GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (June 2018)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Unit A: Transformations of Functions (provincial mean: 73.3%)

Conceptual Knowledge

Many students did not recognize the notation of composition of functions. Some students did not understand that when transforming a function, the answer should still be a function. They missed the f in the transformed function. Other students mixed up horizontal stretch with horizontal compression. Most students knew how to multiply functions, although some divided instead of multiplying. Some students did not understand the meaning of $\left(\frac{f}{g}\right)(x)$. A number of students did not know that $f^{-1}(x)$ is the inverse of $f(x)$ and instead treated it as the reciprocal.

Procedural Skill

When students were asked to write $g(x)$ as a transformation of $f(x)$, they often included g in the function being transformed. Some students did not correctly restrict the domain of composite functions. Many students had difficulty isolating variables, and sometimes omitted brackets, leading to algebraic mistakes.

Communication

Students made many notation errors when the answer involved functions. They were able to describe in words how to sketch the graph of the absolute value function of a given graph, but often lacked clarity in their answers.

Unit B: Trigonometric Functions (provincial mean: 69.0%)

Conceptual Knowledge

Many students were able to use an angle in standard position and the length of a radius to find the arc length. Some students did not convert the angle from degrees to radians before substituting into the formula or did not convert correctly. Other students had difficulty with an angle that was larger than 360 degrees and found a coterminal angle before solving for the arch length. The majority of students were able to find the value of sine using the Pythagorean identity. However, some students did not consider the quadrant in order to write a negative answer. Others did not recognize that the Pythagorean identity was needed to solve for sine and instead gave an exact value from the unit circle. When sketching an angle in standard position, some students confused radian with π. Most students were able to sketch the shape of a sinusoidal function but some struggled with finding the period given the equation. Students understood the concept of reciprocals when determining exact values; however, some confused cosecant with secant.

Procedural Skill

When solving for the arc length, some students forgot to multiply by π when changing their exact value into approximate radians, which resulted in an incorrect final answer. Many students sketched a triangle in order to solve for sine when given the value of cosine. When asked to solve for a coordinate of a given point on the unit circle, some students gave their answer equal to the variable y-value. A few students made arithmetic errors when using the Pythagorean identity. When sketching an angle in standard position most students were able to sketch an angle in the correct quadrant but some students did not consider an appropriate angle. When sketching a sinusoidal function, many students forgot to consider the restricted domain, failing to sketch enough of the graph or including arrowheads and sketching more than was required. Some students were able to find the period but incorrectly included π in the horizontal scale.

Communication

Many students had difficulty rounding the arc length to the correct number of decimal places, with most students only writing two decimal places. As well, some students forgot to include units for the arc length and a few students incorrectly stated the units as degrees. When sketching an angle in standard position many students did not draw the directional arrow to indicate a positive rotation or forgot to include the arrowhead. A few students included vertical asymptotes with their sinusoidal graph. When stating exact values on the unit circle, some students omitted the variable theta from their answers by simply stating cosine equals the value. When substituting into the Pythagorean identity, some students did not include brackets around negative values.

Unit C: Binomial Theorem (provincial mean: 69.9%)

Conceptual Knowledge

Generally, the binomial theorem questions were well done. Most students knew when to use combinations and when to use the fundamental counting principle. However, many students did not know what to do when a restriction was involved in the problem. Most students were able to determine the next row of Pascal's triangle, but a few either missed a middle term or forgot 1 at the beginning and end of the row. Most students could use the formula to determine a specific term of a binomial expansion, but some struggled to simplify the equation or simply forgot to simplify it. When asked to simplify a combination, many students were confused as they were expecting an equation. Some even changed the problem into an equation.

Procedural Skill

Some students added combinations instead of multiplying, and added or factorialized the elements of the fundamental counting principle instead of multiplying. When expanding factorials, some students counted up instead of counting down and used incorrect notation. Some students did not know the power rules for exponents, so while they could substitute into the formula to determine a term correctly, they could not simplify it. When there was a negative variable term, many students did not know what to do with the negative sign. Students made numerous arithmetic errors, even with access to calculators.

