GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (June 2014)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at <www.edu.gov.mb.ca/k12/assess/support/results/index.html>.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2009).

Unit A: Transformations of Functions

Conceptual Knowledge

Most students were able to correctly apply transformations to an equation or graph. Those who committed errors on transformations tended to confuse vertical and horizontal shifts with stretches. When drawing graphs of reciprocal functions, many students drew the inverse instead. Some also confused the inverse with a reflection in the x-axis. A fair number of students performed a horizontal reflection in the line $x = 1$ rather than the y-axis. When writing the reflection of a function, some students changed the function itself while others wrote the function in place of x in $f(x)$.

Many students had difficulty properly restricting the domain of a function so that the inverse was also a function. Instead of writing the restricted domain, many wrote non-permissible values.

When working with operations on functions, students did not restrict the new function to the common domain. Others multiplied the functions together instead of performing the composition of the functions.

Procedural Skill

When graphing functions, asymptotes were frequently omitted. Students also forgot to plot points, especially invariant points, for graphs to pass through.

When performing a vertical reflection on the function equation, many students did very well, but a number of students failed to multiply the negative through the entire equation.

Some students used tables of values to perform transformations on functions. This assisted in mapping points but led to some very unusual shapes as students made arithmetic errors or transcription errors.

Many students demonstrated poor function notation when applying transformations of functions.

Communication

Students seemed to know proper techniques for graphing with the most common errors being the omission of asymptotes and the failure to provide arrowheads or endpoints. When asymptotes were drawn in, a number of students did not ensure the graph demonstrated proper asymptotic behaviour.

Students also frequently drew incorrect shapes for graphs either by confusing the function with a different type of function or by failing to draw it in the right location using points that should appear on the graph.

Notation also caused difficulties for many students, specifically when they were working with function notation.

Unit B: Trigonometric Functions

Conceptual Knowledge

Given a trigonometric equation, most students were able to graph amplitude and period correctly, but some students forgot to account for the vertical reflection. Given a graph, they were able to identify the amplitude but some students had trouble accounting for the vertical shift. Students knew how to find co-terminal angles, calculate arc lengths, and determine exact values. Many students did not understand how to determine whether a given point was on the unit circle or not. They did not make the connection between the Pythagorean theorem and the unit circle.

Procedural Skill

Students struggled to provide their answers in the correct domain. Students also made many arithmetic errors. Some students mixed up sine and cosine when graphing trigonometric functions and when using trigonometric ratios. Students had difficulty using the quadrant to determine the sign of the exact value they were using. When solving for arc lengths, some students forgot to convert the given angle from degrees to radians. Students were able to use reciprocal functions but had trouble explaining in words why some had no solutions.

Communication

When graphing a trigonometric function in a restricted domain, some students used arrowheads instead of endpoints or graphed outside the given domain. Some students forgot to indicate a scale on the y-axis when graphing a trigonometric function. When solving for arc lengths, some students forgot to include final units or made rounding errors. Some students made notation errors when using trigonometric functions without including the variable θ.

Unit C: Binomial Theorem

Conceptual Knowledge

When solving questions related to binomial theorem, the majority of students were able to substitute correctly into the given formula. Some students had trouble solving the formula and made arithmetic errors. Students were confused with the substitution of $n-1$ for n in the formula for ${}_nP_r$ and often could not proceed correctly beyond substitution. Students sometimes failed to reject an extraneous root.

When solving a binomial expansion question, most students were able to substitute correctly into the formula. Some students attempted to solve for a term instead of solving for the unknown exponent of the binomial expansion. Students struggled to correctly set up the solution when looking for the exponent.

Most students were able to explain the difference between a combination and a permutation, knowing that order matters with a permutation. Some were confused about repetitions and considered repetition a permutation.

When solving a fundamental counting principle question, students did not understand the proper grouping and restrictions and some did not apply all of the restrictions.

Procedural skill

When solving questions related to binomial theorem, students were challenged by the algebra and the factorial expansion. In attempting to solve the question, students would make factoring errors that would lead to incorrect solutions. Some students used trial and error to attempt to solve binomial theorem questions.

