Manitoba Education Cataloguing in Publication Data

Grade 12 pre-calculus mathematics achievement test.
ISBN: 978-0-7711-5569-7

1. Mathematics—Examinations, questions, etc.
2. Educational tests and measurements—Manitoba.
3. Mathematics—Study and teaching (Secondary)—Manitoba.
4. Calculus—Study and teaching (Secondary)—Manitoba.
I. Manitoba. Manitoba Education.
515.076

Manitoba Education
School Programs Division
Winnipeg, Manitoba, Canada

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Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.
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General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the Answer/Scoring Sheet are identical
- **students and markers use only a pencil to complete the Answer/Scoring Sheets**
  - the totals of each of the four parts are written at the bottom
  - each student’s final result is recorded, by booklet number, on the corresponding Answer/Scoring Sheet
  - the Answer/Scoring Sheet is complete
  - a photocopy has been made for school records

Once marking is completed, please forward the Answer/Scoring Sheets to Manitoba Education in the envelope provided (for more information see the administration manual).

**Marking the Test Questions**

The test is composed of short-answer questions, long-answer questions, and multiple-choice questions. Short-answer questions are worth 1 or 2 marks each, long-answer questions are worth 3 to 5 marks each, and multiple-choice questions are worth 1 mark each. An answer key for the multiple-choice questions can be found at the beginning of the section “Booklet 2 Questions.”

To receive full marks, a student’s response must be complete and correct. Where alternative answering methods are possible, the Marking Guide attempts to address the most common solutions. For general guidelines regarding the scoring of students’ responses, see Appendix A.

**Irregularities in Provincial Tests**

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an Answer/Scoring Sheet is marked with “0” and/or “NR” only (e.g., student was present but did not attempt any questions) please document this on the Irregular Test Booklet Report.
Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Youyi Sun
Assessment Consultant
Grade 12 Pre-Calculus Mathematics
Telephone: 204-945-7590
Toll-Free: 1-800-282-8069, extension 7590
Email: youyi.sun@gov.mb.ca
Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Answer/Scoring Sheet that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the Answer/Scoring Sheet in a separate section. There is a $\frac{1}{2}$ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors ($\frac{1}{2}$ mark deduction), four E7 errors ($\frac{1}{2}$ mark deduction), and one E8 error ($\frac{1}{2}$ mark deduction). Although seven communication errors were committed in total, there is a deduction of only $1\frac{1}{2}$ marks.

<table>
<thead>
<tr>
<th>COMMUNICATION ERRORS / ERREURS DE COMMUNICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shade in the circles below for a maximum total deduction of 5 marks ($0.5$ mark deduction per error).</td>
</tr>
<tr>
<td>Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
<th>E5</th>
</tr>
</thead>
<tbody>
<tr>
<td>●</td>
<td>○</td>
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<td>○</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E6</th>
<th>E7</th>
<th>E8</th>
<th>E9</th>
<th>E10</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>●</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
</tbody>
</table>

**Mark assigned to the student / Note accordée à l’élève**

\[
\begin{align*}
\text{Booklet 1 / Cahier 1} & + \quad \text{Multiple Choice / Chois multiple} & + \quad \text{Booklet 2 / Cahier 2} & - \quad \text{Communication Errors / Erreurs de communication} \\
\hline
25 & + & 7 & + & 40 & - & \frac{1}{2} \\
36 & 9 & 45 & & & & 90
\end{align*}
\]

maximum deduction of 5 marks / déduction maximale de 5 points
Booklet 1 Questions
Question 1

Find the coterminal angle to $\frac{27\pi}{5}$ over the interval $[-360^\circ, 0^\circ]$.

Solution

Method 1

\[
\frac{27\pi}{5} \left(\frac{180^\circ}{\pi}\right) = 27\left(36^\circ\right) = 972^\circ
\]

$\frac{972^\circ}{360^\circ} = 3 \, \text{or} \, \frac{972^\circ}{2} - 108^\circ$

1 mark for conversion to degrees

Method 2

$\frac{-3\pi}{5}$ is a coterminal angle to $\frac{27\pi}{5}$.

$\frac{-3\pi}{5} \cdot \frac{180^\circ}{\pi} = -108^\circ$

1 mark for conversion to degrees

2 marks
Exemplar 1

\[
\frac{10\pi}{5} = \frac{27\pi}{5} - \frac{10\pi}{5} = \frac{17\pi}{5} - \frac{10\pi}{5} = \frac{7\pi}{5} - \frac{10\pi}{5} = -\frac{3\pi}{5}
\]

1 out of 2

+ ½ mark for coterminal angle
+ ½ mark for correct domain
E7 (notation error in line 1)

Exemplar 2

\[
\frac{\frac{27\pi}{5}}{\frac{\pi}{5}} = \frac{-4860}{5} = -972 = 252^\circ
\]

1½ out of 2

+ 1 mark for conversion to degrees
+ ½ mark for coterminal angle
E7 (notation errors in line 1: \(\frac{27}{5} = \frac{4860}{5}\) and \(972^\circ = 252^\circ\))
Solve the following equation over the interval $0 \leq \theta < 2\pi$.

$$(\tan \theta - 3)(\tan \theta + 1) = 0$$

**Solution**

$$(\tan \theta - 3)(\tan \theta + 1) = 0$$

$\tan \theta = 3 \quad \tan \theta = -1$

$\theta = 1.249\ 046 \quad \theta = \frac{\pi}{4}$

$\theta = 4.390\ 639 \quad \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

or

$\theta = 1.249, 4.391 \quad \theta = 2.356\ 194$

$\theta = 5.497\ 787$

$\therefore \theta = 1.249, \frac{3\pi}{4}, 4.391, \frac{7\pi}{4}$

or

$\theta = 1.249, 2.356, 4.391, 5.498$
Exemplar 1

\[
\begin{align*}
\text{Exemplar 1} \\
\tan \theta &= 3 \\
\text{no sols} \\
\tan \theta &= -1 \\
&= \tan^{-1}(1) = 0.78539 \\
&= \pi - 0.78539 \\
&= 2.35619 \\
\end{align*}
\]

2 out of 3

+ 1 mark (½ mark for each branch) 
+ 1 mark (½ mark for correct answer) 
E7 (notation error in line 2)

Exemplar 2

\[
\begin{align*}
\text{Exemplar 2} \\
\tan \theta - 3 &= 0 \\
\tan \theta + 1 &= 0 \\
\text{tan} \theta &= 3 \\
\text{tan} \theta &= -1 \\
\theta &= 71.565^\circ \\
\theta &= 251.565^\circ \\
\theta &= 3\pi/4 \\
\theta &= 7\pi/4
\end{align*}
\]

3 out of 3

award full marks
E3 (variable omitted in line 1) 
E5 (answer stated in degrees instead of radians in lines 3 and 4) 
E7 (notation error in line 3)
An earthquake in Vancouver had a magnitude of 6.3 on the Richter scale. An earthquake in Japan had a magnitude of 8.9 on the Richter scale.

How many times more intense was the Japan earthquake than the Vancouver earthquake?

You may use the formula below:

\[ M = \log \left( \frac{A}{A_0} \right) \]

where \( M \) is the magnitude of the earthquake on the Richter scale
\( A \) is the intensity of the earthquake
\( A_0 \) is the intensity of a standard earthquake

Express your answer as a whole number.

