Grade 12
Pre-Calculus Mathematics
Acheivement Test

Marking Guide

June 2025

Manitoba

Grade 12 pre-calculus mathematics achievement test.
Marking Guide. June 2025

This document is available in print and electronic formats.
ISBN: 978-0-7711-6710-2 (print)
ISBN: 978-0-7711-6723-2 (pdf)

Manitoba Education and Early Childhood Learning
Winnipeg, Manitoba, Canada

This document will be available on the Manitoba Education and Early Childhood Learning website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Contents

General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Early Childhood Learning in the envelope provided. For more information see the administration manual.

Marking the Test Questions

The test is composed of constructed response questions and selected response questions. Constructed response questions are worth 1 to 5 marks each, and selected response questions are worth 1 mark each. An answer key for the selected response questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. The appendix provides examples of such irregularities as well as procedures to follow to report irregularities.

If a *Answer/Scoring Sheet* is marked with "0" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Early Childhood Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Samuel Tougas
Assessment Consultant
Grade 12 Pre-Calculus Mathematics
Telephone: 204-390-6650
Email: samuel.tougas@gov.mb.ca

Communication Errors

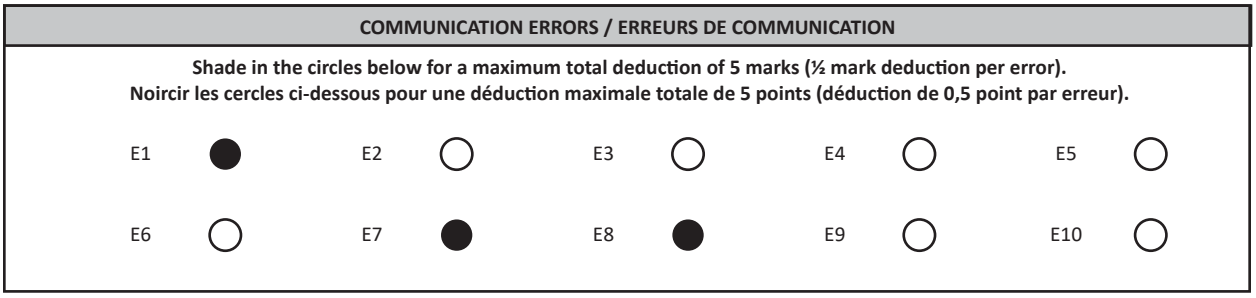
The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

When a given response includes multiple types of communication errors, deductions are indicated in the order in which the errors occur in the response. No communication errors are recorded for work that has not been awarded marks. The total deduction may not exceed the marks awarded.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.



Example: Marks assigned to the student

Marks Awarded	Booklet 1 25	Selected Response 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**Maximum deduction of 5 marks**	**90**

Scoring Guidelines for Booklet 1 Questions

Question 1 P1

Manitoba postal codes consist of three letters and three digits. Determine the total number of possible postal codes in Manitoba if the following characteristics must be met:

- The postal code must begin with the letter R.
- Only 18 letters of the alphabet can be used.
- The letters and digits must alternate.
- The letters and digits can be repeated.

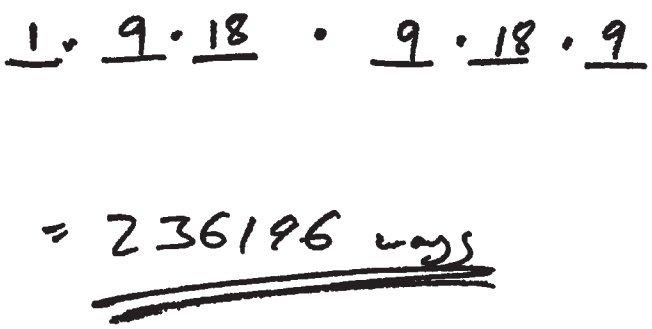
Solution

$\underline{1}_{R} \cdot \underline{10} \cdot \underline{18} \cdot \underline{10} \cdot \underline{18} \cdot \underline{10} = 324\,000$

1 mark

Note:
The restriction of the first letter does not need to be shown.

Exemplar 1



½ out of 1

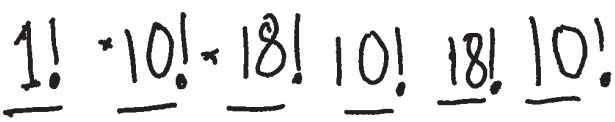
award full marks
– ½ mark for procedural error (9 digits instead of 10)

Exemplar 2

½ out of 1

award full marks
– ½ mark for procedural error (non-repeating digits and letters)

Exemplar 3



0 out of 1

award full marks
– 1 mark for concept error (applying factorials)

Question 2 R10

The percent of people who click on a link, in response to seeing an advertisement on YouTube, can be modeled by the formula:

$$R(t) = -(0.8)^{0.2t} + 0.6$$

where $R(t)$ is the percentage, in decimal form, of people who click on the link

t is the time in days.

Determine, algebraically, the number of days it will take for 45% of people who see the advertisement to click on the link.

Solution

$$0.45 = -(0.8)^{0.2t} + 0.6$$
$$-0.15 = -(0.8)^{0.2t}$$
$$0.15 = (0.8)^{0.2t}$$
$$\log(0.15) = \log(0.8)^{0.2t}$$ ½ mark for applying logarithms
$$\log(0.15) = 0.2t \bullet \log(0.8)$$ 1 mark for power law
$$\frac{\log(0.15)}{0.2 \bullet \log(0.8)} = t$$
$$t = 42.508\,958...$$ ½ mark for evaluating quotient of logarithms
$$t = 42.509 \text{ days}$$

2 marks

Exemplar 1

$45 = -(0.8)^{0.2t} + 0.6$

$44.4 = -(0.8)^{0.2t}$

$\log 44.4 = -\log(0.8)^{0.2t}$

$\frac{\log 44.4}{-\log(0.8)} = \frac{-0.2t \log(0.8)}{-\log(0.8)}$

$\frac{0.2t}{0.2} = \frac{16.999}{0.2}$

$t = 8.4$ days

½ out of 2

+ 1 mark for power law
+ ½ mark for evaluating quotient of logarithms
– ½ mark procedural error in line 1 (substituting percentage as a whole number)
– ½ arithmetic error in line 6

Exemplar 2

$0.45 = -(0.8)^{0.2t} + 0.6$

$\log 0.45 = -0.2t \log 0.8 + 0.6$

$t = 5.964$ days

1 out of 2

+ 1 mark for power law

Exemplar 3

$(0,8)^{0,2t} = 0,45 - 0,6$

$0.2t \log(0.8) = \log(-0,15)$

$t = \frac{\log(-0.15)}{0.2 \log(0.8)}$

1 out of 2

+ ½ mark for applying logarithms
+ 1 mark for power law
– ½ mark for procedural error line 1

Question 3 P4

Determine and simplify the middle term in the binomial expansion of $\left(\frac{5}{x}+4x^3\right)^6$.

Solution

$t_4 = {}_6\mathrm{C}_3\left(\frac{5}{x}\right)^3\left(4x^3\right)^3$ 2 marks (1 mark for ${}_6C_3$; ½ mark for each consistent factor)

$= 20\left(\frac{125}{x^3}\right)\left(64x^9\right)$

$= 160\,000x^6$ 1 mark for simplification (½ mark for coefficient; ½ mark for exponent)

3 marks

Exemplar 1

$t_{3+1} = {}_6C_3 \left(\frac{5}{x}\right)^{6-3} (4x^3)^3$ $\quad \frac{6!}{(6-3)!3!} = 20$

$t_4 = 20 \left(\frac{5}{x}\right)^3 (4x^3)$

$t_4 = \frac{20}{1} \left(\frac{125}{x^3}\right) \left(\frac{262144x^6}{1}\right)$

$= \boxed{655360000x^3}$

2½ out of 3

+ 1 mark for ${}_6C_3$
+ 1 mark for consistent factors
+ ½ mark for exponent

Exemplar 2

$t = 4$
$k = 3$
$n = 7$
$a = \frac{5}{x}$
$b = 4x^3$

$t_{3+1} = {}_nC_3 \left(\frac{5}{x}\right)^{n-3} (4x^3)^3$

$t_4 = {}_nC_3 \left(\frac{5}{x}\right)^{n-3} (64x^9)$

$t_4 = {}_7C_3 \left(\frac{5}{x}\right)^{7-3} (64x^9)$

$t_4 = 35 \left(\frac{5}{x}\right)^4 (64x^9)$

$t_4 = 35 \left(\frac{625}{x^4}\right) (64x^9)$

n = number of terms

n

$t_4 = \frac{1400000x^9}{x^4}$

$t_4 = \frac{1400000}{x^{-5}}$

2 out of 3

+ 1 mark for consistent factors
+ 1 mark for simplification

Question 4 T5

Solve $3\tan^2 x + 5\tan x - 6 = 0$, over the interval $[0, 2\pi]$.

Solution

$$\tan x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-6)}}{2(3)}$$

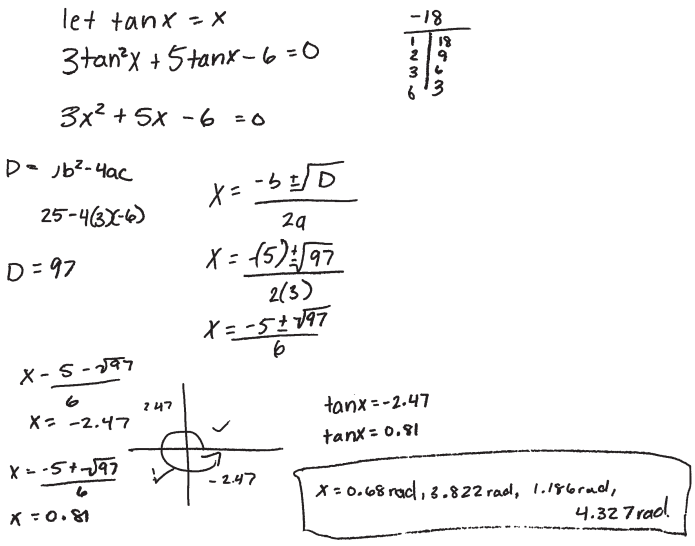
$$\tan x = \frac{-5 \pm \sqrt{97}}{6}$$ 1 mark for solving for tan *x*

$\tan x = 0.808\,142...$	$\tan x = -2.474\,809...$
$x_r = 0.679\,686...$	$x_r = 1.186\,784...$
$x = 0.680, 3.821$	$x = 1.955, 5.096$

2 marks for solving for *x* (½ mark for each value)

3 marks

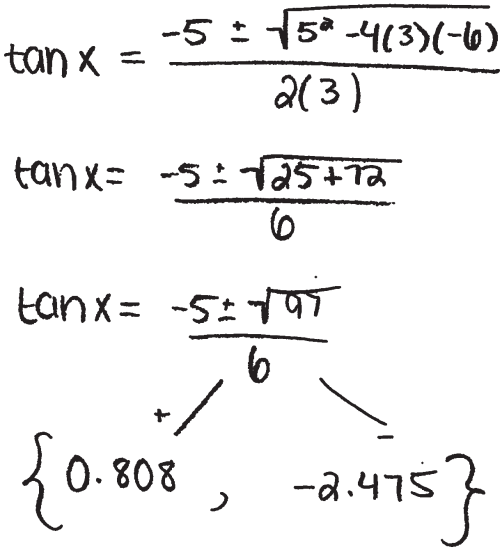
Exemplar 1



2 out of 3

+ 1 mark for solving for tan *x*
+ 1 mark for solving for *x*
E7 (notation error in line 1)
E6 (rounding too early)

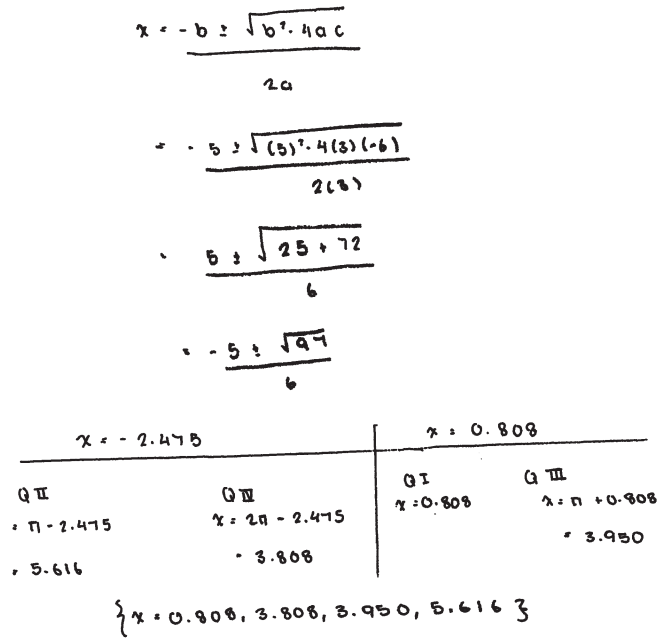
Exemplar 2



1 out of 3

+ 1 mark for solving for tan *x*

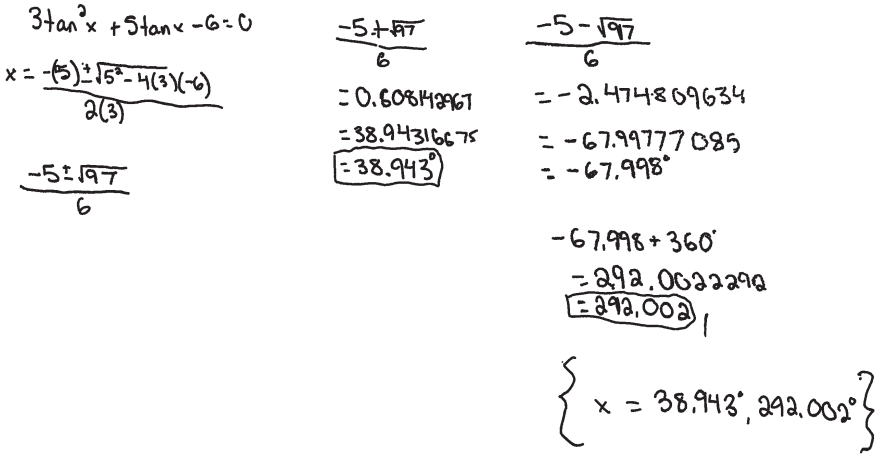
Exemplar 3



1 out of 3

+ 1 mark for solving for tan *x*
+ 1 mark for solving for *x* (consistent with their reference angle in right branch)
– 1 mark for concept error (using tan *x* as a reference angle)

Exemplar 4



2 out of 3

+ 1 mark for solving for tan *x*
+ 1 mark for solving for *x* (consistent with their answer)
E2 (changing an equation into an expression)
E5 (answer stated in degrees instead of radians)
E7 (notation errors)

Exemplar 5

$(3\tan x + 2)(\tan x - 3) = 0$

Q2,4 $\tan x = -\frac{2}{3}$ $\quad$ Q1 and 3 $\tan x = 3$

$X_R = 0.588$ $\quad$ $X_R = 1.249$

$x = \pi - 0.588$

$x = 2.554$

$x = 1.249$

$x = 2\pi - 0.588$

$x = 5.695$

$x = \pi + 1.249$

$x = 4.391$

2 out of 3

+ 2 marks for solving for *x*

Question 5 T1

Given $\theta = -80°$,

a) determine the measure of θ, in radians.

b) state all the coterminal angles of θ.

