Question 34 P2

Solve algebraically:

$${}_nP_3 = 4!(n-1)$$

Solution

$$\frac{n!}{(n-3)!} = 4!(n-1)$$ ½ mark for substitution

$$\frac{n\,\cancel{(n-1)}\,(n-2)\,\cancel{(n-3)!}}{\cancel{(n-3)!}} = 4!\,\cancel{(n-1)}$$ 1 mark for factorial expansion
½ mark for simplification of factorials

$$n(n-2) = 24$$

$$n^2 - 2n - 24 = 0$$

$$(n-6)(n+4) = 0$$

$$n = 6 \quad \cancel{n = -4}$$ ½ mark for rejecting extraneous root
½ mark for the value of n

3 marks

Exemplar 1

$$\frac{n!}{n-3!} = 4!(n-1)$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 4!(n-1)$$

$$n(n-1)(n-2) = 4!(n-1)$$

$$(n^2-n)(n-2) = 24(n-1)$$

$$n^3-2n^2-n^2+2n = 24n-24$$

$$n^3-3n^2+2n = 24n-24$$

$$n^3-3n^2-22n+24 = 0$$

$(n-1)$

1	1	−3	−22	+24
		1	−2	−24
	1	−2	−24	0

$$(n-1)(n^2-2n-24)$$

$$(n-1)(n-6)(n+4)$$

$n = 1$

$n = 6$

~~$n = -4$~~

2½ out of 3

+ ½ mark for substitution
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for values for n

E4 (missing brackets but still implied in line 1)
E2 (changing an equation to an expression in lines 9 and 10)

Exemplar 2

$$\frac{n!}{(n-3)!} = 4!(n-1)$$

$$\frac{(n)(n-1)(n-2)(n-3)!}{(n-3)!} = 4!(n-1)$$

$$\frac{(n)(n-1)(n-2)}{(n-1)} = 24$$

$$(n)(n-2) = 24$$

$$(n)(n-2) = 6 \times 4$$

$$n = 6$$

2½ out of 3

+ ½ mark for substitution
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for the value of n

Exemplar 3

$$= \frac{n!}{(n-3)!}$$

$$= \frac{(n)(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}}$$

$$= n(n-1)(n-2)$$

$$= (n^2-n)(n-2)$$

$$= \boxed{n^3-2n^2-n^2+2n}$$

1½ out of 3

+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials

Given $f(x) = \dfrac{2}{x-1}$, determine the equation of the inverse, $f^{-1}(x)$.

Solution

Method 1

let $y = f(x)$

$$f(x) = \frac{2}{x-1}$$

$$y = \frac{2}{x-1}$$

$$x = \frac{2}{y-1}$$

1 mark for switching x and y values

$$y - 1 = \frac{2}{x}$$

$$y = \frac{2}{x} + 1$$

½ mark for solving for y

$$f^{-1}(x) = \frac{2}{x} + 1$$

½ mark for writing equation of $f^{-1}(x)$

2 marks

Method 2

let $y = f(x)$

$$f(x) = \frac{2}{x-1}$$

$$y = \frac{2}{x-1}$$

$$x = \frac{2}{y-1}$$

1 mark for switching x and y values

$$x(y-1) = 2$$

$$xy - x = 2$$

$$xy = 2 + x$$

$$y = \frac{2+x}{x}$$

½ mark for solving for y

$$f^{-1}(x) = \frac{2+x}{x}$$

½ mark for writing equation of $f^{-1}(x)$

2 marks

Exemplar 1

$f(x) = \frac{2}{x-1}$

let $f(x) = y$

$y = \frac{2}{x-1}$

$x = \frac{2}{y-1}$

$x(y-1) = 2$

$yx - x = 2$

$yx = 2x$

$y = \frac{2x}{x}$

$y = x$

$f(x)^{-1} = x$

1 out of 2

Method 2
+ 1 mark for switching x and y values

Exemplar 2

$x = \frac{2}{y-1}$

$y-1 = \frac{2}{x}$

$y = \frac{2}{x} + 1$

1½ out of 2

Method 1
+ 1 mark for switching x and y values
+ ½ mark for solving for y

Solve:

$$4\log_3 2 - \frac{1}{3}\log_3 8 = \log_3 a$$

Solution

Method 1

$$\log_3 2^4 - \log_3 8^{\frac{1}{3}} = \log_3 a$$ 1 mark for power law (½ mark for each)

