GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (January 2019)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Unit A: Transformations of Functions (provincial mean: 70.1%)

Conceptual Knowledge

Generally, students demonstrated a good knowledge of this unit; however, explaining or describing transformations seemed to be more difficult. When asked to write the equation of a transformation of an $f(x)$ graph into a $g(x)$ graph, students failed to include the f notation in their $g(x)$ equation. They also often used incorrect notation for a horizontal translation. Many also misunderstood the vertical reflection as a reflection across a line other than the x-axis. When asked to transform a graph by applying an absolute value and horizontal reflection, students could do one correctly, but not both. They seemed to understand absolute value, but struggled with the subsequent reflection. When asked to complete an operation on two functions $f(x)$ and $g(x)$, students were confused by the notation $(f-g)(x)$ and tried to multiply by x.

Determining the inverse function was very well done with only a few trying to find the y-intercept or reciprocal instead of the inverse. When asked to state the coordinates of a point as a reflection across $y=x$, many students wrote a vertical reflection, a horizontal reflection, or both.

Procedural Skill

When writing a transformational equation, students did not know what notation to use to express horizontal translations. In describing a horizontal compression, students used the phrases "compressed by a factor" or "stretched by a factor", yet they would show correctly transformed x-values. There was great confusion with the use of the terms "stretch" and "compression". Other errors included the arithmetic error of subtracting a negative value incorrectly, and arithmetic errors when solving for y on an inverse. While students know that denominators cannot equal zero, they are not able to explain why.

Communication

Students had difficulty explaining why there was a restriction for an equation that had a denominator. Some students used improper notation for an inverse $\left(f(x)^{-1}\right.$ instead of $\left.f^{-1}(x)\right)$. Students were also missing arrowheads and endpoints on graphs.

Unit B: Trigonometric Functions (provincial mean: 74.0%)

Conceptual Knowledge

Most students knew when to use the arc length formula but many did not convert degrees to radians. When asked to sketch an angle in standard position, most students found the correct quadrant but many failed to show the correct number of revolutions. Students knew the quadrant in which secant is positive but were not able to justify their answer.

Students knew how to find the period of a sinusoidal function but some had difficulty graphing the sinusoidal with the correct period. Most students could use identities to find the values of circular functions, though some thought $\sec x$ was the reciprocal of $\sin x$.

Procedural Skill

Students made calculator errors when dividing by 2π. When asked to sketch an angle in standard position, many students found the correct quadrant but drew an inappropriate angle. Some students used π instead of 2π for the period of a sinusoidal. Some students made algebraic errors when simplifying.

Generally, students could graph the correct shape of a sinusoidal function and correctly showed the vertical translation. However, students had difficulty graphing the correct period.

Communication

Many students forgot to include units in the answer. Some drew a triangle when asked for an angle in standard position. Many students answered the justification question with the correct quadrant but did not provide a justification or provided an unclear justification. Some students omitted variables or changed equations to expressions. Students graphed the sinusoidal outside the given domain. Sometimes, scales were missing or inaccurate.

Unit C: Binomial Theorem (provincial mean: 68.2%)

Conceptual Knowledge

In general, most students did well on questions involving the binomial theorem. Most students knew how to use the formula to determine a specific term of a binomial expansion, but some had difficulty using exponent laws to simplify. Some students used the incorrect combination but used consistent factors. When asked to justify the number of positive terms in a binomial expansion, most students knew to refer to the expansion and understood that it is the coefficients that determine whether a term is positive or negative. Many students knew how to use the fundamental counting principle, but a few were confused when there was a restriction. As a result, some simply ignored the restrictions while others added their permutations. Some students were not able to expand factorials correctly when the larger one was in the denominator to simplify factorials but knew what to do when it was in the numerator to solve an equation involving the combinations formula.

Procedural Skill

When asked to determine the number of possible arrangements with two restricted spaces, some students only restricted one space. While trying to use the fundamental counting principle, some students incorrectly used factorials instead of numbers. Students could substitute into the formula to determine a term of a binomial expansion correctly, but could not simplify it because they did not know the power rules for exponents. When solving an equation involving a combination, some students did not include the negative sign for the second term of the binomial expansion. Other students made arithmetic or procedural errors. As well, some students did not reject the extraneous root in their solution.