Solution

a) $\theta = -80°\left(\frac{\pi}{180°}\right)$

$\theta = -\frac{4\pi}{9}$

or

$\theta = -1.396$

1 mark

b) $\theta = -80° + 360°k,\ k \in \mathbb{Z}$

or

$\theta = -\frac{4\pi}{9} + 2k\pi,\ k \in \mathbb{Z}$

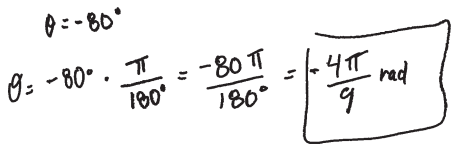
or

$\theta = -1.396 + 2k\pi,\ k \in \mathbb{Z}$

1 mark

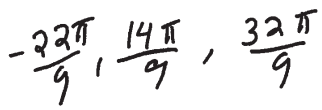
Exemplar 1

a)



1 out of 1

b)



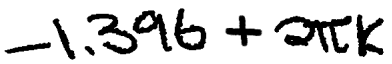
0 out of 1

Exemplar 2

a)

1 out of 1

b)

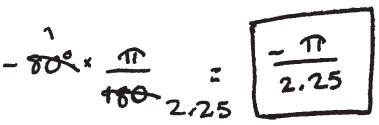


½ out of 1

award full marks
– ½ mark for procedural error

Exemplar 3

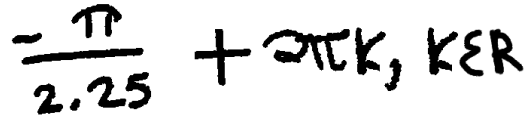
a)



1 out of 1

award full marks
E1 (final answer given as a complex fraction)

b)



½ out of 1

award full marks
– ½ mark for procedural error

Question 6 R12

State a possible value of n if the polynomial function $p(x)=(x+1)^n(x-4)^2$ has a range of $(-\infty,\infty)$.

Solution

$n = 1$

or

$n = 3$

1 mark

Note:
Accept any odd whole number for n.

Exemplar 1

NO even numbers

3, 9 or others

1 out of 1

Exemplar 2

$(x-4)^2 = x^2 - 8x + 16$

this gives $p(x)$ a range of $[0, +\infty)$

in order for it to have a range of $(-\infty, \infty)$
we need to have x to an odd degree.

$(x+1)^{n=1}(x^2 - 8x + 16)$

$x^3 - 8x^2 + 16x + x^2 - 8x + 16$

$x^3 - 7x^2 + 8x + 16$

this gives us a polynomial function where the range is $(-\infty, +\infty)$, therefore a possible value for $n = 1$.

1 out of 1

Question 7 R2, R3, R5

Describe the transformations used to obtain the graph of the function $y = -2f\left(x+8\right)-5$ from the graph of $y = f\left(x\right)$.

Solution

The graph of $y = f\left(x\right)$ is reflected over the *x*-axis, vertically stretched by a factor of 2, translated left 8 units and down 5 units.

1 mark for vertical reflection
1 mark for vertical stretch
1 mark for horizontal translation
1 mark for vertical translation

4 marks

Note:
Deduct a maximum of 1 mark for concept error of incorrect order of vertical transformations.

Exemplar 1

8 left

5 down

vertical ~~movement~~ ~~the~~ ⇕

vertical ~~the~~ change ⇕

double ⇕

2 out of 4

+ 1 mark for horizontal translation
+ 1 mark for vertical translation

Exemplar 2

The graph had a;

vertical stretch of -2,
horizontal shift 8 spaces to the left
vertical shift 5 spaces down

3 out of 4

+ 1 mark for vertical stretch
+ 1 mark for horizontal translation
+ 1 mark for vertical translation

Exemplar 3

$$Y = a(b(x \pm h)) + k$$

$$(x, y) \longmapsto (x+8, -2y-5)$$

horizontal translation 8 units positive/right
vertical reflection by factor of -2 over the x axis
and vertical translation of 5 units down/neg

2 out of 4

+ 1 mark for vertical reflection
+ 1 mark for vertical translation

Question 8 R14

State the domain and range of the function $f(x) = \frac{-2}{x^2}$.

Solution

Domain: $\{x | x \neq 0, x \in \mathbb{R}\}$ or $(-\infty, 0) \cup (0, \infty)$ 1 mark for domain

Range: $\{y | y < 0, y \in \mathbb{R}\}$ or $(-\infty, 0)$ 1 mark for range

2 marks

Exemplar 1

Domain: $\{x \in R \mid x \neq 0\}$

Range: $\{y \in R \mid y \neq -2\}$

1 out of 2

+ 1 mark for domain

Exemplar 2

Domain: $\{x \in R, x \neq -2\}$

Range: $\{y \in R, y \neq 0\}$

0 out of 2

Exemplar 3

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

1 out of 2

+ 1 mark for domain

Question 9 **P4**

Given the following row of Pascal's Triangle, state the values in the next row.

1 5 10 10 5 1

Solution

1 6 15 20 15 6 1

1 mark

Exemplar 1

0 out of 1

Exemplar 2

0 out of 1

Question 10 R6

Given $f(x) = 3 - 2x$, determine the equation of $f^{-1}(x)$.

Solution

Let $y = f(x)$

$y = 3 - 2x$

To determine the inverse of $f(x)$, switch x and y.

$x = 3 - 2y$ — 1 mark for switching x and y

$2y = 3 - x$

$y = \frac{3 - x}{2}$ — ½ mark for solving for y

$f^{-1}(x) = \frac{3 - x}{2}$ — ½ mark for writing equation in terms of $f^{-1}(x)$

2 marks

Exemplar 1

$f(x) = 3 - 2x$

$y = 3 - 2x$

$\frac{x}{-3} = \frac{3}{-3} - \frac{2y}{-3}$

$\frac{-3x}{-2} = \frac{-2y}{-2}$

$\frac{3}{2}x = y$

$f^{-1}(x) = \frac{3}{2}x$

1½ out of 2

+ 1 mark for switching *x* and *y*
+ ½ mark for writing the equation in terms of $f^{-1}(x)$

Exemplar 2

$f(x) = y$

$y = 3 - 2x$

$x = 3 - 2y$

$\frac{x+2}{-3} = y$

1 out of 2

+ 1 mark for switching *x* and *y*

Exemplar 3

$Y = 3 - 2x$

$x = \frac{-}{3} - 2y$

$\frac{x-3}{-2} = \frac{-2y}{-2}$

$\frac{x-3}{-2} = f^{-1}(x)$

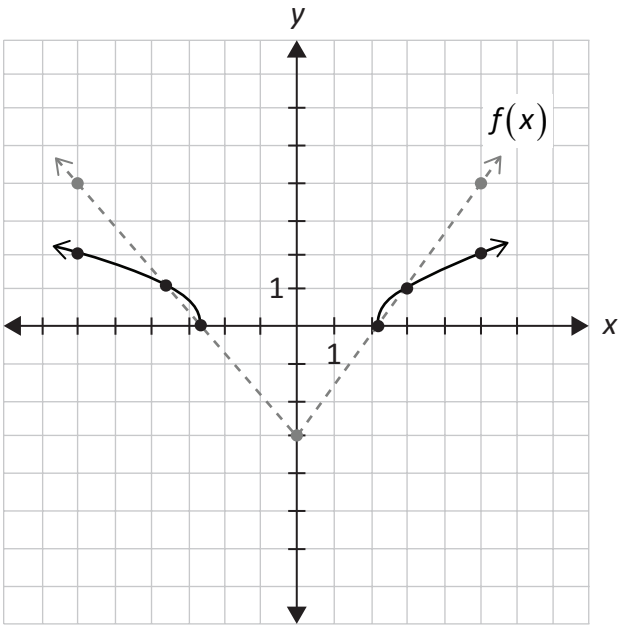
2 out of 2

award full marks
E7 (notation error in line 2)

Question 11 R13

Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$.

Solution



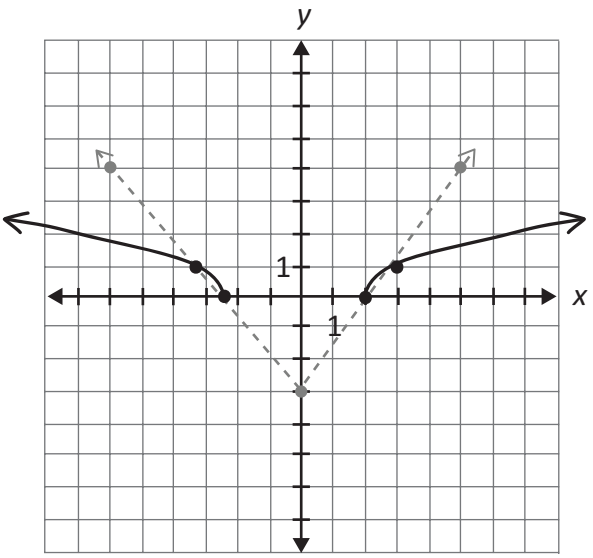
1 mark for restricting domain

½ mark for shape between invariant points, $\{0 \le y \le 1\}$

½ mark for shape above invariant points, $\{y \ge 1\}$

2 marks

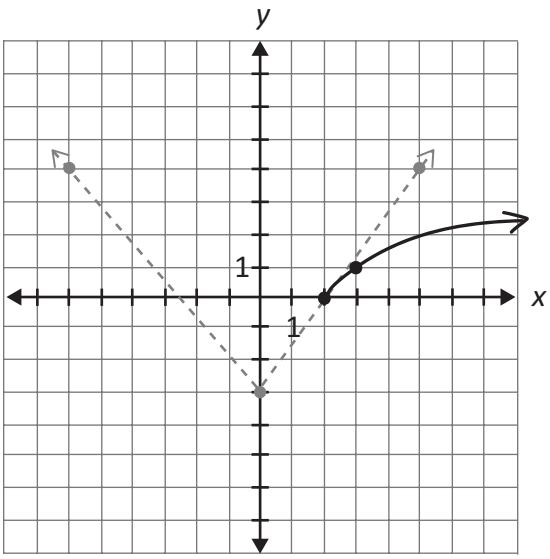
Exemplar 1



1½ out of 2

+ 1 mark for restricting domain
+ ½ mark for shape between invariant points

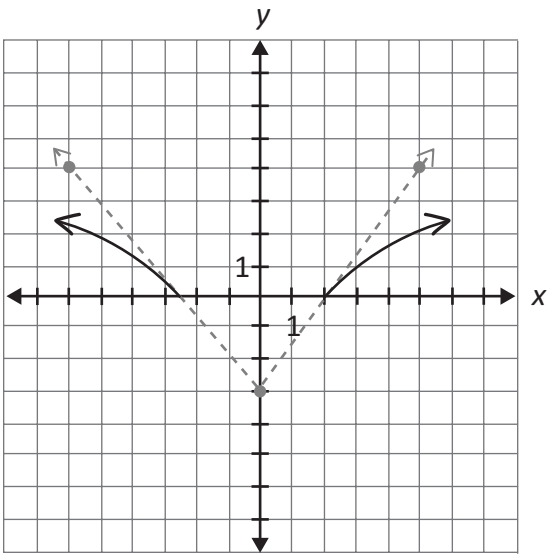
Exemplar 2



½ out of 2

+ ½ mark for shape above invariant points

Exemplar 3



1 out of 2

+ 1 mark for restricting domain

THIS PAGE WAS INTENTIONALLY LEFT BLANK.

Question 12 T6

Prove the identity for all permissible values of x.

$$\frac{\csc^2 x + \sec^2 x}{\tan x + \cot x} = \csc x \sec x$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\dfrac{\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}}{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}}$	$\csc x \sec x$
$\dfrac{\dfrac{\cancel{\cos^2 x + \sin^2 x}}{\sin^2 x \cos^2 x}}{\dfrac{\cancel{\cos^2 x + \sin^2 x}}{\cos x \sin x}}$	
$\dfrac{\cancel{\cos x}\ \cancel{\sin x}}{\sin^{\cancel{2}} x \cos^{\cancel{2}} x}$	
$\dfrac{1}{\sin x \cos x}$	
$\csc x \sec x$	

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Method 2

Left-Hand Side	Right-Hand Side
$\dfrac{\dfrac{1}{\sin^2 x}+\dfrac{1}{\cos^2 x}}{\dfrac{\tan x}{1}+\dfrac{1}{\tan x}}$	$\csc x \sec x$
$\dfrac{\dfrac{\cos^2 x+\sin^2 x}{\sin^2 x \cos^2 x}}{\dfrac{\tan^2 x+1}{\tan x}}$	
$\dfrac{\dfrac{1}{\sin^2 x \cos^2 x}}{\dfrac{\sec^2 x}{\tan x}}$	
$\dfrac{\tan x}{\sin^2 x \cancel{\cos^2 x}\ \cancel{\sec^2 x}}$	
$\dfrac{\dfrac{\cancel{\sin x}}{\cos x}}{\dfrac{\sin^{\cancel{2}} x}{1}}$	
$\dfrac{1}{\sin x \cos x}$	
$\csc x \sec x$	