**Solution**

**Method 1**

Vancouver: substitute \( M = 6.3 \)

\[ 6.3 = \log \left( \frac{A}{A_0} \right) \]

\[ 10^{6.3} = \frac{A}{A_0} \]

\[ A = 10^{6.3} A_0 \]

Japan: substitute \( M = 8.9 \)

\[ 8.9 = \log \left( \frac{A}{A_0} \right) \]

\[ 10^{8.9} = \frac{A}{A_0} \]

\[ A = 10^{8.9} A_0 \]

To compare the two earthquakes divide their intensities.

\[ \frac{\text{the intensity of Japan}}{\text{the intensity of Vancouver}} = \frac{10^{8.9} A_0}{10^{6.3} A_0} \]

\[ = 398.107 \]

\[ = 398 \]

2 marks
Question 3

Method 2

\[ \frac{I_J}{I_V} = \frac{10^{8.9}}{10^{6.3}} = 398 \]

1 mark for comparison
½ mark for exponential form
½ mark for exponential form

2 marks
Exemplar

\[ 8.9 - 6.3 = \log\left(\frac{A}{A_0}\right) \]

\[ 2.6 = \log\left(\frac{A}{A_0}\right) \]

\[ 10^{2.6} = \frac{A}{A_0} \]

\[ 39.8 \times 10^7 \times 0.6 = \frac{A}{A_0} \]

The earthquake in Japan was 39.8 times more intense than the earthquake in Vancouver.

1½ out of 2

award full marks

– ½ mark for procedural error
This page was intentionally left blank.
Find and simplify the last term in the expansion of \((2y - 3x)^7\).

**Solution**

\[ t_8 = C_7 (2y)^0 (-3x)^7 \]
\[ = -2187x^7 \]

½ mark for \(C_7\)
½ mark for \((2y)^0\)
1 mark for \((-3x)^7\) which is equal to \(-2187x^7\)

2 marks

---

**Note(s):**
- no deduction if \(C_7\) and \((2y)^0\) are not shown
Exemplar 1

\[ \gamma C_7 (-3x)^6 (2)^7 \]

\[ = \frac{7!}{7!} (1)(128) \]

\[ = 128 \]

\[ \frac{1}{2} \text{ out of 2} \]
+ \( \frac{1}{2} \) mark for \( \gamma C_7 \)

Exemplar 2

\[ \text{Let } \sum_{k=1}^{n} \binom{a^{n-k} b^k}{c_{6+1}} \]

\[ t_{6+1} = \sum_{k=7}^{6} (2x)^7 (2x)^6 \]

\[ 7(2x)(+729x^6) \]

\[ 10206x^6y \]

\[ 1 \text{ out of 2} \]

award full marks
– 1 mark for concept error (incorrect term)
Given $\log_a 9 = 1.129$ and $\log_a 4 = 0.712$, find the value of $\log_a 12$.

**Solution**

**Method 1**

\[
\log_a 9 = 1.129 \\
\log_a 3^2 = 1.129 \\
2 \log_a 3 = 1.129 \\
\log_a 3 = 0.5645
\]

\[
\log_a 12 = \log_a (4 \cdot 3) \\
= \log_a 4 + \log_a 3 \\
= 0.712 + 0.5645 \\
= 1.2765 \\
= 1.277
\]

1 mark for power rule

1 mark for writing 12 as a product

1 mark for product rule

3 marks

**Method 2**

\[
\log_a 12 = \log_a (\sqrt{9} \cdot 4) \\
= \frac{1}{2} \log_a 9 + \log_a 4 \\
= \frac{1}{2} (1.129) + 0.712 \\
= 1.2765 \\
= 1.277
\]

1 mark for writing 12 as a product

1 mark for power rule

1 mark for product rule

3 marks

**Method 3**

\[
\log_a 9 = 1.129 \\
a^{1.129} = 9 \\
\frac{1}{a} = 9^{1.129} \\
a = 9^{0.129} \\
a = 7
\]

1 mark for exponential form

1 mark for solving for $a$

\[
\log_a 12 = \frac{\log 12}{\log 7} \\
= 1.276\,989 \\
= 1.277
\]

1 mark for value of $\log_a 12$

3 marks
Exemplar 1

\[
\begin{align*}
\alpha^{1.129} &= 9 \\
\alpha &= 6.98976 \\
\alpha^{0.712} &= 4 \\
\alpha &= 7.0079
\end{align*}
\]

\[
(\log_\alpha 4)(\log_\alpha 4)(\log_\alpha 4) = \log_\alpha 12
\]

\[
(0.712)(3) = 2.136
\]

\[
\Box \log_\alpha 12 = 2.13
\]

2 out of 3

Method 3
+ 1 mark for exponential form
+ 1 mark for solving for \(a\)

Exemplar 2

\[
\sqrt{9 \times 4} = 12
\]

\[
\sqrt{\log_\alpha 9 + \log_\alpha 4} = \log_\alpha 12
\]

\[
\sqrt{1.129 + 0.712} = 1.7745
\]

2 out of 3

Method 2
+ 1 mark for writing 12 as a product
+ 1 mark for product rule
How many different ways can 4 girls and 4 boys be arranged in a row if the girls and the boys must alternate?

**Solution**

**Method 1**

\[ 8 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 1152 \text{ ways} \]

1 mark for beginning with either gender
1 mark for alternating genders

2 marks

**Method 2**

Case 1: \[ 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 576 \]

\[
\begin{array}{cccccc}
B & G & B & G & B & G \\
\end{array}
\]

1 mark for arrangement of alternating genders

½ mark for two cases

Case 2: \[ 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 576 \]

\[
\begin{array}{cccccc}
G & B & G & B & G & B \\
\end{array}
\]

½ mark for addition of cases

2 marks
Exemplar

\[
4 \times 3 \times 2 \times 1 \times 1 = 576
\]

\[
576 \text{ options!}
\]

1 out of 2

Method 2
+ 1 mark for arrangement of alternating genders
Question 7  

Solve the following equation over the interval \([0, 2\pi]\).

\[2 \cos 2\theta - 1 = 0\]

**Solution**

**Method 1**

\[2 \cos 2\theta - 1 = 0\]
\[2 \left(2 \cos^2 \theta - 1\right) = 0\]
\[4 \cos^2 \theta - 3 = 0\]
\[\cos^2 \theta = \frac{3}{4}\]
\[\cos \theta = \pm \frac{\sqrt{3}}{2}\]
\[\theta = \frac{\pi}{6}\]
\[\theta = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\]

1 mark for identity
1 mark for solving for \(\cos \theta\)
2 marks for solutions (½ mark for each consistent solution)

**Method 2**

\[y = 2 \cos 2\theta - 1\]

Find all zeros over the interval \([0, 2\pi]\).
\[\theta = 0.524, 2.618, 3.665, 5.760\]

2 marks for solutions
Solution

Method 3

\[ 2 \cos 2\theta - 1 = 0 \]

\[ \cos 2\theta = \frac{1}{2} \]

\[ \text{½ mark for solving for } \cos 2\theta \]

let \( x = 2\theta \)

\[ \therefore \cos x = \frac{1}{2} \]

\[ x_r = \frac{\pi}{3} \]

\[ \text{½ mark for reference angle} \]

\[ x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3} \]

\[ \text{1 mark for correct angles} \]

\[ \therefore 2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3} \]

\[ \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \]

\[ \text{1 mark for coterminal angles} \]

\[ \text{1 mark for all solutions over the interval } [0, 2\pi]. \]

4 marks

Note(s):