Communication

Students displayed a lack of clarity or made terminology errors in their justification of the number of positive terms in a binomial expansion. Some made bracket errors and/or notation errors when expanding factorials, such as misplacing the factorial sign inside the brackets or completely forgetting the brackets. When asked to solve an equation involving a combination, some students changed the equation to an expression.

Unit D: Polynomial Functions (provincial mean: 72.7%)

Conceptual Knowledge

When factoring to determine the zeros, students often did not factor the greatest common factor of x. When asked to solve, students often stated factors, not zeros. To determine the equation from the given graph of $P(x)$, most students were able to find the factors and the multiplicity, but had difficulty finding the value of a. For the questions where students were asked to justify their answer, students had difficulty using the correct terminology when justifying the behaviour of the graph at the multiplicity. When asked to sketch the graph of a polynomial function, most students were able to graph x-intercepts, but many had difficulty determining and/or graphing the y-intercept and the end behaviour.

Procedural Skill

Students often set up synthetic division incorrectly and had the incorrect degree for the quotient after synthetic division. Students often used incorrect terminologies such as *slide, stutter step, swoosh,* and *wiggle* when justifying the behaviour of the graph at the multiplicity. Students did not show any work to determine the value of a. When graphing, students often missed the y-intercept or had the x-intercepts reversed.

Communication

When graphing, the scales on the axis were often not included. Arithmetic errors were often made when students multiplied the factor to find the expanded form of $P(x)$. Students often did not set the function equal to zero when they were asked to determine the zeros.

Unit E: Trigonometric Equations and Identities (provincial mean: 62.9%)

Conceptual Knowledge

When proving identities, most students did well in the substitution part but not well in the algebraic or logic part. Students understood identities, but could not produce enough correct algebraic work to get to the end. Most students realized that the two sides had to equal each other but after an algebraic error, they had to contrive steps to get to the end. Most students were able to use $\csc\theta$ to determine $\sin\theta$ but were unable to use the $\sin\theta$ to determine $\cos 2\theta$. When solving a quadratic trigonometric equation, many students were unable to factor or had many arithmetic errors in their factoring. Some students did not include the general solution in their answer. Most students recognized to use sum/difference identities to solve for an angle that is not one of their common reference angles.

Procedural Skills

When proving identities, students made algebraic errors, which made it impossible to prove the identity. Students also did not recognize when to factor in the identity so they were unable to reduce correctly. They had difficulty finding common denominators. In questions that involved trigonometric ratios, students often did not include a sign to indicate the quadrant where the angle was located. When solving trigonometric equations, some students were not aware that they had to show the rejection of a solution that does not exist or did not know what to do with the solution without a calculator. For the sum/difference identities, students often chose the incorrect identity and many had arithmetic and/or sign errors.

Communication

Many students gave solutions that were missing variables (i.e., writing sine, cosine, or tangent without a variable following it). When solving equations they often changed an equation to an expression. In some cases, students did not define new variables that they used for substitution in their solutions. For general solutions, students would sometimes state $k \in$ Reals instead of $k \in$ Integers.

Unit F: Exponents and Logarithms (provincial mean: 65.6%)

Conceptual Knowledge

Students had difficulty solving the logarithmic word problem involving compound interest. They did not recognize that they had to use logarithms to solve for the exponent. Many did not round their final answer correctly to include the partial withdrawal amount. When presented with an exponential equation, most students were able to apply logarithms and power law but then had difficulty solving it. Many students were able to expand a logarithmic expression; however, some separated the binomial argument thinking it was similar to quotient law, and some forgot to use the power law. Most students had difficulty sketching the graph of a logarithmic function. They used the wrong parent graph. When asked to explain why a value is an extraneous root, students generally understood that zero could not be the argument of a logarithmic expression; however, they were unable to explain using proper terminology. When asked to solve a logarithmic equation with an argument of a logarithmic expression, most students did not correctly change the equation into exponential form. Some students solved by substitution instead of algebraically.