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

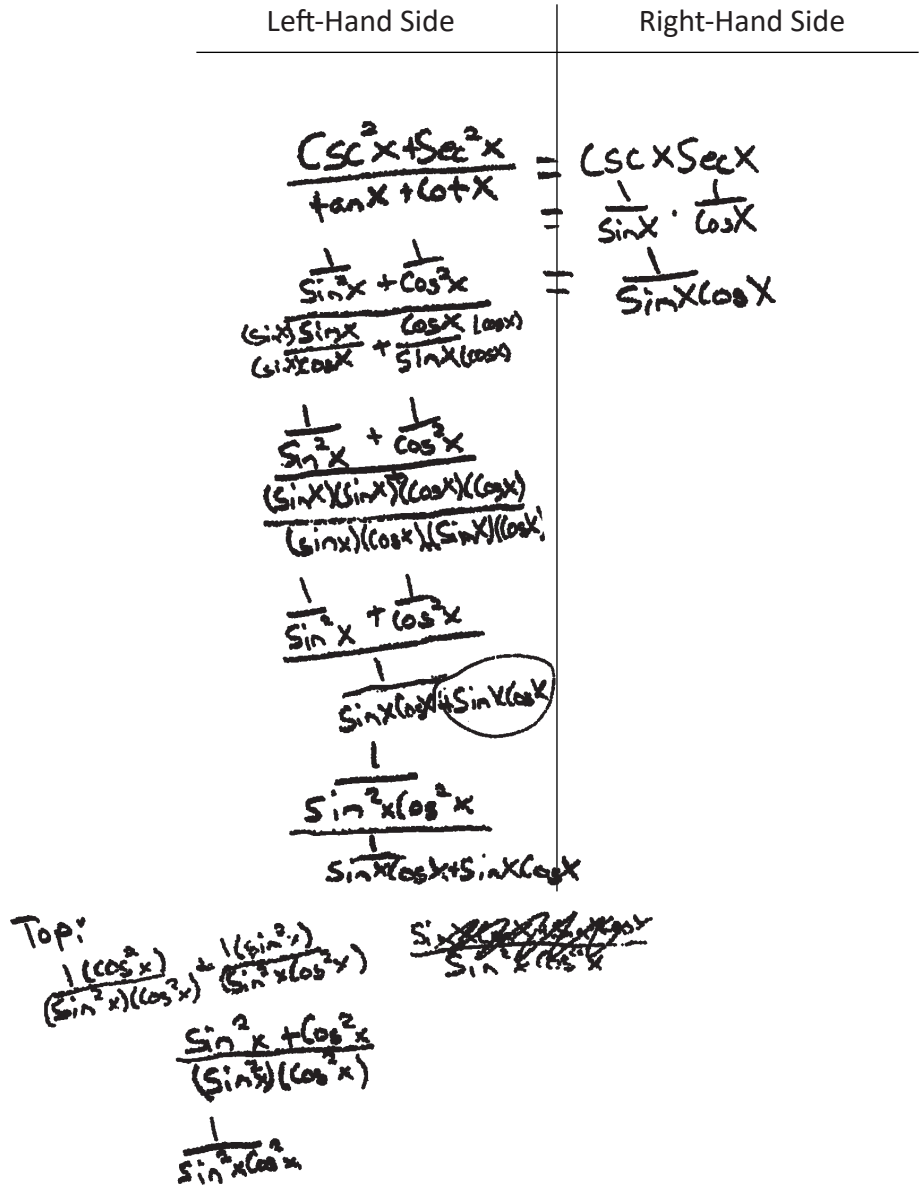
Exemplar 1

Left-Hand Side	Right-Hand Side
$\frac{\csc^2 x + \sec^2 x}{\tan x + \cot x}$	
$\frac{1 + \cot^2 x + \tan^2 x + 1}{\frac{\sin x}{\cos x} + \frac{1}{\frac{\sin x}{\cos x}}}$	
$\frac{(1 + \cot^2 \theta) + (\tan^2 \theta + 1)}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$	
$\frac{\frac{1^2}{\tan^2 x} + \frac{\sin^2 x}{\cos^2 x} + 2}{\frac{\sin^1 x}{\cos x} + \frac{\cos x}{\sin x}}$	
$\frac{\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\cos^2 x} + 2}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$	$= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} + 2$ $= \cot x + \tan \theta + 2$

1 out of 3

+ 1 mark for correct substitution of identities
E3 (variable being introduced without being defined in line 3)

Exemplar 2



1 out of 3

+ 1 mark for correct substitution of identities
E2 (equating the two sides when proving an identity)

Question 13 R9

Justify that the graph of $f(x) = e^{x+1} + 5$ does not have an *x*-intercept.

Solution

$0 = e^{x+1} + 5$

$-5 = e^{x+1}$

$e^{x+1} > 0$, therefore there is no solution.

or

The range of the graph is $\{y | y > 5, y \in \mathbb{R}\}$.

or

The horizontal asymptote is $y = 5$ and it is an increasing exponential function above the asymptote.

1 mark

Exemplar 1

Because if $f(x) = e^{x+1}$ graph has horizontal asymptote on the x-axis, which means your can't touch the x-axis.

½ out of 1

award full marks
– ½ mark for lack of clarity in the explanation

Exemplar 2

the graph starts at (0, 5) and trends upwards

0 out of 1

Exemplar 3

because it's an exponential graph and the asymtote on $y=5$, doesn't let the line pass through the x-axis

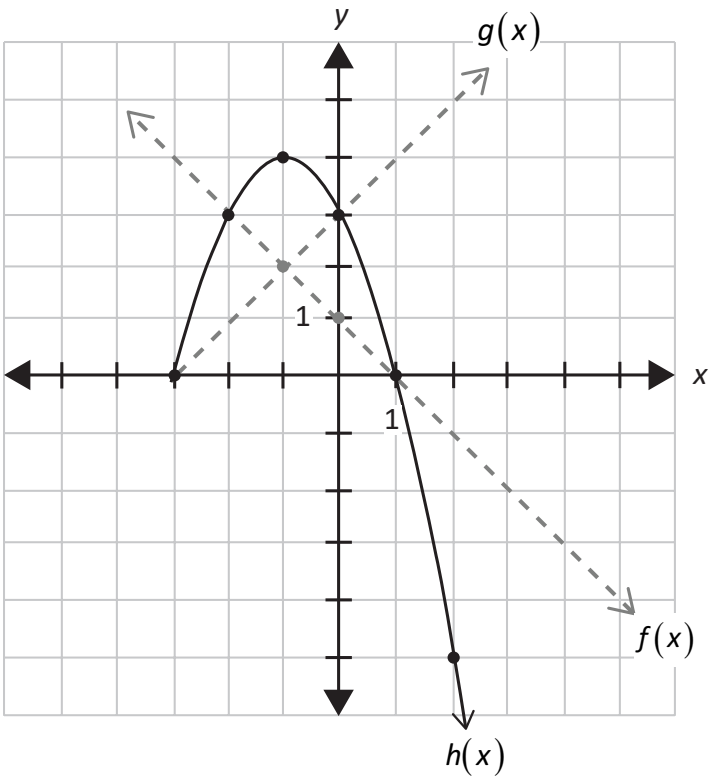
½ out of 1

award full marks
– ½ mark for lack of clarity in the explanation

Question 14 R1

Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $h(x) = f(x) \bullet g(x)$.

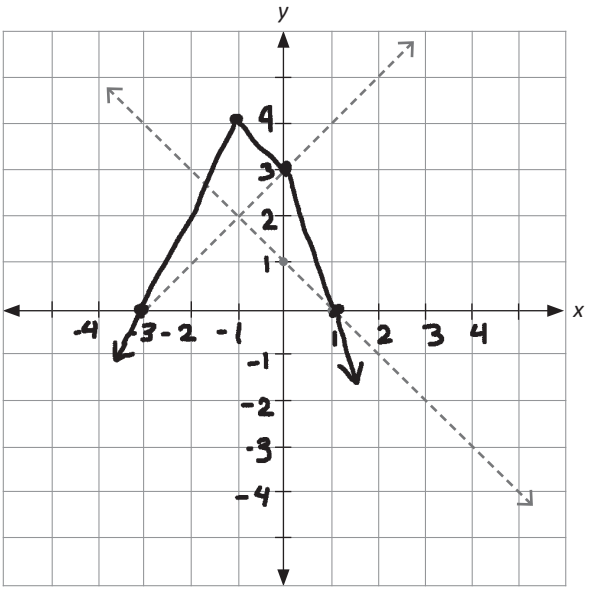
Solution



1 mark for operation of multiplication
1 mark for restricted domain

2 marks

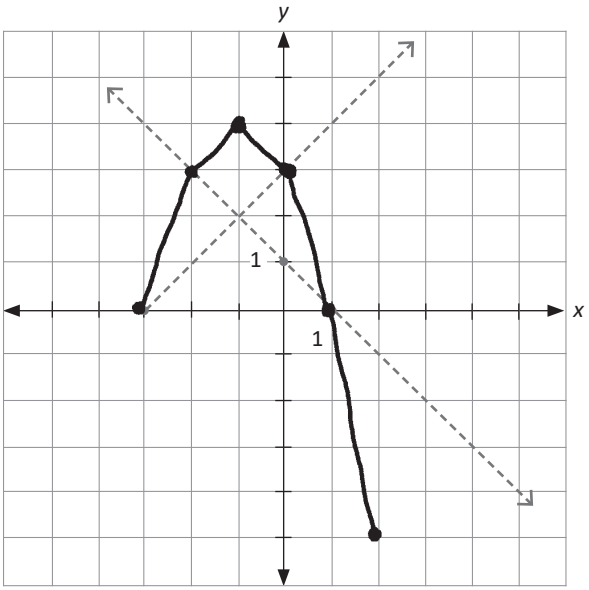
Exemplar 1



½ out of 2

+ 1 mark for operation of multiplication
– ½ mark for incorrect shape of the graph

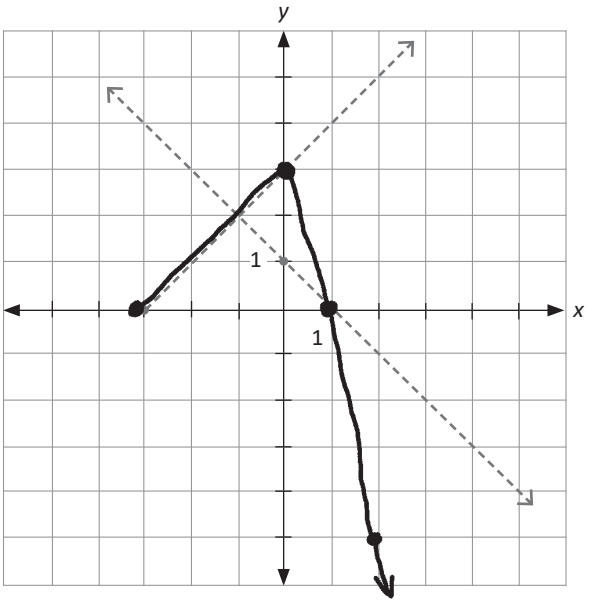
Exemplar 2



2 out of 2

award full marks
E9 (arrowhead omitted on RHS of graph)

Exemplar 3



1 out of 2

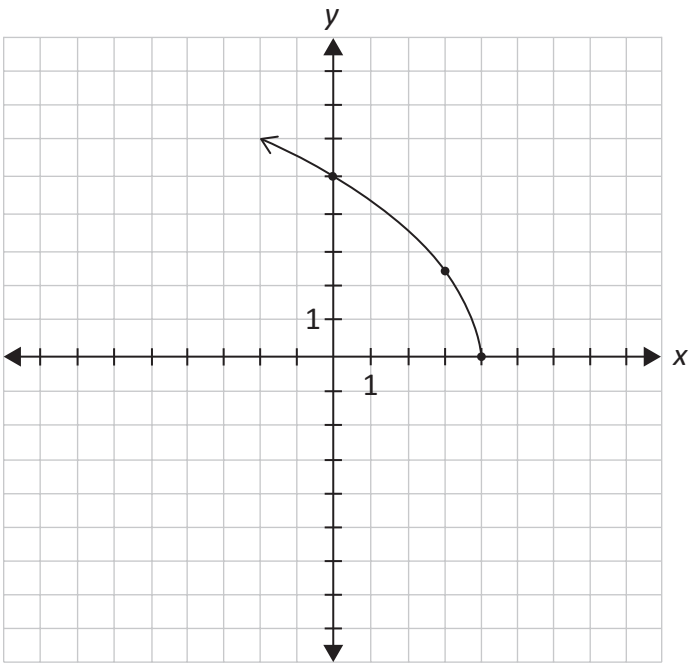
+ 1 mark for restricted domain

THIS PAGE WAS INTENTIONALLY LEFT BLANK.

Question 15 R13

Sketch the graph of $f(x) = \frac{5}{2}\sqrt{-(x-4)}$.

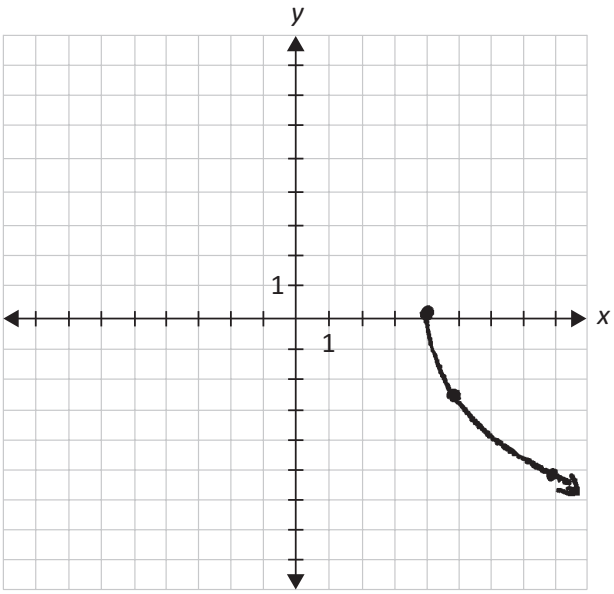
Solution



1 mark for shape of a radical function
1 mark for vertical stretch
1 mark for horizontal reflection
1 mark for horizontal translation

4 marks

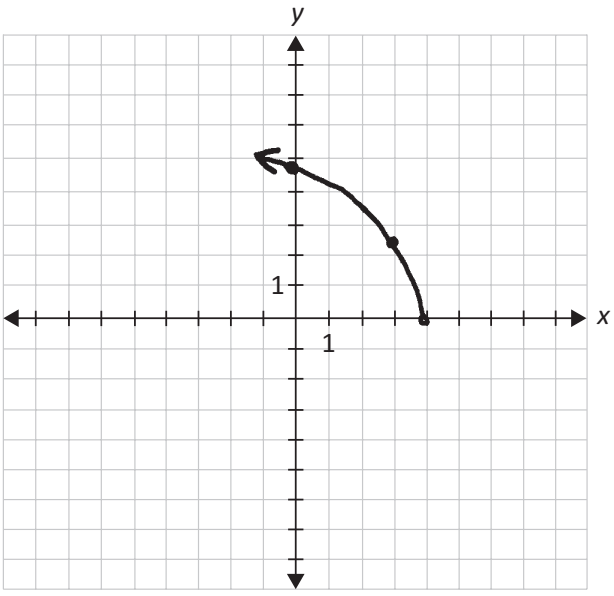
Exemplar 1



3 out of 4

+ 1 mark for shape of a radical function
+ 1 mark for vertical stretch
+ 1 mark for horizontal translation

Exemplar 2



3½ out of 4

award full marks
– ½ mark for procedural error (one incorrect point)

THIS PAGE WAS INTENTIONALLY LEFT BLANK.

