GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (January 2020)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Unit A: Transformations of Functions (provincial mean: 75.6%)

Conceptual Knowledge

Some students did not understand the concept that a vertical reflection is a reflection over the *x*-axis, not the *y*-axis. When asked to do the absolute value of $f(x)$, some students thought that they should do an absolute value graph rather than take the absolute value of an already existing function. Students applied transformations in an incorrect order. They used the terms "compression" and "stretch" incorrectly. When given an equation and asked to describe the transformations, students sometimes mixed vertical transformations with horizontal transformations and had the incorrect order when describing more than one vertical transformation. When asked to do a composite function, some students multiplied the functions. When asked to explain why the domain did not have a restriction, students described the graph rather than discuss the domain or restrictions of the graph. When asked to reflect a graph over the line $y = x$, some students did a reflection across the *y*-axis.

Procedural Skill

Students had difficulty sketching reflection graphs, not always following the shadow graph. Students did not know how to describe a horizontal stretch by a factor, and would describe transformations in the incorrect order. Where arithmetic was necessary to simplify a composite

function, some made errors with their simplification. Students knew that a reflection must have a negative sign, but did not know where to put it in the final statement. When given a random shape to reflect across $y = x$, students would move the individual points but not connect them in the same order as the original graph, so did not end up with an inverse relation.

Communication

Students needed to pay more attention to details such as arrowheads and endpoints. When asked to state $g(x)$ after a reflection of $f(x)$, students used inappropriately both f and the $\sqrt{\ }$ sign from the function of $f(x)$ in their statement of $g(x)$.

Unit B: Trigonometric Functions (provincial mean: 70.6%)

Conceptual Knowledge

Students made a good connection to trigonometric functions for all of the questions. They were able to correctly convert degrees to radians. Students were able to identify asymptotes of a tangent function, but many had difficulty putting into words the notion of an infinite range. Students evaluated the trigonometric function and substituted this for theta. Although this is a concept error, it was treated as a procedural error if students corrected their mistakes in subsequent lines. Students had difficulty understanding period when it was expressed in radians without π and missed π in their calculation of the B value. Students also omitted the angle variable in many cases when a B value was given. Students had difficulty correctly calculating D in the case of transformations of trigonometric functions when a minimum value and amplitude were given.

Procedural Skill

Students equated the change of slope at the zeros of the tangent function to a cubic function. For the double angle identity, many students did not know to plug a combination of values into the double angle identity from the formula sheet. In many cases, however, students knew that they needed to find exact trigonometric values. There were a few mistakes made equating trigonometric function values to the exact ratio. Students had a good understanding of both value and quadrant. Some errors were noted with regards to adding and subtracting fractions. Students had difficulty understanding that to find an unknown angle in a triangle they needed to subtract the known angles from $180°$. Many students thought that the sum of interior angles of a triangle equals $360°$, $270°$ or something other than $180°$.

Communication

It was sometimes difficult for students to articulate the mathematical explanation without the use of a graph or math. Students made communication errors equating an expression to an equation and vice versa when substituting exact trigonometric values. Students had difficulty understanding the interval given, with arrows or part of the graph outside of that interval. Students gave co-terminal angles outside of the interval given.

Unit C: Binomial Theorem (provincial mean: 69.7%)

Conceptual Knowledge

In general, students did not use the binomial expansion formula correctly. They did well determining the coefficient and the first consistent factor, but not the second factor. Many students missed the negative sign when substituting for the second term.

When attempting the combinations question, many students had difficulty with the concept of different cases. Students struggled to deal with the restrictions or tried to use permutations. Some students added the number of groups in each case instead of multiplying them, or they multiplied the outcomes of both cases instead of adding them up.

Many students were trying to solve the permutation question by guess and check, not algebraically. Some students did not understand the definition of permutation, or they could not substitute and/or expand factorials.

Procedural Skill

In general, simplifying exponents was done poorly. Many students made algebraic errors when simplifying binomial expansion terms, (e.g., dropping negative sign).

When solving the permutation/combination question, a significant number of students made arithmetic errors or had difficulty simplifying after substituting into the correct formula (e.g., factorial being expanded in the wrong way). Some students did not reject the extraneous value. Many students had difficulty factoring, and some tried to use the quadratic formula to factor, but did not realize that non-whole numbers would not be permissible values.

Communication

Students made many communication errors. For example, they changed an equation to an expression or vice versa (E2) and made a notation error by dropping the "!" symbol (E7). Some missed brackets while trying to solve algebraically (E4).

Unit D: Polynomial Functions (provincial mean: 82.8%)

Conceptual Knowledge

Students could factor a polynomial function with a variety of strategies but the most common error was that students missed the zero in their work. The a values of polynomial functions and equations were often misused and/or not calculated. This led to many polynomial function graphs having incorrect y-intercepts and many equations stated without proper a values in their factored form. Multiplicities were generally well represented in both function and equation form.

Procedural Skill

Although students were able to use a variety of strategies to find the factored form of a polynomial function, some had small arithmetic errors in their synthetic division. Some students identified a factor, but used it incorrectly when performing the remainder theorem by reversing the factor's sign. When graphing polynomial functions, some students reversed the x-intercepts and, overall, the shape of the multiplicities was inconsistent, some having smooth curves and

others having sharp corners. Small arithmetic errors were found in students' work when finding a values.

Communication

Some students misrepresented their answer by setting their entire polynomial to zero and not stating the fully factored form. Others just didn't indicate a final answer even though they had the correct procedure and work above. When graphing a polynomial function, many students did not indicate their scale axis and/or forgot arrowheads on their curves.