- in Method 1, deduct a maximum of 1 mark if missing \( \cos \theta = -\frac{\sqrt{3}}{2} \)
Exemplar 1

\[ 2\left(2\cos^2\theta - 1\right) = 0 \]

\[ 4\cos^2\theta - 1 = 0 \]

\[ 4\cos^2\theta = 2 \]

\[ \cos^2\theta = \frac{1}{2} \]

\[ \cos\theta = \pm \frac{1}{\sqrt{2}} \]

\[ \cos\theta = \frac{\pi}{4}, \frac{7\pi}{4} \]

\[ 2\frac{1}{2} \text{ out of 4} \]

award full marks

− 1 mark for missing \( \cos \theta = \frac{1}{\sqrt{2}} \)

− \( \frac{1}{2} \) mark for arithmetic error in line 2

E7 (notation error in line 8)
Exemplar 2

\[ y \cos^3 \theta - 2 - 1 \]
\[ y \cos \theta = 3 \]
\[ \sqrt{3} \cos \theta = \sqrt{3} \]
\[ \frac{x \cos \theta}{2} = \frac{\sqrt{3}}{2} \]
\[ \cos \theta = \frac{\sqrt{3}}{2} \]

\[ \theta = \frac{\pi}{6}, \frac{11\pi}{6} \]

3 out of 4

award full marks

− 1 mark for missing \( \cos \theta = -\frac{\sqrt{3}}{2} \)

E2 (changing an equation to an expression in lines 1 and 2)
Alex incorrectly explains to Rashid that the graph of \( y = 2 f(x) + 5 \) means you first move the graph of \( y = f(x) \) up 5 units and then multiply the y-values by 2.

Explain to Rashid the correct way to transform the graph.

**Solution**

Alex explains the transformations correctly, but not in the correct order. First multiply the y-values by 2, then move the graph up 5 units.

1 mark for explanation
Exemplar 1

First you have to do the compression/stretch, which would be multiply by 2.
Then you can do the translation of 5 units up.

1 out of 1
+ 1 mark for explanation

Exemplar 2

You need to do stretches before shifts

½ out of 1
award full marks
- ½ mark for lack of clarity in explanation
Sketch the angle of 5 radians in standard position.

**Solution**

1 mark for angle drawn in Quadrant IV
Exemplar 1

1 out of 1
award full marks
E1 (final answer not stated)

Exemplar 2

1 out of 1
award full marks
E1 (final answer not stated)
Question 10

Given the graphs of $f(x)$ and $(f - g)(x)$, sketch the graph of $g(x)$.

**Solution**

1 mark for subtraction of $f(x) - (f - g)(x)$
1 mark for shape representing the operation given

2 marks
Exemplar 1

1½ out of 2
award full marks
− ½ mark for arithmetic error

Exemplar 2

0 out of 2
Question 11

A particular math class has a large number of students. From this class, you are to create a committee of 4 students that has at least 1 girl.

Without actually solving the problem, explain the strategy you would use to find the total number of ways to select this committee.

**Solution**

**Method 1**

Find all ways to form the committee and subtract the number of ways with no girls on the committee.

1 mark for identifying cases
1 mark for operation on these cases

2 marks

**Method 2**

Find all scenarios with girls on the committee: 1 girl on the committee, 2 girls on the committee, 3 girls on the committee, and a committee with all girls. Then add all scenarios together to get the total number of ways to create a committee with at least 1 girl.

1 mark for identifying cases
1 mark for operation on these cases

2 marks

**Note(s):**

- in Method 2, deduct ½ mark if one of the cases is missing
Exemplar 1

There are 4 cases: 3 boys and 1 girl, 2 boys and 2 girls and 1 boy and 3 girls. I would find the number of options for each case and add them together.

1½ out of 2
+ 1½ marks (see note on previous page)

Exemplar 2

To know all the possibilities, first I would use 1 girl and 3 boys on the committee. The next time, I would put 2 girls and 2 boys on the committee to find different ways to choose the committee. Then, I would put 3 girls and 1 boy on the committee, followed by 4 girls and 0 boys on the committee. After multiplying each option (ex. 2 girls, 2 boys) I would add them up to find the solution.

Diagram

\[
\begin{align*}
\text{G: girls} & \quad \frac{1}{G} \cdot \frac{3}{B} \cdot \frac{2}{B} \cdot \frac{1}{B} = \\
\text{B: boys} & \quad \frac{2}{G} \cdot \frac{1}{G} \cdot \frac{2}{B} \cdot \frac{1}{B} = \\
& \quad \frac{2}{G} \cdot \frac{2}{G} \cdot \frac{1}{B} = \\
& \quad \frac{3}{G} \cdot \frac{3}{G} \cdot \frac{2}{G} = 4 \\
\end{align*}
\]

1 out of 2
award full marks
– 1 mark for concept error (permutations/combinations)
a) Prove the identity below for all permissible values of $\theta$.

\[
\frac{1 + 2\cos^2\theta}{\cos^2\theta} = \tan^2\theta + 3
\]

b) Determine all the non-permissible values for $\theta$.

**Solution**

**Method 1**

<table>
<thead>
<tr>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{\cos^2\theta} + \frac{2\cos^2\theta}{\cos^2\theta}$</td>
<td>$\tan^2\theta + 3$</td>
</tr>
<tr>
<td>$\sec^2\theta + 2$</td>
<td>$\tan^2\theta + 1 + 2$</td>
</tr>
<tr>
<td>$\tan^2\theta + 3$</td>
<td>$\cdot$ LHS = RHS</td>
</tr>
</tbody>
</table>

1 mark for appropriate algebraic strategy
1 mark for appropriate identity substitutions

2 marks

b) $\cos^2\theta = 0$
\[\cos \theta = 0\]
\[\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots\]

$\therefore$ the non-permissible values of $\theta$
are $\frac{\pi}{2} + k\pi, \ k \in I$ or
$90^\circ + 180k, \ k \in I$.  

½ mark for $\cos^2\theta = 0$

½ mark for any non-permissible value of $\theta$

1 mark for all non-permissible values of $\theta$

2 marks
Question 12

Solution

Method 2

a)\[
\begin{array}{c|c}
\text{LHS} & \text{RHS} \\
\frac{1 + 2 \cos^2 \theta}{\cos^2 \theta} & \tan^2 \theta + 3 \\
& \sec^2 \theta - 1 + 3 \\
& \sec^2 \theta + 2 \\
& \frac{1}{\cos^2 \theta} + 2 \\
& \frac{1 + 2 \cos^2 \theta}{\cos^2 \theta} \\
\end{array}
\]

1 mark for appropriate identity substitutions
1 mark for appropriate algebraic strategy

\[ \therefore \text{LHS} = \text{RHS} \]

b) \[ \cos^2 \theta = 0 \]
\[ \cos \theta = 0 \]
\[ \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \]
\[ \therefore \text{the non-permissible values of } \theta \]
\[ \text{are } \frac{\pi}{2} + k\pi, \ k \in \mathbb{I} \]
\[ \text{or} \]
\[ 90^\circ + 180k, \ k \in \mathbb{I}. \]

\[
\frac{\pi}{2} + k\pi \\
\frac{3\pi}{2} + k\pi \\
\]

\[ 90^\circ + 180k, \ k \in \mathbb{I}. \]

\[ \frac{\pi}{2} \]

\[ \frac{3\pi}{2} \]

\[ 90^\circ + 180k, \ k \in \mathbb{I}. \]

\[ \frac{\pi}{2} \]

\[ \frac{3\pi}{2} \]

\[ 90^\circ + 180k, \ k \in \mathbb{I}. \]

\[ \frac{\pi}{2} \]

\[ \frac{3\pi}{2} \]

\[ 90^\circ + 180k, \ k \in \mathbb{I}. \]
Exemplar 1

a)  