Scoring Guidelines for Booklet 2 Questions

Answer Key for Selected Reponse Questions

Question	Answer	Learning Outcome
16	B	R11
17	D	R5
18	B	R7
19	C	R14
20	C	P2
21	B	R11
22	D	T4
23	C	R8
24	A	T1

Question 16 R11

Identify the number of *x*-intercepts on the graph of the polynomial function, $p(x) = 3x(x-8)(x^2+5)$.

a. 1

b. 2

c. 3

d. 4

Question 17 R5

Identify the equation that represents the graph of $y = x^2 - 3$ after a reflection over the *x*-axis.

a. $y = -x^2 - 3$

b. $y = x^2 - 3$

c. $y = x^2 + 3$

d. $y = -x^2 + 3$

Question 18 R7

Given $2^{\log_2 3} = x$, identify the value of *x*.

a. 1

b. 3

c. 6

d. 8

Question 19 R14

Identify the *y*-intercept on the graph of $y = \dfrac{x-3}{x^2-5x+6}$.

a. -3

b. -2

c. $-\dfrac{1}{2}$

d. 3

Question 20 P2

Identify the total number of arrangements for 10 students and 2 teachers to sit in a row if the teachers must sit together.

a. 10!2!

b. 11!

c. 11!2!

d. 12!

Question 21 R11

Identify which expression represents the remainder when the polynomial $P(x)$ is divided by $(x-4)$.

a. $P(-4)$

b. $P(4)$

c. $P(x-4)$

d. $P(x+4)$

Question 22 T4

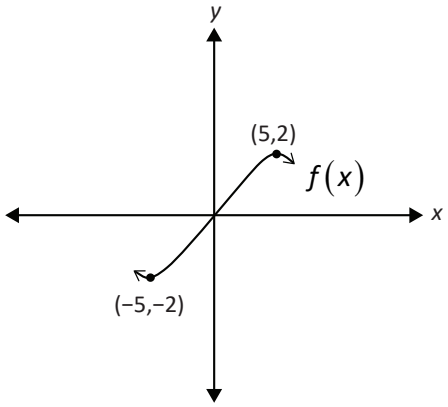
Identify the period of the graph of the sinusoidal function.

a. 10π

b. 10

c. 20π

d. 20



Question 23 R8

Identify an equivalent expression for $\log_3 9 + \log_3 5$.

a. $\log_3 14$

b. $2\log_3 5$

c. $\log_3 45$

d. $\log_3\left(\frac{9}{5}\right)$

Question 24 T1

Identify the coterminal angle(s) of $\theta = -\frac{11\pi}{10}$ over the interval $[-3\pi, \pi]$.

a. $\theta = \frac{9\pi}{10}$

b. $\theta = -\frac{31\pi}{10}, -\frac{21\pi}{10}$

c. $\theta = -\frac{31\pi}{10}, \frac{9\pi}{10}$

d. $\theta = -\frac{21\pi}{10}, \frac{9\pi}{10}$

Question 25 R11

Express $p(x) = x^3 - 13x - 12$ in completely factored form.

Solution

$p(-1) = (-1)^3 - 13(-1) - 12$

$p(-1) = 0$

$\therefore (x+1)$ is a factor

1 mark for identifying one possible zero of $P(x)$

-1	1	0	-13	-12
		-1	1	12
	1	-1	-12	0

1 mark for synthetic division (or equivalent strategy)

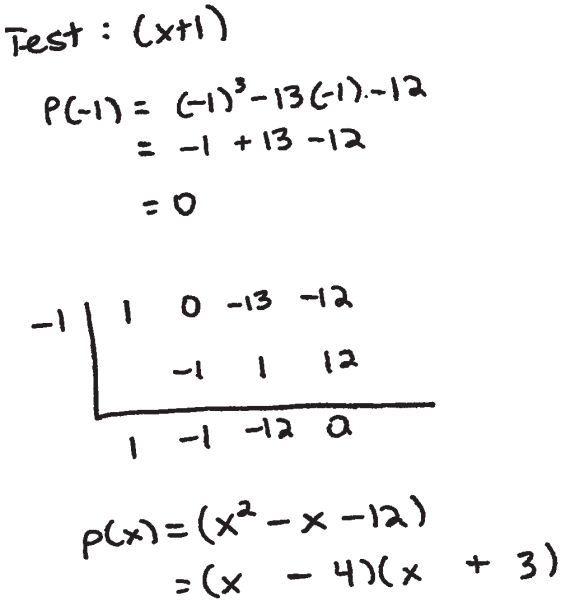
$p(x) = (x+1)(x^2 - x - 12)$

$p(x) = \underline{(x+1)(x-4)(x+3)}$

1 mark for consistent product of factors

3 marks

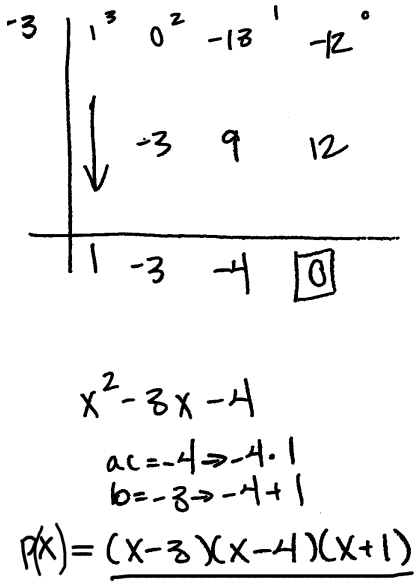
Exemplar 1



2½ out of 3

award full marks
– ½ mark for procedural error (not including the first factor in final answer)

Exemplar 2



2½ out of 3

award full marks
– ½ mark for procedural error (using the wrong sign when identifying the first factor)

Question 26 T5

Verify, by substitution, that the equation $\frac{\cos^2\theta}{\sin^2\theta} = \frac{2\csc\theta}{\sec^2\theta}$ is true for $\theta = \frac{\pi}{6}$.

Solution

Left-Hand Side	Right-Hand Side	
$\frac{\cos^2\left(\frac{\pi}{6}\right)}{\sin^2\left(\frac{\pi}{6}\right)}$	$\frac{2\csc\left(\frac{\pi}{6}\right)}{\sec^2\left(\frac{\pi}{6}\right)}$	
$\frac{\left(\frac{\sqrt{3}}{2}\right)^2}{\left(\frac{1}{2}\right)^2}$	$\frac{2(2)}{\left(\frac{2}{\sqrt{3}}\right)^2}$	2 marks for exact values (½ mark for each)
$\frac{\frac{3}{4}}{\frac{1}{4}}$	$\frac{2(2)}{\left(\frac{4}{3}\right)}$	
	$\frac{12}{4}$	
3	3	1 mark for simplification (½ mark for LHS; ½ mark for RHS)

3 marks

Exemplar 1

Left-Hand Side	Right-Hand Side
$\frac{\cos^2\theta}{\sin^2\theta}$ $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\frac{2\sqrt{3}}{2}$	$\frac{2\frac{1}{\sin\theta}}{\frac{1}{\cos^2\theta}}$ $2\frac{\cos^2\theta}{\sin\theta}$ $\frac{2\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\frac{4\sqrt{3}}{2}$

$LS \neq RS$

2½ out of 3

award full marks
– ½ mark for procedural error (not squaring values)

Exemplar 2

Left-Hand Side	Right-Hand Side
$\frac{\cos^2\left(\frac{\sqrt{3}}{2}\right)}{\sin^2\left(\frac{1}{2}\right)}$	$\frac{2\csc\left(\frac{2}{1}\right)}{\sec^2\left(\frac{2}{\sqrt{3}}\right)}$
$\frac{\left(\frac{\sqrt{3}}{2}\right)^2}{\left(\frac{1}{2}\right)^2}$	$\frac{2\left(\frac{2}{1}\right)}{\left(\frac{2}{\sqrt{3}}\right)^2}$
$\frac{\frac{3}{4}}{\frac{1}{4}}$	$\frac{\frac{4}{1}}{\frac{4}{3}}$
$\frac{3}{4} \times \frac{4}{1} = \frac{12}{4}$	$\frac{4}{1} \times \frac{3}{4} = \frac{12}{4}$
$\frac{3}{1}$	$\frac{3}{1}$
3	3

2½ out of 3

award full marks
– ½ mark for procedural error in line 1

THIS PAGE WAS INTENTIONALLY LEFT BLANK.

Question 27 R9

Given $f(x) = \log_3 x + 2$,

a) determine the *x*-intercept of $f(x) = \log_3 x + 2$.

b) sketch the graph of $f(x) = \log_3 x + 2$.

Solution

a)

$$0 = \log_3 x + 2$$

$$-2 = \log_3 x$$

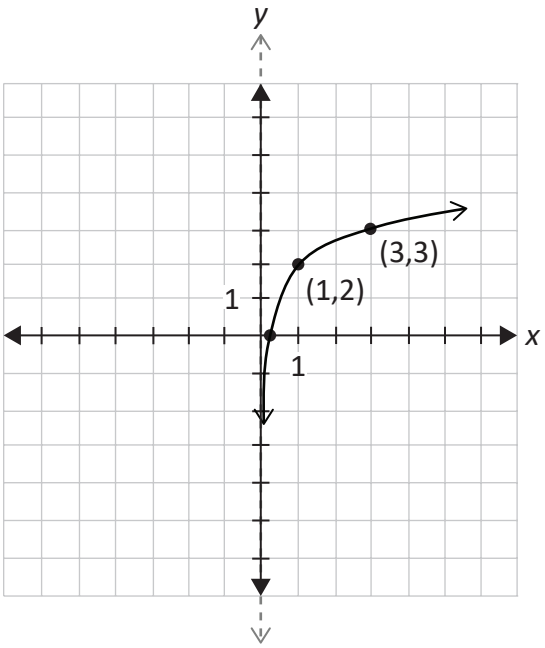
$$3^{-2} = x$$

$$\frac{1}{3^2} = x$$

$$\frac{1}{9} = x$$

1 mark

b)



1 mark for shape of a logarithmic function
1 mark for vertical translation

2 marks

Exemplar 1

a)

$0 = \log_3 x + 2$

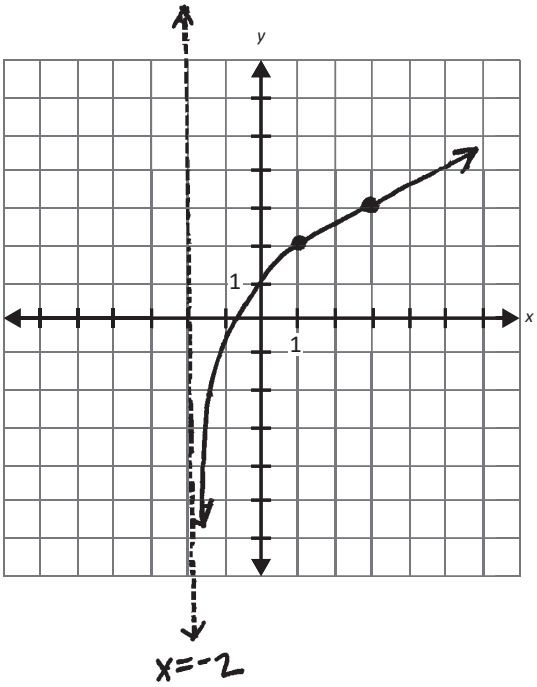
$-2 = \log_3 x$

$3^{-2} = x$

$x = \frac{1}{9}$

1 out of 1

b)



1 out of 2

award full marks
– 1 mark for concept error (shifting the asymptote left 2)

Exemplar 2

a)

$0 = \log_3 x + 2$

$-2 = \log_3 x$

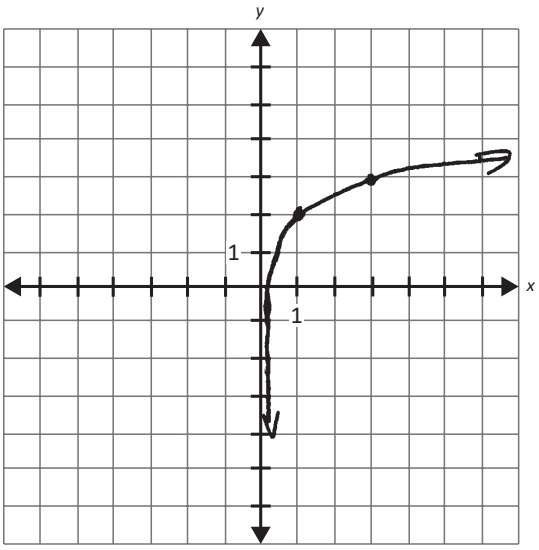
$3^{-2} = x$

$\frac{1}{6} = x$

½ out of 1

award full marks
– ½ mark for arithmetic error

b)



2 out of 2

award full marks
E10 (asymptote omitted but still implied)

Exemplar 3

a)

For x-int : $y = 0$

$\log_3 x + 2 = 0$

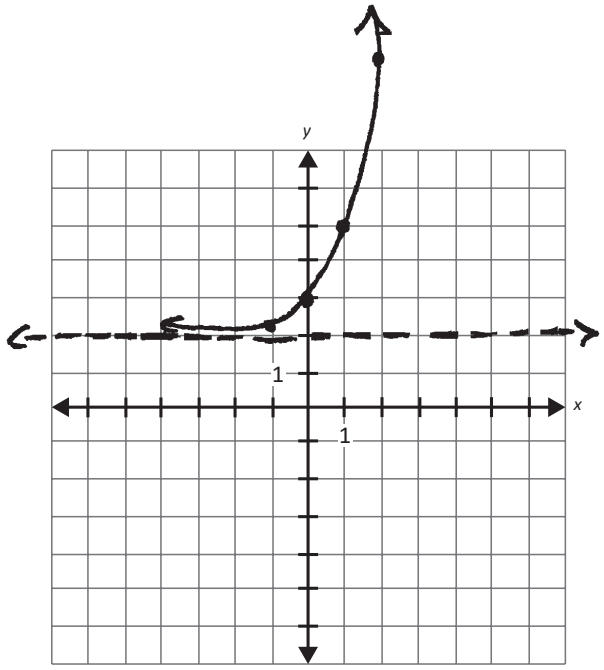
$\log_3 x = -2$

log can not be negative

so, no x-intercept.

