GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (June 2017)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, and changes to demographics. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Summary of Test Results (Province)

June 2017	January 2017	June 2016	January 2016	June 2015	January 2015
67.5%	68.8%	66.6%	66.0%	69.5%	64.5%

Unit A: Transformations of Functions (provincial mean: 67.1%)

Conceptual Knowledge

In general, students knew how to apply transformations on functions. However, they were confused between reciprocal and inverse functions. When asked to graph $y = \frac{1}{f(x)}$, many students would graph $y = f^{-1}(x)$. When students were asked to determine algebraically if two given functions were inverses of each other, they plugged in points to test instead.

Procedural Skill

Students generally knew stretches and reflections but did not know how to do them in the correct order. When sketching the reciprocal graphs, students often forgot to pass through invariant points. When sketching the graph of a given function, arrowheads were often missing. Students made many algebraic errors when trying to solve for *y*.

Communication

Students were generally able to describe the transformations using appropriate vocabulary. There were many communication errors made when determining the composite of functions (e.g., missing brackets). Some students did not use words to answer questions that asked them to either describe or explain. Many answers lacked clarity (e.g., when identifying coordinate values, students often used the word "everything", without specifying which value they were describing—*x*-value or *y*-value).

Unit B: Trigonometric Functions (provincial mean 66.5%)

Conceptual Knowledge

In general, students were able to use the equation, $s = \theta r$; however, some students confused radius and diameter and others did not convert degrees to radians when using this equation. When determining coterminal angles or reciprocal functions, students gave many incorrect values. Some tried to use sum/difference identities to determine coterminal values. When solving the sum/difference identities question, students substituted correct values into incorrect equations. Confronted with an obvious Pythagorean Theorem question, most students could find the missing side and write trigonometric functions correctly using the side lengths; however, many failed to consider the quadrant for which they were answering. When given a value such as 3 radians, students misunderstood this to be 3 rotations, or 3π radians. Students had difficulty graphing trigonometric functions, particularly with the horizontal shift, period, and amplitude.

Procedural Skill

Most students knew how to use the Pythagorean Theorem to determine the missing side. They also knew that $\csc\theta$ was the reciprocal of $\sin\theta$. They did not remember to check the quadrant signs. Arithmetic errors were numerous, especially when dividing a fraction by a fraction. When graphing trigonometric functions, some students had the correct work shown, but could not graph correctly using that work, especially the period of the graph. They also had difficulty creating a correct *x*-axis scale on the graph.

Communication

When students determined the arc length, they often forgot to include units of measure, and some rounded incorrectly. When writing trigonometric functions, they still had notation errors, for example, writing $\sin$ instead of $\sin\theta$. Negative signs for quadrants appeared and disappeared randomly. Students often changed an equation to an expression, and did not use brackets correctly. Students did not always simplify their final answer. When drawing a graph, many students forgot their scales on the axes. Many graphs were not very accurate and did not stay within the correct range once translated.

Unit C: Binomial Theorem (provincial mean 74.7%)

Conceptual Knowledge

Most students were able to correctly solve a combination question, but some used permutations instead of combinations. For an "and" situation, some students added the cases instead of using multiplication. When required to solve an equation involving a permutation, some students were unsure how to expand a factorial and/or how to cancel factorial notations. Some students did not know how to determine the number of ways that people can sit together. Instead, they solved for the number of ways that people cannot sit together. When solving questions involving binomial theorem expansion, students were able to substitute correctly into the given formula but many were unable to identify the correct term they were solving for. Overall, students did very well when required to give a row in Pascal's Triangle.

Procedural Skill

When using algebra to determine a term in a binomial expansion, some students failed to apply the exponent laws correctly, which led to the incorrect exponents. Some students made algebraic errors when trying to simplify their answers. When required to give the next row of Pascal's Triangle, some students multiplied the numbers in the previous row rather than adding them up. When plugging into the Binomial Theorem, some students did not raise the coefficients inside the brackets to the exponent, which led to the incorrect coefficient in their final answer.

Communication

When expanding factorials, some students made notation errors such as misplacing the factorial sign inside the brackets or forgetting the brackets altogether. When solving factorial questions, some students did not reject the extraneous value of n because they cancelled off the n without accounting for the non-permissible value. When using the binomial theorem, some students did not completely simplify their final answer by multiplying all parts of the expansion together. When solving a problem involving the permutation formula, many students changed an equation to an expression. When asked to give the next row of Pascal's Triangle, some students listed too many rows and did not indicate their final answer.