When solving binomial expansion questions, students had trouble with exponent laws and obtained incorrect answers.

When using the new concept of Pascal's triangle, students made arithmetic errors and would sometimes be missing values.

When using the fundamental counting principle, students multiplied numbers that they had set up.

Communication

When solving questions related to binomial theorem, students had good communication. For fundamental counting questions, most students used fundamental counting principles to show their thinking. Other common communication errors included changing an equation to an expression (especially when factoring), bracket errors, and notation errors (missing factorial sign).

Unit D: Polynomial Functions

Conceptual Knowledge

Students did quite well on this unit. Some students used zero as a placeholder value for missing powers of x when doing polynomial division. A second, but less frequent, error was the failure to know how to use the given root when asked to determine the remainder of a polynomial division problem. For those students who did know what to do, about 50% used the remainder theorem while the other 50% used synthetic division to determine the remainder.

Procedural skill

When asked to write the equation of a polynomial function, most students could convert zeroes into factors and understood multiplicity and the degree of the polynomial. They did not, however, know how to incorporate the *y*-intercept into the equation of the polynomial and they failed to account for the "*a*" value in their polynomial function. Graphically, some students did not know the shape of a graph of multiplicity of 3. While most students did know what to do on each question for this unit, their work was riddled with arithmetic errors, notably adding instead of subtracting, and vice versa, in polynomial division (synthetic or otherwise), and expanding polynomials such as $(x+1)^2$ incorrectly.

Communication

A common error in this unit was students just not knowing how to express their answers, whether asked for a remainder (some wrote $\frac{\text{remainder}}{\text{divisor}}$), or a quotient (some wrote (divisor) (quotient) or even (divisor) $(\text{quotient}) = 0$). There were also some terminology errors, such as "inverses", and "factors", and just knowing to multiply factors rather than adding factors in a polynomial equation. Students also continued to change equations to expressions and vice versa.

Unit E: Trigonometric Equations and Identities

Conceptual knowledge

When solving trigonometric equations using the quadratic formula, students did not know if they were solving for θ or for $\tan\theta$. Students did very well when solving the equation and understood how to use the quadratic formula. The concept of solving an identity was well captured, but students had trouble with the algebra needed to obtain a final answer. The poor algebra or arithmetic work hindered the students' success on the identity. Overall, students did very well verifying a variable in a trigonometric equation. Students chose two methods. Some solved the equation and then compared their final answers with θ given in the question; others inserted θ into the formula and showed how the RHS was equal to the LHS. Students had tremendous difficulty finding the error in the trigonometric equation that was given on the exam. Students did not know that they could not divide by $\cos\theta$ (e.g., because $\cos\theta$ could equal 0). When finding solutions graphically, students wanted to use the amplitude or period to solve the question instead of $y = \frac{1}{2}$. Some students used the vertical translation to find the new *x*-intercepts, which is correct, but not many did this. When determining values of trigonometric ratios to substitute into sum and difference identities, most students determined the correct values but some failed to place the angles in the correct quadrants. Many determined the correct sum or difference identity to use, but made arithmetic errors. The concept of sum and difference was well captured.

Procedural Skill

When using the quadratic formula, some students made substitution errors with the formula. When working with the identity, students had trouble factoring a binomial when there was a 1 remaining. When looking for errors in the question where work was given, students guessed on the error and tried to explain their choice. When solving a solution graphically, many students tried to explain how to transform the graph instead of actually finding the solutions. The procedure of the sum and difference identity was done well, but arithmetic errors were the main problem with the students' work.

Communication

When communicating final answers, students were including all answers instead of answers in the interval given. There were some domain errors and rounding errors. Changing an equation to an expression was noticeable in the verification questions when students factored. The explanation questions were well done when students were able to find an error. They could use words to explain their mathematical reasoning. Terminology was always an issue when students were trying to explain mathematically. Students could understand the idea but would use illogical words within their explanation.