0 out of 1

b)



1 out of 2

+ 1 mark for vertical translation

Question 28 T3

Evaluate.

$$\sec^2\left(\frac{5\pi}{4}\right) \cdot \tan^2\left(-\frac{2\pi}{3}\right)$$

Solution

$\left(-\frac{2}{\sqrt{2}}\right)^2\left(\frac{\sqrt{3}}{1}\right)^2$

$\left(\frac{4}{2}\right)\left(\frac{3}{1}\right)$

6

1 mark for $\sec\left(\frac{5\pi}{4}\right)$ (½ mark for quadrant; ½ mark for value)

1 mark for $\tan\left(-\frac{2\pi}{3}\right)$ (½ mark for quadrant; ½ mark for value)

2 marks

Exemplar 1

$\left(\frac{2}{\sqrt{2}}\right)^2 \cdot -\sqrt{3}$

$\frac{4}{2}$

$2 \cdot -\sqrt{3}$

$-2\sqrt{3}$

1 out of 2

+ ½ mark for value of $\sec\left(\frac{5\pi}{4}\right)$

+ ½ mark for value of $\tan\left(-\frac{2\pi}{3}\right)$

E7 (transcription error in line 1)

Exemplar 2

$\cos^2\left(\frac{2}{\sqrt{2}}\right) \cdot \tan^2\left(\frac{\sqrt{3}}{1}\right)$

$\left(\frac{2}{\sqrt{2}}\right)^2 \cdot \left(\frac{\sqrt{3}}{1}\right)^2$

$\frac{4}{2} \cdot \frac{3}{1} = \frac{12}{2} = \boxed{6}$

1 out of 2

+ ½ mark for value of $\sec\left(\frac{5\pi}{4}\right)$

+ 1 mark for $\tan\left(-\frac{2\pi}{3}\right)$

– ½ mark for procedural error in line 1

Question 29 R1

Given the functions $f(x) = x - 3$ and $g(x) = \sqrt{x} + 2$, state the domain of $g(f(x))$.

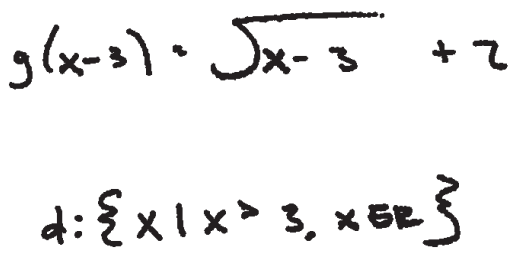
Solution

$\{x | x \geq 3, x \in \mathbb{R}\}$ **1 mark**

or

$[3, \infty)$

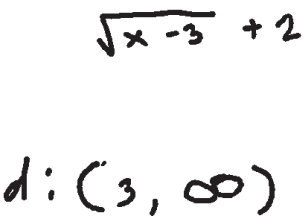
Exemplar 1



1 out of 1

award full marks
E8 (bracket error when stating domain)

Exemplar 2



1 out of 1

award full marks
E8 (bracket error when stating domain)

Question 30 T6

Evaluate.

$$\sin 70^\circ \cos 25^\circ - \cos 70^\circ \sin 25^\circ$$

Solution

$$\begin{aligned}\sin 70^\circ \cos 25^\circ - \cos 70^\circ \sin 25^\circ &= \sin\left(70^\circ - 25^\circ\right) \\ &= \sin 45^\circ \\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

½ mark for substitution into correct identity

½ mark for exact value

1 mark

Exemplar 1

½ out of 1

+ ½ mark for substitution into correct identity
E5 (units of measure omitted in final answer)

Question 31 R10

Solve, algebraically.

$$\log_{15}\left(x^2-1\right)=1$$

Solution

Method 1

$15^1 = x^2 - 1$ — 1 mark for exponential form

$0 = x^2 - 16$

$0 = (x+4)(x-4)$

$x = -4, x = 4$ — 1 mark for solving for x (½ mark for each value)

2 marks

Method 2

$\log_{15}\left(x^2-1\right) = \log_{15} 15$ — ½ mark for logarithmic form

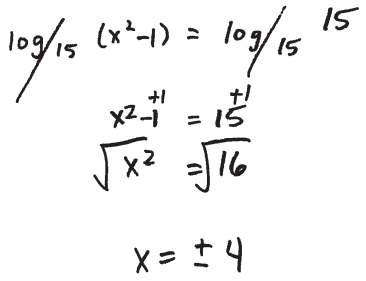
$x^2 - 1 = 15$ — ½ mark for equating arguments

$x^2 = 16$

$x = \pm 4$ — 1 mark for solving for x (½ mark for each value)

2 marks

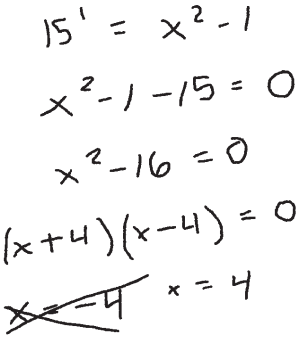
Exemplar 1



1½ out of 2

award full marks
– ½ mark for procedural error in line 1

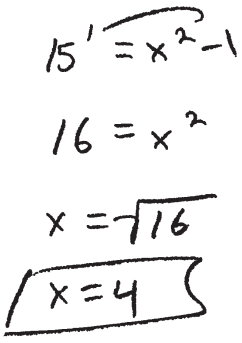
Exemplar 2



1½ out of 2

award full marks
– ½ mark for procedural error (incorrectly rejecting correct answer)

Exemplar 3



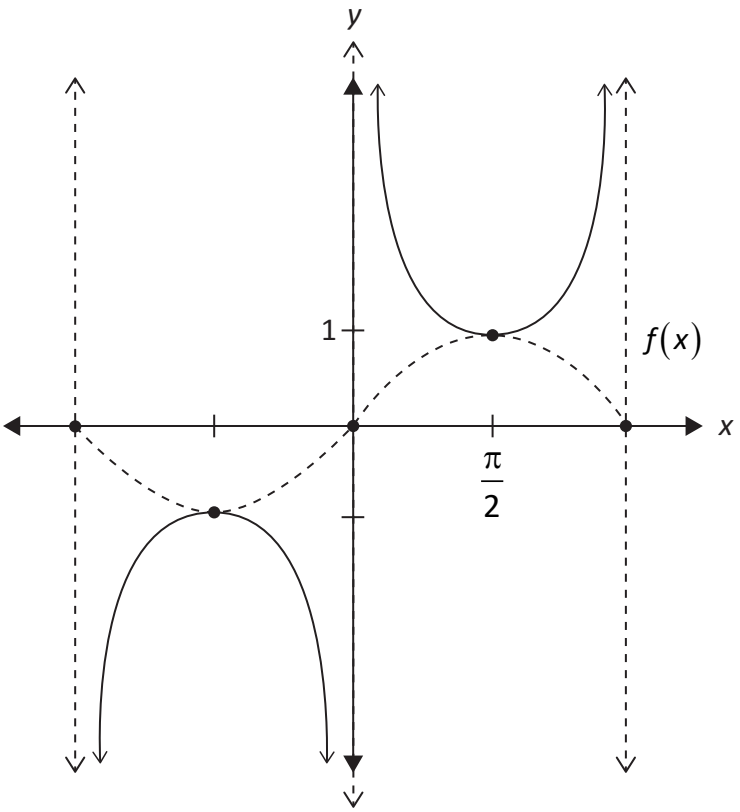
1½ out of 2

+ 1 mark for exponential form
+ ½ mark for solving for x

Question 32 R1

Given the graph of $y = f(x)$, sketch the graph of $y = \dfrac{1}{f(x)}$.

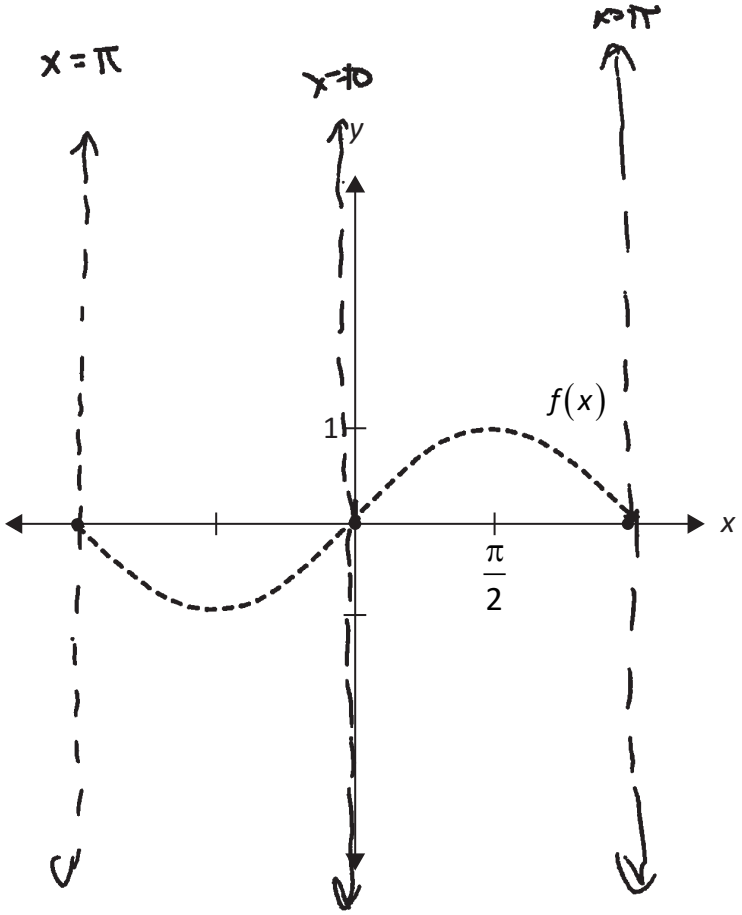
Solution



1 mark for asymptotic behaviour approaching $x = -\pi$, $x = 0$, and $x = \pi$
1 mark for invariant points (½ mark for $y = -1$; ½ mark for $y = 1$)

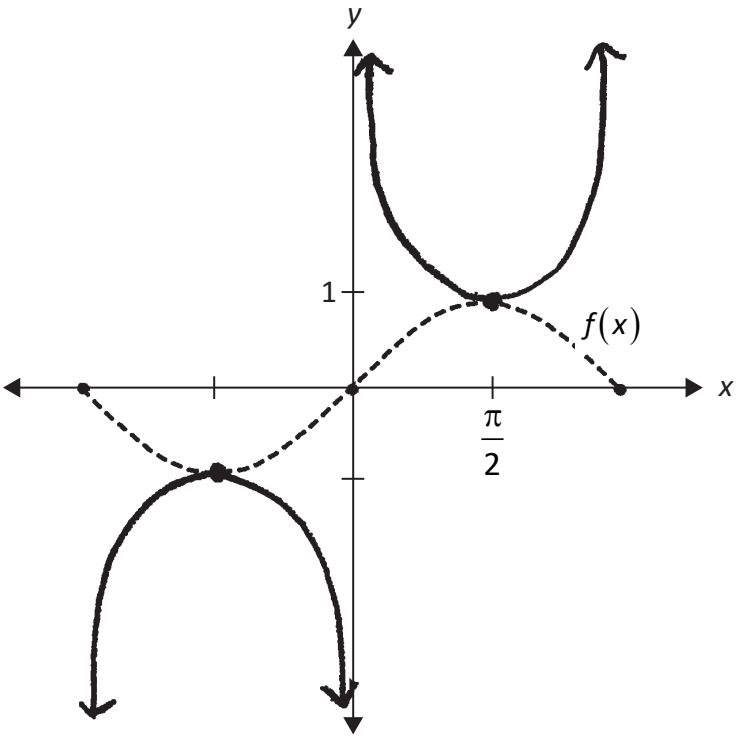
2 marks

Exemplar 1



0 out of 2

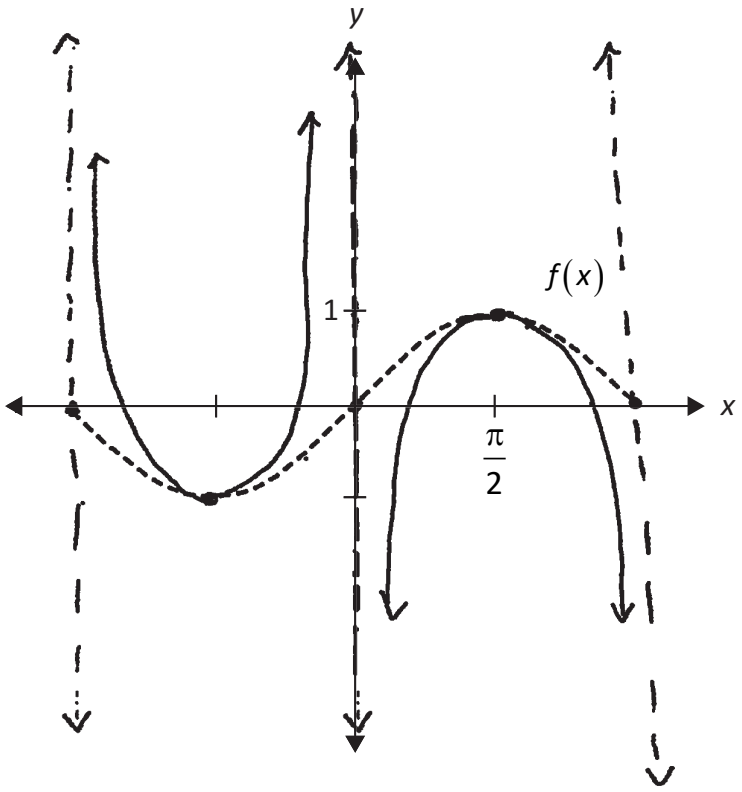
Exemplar 2



2 out of 2

award full marks
E10 (asymptotes omitted but still implied)

Exemplar 3



1 out of 2

award full marks
– 1 mark concept error (incorrect reciprocal values)

Question 33 T4

A light is attached to the end of a blade of a wind turbine. The height of the light, with respect to the ground, follows the sinusoidal equation:

$$h(t) = 80\cos\left(\frac{\pi}{2}t\right) + 240$$

where $h(t)$ is the height of the light with respect to the ground, measured in feet and,

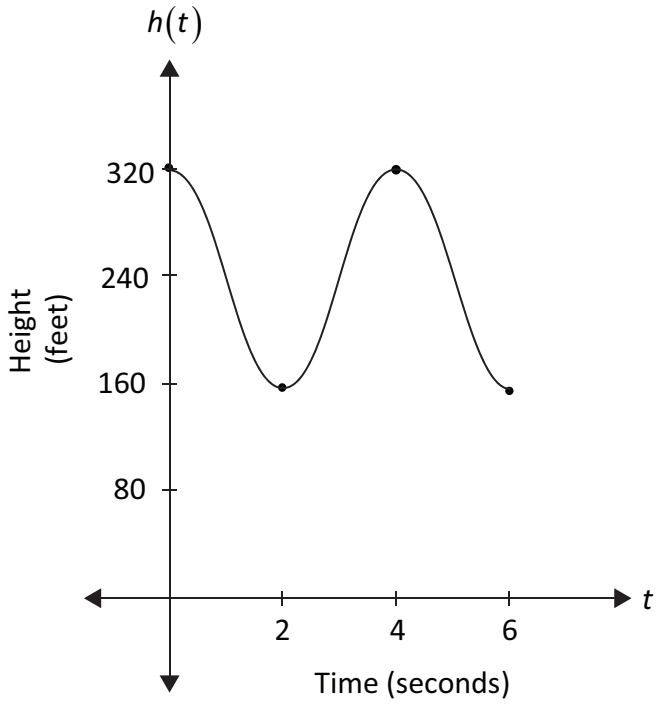
t is the time, measured in seconds.

a) Sketch the graph of the height of the light with respect to the ground, over an interval of 6 seconds.

b) If the turbine begins to rotate faster, describe the resulting effect on the period of the graph of $h(t)$.

