GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (January 2017)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Summary of Test Results (Province)

January 2017	June 2016	January 2016	June 2015	January 2015	June 2014
68.8%	66.6%	66.0%	67.0%	69.5%	64.5%

Unit A: Transformations of Functions (provincial mean: 73.4%)

Conceptual Knowledge

In general, students knew how to apply transformations on functions. Students had difficulty with the absolute value when sketching the graph. When asked to state the equation in terms of another function, many students identified transformations on only one point and not the graph as a whole. Also, students did not include the function notation with the transformations. Students often made mistakes restricting the domain on a graph when combining functions. They did not understand that if one function had a restricted domain, then the resulting function should also be restricted.

Procedural Skill

Students generally followed the correct order of operations on transformations. When transforming a graph, some students did not transform the whole graph, ignoring sections of the graph, which led to an incorrect final graph. When sketching composite functions, generally the operation on the functions was done correctly, but students would often make a mistake on one point. Sometimes they added the x-values together to create the new graph.

Communication

When describing transformations, students were generally able to describe the appropriate transformations using appropriate vocabulary. Many communication errors were made when determining the composition of functions (e.g., missing brackets and sometimes even equating to zero). Some students did not use words when answering questions which required a description or explanation answer. When identifying a function, students often used the word "it", without specifying which function they were describing.

Unit B: Trigonometric Functions (provincial mean: 63.4%)

Conceptual Knowledge

Most students were able to convert angles in degrees to angles in radians. But when asked to write the general solution for all of the coterminal angles, most students did not know what to do. They either gave related angles in the other three quadrants in the general solution or just found a coterminal angle, without the general solution. Students were able to use the arc length equation correctly. They were able to draw angles in standard position. Students were generally able to use unit circle values correctly when the substitution was for sine, cosine, or tangent. They were less skillful at substituting correct values for the reciprocal functions of secant and cosecant. Students were able to use the Pythagorean Theorem to calculate the length of the terminal arm given a point on it, but did not know what to do with it in order to determine the corresponding coordinates of $P\left(\theta\right)$ on the unit circle. When asked to write the equation of a sinusoidal graph, some students used sine with a horizontal shift (appropriate for the cosine graph) when no shift was needed. They also struggled to determine the value of b for their equation.

Procedural Skills

Some students misunderstood 3 radians as 3π radians. When writing the general solution for the coterminal angles, students often used π instead of 2π. They also used the set of real numbers rather than the set of integers (or other appropriate number set given the context). Students also made many arithmetic errors (e.g., incorrect reduction; dropping the squared function without squaring their value; changing multiplication to addition or subtraction). Students made arithmetic errors when using the Pythagorean Theorem. When finding the b value for the sinusoidal function, students sometimes wrote $\frac{4}{\pi}$ instead of $\frac{\pi}{4}$.

Communication

Students made rounding errors when solving arc length and conversion problems, using only 1 or 2 decimal places rather than 3 decimal places. They sometimes forgot the unit in the arc length problem. In explanation or description, students' answers were often vague. They attempted to use a diagram as an illustration instead of explaining in words. When evaluating $\sec\theta$ or $\csc\theta$ with exact values from the unit circle, some students continued to write sec or csc in front of the exact values. When writing the equation of a trigonometric function, students did not use the dependent and independent variables indicated on the graph, but reverted to using x or θ and y without defining the change. When asked to write the coordinates of a point, $P(\theta)$, they did not write the final answer as an ordered pair. They also made many notation errors, such as -2^2 instead of $(-2)^2$.

Unit C: Binomial Theorem (provincial mean: 70.7%)

Conceptual Knowledge

Most students were able to correctly solve for a combination question, but some used permutations instead of combination. When arranging different groups, students were able to use permutations within each group but many forgot to account for the arrangement of all the groups. Students were able to justify the number of permutations with repetition. They knew how to work with permutations with restrictions but some had trouble identifying all the cases involved in the restrictions. When solving questions related to binomial theorem expansion, students were able to substitute correctly into the given formula but many were unable to identify the correct term for which they were to solve.