Unit F: Exponents and Logarithms

Conceptual Knowledge

When asked to use laws of logarithms, students generally did a good job with the product and quotient rules but had more difficulty using the power law. Also, some students did not know how to change into exponential form after substituting zero for x and y to solve for intercepts. Students could sketch the graph of an exponential function but did not include the horizontal asymptote or just automatically put it along the y-axis, ignoring the vertical shift. Some students did not recognize that it was an exponential function or did not know the shape and instead drew a linear function. Some students were able to solve an application of logarithmic function question but most didn't know what to do after substituting values into the formula.

Procedural Skill

When solving for intercepts, many students did not know how to evaluate logarithms properly. They were able to apply the laws of logarithms but used incorrect order of operations. When changing into an exponential form, some students did so incorrectly. Many students were unable to correctly substitute values into the future value formula. Students incorrectly equated the arguments of a logarithmic equation before using laws of logarithms to simplify to single logarithms. After solving a logarithmic equation, some students did not reject the extraneous root.

Communication

Communication errors were minimal in the questions from this unit. Students made some notation errors while solving a logarithmic equation and some students changed an equation to an expression. When graphing an exponential function, some students correctly drew the graph with correct asymptotic behaviour but failed to include the horizontal asymptote. A few students did not state the number of monthly investments as a whole number even though this was stated in the question.

Unit G: Radicals and Rationals

Conceptual Knowledge

When asked to graph a radical function given an equation, students had various shapes that were not radical functions. Students most commonly mixed horizontal and vertical stretches and some students mixed horizontal and vertical reflections. When graphing a rational function, students had difficulty finding the horizontal asymptote. When using a table of values to graph rational functions, students had various shapes that were not rational functions. Some students confused domain with range.

Procedural Skill

When stating the domain of a radical function with multiple parts, students often only answered part of the domain instead of stating both answers. When graphing rational functions, many students knew how to draw a radical shape but some students did not include one point in each section on their graph.

Communication

When explaining how to graph a radical function given a linear function, students did not know how to explain that the graph must be above the original function over the interval $[0, 1]$. Many students simply said that the graph must "curve" but were not specific to the interval between the invariant points. Some students made bracket errors or inequality symbol errors when stating domain or range. Students also made notation errors in missing a union symbol when stating a domain with multiple parts.

Communication Errors

Errors that are not related to the concepts within a question are called "Communication Errors" and these were indicated on the *Answer/Scoring Sheet* in a separate section. There was a maximum of ½ mark deduction for each type of communication error committed, regardless of the number of errors committed for a certain type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1	§ answer given as a complex fraction § final answer not stated	22.0%
E2	§ changing an equation to an expression § equating the two sides when proving an identity	29.8%
E3	§ variable omitted in an equation or identity § variables introduced without being defined	16.5%
E4	§ "$\sin x^2$" written instead of "$\sin^2 x$" § missing brackets but still implied	7.8%
E5	§ missing units of measure § incorrect units of measure § answer stated in degrees instead of radians or vice versa	21.2%
E6	§ rounding error § rounding too early	20.7%
E7	§ notation error § transcription error	39.1%
E8	§ answer included outside the given domain § bracket error made when stating domain or range § domain or range written in incorrect order	10.2%
E9	§ incorrect or missing endpoints or arrowheads § scale values on axes not indicated § coordinate points labelled incorrectly	35.8%
E10	§ asymptotes drawn as solid lines § asymptotes missing but still implied § graph crosses or curls away from asymptotes	13.4%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at <www.edu.gov.mb.ca/k12/assess/support/results/index.html>.

These reports include a chart comparing the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 27.5% of the test booklets sampled resulted in a higher score locally than those given at the department; in 17.8% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 54.7% of the booklets sampled and marked by the department received a central mark within $\pm 2\%$ of the local mark and 94.1% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 0.3% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in June 2014 were invited to provide comments regarding the test and its administration. A total of 111 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 91.3% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 93.4% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document. 97.1% of teachers indicated that the reading level of the test was appropriate and 94.4% of them thought the test questions were clear.
- 89.4% and 82.9% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 96.3% of the teachers indicated that their students used a formula sheet throughout the semester and 99.9% of teachers indicated that their students used the formula sheet during the test.
- 52.6% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 78.9% of teachers indicated that the use of a scientific calculator was sufficient for the test.