Solution

a)



1 mark for shape of a sinusoidal function with correct amplitude
1 mark for period
1 mark for vertical translation

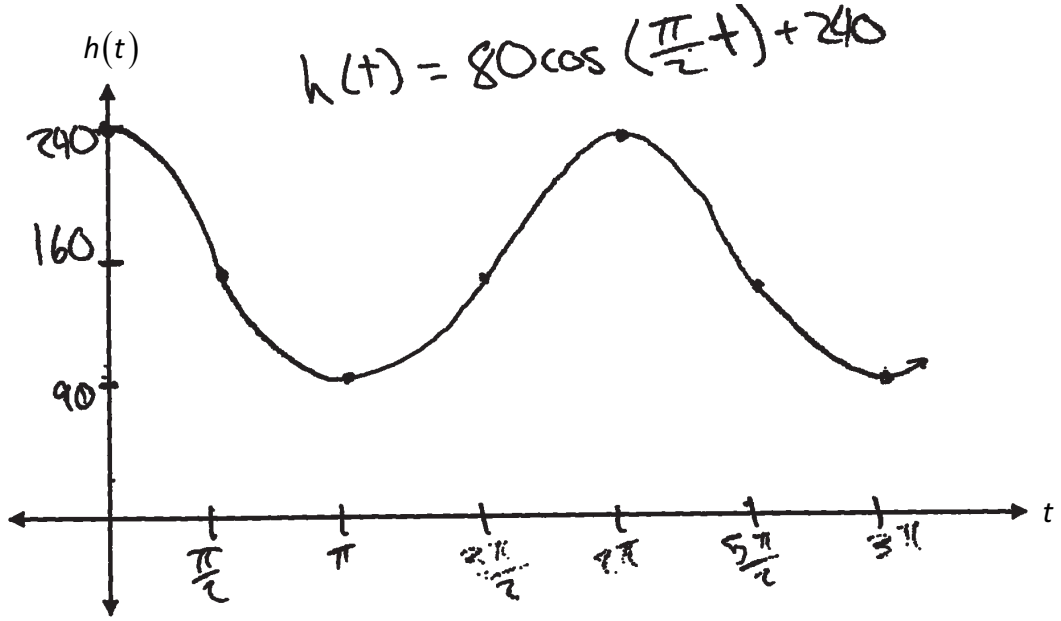
3 marks

1 mark

b) The period would decrease.

Exemplar 1

a)



½ out of 3

+ 1 mark for shape of a sinusoidal function with correct amplitude
– ½ mark for arithmetic error when calculating the amplitude

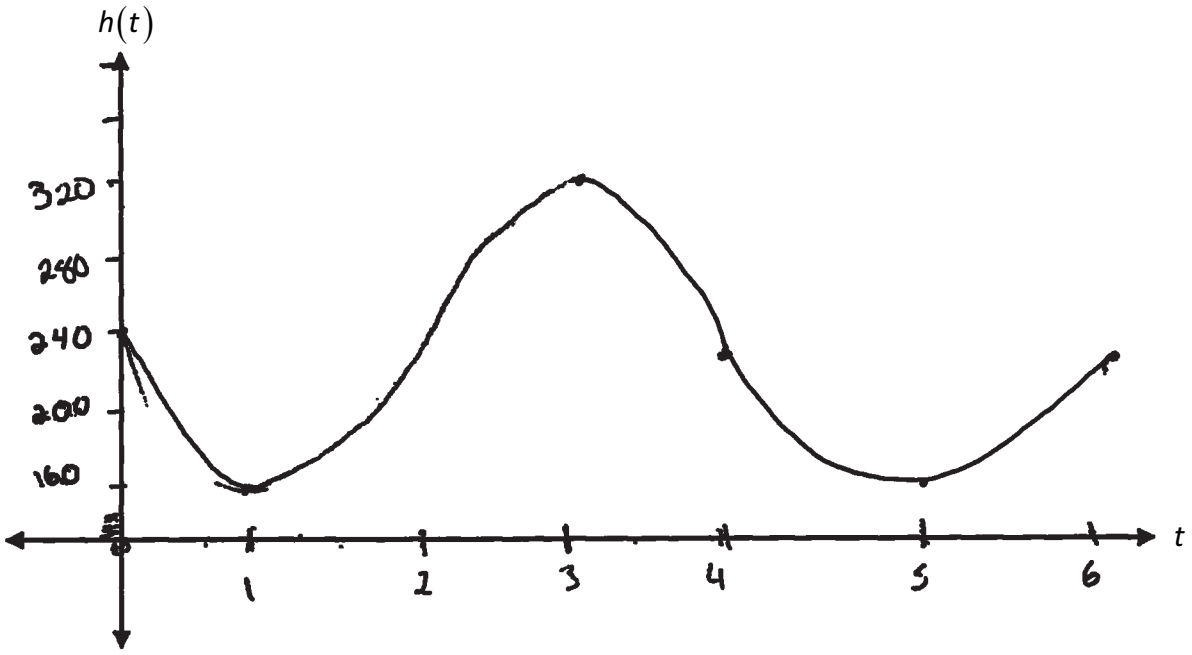
b)

If the turbine begins to rotate faster, the period of the graph will shorten, and the graph will begin to move up + down faster, or over less intervals.

1 out of 1

Exemplar 2

a)



2 out of 3

award full marks

– 1 mark for concept error (sketching $y = -\sin t$ instead of $y = \cos t$)

b)

The period will become smaller

1 out of 1

THIS PAGE WAS INTENTIONALLY LEFT BLANK.

Question 34 P2

Elodie was asked to determine the total number of arrangements of the letters in the word EXCELLENCE.

Her solution:

$10! = 3628\ 800$

Describe her error.

Solution

She did not consider the repeating letters.

1 mark

Exemplar 1

There are letters which repeat that she counted as all separate. The proper answer is 5!.

0 out of 1

Exemplar 2

She didn't factor in that there are letter repeats in the word. The correct solution is $\frac{10!}{4!2!}$

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 35 T6

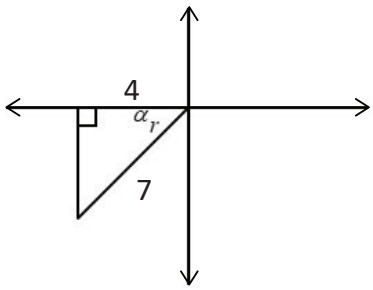
Given that $\cos\alpha = -\frac{4}{7}$ where α is in quadrant III and $\sin\beta = \frac{5}{13}$ where β is in quadrant II, determine the exact value of:

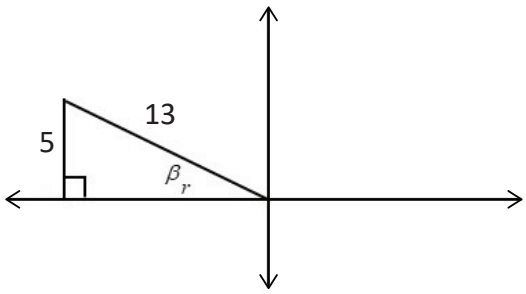
a) $\cos(\alpha+\beta)$

b) $\sec(\alpha+\beta)$

Solution

a)





$x^2 + y^2 = r^2$

$16 + y^2 = 49$

$y^2 = 33$

$y = \pm\sqrt{33}$

$x^2 + y^2 = r^2$

$x^2 + 25 = 169$

$x^2 = 144$

$x = \pm 12$

½ mark for value of x
½ mark for value of y

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(-\frac{4}{7}\right)\left(-\frac{12}{13}\right) - \left(-\frac{\sqrt{33}}{7}\right)\left(\frac{5}{13}\right)$

$= \frac{48 + 5\sqrt{33}}{91}$

½ mark for consistent value of $\cos\beta$
½ mark for consistent value of $\sin\alpha$
1 mark for substitution into correct identity

3 marks

b) $\sec(\alpha+\beta) = \frac{91}{48 + 5\sqrt{33}}$

1 mark for consistent value of $\sec(\alpha+\beta)$

1 mark

Note:

Accept any of the following values for x : $x = \pm 12$, $x = -12$, or $x = 12$.

Accept any of the following values for y : $y = \pm\sqrt{33}$, $y = -\sqrt{33}$, or $y = \sqrt{33}$.

Exemplar 1

a)

$\sin\beta = \frac{5}{13}$ $\quad \cos\alpha = -\frac{4}{7}$

$\cos\beta = \frac{-12}{13}$ $\quad \sin\alpha = -\frac{\sqrt{33}}{7}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{-4}{7}\right)\left(\frac{-12}{13}\right) - \left(\frac{-\sqrt{33}}{7}\right)\left(\frac{5}{13}\right)$

$= \frac{-48}{91} - \frac{5\sqrt{33}}{91}$

$= \frac{-48-5\sqrt{33}}{91}$

2½ out of 3

award full marks
– ½ mark for arithmetic errors in line 5

b)

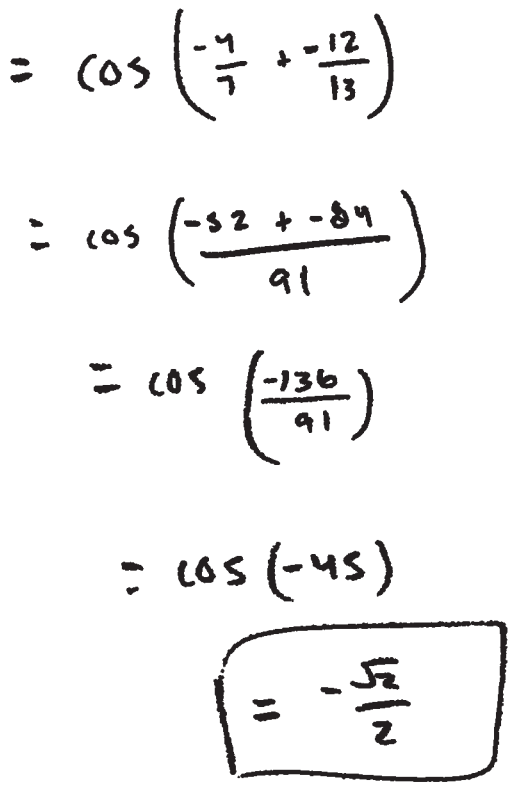
$\sec(\alpha+\beta) = \frac{1}{\cos(\alpha+\beta)}$

$= \frac{91}{(-48-5\sqrt{33})}$

1 out of 1

Exemplar 2

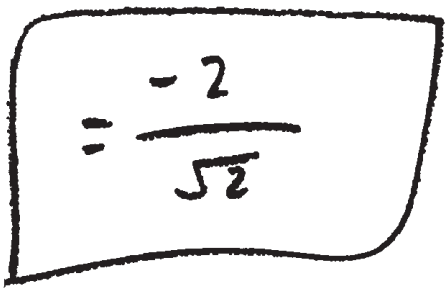
a)



½ out of 3

+ ½ mark for value of *x*

b)

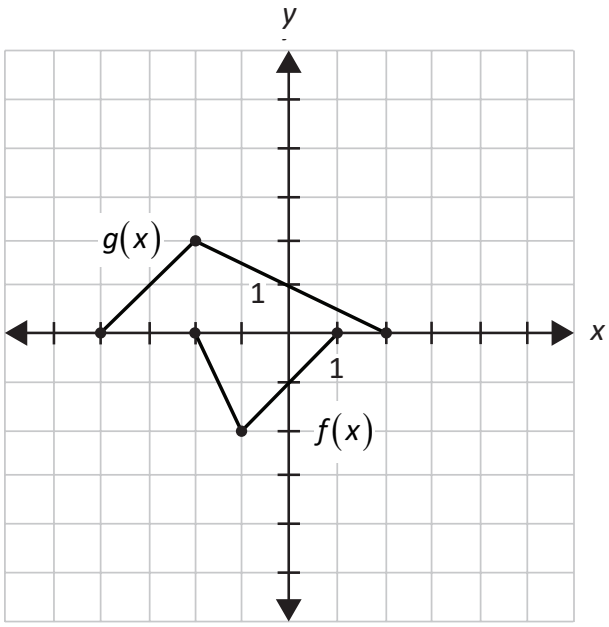


1 out of 1

THIS PAGE WAS INTENTIONALLY LEFT BLANK.

Question 36 R4, R5

Determine the equation for $g(x)$, in terms of $f(x)$.



Solution

$g(x) = -f\left(\frac{x}{2}\right)$

1 mark for vertical reflection
1 mark for horizontal stretch

2 marks

Exemplar 1

$g(x) =$ $-f(2x)$

1 out of 2

+ 1 mark for vertical reflection

Exemplar 2

$g(x) =$ $f(\frac{1}{2}x)$

1 out of 2

+ 1 mark for horizontal stretch

Exemplar 3

$g(x) =$ $-g(\frac{1}{2}x)$

1½ out of 2

award full marks
– ½ mark for procedural error (*g* instead of *f*)

Exemplar 4

$g(x) =$ $(\frac{-1}{2}x)$

1 out of 2

award full marks
– 1 mark for concept error (not writing an equation in term of $f(x)$)

Question 37 R7

State possible values for m and n that satisfy the equation, $\log_m n = 5$.

Solution

Some possible solutions are:

$m = 2 \quad n = 32$

or

$m = 3 \quad n = 243$

or

$m = 4 \quad n = 1024$

or

$m = 5 \quad n = 3125$

1 mark

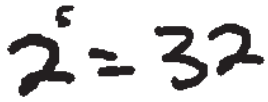
Notes:

Other values for m and n are possible.