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
</table>
| \[
\frac{1 + 2(1 - \sin^2 \theta)}{\cos^2 \theta}
\] | \[
\frac{\sin^2 \theta}{\cos^2 \theta} + 3
\] |
| \[
\frac{1 + 2 \sin \theta}{1 - \sin \theta}
\] | \[
\frac{\sin \theta}{1 - 2 \sin \theta} + 3
\] |
| \[
\frac{3}{1}
\] | \[
3
\] |
| \[
= \frac{3}{1}
\] | \[
= 3
\] |

1 out of 2  
+ 1 mark for appropriate identity substitutions

b)  
\[
\cos \theta \neq 0
\]  
\[
\frac{\pi}{3} \neq 0
\]  
\[
\frac{3\pi}{2} \neq 0
\]

1 out of 2  
+ ½ mark for \( \cos^2 \theta = 0 \)  
+ ½ mark for non-permissible values  
E7 (notation error)
Exemplar 2

a) 

<table>
<thead>
<tr>
<th>Left-Hand Side</th>
<th>Right-Hand Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1 + 2 \cos^2 \theta}{\cos^2 \theta}$</td>
<td>$\frac{3 \sin^2 \theta}{\cos^2 \theta} + 3$</td>
</tr>
<tr>
<td>$\frac{1 \cdot 2 (1 - \sin^2 \theta)}{\cos^2 \theta}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1 + 2 - 2 \sin^2 \theta}{\cos^2 \theta}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{3 - 2 \sin^2 \theta}{\cos^2 \theta}$</td>
<td></td>
</tr>
</tbody>
</table>

1 out of 2
+ 1 mark for appropriate identity substitutions

b) 

$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi k, 4\pi k, k \in \mathbb{I}$

1 out of 2
+ $\frac{1}{2}$ mark for $\cos^2 \theta = 0$
+ $\frac{1}{2}$ mark for non-permissible value
Given the graph of $f(x)$ below, sketch the graph of $g(x) = f(x - 2) + 3$.

**Solution**

1 mark for horizontal shift
1 mark for vertical shift

2 marks
Exemplar

1 out of 2

- award full marks
- 1 mark for concept error \( x \leftrightarrow y \)
- E9 (missing arrowhead)
Given that $\sin \alpha = \frac{5}{13}$, where $\alpha$ is in Quadrant II, and $\cos \beta = \frac{2}{5}$, where $\beta$ is in Quadrant IV, find the exact value of:

a) $\cos (\alpha + \beta)$

b) $\sin 2\alpha$

**Solution**

a) 

\[
\text{Given: } \sin \alpha = \frac{5}{13}, \quad \text{and } \cos \beta = \frac{2}{5}
\]

\[
x^2 + y^2 = r^2
\]

\[
x^2 + 25 = 169
\]

\[
x^2 = 144
\]

\[
x = \pm 12, \quad x = -12
\]

\[
\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
\]

\[
= \left( -\frac{12}{13} \right) \left( \frac{2}{5} \right) - \left( \frac{5}{13} \right) \left( -\frac{\sqrt{21}}{5} \right)
\]

\[
= -\frac{24}{65} + \frac{5\sqrt{21}}{65}
\]

\[
= \frac{5\sqrt{21} - 24}{65}
\]

½ mark for value of $x$

½ mark for value of $y$

½ mark for $\cos \alpha$

½ mark for $\sin \beta$

1 mark for substitution into correct identity

3 marks

b) $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

\[
= 2 \left( \frac{5}{13} \right) \left( -\frac{12}{13} \right)
\]

\[
= -\frac{120}{169}
\]

1 mark for substitution into correct identity

1 mark

**Note(s):**

- accept any of the following values for $x$: $x = \pm 12, x = -12$ or $x = 12$
- accept any of the following values for $y$: $y = \pm \sqrt{21}, y = -\sqrt{21}$ or $y = \sqrt{21}$
Exemplar 1

a) 

\[
\cos\left(\frac{12}{13}\right) \cos \left(\frac{4}{3}\right) - \sin\left(\frac{5}{13}\right) \sin \left(\frac{\sqrt{21}}{5}\right)
\]

\[
\left( -\frac{2\sqrt{3}}{65} \right) - \frac{-5\sqrt{21}}{65}
\]

\[
-\frac{2\sqrt{3} + 5\sqrt{21}}{65}
\]

3 out of 3
award full marks
E7 (notation error in line 1)

b) 

\[
2 \left( \frac{5}{13} \right) \left( -\frac{14}{13} \right) - 120 \left( \frac{169}{169} \right)
\]

1 out of 1
Exemplar 2

a)

\[ \cos(a+b) = \cos a \cos b - \sin a \sin b \]

\[ -12 \cdot 2 = -5 \cdot (-\sqrt{21}) \]

\[ -24 \quad + \quad 5\sqrt{21} \]

\[ -10\sqrt{21} \]

\[ x^2 + y^2 = 25 \]

\[ x = 12 \]

\[ x^2 + y^2 = 13^2 \]

\[ 25 + x^2 = 160 \]

\[ -24^2 \]

\[ \sqrt{144} \]

\[ \frac{4}{12} \]

1½ out of 3

+ ½ mark for \( x \)

+ ½ mark for \( y \)

+ 1 mark for substitution into the correct identity

− ½ mark for arithmetic error

E7 (notation error in lines 6 and 7)

b)

\[ -2 \sin a \cos a \]

\[ 2 \cdot 5 \cdot (-12) \]

\[ -120 \]

1 out of 1

+ 1 mark [marked consistently with a)]
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Question 15

If \( f(x) = x^3 \) and \( g(x) = 2x - 3 \), what is the value of \( \left(\frac{f}{g}\right)(-1) \)?

Solution

\[
\begin{align*}
f(-1) &= (-1)^3 \\
&= -1 \\
\quad \text{½ mark for substituting into } f(x) \text{ and } g(x)
\end{align*}
\]

\[
\begin{align*}
g(-1) &= 2(-1) - 3 \\
&= -5
\end{align*}
\]

\[
\left(\frac{f}{g}\right)(-1) = \frac{-1}{-5} \\
&= \frac{1}{5} \\
\quad \text{½ mark for evaluating } \left(\frac{f}{g}\right)(-1)
\]

1 mark
Exemplar 1

\( f(-1) = (-1)^3 = -1 \)

\( g(-1) = 2(-1) - 3 \)

\[ = -2 - 3 \]

\[ = -5 \]

\[ \left( \frac{f}{g} \right)(-1) = -5 \]

\( \frac{1}{2} \) out of 1

+ \( \frac{1}{2} \) mark for substitution into \( f(x) \) and \( g(x) \)

Exemplar 2

\( \left( \frac{f(x)}{g(x)} \right)(-2) = \frac{x^3}{2x - 3} \)

\[ = \frac{-7}{-2 - 3} \]

\[ = \frac{1}{5} \]

1 out of 1

award full marks

E7 (notation error)
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## Answer Key for Multiple-Choice Questions