$$\log_3 16 - \log_3 2 = \log_3 a$$

$$\log_3\left(\frac{16}{2}\right) = \log_3 a$$ 1 mark for quotient law

$$\log_3 8 = \log_3 a$$

$$a = 8$$ 1 mark for equating arguments

3 marks

Method 2

$$\log_3 2^4 - \log_3 8^{\frac{1}{3}} - \log_3 a = 0$$ 1 mark for power law (½ mark for each)

$$\log_3 16 - \log_3 2 - \log_3 a = 0$$

$$\log_3\left(\frac{16}{2a}\right) = 0$$ 1 mark for quotient law

$$\log_3\left(\frac{8}{a}\right) = 0$$

$$3^0 = \frac{8}{a}$$ 1 mark for exponential form

$$1 = \frac{8}{a}$$

$$a = 8$$

3 marks

Exemplar 1

$$\log_3\left(\frac{2^4}{\sqrt[3]{8}}\right) = \log_3 a$$

$$\log_3\left(\frac{16}{2}\right) = \log_3 a$$

$$\log_3(8) = \log_3 a$$

$$8 = a$$

2½ out of 3

Method 1
award full marks
– ½ mark for procedural error in line 3

Exemplar 2

$$2^4 - 8^{1/3} = a$$

$$16 - 2 = a$$

$$14 = a$$

---

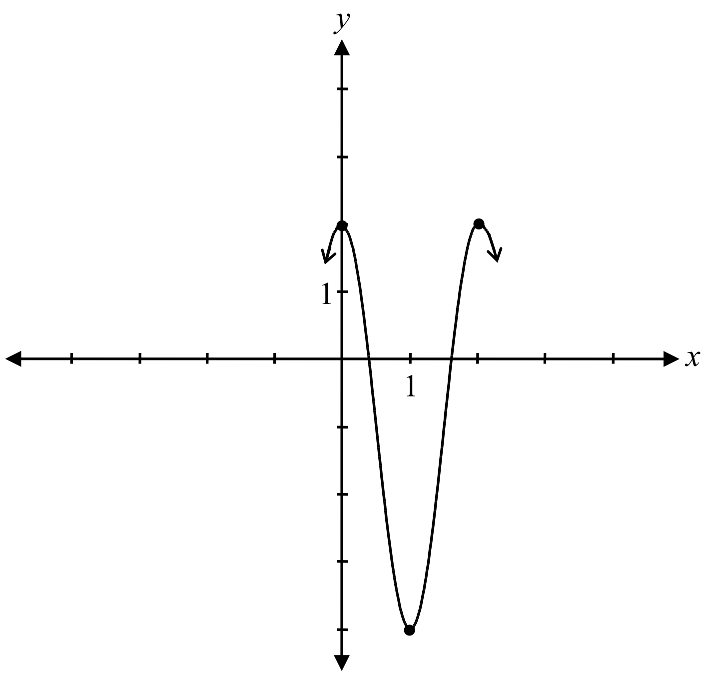
0 out of 3

+ 1 mark for power law
– 1 mark for concept error (using power law without logarithms)

Question 37 T4

Sketch the graph of at least one period of the function $y = 3\cos(\pi x) - 1$.