Exemplar 1

1 out of 1

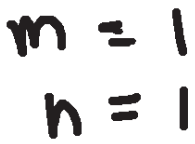
Exemplar 2



1 out of 1

award full marks
E1 (final answer not stated)

Exemplar 3



0 out of 1

Question 38 R14

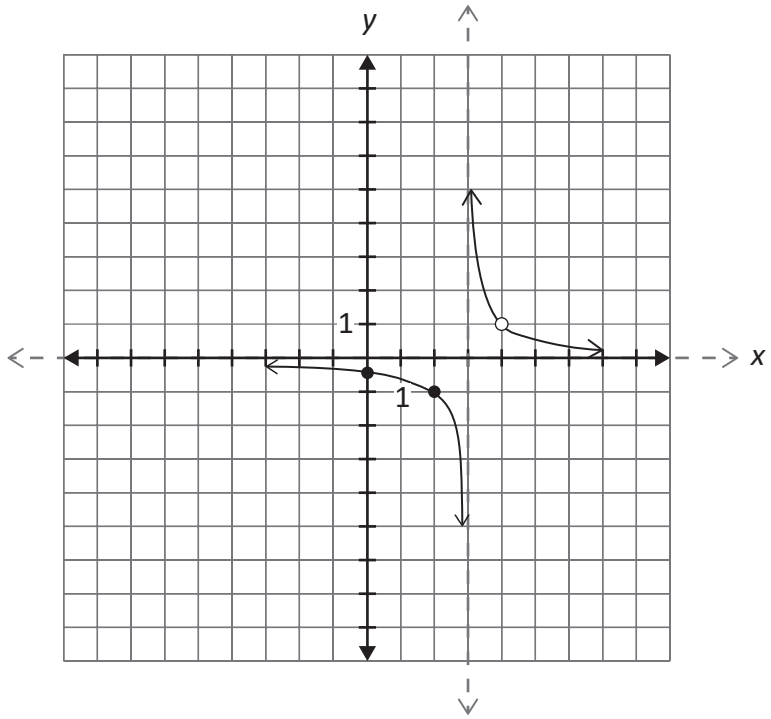
Sketch the graph of $f(x) = \dfrac{x-4}{x^2-7x+12}$.

Solution

$$f(x) = \frac{\cancel{(x-4)}}{\cancel{(x-4)}(x-3)}$$

$$= \frac{1}{x-3}, \quad x \neq 4$$

$\therefore$ there is a point of discontinuity (hole) at (4,1)



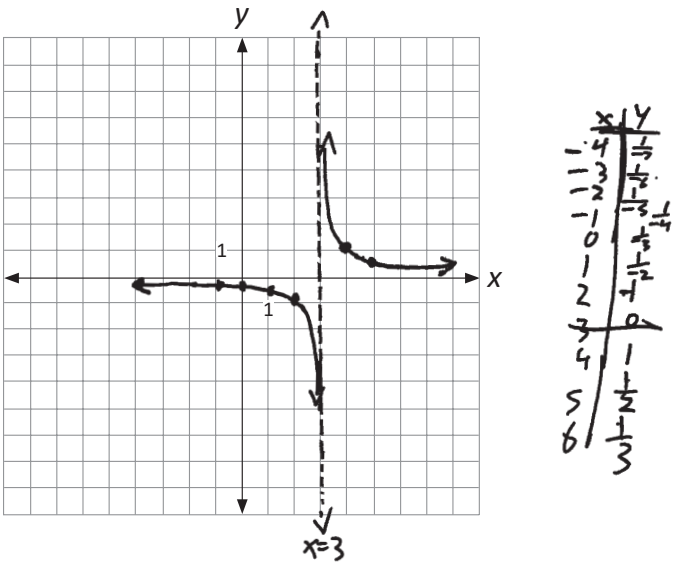
1 mark for asymptotic behaviour approaching $x = 3$
1 mark for asymptotic behaviour approaching $y = 0$
1 mark for point of discontinuity (hole) at $x = 4$
1 mark for shape of rational function (½ mark for graph left of $x = 3$, ½ mark for graph right of $x = 3$)

4 marks

Notes:

Deduct ½ mark for procedural error (incorrect *y*-value for point of discontinuity (hole)).

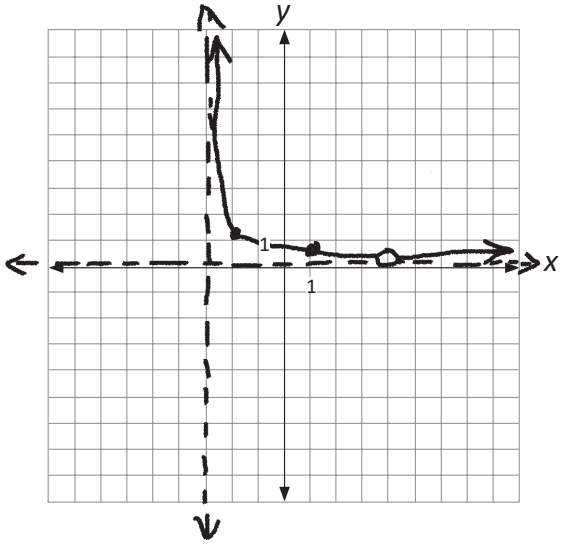
Exemplar 1



3 out of 4

+ 1 mark for asymptotic behaviour approaching $x = 3$
+ 1 mark for asymptotic behaviour approaching $y = 0$
+ 1 mark for shape of rational function
E10 (asymptote omitted but still implied)

Exemplar 2



$$f(x) = \frac{x-4}{(x-4)(x+3)}$$

$$f(x) = \frac{1}{x+3}$$

3 out of 4

+ 1 mark for asymptotic behaviour approaching consistent vertical asymptote
+ 1 mark for asymptotic behaviour approaching $y = 0$
+ 1 mark for point of discontinuity (hole) at $x = 4$
+ ½ mark for consistent shape of graph right of the vertical asymptote
– ½ mark for arithmetic error (incorrect factoring)

Question 39 P2

Justify that the value of ${}_3P_4$ does not exist.

Solution

Method 1

When using ${}_nP_r$, $n \geq r$.

Method 2

$${}_3P_4 = \frac{3!}{(3-4)!}$$

$${}_3P_4 = \frac{3!}{(-1)!}$$

$(-1)!$ does not exist, therefore ${}_3P_4$ does not exist.

1 mark

Exemplar 1

Selection must be less than or equal to the total.

1 out of 1

Exemplar 2

In $_nP_r$, n must be greater than r.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 40 T4

State the range of the sinusoidal function that has the following characteristics:

- an amplitude of 3
- a maximum at $\left(\frac{\pi}{2}, 1\right)$

Solution

$\{y \mid -5 \leq y \leq 1, y \in \mathbb{R}\}$

or

$[-5, 1]$

1 mark

Exemplar 1

Range: [1, -5]

1 out of 1

award full marks
E8 (range written in incorrect order)

Exemplar 2

Range: (-5, 1)

1 out of 1

award full marks
E8 (bracket error when stating range)

Exemplar 3

Range: [-2, 1]

0 out of 1

Question 41 P3

Solve, algebraically.

$${}_{n}C_{2} = 15$$

Solution

$$\frac{n!}{2!(n-2)!} = 15$$

½ mark for substitution

$$\frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}} = 15$$

1 mark for factorial expansion
½ mark for simplification of factorials

$$n(n-1) = 30$$

$$n^2 - n - 30 = 0$$

$$(n-6)(n+5) = 0$$

$$n = 6 \quad \cancel{n = -5}$$

½ mark for the permissible value of n
½ mark for showing the rejection of the extraneous root

3 marks

Exemplar 1

$$\frac{n!}{2!(n-2)!} = 0$$

$$\frac{(n)(n-1)(n-2)!}{2(n-2)!} = 0$$

$$\frac{(n)(n-1)}{2} = 0$$

$$\frac{n(n-1)}{n} = \frac{0}{n}$$

$$n-1 = 0$$

$$\boxed{n = 1}$$

2 out of 3

+ ½ mark for substitution
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
E7 (transcription error in line 1)

Exemplar 2

$$\frac{n!}{2!(n-2)!} = 15$$

$$\frac{(n)(n-1)(n-2)!}{2! \cdot 2!(n-2)!} = 15 \cdot 2!$$

$$n(n-1) = 30$$

$$6(6-1) = 30$$
$$6(5) = 30$$
$$n = 6$$

2½ out of 3

+ ½ mark for substitution
+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for the permissible value of *n*

E7 (notation error in line 2)

Exemplar 3

$$_nC_2 = \frac{n!}{2!(n-2)!} = 15$$

$$_nC_2 = \frac{(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}} = 15$$

$$\frac{(n-1)}{2!} = 15$$

$$\frac{(n-1)\cdot 2}{2} = 15 \cdot 2$$

$$n - 1 = 30$$
$$+1 \quad +1$$
$$n = 31$$

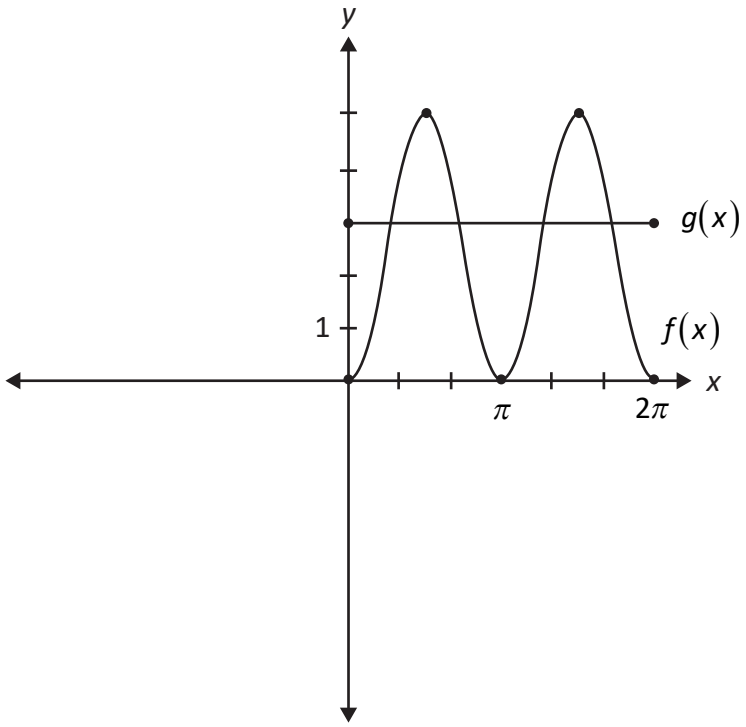
1½ out of 3

+ ½ mark for substitution
+ ½ mark for simplification of factorials
+ ½ mark for the permissible value of *n*

E4 (missing brackets but still implied in line 2)

Question 42 T5

Describe how to use the graphs of $f(x) = 5\sin^2 x$ and $g(x) = 3$ to solve the equation, $5\sin^2 x = 3$, over the interval $[0, 2\pi]$.



Solution

The solution will be the *x*-values where the two graphs intersect.

1 mark

Exemplar 1

The points where the two graphs intercept. are the solutions to $5\sin^2 x = 3$.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

The points of intersection will be the solution.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 43 R8

Solve, algebraically.

$$2\log_3 5 - \frac{1}{3}\log_3 125 = \log_3 a$$

Solution

Method 1

$\log_3 5^2 - \log_3 125^{\frac{1}{3}} = \log_3 a$ — 1 mark for power law (½ mark for each)

$\log_3 25 - \log_3 5 = \log_3 a$

$\log_3\left(\frac{25}{5}\right) = \log_3 a$ — 1 mark for quotient law

$\log_3(5) = \log_3 a$

$a = 5$ — 1 mark for equating arguments

3 marks

Method 2

$\log_3 5^2 - \log_3 125^{\frac{1}{3}} - \log_3 a = 0$ — 1 mark for power law (½ mark for each)

$\log_3\left(\frac{25}{5a}\right) = 0$ — 1 mark for quotient law

$3^0 = \frac{25}{5a}$ — 1 mark for exponential form

$5a = 25$

$a = 5$

3 marks

Exemplar 1

$$\log_3 25 - \log_3 \frac{1}{5} = \log_3 9$$

$$\log_3 5 = \log_3 9$$

$$5 = 9$$

1½ out of 3

+ ½ mark for power law
+ 1 mark for equating arguments

Exemplar 2

$$\log_3 5^2 - \log_3 125^{\frac{1}{3}} = \log_3 a$$

$$\log_3 \left(\frac{5^2}{\sqrt[3]{125}} \right) = \log_3 a$$

$$\frac{25}{5} = a$$

$$a = 5$$

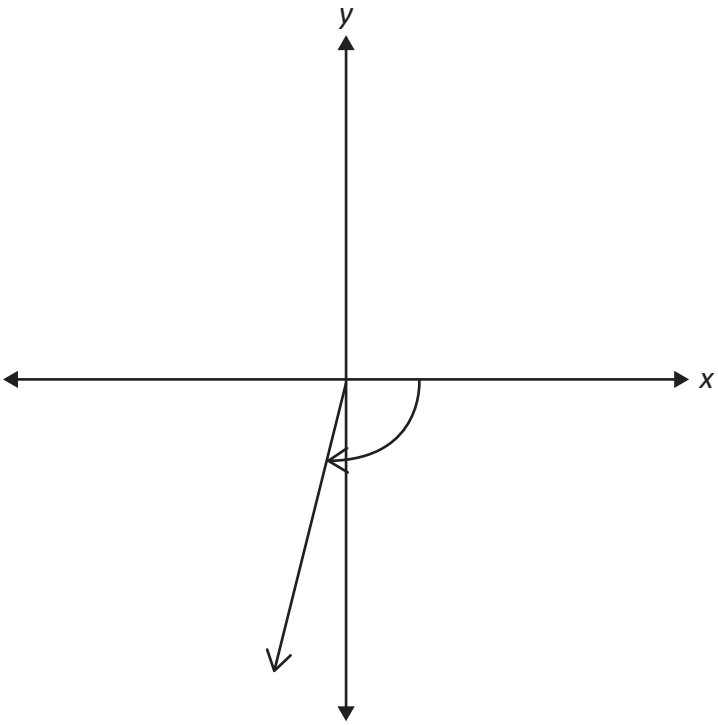
2½ out of 3

award full marks
– ½ mark for procedural error (crossing out the logarithms)

Question 44 T1

Sketch the angle, $\frac{-7\pi}{12}$, in standard position.