Unit D: Polynomial Functions (provincial mean: 69.1%)

Conceptual Knowledge

When asked to determine the value of the leading coefficient of the graph of a polynomial function, most students were able to correctly identify the binomial factors from the zeros of the function but did not include the multiplicity when the graph flattened out and crossed the x-axis. Most students substituted 0 for x when replacing y with the value of the y-intercept. When asked to express a polynomial function as a product of factors, many students were able to correctly identify one possible value for x, using the remainder theorem. Most students were then able to use the process of synthetic division; however, some did not include their first value in the product of factors. Other students were able to correctly use alternate strategies such as long division and/or factor theorem to identify the zeros of the function, which enabled them to express the polynomial function as a product of factors. When asked to describe a difference between two cubic functions with the same binomial factors (same multiplicity) and same lead coefficients with opposite signs, most students were able to describe that they had different end behaviours, different y-intercepts, or that one graph was the vertical reflection of the other. Some students incorrectly stated that one graph was a reflection over the y-axis or that one graph opened up and the other opened down. When asked to describe the relationship between the zeros of a function, roots of the corresponding equation, and x-intercepts of the corresponding graph, some students only mentioned the relationship between two of the three. Other students only focused on the multiplicity within the given polynomial function.

Procedural Skill

Some students were able to correctly identify the leading coefficient of the graph of a polynomial function but did not know how to solve algebraically for this value. Some students struggled with the synthetic division procedures, making numerous procedural and/or arithmetic errors. Numerous students stated that the polynomial function was equal to the quotient of their synthetic division. Some students solved for the zeros of the function rather than describe the relationship with the roots and x-intercepts.

Communication

Some students changed an equation to an expression when trying to solve for a leading coefficient. There were many notation errors when using the remainder theorem as students forgot to substitute the value of x into $p(x)$ as well as into the equation for $p(x)$. Many students described the difference between two polynomial functions by referring to the quadrants in which they would be sketched; however, some of them incorrectly stated in which quadrants the graphs would be sketched.

Unit E: Trigonometric Equations and Identities (provincial mean: 64.8%)

Conceptual Knowledge

Students generally had difficulty solving a trigonometric equation algebraically. Overall, students were able to prove the identity by correctly substituting the double angle identities. Some students were able to identify the error in an incorrectly solved trigonometric equation. When students were required to solve a trigonometric equation using the substitution of an identity, most were able to use the appropriate identity to solve but omitted the general solution. Most students experienced difficulty when required to verify that a specific angle was a solution to an equation. They attempted to solve the question or gave an answer without any supporting work, rather than show verification. When students were required to solve a trigonometric equation with a reciprocal identity, most students understood the concept of solving for the reciprocal function but experienced difficulty determining the correct reciprocal and, as a result, the correct solution. Most students had difficulty when required to determine the exact value of an angle that required using a sum/difference identity. In this case, most students were able to determine the correct combination required but struggled to use these values correctly in the equation.

Procedural Skills

When solving trigonometric equations students had difficulty factoring the equation in order to solve. Many students were unsure how to work with the branch that was not an exact value on the unit circle. When proving the identity, many students omitted the brackets when substituting the identity and, as a result, were unable to complete the proof. Students had difficulty with appropriate algebraic strategies. When solving a trigonometric equation that required taking the square root to solve for the trigonometric function, many students did not include two branches ($+$ and $-$) and only provided half of the solutions. Students made many arithmetic errors in verifying a solution for an equation, incorrectly substituting exact values without accounting for the quadrant of the angle.

Communication

Students commonly interchanged θ and x when solving equations or omitted the variable throughout their work. They made many notation errors in solving trigonometric equations. Some students did not state their solutions as an equation. Students had difficulty when required to describe the error in an incorrectly solved equation. When determining the exact value of an angle not on the unit circle, students often changed from an equation to an expression, did not use brackets correctly, and did not simplify their final answer.

Unit F: Exponents and Logarithms (provincial mean: 73.2%)

Conceptual Knowledge

When asked to solve a logarithm problem involving an exponential formula, many students did not substitute correctly into the given equation. Some students used a guess and check method to find the solution instead of correctly applying laws of logarithms and using algebraic strategies. When asked to describe how a value that is added or subtracted from the argument in a logarithmic equation affects the asymptote, some students described the domain of the graph instead of the behaviour of the asymptote. When solving an exponential equation algebraically, some students failed to recognize that the bases needed to be changed to a common base. Instead, they applied logs to the exponential equation but then were unable to simplify the equation or incorrectly divided out the logarithms to cancel them. When solving a logarithmic equation algebraically to find an unknown base, most students understood how to apply the product law and how to change the logarithmic equation to exponential form. Some students did not understand the power law and instead divided the coefficient to simplify the equation. When finding the x-intercept of an exponential equation with a base of e, some students incorrectly solved for the y-intercept. Other students were able to correctly substitute 0 for y but did not know how to evaluate the natural logarithm of 1. Some students were confused by the base of e and did not recognize that any base to an exponent of zero would always result in an answer of 1.