Procedural Skill

When solving the logarithmic word problem involving compound interest, many students misunderstood the term "quarterly" and incorrectly substituted into the equation. Many students made algebraic errors, applied logarithms to negative values, and incorrectly used the laws of logarithms. When presented with an exponential equation, many students forgot to use brackets when applying the power law, and had difficulty evaluating the quotient of logarithms. When asked to sketch the graph of a logarithmic function, many students put the asymptote in the wrong place or shifted the graph incorrectly. Most students forgot to include the x-intercept. When asked to solve a logarithmic equation, some students tried to divide out the "log" in order to solve for the variable.

Communication

When solving the logarithmic word problem involving compound interest, some students did not reject their impossible solution (negative value). When solving an exponential equation, some students were missing brackets but they were still implied. When completely expanding a logarithmic expression, some students were missing brackets around the binomial argument. When graphing the logarithmic function, some students were missing the asymptote but it was still implied.

Unit G: Radicals and Rationals (provincial mean: 70.0%)

Conceptual Knowledge

When asked to state an equation of a rational function that has a vertical and horizontal asymptote at a specific value, many students had difficulty with the horizontal asymptote. Some students applied an equation of a radical function instead of a rational function. When asked to find the range, some students mixed up domain and range. Students stated domain instead of range. Some students stated range with the x variable. When asked to sketch the graph of a radical function, some students did a horizontal stretch instead of a horizontal compression. Some students thought the graph was linear. Some students did a horizontal translation instead of a vertical translation and included asymptotes. When asked to describe the error in sketching the graph of $y = \sqrt{f(x)}$ based on the graph of $= f(x)$, which has a restricted domain, many students thought the error was that it was not possible to take the square root of a negative number or that $x \geq 0$. Some thought that it was the direction that was the error and others confused the graph of $f(x)$ as part of the work. Students had difficulty determining the equation of a horizontal asymptote of a rational function. Students did not understand the difference between a point of discontinuity and vertical asymptote. They sketched a vertical asymptote instead of a point of discontinuity to their graph creating incorrect shapes when sketching the graph of a rational function.

Procedural Skill

When sketching the graph of a given radical function, some students had one incorrect point. This is often a result of using a table of values. Students did not state the equation of the horizontal asymptote properly. Errors included the abbreviation of HA or only stated the value of the horizontal asymptote and not as an equation. When sketching the rational function, some students included at least one incorrect point on each section of the graph. Students were able to state the correct value of x for the point of discontinuity but did not state the correct *y*-value.

Communication

When asked to state an equation, some students wrote an expression rather than an equation. Many students made bracket errors when stating range. Students could not describe in words the error in sketching the graph of $y = \sqrt{f(x)}$ based on the graph of $y = f(x)$ with a restricted domain. Students mixed up $f(x)$ and $\sqrt{f(x)}$. Some students knew that the graph should approach a horizontal asymptote at the *y*-axis but did not sketch the asymptote when sketching the graph of a rational function. Others missed arrow heads on their graphs.

Communication Errors

Errors that are not related to the concepts or procedures are called "Communication Errors" and these were tracked on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in steps leading to final answer	25.6%
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity	33.7%
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	15.4%
E4 brackets	▪ " $\sin x^2$ " written instead of " $\sin^2 x$ " ▪ missing brackets but still implied	27.3%
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	15.4%
E6 rounding	▪ rounding error ▪ rounding too early	23.0%
E7 notation/transcription	▪ notation error ▪ transcription error	34.9%
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	14.5%
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	21.1%
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes	17.5%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

These reports compare the local marking results to the results from the departmental re-marking of sample test booklets.

Provincially, 36.6% of the test booklets sampled resulted in a higher score locally than those given at the department; in 8.8% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 54.6% of the booklets sampled and marked by the department received a central mark within $\pm 2.0\%$ of the local mark and 94.7% of the sampled booklets were within $\pm 6.0\%$. Scores awarded at the local level were, on average, 1.2% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in January 2019 were invited to provide comments regarding the test and its administration. A total of 122 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 98.1% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 100% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document, that the reading level of the test was appropriate, and that the test questions were clear.
- 95.5% and 91.9% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 97.4% of the teachers indicated that their students used a formula sheet throughout the semester and 99.1% of teachers indicated that their students used the formula sheet during the test.
- 39.1% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 97.4% of teachers indicated that the use of a scientific calculator was sufficient for the test.