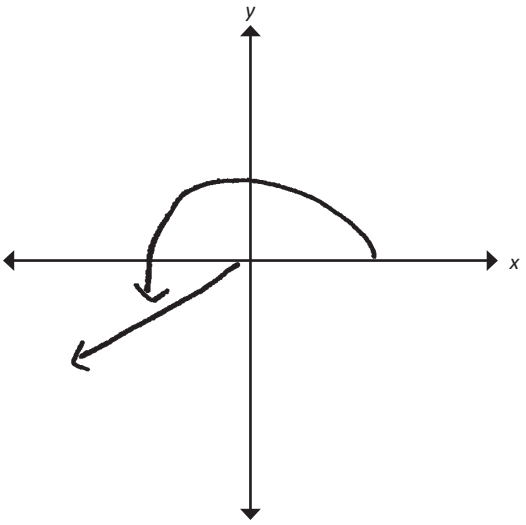
Solution



½ mark for an appropriate angle in quadrant III
½ mark for correct direction

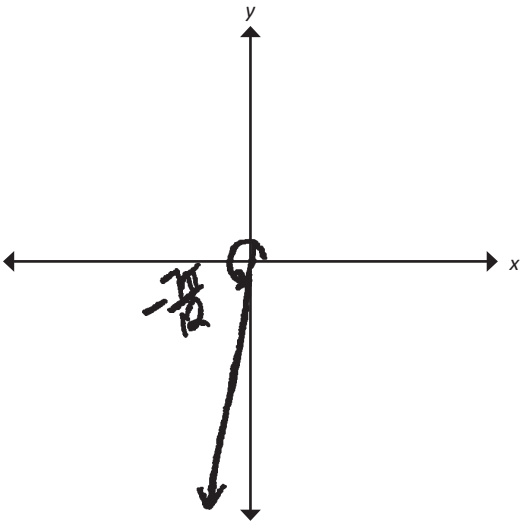
1 mark

Exemplar 1



0 out of 1

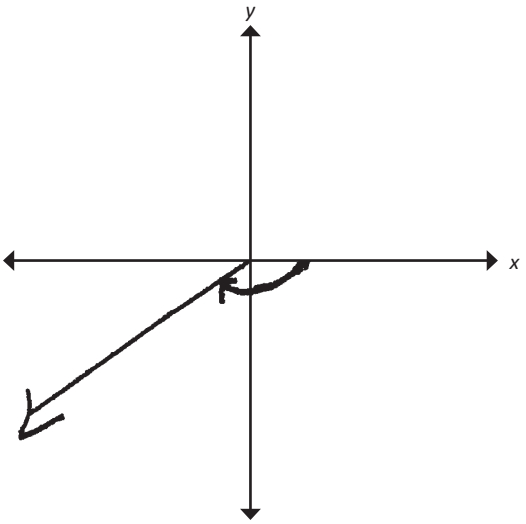
Exemplar 2



½ out of 1

+ ½ mark for an appropriate angle in quadrant III

Exemplar 3



½ out of 1

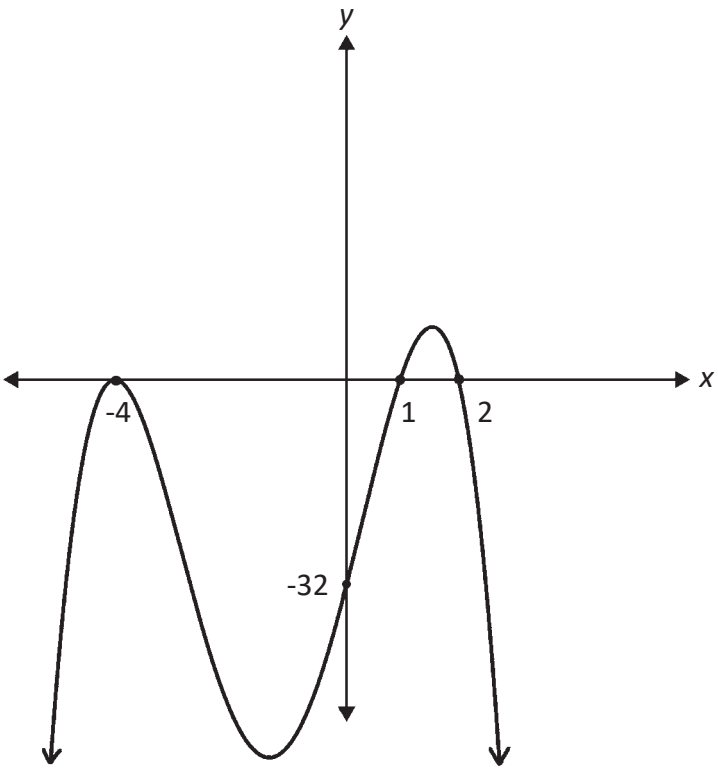
+ ½ mark for correct direction

THIS PAGE WAS INTENTIONALLY LEFT BLANK.

Question 45 R12

Sketch the graph of $p(x) = -(x-1)(x-2)(x+4)^2$.

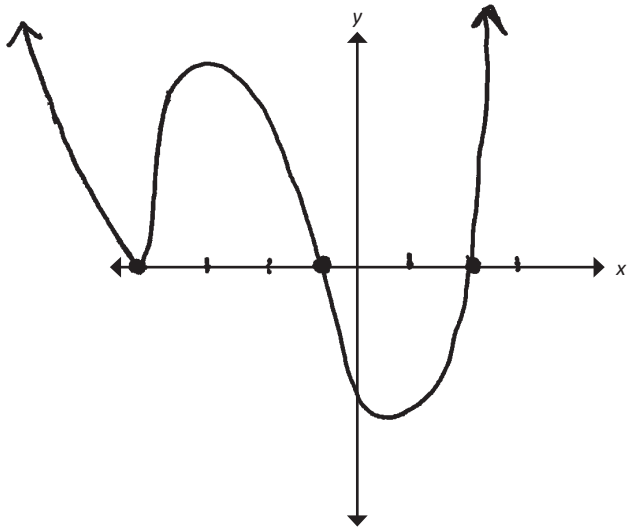
Solution



1 mark for x-intercepts
½ mark for y-intercept
1 mark for multiplicity of 2 at $x = -4$
½ mark for end behaviour

3 marks

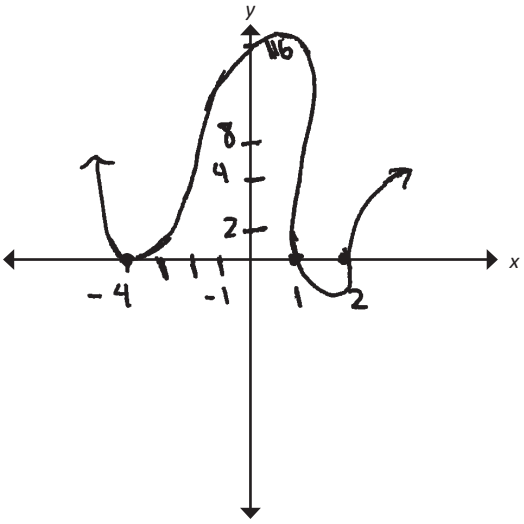
Exemplar 1



1½ out of 3

+ 1 mark for *x*-intercepts
+ 1 mark for multiplicity of 2 at $x = -4$
– ½ mark for procedural error (one incorrect *x*-intercept)
E9 (scale values on axes not indicated)

Exemplar 2



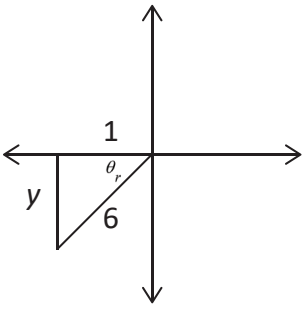
1½ out of 3

+ 1 mark for *x*-intercepts
+ 1 mark for multiplicity of 2 at $x = -4$
– ½ mark for incorrect shape of graph
E9 (scale values on axes incorrectly spaced)

Question 46 T3

The point, $P(\theta) = \left(-\frac{1}{6}, y\right)$, lies on the unit circle and is located in Quadrant III. Determine the exact value of $\csc\theta$.

Solution



$(-1)^2 + y^2 = (6)^2$ ½ mark for substitution

$1 + y^2 = 36$

$y^2 = 35$

$y = \pm\sqrt{35}$ ½ mark for solving for *y*

$\csc\theta = -\frac{6}{\sqrt{35}}$ 1 mark for consistent value of $\csc\theta$
(½ mark for quadrant; ½ mark for value)

2 marks

or

$\csc\theta = -\frac{6\sqrt{35}}{35}$

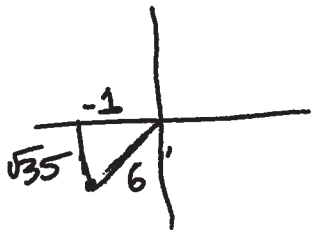
Notes:

Accept any of the following values for y : $y = \pm\sqrt{35}$, $y = \sqrt{35}$, or $y = -\sqrt{35}$.

Exemplar 1

$\frac{1}{\sin\theta}$

$\csc\theta = \frac{6}{\sqrt{35}}$



1½ out of 2

+ ½ mark for substitution
+ ½ mark for solving for *y*
+ ½ mark for value of $\csc\theta$

Exemplar 2

$(-\frac{1}{6})^2 + y^2 = 1^2$

$\frac{1}{36} + y^2 = 1$

$y^2 = \frac{35}{36}$

$y = -\sqrt{\frac{35}{36}}$

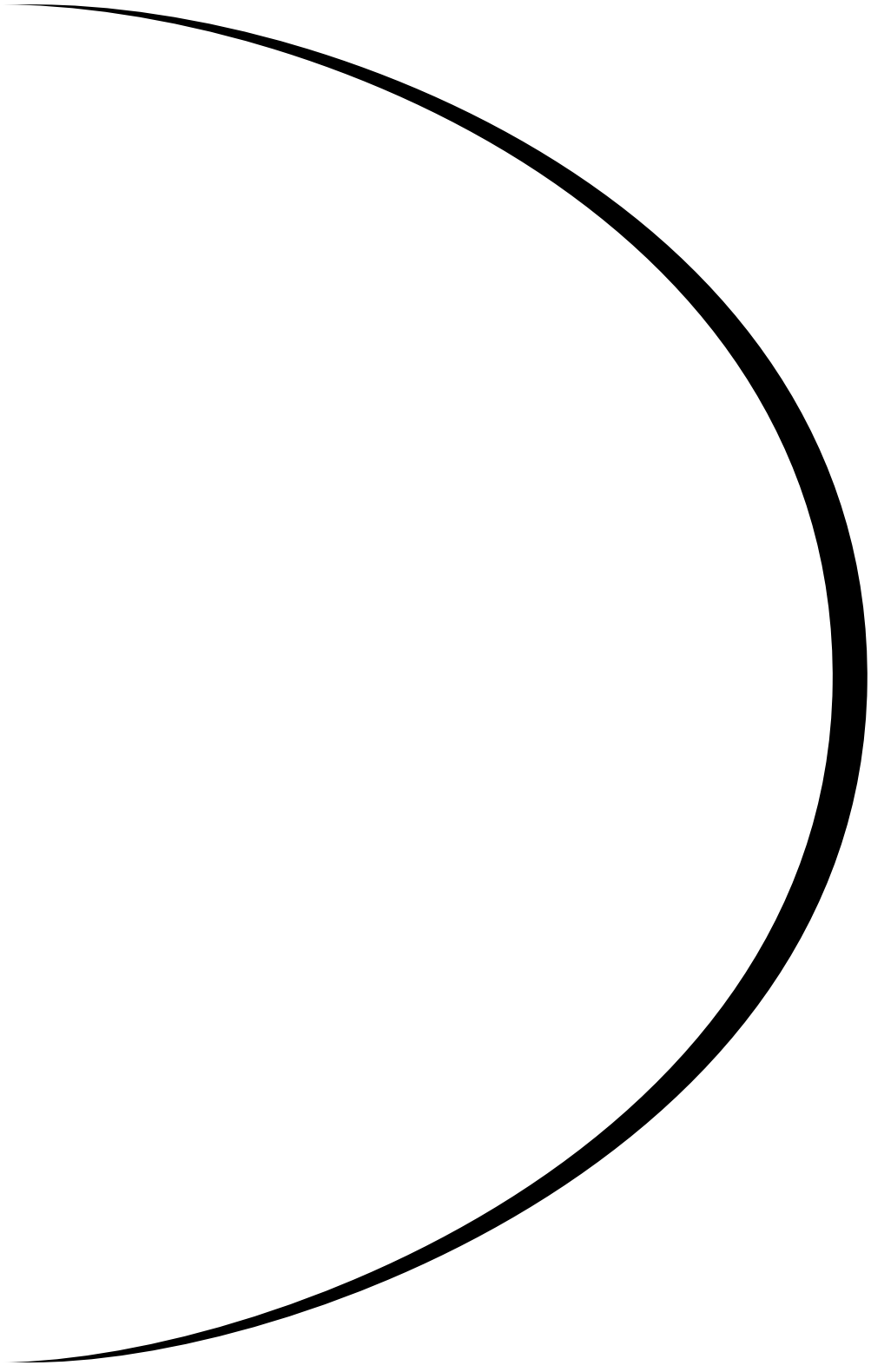
$\csc\theta = \frac{1}{\sin\theta}$

$\csc\theta = \frac{1}{-\sqrt{\frac{35}{36}}}$

1½ out of 2

award full marks
– ½ mark for procedural error (not taking the square root of both the numerator and denominator)
E1 (answer given as a complex fraction)

Appendices



Appendix A

Marking Guidelines

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply:

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation, description, or justification
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet.*

E1 **final answer**	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in step leading to final answer
E2 **equation/expression**	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3 **variables**	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4 **brackets**	▪ "sin x^2" written instead of "$\sin^2 x$" ▪ missing brackets but still implied
E5 **units**	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa
E6 **rounding**	▪ rounding error ▪ rounding too early
E7 **notation/transcription**	▪ notation error ▪ transcription error
E8 **domain/range**	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9 **graphing**	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated or incorrectly spaced ▪ coordinate points labelled incorrectly
E10 **asymptotes**	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes

Appendix B

Irregularities in Provincial Tests

A Guide for Local Marking

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report.*

Except in the case of cheating or plagiarism where the result is a provinical test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted and the follow-up. The originial copy of this report is to be returned by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: _____

Date marked: _____

Booklet No.: _____

_____

Problem(s) noted: _____

_____

_____

_____

_____

_____

Question(s) affected: _____

_____

_____

Action taken or rationale for assigning marks: _____

_____

_____

_____

_____

_____

_____

_____

Follow up: _____

_____

_____

_____

_____

_____

Decision: _____

_____

_____

_____

_____

_____

Marker's Signature: _____

Principal's Signature: _____

For Department Use Only - After Marking Complete

Consultant: _____

Date: _____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
7	R2, R3, R5	4
10	R6	2
14	R1	2
17	R5	1
29	R1	1
32	R1	2
36	R4, R5	2
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
5a)	T1	1
5b)	T1	1
22	T4	1
24	T1	1
28	T3	2
33a)	T4	3
33b)	T4	1
40	T4	1
44	T1	1
46	T3	2
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
1	P1	1
3	P4	3
9	P4	1
20	P2	1
34	P2	1
39	P2	1
41	P3	3
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
6	R12	1
16	R11	1
21	R11	1
25	R11	3
45	R12	3

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
4	T5	3
12	T6	3
26	T5	3
30	T6	1
35a)	T6	3
35b)	T6	1
42	T5	1
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
2	R10	2
13	R9	1
18	R7	1
23	R8	1
27a)	R9	1
27b)	R9	2
31	R10	2
37	R7	1
43	R8	3
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
8	R14	2
11	R13	2
15	R13	4
19	R14	1
38	R14	4