Unit E: Trigonometric Equations and Identities (provincial mean: 63.9%)

Conceptual Knowledge

Students used good logical process and correct identities to prove the identity. They were able to substitute correct values into the verification question, but sometimes had the incorrect quadrant. Students could indicate on a graph where to find at least one solution to a trigonometric equation. Students struggled with solving the trigonometric equation when they had to use the quadratic formula to solve for $\sin\theta$. Students were so used to being able to factor that, when the equation was not factorable, they did not know what to do. Some students who used the quadratic equation thought that they were solving for θ rather than $\sin\theta$. Some students did not use the addition identity to determine the exact value of a trigonometric ratio, but instead tried using Pythagorean theorem or the guess and check strategy. Even though they were asked for the coterminal value, they didn't recognize that they could just use this value to determine the exact value of the trigonometric ratio.

Procedural Skills

Students made many algebraic errors. They did not know how to multiply or simplify radicals in the addition identities and in the verification question. They had difficulty simplifying the quadratic formula with its radical. Some students did not know how to use the quadratic formula for a trigonometric equation. They would try to factor instead and made errors. They also simplified the quadratic formula incorrectly. If students' algebraic strategies in proving the identity could have been executed correctly, they would have been far more successful. Students had excellent identity substitutions and showed good logical process, but could not get their left-hand side to equal their right-hand side because of algebraic errors. Some even forced the two sides to be equal without correct proof. Students struggled with the algebra needed to verify an equation when it had fractions and radicals that had to be simplified. Students added when they should have multiplied and just did not know what to do with the radicals in the addition identity.

Communication

Students switched variables without defining them, such as changing θ to x. Most students remembered to use θ with "sin" and "cos" on the proof, but forgot to do so on other questions. Some students gave final answers in degrees instead of radians.

Unit F: Exponents and Logarithms (provincial mean: 66.7%)

Conceptual Knowledge

When solving an exponential equation, students did not recognize that they had to use logarithms when they were unable to change to a common base. They did not recognize that they could

change to a common base when they were able to do so. When asked to express a logarithmic expression in variable terms, students generally understood the concept of product and power law but struggled to express their final answer in variable terms. When asked to express a logarithmic expression as a simple logarithm, students incorrectly used the quotient law by separating the difference of two terms in an argument. Students struggled to differentiate between the shape of a logarithmic graph and an exponential graph. When given an equation of an exponential function, students had difficulty stating the range and the equation of the asymptote.

Procedural Skill

When solving an exponential equation using logarithms, students did not properly apply the order of operations. Students also incorrectly applied the order of operations when asked to express a logarithmic expression as a simple logarithm, as well as incorrectly simplified their final answer by cancelling the variables in the numerator and denominator. When solving an exponential equation by changing to a common base, students made many arithmetic errors and did not understand how to work with a negative exponent. When sketching a logarithmic graph, students made arithmetic errors in their table of values.

Communication

When solving a logarithmic equation, students made rounding errors. When solving an exponential equation with a common base, many students applied logarithms which did not allow them to reach a final answer without a calculator. When sketching a logarithmic graph, students labelled points incorrectly or did not show the vertical asymptote. When stating the range of an exponential function, students made bracket errors.

Unit G: Radicals and Rationals (provincial mean: 66.6%)

Conceptual Knowledge

Students were generally able to sketch the graph of a given radical function and its transformations. They were also able to determine the restrictions on the graph of a radical function. When asked to sketch a rational function, students were able to identify the required shape. However, some students had difficulty determining the horizontal asymptote. Students were able to clearly explain the difference between an asymptote and a point of discontinuity. Students were generally able to determine the vertical asymptote of a rational function. Some students used a table of values to graph, showing no knowledge of graphing using transformations.

Procedural Skill

Some students had difficulty sketching the transformation in the correct order; others did not understand how to apply a horizontal reflection or missed it entirely. When asked to sketch a rational function, students struggled to determine the correct points due to arithmetic errors particularly with fractions. When graphing rational functions, many students knew how to draw a radical shape but some students did not include one point in each section on their graph. Many students wrote their domain and range backwards; others mixed up the domain and range, and some students combined set notation with interval notation.

Communication

Many students did not show two points on a graph as required. Students did not label their axes with a scale on their graph of a rational function and did not indicate a scale through labeling points and/or asymptotes. Some students knew the graph should approach the asymptote but did not show proper asymptotic behaviour when their graphs were supposed to approach the asymptote. Some students made bracket errors or inequality symbol errors when stating domain or range. Students also made notation errors in missing a union symbol when stating a domain and/or range with multiple parts.

Communication Errors

Errors that are not related to the concepts or procedures are called "Communication Errors" and these were tracked on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in steps leading to final answer	15.0%
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity	17.4%
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	17.1%
E4 brackets	▪ "$\sin x^2$" written instead of "$\sin^2 x$" ▪ missing brackets but still implied	12.9%
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	10.1%
E6 rounding	▪ rounding error ▪ rounding too early	22.1%
E7 notation/transcription	▪ notation error ▪ transcription error	29.3%
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	35.7%
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	37.6%
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes	12.4%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/.

These reports compare the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 41.9% of the test booklets sampled resulted in a higher score locally than those given at the department; in 6.9% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 51.2% of the booklets sampled and marked by the department received a central mark within $\pm 2.0\%$ of the local mark and 94.5% of the sampled booklets were within $\pm 6.0\%$. Scores awarded at the local level were, on average, 1.7% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in January 2020 were invited to provide comments regarding the test and its administration. A total of 108 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 99.0% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 98.1% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document and that the reading level of the test was appropriate. 96.2% of teachers indicated that the test questions were clear.
- 97.2% and 91.3% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 96.3% of the teachers indicated that their students used a formula sheet throughout the semester and 100% of teachers indicated that their students used the formula sheet during the test.