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Learning Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>B</td>
<td>R3</td>
</tr>
<tr>
<td>17</td>
<td>C</td>
<td>R9</td>
</tr>
<tr>
<td>18</td>
<td>A</td>
<td>T3</td>
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<tr>
<td>19</td>
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<tr>
<td>24</td>
<td>B</td>
<td>R13</td>
</tr>
<tr>
<td>25</td>
<td>A</td>
<td>R10</td>
</tr>
</tbody>
</table>
Question 16

If the point \((4, -3)\) lies on the graph of \(f(x)\), which point must lie on the graph of \(2f(2x)\)?

a) \((8, -6)\)

b) \((2, -6)\)

c) \(\left(8, -\frac{3}{2}\right)\)

d) \(\left(2, -\frac{3}{2}\right)\)

Question 17

The graph of \(y = \log_2(2x + 6)\) intersects the graph of \(y = 4\) at:

a) \(x = -1\)

b) \(x = 1\)

c) \(x = 5\)

d) \(x = 14\)

Question 18

Given the point \(A(-3, 5)\) on the terminal arm of an angle \(\theta\), identify the value of \(\cot \theta\).

a) \(-\frac{3}{5}\)

b) \(-\frac{5}{3}\)

c) \(-\frac{4}{5}\)

d) \(-\frac{5}{4}\)
Question 19

The graph of \( y = \left(\frac{1}{2}\right)^x \) compared to the graph of \( x = \left(\frac{1}{2}\right)^y \) is a:

a) reflection in the \( x \)-axis

b) reflection in the \( y \)-axis

c) reflection in the line \( y = x \)

d) reciprocal function

Question 20

Given the above graph of a polynomial function, which one of the following statements can be true?

a) The function has a degree of 4 with a positive leading coefficient.

b) The function has a degree of 4 with a negative leading coefficient.

c) The function has a degree of 3 with a positive leading coefficient.

d) The function has a degree of 3 with a negative leading coefficient.
Question 21

Given that \((x + 3)\) is a factor of polynomial \(P(x)\), which of the following is true?

a) \(P(-3) = 0\)  
b) \(P(0) = -3\)  
c) \(P(0) = 3\)  
d) \(P(3) = 0\)

Question 22

Which of the following is a reasonable estimate for the value of \(\log 350\)?

a) 2  
b) 2.5  
c) 2.8  
d) 3

Question 23

The graph of the function \(f(x)\) shown below is best described by the equation:

a) \(f(x) = 2^{x+3}\)  
b) \(f(x) = 2^x + 3\)  
c) \(f(x) = 2^{x-3}\)  
d) \(f(x) = 2^x - 3\)
Question 24

Given the graph of $y = f(x)$, what is the domain of $\sqrt{f(x)}$?

- $x \in \mathbb{R}$
- $-2 \leq x \leq 2$
- $x \leq -2$ or $x \geq 2$
- $0 \leq x \leq 4$

Question 25

Solve:

$e^{\ln(5-x)} = 7$

- $-2$
- $-\ln 2$
- $\ln 7 - \ln 5$
- $\frac{7}{5}$
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Question 26

One of the factors of \( P(x) = x^3 - kx^2 - 7x + 10 \) is \((x - 2)\).

Find the value of \( k \).

**Solution**

**Method 1**

\[ x = 2 \quad \frac{1}{2} \text{ mark for } x = 2 \]

\[ 0 = (2)^3 - k(2)^2 - 7(2) + 10 \]
\[ 0 = 8 - 4k - 14 + 10 \]
\[ 0 = 4 - 4k \]

\[ 4k = 4 \]
\[ k = 1 \quad \frac{1}{2} \text{ mark for solving for } k \]

**Method 2**

\[ x = 2 \quad \frac{1}{2} \text{ mark for } x = 2 \]

\[
\begin{array}{c|cccc}
2 & 1 & -k & -7 & 10 \\
 & 2 & -2k + 4 & -4k - 6 & \\
1 & 1 & -k + 2 & -2k - 3 & -4k + 4 \\
\end{array}
\]

\[ -4k + 4 = 0 \]
\[ 4k = 4 \]
\[ k = 1 \quad \frac{1}{2} \text{ mark for equating the remainder to zero} \]

\[ 2 \text{ marks} \]
Exemplar 1

\[ \begin{array}{cccc}
1 & -1 & \frac{1}{2} & +a \\
\hline
1 & 1 & -5 & -10 \\
\hline
1 & 1 & -5 & 0 \\
\end{array} \]

Used guess and check to find \( k \)

\[ -\frac{1}{2} + a = 1 \]

\[ k = 1 \]

2 out of 2

Exemplar 2

\( P(2): \alpha^2 - K(2)^2 - 7(2) + 10 \)

\[ = 3 - K(9) - 14 + 10 \]

\[ = \frac{3}{9} - 4K - 4 \]

\[ = -4K + 4 \]

\[ -4 = \frac{4K}{4} \]

\[ -1 = K \]

2 out of 2

award full marks
E7 (notation error in line 2 [left side should equal 0])
E7 (transcription error in line 5)
a) Sketch the graph of the function $y = \sqrt{-x} + 1$.

**Solution**

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{graph.png}
\end{figure}

1 mark for general shape
1 mark for horizontal reflection
1 mark for vertical shift

3 marks

b) Determine the value of $x$ when $y = 3$.

**Method 1**

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{method1.png}
\end{figure}

\[
(-4, 3)
\]

\[
y = 3
\]

\[
\therefore x = -4
\]

1 mark for consistent value of $x$

**Method 2**

\[
y = \sqrt{-x} + 1
\]

\[
3 = \sqrt{-x} + 1
\]

\[
2 = \sqrt{-x}
\]

\[
4 = -x
\]

\[
x = -4
\]

1 mark for consistent value of $x$
2 out of 3
+ 1 mark for general shape
+ 1 mark for vertical shift

b)
\[ x = 4 \]

1 out of 1
+ 1 mark for consistent value of \( x \)
This page was intentionally left blank.
Solve the following equation:

\[ nP_2 = nC_3 \]

**Solution**

**Method 1**

\[
\begin{align*}
\frac{n!}{(n-2)!} &= \frac{n!}{(n-3)!3!} \\
\frac{n!}{(n-3)!3!} &= n!(n-2)! \\
6 &= \frac{(n-2)!}{(n-3)!} \\
6 &= \frac{(n-2)(n-3)!}{(n-3)!} \\
6 &= n-2 \\
8 &= n
\end{align*}
\]

½ mark for substituting for \( nP_2 \)
½ mark for substituting for \( nC_3 \)
1 mark for simplification
1 mark for expansion of \((n-2)!\)

**Method 2**

\[
\begin{align*}
nP_2 &= nC_3; \text{ we know } n \geq 3 \\
\frac{n!}{(n-2)!} &= \frac{n!}{(n-3)!3!} \\
\frac{n(n-1)(n-2)!}{(n-2)!} &= \frac{n(n-1)(n-2)(n-3)!}{(n-3)!6} \\
6 &= n(n-1)(n-2) \\
6 &= n-2 \\
8 &= n
\end{align*}
\]

½ mark for substituting for \( nP_2 \)
½ mark for substituting for \( nC_3 \)
½ mark for expansion of \( nP_2 \)
½ mark for expansion of \( nC_3 \)
½ mark for substituting for \( nP_2 \)
½ mark for substituting for \( nC_3 \)
1 mark for simplification