Solution

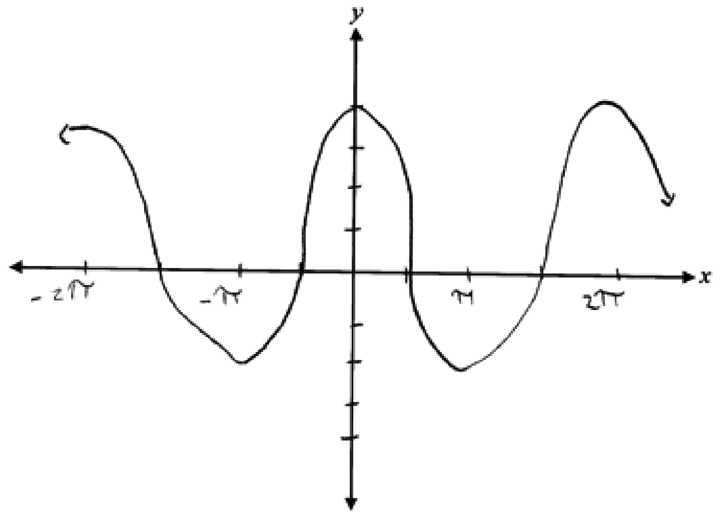


$$\text{period} = \frac{2\pi}{\pi}$$
$$= 2$$

1 mark for amplitude
1 mark for period
1 mark for vertical translation

3 marks

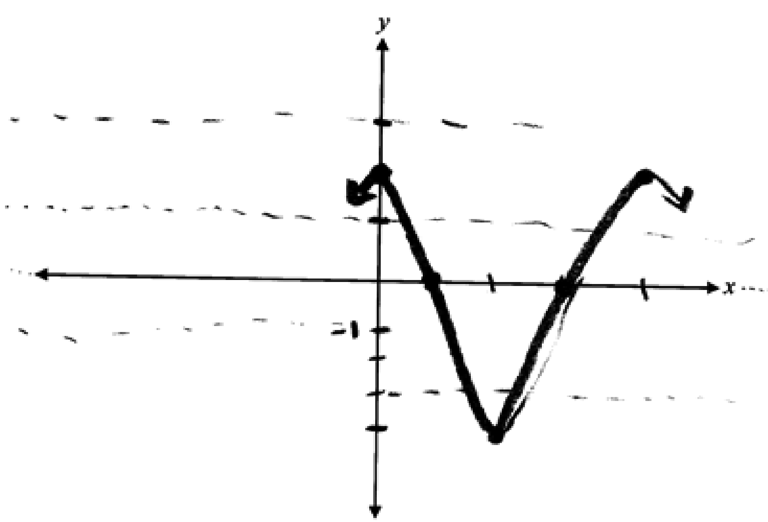
Exemplar 1



1 out of 3

+ 1 mark for amplitude
E9 (scale values on y-axis not indicated)

Exemplar 2



2 out of 3

+ 1 mark for amplitude
+ 1 mark for vertical translation

Question 38 R8

Using the laws of logarithms, fully expand the expression:

$$\log_a\left(\frac{x^3}{y\sqrt{z}}\right)$$

Solution

$$\log_a\left(\frac{x^3}{y\sqrt{z}}\right) = \log_a x^3 - \left(\log_a y + \log_a \sqrt{z}\right)$$

1 mark for quotient law
1 mark for product law

$$= 3\log_a x - \left(\log_a y + \frac{1}{2}\log_a z\right)$$

1 mark for power law (½ mark for each)

$$= 3\log_a x - \log_a y - \frac{1}{2}\log_a z$$

3 marks

Exemplar 1

$$\log_a x^3 - \log_a y\sqrt{z}$$

$$3\log_a x - \log_a y\sqrt{z}$$

1½ out of 3

+ 1 mark for quotient law
+ ½ mark for power law

Exemplar 2

$$= \log_a x^3 - (\log_a y + \log_a \sqrt{z})$$

$$= 3\log_a x - \log_a y + \frac{1}{2}\log_a z$$

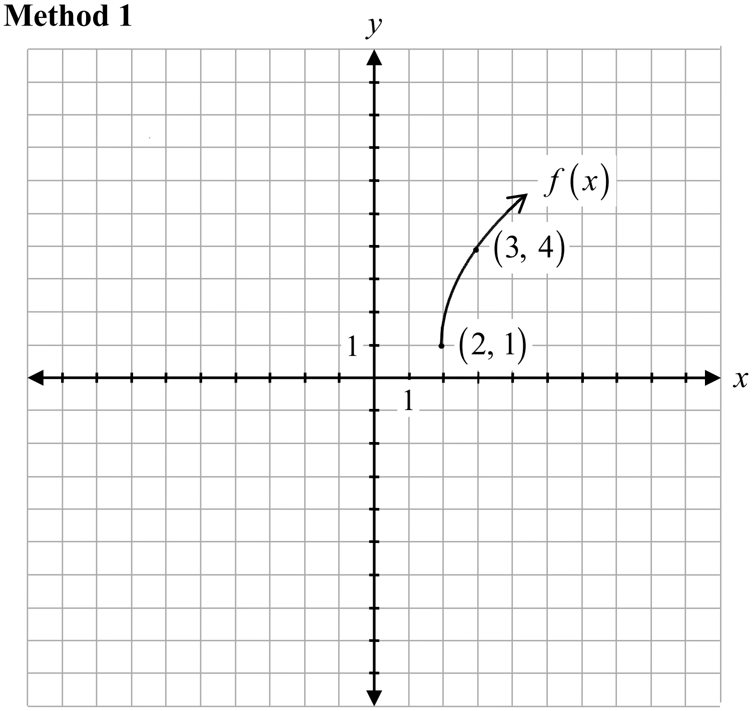
2½ out of 3

award full marks
– ½ mark for arithmetic error in line 2

Sketch the graph of $f(x) = 3\sqrt{x-2} + 1$.