Communication

A few students made bracket errors and notation errors when expanding factorials and simplifying the coefficient. A few students forgot to identify their answer, especially when work was scattered on the page or when going further than necessary. While students seemed to understand the concept of arrangements with identical elements, they lacked clarity in their explanations of the concept. When simplifying a combination, a few students changed the expression to an equation in an attempt to "solve" rather than "simplify".

Unit D: Polynomial Functions (provincial mean: 67.5%)

Conceptual Knowledge

When asked to factor, students often solved for the zeros. In explain/describe questions, some students solved the problem rather than using words to explain/describe. Many also simply drew the graph rather than using words to explain. Many students were not able to sketch the graph of a quartic polynomial function. Some students did not realize that they needed to reflect the graph and include an x-intercept and y-intercept of zero when sketching the graph of the function beginning with "$-x$".

Procedural Skill

Students often set up synthetic division incorrectly. A common error was to miss a zero term. Students struggled when factoring quadratics. Students often used incorrect terminology. They simply stated that the zero was the opposite instead of solving when the factor is equal to zero. Students were unable to reflect a graph of a polynomial and did not demonstrate a restricted domain correctly. Students were often unable to read information of the graph correctly in a contextual problem.

Communication

Students did not fully factor a function as required. They set the function equal to zero when they were not asked to solve. For explain/describe questions, students often answered the questions by solving or graphing rather than using words to explain or describe. When asked to describe the end behaviour of a polynomial function, many students only described one side of the graph. When graphing, the scale on the axis were often not included and the domain and range were not restricted correctly.

Unit E: Trigonometric Equations and Identities (provincial mean: 62.7%)

Conceptual Knowledge

Many students had difficulty recognizing the correct identity. They were not able to make the correct combination of angles. Many students confused $\sin(2x)$ with $\sin^2 x$. Some struggled with finding the solutions in the given domain. Others were not able to use appropriate strategies to prove the identity. Some students did not know the exact values of special angles in appropriate quadrants and the reciprocal of $\sec(x)$.

Procedural Skills

Students had difficulty recognizing the factored form of a quadratic trigonometric equation. When proving trigonometric identities, some students had difficulty with the algebraic strategies, such as making a common denominator, splitting an angle into two, or cancelling trigonometric ratios.

Communication

Many students gave solutions outside the domain. When solving equations, they often changed an equation to an expression, and omitted or interchanged variables. Some students did not simplify their final answers or stated the final answer in degrees rather than an equation.

Unit F: Exponents and Logarithms (provincial mean: 67.2%)

Conceptual Knowledge

When presented with an exponential equation, many students understood when to use logarithms and when to change to a common base in order to solve. Some students did not apply logarithms and tried a “guess and check” method or calculator strategy. Many students were able to correctly solve a logarithmic word problem involving compound interest. Some students did not know how to take the logarithm of both sides of an equation. When working with logarithmic expressions and equations, many students had difficulty using the product and quotient laws. Some students tried to solve a logarithmic expression as if it were an equation. Some students did not know how to equate arguments when solving a logarithmic equation. When students were presented with an equation of an exponential function with a vertical translation, many mistook the horizontal asymptote for the y-intercept. When asked to sketch the graph of a logarithmic function with a horizontal translation, many students were missing the vertical asymptote.

Procedural Skill

When solving an exponential equation using logarithms, some students did not show all of their work, which made it difficult to recognize whether or not they understood the concept. A few students used poor algebraic strategies when solving for the variable. When solving a logarithmic word problem involving compound interest, some students were unable to properly substitute the interest rate into the equation. A few students did not recognize what to use for the compounding period. Some students made arithmetic errors when simplifying logarithmic expressions. When asked to explain the difference in solving two exponential equations, many students were able to state the difference but used incorrect terminology and lacked clarity in their explanation. When graphing a logarithmic function, many students did not show two correct points, and some did not recognize that the function should be increasing. Some students cancelled or divided out "log", and made arithmetic errors when solving a logarithmic equation.