3 marks
Solution

Method 3

\[ n P_2 = n C_3 \]

\[ \frac{n!}{(n-2)!} = \frac{n!}{(n-3)!3!} \]

\[ \frac{n(n-1)(n-2)!}{(n-2)!} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!3!} \]

\[ 6n(n-1) = n(n-1)(n-2) \]

\[ 6n(n-1) - n(n-1)(n-2) = 0 \]

\[ n(n-1)(6-(n-2)) = 0 \]

\[ n(n-1)(8-n) = 0 \]

\[ n = 8 \]

Note(s):
- deduct \( \frac{1}{2} \) mark for not rejecting non-permissible values
\[
\frac{n!}{(n-3)!} = \frac{n!}{3!(n-3)!} \\
\frac{n(n-1)(n-2)!}{(n-3)!} = \frac{n(n-1)(n-2)(n-3)}{3!(n-3)!} \\
n(n-1) = \frac{n(n-1)(n-2)}{3} \\
n^2 - n = \frac{n(n^2 - 3n + 2)}{3} \\
(2)n^2 - n = \frac{n^3 - 3n^2 + 2n}{3} \\
2n^2 - 3n = \frac{n^3 - 3n^2 + 2n + 3n}{3} \\
0 : n^3 - 6n^2 + 5n \\
0 : n(n^2 - 6n + 5) \\
0 : n(n-1)(n-5) \\
n = 0, 1, 5
\]

**2 out of 3**

award full marks
- ½ mark for arithmetic error in line 3
- ½ mark for non-permissible values of \( n \)
- E7 (notation error in line 2)
- E4 (missing brackets but still implied in line 5)
This page was intentionally left blank.
Given \( f(x) = x^2 - 1 \) and \( g(x) = \sqrt{x+1} \), sketch the graph of \( y = f(g(x)) \) and state its domain.

**Solution**

**Method 1**

\[
f(x) = x^2 - 1 \quad \quad \quad g(x) = \sqrt{x+1}
\]

Domain: \( x \in \mathbb{R} \) \quad Domain: \( [-1, \infty) \)

\[
f(g(x)) = (\sqrt{x+1})^2 - 1 = x + 1 - 1 = x
\]

1 mark for determining the function \( f(g(x)) \)

[Graph of \( f(g(x)) \)]

1 mark for graph of composite function

Domain of \( f(g(x)) : [-1, \infty) \) \quad \text{or} \quad \{x \mid x \geq -1, x \in \mathbb{R}\}

1 mark for restricted domain

3 marks
Question 29

Solution

Method 2

<table>
<thead>
<tr>
<th>$x$</th>
<th>$g(x)$</th>
<th>$f(g(x))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

1 mark for table of values

1 mark for graph of composite function

Domain of $f(g(x))$: $[-1, \infty)$ or $\{x \mid x \geq -1, x \in \mathbb{R}\}$

1 mark for restricted domain

3 marks
Exemplar

\[ f(x) = x^2 \]
\[ g(x) = \sqrt{x+1} \]
\[ f(g(x)) = (\sqrt{x+1})^2 - 1 \]
\[ = (x+1) - 1 \]

Domain: \( \{ x \in \mathbb{R} \} \)

2 out of 3

award full marks
– 1 mark (concept error for not restricting domain)
Write the equation of the horizontal asymptote for the function \( f(x) = \frac{x - 3}{x - 2} \).

**Solution**

\[ y = 1 \]

1 mark for equation of horizontal asymptote

1 mark
$HA = 1 \div 1$

$HA = 1$

**1 out of 1**

award full marks
E7 (notation error)
The $x$-intercept of $f(x)$ is 4 and the $x$-intercept of $g(x)$ is 4.
Benjamin concludes that the $x$-intercept of $f(x) + g(x)$ is 8.
Do you agree with Benjamin? Justify your answer.

**Solution**

No, I do not agree with Benjamin.

Benjamin added the $x$-values instead of adding the $y$-values.

If the $x$-intercept of $f(x)$ is 4, then $y = 0$.

If the $x$-intercept of $g(x)$ is 4, then $y = 0$.

\[
\therefore \text{the } x\text{-intercept of } f(x) + g(x) \text{ is 4.}
\]
Exemplar 1

No the $x$-intercept would still be 4.

Exemplar 2

$\text{NO}$
Solve the following equation:

\[ 2 \log_4 x - \log_4 (x + 3) = 1 \]

**Solution**

\[ 2 \log_4 x - \log_4 (x + 3) = 1 \]

\[ \log_4 \left( \frac{x^2}{x + 3} \right) = 1 \]

1 mark for power rule

1 mark for quotient rule

\[ 4^1 = \left( \frac{x^2}{x + 3} \right) \]

1 mark for exponential form

\[ 4(x + 3) = x^2 \]

\[ x^2 - 4x - 12 = 0 \]

\[ (x - 6)(x + 2) = 0 \]

\[ x = 6 \]

\[ \frac{1}{2} \text{ mark for solving for } x \]

\[ \frac{1}{2} \text{ mark for rejecting extraneous root} \]

4 marks
Exemplar 1

\[ \frac{1}{\log_4 2} = \frac{\log_4 (x+3)}{x} = 1 \]

\[ \frac{2x}{x+3} = 4 \]

\[ 2x = 4(x+3) \]

\[ 2x = 4x + 12 \]

\[ -12 = 2x \]

\[ x = -6 \]

2½ out of 4
+ 1 mark for quotient rule
+ 1 mark for exponential form
+ ½ mark for solving for \( x \)

Exemplar 2

\[ 2 \log_4 x - \log_4 (x+3) = 1 \]

\[ \log_4 x^2 - \log_4 (x+3) = 1 \]

\[ \log_4 \frac{x^2}{x+3} = 1 \]

\[ \frac{x^2}{x+3} = 4 \]

\[ x^2 = 4x + 12 \]

\[ x^2 - 4x - 12 = 0 \]

\[ (x-6)(x+2) = 0 \]

\[ x = 6 \]

3 out of 4
+ 1 mark for power rule
+ 1 mark for quotient rule
+ 1 mark for exponential form
The following graph represents tidal levels in the Bay of Fundy over a 25-hour period.

a) What is the average height of the water?

b) What is the period of the graph above?

   Explain what the period represents in this situation.

**Solution**

a) 6 metres  

b) Period = \( \frac{25}{2} \)  

   = 12.5 hours

The period represents the time to complete one cycle of tidal levels in the Bay of Fundy.
Exemplar

a)

\[ \text{amplitude} = \frac{y_{\text{max}} - y_{\text{min}}}{2} = \frac{10 - 2}{2} = 4 \]

\[ \therefore \text{average height of the water is 4 metres} \]

0 out of 1

b)

the period is 12.5

it represents the amount of time it takes the water to go from its highest point to its lowest and then back

2 out of 2

award full marks

E5 (missing unit of measure in line 1)
Given the graph of \( y = f(x) \), sketch the graph of \( y = \sqrt{f(x)} \).

**Solution**

1 mark for restricting the domain

½ mark for graph above \( y = f(x) \) over the range \([0, 1]\)

½ mark for graph below \( y = f(x) \) over the range \([1, 2]\)

2 marks
Exemplar 1

1½ out of 2
+ 1 mark for restricting the domain
+ ½ mark for the graph below $y = f'(x)$ over the range $[1, 2]$

Exemplar 2

½ out of 2
+ ½ mark for graph below $y = f'(x)$ over the range $[1, 2]$
E9 (incorrect endpoint)
When $P(x)$ is divided by $x - 3$, it has a quotient of $2x^2 + x - 6$ and a remainder of 4. Determine $P(x)$.