Procedural Skill

When using algebra to determine a term in a binomial expansion, some students failed to apply the exponent laws correctly, which led to stating the incorrect term. Some students made algebraic errors when trying to simplify their answers. When arranging objects of different groups, some students added the groups together instead of multiplying them. When using the fundamental counting principle, some students incorrectly used factorials instead of numbers.

Communication

When expanding factorials, some students made notation errors such as forgetting to include the factorial sign or misplacing it inside the brackets. When solving factorial questions, some students forgot to reject the extraneous value of n or failed to properly communicate their understanding of the rejection. When solving a problem involving the combination formula, many students changed an equation to an expression.

Unit D: Polynomial Functions (provincial mean: 79.2%)

Conceptual Knowledge

Most students were able to correctly use the process of synthetic division, but some forgot to use a placeholder of zero for the missing term or incorrectly identified the missing coefficient as 1. Other students were able to correctly use alternate strategies such as long division and/or factor theorem. Students, in general, were able to explain that all of the exponents of a function should be whole numbers for it to be a polynomial function. Some students knew that the function should not have a rational exponent but did not fully explain why this would not make it a polynomial function. When asked to match a set of equations with their graphs, some students did not take into account the difference between the function with a lead coefficient of one and the function with a vertical stretch by a factor of two. Given a set of conditions that a polynomial function must satisfy, students were able to sketch a polynomial function with correct x-intercepts and multiplicity but forgot to include the y-intercept. Some students included an incorrect y-intercept and/or sketched a polynomial function with incorrect end behaviour. A few students plotted the x-intercepts with opposite signs and/or included an extra x-intercept.

Procedural Skill

Some students struggled with the synthetic division procedures. When graphing a polynomial function, some students had difficulty graphing the correct multiplicity of two and included an extra x-intercept in order to make the end behaviour correct. Some students sketched a polynomial function with a bounce at both x-intercepts even though only one factor had a multiplicity of two. When asked to solve for the zeros of a polynomial function, some students did not equate the function to zero before solving the equation. Other students left the function in factored form without solving for the zeros.

Communication

When graphing polynomial functions, sometimes scales were not indicated on axes and/or arrowheads were omitted. When asked to solve for the zeros of a polynomial function, some students changed the equation into an expression. Students used poor terminology and/or demonstrated a lack of clarity when explaining why a function was not a polynomial function.

Unit E: Trigonometric Equations and Identities (provincial mean: 70.0%)

Conceptual Knowledge

When solving trigonometric equations, students generally understood that they were solving for θ, though some struggled to determine which answers were on the unit circle and how to reject the branch with no solution. Students also struggled when determining which side to work with when solving the identity. Some students had difficulty determining the correct quadrants when solving trigonometric equations.

Procedural Skill

Some students made arithmetic errors when solving trigonometric equations. Students struggled to demonstrate correct algebraic procedures when solving a proof question. They had difficulty with the strategy of using common denominators and reducing fractions. When solving a trigonometric identity, students did not insert the correct values for α and β, and had difficulty reducing to the final answer. Students did very well in factoring trigonometric equations and understood how to solve for non-permissible values. Some students made transcription errors when copying their values from one part of the question into other formulas.

Communication

When solving proof questions and trigonometric equations, students missed variables after sine or cosine. When solving a trigonometric equation, students changed the equation to an expression by omitting the equal sign. Some students also omitted the $\theta =$ when solving for a variable.

Unit F: Exponents and Logarithms (provincial mean: 66.4%)

Conceptual Knowledge

When asked to determine the number of monthly investments to find a future value, many students did not substitute correctly into the given equation. They did not correctly substitute the annual interest rate and/or did not divide the interest rate by the number of compounding periods. Some students did not apply logs to solve the question. Many students were able to correctly apply the laws of logarithms to expand a logarithmic expression. They were able to use the quotient law to simplify a logarithmic equation, but struggled to expand logarithms when having to manipulate a number into a product of numbers and therefore were unable to apply laws of logarithms. Some students were unable to justify an estimate of a logarithmic expression that was not a whole number value, and lack of clarity was very common in their justification. When graphing an exponential function, many students did not recognize the shape of the base graph and instead drew logarithmic functions, radical functions, or polynomial type functions. When describing why a logarithmic function must have a positive argument, many students simply described features of the graph but not how they were related to the domain.