Solution

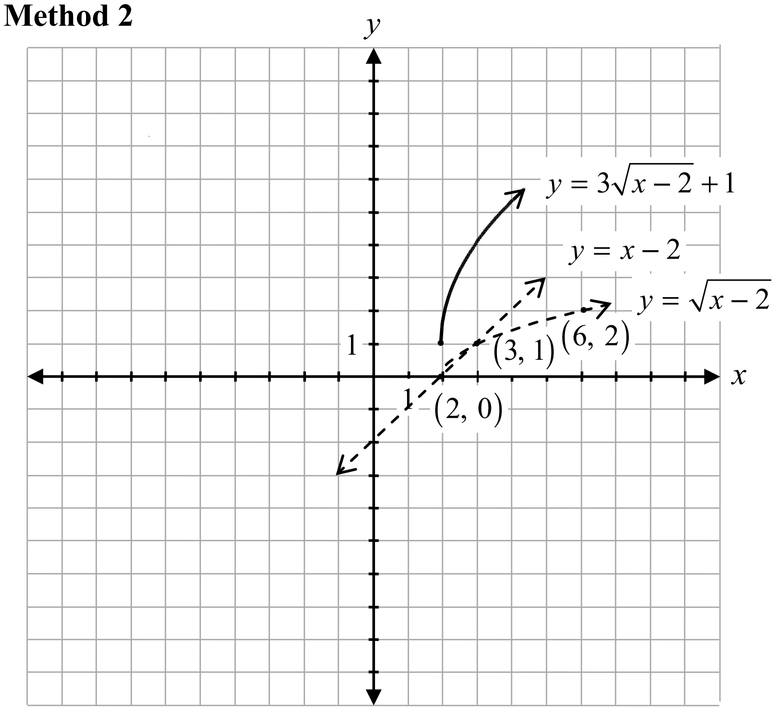
Method 1



1 mark for horizontal translation
1 mark for vertical translation
1 mark for shape of a radical function
1 mark for vertical stretch

4 marks

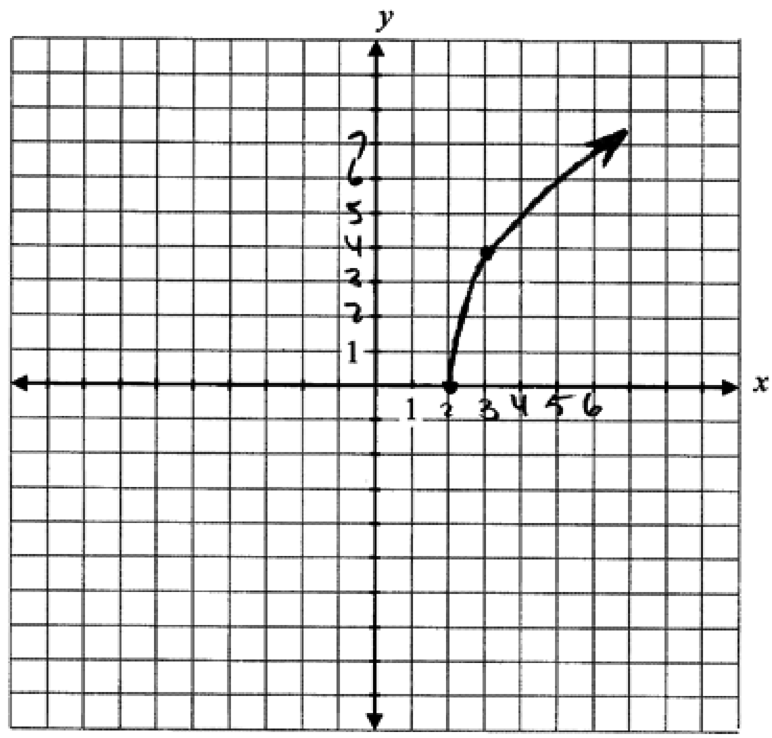
Method 2



1 mark for invariant points where $y = 0$ and $y = 1$ (½ mark for each point)
1 mark for domain $[2, \infty)$
½ mark for shape between invariant points
½ mark for shape to the right of the invariant points
1 mark for applying transformations (½ mark for vertical stretch, ½ mark for vertical translation)