**Solution**

$$P(x) = (x - 3)(2x^2 + x - 6) + 4$$

or

$$P(x) = 2x^3 - 5x^2 - 9x + 22$$
0 out of 1

award full marks

– 1 mark for concept error of not including the remainder
Identify the domain and range of the following function:

\[ f(x) = \frac{3}{x^2 + 1} \]

**Solution**

Domain: \( \{ x \in \mathbb{R} \} \) or \((-\infty, \infty)\)

1 mark for domain

Range: \( \{ y \in \mathbb{R} \mid 0 < y \leq 3 \} \) or \((0, 3]\)

1 mark for range

2 marks
Exemplar

\[ x^2 \text{ is always greater than zero} \]

D: \([0, \infty)\)

R: \((0, 3]\)

1 out of 2

+ 1 mark for range
Evaluate:

\[ \csc \left( \frac{11\pi}{6} \right) + \sin^2 \left( -\frac{3\pi}{4} \right) + \cos \left( \frac{23\pi}{3} \right) \]

Solution

\[ (-2) + \left( -\frac{\sqrt{2}}{2} \right)^2 + \frac{1}{2} \]

\[ = -2 + \frac{1}{2} + \frac{1}{2} \]

\[ = -1 \]

1 mark for \( \csc \left( \frac{11\pi}{6} \right) \) (½ mark for quadrant, ½ mark for value)

1 mark for \( \sin^2 \left( -\frac{3\pi}{4} \right) \) (½ mark for quadrant, ½ mark for value)

1 mark for \( \cos \left( \frac{23\pi}{3} \right) \) (½ mark for quadrant, ½ mark for value)

3 marks
\[
\csc\left(-\frac{\sqrt{3}}{2}\right) + \sin^2\left(-\frac{1}{\sqrt{2}}\right) + \cos\left(\frac{1}{2}\right)
\]

\[
= \left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)
\]

\[
= -\frac{\sqrt{3}}{\sqrt{2}}
\]

**1 out of 3**

+ 1 mark for \(\sin^2\left(-\frac{3\pi}{4}\right)\)

+ 1 mark for \(\cos\left(\frac{23\pi}{3}\right)\)

- ½ mark for arithmetic error in line 2

- ½ mark for arithmetic error in line 3

- E7 (notation error in line 1)
Evaluate the coefficient of the term containing $x^3$ in the expansion of $(1 + x)^7$.

Justify your answer.

**Solution**

**Method 1**

\[
\begin{array}{cccccc}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1 \\
1 & 6 & 15 & 20 & 15 & 6 & 1 \\
1 & 7 & 21 & 35 & 35 & 21 & 7 & 1
\end{array}
\]

The coefficient of $x^3$ is 35.

1 mark for justification

1 mark for identifying coefficient

2 marks

**Method 2**

\[
\begin{align*}
t_4 &= \binom{7}{3} (1)^4 (x)^3 \\
\binom{7}{3} &= \frac{7!}{3!4!} \\
&= \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3!} \\
&= 35
\end{align*}
\]

The coefficient of $x^3$ is 35.

1 mark for justification

1 mark for evaluating coefficient

2 marks
Exemplar 1

\[ \begin{align*}
\text{-}44 & \quad \frac{7 \times 6 \times 5}{3 \times 2} \\
& = 35
\end{align*} \]

2 out of 2

Exemplar 2

\[ \begin{align*}
1 & \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1 \\
1 & 6 & 15 & 20 & 15 & 6 & 1 \\
1 & 7 & 21 & 35 & 35 & 21 & 7 & 1
\end{align*} \]

[35(x)^3]^4

35x^3

2 out of 2
Question 39

Which of the following equations could be solved without the use of logarithms?
Without actually solving the problem, explain your choice.

\[ 4^x = 10^{3x+1} \quad \text{or} \quad \left( \frac{1}{3} \right)^{2x+1} = 27^{4x-1} \]

**Solution**

\[ \left( \frac{1}{3} \right)^{2x+1} = 27^{4x-7} \] can be solved without the use of logarithms because \( \frac{1}{3} \) and 27 can both be changed to a base of 3.

1 mark for explanation
the second one because you know 3 is a factor of 27 and since its below 1 you know the exponent is negative. All you have to do is make the exponent $= -3$. 

1 out of 1
Sketch the graph of \( y = x^3 + x^2 - 5x + 3 \) given that one of the \( x \)-intercepts is 1.

Identify the \( x \)-intercepts and \( y \)-intercept.

**Solution**

\( x = 1 \)

\[
\begin{array}{c|cccc}
  & 1 & 1 & -5 & 3 \\
\hline
 1 & 1 & 2 & -3 \\
\end{array}
\]

1 mark for synthetic division

\[
y = (x-1)(x^2 + 2x - 3)
\]

\[
y = (x-1)(x+3)(x-1)
\]

\[
y = (x+3)(x-1)^2
\]

1 mark for identifying the factors

2 marks for graph (½ mark for \( x \)-intercepts, ½ mark for multiplicity, ½ mark for \( y \)-intercept, ½ mark for end behaviour)

4 marks
Exemplar

\[
\begin{array}{c|ccc}
\times-1 & 1 & 1 & -5 \\ \hline & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & 0 \\
\end{array}
\]

\((x-1)(x^2-2x-3)=0\)

\((x-1)(x-3)(x+1)=0\)

\(x=1\) \(x=3\) \(x=-1\)

3 out of 4

+ 1 mark for synthetic division
+ 1 mark for identifying factors
+ \(\frac{1}{2}\) mark for \(x\)-intercepts
+ \(\frac{1}{2}\) mark for end behaviour
+ \(\frac{1}{2}\) mark for multiplicity
- \(\frac{1}{2}\) mark for incorrect shape (not a function)
E7 (transcription error in line 3)
If \( f(x) = \frac{1}{x - 2} \) and \( g(x) = x - 2 \), what is the domain of \( f(x) \cdot g(x) \)?

**Solution**

\[
f(x) = \frac{1}{x - 2} \quad \text{and} \quad g(x) = x - 2
\]

Domain: \( x \in \mathbb{R}, x \neq 2 \) \quad Domain: \( x \in \mathbb{R} \)

Domain of \( f(x) \cdot g(x) \): \( \{ x \in \mathbb{R} | x \neq 2 \} \) \quad 1 mark for domain of \( f(x) \cdot g(x) \)
Exemplar

\[ f(x) = \frac{1}{x-2} \]
\[ g(x) = x - 2 \text{  D: } x \neq 2 \]

1 out of 1
Question 42

Given \( f(x) = (x + 1)^2 \) for \( x \leq -1 \), write the equation of \( y = f^{-1}(x) \).

