GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (January 2014)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at <www.edu.gov.mb.ca/k12/assess/support/results/index.html>.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2009).

Unit A: Transformations of Functions

Conceptual Knowledge

Most students were able to correctly apply transformations to an equation or graph. When solving for inverse equations, most students understood that switching x and y was required to determine $f^{-1}(x)$, but very few students understood how to carry through the original restriction on $f(x)$ for $f^{-1}(x)$. Most students were able to correctly graph operations on functions when two graphs were given. Most students did not understand the concept of operations on functions when given equations. Instead of evaluating functions for a given value (e.g., -1), most students incorrectly interpreted the value for a vertical reflection. When determining composite functions, some students made the mistake of confusing $f(g(x))$ for $f(x) \bullet g(x)$. When identifying the domain from a composition of functions or operations on functions, most students did not know how to restrict the domain.

Procedural Skill

When solving for inverse equations, many students forgot to consider both the positive and negative values when isolating y^2. Even though students understood the concept of graphing operations on functions, some students made arithmetic errors which resulted in an incorrect coordinate on their graph. When evaluating functions, some students made arithmetic errors that lead to incorrect final answers even though they understood the concept. When doing operations on functions, some students incorrectly used the x-values instead of the y-values.

Communication

When graphing, some students had missing arrowheads while other students made transcription errors by incorrectly plotting one point or coordinate even though their overall graph was correct. When stating the domain of a graph, some students made bracket errors or gave a domain that was incorrect for their graph.

Unit B: Trigonometric Functions

Conceptual Knowledge

Given a trigonometric equation, most students were able to graph amplitude and horizontal shift. Given a graph, they were able to identify the vertical shift, but they were not able to identify the period of the graph. Students could draw the terminal arm of a given angle very well, and most students could find a coterminal angle and convert between radians and degrees. Some students found having to do two concepts (e.g., angle conversion and finding a coterminal angle) in the same question difficult. Students knew when to use their unit circle to find exact values. They did not, however, know how to solve for a double angle very well. They were able to get the correct identity but did not know how to solve the trigonometric equation the rest of the way. When solving the square root of a trigonometric function, they often missed the negative root. When trying to solve for a double angle algebraically, they would often forget the second rotation solutions.

Procedural Skill

Students struggled to provide their answers in the correct domain. Students also made many arithmetic errors and incorrectly used the distributive law. Some students also read the graph incorrectly, which then led to incorrect answers. Students had difficulty using the quadrant to determine the sign of the exact value they were using. Students had difficulty drawing a sine graph shifted left of the y-axis, especially if they had trouble calculating the period.

Communication

Some students changed equations to expressions and then later in their work changed them back into equations. Some students made notation errors, especially when finding coterminal angles. They would start with one angle, subtract 2π, find a coterminal angle, then continue in the same line, subtract 2π again, find a new coterminal angle, and so on. Some students put the trigonometric function term in front of the exact value as they did their substitution into a trigonometric expression, demonstrating that they may not understand the conceptual difference between $\cos 30^\circ$ and its exact value, $\frac{\sqrt{3}}{2}$, for example. Students dropped their units of measure and forgot the arrowhead on their terminal arm of a given angle. They also did not show the direction of rotation for a given angle in standard position. Some students had difficulty understanding everyday contexts well enough to answer "explanation" questions.

Unit C: Binomial Theorem

Conceptual Knowledge

When solving questions related to binomial theorem, the majority of students were able to substitute correctly into the given formula. Some students had trouble solving the formula, making arithmetic errors. The students who did not substitute correctly had trouble solving because they dropped negative values before using exponent laws. Students did understand cases quite well when working with a combination. They were able to set up the question well for better understanding.

Procedural skill

When solving questions related to binomial theorem, students were challenged by the algebra and the factorial expansion. Students had trouble with exponent laws which gave them answers with extra variables that were not in the question. Students did not use the new concept of Pascal's triangle when solving for terms in a binomial expansion; they used the formula method more frequently. When used, the fundamental counting principle was well done. A common error among students was to multiply their cases at the end of their question instead of adding them. Factorial expansion was understood but students did forget terms from the expansion, which caused them to obtain incorrect final answers. Students did very well overall explaining their procedures and cases in a combination question.

Communication

When solving questions related to binomial theorem, students had good communication. Some students did not read a specific question carefully and made a communication error of not stating the final answer. Other common communication errors included changing an equation to an expression and bracket errors.

Unit D: Polynomial Functions

Conceptual knowledge

Students generally had a good grasp of the concepts connected to polynomial functions. They understood that finding the x-intercepts and understanding the end behaviour were critical for visually representing the function. They demonstrated a good understanding of how multiplicity affects the graph. Students also understood how synthetic division was required in working with various components of polynomial functions.

Procedural Skill

Students had some difficulty following through with some procedures. Incorrect synthetic division resulted in sign errors while determining the quotient and the factors, and solving for k . These errors were often compounded in the graphs as they were unable to utilize the information from the y-intercept and the end behaviour to identify and correct their mistakes.

Communication

Students generally communicated their understanding well in this unit and used proper notation, although some graphs were missing the scales on the y-axis.