4 marks

Exemplar 1



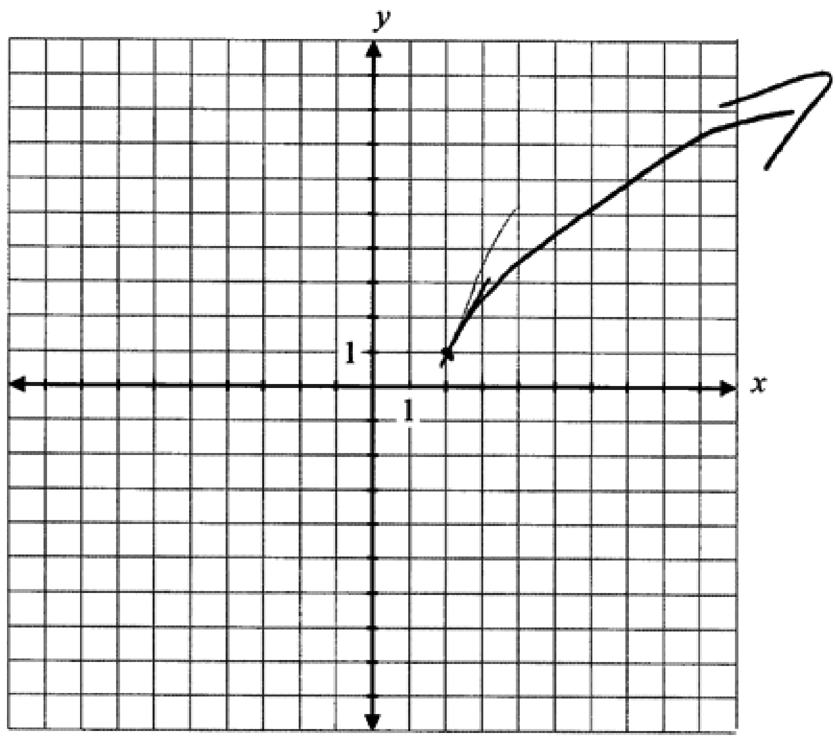
(0,0) → (2,0)
(1,1) → (3,4)
(4,2) → (6,7)

y · 3
x + 2
y + 1

3½ out of 4

Method 1
award full marks
– ½ mark for arithmetic error in line 1

Exemplar 2



2½ out of 4

Method 1

+ 1 mark for horizontal translation
+ 1 mark for vertical translation
+ 1 mark for shape of a radical function
– ½ mark for procedural error (not including a minimum of 2 points on the graph)

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Question 40 R13

a) Determine the domain of the graph of the function $f(x) = \sqrt{x^2 - 4}$.

b) Explain why the domain of $f(x) = \sqrt{x^2 - 4}$ is restricted.

Solution

a) $\{x \mid x \leq -2 \quad \cup \quad x \geq 2\}$

or

D: $(-\infty, -2] \cup [2, \infty)$

1 mark for domain (½ mark for $x \leq -2$, ½ mark for $x \geq 2$)

1 mark

b) The domain is restricted because you cannot take the square root of a negative number.

1 mark

Exemplar 1

a)

$\sqrt{x^2 - 4}$ $\sqrt{2^2 - 4}$

$\{x/x \in R; x > 2\}$

½ out of 1

+ ½ mark for domain
E8 (bracket error made when stating domain)

b)

because we cannot with a negative root, ex $(\sqrt{-2})$ is impossible,

1 out of 1

Exemplar 2

a)

$x > 0$

$(-9)^2 = \sqrt{81} = 9$

0 out of 1

b)

you cannot √ a negative number.
Once you square a negative number the √ will come out positive, changing the question.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3

a)

domain : (∞, ∞)

0 out of 1

b)

Because the domain should be only positive and not negative due to the radical.

0 out of 1

Question 41 R1, R2, R5

Given the point $(-12, -18)$ on the graph of $f(x)$, determine the new points after the following transformations of $f(x)$.

a) $\dfrac{1}{f(x)}$

b) $f(-x)+10$

Solution

a) $\left(-12, \dfrac{-1}{18}\right)$

1 mark

b) $(12, -8)$

1 mark (½ mark for x-value, ½ mark for y-value)

1 mark

Exemplar 1

a)

(x, y)

(y, x)

$(-18, -12)$

0 out of 1

b)

$(22, -8)$

½ out of 1

+ ½ mark for *y*-value

Exemplar 2

a)

$\left(-\frac{1}{12}, -\frac{1}{18}\right)$

0 out of 1

b)

reflect over y axis

$f(-(-12)) + 10 - 18$

$(12, -8)$

1 out of 1

Question 42 T3

Explain why there is no solution for the equation $\csc\theta = -\frac{1}{2}$.

Solution

The value of $\csc\theta$ cannot be between -1 and 1.

or

The value of $\sin\theta$ cannot be less than -1.

