GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (June 2019)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Unit A: Transformations of Functions (provincial mean: 73.7%)

Conceptual Knowledge

In general, students were able to demonstrate their understanding of transformations of functions. Students were further able to determine the correct equation given a list of transformations; however, they sometimes altered the type of function in doing so. Some students had difficulty understanding composite functions, treating them instead as the multiplication of two functions. Performing operations on functions was well done overall, though some students had trouble determining the correct resulting domain. When asked to state a property of a reciprocal function given the graph of the original function, most students faltered and were unable to show their knowledge, often mixing up reciprocal and inverse functions.

Procedural Skill

When asked to state an equation after a series of transformations, students often included functional notation in their answer, despite the situation not being appropriate for it. Students were able to follow the correct procedure for showing transformations in an equation but, in many cases, they had problems with horizontal stretches and compressions. When composing functions, students sometimes performed the opposite composition, or evaluated an x-value in one function without proceeding to substitute the answer in the second function. Some students were not able to state the correct domain of a sum function or a composite function. There were some arithmetic errors and factoring errors that prevented students from showing their knowledge. When stating the equation of the asymptote for a reciprocal function, some students used the y-intercept instead of the x-intercept.

Communication

Students made some notation errors when expressing answers in this unit. When stating the equation of a function after some transformations, many students were unable to correctly use the absolute value notation. When evaluating a composite function for a given x-value, a few students expressed the value as a coordinate pair, and still others made transcription errors. Some students had trouble correctly expressing the equation of an asymptote, using the incorrect variable or incorrect notation.

Unit B: Trigonometric Functions (provincial mean: 71.1%)

Conceptual Knowledge

When asked to describe the error in an arc length formula question, most students were able to explain that the angle must be in radians. Some students mixed "radius" with "radians" or explained that the error was that the degree sign was missing from the angle. When asked to sketch an angle in standard position, students often found the correct quadrant but drew an incorrect size of angle or drew a coterminal angle. Most students understood how to find a coterminal angle. Students had difficulty analyzing trigonometric ratios. Some thought that $\csc x$ was the reciprocal of $\cos x$. When asked to evaluate a trigonometric expression, students had difficulty with reference angles, quadrant values, and the value of $\sec x$. When asked to sketch a trigonometric function, students did the opposite horizontal translation and often had an incorrect/inconsistent period. Most students were able to identify the missing x-value of a point on a sinusoidal graph.

Procedural Skill

When asked to find a coterminal angle, students had difficulty combining fractions. Students made many arithmetic errors when using the Pythagorean theorem. When asked to evaluate a trigonometric expression, students made many arithmetic errors. When asked to determine the missing x-value of a point on a sinusoidal graph, students often attempted to solve algebraically as opposed to using the given graph.

Communication

When asked to sketch an angle in standard position, some students forgot to indicate the directional arrow. When asked to evaluate a trigonometric expression, students made transcription errors by changing multiplication to addition or forgetting to square a value. When asked to sketch a trigonometric function, students forgot to label the axes or had inconsistent scale values.

Unit C: Binomial Theorem (provincial mean: 67.1%)

Conceptual Knowledge

In general, most students used the correct formula for the binomial expansion question. Students did well in using consistent factors, but some had difficulty substituting the correct combination for the coefficient. Many students missed the negative sign when substituting for the term and many used an incorrect permutation formula. When attempting a question involving factorials with restrictions, students had difficulty with the concept of grouping and/or identifying multiple cases. Many students missed the number of groups. Some students added the group arrangements rather than multiplying and some missed the factorial signs. When answering explanation questions involving binomial theorem expansion, many students used examples to explain and some made terminology errors.

Procedural Skill

In general, many students made algebraic errors when simplifying binomial expansion terms. While solving the permutation/combination question a significant number of students made arithmetic errors or had difficulty simplifying after substituting into the correct formula. Some students did not reject the impossible solution.

Communication

In general, many students made bracket errors when expanding binomial terms and/or in the permutation/combination question. Some students made notation errors, such as misplacing the factorial sign inside the brackets or completely forgetting the brackets.

Unit D: Polynomial Functions (provincial mean: 84.3%)

Conceptual Knowledge

Generally students performed well in this area. All students had a viable strategy for arriving at the solution to questions involving polynomial functions and factors. However, students commonly forgot to equate the function to zero when determining the zeros of the function. Students were able to express a polynomial function in completely factored form instead of giving zeros. In graphing a polynomial function students had difficulty finding the y-intercept and often used the a value as the y-intercept or simply omitted the y-intercept all together. Some students did not know how to determine correct zeros when given factors. Multiplicities were misrepresented on the graph, which led to incorrect end behaviour.

Procedural Skill

Both synthetic division and the remainder theorem were used appropriately to solve polynomial functions; however, many students made arithmetic errors in using synthetic division. Some students substituted an incorrect value into the remainder theorem, forgetting to take the additive inverse of the given factor. When asked to express the function in completely factored form, students did not factor the remaining trinomial either due to not recognizing that it is factorable or not even attempting to factor. Students forgot to write the initial factor given in the question along with the remaining factors. In graphing polynomial functions, students had difficulty calculating the y-intercept.

Communication

Students demonstrated many versions of synthetic division, at times combining long division and synthetic division, but obtained a correct solution. Although there were times when the zero was not declared as the additive inverse, if the students then subtracted the coefficients and products, a correct answer was achieved. A marked improvement on indicating factors and not zeros was noted when factors were required. When graphing, students often forgot to indicate the scale specifically on the y-axis. The y-intercept was often omitted.