Procedural Skill

Some students were able to correctly substitute into logarithmic equations but struggled when applying logarithms and using algebra to isolate the unknown variable. Many students understood how to solve an exponential equation by changing to a common base and were able to correctly apply laws of exponents to find a solution. Some students made arithmetic or procedural errors when changing the equation to a common base, which led to incorrect final answers. When solving a logarithmic equation to find an unknown base, some students made arithmetic errors in their work that resulted in impossible bases. Some students did not recognize how to evaluate a logarithmic expression and simply applied the quotient law to write the expression as a single logarithm. Other students struggled with applying the quotient law and subtracted the arguments instead of dividing them.

Communication

Some students struggled to round their answers correctly to a whole number value in the logarithm word problem. Some students did not understand the concept of rounding up, regardless of the decimal, in order to ensure the “minimum” requirement of the problem was met. Many students were unclear when describing the behaviour of an asymptote in a logarithmic equation. Students did not describe the asymptote in relation to the base graph, without including the movement left or right–some students simply stated that the asymptote would only move to the right. Other students did not mention that the asymptote in a logarithmic equation would be vertical. Some students were missing brackets around the argument of a logarithm when simplifying a logarithmic equation using the quotient law but still were able to correctly solve to find the missing base. When finding the x-intercept of an exponential equation with a base of e, some students did not realize they needed to give a numerical answer and their final answer was incorrectly stated as a natural logarithm. When evaluating a logarithmic expression, some students introduced a variable to change the expression to an equation without defining the variable.

Unit G: Radicals and Rationals (provincial mean: 68.4%)

Conceptual Knowledge

When asked to identify a solution from a graph, many students included both the x- and y-values instead of just the x-value. Students frequently mixed up vertical and horizontal reflections when matching radical graphs to their equations. When asked to sketch a radical function from an existing graph, many students were not able to restrict the domain or to properly sketch the resulting radical function. When asked to graph a rational function from an equation, most students were able to draw proper vertical and horizontal asymptotic behaviour but many included an extra vertical asymptote, resulting in an improper shape of graph. Most students were able to correctly state the range of the graph they had drawn. When asked to identify the transformations applied to a graph, most students were able to identify the correct transformations but wrote them in the incorrect order.

Procedural Skill

When graphing a rational function, many students did not identify correct points on the graph or did not include one point in each section of the rational graph. When graphing radical functions, some students incorrectly included arrowheads in their final graph and many students had an incorrect shape of graph between the invariant points.

Communication

When graphing a rational function, students often did not show their horizontal asymptotes, especially when the asymptotes were on the x-axis, and many did not indicate scale values on the axes. Some students had difficulty using the correct brackets when stating the range of a rational function. When explaining how to determine the horizontal asymptote of a rational graph, students' responses lacked clarity and many had terminology errors. Some students gave examples instead of using words when answering questions that required explaining or describing.

Communication Errors

Errors that are not related to the concepts or procedures are called "Communication Errors" and these were tracked on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated	20.7%
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity	36.4%
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	22.2%
E4 brackets	▪ " $\sin x^2$ " written instead of " $\sin^2 x$ " ▪ missing brackets but still implied	14.6%
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	15.2%
E6 rounding	▪ rounding error ▪ rounding too early	53.4%
E7 notation/transcription	▪ notation error ▪ transcription error	36.5%
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	12.8%
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	22.8%
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes	18.5%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

These reports include a chart comparing the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 43.8% of the test booklets sampled resulted in a higher score locally than those given at the department; in 13.2% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 43.0% of the booklets sampled and marked by the department received a central mark within $\pm 2\%$ of the local mark and 90.5% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 1.5% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in June 2017 were invited to provide comments regarding the test and its administration. A total of 114 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 94.6% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 99.1% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document. 95.4% of teachers indicated that the reading level of the test was appropriate and 94.4% of them thought the test questions were clear.
- 94.5% and 93.9% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 98.2% of the teachers indicated that their students used a formula sheet throughout the semester and 98.2% of teachers indicated that their students used the formula sheet during the test.
- 52.3% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 92.7% of teachers indicated that the use of a scientific calculator was sufficient for the test.