1 mark

Exemplar 1

Because $\csc\theta = -\frac{1}{2}$ is equal to
$\sin\theta = -\frac{2}{1}$ therefore $\sin\theta = -2$.
$\sin\theta = -2$ is not on the unit circle.

1 out of 1

Exemplar 2

There is no value for sin −2 or 2
sin 2 be cause the function
of sin only ranges from (−1, 1)

1 out of 1

Exemplar 3

Because the value of cosine can't be less than −1.

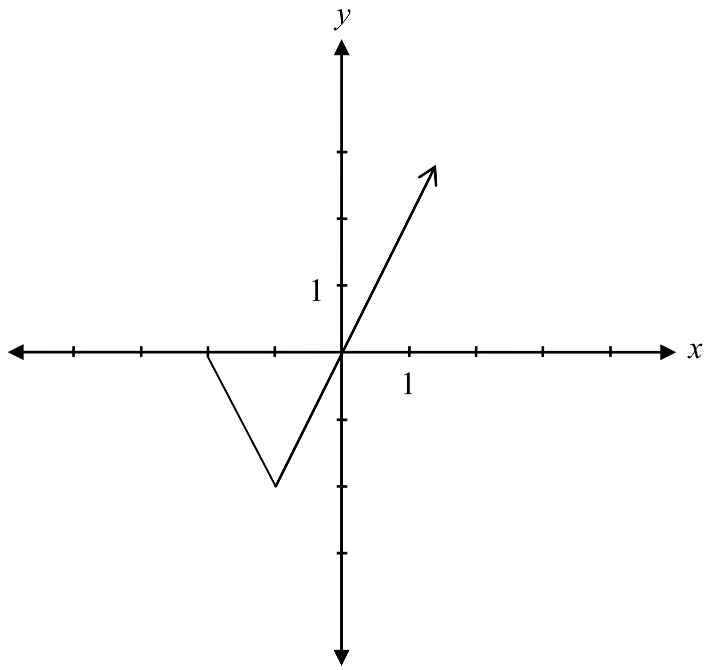
0 out of 1

award full marks

– 1 mark for concept error $\left(\csc\theta \neq \dfrac{1}{\cos\theta}\right)$

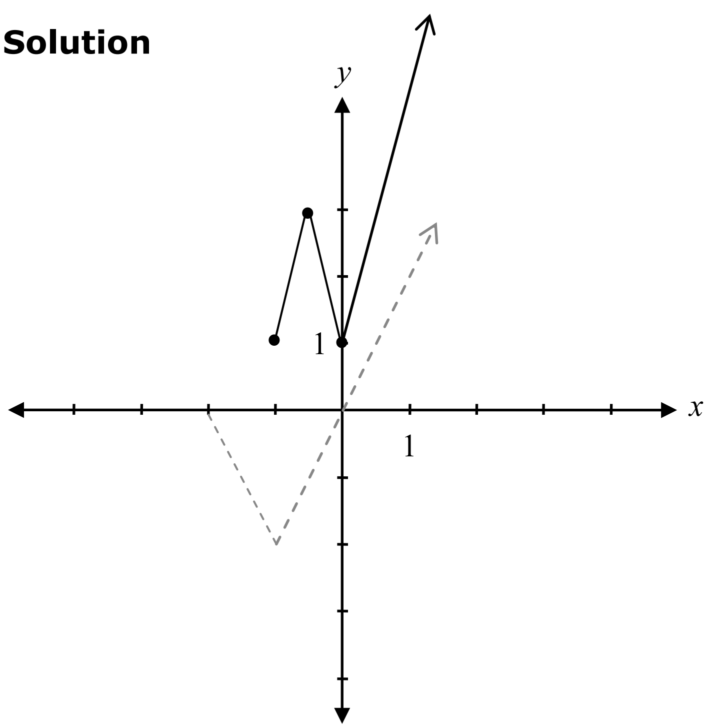
Question 43 R1, R4

Given the graph of $y = f(x)$,



sketch the graph of $y = |f(2x)| + 1$.