Unit E: Trigonometric Equations and Identities (provincial mean: 63.1%)

Conceptual Knowledge

When asked to solve a trigonometric equation over the interval $[0, 2\pi]$, many students included the general solution in their answer. Other students gave their solutions in degrees instead of radians. Overall, students were able to substitute the correct Pythagorean identity as required but made algebraic errors when isolating the trigonometric function. When asked to describe an error in a solution that did not include the general solution, most students knew conceptually what the error was but had difficulty using words to describe it. Many students substituted appropriate identities in order to prove a trigonometric identity, but some students struggled with a logical process to prove it. When asked to verify that a trigonometric equation was true for a specific angle, many students proved the identity for all permissible values.

Procedural Skills

Students had difficulty factoring when solving quadratic trigonometric equations. Some students did not solve for values of θ after solving for a value of $\cos\theta$ that was not the value for a special angle. When asked to describe an error in a solution that did not include the general solution, some students incorrectly stated that k was an element of the real number set rather than an element of the set of integers. Many students knew they needed to substitute identities in order to prove a trigonometric identity but did not substitute correct quotient and/or reciprocal identities. When proving trigonometric identities, some students had difficulty with algebraic strategies. Students also did not recognize a common factor in the identity and were unable to reduce correctly. Some students had difficulty using the Pythagorean Theorem when asked to determine the exact value of a trigonometric function using a difference identity. Other students substituted the trigonometric ratio in for the angle rather than the trigonometric function.

Communication

Students often changed an equation to an expression when solving equations and/or using a difference identity. Some omitted variables while solving a trigonometric equation and/or proving a trigonometric identity. Solutions to a trigonometric equation were not always presented correctly to three decimal places. When asked to verify that a trigonometric equation was true for a specific angle, many students did not state a final answer by equating both sides at the end of their verification. There were also various notation errors throughout the questions in this unit.

Unit F: Exponents and Logarithms (provincial mean: 67.6%)

Conceptual Knowledge

When asked to use laws of logarithms, students generally did well with the product, quotient, and power laws. However, many students had trouble working with the *e* value in the logarithmic word problem. They did not use logs and were unable to apply the law correctly to solve the problem. They also mixed up an argument that was a sum of terms with an argument that was a product of terms. When solving a logarithmic equation, some students did not apply the exponential theorem correctly. Also, when solving an exponential equation that can easily be switched to common bases, some students used logarithms, which made it very difficult to find the final answer correctly. Students were unable to relate the domain of a logarithmic equation to the *y*-intercept.

Procedural Skill

Students did not know what to do with the *e* in the equation and many simply omitted it when they used their calculators. When graphing an exponential equation, students appeared to know how to use transformations but, in many cases, they shifted left instead of right or they did not shift the horizontal asymptote. When stating the domain, many students included the asymptote in their answer. With exponential equations, students made some arithmetic errors with the bases and distribution of the brackets.

Communication

Communication errors were minimal. Students made some rounding errors and some did not include units. When graphing an exponential function, some students correctly drew the graph with correct asymptotic behaviour but failed to include the horizontal asymptote and did not label the *x* and *y* axes. Some students were unclear in their explanations leading to a deduction for lack of clarity.

Unit G: Radicals and Rationals (provincial mean: 67.8%)

Conceptual Knowledge

Most students knew to restrict the domain when graphing the radical function, but many had incorrect points on the graph. When graphing the rational function, many students gave incorrect shapes, not including a point in each portion of the graph. Many students had difficulty using correct terminology when answering the explanation question. They referred to transformations instead of considering rational functions. When asked to determine the coordinates of the point of discontinuity (hole) on the graph of a rational function, students just gave the *x*-value and missed the *y*-value in their answers.

Procedural Skill

When asked to determine an equation based on the given graph of a radical function $f(x)$, students were able to identify the transformations, but often included *f* in their answers or made a concept error by giving an incorrect function without a radical. When asked to describe how to use transformations to determine the domain of the transformed function, students simply described the transformations without describing the effect on the domain.

Communication

Many students missed the horizontal asymptotes when graphing the rational function. In answering the description question, students used vague wording resulting in a loss of half a mark for lack of clarity.

Communication Errors

Errors that are not related to the concepts or procedures are called "Communication Errors" and these were tracked on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in steps leading to final answer	24.3%
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity	18.5%
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	8.0%
E4 brackets	▪ " $\sin x^2$ " written instead of " $\sin^2 x$ " ▪ missing brackets but still implied	11.3%
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	12.7%
E6 rounding	▪ rounding error ▪ rounding too early	25.7%
E7 notation/transcription	▪ notation error ▪ transcription error	32.6%
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	9.4%
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	17.8%
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes	16.8%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

These reports compare the local marking results to the results from the departmental re-marking of sample test booklets.

Provincially, 40.9% of the test booklets sampled resulted in a higher score locally than those given at the department; in 8.4% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 50.8% of the booklets sampled and marked by the department received a central mark within $\pm$ 2.0% of the local mark and 93.1% of the sampled booklets were within $\pm$ 6.0%. Scores awarded at the local level were, on average, 1.6% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in June 2019 were invited to provide comments regarding the test and its administration. A total of 122 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 95.7% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 100% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document and that the reading level of the test was appropriate. 98.3% of the teachers thought that the test questions were clear.
- 96.7% and 91.2% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 98.3% of the teachers indicated that their students used a formula sheet throughout the semester and 99.2% of teachers indicated that their students used the formula sheet during the test.
- 39.7% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 96.7% of teachers indicated that the use of a scientific calculator was sufficient for the test.