Unit E: Trigonometric Equations and Identities

Conceptual knowledge

When solving trigonometric equations involving the tangent function, many students rejected the branch involving $\tan\theta > 1$ thinking there was no solution. When stating general solutions, students had many problems determining the period for the solution, stating $k\pi$ instead of $2k\pi$, or vice versa. In determining non-permissible values, most students managed to determine the required values but were unable to extend these values to a general solution. Some did not understand which expressions provided non-permissible values and so included as many expressions as possible to cover all options. When determining values of trigonometric ratios to substitute into sum and difference identities, most students determined the correct values but failed to place the angles in the correct quadrants. Many determined the correct sum or difference identity to use, but failed to understand how to make use of that identity. Working further with these trigonometric ratios in a double-angle identity, most student errors were a result of carry-over errors from their first calculation of the trigonometric ratios.

Procedural Skill

Some students required the use of a calculator because they had difficulty finding the reference angle for $\tan\theta = -1$. When proving an identity, many students engaged in random cancelling of terms in order to make the expression fit the proof. Many others substituted in identities and then stopped too early, preventing them from receiving the mark for the algebraic strategy. Others changed the expression to a double-angle identity and then had difficulty in the proof process. When applying sum and difference identities, students generally had difficulties with the entire process, failing to understand how to make use of the identity. Students also had significant difficulty with arithmetic errors in multiplying fractions.

Communication

When expressing final answers, many students had difficulty with rounding at the appropriate time to the required three decimal places. Many students forgot the variables in trigonometric functions or incorrectly wrote squared trigonometric functions as functions with double angles. When solving an equation, students changed equations into expressions and back again. Many students forgot to state $k \in \mathrm{I}$ for general solutions. Students generally substituted identities into proofs well but then had difficulties with the algebraic strategies, specifically with trying to simplify expressions. Many attempted to simplify by incorrectly dividing out what appeared to them as common factors. When working with trigonometric functions in identities, many students made the error of substituting the trigonometric ratio in place of the angle in the trigonometric function. This prevented them from continuing with the solution.

Unit F: Exponents and Logarithms

Conceptual Knowledge

When asked to solve a logarithmic equation, many students used the laws of logarithms incorrectly. Also, some students did not know how to change into exponential form. After solving, some students did not reject the extraneous root. Many students did not understand how to use two logarithms to solve for an unknown logarithm in the same base. Students could not explain the concept of changing to a common base. Some students were able to solve an application of logarithmic function question but most didn’t know what to do after substituting values into the formula.

Procedural Skill

When solving a logarithmic equation, many students distributed the logarithm through the brackets and then applied the laws of logarithms. Some students ignored logarithms altogether and just solved the equation after “dropping” the logarithms incorrectly. When changing into exponential form, some students did so incorrectly. While trying to use an exponent of ½, some students took the square root of the logarithm, rather than just the value.

Communication

Some students didn’t show both solutions of a logarithmic equation and then reject the extraneous one. Instead they only showed the correct value. Students made numerous notation errors while solving a logarithmic equation and students often changed an equation to an expression. Students did not know how to explain their reasoning or used the wrong terminology in their explanations.

Unit G: Radicals and Rationals

Conceptual Knowledge

When given a function and asked to graph its radical, students had various shapes, including the inverse. Some students had difficulty finding the range of a rational function with non-permissible values. Some students mixed horizontal and vertical shifts, as well as horizontal and vertical reflections. Also, students made no connection between the graph and the answer given.

Procedural Skill

When graphing a radical function, students did not graph it above the original function over the $[0, 1]$. Some incorrectly factored the denominator of a rational function or mistakenly simplified the expression to a linear function. Students knew shape for the most part, but some included asymptotes.

Communication

Some students made notation errors on horizontal asymptotes and also made many errors involving incorrect end points.

Communication Errors

Errors that are not related to the concepts within a question are called "Communication Errors" and these were indicated on the *Answer/Scoring Sheet* in a separate section. There was a maximum of ½ mark deduction for each type of communication error committed, regardless of the number of errors committed for a certain type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1	▪ answer given as a complex fraction ▪ final answer not stated	19.9%
E2	▪ changing an equation to an expression ▪ equating the two sides when proving an identity	19.6%
E3	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	7.6%
E4	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied	4.7%
E5	▪ missing units of measure ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	14.9%
E6	▪ rounding error ▪ rounding too early	19.2%
E7	▪ notation error ▪ transcription error	57.3%
E8	▪ answer included outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	13.4%
E9	▪ incorrect or missing endpoints or arrowheads ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	40.0%
E10	▪ asymptotes drawn as solid lines ▪ asymptotes missing but still implied ▪ graph crosses or curls away from asymptotes	5.5%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at <www.edu.gov.mb.ca/k12/assess/support/results/index.html>.

These reports include a chart comparing the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 39.9% of the test booklets sampled resulted in a higher score locally than those given at the department; in 9.7% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 50.4% of the booklets sampled and marked by the department received a central mark within $\pm 2\%$ of the local mark and 94.8% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 1.5% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in January 2014 were invited to provide comments regarding the test and its administration. A total of 118 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 88.6% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 93.5% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document. 94.5% of teachers indicated that the reading level of the test was appropriate and 91.9% of them thought the test questions were clear.
- 78.6% and 75.0% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 93.9% of the teachers indicated that their students used a formula sheet throughout the semester and 97.4% of teachers indicated that their students used the formula sheet during the test.
- 52.2% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 89.4% of teachers indicated that the use of a scientific calculator was sufficient for the test.