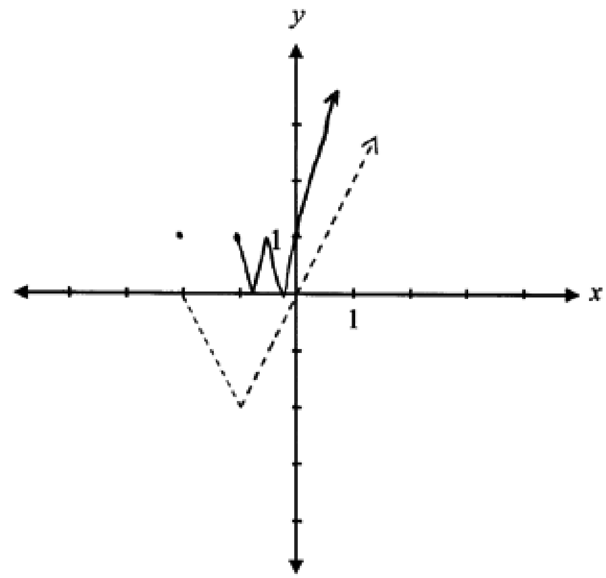
Solution



1 mark for absolute value
1 mark for horizontal compression
1 mark for vertical translation

3 marks

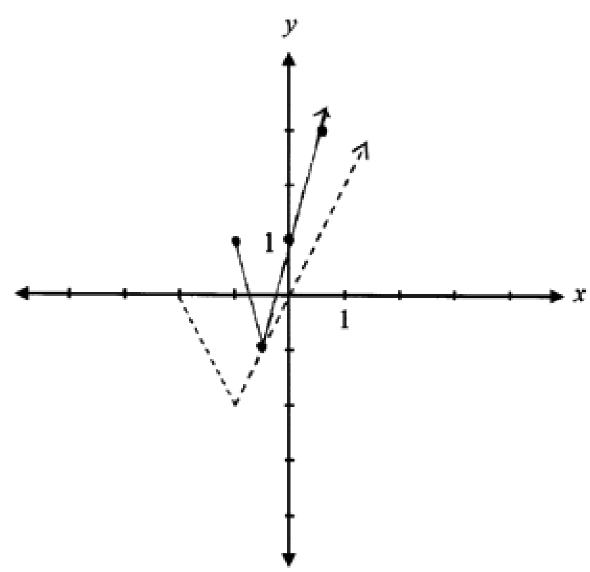
Exemplar 1



2 out of 3

award full marks
– 1 mark for concept error (wrong order)

Exemplar 2



2 out of 3

+ 1 mark for horizontal compression
+ 1 mark for vertical translation

Question 44 R9

Given $f(x) = 2^x + 1$, state the equation of the horizontal asymptote.

Solution

$y = 1$

1 mark

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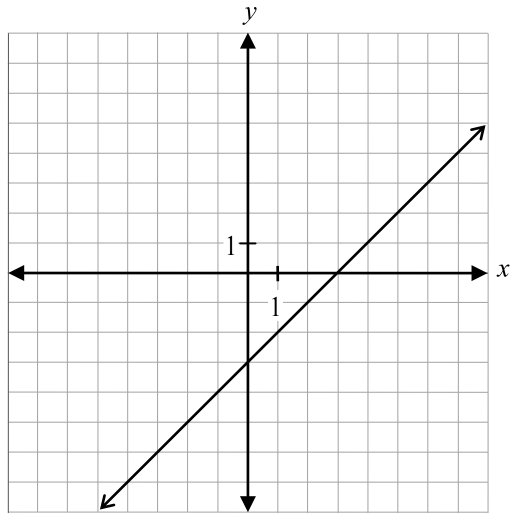
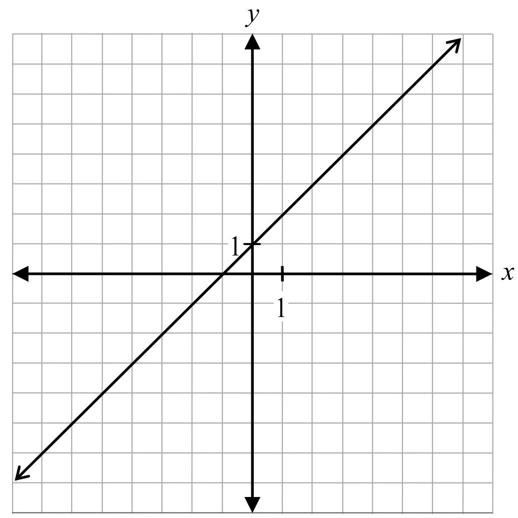
Given the following graphs of $f(x) = x - 3$ and $g(x) = x + 1$,

$f(x)$

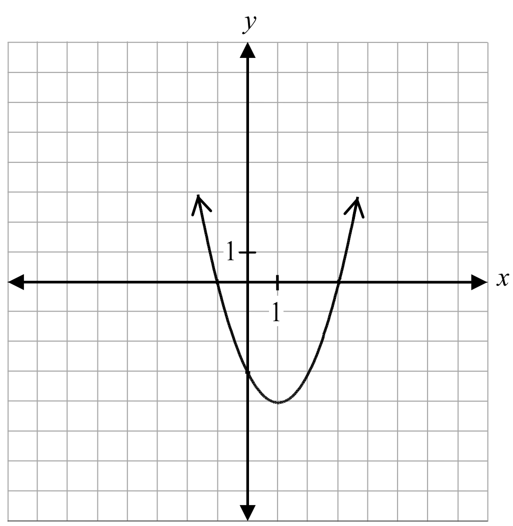


$g(x)$



sketch the graph of $h(x) = (f \cdot g)(x)$.