Communication

When solving a logarithmic word problem involving time as a final answer, some students did not recognize that their solution must be rounded to three decimal places and, as a result, rounded down to the nearest whole number. Some students incorrectly showed the division of both sides of a logarithmic equation by putting a line under the equal sign. When expressing logarithms, some students made bracket errors. On their graph of a logarithmic function, many students were missing an arrowhead. After incorrectly solving a logarithmic equation, some students did not reject their impossible solution, which resulted in a negative argument or argument of zero.

Unit G: Radicals and Rationals (provincial mean: 68.1%)

Conceptual Knowledge

When asked to sketch the graph of a radical function, many students applied a vertical stretch rather than a compression. Some students incorrectly applied a vertical reflection over the x-axis as opposed to a horizontal reflection over the y-axis. In a few cases, students used a table of values to graph, rather than demonstrating knowledge of graphing using transformations. Other students did not understand what a radical graph should look like and sketched a variety of other shapes. When asked to state the equation of the horizontal asymptote of a rational function, some students stated the equation of the vertical asymptote(s) and stated the equation of the horizontal asymptote when asked to state the equation of the vertical asymptote. Other students confused asymptotes with intercepts. Most students understood that vertical asymptotes occur when the denominator is equal to zero, but many had difficulty understanding how to determine the equation of a horizontal asymptote. When asked to describe what the non-permissible value of a rational function (determined from an operation of division on two functions) represents on the graph, many students gave a general definition of non-permissible value, rather than describing the point of discontinuity. When sketching the graph of a rational function, some students sketched a horizontal asymptote but also incorrectly added a vertical asymptote to their graph creating incorrect shapes. Given the domain and range and asked to determine the equation of a radical function, some students forgot to include the radical symbol or incorrectly used functional notation.

Procedural Skill

When sketching the graph of a given radical function, some students did not include two correct points while others exaggerated the shape between invariant points that resulted from incorrect transformations. Students also did not show two correct points while sketching the graph of a rational function. Some students did not state the equations of vertical and/or horizontal asymptotes properly. This included errors such as using abbreviations HA or VA.

Communication

When sketching the graph of a rational function, some students knew the graph should approach a horizontal asymptote at the y-axis but did not sketch the asymptote. A few students displayed improper asymptotic behaviour when their graphs crossed or curled away from the asymptote. Some students used poor notation when stating the equations of vertical and/or horizontal asymptotes. This included errors such as $x \neq$ value and $y \neq$ value rather than using the equal sign.

Communication Errors

Errors that are not related to the concepts or procedures are called "Communication Errors" and these were tracked on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in steps leading to final answer	21.8%
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity	12.4%
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	12.2%
E4 brackets	▪ " $\sin x^2$ " written instead of " $\sin^2 x$ " ▪ missing brackets but still implied	20.8%
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	13.3%
E6 rounding	▪ rounding error ▪ rounding too early	32.6%
E7 notation/transcription	▪ notation error ▪ transcription error	39.3%
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	48.7%
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	23.5%
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes	11.1%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

These reports compare the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 41.0% of the test booklets sampled resulted in a higher score locally than those given at the department; in 9.1% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 59.1% of the booklets sampled and marked by the department received a central mark within $\pm 2\%$ of the local mark and 92.6% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 1.6% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in June 2018 were invited to provide comments regarding the test and its administration. A total of 122 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 95.0% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 97.5% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document. 99.1% of teachers indicated that the reading level of the test was appropriate and 96.6% of them thought the test questions were clear.
- 94.8% and 82.9% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 96.7% of the teachers indicated that their students used a formula sheet throughout the semester and 98.4% of teachers indicated that their students used the formula sheet during the test.
- 45.5% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 94.8% of teachers indicated that the use of a scientific calculator was sufficient for the test.