Procedural Skill

Some students were able to correctly substitute into logarithmic equations but struggled when applying logarithms and using algebra to isolate the unknown variable. When using laws of logarithms to simplify an equation, some students incorrectly cancelled the logs before applying the laws. Other students were able to apply the quotient law correctly but used incorrect division of binomials to cancel the variable instead of multiplying by both sides and using algebra to solve. Some students correctly multiplied both sides of the equation by a binomial, but did not fully distribute the multiplication of the monomial and binomial before combining like terms of the equation. When graphing exponential functions, some students drew a vertical asymptote instead of a horizontal one. Other students confused a reflection over the x-axis for a reflection over the y-axis.

Communication

When asked to round the number of investments to a whole number value, some students struggled to round their answers correctly or misread the question. Some students did not understand the concept of rounding up, regardless of the decimal, in order to ensure the minimum future value was met. Some students changed logarithmic expressions to equations to expand or solve. Other students tried to change the base of a logarithmic expression when this was not necessary to solve the problem. When graphing exponential functions, students commonly forgot to sketch the horizontal asymptote but had the correct asymptotic behaviour. Students had difficulty using the correct terminology when explaining or describing logarithmic functions. Many confused the argument of a logarithm with a negative logarithm.

Unit G: Radicals and Rationals (provincial mean: 72.1%)

Conceptual Knowledge

Students were generally able to sketch the graph of a given radical function and its transformations. They were also able to determine the domain and range of a radical function. When asked to sketch a rational function, students were able to identify the required shape. However, some students did not draw both branches of the function. When sketching a rational function with a point of discontinuity, students often mistakenly placed an asymptote in its position. Students were generally able to determine the vertical asymptote of a rational function. However, many of them were unable to identify a horizontal asymptote and some found the x-intercept and labeled it as an additional vertical asymptote.

Procedural Skill

Some students had difficulty sketching the horizontal compression and reflection of the radical graph from the given function. When asked to sketch a rational function with a point of discontinuity, students struggled to determine the correct y value of the hole. Students had difficulty writing equations for the asymptotes of a rational function. Many did not write the solution as an equation or interchanged the variables on the vertical and horizontal asymptote equations.

Communication

Some students had difficulty using the correct brackets when stating the domain and range of a radical function. Students often did not draw the horizontal asymptote at $y = 0$ when sketching a rational function.

Communication Errors

Errors that are not related to the concepts or procedures are called "Communication Errors" and these were tracked on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated	19.1%
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity	37.2%
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	52.4%
E4 brackets	▪ " $\sin x^2$ " written instead of " $\sin^2 x$ " ▪ missing brackets but still implied	12.4%
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	27.5%
E6 rounding	▪ rounding error ▪ rounding too early	16.2%
E7 notation/transcription	▪ notation error ▪ transcription error	47.8%
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	16.4%
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	15.5%
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes	13.6%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at www.edu.gov.mb.ca/k12/assess/support/results/index.html.

These reports include a chart comparing the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 34.2% of the test booklets sampled resulted in a higher score locally than those given at the department; in 7.4% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 58.5% of the booklets sampled and marked by the department received a central mark within $\pm 2\%$ of the local mark and 95.3% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 1.3% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in January 2017 were invited to provide comments regarding the test and its administration. A total of 113 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 93.7% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 100% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document. 100% of teachers indicated that the reading level of the test was appropriate and 98.1% of them thought the test questions were clear.
- 98.2% and 93% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 97.3% of the teachers indicated that their students used a formula sheet throughout the semester and 99.1% of teachers indicated that their students used the formula sheet during the test.
- 41.1% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 93.6% of teachers indicated that the use of a scientific calculator was sufficient for the test.