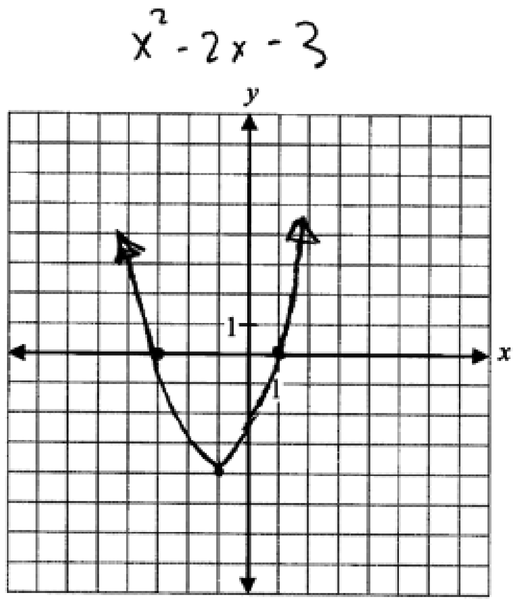
Solution



1 mark for operation of multiplication
1 mark for shape representing the given operation

2 marks

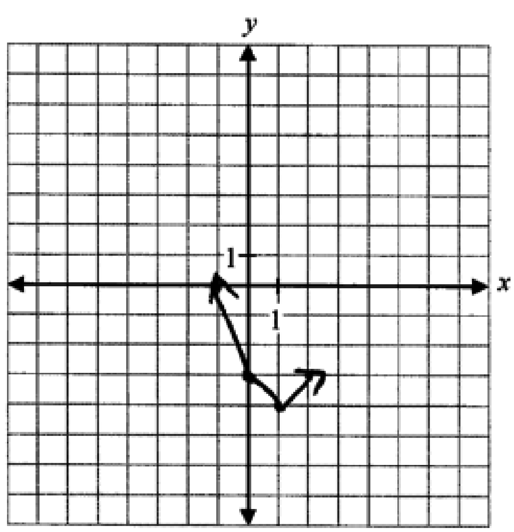
Exemplar 1



1½ out of 2

award full marks
– ½ mark for procedural error

Exemplar 2



1 out of 2

+ 1 mark for operation of multiplication

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Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated
E2 equation/expression	▪ changing an equation to an expression ▪ equating the two sides when proving an identity
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 units	▪ missing units of measure ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 rounding	▪ rounding error ▪ rounding too early
E7 notation/transcription	▪ notation error ▪ transcription error
E8 domain/range	▪ answer given outside the domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 graphing	▪ incorrect or missing endpoints or arrowheads ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes missing but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: ____________________

Date marked: ____________________

Booklet No.: ____________________

Problem(s) noted: ____________________

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Question(s) affected: ____________________

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Action taken or rationale for assigning marks: ____________________

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Follow-up: ____

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Decision: ____

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Marker's Signature: ____

Principal's Signature: ____

For Department Use Only—After Marking Complete

Consultant: ____

Date: ____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
7	R2, R3	2
9	R2, R5	2
15	R3	1
27	R1	1
35	R6	2
41 a)	R1	1
41 b)	R2, R5	1
43	R1, R4	3
45	R1	2
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
16	T1	1
24	T3	2
26	T2	2
33	T4	4
37	T4	3
42	T3	1
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
2 a)	P3	1
2 b)	P3	2
12	P1	1
18	P4	1
32	P4	2
34	P2	3
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
6	R12	1
8	R11	1
13	R11	2
14	R12	1
19	R12	1
22	R12	3

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
3	T5	3
5	T5	4
10	T6	3
11	T5	1
17	T6	1
20	T6	1
30	T6	2
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
4	R10	4
23	R7	1
25	R7	1
28	R9	3
36	R8	3
38	R8	3
44	R9	1
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
21	R14	1
29	R14	1
31	R14	4
39	R13	4
40 a)	R13	1
40 b)	R13	1