Solution

Method 1

\[
\begin{align*}
y &= (x + 1)^2 \\
x &= (y + 1)^2 \\
y &= \pm \sqrt{x - 1}
\end{align*}
\]

1 mark for inverse

\( \frac{1}{2} \) mark for solving for \( y \)

Since the domain of \( f(x) \) is \( x \leq -1 \),
the range of the inverse is \( y \leq -1 \).

\[
\therefore y = -\sqrt{x - 1}
\]

\( f^{-1}(x) = -\sqrt{x - 1} \)

\( \frac{1}{2} \) mark for rejecting \( y = \sqrt{x} \)

2 marks

Method 2

\[
y = (x + 1)^2 \\
(\text{where } x \leq -1)
\]

1 mark for reflection over the line \( y = x \)

\[
y = -\sqrt{x - 1}
\]

1 mark for correct equation

2 marks
\[ y = (x+1)^2 \]
\[ x = (y+1)^2 \]
\[ \sqrt{x} = y+1 \]
\[ y = \sqrt{x} - 1 \]

\[ f(x) = \sqrt{x} - 1 \]

Check: \[ f(f^{-1}(x)) = (\sqrt{x} - 1 + 1)^2 = \sqrt{x} = x \checkmark \]

**1 out of 2**

*Method 1*

+ 1 mark for inverse
Sketch a graph of at least one period of the function \( y = 5 \sin \left[ \frac{\pi}{2} (x + 1) \right] \).

Clearly indicate the \( x \)-intercepts.

**Solution**

\[ b = \pi \]

\[ \therefore \text{period} = \frac{2\pi}{\pi} = 2 \]

1 mark for amplitude
1 mark for horizontal shift
1 mark for period
1 mark for clearly indicating at least two \( x \)-intercepts consistent with graph

4 marks
2½ out of 4

+ 1 mark for amplitude
+ 1 mark for period
+ 1 mark for clearly indicating at least two $x$-intercepts consistent with graph
− ½ mark for incorrect shape of graph (solid lines at top and bottom)
Sketch the graph of the following function:

\[ f(x) = \frac{x - 2}{(2x - 3)(x - 2)} \]

**Solution**

\[ f(x) = \frac{x - 2}{(2x - 3)(x - 2)} = \frac{1}{2x - 3} \text{ with a point of discontinuity at } x = 2 \]

point of discontinuity: \( f(2) = 1 \)

\[ \therefore \text{ there is a point of discontinuity at } (2, 1) \]

\[ y\text{-intercept: } f(0) = \frac{0 - 2}{(2(0) - 3)(0 - 2)} = \frac{-2}{-6} = \frac{1}{3} \]

1 mark for horizontal asymptote at \( y = 0 \)
1 mark for vertical asymptote at \( x = \frac{3}{2} \)
½ mark for graph left of vertical asymptote
½ mark for graph right of vertical asymptote
1 mark for point of discontinuity at \((2, 1)\); (½ mark for \(x = 2\), ½ mark for \(y = 1\))

4 marks
Exemplar 1

\[
f(x) = \frac{x - 2}{(2x - 3)(x - 2)}
\]

\[
x \neq \frac{3}{2}, \quad x = 2
\]

\[
y \text{- intercept: } -\frac{1}{3}
\]

\[
x \text{- intercept: } 2
\]

\[
\text{Vertical asymptote: } x = \frac{3}{2}
\]

\[
\text{Horizontal asymptote: } y = 0
\]

3 out of 4

+ 1 mark for horizontal asymptote
+ 1 mark for vertical asymptote
+ \( \frac{1}{2} \) mark for graph left of vertical asymptote
+ \( \frac{1}{2} \) mark for point of discontinuity when \( x = 2 \)
Exemplar 2

\[
\frac{x - 2}{(2x - 3)(x - 2)} \quad \text{hole}
\]

\[
X \neq 2; \quad \frac{3}{2}
\]

\[
Y = \frac{1}{(2x - 3)}
\]

\[
X = 2
\]

+ \frac{1}{2} \text{ mark for graph right of vertical asymptote}
+ \frac{1}{2} \text{ mark for point of discontinuity when } x = 2
MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.
- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the Answer/Scoring Sheet.

<table>
<thead>
<tr>
<th>Error</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>answer given as a complex fraction</td>
</tr>
<tr>
<td></td>
<td>final answer not stated</td>
</tr>
<tr>
<td>E2</td>
<td>changing an equation to an expression</td>
</tr>
<tr>
<td></td>
<td>equating the two sides when proving an identity</td>
</tr>
<tr>
<td>E3</td>
<td>variable omitted in an equation or identity</td>
</tr>
<tr>
<td></td>
<td>variables introduced without being defined</td>
</tr>
<tr>
<td>E4</td>
<td>“sin x^2” written instead of “sin^2 x”</td>
</tr>
<tr>
<td></td>
<td>missing brackets but still implied</td>
</tr>
<tr>
<td>E5</td>
<td>missing units of measure</td>
</tr>
<tr>
<td></td>
<td>incorrect units of measure</td>
</tr>
<tr>
<td></td>
<td>answer stated in degrees instead of radians or vice versa</td>
</tr>
<tr>
<td>E6</td>
<td>rounding error</td>
</tr>
<tr>
<td></td>
<td>rounding too early</td>
</tr>
<tr>
<td>E7</td>
<td>notation error</td>
</tr>
<tr>
<td></td>
<td>transcription error</td>
</tr>
<tr>
<td>E8</td>
<td>answer included outside the given domain</td>
</tr>
<tr>
<td></td>
<td>bracket error made when stating domain or range</td>
</tr>
<tr>
<td></td>
<td>domain or range written in incorrect order</td>
</tr>
<tr>
<td>E9</td>
<td>incorrect or missing endpoints or arrowheads</td>
</tr>
<tr>
<td></td>
<td>scale values on axes not indicated</td>
</tr>
<tr>
<td></td>
<td>coordinate points labelled incorrectly</td>
</tr>
<tr>
<td>E10</td>
<td>asymptotes drawn as solid lines</td>
</tr>
<tr>
<td></td>
<td>asymptotes missing but still implied</td>
</tr>
<tr>
<td></td>
<td>graph crosses or curls away from asymptotes</td>
</tr>
</tbody>
</table>
During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the Department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the Department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Department along with the test materials.
Irregular Test Booklet Report

Test: ________________________________________________________________

Date marked: __________________________________________________________

Booklet No.: __________________________________________________________

Problem(s) noted: ______________________________________________________

______________________________________________________________________

______________________________________________________________________

______________________________________________________________________

Question(s) affected: ____________________________________________________

______________________________________________________________________

______________________________________________________________________

Action taken or rationale for assigning marks: ____________________________

______________________________________________________________________

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Follow-up: ______________________________________________________________
________________________________________________________________________
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Decision: __________________________________________________________________
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________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Marker’s Signature: ______________________________________________________
Principal’s Signature: ____________________________________________________

For Department Use Only—After Marking Complete
Consultant: ______________________________________________________________
Date: ___________________________________________________________________
## Table of Questions by Unit and Learning Outcome

### Unit A: Transformations of Functions

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<td>R4</td>
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<tr>
<td>10</td>
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<tr>
<td>13</td>
<td>R2</td>
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<td>16</td>
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<td>R1</td>
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</tr>
<tr>
<td>31</td>
<td>R1</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>R1</td>
<td>1</td>
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<td>R5, R6</td>
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<tr>
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<tr>
<td>9</td>
<td>T1</td>
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<tr>
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<td>T3</td>
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<td>43</td>
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### Unit C: Binomial Theorem

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<td>38</td>
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<tr>
<td>40</td>
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<td>T5</td>
<td>3</td>
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<tr>
<td>14</td>
<td>T6</td>
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<td>12 a)</td>
<td>T6</td>
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<td>12 b)</td>
<td>T5, T6</td>
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### Unit F: Exponents and Logarithms

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<tbody>
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<td>R8</td>
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### Unit G: Radicals and Rationals

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<td>27 a)</td>
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<td>27 b)</td>
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<tr>
<td>44</td>
<td>R14</td>
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